Exercise 5F-2. VCGen Do-While [8 points]. The VC rule for $\mathsf{do}_{Inv}\ c\ \mathsf{while}\ b$ is as follows: $\mathrm{VC}(\mathsf{do}_{Inv}\ c\ \mathsf{while}\ b) =$
$\mathrm{Inv}\ \wedge (\forall x_1, \ldots, x_n.\ \mathrm{Inv}\ \ {=}{>}\ (VC(c, b \implies \mathrm{VC}(c,\ \mathrm{Inv}\ ) \wedge (\neg b \implies P)))$

This rule follows similar to the rule for VC(while), except that we run the command once before evaluating the loop guard (b). We start with the loop invariant (Inv), which forms the first clause of the conjunct. For later iterations $x_i \in [1, n]$, if we can establish the loop invariant again, then we get recursively call (VC), with the command (c), the VC condition with the guard established, and the ability to infer P, when we know b is false.

Exercise 5F-3. VCGen Mistakes [20 points]. We will now demonstrate that the stark rule is sound but incomplete.

$$\frac{\vdash \{X \ \wedge \ b\} \ c \ \{X\}}{\vdash \{X\} \ \textsf{while} \ b \ \textsf{do} \ c \ \{X\}} \ \textsf{stark}$$

1. the name of the rule - stark
2. A - x $=$ 5
3. B and - x $=$ 5
4. σ and - $\sigma(x) = 5$
5. σ' and - $\sigma'(x) = 5$
6. c such that - while (x $<$ 5) do (skip)
7. $\langle c, \sigma \rangle \Downarrow \sigma'$ - loop guard is false on entry
8. $\sigma \models A$ and
9. $\sigma' \models B$ but
10. it is not possible to prove $\vdash \{A\} \ c \ \{B\}$.

Using the rule, we would try to prove that $\{x = 5\}c\{x = 5\}$. However, since the guard is false on entry already, we cannot apply the premise to show that the post-condition (which is *trivial*) in this case. So, we are unable to extract the information we know is true to begin with, reflecting on the incompleteness of the rule.

We will now demonstrate that the **targaryen** rule is sound but incomplete.

$$\frac{\vdash \{X\}\ c\ \{X\}}{\vdash \{X\}\ \mathsf{while}\ b\ \mathsf{do}\ c\ \{X\ \wedge\ \neg b\}}\ \mathsf{targaryen}$$

1. the name of the rule - targaryen
2. A - x = 0
3. B - x = 5
4. σ - $\sigma(x) = 0$
5. σ' - $\sigma'(x) = 5$
6. c - (x := x + 1)
7. $\langle c, \sigma\rangle \Downarrow \sigma'$ - loop terminates when x is incremented till it is 5.
8. $\sigma \models A$ and
9. $\sigma' \models B$ but
10. it is not possible to prove $\vdash \{A\}\ c\ \{B\}$.

To apply Targaryen, we would need to show the premise $\{X\}c\{X\}$ for some invariant X. But here, c changes x from 0 to 1, so it does not preserve x = 0. Even for other X, the rule can only conclude additionally that the guard is no longer true ($\neg b$). We do eventually reach x = 5, which is stronger than $X \wedge \neg(x < 5)$. Thus the rule is incomplete: it fails to prove some perfectly correct triples, including this simple loop that increments x from 0 to 5.