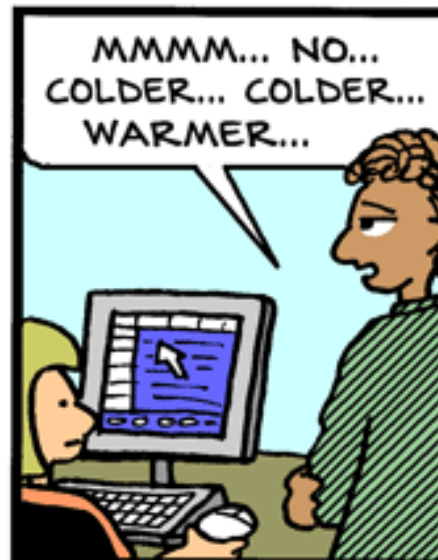
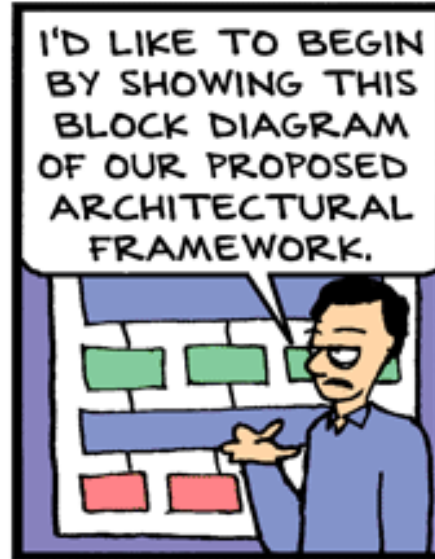


In Our Last Exciting Episode



Lessons From Model Checking

- To find **bugs**, we need **specifications**
 - What are some good specifications?
- To **convert** a program into a **model**, we need **predicates**/invariants and a **theorem prover**.
 - Which are the important predicates? Invariants?
 - What should we track when reasoning about a program and what should we abstract?
 - How does a theorem prover work?
- **Simple** algorithms (e.g., depth first search, pushing facts along a CFG) can work well
 - ... under what circumstances?

The Big Lesson



- To reason about a program
(= “is it doing the right thing? the wrong thing?”)
we must *understand what the program means!*

