Logit Arithmetic Elicits Long Reasoning Capabilities Without Training

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Abstract

Large reasoning models exhibit long chain-of-thought reasoning with strategies such as back-tracking and self-correction, though recent studies (Muennighoff et al., 2025; Zeng et al., 2025) suggest that these abilities typically require additional training. We first investigate whether such behaviors can be elicited without *any* training. To this end, we propose a decoding-time approach, **ThinkLogit**, which utilizes logit arithmetic (Liu et al., 2024) to tune a target large non-reasoning model for long reasoning using a substantially smaller reasoning model as the guider. We then show that we can further boost its performance by training the guider model with preference optimization over correct/incorrect reasoning pairs sampled from both the target and guider model, a setup we refer to as **ThinkLogit-DPO**. Our experiments demonstrate that ThinkLogit and ThinkLogit-DPO achieve a relative improvement in average accuracy by 24.5% and 29.1%, respectively, over five reasoning benchmarks using the Qwen2.5-32B guided by R1-Distill-Qwen-1.5B, a model 21x smaller. Moreover, we find that ThinkLogit remains effective when the guider and target come from different model families. It is also orthogonal to post-training methods for small models, as guiders improved through supervised distillation or reinforcement learning can be directly plugged in to yield stronger large models, offering a practical path to unlock long reasoning in large-scale models without costly post-training.[1]

1 Introduction

Large reasoning models (LRMs), such as DeepSeek-R1 (DeepSeek-AI et al., 2025), OpenAI o1 (OpenAI, 2024), and Qwen3 (Qwen Team, 2025), have significantly advanced reasoning by leveraging inference-time compute (Snell et al., 2024; Brown et al., 2024). These models generate very long chain-of-thought (CoT) traces involving planning, reflection, and self-correction (Gandhi et al., 2025). It is widely believed that such behaviors require specialized training, either through reinforcement learning (RL) with verifiable rewards (DeepSeek-AI et al., 2025; Lambert et al., 2024; Shao et al., 2024) or supervised distillation (Muennighoff et al., 2025; Li et al., 2025) from other LRMs. However, such training is costly for large models with long generations and many parameters. Meanwhile, existing *training-free* long CoT elicitation methods (Pang et al., 2025; Muennighoff et al., 2025; Zou et al., 2023; Tang et al., 2025; Zhao et al., 2025) still remain limited, as they often lengthen outputs without reliably inducing genuine long reasoning and further require domain-specific supervision or white-box access. While the training costs are often *prohibitive* for large models, small models can be trained with modest compute (Dang and Ngo, 2025; Luo et al., 2025a). This observation motivates our central research question: ***Can a small reasoning model elicit long CoT behavior in a large non-reasoning model at inference time, without training the large model?***

We address this question with a decoding-time method, **ThinkLogit**, which elicits long CoT reasoning in a large non-reasoning model as the target. At each decoding step, we use logit arithmetic (Liu et al., 2021, 2024; Mitchell et al., 2024) by computing the logit difference between a small guider model trained for long reasoning and its base version, and add the resulting shift to the target logits. This token-by-token guidance transfers long reasoning signals from the small model to the large one without requiring any training of the target. Furthermore, since the output distributions of long and short CoTs differ substantially, we align them by training the small guider to correct errors made by the target model while maintaining the strengths of the target model. This training pro-

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[1]Our code is publicly avaiable at https://github.com/yunx-z/ThinkLogit.

cess uses Direct Preference Optimization (DPO; Rafailov et al., 2023) on mixed preference pairs sampled from both the guider and target models, thereby making THINKLOGIT more *on-policy*, and then applies logit arithmetic using the fine-tuned guider. We refer to this approach as **THINKLOGIT-DPO** and show that such training can further boost performance compared to THINKLOGIT.

We evaluate our methods on five challenging benchmarks covering mathematical and scientific reasoning. Our results show that fusing the logits of a small reasoning model (DeepSeek-R1-Distill-Qwen-1.5B) with those of a large target (Qwen2.5-32B) yields 24.5% and 29.1% relative accuracy gains with THINKLOGIT and THINKLOGIT-DPO, respectively, over the frozen target baseline. We further show that our method remains effective even when the target and guider models come from different families. Specifically, we show that a Qwen-based guider can drive long reasoning from a Llama target, highlighting *cross-family* generalization. We further demonstrate that THINKLOGIT can emulate reinforcement learning in large frozen models by leveraging small RL-trained guiders, sidestepping the prohibitive cost of direct RL at scale. Our ablation study shows that gains of THINKLOGIT-DPO arise only when preference pairs combine the strengths of both the guider and target models.

Our approach is particularly favorable when training the target is infeasible: *extreme-scale* models beyond practical fine-tuning budgets (Dettmers et al., 2023), *black-box* models limited to logit access (Ormazabal et al., 2023),[2] or *privacy-preserving* settings where an on-device guider steers a centralized model without exposing private data (McMahan et al., 2017; Xu et al., 2024). In these important scenarios, it enables efficient reasoning transfer without full-model fine-tuning.

2 Related Work

Eliciting Long Chain-of-Thought Reasoning. Large reasoning models, such as OpenAI's o1 and o3 (OpenAI, 2024, 2025), DeepSeek-R1 (DeepSeek-AI et al., 2025), and QwQ (Team, 2025), outperform standard LLMs on logic, math, and programming tasks, with the ability to revisit earlier steps and leverage extra test-time computation to generate longer reasoning chains. A common way to elicit long reasoning is **reinforcement learning** (Lambert et al., 2024; Shao et al., 2024; Yu et al., 2025; Liu et al., 2025), which optimizes the model for outcome-based correctness rewards. A complementary line of work demonstrates that the same capability can be acquired with **data-efficient supervised fine-tuning (SFT)**. Distilled long CoTs from stronger teacher models allow a student to extend its reasoning length and improve accuracy with as few as one thousand training examples (Muennighoff et al., 2025; Xu et al., 2025; Ye et al., 2025; Li et al., 2025). However, applying either RL or SFT to large models requires significant resources (e.g., 16 H100 GPUs for SFT a 32B model reported by Muennighoff et al. (2025)), making these approaches infeasible for many practitioners. Our THINKLOGIT framework emulates the benefits of SFT or RL on large targets without incurring this heavy training cost. Finally, **training-free** methods exploit the fact that pretrained LLMs already exhibit long CoT behaviours (Liu et al., 2025; Gandhi et al., 2025). Tang et al. (2025) inject contrastive long-versus-short CoT representations into hidden states via representation engineering (Zou et al., 2023), whereas Zhao et al. (2025) amplify a handful of key neurons at inference. Both techniques, however, require domain-specific long/short CoTs and white-box access, limiting their applicability in out-of-domain or black-box settings. In contrast, our flexible methods do not require domain-specific supervision, or parameter access to the target model.

Logit Arithmetic. Logit arithmetic is a decoding-time technique that blends token-level output distributions from multiple models (Liu et al., 2021; Ormazabal et al., 2023; Fan et al., 2024; Shi et al., 2024), and has been applied to emulate pre-training (Mitchell et al., 2024), task-specific fine-tuning (Liu et al., 2024; Fan et al., 2024), knowledge unlearning (Huang et al., 2025), and overriding safety filters (Zhao et al., 2024). In these applications, the guiding signal typically induces relatively shallow adjustments, such as shifting style, steering toward domain-specific vocabulary, or suppressing certain behaviors, with only modest distributional divergence from the target. In contrast, our setting demands bridging a larger gap: the guider produces long CoTs that not only exceed the target's short reasoning in length but also introduce substantially more complex cognitive behaviors, including systematic backtracking, verification, and self-reflection (Gandhi et al., 2025). To our knowl-

[2]For example, the OpenAI API exposes a `logit_bias` parameter that lets users alter token probabilities at inference by boosting or suppressing specific tokens (OpenAI, 2025).

