

Model Checking

One-Slide Summary

- **Model Checking** is an **algorithmic** method in **software engineering** used to verify if a finite-state model of a system meets specific requirements.
- **Formal Verification** refers to the process of using **mathematical techniques** to prove or disprove the correctness of a system concerning a formal specification.
- **Formal Methods** utilize **mathematical techniques** to specify, develop, analyze, and verify software and hardware systems.
- **Theoretical Aspects of Software Engineering (TASE)** involve studying and applying **mathematical** and **logical foundations** to understand, model, and enhance **software engineering** processes and systems.

Model Checking \subset Formal Verification \subset Formal Methods \subset TASE

Learning Objectives: by the end of today's lecture, you should be able to...

1. (*Knowledge*) Review the foundations of software engineering
2. (*Value*) Understand the concept of formal methods and its relation to software engineering
3. (*Skill*) Review formal verification and model checking

Overview

- Motivation
- Background and Basic Concepts
- Formal Verification and Model Checking
- Abstract (Semantic) Models
- Linear Time Logic (LTL)

Motivation

What is Model Checking?

- **Model checking** is a **formal verification** technique in **software engineering** that **algorithmically** verifies if a **finite-state model** of a system satisfies a given **specification**, usually expressed in **temporal logic**.
- It systematically explores **all possible states** of the system to ensure **correctness** and identify **potential errors**.

The potential of model checking

- **Model checking** is particularly valuable for **safety-critical** systems because it can rigorously verify that these systems meet their specifications and do not exhibit undesirable behaviors.
- **Model checking** helps ensure the **reliability** and **safety** of these systems by exhaustively exploring all possible states and transitions to detect errors early in the development process.

Model checking Prospects

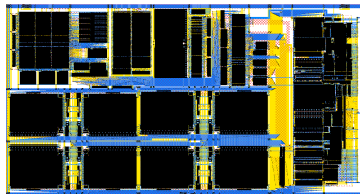
- Theoretically speaking, **model checking** of **large language models (LLMs)** may even be possible!!!
- The **operational semantics** of an **LLM** may be represented as a **transition system**.
- In this framework, each **state** represents a specific configuration of the **model's parameters** and **memory**, while **transitions** correspond to the **model's responses** to inputs or internal updates.

Too big a transition system! Why?

- **Parameter Space:** LLMs have **billions** of parameters, each of which can take on a wide range of values.
- **Input Combinations:** The variety of possible inputs (words, sentences, contexts) further **multiplies** the number of potential states.
- **Internal Memory:** The model's internal memory and context tracking add another layer of complexity.
- Given these factors, the **total number of states** can be on the order of **(10^{100})** or more, depending on the specific architecture and application of the **LLM**.

Symbolic Model Checking

- **Symbolic Model Checking** using data structures such as **OBDDs** has been able to verify **transition systems** with more than 10^{120} states.



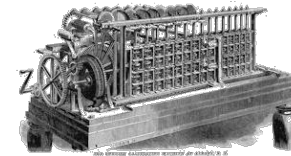
10^{120} states



10^{51} atoms

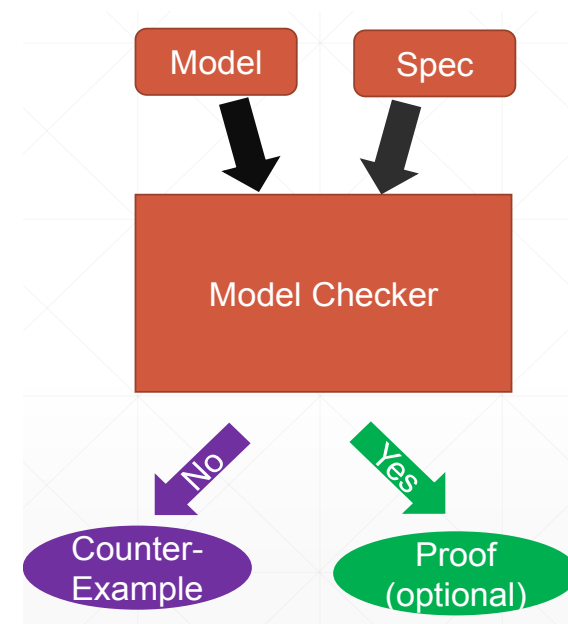
<https://sciencenotes.org/how-many-atoms-are-in-the-world/>
https://education.jlab.org/qa/mathatom_05.html

- **Model Checking** =
 - mechanical, push-button technology
 - performed without human intervention



