

## 15F-1 Bookkeeping

- 0 pts Correct

$$\begin{aligned}
& \text{vc}(\text{do}_{\text{inv}} c \text{ while } b, P) = \\
& = \text{vc}(c; \text{while } b \text{ do } c, P) \\
& = \text{vc}(c, \text{vc}(\text{while } b \text{ do } c, P)) \\
& = \\
& \text{vc}(c, \text{inv}) \wedge (\forall x_1, \dots, x_n. \text{inv} \Rightarrow (b \Rightarrow \text{vc}(c, \text{inv}) \\
& \quad \wedge \neg b \Rightarrow P))
\end{aligned}$$

The only change will be in the entry condition. Instead of checking the invariant as we do in `while ... do ..`, we should check  $\text{vc}(c, \text{inv})$  because the the loop body  $c$  will already have been executed once before reaching the invariant for the first time. The condition for checking the invariant on an arbitrary iteration should remain the same.

## 2 5F-2 VCGen Do-While

- 0 pts Correct

1.  $\text{stack}$ 2.  $x = 0$ 3.  $x = 5$ 4.  $\sigma[x] = 0$ 5.  $\sigma'[x] = 5$ 6.  $\text{while } x < 5 \text{ do } x := x + 1$ 7.  $\langle \text{while } x < 5 \text{ do } x := x + 1, \sigma[x=0] \rangle \Downarrow \sigma'[x=5]$ 8.  $\sigma \vDash A = \sigma \vDash x=0 = \text{true}$ 9.  $\sigma' \vDash B = \sigma' \vDash x=5 = \text{true}$ 

10.

$$\vdash \{A\} \subset \{B\}$$

$$= \vdash \{x=0\} \text{ while } x < 5 \text{ do } x := x + 1 \{x=5\}$$

To show the provability of the above statement, we need to find an intermediate condition  $Z$  such that:

$$\textcircled{1} \{x=0\} \Rightarrow \{Z\}$$

$$\textcircled{2} \vdash \{Z \wedge x < 5\} \subset \{Z\}$$

$$\textcircled{3} \{Z\} \Rightarrow \{x=5\}$$

Combining  $\textcircled{1}$  and  $\textcircled{3}$

$$\{x=0\} \Rightarrow \{x=5\}$$

which is clearly false. Thus, the system  $\{\textcircled{1}, \textcircled{2}, \textcircled{3}\}$  is unsatisfiable. Thus, such a  $Z$  does not exist.

1.  $\text{targaryen}$

2.  $x = 0$

3.  $x = 5$

4.  $\sigma[x] = 0$

5.  $\sigma'[x] = 5$

6.  $\text{while } x < 5 \text{ do } x := x + 1$

7.  $\langle \text{while } x < 5 \text{ do } x := x + 1, \sigma[x=0] \rangle \Downarrow \sigma'[x=5]$

8.  $\sigma \vDash A = \sigma \vDash x=0 = \text{true}$

9.  $\sigma' \vDash B = \sigma' \vDash x=5 = \text{true}$

10.

$\vdash \{A\} \subset \{B\}$

$= \vdash \{x=0\} \text{ while } x < 5 \text{ do } x := x + 1 \{x=5\}$

To show the provability of the above statement, we need to find an intermediate condition  $Z$  such that:

①  $\{x=0\} \Rightarrow \{Z\}$

②  $\vdash \{Z\} \subset \{Z\}$

③  $\vdash \{Z \wedge x > 5\} \Rightarrow \{x=5\}$

By ③  $Z = \{x=5\}$  or  $Z = \{x < 6\}$

Case 1:  $Z = \{x=5\}$

This contradicts with ① because

$\{x=0\} \Rightarrow \{x=5\}$

is clearly false

Case 2:  $Z = \{x > 6\}$

This contradicts with (2) because

$$\{x > 6\} \quad x := x + 1 \quad \{x > 6\}$$

is clearly not provable

Thus, the system  $\{①, ②, ③\}$  is unsatisfiable.

Thus, such a  $Z$  does not exist.

$\{A\} \subset \{B\}$  is not provable by rule targaryen

### 3 5F-3 VCGen Mistakes

- 0 pts Correct