1 4F-1 Bookkeeping

- 0 pts Correct

Exercise 4F-2

The original let rule doesn't consider the scope where x is assigned to Aexp e. We can deduct a correct let rule by creating a new variable $temp$, assigning $temp$ to the original value of x first, and reassigning x back to its original value $temp$ after executing c. This means evaluating $VC(let\ x = e\ in\ c,\ B)$ is equivalent to evaluating $VC(temp := \ x;\ x := e;\ c;\ x := temp,\ B)$. Applying other given rules, then we have:

$$\begin{aligned} VC(temp := \ x;\ x := e;\ c;\ x := temp,\ B) &= [x/temp]\, VC(x := e;\ c;\ x := temp,\ B) \\ &= [x/temp]\,[e/x]\, VC(\ c;\ x := temp,\ B) \\ &= [x/temp]\,[e/x]\, VC(\ c,\ VC(x := temp,\ B)) \\ &= [x/temp]\,[e/x]\, VC(\ c,\ [temp/x]\, B) \end{aligned}$$

Thus, $VC(let\ x = e\ in\ c,\ B) = [x/temp]\,[e/x]\, VC(c,\ [temp/x]\, B)$

Exercise 4F-3

To show that the buggy let rule is unsound, we set:

1. $c\ ==\ let\ x = 3\ in\ y := y - x$
2. $B\ ==\ x = y$
3. $\sigma(x) = 1\ and\ \sigma(y) = 6$
4. We know that $\sigma \vDash VC(c,\ B)$ because

$$\begin{aligned} VC(let\ x = 3\ in\ y := y - x,\ x = y) &= [3/x]\, VC(\ y := y - x,\ x = y) \\ &= [3/x]\,[y - x/y]\, x = y \\ &= [3/x]\ x = y - x \\ &= (3 = y - 3) \end{aligned}$$

since $\sigma(y) = 6$, we have $3 = 3$. Thus, $VC(c,\ B)$ is true in σ.

5. According to $< c,\ \sigma > \Downarrow \sigma'$, we have $\sigma'(x) = 1\ and\ \sigma'(y) = 3$
6. $\sigma' \nvDash B$because in σ' we have $\sigma'(x) = 1,\ \sigma'(y) = 3$, but $1 \neq 3$.

Hence, we demonstrate that the buggy let rule can prove a false thing, and thus it is unsound.

2 4F-2 VCGen for Let

- 0 pts Correct

Exercise 4F-2

The original let rule doesn't consider the scope where x is assigned to Aexp e. We can deduct a correct let rule by creating a new variable $temp$, assigning $temp$ to the original value of x first, and reassigning x back to its original value $temp$ after executing c. This means evaluating $VC(let\ x = e\ in\ c,\ B)$ is equivalent to evaluating $VC(temp := x;\ x := e;\ c;\ x := temp,\ B)$. Applying other given rules, then we have:

$$
\begin{aligned}
VC(temp := x;\ x := e;\ c;\ x := temp,\ B) &= [x/temp]\, VC(x := e;\ c;\ x := temp,\ B) \\
&= [x/temp]\, [e/x]\, VC(\ c;\ x := temp,\ B) \\
&= [x/temp]\, [e/x]\, VC(\ c,\ VC(x := temp,\ B)) \\
&= [x/temp]\, [e/x]\, VC(\ c,\ [temp/x]\, B)
\end{aligned}
$$

Thus, $VC(let\ x = e\ in\ c,\ B) = [x/temp]\, [e/x]\, VC(c,\ [temp/x]\, B)$

Exercise 4F-3

To show that the buggy let rule is unsound, we set:

1. $c == let\ x = 3\ in\ y := y - x$
2. $B == x = y$
3. $\sigma(x) = 1\ and\ \sigma(y) = 6$
4. We know that $\sigma \vDash VC(c,\ B)$ because

$$
\begin{aligned}
VC(let\ x = 3\ in\ y := y - x,\ x = y) &= [3/x]\, VC(\ y := y - x,\ x = y) \\
&= [3/x]\, [y - x/y]\, x = y \\
&= [3/x]\ x = y - x \\
&= (3 = y - 3)
\end{aligned}
$$

since $\sigma(y) = 6$, we have $3 = 3$. Thus, $VC(c,\ B)$ is true in σ.

5. According to $< c,\ \sigma > \Downarrow \sigma'$, we have $\sigma'(x) = 1\ and\ \sigma'(y) = 3$
6. $\sigma' \nvDash B$because in σ' we have $\sigma'(x) = 1,\ \sigma'(y) = 3$, but $1 \neq 3$.

Hence, we demonstrate that the buggy let rule can prove a false thing, and thus it is unsound.

3 4F-3 VCGen Mistakes

- 0 pts Correct

Exercise 4F-4

We know c is executed once before b is evaluated. Thus, we can derive Hoare rule for $do\ while$ according to the $while$ rule:

$$\frac{\vdash \{A\}\ c\ \{B\} \quad \vdash \{B\}\ while\ b\ do\ c\ \{C\}}{\vdash \{A\}\ do\ c\ while\ b\ \{C\}}$$

Then, we can further derive it to:

$$\frac{\vdash \{A\}\ c\ \{B\} \quad \vdash \{B\}\ while\ b\ do\ c\ \{B \land \neg\ b\}}{\vdash \{A\}\ do\ c\ while\ b\ \{B \land \neg\ b\}}$$

Finally, we will have:

$$\frac{\vdash \{A\}\ c\ \{B\} \quad \vdash \{B \land b\}\ c\ \{B\}}{\vdash \{A\}\ do\ c\ while\ b\ \{B \land \neg\ b\}}$$

4 4F-4 Axiomatic Do-While

- 0 pts Correct

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