A Simple Imperative Language

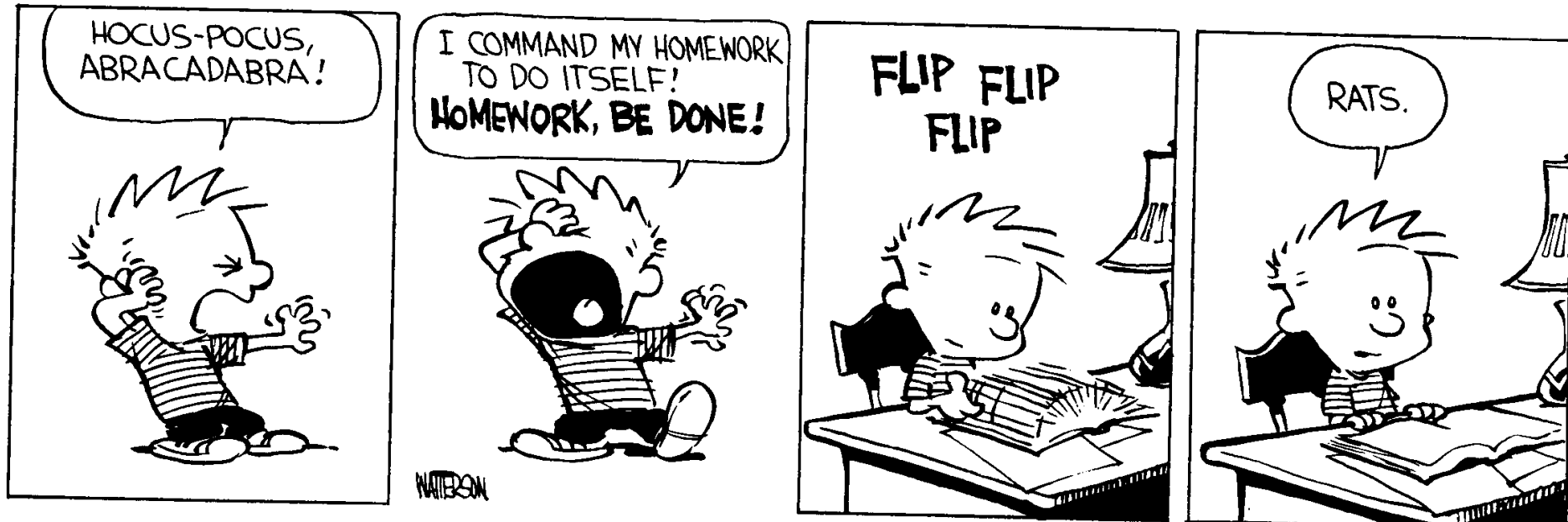
Operational Semantics

(= “**meaning**”)



Homework Peer Review Today

- We will cover some aspects during the discussion section



One-Slide Summary

- We use **IMP**, a simple language, to study semantics. It has imperative **commands** (such as assignments to variables and while loops).
- **Operational semantics** abstracts the execution of a concrete interpreter. It is one way of reasoning about what programs **mean**. We use a **state** mapping to model storage (variables).
- **Judgments** are assertions about programs that can be true or false. We use **rules of inference** (**evaluation rules**) to help **derive** when judgments are true.

Medium-Range Plan

- Study a *simple imperative* language **IMP**
 - Abstract syntax (today)
 - Operational semantics (today)
 - Axiomatic semantics
 - Symbolic execution
 - ... and relationships between various semantics (with proofs, peut-être)
 - Today: operational semantics
 - Follow along in Chapter 2 of Winskel

Syntax of IMP

- Concrete syntax: The rules by which programs can be expressed as strings of characters
 - Keywords, identifiers, statement separators vs. terminators (Niklaus!?), comments, indentation (Guido!?)
- Concrete syntax is important in practice
 - For readability (Larry!?), familiarity, parsing speed (Bjarne!?), effectiveness of error recovery, clarity of error messages (Robin!?)
- Well-understood principles
 - Use finite automata and context-free grammars
 - Automatic lexer/parser generators

(Note On Post-LALR Advances)

- If-as-and-when you find yourself making a new language, consider GLR (**elkhound**) instead of LALR(1) (**bison**)
- Scott McPeak, George G. Necula:
Elkhound: A Fast, Practical GLR Parser Generator. CC 2004: pp. 73-88
- As fast as LALR(1), more natural, handles basically all of C++, etc.



Abstract Syntax

- We **ignore** parsing issues and study programs given as **abstract syntax trees**
 - I provide the parser in the homework ...
- An abstract syntax tree is (a subset of) the parse tree of the program
 - Ignores issues like comment conventions
 - More convenient for formal and algorithmic manipulation
 - All research papers use ASTs, etc.

IMP Abstract Syntactic Entities

- **int** integer constants ($n \in \mathbb{Z}$)
- **bool** bool constants (true, false)
- **L** locations of variables (x, y)
- **Aexp** arithmetic expressions (e)
- **Bexp** boolean expressions (b)
- **Com** commands (c)

- (these also encode the types)

Abstract Syntax (Aexp)

- Arithmetic expressions (Aexp)

$e ::= n$ for $n \in \mathbb{Z}$
| x for $x \in L$
| $e_1 + e_2$ for $e_1, e_2 \in \text{Aexp}$
| $e_1 - e_2$ for $e_1, e_2 \in \text{Aexp}$
| $e_1 * e_2$ for $e_1, e_2 \in \text{Aexp}$

- Notes:

- Variables are not declared
- All variables have integer type
- No side-effects (in expressions)

Abstract Syntax (Bexp)

- Boolean expressions (Bexp)

$b ::= \text{true}$

| false

| $e_1 = e_2$ for $e_1, e_2 \in \text{Aexp}$

| $e_1 \leq e_2$ for $e_1, e_2 \in \text{Aexp}$

| $\neg b$ for $b \in \text{Bexp}$

| $b_1 \wedge b_2$ for $b_1, b_2 \in \text{Bexp}$

| $b_1 \vee b_2$ for $b_1, b_2 \in \text{Bexp}$

“Boolean”

- George Boole
 - 1815-1864
- I’ll assume you know **boolean algebra** ...

