

14F-1 Bookkeeping

- 0 pts Correct

Exercise 4F-2. VCGen for Let [6 points]

Let x_2 be a fresh variable in the following rule.

$$\begin{aligned} \text{VC}(\text{let } x = e \text{ in } c, B) &= \text{VC}(x_2 := x; x := e; c; x := x_2, B) \\ &= \text{VC}(x_2 := x; x := e; c, \text{VC}(x := x_2, B)) \\ &= \text{VC}(x_2 := x; x := e; c, [x_2/x]B) \\ &= \text{VC}(x_2 := x; x := e, \text{VC}(c, [x_2/x]B)) \\ &= \text{VC}(x_2 := x, \text{VC}(x := e, \text{VC}(c, [x_2/x]B))) \\ &= \text{VC}(x_2 := x, [e/x]\text{VC}(c, [x_2/x]B)) \\ &= [x/x_2][e/x]\text{VC}(c, [x_2/x]B) \end{aligned}$$

Exercise 4F-3. VCGen Mistakes [6 points]

Considering the following example:

1. c : **let** $x = 1$ **in skip**
2. B : $x > 0$
3. σ : $x = 0$

Consider the following situation where we evaluate $\text{VC}(c, B)$ using σ

$$\begin{aligned} \text{VC}(c, B) &= \text{VC}(\text{let } x = 1 \text{ in skip}, B) \\ &= [1/x]\text{VC}(\text{skip}, B) \\ &= [1/x]B \\ &= [1/x](x > 0) \\ &= \mathbf{True} \end{aligned}$$

Thus, $\sigma \models \text{VC}(c, B)$ as desired.

Now, let's evaluate σ' .

$$\begin{aligned} \langle c, \sigma \rangle &\Downarrow \sigma' \\ \langle \text{let } x = 1 \text{ in skip};, \sigma \rangle &\Downarrow \sigma' \end{aligned}$$

This, consequently gives us $\sigma' = \sigma$ as the command in the *let* does not modify anything. Thus $\sigma' : x = 0$.

Now, let's evaluate and show $\sigma' \not\models B$. We know that $\sigma'[x] = 0$. Then, when B checks $x > 0$ it compares $0 > 0$, which is clearly false.

Thus, our *let* rule is unsound as we have proven a statement that we have just shown is false.

2 4F-2 VCGen for Let

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3 4F-3 VCGen Mistakes

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Exercise 4F-4. Axiomatic Do-While [6 points]

We just want to somehow encode the information of running c once before we run the while loop stuff. Thus, we have a statement that looks very similar to the while loop rule presented in lecture.

$$\frac{\vdash \{A\} c \{B\} \quad \vdash \{B \wedge b\} c \{B\}}{\vdash \{A\} \mathbf{do} c \mathbf{while} b \{B \wedge \neg b\}}$$

4 4F-4 Axiomatic Do-While

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