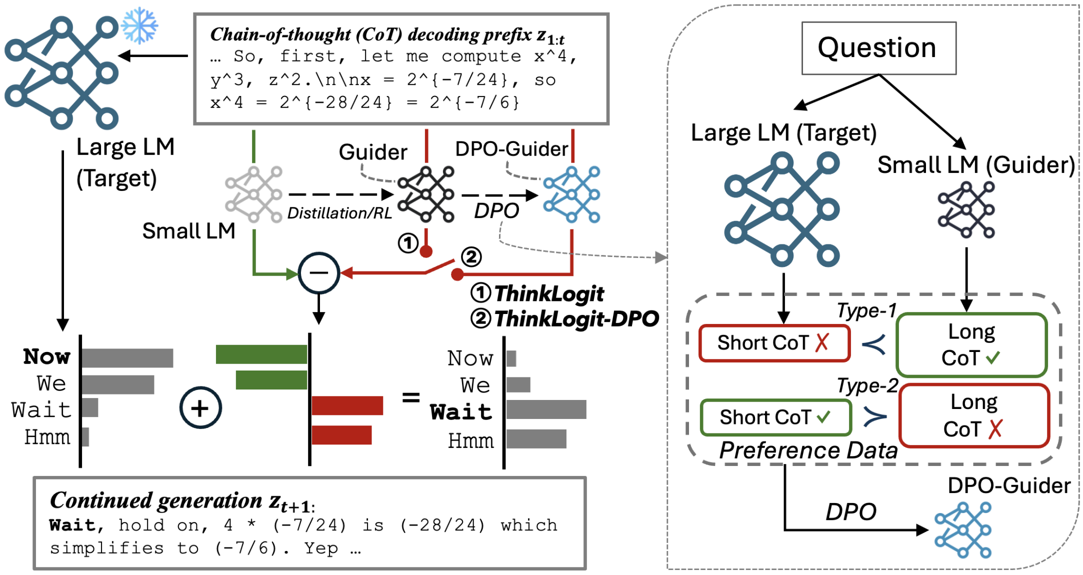

Figure 1: Overview of our proposed THINKLOGIT and THINKLOGIT-DPO approaches to elicit long chain-of-thought reasoning from a large non-reasoning model that is frozen.

edge, THINKLOGIT is the first to demonstrate that logit arithmetic can elicit such behaviors in large non-reasoning models. THINKLOGIT-DPO further introduces a novel alignment strategy that mitigates distribution mismatch between long- and short-CoT reasoning for better performance.

3 Methodology

Our goal is to elicit long CoT reasoning capabilities from a large, frozen, non-reasoning language model without expensive training. We introduce two lightweight decoding-time techniques (see Figure 1): **THINKLOGIT**, which transfers long CoT behaviors from a small guider via simple logit arithmetic (Liu et al., 2024), and **THINKLOGIT-DPO**, which further refines the guider using Direct Preference Optimization (DPO; Rafailov et al., 2023) to align its guidance with the target model.

3.1 THINKLOGIT

Let $\mathbf{z}_{1:t} = z_1, \ldots, z_t$ be the partially decoded sequence of reasoning tokens at step t. For any language model f, denote its pre-softmax logits at the next step by $\ell^{(f)}_{t+1} = f(\mathbf{z}_{1:t}) \in \mathbb{R}^{|\mathcal{V}|}$, where $\mathcal{V}$ is the vocabulary. We assume three models during inference:

- **large (target)** L, a pre-trained LLM lacking long CoT capability;
- **small base** S_0, a pre-trained model without long reasoning fine-tuning;
- **small reasoning (guider)** S, obtained via long CoT post-training to S_0.

At decoding step $t+1$, the fused logits are computed as $\tilde{\ell}_{t+1} = \ell^{(L)}_{t+1} + \alpha\big(\ell^{(S)}_{t+1} - \ell^{(S_0)}_{t+1}\big)$, where $\alpha \geq 0$ controls the guidance strength. The delta term $\ell^{(S)} - \ell^{(S_0)}$ encodes the probability shift that turns a short-CoT model into a long-CoT one. Intuitively, adding this delta to L induces analogous long reasoning behaviors without altering its weights.

Warm-up for Stable Decoding. We empirically observe that directly applying logit arithmetic at each decoding step would cause many repetitive generations in the long CoT outputs. To stabilize generations, we defer inference-time guidance for long CoT until a prefix of length T:

$$\tilde{\ell}_{t+1} = \begin{cases} \ell^{(L)}_{t+1}, & t+1 \leq T, \\ \ell^{(L)}_{t+1} + \alpha\big(\ell^{(S)}_{t+1} - \ell^{(S_0)}_{t+1}\big), & t+1 > T, \end{cases} \tag{1}$$

3.2 THINKLOGIT-DPO

The effectiveness of THINKLOGIT can be constrained by distribution mismatches between the guider and target. To address this, we further train the small model as a stronger guider that corrects target reasoning errors while retaining the target strengths, using a mixture of two types of preference pairs sampled from both the target L and the guider S outputs:

Type-1: $(x,\, y^{L\checkmark},\, y^{S\times})$ — The *large* model's correct (short) CoT is preferred over the *small* model's incorrect (long) one. This encourages the guider to preserve the correctness of the target model and avoid introducing new errors.

Type-2: $(x,\, y^{S\checkmark},\, y^{L\times})$ — The *small* model's correct (long) CoT is preferred over the *large* model's incorrect (short) one, teaching the guider to be more confident at fixing the large model's reasoning errors.

We gather these pairs from training queries x by independently sampling CoTs from L and S and labeling correctness based on the final answer. Let θ denote the parameters of the preference-optimized guider, initialized from S. We train θ with a DPO objective function that mixes the two pair types:

$$\mathcal{L}_{\text{DPO}}(\theta) = \lambda\, \mathbb{E}_{(x,y^{L\checkmark},y^{S\times})\sim\mathcal{D}_1}\, \ell_\theta\big(x;\, y^{L\checkmark}, y^{S\times}\big) + (1-\lambda)\, \mathbb{E}_{(x,y^{S\checkmark},y^{L\times})\sim\mathcal{D}_2}\, \ell_\theta\big(x;\, y^{S\checkmark}, y^{L\times}\big), \tag{2}$$

where $\ell_\theta(x;\, y^+, y^-) = \log\sigma\big(r_\theta(x,y^+) - r_\theta(x,y^-)\big)$, σ is the sigmoid function, $r_\theta(x,y) = \beta[\log\pi_\theta(y \mid x) - \log\pi_{\text{ref}}(y \mid x)]$ is the implicit reward of output y, and $\lambda \in [0,1]$ balances the two datasets $\mathcal{D}_1$ (Type-1) and $\mathcal{D}_2$ (Type-2). We use $\lambda = \frac{|\mathcal{D}_1|}{|\mathcal{D}_1|+|\mathcal{D}_2|}$ by default, directly concatenating two datasets as DPO training data without further rebalancing. After fine-tuning, we replace S in ThinkLogit with the optimized guider to obtain ThinkLogit-DPO.

4 Experimental Setup

Benchmarks. We evaluate models on five widely used reasoning benchmarks for LRMs. Four of them are competition math problems sources from AIME2024 (30 problems), AIME2025 (30 problems), AMC23 (40 problems), and a subset of 134 hard problems (level 5) from MATH500 (Lightman et al., 2024). We also evaluate on another scientific reasoning dataset GPQA Diamond (Rein et al., 2023), consisting of 198 PhD-level science questions in Biology, Chemistry, and Physics. For each dataset, we independently sample 8 completions with a decoding temperature of 0.6 and maximum output length of 8192, and then compute their average accuracy as **Avg@8** for our primary metric.