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F



Abstract Syntax (Com)



- Commands (Com)

$c ::= \text{skip}$

| $x := e$

$x \in L \wedge e \in Aexp$

| $c_1 ; c_2$

$c_1, c_2 \in Com$

| **if b then c_1 else c_2**

$c_1, c_2 \in Com \wedge b \in Bexp$

| **while b do c**

$c \in Com \wedge b \in Bexp$

- Notes:

- The typing rules are embedded in the syntax definition
- Other parts are not context-free and need to be checked separately (e.g., all variables are declared)
- Commands contain all the side-effects in the language
- Missing: pointers, function calls, what else?

Why Study Formal Semantics?

- Language design (denotational)
- Proofs of correctness (axiomatic)
- Language implementation (operational)
- Reasoning about programs
- Providing a clear behavioral specification
- “All the cool people are doing it.”
 - You need this to understand PL research
- “First one’s free.”

Consider This Legal Java

```
x = 0;  
try {  
    x = 1;  
    break mygoto;  
} finally {  
    x = 2;  
    raise  
        NullPointerException;  
}  
x = 3;  
mygoto:  
x = 4;
```

- What happens when you execute this code?
- Notably, which assignments are executed?

14.20.2 Execution of try-catch-finally

- A try statement with a finally block is executed by first executing the try block. Then there is a choice:
- If execution of the try block completes normally, then the finally block is executed, and then there is a choice:
 - If the finally block completes normally, then the try statement completes normally.
 - If the finally block completes abruptly for reason *S*, then the try statement completes abruptly for reason *S*.
- If execution of the try block completes abruptly because of a throw of a value *V*, then there is a choice:
 - If the run-time type of *V* is assignable to the parameter of any catch clause of the try statement, then the first (leftmost) such catch clause is selected. The value *V* is assigned to the parameter of the selected catch clause, and the *Block* of that catch clause is executed. Then there is a choice:
 - If the catch block completes normally, then the finally block is executed. Then there is a choice:
 - If the finally block completes normally, then the try statement completes normally.
 - If the finally block completes abruptly for any reason, then the try statement completes abruptly for the same reason.
 - If the catch block completes abruptly for reason *R*, then the finally block is executed. Then there is a choice:
 - If the finally block completes normally, then the try statement completes abruptly for reason *R*.
 - If the finally block completes abruptly for reason *S*, then the try statement completes abruptly for reason *S* (and reason *R* is discarded).
 - If the run-time type of *V* is not assignable to the parameter of any catch clause of the try statement, then the finally block is executed. Then there is a choice:
 - If the finally block completes normally, then the try statement completes abruptly because of a throw of the value *V*.
 - If the finally block completes abruptly for reason *S*, then the try statement completes abruptly for reason *S* (and the throw of value *V* is discarded and forgotten).
- If execution of the try block completes abruptly for any other reason *R*, then the finally block is executed. Then there is a choice:
 - If the finally block completes normally, then the try statement completes abruptly for reason *R*.
 - If the finally block completes abruptly for reason *S*, then the try statement completes abruptly for reason *S* (and reason *R* is discarded).

Can't we just nail this somehow?



Ouch! Confusing.