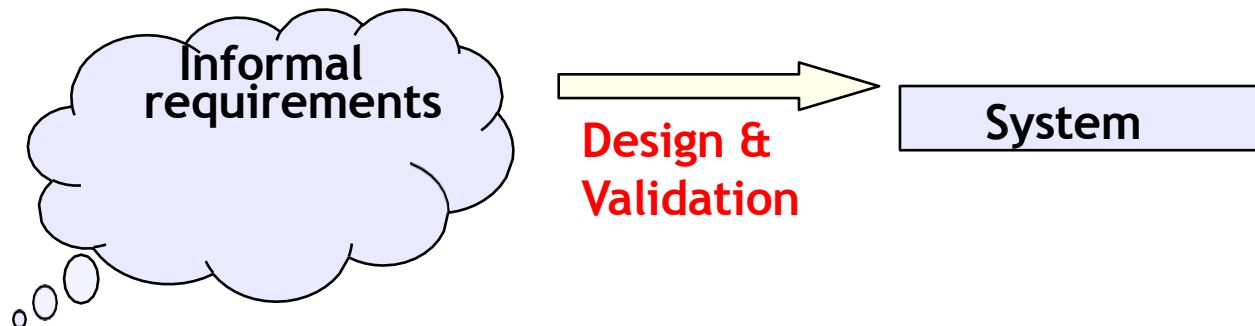
Model Checking Approach

- An approach for verifying the **temporal** behavior of a system.
- Primarily **fully automated** (“push-button”) techniques.
- **Model**
 - Representation of the system
 - Need to decide the right level of granularity.
- **Specification**
 - High-level desired property of a system
 - Considers infinite sequences.
- **PSPACE-complete** w.r.t. size of the **specification** model and **linear** w.r.t. size of the **transition system** model.



Conventional software engineering

- From requirements to a software system
 - apply design and validation methodologies
 - code directly in a programming language
 - validation mainly via testing, code walkthroughs, etc.



But my program works!

- True, there are many successful large-scale complex computer systems...
 - online banking, electronic commerce
 - information services, online libraries, business processes
 - supply chain management
 - mobile phone networks
- Yet many new potential application domains with far greater complexity and higher expectations
 - automotive drive-by-wire
 - medical sensors: heart rate & blood pressure monitors
 - intelligent buildings and spaces, environmental sensors
- Learning from mistakes is costly...

Toyota Prius

- Toyota Prius
 - first mass-produced hybrid vehicle
- February 2010
 - software “glitch” found in anti-lock braking system
 - in response to numerous complaints/accidents
- Eventually fixed via software update
 - in total 185,000 cars were recalled, at a huge cost
 - handling of the incident prompted much criticism, bad publicity



Ariane 5

- ESA (European Space Agency) Ariane 5 launcher
 - shown here in maiden flight on 4th June 1996
 - 37secs later self-destructs
 - uncaught exception: numerical overflow in a conversion routine results in incorrect altitude sent by the on-board computer
- Expensive, embarrassing...



The London Ambulance Service

- London Ambulance Service computer-aided despatch system
 - Area 600sq miles
 - Population 6.8million
 - 5000 patients per day
 - 2000-2500 calls per day
- Introduced October 1992
- Severe system failure:
 - position of vehicles incorrectly recorded
 - multiple vehicles sent to the same location
 - 20-30 people estimated to have died as a result



Smart Vehicles

- Safety-critical systems
 - Airplanes
 - Space shuttles
 - Railways
- Expensive mistakes
 - Chip design
 - Critical software
- Want to guarantee safe behavior over unbounded time
- [https://web.eecs.umich.edu/~movaghar/Software Model Checking-CACM2010.pdf](https://web.eecs.umich.edu/~movaghar/Software%20Model%20Checking-CACM2010.pdf)



