Exercise 4F-2. VCGen for Let [6 points].

$$\text{VC}(\mathsf{let}\ x = e\ \mathsf{in}\ c, B) = [e/x]\text{VC}(c, [\sigma(x)/x]B)$$

This new rule ensures the restoration of the original value of x.

Exercise 4F-3. VCGen Mistakes [6 points].
Let us define a command c as

$$\mathsf{let}\ x = 1\ \mathsf{in}\ x = y + x$$

a post-condition B as

$$x + y = 3$$

and an initial state σ as

$$\{x = 5, y = 1\}$$

We can see that using the buggy rule for VC($\mathsf{let}\ x = 1\ \mathsf{in}\ x = y + x, x + y = 3$), we get

$$\begin{aligned}&[1/x]\text{VC}(x = y + x, x + y = 3)\\ &\Rightarrow [1/x](2y + x = 3)\\ &\Rightarrow 2y + 1 = 3\\ &\Rightarrow 2y = 2\\ &\Rightarrow y = 1\end{aligned}$$

which is satisfied by our initial state σ. After evaluating $\langle c, \sigma\rangle \Downarrow \sigma'$, we get σ' as

$$\{x = 5, y = 1\}$$

as the original $\sigma(x)$ is preserved after the let command. We can see that $\sigma' \not\models B$, as $x+y \neq 3$. Therefore, we have demonstrated the unsoundness of the buggy let rule.

Exercise 4F-4. Axiomatic Do-While [6 points].

$$\frac{\vdash \{A\}\ c\ \{B\} \quad \vdash \{B \wedge b\}\ c\ \{B\}}{\vdash \{A\}\ \mathsf{do}\ c\ \mathsf{while}\ b\ \{B \wedge \neg b\}}$$