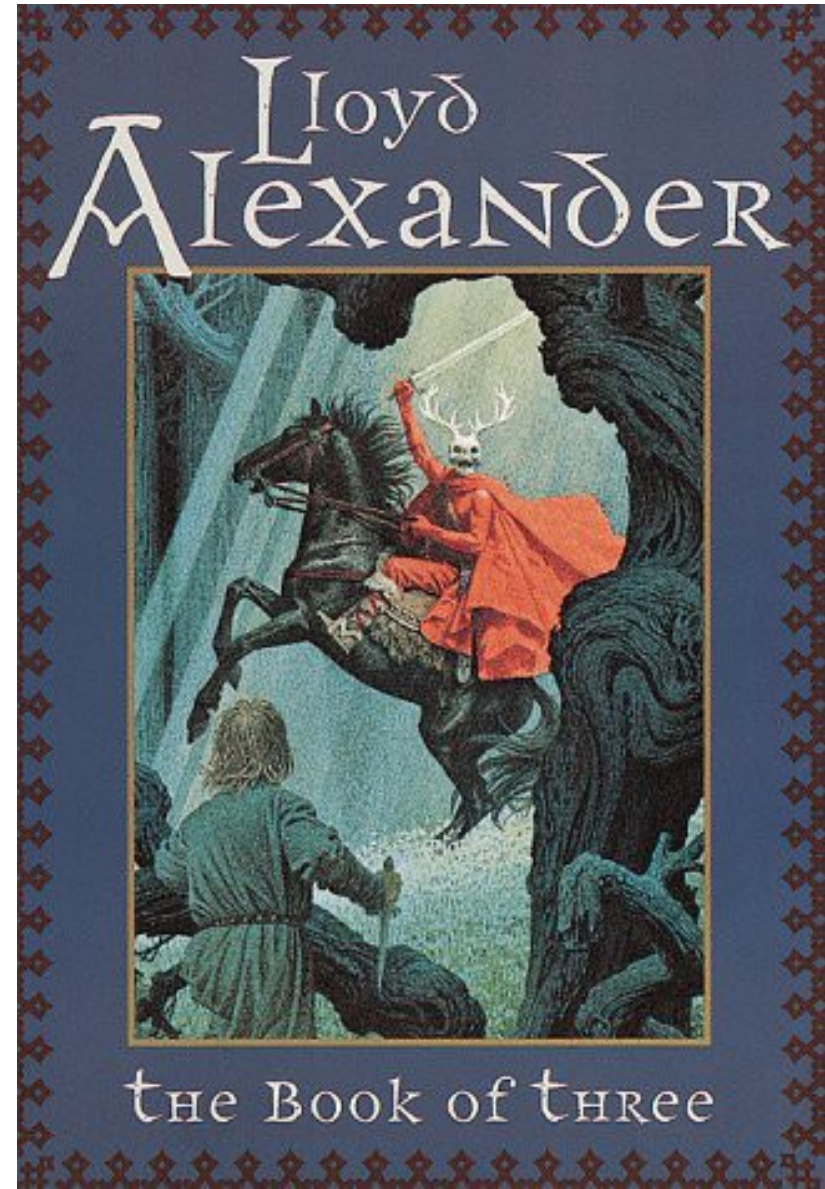
- Wouldn't it be nice if we had some way of describing what a language (feature or program) means ...
 - More **precisely** than English
 - More compactly than English
 - So that you might build a compiler
 - So that you might **prove** things about programs

Analysis of IMP

- Questions to answer:
 - What is the “meaning” of a given IMP expression/command?
 - How would we go about evaluating IMP expressions and commands?
 - How are the evaluator and the meaning related?

Three Canonical Approaches

- Operational
 - How would I execute this?
- Axiomatic
 - What is true after I execute this?
 - Symbolic Execution
- Denotational
 - What is this trying to compute?



An Operational Semantics

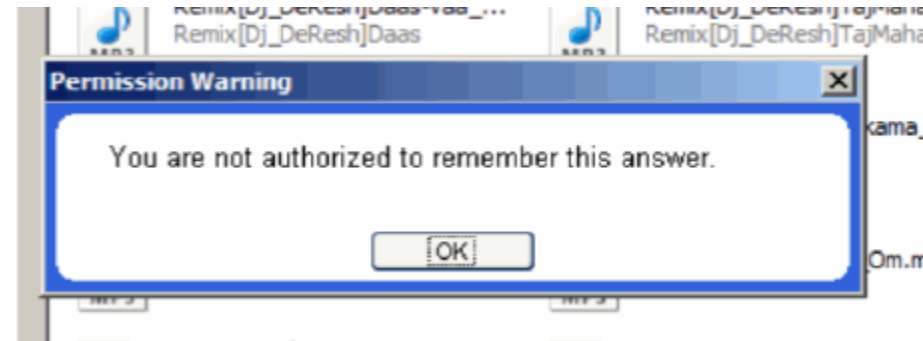
- Specifies how expressions and commands should be evaluated
- Depending on the form of the expression
 - 0, 1, 2, . . . don't evaluate any further.
 - They are normal forms or values.
 - $e_1 + e_2$ is evaluated by first evaluating e_1 to n_1 , then evaluating e_2 to n_2 . (post-order traversal)
 - The result of the evaluation is the literal representing $n_1 + n_2$.
 - Similarly for $e_1 * e_2$
- Operational semantics abstracts the execution of a concrete interpreter
 - Important keywords are colored & underlined in this class.