Models. Our primary target model is **Qwen2.5-32B** (Yang et al., 2024a). We guide it using **R1-Distill-Qwen-1.5B** (DeepSeek-AI et al., 2025), a 1.5B parameter model based on Qwen2.5-Math-1.5B (Yang et al., 2024b) that has been supervised fine-tuned on 800K long CoT examples distilled from DeepSeek-R1. In our main experiment setup (Section 5.1), the target, guider, and its base model all come from the same Qwen family and share an identical tokenizer, allowing their output logits to be directly combined arithmetically.

To showcase the versatility of our approach, we investigate several alternative model configurations. First, to demonstrate robustness across heterogeneous model families, we employ R1-Distill-Qwen-1.5B as the guider of **Llama-3.3-70B-Instruct** (Dubey et al., 2024) for long reasoning (Section 5.3). Second, we show that ThinkLogit can emulate the benefits of RL on larger models without resource-intensive training by evaluating its effectiveness with two guiders produced via RL rather than distillation (Section 6.1): **One-Shot-RLVR-1.5B**[3] (Wang et al., 2025) and **DeepScaleR-1.5B-Preview** (Luo et al., 2025b). Unless otherwise noted, ThinkLogit and ThinkLogit-DPO use T=100 warmup steps and a guidance strength of α=1 as the default hyperparameters.

Preference Data Construction. We use the level 4–5 subset of the MATH training set (Hendrycks et al., 2021) and sample 5 completions for each question from both the guider model (S) and the target model (L). Each completion is checked for final-answer correctness against the gold label.[4] The target model L yields 12,412 correct completions ($y^{L\checkmark}$) and 16,448 incorrect ones ($y^{L\times}$), whereas the guider S produces 18,651 correct ($y^{S\checkmark}$) and 10,209 incorrect ($y^{S\times}$) completions. Forming the Cartesian product for each question gives 11,974 Type-1 preference pairs $\big(y^{L\checkmark}, y^{S\times}\big)$ and 43,209 Type-2 pairs $\big(y^{S\checkmark}, y^{L\times}\big)$, for a total of 55,183 pairs. We then randomly select 10K preference pairs from the total 55K pairs for the DPO training. To save training compute, we apply LoRA (Hu et al., 2022) with a rank size of 64 for parameter-efficient fine-tuning of the guider model. More training details are in Appendix A.

[3] We use the checkpoint released at `https://huggingface.co/ypwang61/One-Shot-RLVR-Qwen2.5-Math-1.5B-pi1_pi13`.

[4] We robustly extract answers from `\boxed{}` and compute exact match with ground-truths based on this script `https://github.com/openai/prm800k/blob/main/prm800k/grading/grader.py` by (Lightman et al., 2024).

Model	# Trainable Params	AIME 2024	AIME 2025	AMC 23	MATH Level 5	GPQA Diamond	Average
`(Guider)` R1-Distill-Qwen-1.5B	-	16.2	18.8	51.2	47.5	28.9	32.5
`(Target)` Qwen2.5-32B	-	14.6	8.3	57.2	44.7	36.9	32.3
No Fine-tuning of the **`Target`**							
`Target` + THINKLOGIT	0	22.5 +6.3	19.2 +0.3	62.2 +5.0	55.3 +7.8	41.8 +4.9	40.2 +7.7
`Target` + THINKLOGIT-DPO	78M	22.1 +5.9	21.7 +2.9	63.7 +6.5	58.5 +11.0	42.4 +5.5	41.7 +9.2
Full Fine-tuning of the **`Target`**							
s1.1-32B	32B	32.9	25.4	70.0	72.2	51.9	44.5
R1-Distill-Qwen-32B	32B	45.8	35.0	76.9	72.7	55.6	57.2

Table 1: Comparison of avg@8 performance across five reasoning benchmarks. The yellow cells highlight the improvement of our methods over the stronger baseline model (**`Target`** or **`Guider`**). We show that THINKLOGIT and THINKLOGIT-DPO provide substantial gains over the baselines and partially recovers the benefits of full-model fine-tuning without any training of the large target model.

5 Experiment Results

5.1 Main Results

Table 1 presents the avg@8 accuracies for all systems. We highlight two key observations. *First,* THINKLOGIT *consistently boosts reasoning accuracy upon both the target and the guider model, and* THINKLOGIT-DPO *raises it further.* Combining the logits of the 32B target with those of the 1.5B guider (THINKLOGIT) raises the average accuracy by 24.5% relative to the frozen target and by 23.7% relative to the guider. Replacing the vanilla guider with the DPO-trained guider (THINKLOGIT-DPO) brings the relative improvement to 29.1% over the target model, without any extra inference cost.

Second, our approach significantly narrows the performance gap with full-parameter fine-tuning while modifying only a small subset of weights. With LoRA, we adjust only 78M adapter parameters, yet THINKLOGIT-DPO closes 77% of the avg@8 gap between the frozen 32B target and the fully fine-tuned s1.1-32B (Muennighoff et al., 2025), which updates all 32B parameters. Although our pipeline uses a 1.5B guider (R1-distill-1.5B) fine-tuned on 800K distilled examples, this training is 21× cheaper in FLOPs than directly tuning a 32B target model (R1-distill-32B). Overall, the cumulative data and compute requirements of our pipeline remain well below those of fully fine-tuning large models, while still delivering substantial accuracy gains.

5.2 Comparison with Training-Free Baselines

A natural question is whether existing training-free techniques are sufficient to elicit long CoTs, or if specialized methods are required. Figure 2 contrasts our approach against two such baselines. First, the budget-forcing heuristic introduced by Muennighoff et al. (2025) replaces the end-of-sentence token with a placeholder string like "Wait" to artificially increase output length. Although this yields longer completions, the gains are limited compared to our method, indicating that *mere verbosity does not translate into deeper reasoning*.[5]

Figure 2: Comparison of THINKLOGIT against two training-free long CoT elicitation baselines: budget forcing and one-shot long CoT in-context learning (ICL). While these approaches increase verbosity, their accuracies are generally lower and can even degrade, whereas THINKLOGIT consistently produces longer reasoning that delivers the best performance.

Second, inserting a single long CoT example in the prompt (sampled from the s1.1-1K dataset (Muennighoff et al., 2025)) for in-context learning (ICL; Brown et al., 2020; Min et al., 2022; Dong et al., 2024) also degrades performance despite longer outputs from the target model. In contrast, THINKLOGIT uses logit-level guidance from a small reasoning model to steer the decoding to-

[5] For budget forcing, we report results using Qwen2.5-32B-Instruct, since applying it to the base Qwen2.5-32B led to low-quality outputs and degraded performance. This choice is consistent with the setup in Muennighoff et al. (2025).

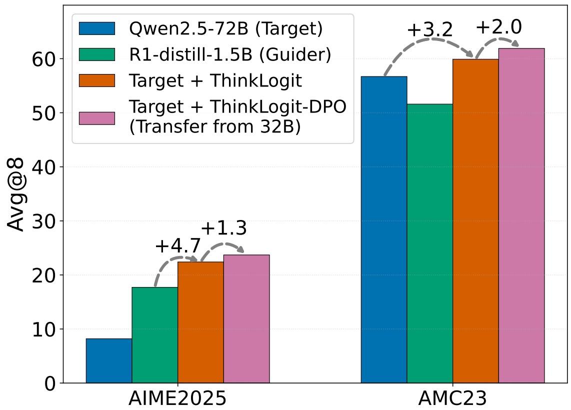

Figure 3: Avg@8 for eliciting long CoT in a 72B target model with our methods. THINKLOGIT-DPO delivers larger performance improvements on AIME2025 and AMC23 compared to THINKLOGIT, demonstrating that preference signals learned on a 32B model transfer effectively to a larger 72B model in the same family.