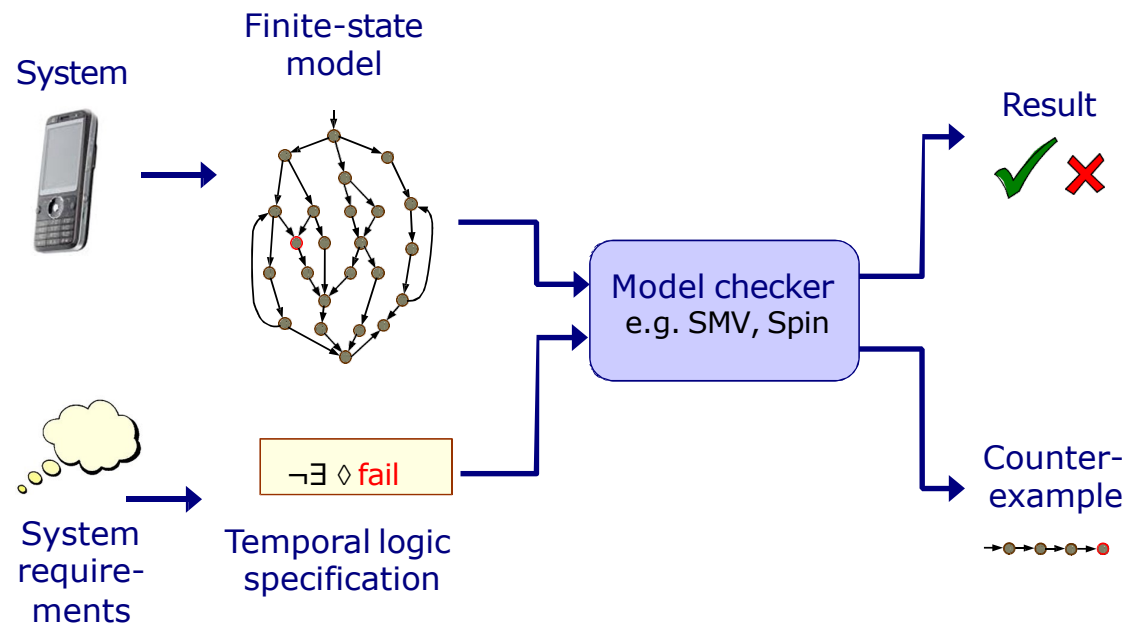
Smart vehicles

What do these stories have in common?

- Programmable computing devices
 - conventional computers and networks
 - software embedded in devices
 - airbag controllers, mobile phones, etc.
- Programming error direct cause of failure
- Software critical
 - for safety
 - for business
 - for performance
- High costs incurred: not just financial
- Failures avoidable...

Model Checking

Automated formal verification for finite-state models



Model checking in practice

- Model checking now routinely applied to real-life systems
 - not just “verification” ...
 - model checkers used as a debugging tool
 - at IBM, bugs detected in arbiter that could not be found with simulations
- Now widely accepted in industrial practice
 - Microsoft, Intel, Cadence, Bell Labs, IBM, ...
- Many software tools, both commercial and academic
 - NuSmv, PRISM, SPIN, SLAM, FDR2, FormalCheck, RuleBase, ...
 - software, hardware, protocols, ...
- Extremely active research area
 - 2008 Turing Award won by Edmund Clarke, Allen Emerson, and Joseph Sifakis for their work on model-checking. <https://web.eecs.umich.edu/~movaghar/ModelChecking-TuringAward2007-CACM.pdf>

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Background and Basic Concepts

Software Engineering

- IEEE, in its standard 610.12-1990, defines **software engineering** as the application of a systematic, disciplined, which is a computable approach for the **development**, **operation**, and **maintenance** of **software**, that is, the application of engineering to **software**.

What is Software?

- We will present a **philosophical** definition of **software** (**hardware**) as follows.
- Accordingly, we find that:
 - **Software** is more **complex** than **hardware**.
 - **Software** is a **complex** system.

Philosophical Definition of Hardware

- **Hardware** refers to the **tangible**, **physical** components of a computer system.
- Philosophically, **hardware** can be seen as the **material substrate** that provides the necessary infrastructure for computational processes.
- It is the **physical** embodiment of the machine's capabilities, constrained by its **material** properties and design.

Philosophical Definition of Software

- **Software**, on the other hand, is **intangible**.
- It consists of the **instructions** and **data** that tell the **hardware** how to perform tasks.
- From a philosophical perspective, **software** represents a computer system's **logical** and **functional** essence.
- It is the **abstract set of rules** and **instructions** that govern the behavior of the **hardware**, enabling it to **perform** complex operations.

Philosophical Distinction

- **Tangibility:** **Hardware** is defined by its physical presence, while **software** is independent of a particular physical form.
- **Functionality:** **Hardware's** primary purpose is physical functions, whereas **software's** is to execute logical functions, manipulating symbols and data.
- **Malleability:** **Hardware** is relatively difficult to change once manufactured, while **software** can be easily modified and updated.

Brain versus Mind

- The philosophical distinction between the **hardware** and the **software** is often compared to the relationship between the **brain** and the **mind** in the **philosophy of mind**, where:
 - the **brain** is the physical **hardware** and
 - the **mind** is the **software** or the set of processes and functions that the **brain** performs.

The theoretical aspects of software engineering (TASE)

- The theoretical aspects of software engineering focus on the underlying principles and formal methods that guide the development, verification, and maintenance of software systems.

Key Areas of TASE (1/2)

- **Formal Methods**: Techniques like **model checking**, **theorem proving**, and **formal verification** to ensure software **correctness** and **reliability**.
- **Program Semantics**: Understanding the **meaning** of programs through **formal languages** and **logic**.
- **Type Systems**: Ensuring program correctness by defining and enforcing **rules** about how **data types** are used.

Key areas of TASE (2/2)

- **Abstract Interpretation**: A **theory** used to analyze programs by **approximating** their behaviors.
- **Automata Theory**: The study of **abstract machines** and the **problems** they can solve, which is **fundamental** in designing compilers and interpreters.

