

**Exercise 4F-2. VCGen for Let [6 points].** The problem here is that the current VC rule for `let` treats it like a literal assignment `x := e`, and doesn't account for scoping.

$$\text{VC}(c[x \mapsto x_{\text{local}}, B][x_{\text{local}} \mapsto e])$$

Here, we rename the fresh *local*  $x$  in  $c$  to a new variable ( $x_{\text{local}}$ ) to avoid capturing the original scoping, and account for possible shadowing. The key idea is that the old  $x$  is restored after the `let`.

Question assigned to the following page: [3](#)

**Exercise 4F-3. VCGen Mistakes [6 points].** Given  $\{A\}c\{B\}$  we desire that  $A \implies \text{VC}(c, B) \implies \text{WP}(c, B)$ . We say that our VC rules are *sound* if  $\models \{\text{VC}(c, B)\} c \{B\}$ . Demonstrate the unsoundness of the buggy let rule by giving the following six things:

1. a command  $c$  - let  $x = 5$  in skip
2. a post-condition  $B$  -  $(x = 5)$
3. a state  $\sigma$  such that -  $\sigma(x) = 10$
4.  $\sigma \models \text{VC}(c, B)$  -  $[5/x]\text{VC}(\text{skip}, B)$ ,  $\text{VC}(\text{skip}, B) = B$ ,  $[5/x](x = 5)$ , which is true and satisfied by every  $\sigma$ . So,  $\sigma \models \text{VC}(c, B)$ .
5.  $\langle c, \sigma \rangle \Downarrow \sigma' - \langle \text{let } x = 5 \text{ in skip}, \sigma[x := 10] \rangle$ . The skip does nothing, and restores  $x$  to 10 after execution.
6.  $\sigma' \not\models B$ . - but  $\sigma' \neq 5$ , so  $\sigma' \not\models B$  as  $\sigma'(x) = 10$

Problem arises from treating let binding like an assignment ( $:=$ ).

Question assigned to the following page: [4](#)

**Exercise 4F-4. Axiomatic Do-While [6 points].** Write a sound and complete Hoare rule for `do c while b`. This statement has the standard semantics (e.g.,  $c$  is executed at least once, before  $b$  is tested).

$$\frac{\text{DO-WHILE} \quad \frac{\{A\}c\{B\} \quad \{B \wedge b\}c\{B\}}{\{A\} \text{ do } c \text{ while } b \{B \wedge \neg b\}}}{\{A\} \text{ do } c \text{ while } b \{B \wedge \neg b\}}$$

The idea is that command  $c$  is first executed unconditionally, which establishes another assertion  $B$ , from the initial pre-condition  $A$ . Then, the loop behaves similar to a `while` loop as discussed in class, which established a loop invariant  $B$  to continue running, and finally exits when the loop guard ( $b$ ) is false (with  $B$  still holding).