Model		Avg@8	# Avg Tokens
`(Guider)` R1-Distill-***Qwen***-1.5B		51.6	5.1K
`(Target)` ***Llama***-3.3-70B-Instruct		53.8	2.4K
`Target`	+ THINKLOGIT	55.8 +2.0	3.2K
`Target`	+ THINKLOGIT-DPO	59.3 +5.5	3.4K

Table 2: Avg@8 and average output tokens for *cross-family* guidance on AMC23. Results show that a small **Qwen**-based guider can elicit long CoTs from a large non-reasoning model from the **Llama** family, demonstrating that our methods work across model families.

Model		AIME 2024	AIME 2025
Emulating RL on a Base **`Target`**			
`(Guider)` One-Shot-RLVR-1.5B		13.3	7.1
`(Target)` Qwen2.5-32B		14.6	8.3
`Target`	+ THINKLOGIT	17.5 +2.9	11.2 +2.9
Emulating RL on a Supervised Fine-Tuned **`Target`**			
`(Guider)` DeepScaleR-1.5B		30.0	23.8
`(Target)` R1-Distill-Qwen-32B		45.8	35.0
`Target`	+ THINKLOGIT	47.5 +1.7	37.9 +2.9

Table 3: Avg@8 performance of THINKLOGIT emulating reinforcement learning (RL) on large target models. The two small guiders are trained via RL. Both emulated RL pipelines deliver consistent performance gains while avoiding the prohibitive cost of applying RL training on large target models.

wards genuine long CoT reasoning, which translates into a clear uplift in the answer accuracy. This shows that our improvements stem from *the quality of the guidance* being applied, rather than *the quantity of tokens* generated.

5.3 Generalization Across Scales and Families

A key strength of our framework is that once trained, a small reasoning guider can be reused far beyond its original training setting. *First, preference signals learned on smaller targets generalize to larger models within the same family.* As shown in Figure 3, a guider optimized on Qwen2.5-32B outputs transfers effectively to Qwen2.5-72B, where the DPO-trained variant (THINKLOGIT-DPO) still outperforms the vanilla THINKLOGIT.

Second, the guider can also be applied to target models from different families. Although logit arithmetic typically assumes a shared tokenizer, we overcome this constraint by aligning vocabularies with a minimum edit distance mapping (Wan et al., 2024). In practice, this is a one-time, offline step that introduces no inference overhead. Using this strategy, we guide **Llama**-3.3-70B-Instruct with the R1-Distill-**Qwen**-1.5B. As shown in Table 2, both THINKLOGIT and THINKLOGIT-DPO improve accuracy over the Llama baseline, with THINKLOGIT-DPO yielding a larger +5.5 point gain. The guided models also produce substantially longer CoTs. Manual inspection of their outputs confirms the emergence of self-correction behaviors such as "*Wait, no. Looking back at my steps, I made an error ...*".

Together, these results highlight the broad applicability of our approach: a single specialized guider can be reused for larger models within or across families, offering a scalable way to unlock long reasoning without retraining each target.

6 Additional Analyses

6.1 Emulating Reinforcement Learning with THINKLOGIT

RL with verifiable rewards is a powerful paradigm for enhancing the reasoning capabilities of language models (Lambert et al., 2024; Shao et al., 2024), but it is often prohibitively expensive. For example, training even a small 1.5B model with RL for long reasoning can require thousands of GPU hours (Luo et al., 2025a), an expense that becomes prohibitive for large-scale models. Table 3 shows that THINKLOGIT enables the emulation of RL effects on a large target model without training it.

First, we simulate a **Zero-RL** (applying RL directly on a base model; DeepSeek-AI et al., 2025; Zeng et al., 2025) pipeline to enhance a large non-reasoning model, Qwen2.5-32B. We apply reasoning from the One-Shot-RLVR-1.5B (Wang et al., 2025) guider, which is RL-trained on only one example from a Qwen2.5-Math-1.5B base. Second, we simulate an **SFT-then-RL** pipeline to further improve an already supervised fine-tuned reasoning model like R1-Distill-Qwen-32B. We apply guidance from the DeepScaleR-1.5B-Preview (Luo et al., 2025a) guider, which is trained via distributed RL from an R1-Distill-Qwen-1.5B SFT base. Both emulated pipelines deliver consistent performance gains over the target large model and RL-trained small guiders, while *avoiding the prohibitive cost of applying RL training directly to the large target model*. This confirms that our method is an *orthogonal* technique that can directly benefit from advances in small-model post-training (e.g., distillation or RL), *offering a flexible and efficient mechanism to transfer the benefits of powerful but expensive training paradigms to larger models*.

Model	Training Data for Guider	Avg@8
ThinkLogit-DPO (ours)	$(x,\, y^{L\checkmark},\, y^{S\times})$, $(x,\, y^{S\checkmark},\, y^{L\times})$	**63.7**
ThinkLogit-DPO		
w/o dual sources	$(x,\, y^{S\checkmark},\, y^{S\times})$	58.8
w/o Type-1 pairs	$(x,\, y^{S\checkmark},\, y^{L\times})$	57.2
w/o Type-2 pairs	$(x,\, y^{L\checkmark},\, y^{S\times})$	51.9
ThinkLogit-SFT		
learning from target	$(x, y^{L\checkmark})$	44.7
self-learning	$(x, y^{S\checkmark})$	55.6
learning from teacher	$(x, y^{R1\checkmark})$	60.9

Table 4: Avg@8 on AMC23 under ablations of guider's training data and objectives in ThinkLogit-DPO. x, $y^{L\checkmark}$, and $y^{S\times}$ denote the question, the correct (✓) response for the large target model (L), and the incorrect ($\times$) response from the small guider model (S), respectively. Our dual-source, mixed-pair DPO performs the best, demonstrating the necessity of complementary preference signals and preference-based alignment.

6.2 Ablation Study of ThinkLogit-DPO

To further investigate the design choices in ThinkLogit-DPO, we ablate both our mixed-pair data construction and preference-based learning objective (DPO) against single-source or supervised fine-tuning alternatives. Results in Table 4 answer the following research questions.

Are preference pairs sourced from both the target and the guider necessary to maximize performance? We construct the same amount of 10K preference pairs using only the guider's correct vs. incorrect outputs, i.e., $(x,\, y^{S\checkmark},\, y^{S\times})$. DPO on this data underperforms markedly on AMC23 (58.8 vs. 63.7 by our ThinkLogit-DPO), confirming that mixing pairs which highlight *both* the target's and guider's strengths is crucial for maximal gains.

Is training on both types of pairs necessary for the effectiveness of ThinkLogit-DPO? We next ablate by training on only one type of preference pairs at a time: using only Type-2 pairs $(x,\, y^{S\checkmark},\, y^{L\times})$ (i.e., $\lambda = 0$ in Equation 2) yields an avg@8 of 57.2, while using only Type-1 pairs $(x,\, y^{L\checkmark},\, y^{S\times})$ (i.e., $\lambda = 1$ in Equation 2) drops further to 51.9. Both are substantially below the 63.7 achieved by the full mixture, indicating that *both* Type-2 pairs (which teach the guider to correct target errors) and Type-1 pairs (which enforce preservation of the correct reasoning of the target) provide complementary signals necessary for better alignment.

Can supervised fine-tuning replace preference-based alignment of the guider? We evaluate SFT against DPO by training the guider on three equally sized sets of high-quality completions: **Option 1:** the target model's correct outputs $y^{L\checkmark}$; **Option 2:** the guider's own correct outputs $y^{S\checkmark}$ (also known as rejection-sampling fine-tuning (Yuan et al., 2023)); **Option 3:** R1-distilled completions $y^{R1\checkmark}$. Although SFT on Options 1 and 2 makes the guider a stronger *standalone* reasoner, none of these variants match the performance of the DPO-aligned guider. This gap highlights that *optimizing with pairwise preference comparisons yields a better guider than optimizing solely for correctness*. While SFT on Option 3 adapts the guider toward the target's short CoT reasoning style and thus reduces the distributional gap, it also tends to overwrite the guider's native strengths of long reasoning. In contrast, DPO *preserves the guider's intrinsic reasoning capabilities* while *selectively aligning it to the target's preferences* through pairwise comparisons.