Formal Methods in Software Engineering

- **Formal methods** in **software engineering** are **mathematically** rigorous techniques used for the **specification, development, analysis, and verification** of software and hardware systems.
- These methods involve the use of **formal languages, logic, and mathematical models** to ensure the **correctness** and **reliability** of software systems.

Formal Methods: Key Areas

- **Formal Specification**: Creating precise and unambiguous descriptions of software behavior and properties.
- **Formal Verification**: Using mathematical proofs to verify that software meets its specifications.
- **Model Checking**: Automatically verifying finite-state models of software against desired properties.

Formal Methods: Applications

- **Formal methods** are particularly valuable in **safety-critical** and **security-critical** systems, such as **avionics** and **medical devices**, where **reliability** is paramount.
- **Formal methods** can be considered as **research** works in software engineering. This **research** contributes to advancing the **theoretical foundations** of **software engineering** and enhancing practical applications.

<https://github.com/ligurio/practical-fm>

<https://web.eecs.umich.edu/~movaghar/Formal Methods Manifesto 2023.pdf>

Amazon

Amazon is actively involved in research related to **formal verification**, particularly through its **Amazon Web Services (AWS)** division. Here are a few notable examples:

- **Cryptographic Software:** Amazon's **Automated Reasoning** group has used **formal verification** to improve the **efficiency** and **security** of cryptographic algorithms, such as **RSA**, on their Graviton2 chips. This ensures that the **cryptographic software** behaves **correctly** and **securely**.
- **Amazon s2n:** This is an open-source implementation of the **TLS (Transport Layer Security)** protocol used by many **Amazon** services. **Formal verification** is continuously applied to **s2n** to ensure its **correctness** and **security** throughout its lifecycle.
- **Cloud Infrastructure:** **Formal verification tools** are used within **AWS** to enhance the security of its **cloud infrastructure**, helping to **secure** both the infrastructure itself and the customers using it.

Microsoft

Microsoft is actively involved in research related to formal verification across various projects and divisions. Here are a few notable examples:

- The Research in Software Engineering (RISE) group at Microsoft focuses on formal methods, automated reasoning, and proof-oriented programming to ensure the correctness and security of their systems.
- VeriSol: This is a formal verification tool developed by Microsoft Research for verifying smart contracts written in the Solidity programming language. VeriSol helps ensure the correctness and security of smart contracts used in Azure Blockchain.
- Verus: This project focuses on creating a practical foundation for systems verification. It aims to eliminate bugs at compile time, ensuring that software is correct before it ships.
- Practical System Verification: This initiative explores methods to make formal verification more practical and scalable for system software. It addresses the challenges of verifying complex systems with minimal developer effort.

Meta (formerly Facebook)

Meta is involved in research related to **formal verification**.

- One notable project involves applying **formal verification** to microkernel **inter-process communication (IPC)**. This research uses Iris, a **concurrent separation logic** implemented in the **Coq proof assistant**, to **verify** queue data structures used for **IPC** in an operating system under development at **Meta**. The project has successfully identified and corrected **bugs**, leading to more **reliable** and **efficient** code.
- **Meta**'s efforts in **formal verification** are part of a broader initiative to enhance the **reliability** and **security** of its **software systems**. This includes using **formal methods** to verify the **correctness** of algorithms and improve the overall **robustness** of their infrastructure.

Airbus

Airbus is actively involved in research related to **formal verification**, particularly for its **avionics software**. Here are some key points:

- **Avionics Software**: Airbus has been integrating **formal verification** techniques into the development process of **avionics software** since 2001. These techniques include **abstract interpretation**, **theorem proving**, and **model-checking**.
- **DO-178B Compliance**: The formal verification methods used by **Airbus** comply with the stringent requirements of the **DO-178B** standard, which governs the development of **avionics software**.
- **Collaborations**: **Airbus** collaborates with academic and industrial labs, such as **ONERA** (the French aerospace lab), to advance **formal verification** methods and their application in critical embedded systems.
- **Tools and Techniques**: Airbus has developed and transferred several **formal verification tools** to its operational teams, including **Caveat**, **aiT**, and **Stack analyzer**, which are used to achieve **DO-178B verification** objectives.

NASA

NASA is deeply involved in research related to **formal verification**, particularly to ensure the **safety** and **reliability** of its **aerospace systems**. Here are some key areas of their work:

- **Flight Critical Software:** NASA has applied **formal verification** techniques to **flight critical software**, such as the **Flight Control Systems (FCS)** used in aircraft. This involves using **formal methods** to **verify** the behavior of system components and ensure they meet **safety** requirements.
- **Langley Formal Methods Program:** The **Formal Methods** group at NASA's Langley Research Center develops and maintains the NASA PVS Library, which includes a wide range of **formal verification tools** and **frameworks**. These **tools** are used for **verifying** air traffic systems, fault-tolerant protocols, and other critical systems.
- **Autonomous Systems:** NASA also explores **formal verification** approaches for autonomous robotic systems. This includes **specifying** and **verifying** the behavior of **autonomous systems** to ensure they operate **safely** and **reliably** in space missions..

