1 2F-1 Bookkeeping

- 0 pts Correct

Exercise 2F-2. Mathematical Induction [5 points]. Find the flaw in the following inductive proof that "All flowers smell the same". Please indicate exactly which sentences are wrong in the proof via highlighting or underlining.

Proof: Let F be the set of all flowers and let $\mathsf{smells}(f)$ be the smell of the flower $f \in F$. (The range of smells is not so important, but we'll assume that it admits equality.) We'll also assume that F is countable. Let the property $P(n)$ mean that all subsets of F of size at most n contain flowers that smell the same.

$$P(n) \stackrel{\text{def}}{=} \forall X \in \mathcal{P}(F).\ |X| \le n \implies (\forall f, f' \in X.\ \mathsf{smells}(f) = \mathsf{smells}(f'))$$

(the notation $|X|$ denotes the number of elements of X)

One way to formulate the statement to prove is $\forall n \ge 1.P(n)$. We'll prove this by induction on n, as follows:

Base Case: $n = 1$. Obviously all singleton sets of flowers contain flowers that smell the same (by the definition of $P(n)$).

Induction Step: Let n be arbitrary and assume that all subsets of F of size at most n contain flowers that smell the same. We will prove that the same thing holds for all subsets of size at most $n+1$. Pick an arbitrary set X such that $|X| = n+1$. Pick two distinct flowers $f, f' \in X$ and let's show that $\mathsf{smells}(f) = \mathsf{smells}(f')$. Let $Y = X - \{f\}$ and $Y' = X - \{f'\}$. Obviously Y and Y' are sets of size at most n so the induction hypothesis holds for both of them. Pick any arbitrary $x \in Y \cap Y'$. Obviously, $x \neq f$ and $x \neq f'$. We have that $\mathsf{smells}(f') = \mathsf{smells}(x)$ (from the induction hypothesis on Y) and $\mathsf{smells}(f) = \mathsf{smells}(x)$ (from the induction hypothesis on Y'). Hence $\mathsf{smells}(f) = \mathsf{smells}(f')$, which proves the inductive step, and the theorem.

(One indication that the proof might be wrong is the large number of occurrences of the word "obviously" :-))

2 2F-2 Mathematical Induction

- 0 pts Correct

2F-3. Hints from GSI's discussion

Base Case: while false.

Case: the last rule used in D was the one for while false

$$\frac{\langle b, \sigma \rangle \Downarrow false}{\langle \text{while } b \text{ do } x := x+2, \sigma \rangle \Downarrow \sigma}$$

so $\sigma'(x) = \sigma(x)$, even.

Induction Case: while true

Case: the last rule used in D was the one for while true

$$\frac{\langle b, \sigma \rangle \Downarrow true \quad \langle x := x+2 \rangle \Downarrow \sigma'' \quad D' :: \langle \text{while } b \text{ do } x := x+2, \sigma'' \rangle \Downarrow \sigma'}{\langle \text{while } b \text{ do } x := x+2, \sigma \rangle \Downarrow \sigma'}$$

Then, observe that our hypothesis is if $\sigma''(x)$ is even in D', then $\sigma'(x)$ will also be even.

Then let's analysis $\sigma''(x)$.

we have
$$\frac{\dfrac{\langle 2, \sigma \rangle \Downarrow 2 \quad \langle x, \sigma \rangle \Downarrow \sigma(x)}{\langle x+2 \rangle \Downarrow \sigma(x)+2}}{\langle x := x+2, \sigma \rangle \Downarrow \sigma[x := \sigma(x)+2]}$$

so, $\sigma''(x) = \sigma[x := \sigma(x)+2]$

and if $\sigma(x)$ is even. then $\sigma''(x)$ is also even.

Then since if $\sigma(x)$ is even, then $\sigma''(x)$ is even, and if $\sigma''(x)$ is even, then $\sigma(x)$ is even, we are done.

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3 2F-3 While Induction

- 0 pts Correct

2F-4

Throw

$$\frac{\langle e, \sigma \rangle \Downarrow n}{\langle \text{throw } e, \sigma \rangle \Downarrow \sigma \text{ exc } n}$$

Try c_1 catch x $c2$

① if c_1 normally terminates

$$\frac{\langle c_1, \sigma \rangle \Downarrow \sigma'}{\langle \text{try } c_1 \text{ catch } x\ c_2, \sigma \rangle \Downarrow \sigma'}$$

② if c_1 has exception value e

$$\frac{\langle c_1, \sigma \rangle \Downarrow \sigma' \text{ exc } n \quad \langle x := n; c_2, \sigma' \rangle \Downarrow t}{\langle \text{try } c_1 \text{ catch } x\ c_2, \sigma \rangle \Downarrow t}$$

After c_1 finally c_2

① if c_1 normally terminates

$$\frac{\langle c_1, \sigma \rangle \Downarrow \sigma' \quad \langle c_2, \sigma' \rangle \Downarrow t}{\langle \text{after } c1 \text{ finally } c_2, \sigma \rangle \Downarrow t}$$

② if c_1 has exception value e, and c_2 normally terminate

$$\frac{\langle c_1, \sigma \rangle \Downarrow \sigma' \text{ exc } n_1 \quad \langle c_2, \sigma' \rangle \Downarrow \sigma''}{\langle \text{after } c1 \text{ finally } c_2, \sigma \rangle \Downarrow \sigma'' \text{ exc } n_1}$$

③ if both c_1, c_2 have exceptions with value e_1, e_2 respectively.

$$\frac{\langle c_1, \sigma \rangle \Downarrow \sigma' \text{ exc } n_1 \quad \langle c_2, \sigma' \rangle \Downarrow \sigma'' \text{ exc } n_2}{\langle \text{after } c1 \text{ finally } c_2, \sigma \rangle \Downarrow \sigma'' \text{ exc exc } n_2}$$

4 2F-4 Language Features, Large Step

- 0 pts Correct

2F-5. (By GSI's comment on HW1, the following paragraph is checked by Grammarly)

I agree with the statement that it is more natural to describe IMP with exceptions using small-step contextual semantic. I believe in small-steps, we will show all the conditions to handle all the possible cases, like which part throws which exception. However, in the big-steps, for instance, like the after-finally case in previous question, we use the t in the big-steps, to combine all other uncertainties.

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5 2F-5 Language Features, Analysis

- 0 pts Correct