6.3 Inference-Time Scaling Properties

Scaling the number of generations at test time is a well-established strategy to improve reasoning performance (Brown et al., 2024; Snell et al., 2024), as drawing more samples increases the likelihood of finding a correct solution. To quantify this effect, we use the pass@k metric (Chen et al., 2021), which measures the probability that at least one

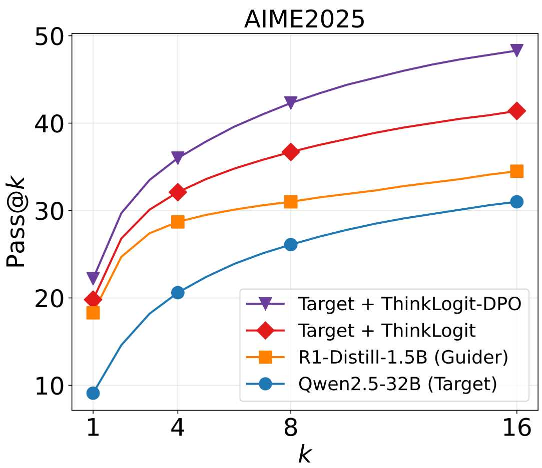

Figure 4: Inference-time scaling on AIME2025. *Pass@* k for $k = 1$–16 comparing the target, guider, their direct logit fusion (THINKLOGIT), and the DPO-aligned fusion (THINKLOGIT-DPO). Our methods demonstrate superior sample efficiency, reaching stronger performance with fewer generations and maintaining larger gains as the sample budget increases.

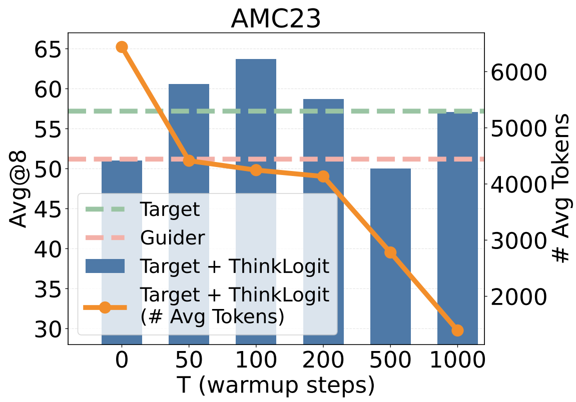

Figure 5: Impact of warm-up T on THINKLOGIT: early guidance (T=0) lowers accuracy and causes over-long, repetitive outputs, while moderate warm-up (T=100) gives the best performance with coherent long CoTs.

of k sampled outputs is correct. Figure 4 plots pass@k for $k = 1$–16 on AIME2025, the benchmark where the 32B target performs weakest and scaling effects are most pronounced. Both THINKLOGIT and THINKLOGIT-DPO exceed the target's pass@16 accuracy with only four samples, representing a fourfold improvement in *sample efficiency*. The gap further widens as k increases: at $k = 16$, the DPO-aligned guider surpasses the target by roughly 17 points. These results demonstrate that inference-time guidance consistently improves reasoning and that the gains compound with larger sampling budgets.

6.4 Impact of Key Hyperparameters

In THINKLOGIT, two hyperparameters play a critical role: the warm-up length T and the guidance strength α (Eq. 1). We evaluate their effects on the AMC23 benchmark, which presents a suitable mix of problem difficulties and clearly exhibits both stability and guidance effects. To assess warm-up, we vary T over $\{0, 50, 100, 200, 500, 1000\}$ with α fixed to 1 (Figure 5). When T=0, guidance is applied from the first decoded token, which empirically leads to more repetitions, lower output quality, and the lowest accuracy. Allowing 50–200 tokens of unguided generation stabilizes the chain-of-thought, improving accuracy over both target and guider models. Increasing T beyond 200 causes the model to revert to the shorter CoTs typically produced by the target model, leading to an accuracy drop and shorter outputs.

With T fixed at 100, we sweep α over $\{0.5, 0.75, 1.0, 1.25, 1.5\}$ to control how strongly the guider's delta-logits modify the target's distribution (Figure 6 in Appendix B). At α=1.0, we observe the highest avg@8 together with moderate generation length, indicating an optimal trade-off between the guider's corrective signal and the target model's own priors. Crucially, all experiments in earlier sections except this hyperparameter study use the same hyperparameters (T=100 and α=1.0) as a robust default, demonstrating that *our method achieves strong performance without extensive hyperparameter tuning*.

7 Conclusion

We present THINKLOGIT, a training-free, decoding-time framework for eliciting long chain-of-thought reasoning in large non-reasoning models through logit-level guidance from a much smaller reasoning model. Its enhanced variant, THINKLOGIT-DPO, leverages preference optimization to align the guider with the target model, achieving stronger improvements. Across five reasoning benchmarks, our approach delivers up to 29.1% relative accuracy gains using a guider 21× smaller than the target. THINKLOGIT demonstrates that a single guider can generalize across target models of different sizes and architectures, while also emulating the benefits of reinforcement learning through the reuse of small RL-trained guiders. Overall, our results establish inference-time guidance as a practical and flexible alternative to costly post-training, opening a path toward modular reasoning systems where small, specialized models can endow large frozen models with advanced reasoning capabilities.

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Limitations

Inference-time Overhead. Table 5 in Appendix B compares the inference efficiency of THINKLOGIT with the frozen target alone. Deploying THINKLOGIT requires hosting the large target together with two smaller models, the base model S_0 and the DPO-aligned guider S. In our primary setup (a 32B target guided by a 1.5B model), this increases the total parameter count by about $1.1\times$ relative to the target alone, while still fitting on the same number of GPUs. Profiling on NVIDIA L40S GPUs shows a moderate slowdown of roughly 25% fewer tokens per second compared to the target in isolation. THINKLOGIT-DPO does not introduce any extra overhead, since it simply replaces the guider in THINKLOGIT with a preference-optimized model of identical size. The observed throughput reduction stems from our prototype implementation, which queries the three models sequentially at each decoding step. In a production environment, logits from all three models can be computed *concurrently* across distributed GPUs. *This would largely mitigate the slowdown, making the throughput, in principle, comparable to that of the target model alone.*

Limited Domains of Evaluation. Our experiments focus on math- and science-oriented reasoning tasks. A broader evaluation suite, including coding (Jimenez et al., 2023; Jain et al., 2025), planning (Zheng et al., 2024a; Xie et al., 2024), and tool-use (Huang et al., 2024; Patil et al., 2025), is needed to understand failure modes that may emerge in less structurally similar settings.

Offline Alignment. The guider is aligned with the target via Direct Preference Optimisation (DPO) on a fixed set of preference pairs. This *offline* formulation cannot adapt once deployment uncovers new error patterns or distribution drift. Incorporating *online* reinforcement learning (Schulman et al., 2017; Shao et al., 2024) that updates the guider from streamed on-policy samples could, in principle, reduce this brittleness. However, on-policy RL introduces training efficiency and stability challenges that remain open research problems.

References

Bradley C. A. Brown, Jordan Juravsky, Ryan Ehrlich, Ronald Clark, Quoc V. Le, Christopher Ré, and Azalia Mirhoseini. 2024. Large language monkeys: Scaling inference compute with repeated sampling. *CoRR*, abs/2407.21787.

Tom B. Brown, Benjamin Mann, Nick Ryder, Melanie Subbiah, Jared Kaplan, Prafulla Dhariwal, Arvind Neelakantan, Pranav Shyam, Girish Sastry, Amanda Askell, Sandhini Agarwal, Ariel Herbert-Voss, Gretchen Krueger, Tom Henighan, Rewon Child, Aditya Ramesh, Daniel M. Ziegler, Jeffrey Wu, Clemens Winter, and 12 others. 2020. Language models are few-shot learners. In *Advances in Neural Information Processing Systems 33: Annual Conference on Neural Information Processing Systems 2020, NeurIPS 2020, December 6-12, 2020, virtual*.