Tesla

Tesla is involved in research related to **formal verification**, particularly in the context of its **autonomous driving systems**.

- **Formal verification** methods are used to ensure the **safety** and **reliability** of the **software** that controls **Tesla**'s vehicles. This includes **verifying** that the **software** behaves **correctly** under all possible conditions, which is crucial for the development of **safe** and **reliable autonomous vehicles**.
- **Tesla**'s **Autopilot** and **Full Self-Driving (FSD)** systems rely heavily on advanced **software engineering** and **formal methods** to **validate** the complex algorithms that enable **autonomous driving**. This research helps in identifying and mitigating potential **risks**, ensuring that the systems operate **safely** in real-world scenarios.

xAI

- xAI is actively involved in research related to **formal verification**.
- According to recent reports, xAI plans to incorporate **formal verification** techniques into its **AI models**.
- This approach aims to ensure that the code generated by their models, such as the **Grok language model**, is free from **bugs** and adheres to specified **safety** and **performance** criteria.
- **Formal verification** is a mathematical method used to prove the **correctness** of systems, and its application in **AI** can significantly enhance the **reliability** and **trustworthiness** of **AI-generated** outputs. This is particularly important for **safety-critical** applications where **errors** can have serious consequences.

SpaceX

- **SpaceX** is involved in research related to **formal verification**, particularly to ensure the **safety** and **reliability** of its spacecraft and rocket systems. **Formal verification** methods are crucial for **validating** the complex **software** that controls these systems, ensuring they perform **correctly** under all possible conditions.
- **SpaceX's software engineering** teams use **formal methods** to **verify** the **correctness** of flight control systems, mission planning **software**, and other critical components. This rigorous approach helps prevent **errors** that could lead to mission **failures**, making it an essential part of their **development process**.

OpenAI

- **Neural Theorem Proving:** OpenAI has developed a **neural theorem prover** for the **Lean proof assistant**, which is used to **solve formal mathematics problems**. This involves using **language models** to generate **proofs** for formal statements, enhancing the **reliability** and **correctness** of **mathematical proofs**.
- **Autoformalization:** OpenAI has explored **autoformalization**, which is the process of translating natural language mathematics into **formal specifications** and **proofs**. This research aims to improve the **accuracy** and **efficiency** of **formal verification** by leveraging **large language models**.
- **Improving Verifiability:** OpenAI has published reports on mechanisms to improve the **verifiability** of **AI** systems. These tools help developers provide evidence that **AI** systems are **safe**, **secure**, **fair**, and **privacy-preserving**.

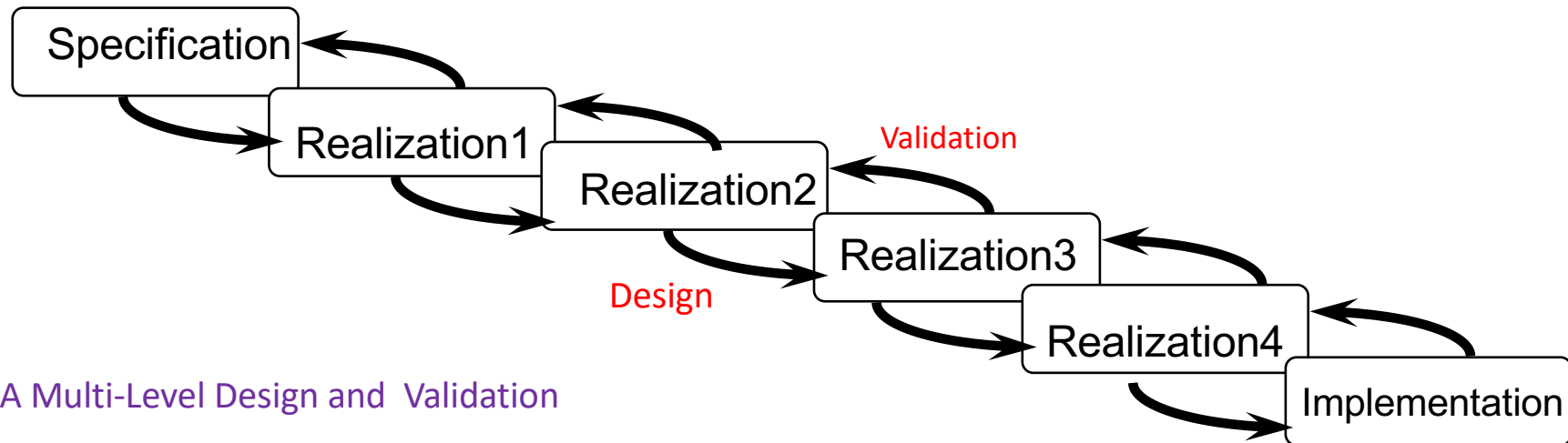
Google

Google is actively involved in research related to **formal verification**. Here are some key areas of their work:

