1 5F-1 Bookkeeping

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Exercise 5F-2

We know c is executed once before b is evaluated. Thus, we know that $do\ c\ while\ b$ is equivalent to $c;\ while\ b\ do\ c$. Chosen the second version in the question $do_{inv1,\,inv2}\ c\ while\ b$, we know that $inv2$is the loop invariant of the while loop. Then, we have:

$VC(do_{inv1,\,inv2}\ c\ while\ b)$

$= \ inv1\ \wedge\ VC(c;\ while_{\ inv2}\ b\ do\ c,\ P)$

$= \ inv1\ \wedge\ VC(c,\ VC(while_{\ inv2}\ b\ do\ c,\ P))$

$= \ inv1\ \wedge\ VC(c,\ inv2\ \wedge\ (\forall\, x_1, \ldots, x_n.\ inv2 \Rightarrow (b \Rightarrow VC(c,\ inv2)\ \wedge\ \neg b\ \Rightarrow P)))$

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2 5F-2 VCGen Do-While

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Exercise 5F-3

1. Stark

2. A == true

3. B == x>=10

4. σ(x) = 0

5. σ'(x) = 11

6. c == while x < 10 do x := 11

7.

$$\frac{\langle x < 10,\ \sigma \rangle \Downarrow true \quad \langle x := 11,\ \sigma \rangle \Downarrow \sigma[x := 11] \quad \dfrac{\langle x < 10,\ \sigma[x := 11] \rangle \Downarrow false}{\langle while\ x < 10\ do\ x := 11,\ \sigma[x := 11] \rangle \Downarrow \sigma'}}{\langle while\ x < 10\ do\ x := 11,\ \sigma \rangle \Downarrow \sigma'}$$

$\sigma[x := 11]$ is equivalent to σ' Thus, we prove that $\langle c, \sigma \rangle \Downarrow \sigma'$

8. $\sigma \vDash A$ as $\sigma \vDash true$

9. $\sigma' \vDash B$ is equivalent to $\sigma' \vDash x >= 10$. It is true because σ'(x) = 11

10. Assume we are able to prove $\vdash \{A\}\ c\ \{B\}$using the Stark rule. Then we have:

D :: $\vdash \{true\}\ while\ x < 10\ do\ x := 11\ \{x >= 10\}$

By inversion, the last rule used by D can only be the Stark rule or the rule of consequence.

1) let the last rule used by D be the Stark rule. We can't proceed as the Stark rule requires the precondition and the postcondition be the same, whereas $true$ is not the same as $x >= 10$.
2) let the last rule used by D be the rule of consequence. Then we have:

$$\frac{\vdash true \Rightarrow C \quad \vdash \{C\}\ while\ x < 10\ do\ x := 11\ \{C\} \quad \vdash C \Rightarrow x >= 10}{\vdash \{true\}\ while\ x < 10\ do\ x := 11\ \{x >= 10\}}$$

However, we are stuck on $\vdash true \Rightarrow C \Rightarrow x >= 10$ as no such Cexists.

Hence, we reach contradictions in both cases. We can't prove $\vdash \{A\}\ c\ \{B\}$that is true so the Stark rule is incomplete.

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1. Targaryen

2. A == x <= 10

3. B == x =10

4. σ(x) = 9

5. σ'(x) = 10

6. c == while x < 10 do x := x + 1

7.

$$\dfrac{<x<10, \sigma> \Downarrow true \quad <x:=x+1, \sigma> \Downarrow \sigma[x:=10] \quad \dfrac{<x<10,\ \sigma[x:=10]> \Downarrow false}{<while\ x<10\ do\ x:=x+1,\ \sigma[x:=10]> \Downarrow \sigma'}}{<while\ x\ <\ 10\ do\ x:=\ x+1,\ \sigma> \Downarrow \sigma'}$$

$\sigma[x:=10]$ is equivalent to σ' Thus, we prove that $<c,\ \sigma>\ \Downarrow \sigma'$

8. $\sigma \vDash A$ is equivalent to $\sigma \vDash x <= 10$, which is true because σ(x) = 9

9. $\sigma' \vDash B$ is equivalent to $\sigma' \vDash x = 10$. It is true because σ'(x) = 10

10. Assume we are able to prove $\vdash \{A\}\ c\ \{B\}$using the Targaryen rule. Then we have:

D :: $\vdash \{x <= 10\}\ while\ x\ <\ 10\ do\ x:=x+1\ \{x\ =\ 10\}$

By inversion, the last rule used by D can only be the Targaryen rule or the rule of consequence.

3) let the last rule used by D be the Targaryen rule. We can't proceed as the Targaryen rule requires the postcondition includes the precondition, whereas $x = 10$ is not the same as $x\ <= 10$.

4) let the last rule used by D be the rule of consequence. Then we have:

$$\dfrac{\vdash x <= 10 \Rightarrow C \quad \vdash \{C\}\ while\ x\ <\ 10\ do\ x:=x+1\ \{C \land x >= 10\} \quad \vdash C \land x >= 10 \Rightarrow x = 10}{\vdash \{x <= 10\}\ while\ x\ <\ 10\ do\ x:=x+1\ \{x = 10\}}$$

we can easily conclude from above that C should be x<=10. However, when we further apply the Targaryen rule on $\vdash \{C\}\ while\ x\ <\ 10\ do\ x:=x+1\ \{C \land x >= 10\}$, we have $\vdash \{C\}\ x:=x+1\ \{C\}$, which can't be true under all possible correct C.

Hence, we reach contradictions in both cases. Since the Targaryen rule can't prove all $\vdash \{A\}\ c\ \{B\}$that are true, it is incomplete.

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3 5F-3 VCGen Mistakes

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