Mark Chen, Jerry Tworek, Heewoo Jun, Qiming Yuan, Henrique Pondé de Oliveira Pinto, Jared Kaplan, Harri Edwards, Yuri Burda, Nicholas Joseph, Greg Brockman, Alex Ray, Raul Puri, Gretchen Krueger, Michael Petrov, Heidy Khlaaf, Girish Sastry, Pamela Mishkin, Brooke Chan, Scott Gray, and 39 others. 2021. Evaluating large language models trained on code. *CoRR*, abs/2107.03374.

Quy-Anh Dang and Chris Ngo. 2025. Reinforcement learning for reasoning in small llms: What works and what doesn't. *arXiv preprint arXiv:2503.16219*.

DeepSeek-AI, Daya Guo, Dejian Yang, Haowei Zhang, Junxiao Song, Ruoyu Zhang, Runxin Xu, Qihao Zhu, Shirong Ma, Peiyi Wang, Xiao Bi, Xiaokang Zhang, Xingkai Yu, Yu Wu, Z. F. Wu, Zhibin Gou, Zhihong Shao, Zhuoshu Li, Ziyi Gao, and 81 others. 2025. Deepseek-r1: Incentivizing reasoning capability in llms via reinforcement learning. *CoRR*, abs/2501.12948.

Tim Dettmers, Artidoro Pagnoni, Ari Holtzman, and Luke Zettlemoyer. 2023. Qlora: Efficient finetuning of quantized llms. In *Advances in Neural Information*

Processing Systems 36: Annual Conference on Neural Information Processing Systems 2023, NeurIPS 2023, New Orleans, LA, USA, December 10 - 16, 2023.

Qingxiu Dong, Lei Li, Damai Dai, Ce Zheng, Jingyuan Ma, Rui Li, Heming Xia, Jingjing Xu, Zhiyong Wu, Baobao Chang, Xu Sun, Lei Li, and Zhifang Sui. 2024. A survey on in-context learning. In *Proceedings of the 2024 Conference on Empirical Methods in Natural Language Processing, EMNLP 2024, Miami, FL, USA, November 12-16, 2024*, pages 1107–1128. Association for Computational Linguistics.

Abhimanyu Dubey, Abhinav Jauhri, Abhinav Pandey, Abhishek Kadian, Ahmad Al-Dahle, Aiesha Letman, Akhil Mathur, Alan Schelten, Amy Yang, Angela Fan, Anirudh Goyal, Anthony Hartshorn, Aobo Yang, Archi Mitra, Archie Sravankumar, Artem Korenev, Arthur Hinsvark, Arun Rao, Aston Zhang, and 82 others. 2024. The llama 3 herd of models. *CoRR*, abs/2407.21783.

Chenghao Fan, Zhenyi Lu, Wei Wei, Jie Tian, Xiaoye Qu, Dangyang Chen, and Yu Cheng. 2024. On giant's shoulders: Effortless weak to strong by dynamic logits fusion. In *Advances in Neural Information Processing Systems 38: Annual Conference on Neural Information Processing Systems 2024, NeurIPS 2024, Vancouver, BC, Canada, December 10 - 15, 2024*.

Kanishk Gandhi, Ayush Chakravarthy, Anikait Singh, Nathan Lile, and Noah D. Goodman. 2025. Cognitive behaviors that enable self-improving reasoners, or, four habits of highly effective stars. *CoRR*, abs/2503.01307.

Dan Hendrycks, Collin Burns, Saurav Kadavath, Akul Arora, Steven Basart, Eric Tang, Dawn Song, and Jacob Steinhardt. 2021. Measuring mathematical problem solving with the MATH dataset. In *Proceedings of the Neural Information Processing Systems Track on Datasets and Benchmarks 1, NeurIPS Datasets and Benchmarks 2021, December 2021, virtual*.

Edward J. Hu, Yelong Shen, Phillip Wallis, Zeyuan Allen-Zhu, Yuanzhi Li, Shean Wang, Lu Wang, and Weizhu Chen. 2022. Lora: Low-rank adaptation of large language models. In *The Tenth International Conference on Learning Representations, ICLR 2022, Virtual Event, April 25-29, 2022*. OpenReview.net.

James Y. Huang, Wenxuan Zhou, Fei Wang, Fred Morstatter, Sheng Zhang, Hoifung Poon, and Muhao Chen. 2025. Offset unlearning for large language models. *Trans. Mach. Learn. Res.*, 2025.

Shijue Huang, Wanjun Zhong, Jianqiao Lu, Qi Zhu, Jiahui Gao, Weiwen Liu, Yutai Hou, Xingshan Zeng, Yasheng Wang, Lifeng Shang, Xin Jiang, Ruifeng Xu, and Qun Liu. 2024. Planning, creation, usage: Benchmarking llms for comprehensive tool utilization in real-world complex scenarios. In *Findings of the Association for Computational Linguistics, ACL 2024, Bangkok, Thailand and virtual meeting, August 11-16, 2024*, pages 4363–4400. Association for Computational Linguistics.

Naman Jain, King Han, Alex Gu, Wen-Ding Li, Fanjia Yan, Tianjun Zhang, Sida Wang, Armando Solar-Lezama, Koushik Sen, and Ion Stoica. 2025. Livecodebench: Holistic and contamination free evaluation of large language models for code. In *The Thirteenth International Conference on Learning Representations, ICLR 2025, Singapore, April 24-28, 2025*. OpenReview.net.

Carlos E. Jimenez, John Yang, Alexander Wettig, Shunyu Yao, Kexin Pei, Ofir Press, and Karthik Narasimhan. 2023. Swe-bench: Can language models resolve real-world github issues? *CoRR*, abs/2310.06770.

Nathan Lambert, Jacob Morrison, Valentina Pyatkin, Shengyi Huang, Hamish Ivison, Faeze Brahman, Lester James V. Miranda, Alisa Liu, Nouha Dziri, Shane Lyu, Yuling Gu, Saumya Malik, Victoria Graf, Jena D. Hwang, Jiangjiang Yang, Ronan Le Bras, Oyvind Tafjord, Chris Wilhelm, Luca Soldaini, and 4 others. 2024. Tülu 3: Pushing frontiers in open language model post-training. *CoRR*, abs/2411.15124.

Dacheng Li, Shiyi Cao, Tyler Griggs, Shu Liu, Xiangxi Mo, Eric Tang, Sumanth Hegde, Kourosh Hakhamaneshi, Shishir G. Patil, Matei Zaharia, Joseph E. Gonzalez, and Ion Stoica. 2025. Llms can easily learn to reason from demonstrations structure, not content, is what matters! *CoRR*, abs/2502.07374.

Hunter Lightman, Vineet Kosaraju, Yuri Burda, Harrison Edwards, Bowen Baker, Teddy Lee, Jan Leike, John Schulman, Ilya Sutskever, and Karl Cobbe. 2024. Let's verify step by step. In *The Twelfth International Conference on Learning Representations, ICLR 2024, Vienna, Austria, May 7-11, 2024*. OpenReview.net.

Alisa Liu, Xiaochuang Han, Yizhong Wang, Yulia Tsvetkov, Yejin Choi, and Noah A. Smith. 2024. Tuning language models by proxy. volume abs/2401.08565.

Alisa Liu, Maarten Sap, Ximing Lu, Swabha Swayamdipta, Chandra Bhagavatula, Noah A. Smith, and Yejin Choi. 2021. Dexperts: Decoding-time controlled text generation with experts and anti-experts. In *Proceedings of the 59th Annual Meeting of the Association for Computational Linguistics and the 11th International Joint Conference on Natural Language Processing, ACL/IJCNLP 2021, (Volume 1: Long Papers), Virtual Event, August 1-6, 2021*, pages 6691–6706. Association for Computational Linguistics.