- **Towards Making Formal Methods Normal: Google Research** has published work on integrating **formal methods** into developers' existing practices and workflows. This research aims to increase the adoption of **formal verification** by making it more accessible and practical for everyday **software development**.
- **Formal Verification Techniques: Google** uses **formal methods** to **verify** critical **software systems**. This involves modeling system **requirements** using specification languages and **validating** these models with tool support to ensure **consistency** and early **verification**.
- **Automated Reasoning: Google's Automated Reasoning** team focuses on developing tools and techniques for **formal verification** to improve the **reliability** and **security** of **software systems**. This includes work on **verifying cryptographic protocols** and other critical components.

Design and Validation

- A **design** is a process of getting a (more detailed) **realization** from a given **specification**.
- **Validation** is a process of ensuring that a **realization** satisfies its **specification**.



- An **implementation** may be viewed as the lowest level of **realization**.

Design

- Design is a process of getting a realization from a given specification.
- The design of a complex system may happen on many levels.
- An implementation may be viewed as the lowest level of realization.

Validation

- **Validation** is a **process** of ensuring that a **realization** satisfies its **specification**.

Validation Methods

- Validation has three main methods:
 - Formal Verification
 - Evaluation
 - Testing

Formal Verification

- **Formal Verification** is a **mathematical method** to prove that a **realization** satisfies its **specification**.

Evaluation

- **Evaluation** is a method for finding how **well** a system behaves.

Testing

- **Testing** is a **method** of proving that a **realization does not** satisfy its **specification**.

Integrated Validation

- Testing, Formal Verification, and Evaluation are usually **complementary**.

Evaluation Methods

- Measurement
- Analytical Modeling
- Simulation Modeling
- Hybrid Modeling

So, why not only test?

- Testing only shows the presence of bugs, not their absence!

Verification and Formal Verification

- In **software engineering**, **verification** and **formal verification** are crucial processes, but they **differ** significantly in their **approaches** and **objectives**.
- Both **methods** are essential for ensuring a system is **reliable**, **functional**, and **meets user needs**.
- They are often used together to provide comprehensive **assurance** of **system quality**.

Verification

- **Verification** is the process of ensuring that the **software** **meets** its specified **requirements**.
- It involves checking that software is built **correctly** according to the design and **specifications**.
- **Verification** aims to catch errors early in the development process and ensure that the software behaves as expected under specified conditions.

Formal Verification

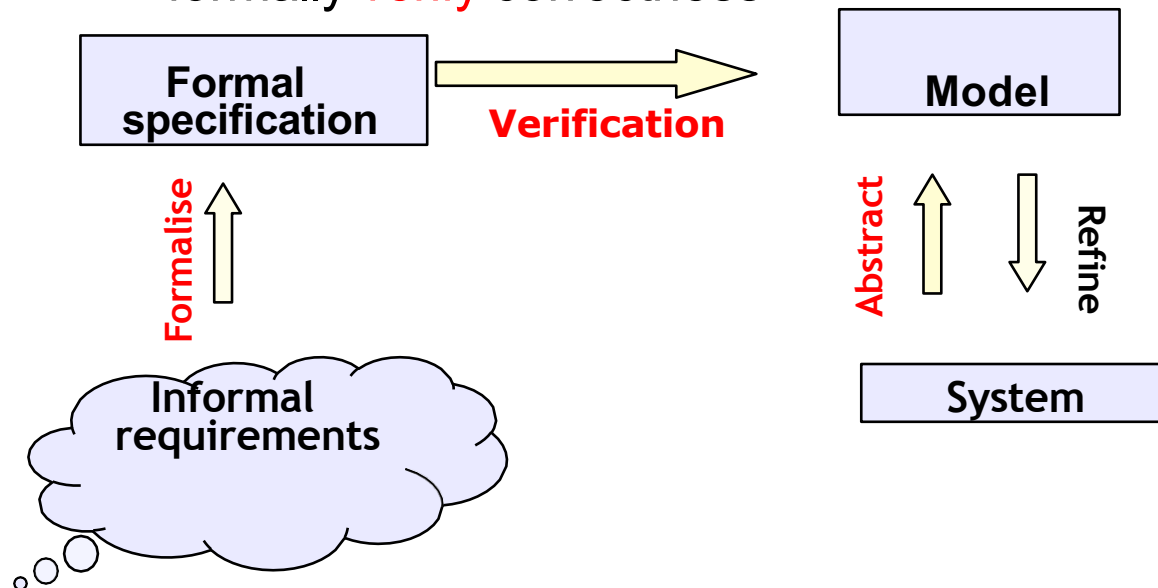
- **Formal Verification**, on the other hand, uses **mathematical methods** to prove the correctness of a system.
- It involves creating **formal software models** and using **logical reasoning** to verify that the software adheres to its specifications under all possible conditions.
- **Formal verification** provides a **higher level** of assurance because it can **prove** the absence of certain errors, rather than just **finding** them through **testing**.

Verification versus Formal Verifications

- **Approach:** **Verification** relies on **testing** and **reviews**, while **formal verification** uses **mathematical** proofs and models.
- **Scope:** **Verification checks** the software against specific scenarios, whereas **formal verification** aims to **prove** correctness under all possible scenarios.
- **Assurance:** **Formal verification** offers a **higher level** of confidence in the correctness of the software, as it can **mathematically** guarantee certain properties.

Formal verification

- From requirements to formal specification
 - formalize specifications, derive a model
 - formally **verify** correctness



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Formal Verification and Model Checking

What are Formal Methods?

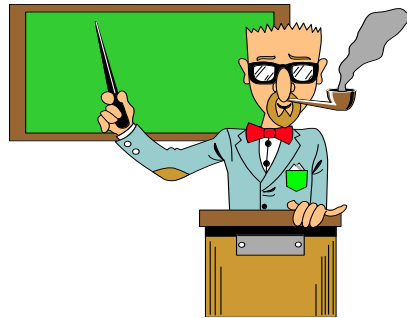
- Techniques for analyzing systems, based on some mathematics.
- This does not mean that the user must be a mathematician.
- Some of the work is done informally, due to complexity.

Formal Methods

- Mathematically-based techniques for describing properties of systems
- Provide framework for
 - Specifying systems (and thus the notion of correctness)
 - Developing systems
 - Verifying correctness
 - Of implementation w.r.t. the specification
 - Equivalence of different implementations
- Reasoning is based on logic
 - Amenable to machine analysis and manipulation

Why aren't FMs used more?

“Formal methods can revolutionize development!”



“Formal methods are difficult, expensive, not widely useful and for safety-critical systems only”



Formal Verification

- **Formal verification** seeks to establish a **mathematical proof** that a system works **correctly**.

<https://web.eecs.umich.edu/~movaghar/Principles of Model Checking-Book-2008.pdf>

Formal Verification Steps

- A **formal verification** is done in three steps:
 - A **system model** to describe the system,
 - A **specification model** to describe the correctness requirement,
 - An **analysis technique** to **verify** that the **system** meets its **specifications**.

Some System Model

- Transition Systems (**Automata**)
 - Communicating Sequential Processes (**CSP**)
 - Reo
 - (High-level) Programming Languages
-
- <https://web.eecs.umich.edu/~movaghar/cspbook.pdf>
 - <https://web.eecs.umich.edu/~movaghar/Reo Arbab 2003.pdf>

Some Specification Models

- Propositional Logic
- First-order Logic
- Linear Temporal Logic (**LTL**)
- Computational Tree Logic (**CTL**)

Formal Verification Methods

- There are **two** major methods for formal verification:
 - **Deductive Method**
 - **Model Checking**

Deductive Method

- In the **deductive method**, the problem is formulated as **proving a theorem** in a **mathematical proof system**.

Model Checking

- In the method of **model checking**, the behavior of the system is **checked algorithmically** through an **exhaustive search** of all reachable states.

<https://web.eecs.umich.edu/~movaghar/Model Checking-Turing Award2007-CACM.pdf>

<https://web.eecs.umich.edu/~movaghar/Model Checking-Q&A-Turing Award2007.CACM.pdf>

https://en.wikipedia.org/wiki/Model_checking

View of Model Checking in Theoretical Settings

- Theoretical Aspects of Software Engineering
 - Formal Methods
 - Formal Verification
 - **Model Checking**

Popular Model-Checking Tools

- NuSMV
- PRISM

<https://nusmv.fbk.eu/index.html>

Companies Using NuSMV

- **Airbus:** Utilizes NuSMV for verifying the correctness and **safety** of their avionics system.
- **Intel:** Employs NuSMV in the verification of hardware designs to ensure **reliability** and **performance**.
- **Siemens:** Uses NuSMV for verifying industrial automation systems and ensuring they meet **safety standards**.
- **NASA:** Applies NuSMV in the verification of critical software systems used in **space missions**.

Companies Using PRISM

- **Google:** Utilizes PRISM for verifying the **reliability** and **performance** of their systems, particularly in areas involving **probabilistic models**.
- **Microsoft:** Employs PRISM in their research and development to ensure the **correctness** and **reliability** of software systems.
- **IBM:** Uses PRISM for formal verification of complex systems, ensuring they meet required **performance** and **reliability** standards.