Zichen Liu, Changyu Chen, Wenjun Li, Penghui Qi, Tianyu Pang, Chao Du, Wee Sun Lee, and Min Lin. 2025. Understanding r1-zero-like training: A critical perspective. *arXiv preprint arXiv:2503.20783*.

Michael Luo, Sijun Tan, Justin Wong, Xiaoxiang Shi, William Y. Tang, Manan Roongta, Colin Cai, Jeffrey Luo, Li Erran Li, Raluca Ada Popa, and Ion Stoica. 2025a. Deepscaler: Surpassing o1-preview with a 1.5b model by scaling rl. Notion Blog. Accessed 2025.

Michael Luo, Sijun Tan, Justin Wong, Xiaoxiang Shi, William Y Tang, Manan Roongta, Colin Cai, Jeffrey Luo, Tianjun Zhang, Li Erran Li, and 1 others. 2025b. Deepscaler: Surpassing o1-preview with a 1.5 b model by scaling rl. *Notion Blog*.

Brendan McMahan, Eider Moore, Daniel Ramage, Seth Hampson, and Blaise Agüera y Arcas. 2017. Communication-efficient learning of deep networks from decentralized data. In *Proceedings of the 20th International Conference on Artificial Intelligence and Statistics, AISTATS 2017, 20-22 April 2017, Fort Lauderdale, FL, USA*, volume 54 of *Proceedings of Machine Learning Research*, pages 1273–1282. PMLR.

Sewon Min, Xinxi Lyu, Ari Holtzman, Mikel Artetxe, Mike Lewis, Hannaneh Hajishirzi, and Luke Zettlemoyer. 2022. Rethinking the role of demonstrations: What makes in-context learning work? In *Proceedings of the 2022 Conference on Empirical Methods in Natural Language Processing, EMNLP 2022, Abu Dhabi, United Arab Emirates, December 7-11, 2022*, pages 11048–11064. Association for Computational Linguistics.

Eric Mitchell, Rafael Rafailov, Archit Sharma, Chelsea Finn, and Christopher D. Manning. 2024. An emulator for fine-tuning large language models using small language models. In *The Twelfth International Conference on Learning Representations, ICLR 2024, Vienna, Austria, May 7-11, 2024*. OpenReview.net.

Niklas Muennighoff, Zitong Yang, Weijia Shi, Xiang Lisa Li, Li Fei-Fei, Hannaneh Hajishirzi, Luke Zettlemoyer, Percy Liang, Emmanuel J. Candès, and Tatsunori Hashimoto. 2025. s1: Simple test-time scaling. *CoRR*, abs/2501.19393.

OpenAI. 2024. Learning to reason with LLMs.

OpenAI. 2025. OpenAI o3 and o4-mini System Card. `https://cdn.openai.com/pdf/2221c875-02dc-4789-800b-e7758f3722c1/o3-and-o4-mini-system-card.pdf`. Accessed: 2025-07-05.

OpenAI. 2025. Using logit bias to alter token probability with the openai api.

Aitor Ormazabal, Mikel Artetxe, and Eneko Agirre. 2023. Comblm: Adapting black-box language models through small fine-tuned models. In *Proceedings of the 2023 Conference on Empirical Methods in Natural Language Processing, EMNLP 2023, Singapore, December 6-10, 2023*, pages 2961–2974. Association for Computational Linguistics.

Bo Pang, Hanze Dong, Jiacheng Xu, Silvio Savarese, Yingbo Zhou, and Caiming Xiong. 2025. BOLT: bootstrap long chain-of-thought in language models without distillation. *CoRR*, abs/2502.03860.

Shishir G. Patil, Huanzhi Mao, Charlie Cheng-Jie Ji, Fanjia Yan, Vishnu Suresh, Ion Stoica, and Joseph E. Gonzalez. 2025. The berkeley function calling leaderboard (bfcl): From tool use to agentic evaluation of large language models. In *Forty-second International Conference on Machine Learning*.

Qwen Team. 2025. Qwen3: Think Deeper, Act Faster | Qwen.

Rafael Rafailov, Archit Sharma, Eric Mitchell, Christopher D. Manning, Stefano Ermon, and Chelsea Finn. 2023. Direct preference optimization: Your language model is secretly a reward model. In *Advances in Neural Information Processing Systems 36: Annual Conference on Neural Information Processing Systems 2023, NeurIPS 2023, New Orleans, LA, USA, December 10 - 16, 2023*.

David Rein, Betty Li Hou, Asa Cooper Stickland, Jackson Petty, Richard Yuanzhe Pang, Julien Dirani, Julian Michael, and Samuel R. Bowman. 2023. GPQA: A graduate-level google-proof q&a benchmark. *CoRR*, abs/2311.12022.

John Schulman, Filip Wolski, Prafulla Dhariwal, Alec Radford, and Oleg Klimov. 2017. Proximal policy optimization algorithms. *CoRR*, abs/1707.06347.

Zhihong Shao, Peiyi Wang, Qihao Zhu, Runxin Xu, Junxiao Song, Mingchuan Zhang, Y. K. Li, Y. Wu, and Daya Guo. 2024. Deepseekmath: Pushing the limits of mathematical reasoning in open language models. *CoRR*, abs/2402.03300.

Weijia Shi, Xiaochuang Han, Mike Lewis, Yulia Tsvetkov, Luke Zettlemoyer, and Wen-tau Yih. 2024. Trusting your evidence: Hallucinate less with context-aware decoding. In *Proceedings of the 2024 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies: Short Papers, NAACL 2024, Mexico City, Mexico, June 16-21, 2024*, pages 783–791. Association for Computational Linguistics.

Charlie Snell, Jaehoon Lee, Kelvin Xu, and Aviral Kumar. 2024. Scaling LLM test-time compute optimally can be more effective than scaling model parameters. *CoRR*, abs/2408.03314.

Xinyu Tang, Xiaolei Wang, Zhihao Lv, Yingqian Min, Wayne Xin Zhao, Binbin Hu, Ziqi Liu, and Zhiqiang Zhang. 2025. Unlocking general long chain-of-thought reasoning capabilities of large language models via representation engineering. *arXiv preprint arXiv:2503.11314*.

Qwen Team. 2025. Qwq-32b: Embracing the power of reinforcement learning.

Fanqi Wan, Xinting Huang, Deng Cai, Xiaojun Quan, Wei Bi, and Shuming Shi. 2024. Knowledge fusion of large language models. In *The Twelfth International Conference on Learning Representations, ICLR 2024, Vienna, Austria, May 7-11, 2024*. OpenReview.net.

Yiping Wang, Qing Yang, Zhiyuan Zeng, Liliang Ren, Lucas Liu, Baolin Peng, Hao Cheng, Xuehai He, Kuan Wang, Jianfeng Gao, and 1 others. 2025. Reinforcement learning for reasoning in large language models with one training example. *arXiv preprint arXiv:2504.20571*.

Jian Xie, Kai Zhang, Jiangjie Chen, Tinghui Zhu, Renze Lou, Yuandong Tian, Yanghua Xiao, and Yu Su. 2024. Travelplanner: A benchmark for real-world planning with language agents. In *Forty-first International Conference on Machine Learning, ICML 2024, Vienna, Austria, July 21-27, 2024*. OpenReview.net.

Haotian Xu, Xing Wu, Weinong Wang, Zhongzhi Li, Da Zheng, Boyuan Chen, Yi Hu, Shijia Kang, Jiaming Ji, Yingying Zhang, Zhijiang Guo, Yaodong Yang, Muhan Zhang, and Debing Zhang. 2025. Redstar: Does scaling long-cot data unlock better slow-reasoning systems? *CoRR*, abs/2501.11284.