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Abstract (Semantic) Models

Abstract (Semantic) Models

- A prerequisite for **model checking** is to provide a **model** of the system.
 - We introduce **transition systems as abstract (semantic) models** to represent hardware and software systems.
 - Using the **Structural Operation Semantics (SOS)** method, we can define the **operational semantics** of any model, including all **(high-level) programming languages**.

What are Transition Systems?

- **Transition Systems (TSs)** are **abstract (semantic)** models that are the **operational semantics** of many models, including all **(high-level) programming languages**.
- **TSs** are directed graphs where nodes and edges represent **states** and **transitions**, respectively.

States of a Transition System

- A state describes **information** about a system at a **certain moment of its behavior**:
 - The current **color** of a **traffic light**.
 - The current **values** of all program **variables** + the program **counter**.
 - The current **value** of the **registers** together with the **values** of the **input bits**.

Transitions of a Transition System

- **Transitions** specify how the system **evolves** from one state to another.
 - A **switch** from one color to another (for traffic light).
 - The **execution** of a program statement.
 - The **change** of the registers and output bits for a new input.

Structural Operation Semantics

- The transition relation of $TS(\text{Model})$ is defined using the so-called **SOS notation**:

$$\frac{\text{premise}}{\text{conclusion}}$$

- This implies if the proposition above the “solid line” holds, then the proposition under the fraction bar holds as well.
- If the premise is a tautology, the rule is called an **axiom**.

Operation Semantics

- The **operational semantics** of many models, including all (high-level) programming languages, are defined using **SOS rules**.
- <https://web.eecs.umich.edu/~movaghar/SOS-1.pdf>
- <https://web.eecs.umich.edu/~movaghar/SOS-2.pdf>

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Linear Time Logic (LTL)

What is Linear Temporal Logic (LTL)?

- Linear Temporal Logic (LTL) is a type of modal temporal logic used to describe sequences of events or states over time.
- LTL is widely used to formally specify safety-critical properties of hardware and software systems.
- For example, it can be used to ensure that a system will never reach an undesirable state or that a certain condition will eventually be met.

LTL Syntax

- **LTL formulas** are built using a set of **propositional variables**, **Propositional operators** (like \neg , \vee , \wedge), and **temporal operators**.

Temporal Operators:

- **X (Next)**: A condition will be true at the next state.
- **F (Eventually)**: A condition will be true at some point in the future.
- **G (Globally)**: A condition will always be true.
- **U (Until)**: One condition will be true until another condition becomes true.

Example

- LTL allows for the specification of the **relative order** of **events**. However, it does not support any means to refer to the **precise timing** of events.
 - “The car stops once the driver pushes the brake”.
 - “The message is received after it has been sent”.

LTL Semantics

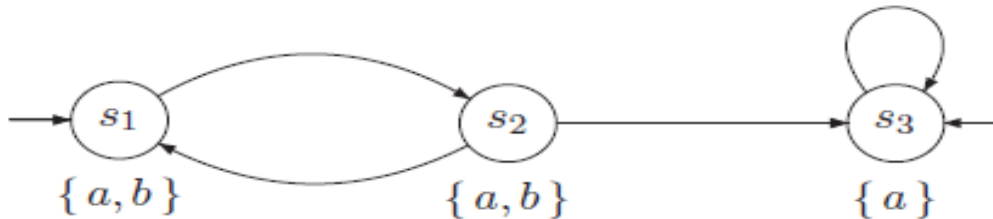
- **LTL formulas** are evaluated over infinite sequences of states (often called **paths**).
- A path satisfies an **LTL formula** if the formula holds for the entire sequence of states.

infinitely often and eventually forever

- By combining **G** and **F**, new temporal modalities are obtained:
 - **GFa** describes the property stating that at any moment j there is a moment $l \geq j$ at which **a**-state is visited. Thus **a**-state is visited **infinitely often**.
 - **FGa** expresses that from any moment j , finally only **a**-state is visited. Thus **a**-state is visited **eventually forever**.

LTL Example (1/2)

- **Example:** Consider the **Transition System (TS)** below with the set of **atomic propositions** $AP=\{a,b\}$



- $TS \models Ga$
- $TS \not\models X(a \wedge b)$
- $TS \models G(\neg b \rightarrow G(a \wedge \neg b))$
- $TS \not\models b \cup (a \wedge \neg b)$

LTL Example (2/2)

- **Example:** Properties for **mutual exclusion problem**:
 - **Safety property** states that two processes P_1 and P_2 never simultaneously have access to their critical section: $G(\neg \text{crit}_1 \vee \neg \text{crit}_2)$.
 - **Liveness property** stating each process P_i is infinitely often in its critical section: $(GF \text{crit}_1) \wedge (GF \text{crit}_2)$.