Jiajun Xu, Zhiyuan Li, Wei Chen, Qun Wang, Xin Gao, Qi Cai, and Ziyuan Ling. 2024. On-device language models: A comprehensive review. *CoRR*, abs/2409.00088.

An Yang, Baosong Yang, Beichen Zhang, Binyuan Hui, Bo Zheng, Bowen Yu, Chengyuan Li, Dayiheng Liu, Fei Huang, Haoran Wei, Huan Lin, Jian Yang, Jianhong Tu, Jianwei Zhang, Jianxin Yang, Jiaxi Yang, Jingren Zhou, Junyang Lin, Kai Dang, and 22 others. 2024a. Qwen2.5 technical report. *CoRR*, abs/2412.15115.

An Yang, Beichen Zhang, Binyuan Hui, Bofei Gao, Bowen Yu, Chengpeng Li, Dayiheng Liu, Jianhong Tu, Jingren Zhou, Junyang Lin, Keming Lu, Mingfeng Xue, Runji Lin, Tianyu Liu, Xingzhang Ren, and Zhenru Zhang. 2024b. Qwen2.5-math technical report: Toward mathematical expert model via self-improvement. *CoRR*, abs/2409.12122.

Yixin Ye, Zhen Huang, Yang Xiao, Ethan Chern, Shijie Xia, and Pengfei Liu. 2025. LIMO: less is more for reasoning. *CoRR*, abs/2502.03387.

Qiying Yu, Zheng Zhang, Ruofei Zhu, Yufeng Yuan, Xiaochen Zuo, Yu Yue, Tiantian Fan, Gaohong Liu, Lingjun Liu, Xin Liu, Haibin Lin, Zhiqi Lin, Bole Ma, Guangming Sheng, Yuxuan Tong, Chi Zhang, Mofan Zhang, Wang Zhang, Hang Zhu, and 16 others. 2025. DAPO: an open-source LLM reinforcement learning system at scale. *CoRR*, abs/2503.14476.

Zheng Yuan, Hongyi Yuan, Chengpeng Li, Guanting Dong, Chuanqi Tan, and Chang Zhou. 2023. Scaling relationship on learning mathematical reasoning with large language models. *CoRR*, abs/2308.01825.

Weihao Zeng, Yuzhen Huang, Qian Liu, Wei Liu, Keqing He, Zejun Ma, and Junxian He. 2025. Simplerl-zoo: Investigating and taming zero reinforcement learning for open base models in the wild. *CoRR*, abs/2503.18892.

Xuandong Zhao, Xianjun Yang, Tianyu Pang, Chao Du, Lei Li, Yu-Xiang Wang, and William Yang Wang. 2024. Weak-to-strong jailbreaking on large language models. *CoRR*, abs/2401.17256.

Zekai Zhao, Qi Liu, Kun Zhou, Zihan Liu, Yifei Shao, Zhiting Hu, and Biwei Huang. 2025. Activation control for efficiently eliciting long chain-of-thought ability of language models. *arXiv preprint arXiv:2505.17697*.

Huaixiu Steven Zheng, Swaroop Mishra, Hugh Zhang, Xinyun Chen, Minmin Chen, Azade Nova, Le Hou, Heng-Tze Cheng, Quoc V. Le, Ed H. Chi, and Denny Zhou. 2024a. NATURAL PLAN: benchmarking llms on natural language planning. *CoRR*, abs/2406.04520.

Yaowei Zheng, Richong Zhang, Junhao Zhang, Yanhan Ye, Zheyan Luo, Zhangchi Feng, and Yongqiang Ma. 2024b. Llamafactory: Unified efficient fine-tuning of 100+ language models. In *Proceedings of the 62nd Annual Meeting of the Association for Computational Linguistics (Volume 3: System Demonstrations)*, Bangkok, Thailand. Association for Computational Linguistics.

Andy Zou, Long Phan, Sarah Li Chen, James Campbell, Phillip Guo, Richard Ren, Alexander Pan, Xuwang Yin, Mantas Mazeika, Ann-Kathrin Dombrowski, Shashwat Goel, Nathaniel Li, Michael J. Byun, Zifan Wang, Alex Mallen, Steven Basart, Sanmi Koyejo, Dawn Song, Matt Fredrikson, and 2 others. 2023. Representation engineering: A top-down approach to AI transparency. *CoRR*, abs/2310.01405.

Haosheng Zou, Xiaowei Lv, Shousheng Jia, and Xiangzheng Zhang. 2024. 360-llama-factory.

A Training Details

Environment. All experiments were conducted using NVIDIA A40/L40S GPUs with 48GB memory. The software environment was configured as follows:

- `360-LLaMA-Factory` (Zou et al., 2024) (A long CoT adapted version of `LLaMA-Factory` 0.9.1 (Zheng et al., 2024b))
- `torch` 2.7.0
- `transformers` 4.51.3
- `accelerate` 1.0.1
- `datasets` 3.1.0
- `trl` 0.9.6
- `peft` 0.12.0
- `deepspeed` 0.14.4

	`Target` (Qwen2.5-32B)	`Target` + THINKLOGIT	`Target` + Full SFT (R1-Distill-Qwen-32B)
Trainable Parameters	–	0	32B
Training Examples	–	0	800K
Inference Parameters	32B	35B (=32B+1.5B+1.5B)	32B
Inference GPU Count (# NVIDIA L40S, 46GB)	2	2	2
Inference Throughput (tokens/second)	10.1	7.6 (–25%)	9.1 (–10%)
Generation Length (tokens)	1166.6	6070.6	5445.5
Inference Latency Per Generation (seconds)	115.3	797.5	597.6

Table 5: Comparison of training and inference efficiency. THINKLOGIT trades increased inference latency for zero training cost.

LoRA Configuration. We applied LoRA (Hu et al., 2022) for parameter-efficient fine-tuning of the guider model:

- Rank: 64
- α_{LoRA}: 128
- Target modules: `q_proj`, `k_proj`, `v_proj`, `o_proj`
- Bias: None

DPO Training. For preference optimization with DPO, we used the following settings:

- Batch size: 32 (4 GPUs * 8 Gradient Accumulation)
- Epoch: 1
- Learning rate: 5e-6
- Optimizer: AdamW
- Learning rate scheduler: cosine with warmup
- Warmup ratio: 0.1
- β (reward scaling): 0.1
- Cutoff length: 8192

B Additional Results

Table 5 compares key generation efficiency metrics across the target model, THINKLOGIT, and a fully fine-tuned target. THINKLOGIT requires no additional training examples or tunable parameters, while full fine-tuning involves 800K examples and 32B trainable parameters. The decoding-time method increases inference latency due to longer reasoning traces but eliminates the need for expensive training and parameter updates. Figure 6 shows the effect of guidance strength α on THINKLOGIT performance, with α=1.0 serving as the robust default.

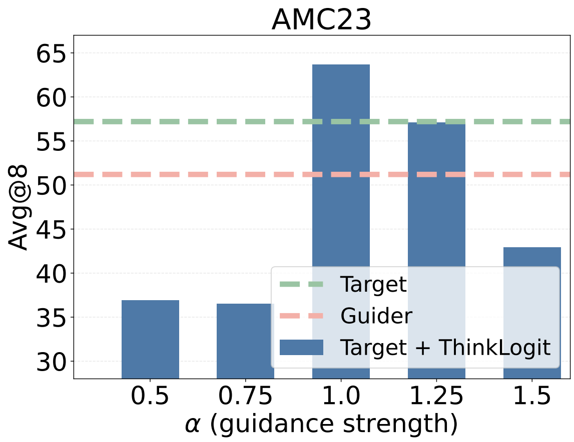

Figure 6: Sweeping the guidance strength α shows that α = 1.0 yields the best trade-off between guider influence and target model priors.