Semantics of IMP

- The meanings of IMP expressions **depend on** the values of variables
 - What does “ $x+5$ ” mean? It depends on “ x ”!
- The value of variables at a given moment is abstracted as a function from L to \mathbb{Z} (a **state**)
 - If $x = 8$ in our state, we expect “ $x+5$ ” to mean **13**
- The set of all states is $\Sigma = L \rightarrow \mathbb{Z}$
- We shall use σ to range over Σ
 - σ , a **state**, maps variables to values

Program State

- The **state** σ is somewhat like “**memory**”
 - It holds the current values of all variables
 - Formally, $\sigma : L \rightarrow \mathbb{Z}$



“Romance” Novels



- This 14th-century Chinese novel (perhaps by Luo Guanzhong) is a dramatic description of hundreds of characters in three main factions during the 3rd century, a bloody period of Chinese history. It is one of the four great classic novels and includes secret decrees, red cliffs, and assassinations. Give its English name.
- Bonus: How many people became sworn brothers via the Peach Garden Oath?

Q: Cartoons (682 / 842)

- Why is Gargamel trying to capture the Smurfs?



Q: Computer Science

- This American Turing Award winner is notable for his work in the theory of algorithms, a max-flow solver, a bipartite graph matcher, a string search algorithm, and “Reducibility Among Combinatorial Problems” in which he proved 21 problems to be NP-complete. He introduced the standard methodology for proving problems to be NP-complete.

Notation: Judgment

- We write:

$$\langle e, \sigma \rangle \Downarrow n$$

- To mean that e evaluates to n in state σ .
- This is a judgment. It asserts a relation between e , σ and n .
- In this case we can view \Downarrow as a function with two arguments (e and σ).

Operational Semantics

- This formulation is called natural operational semantics
 - or big-step operational semantics
 - the \Downarrow judgment relates the expression and its “meaning”

- How should we define

$$\langle e_1 + e_2, \sigma \rangle \Downarrow \dots ?$$

Notation: Rules of Inference

- We express the evaluation rules as rules of inference for our judgment
 - called the derivation rules for the judgment
 - also called the evaluation rules (for operational semantics)
- In general, we have **one rule for each language construct**:

$$\frac{\langle e_1, \sigma \rangle \Downarrow n_1 \quad \langle e_2, \sigma \rangle \Downarrow n_2}{\langle e_1 + e_2, \sigma \rangle \Downarrow n_1 + n_2}$$

This is the *only* rule for $e_1 + e_2$

Rules of Inference

Hypothesis₁ ... Hypothesis_N

Conclusion

$\Gamma \vdash b : \text{bool}$ $\Gamma \vdash e1 : \tau$ $\Gamma \vdash e2 : \tau$

$\Gamma \vdash \text{if } b \text{ then } e1 \text{ else } e2 : \tau$

- For any given proof system, a finite number of rules of inference (or schema) are listed somewhere
- Rule instances should be **easily checked**
- **What is the definition of “NP”?**

Derivation

$$\frac{\frac{\frac{\Gamma(x) = int}{\Gamma \vdash x : int} \text{ var} \quad \frac{\Gamma \vdash 3 : int}{\Gamma \vdash 3 : int} \text{ int}}{\Gamma \vdash x > 3 : bool} \text{ gt} \quad \frac{\frac{\frac{\frac{\Gamma(x) = int}{\Gamma \vdash x : int} \text{ var} \quad \frac{\frac{\Gamma(x) = int}{\Gamma \vdash 1 : int} \text{ var}}{\Gamma \vdash 1 : int} \text{ int sub}}{\Gamma \vdash x - 1 : int} \text{ assign}}{\Gamma \vdash x := x - 1} \text{ while}}{\Gamma \vdash \text{while } x > 3 \text{ do } x := x - 1 \text{ done}}$$

- Tree-structured (conclusion at bottom)
- May include multiple sorts of rules-of-inference
- Could be constructed, typically are not
- Typically verified in polynomial time

Evaluation Rules (for Aexp)

$$\frac{}{\langle n, \sigma \rangle \Downarrow n}$$

$$\frac{}{\langle x, \sigma \rangle \Downarrow \sigma(x)}$$

$$\frac{\langle e_1, \sigma \rangle \Downarrow n_1 \quad \langle e_2, \sigma \rangle \Downarrow n_2}{\langle e_1 + e_2, \sigma \rangle \Downarrow n_1 + n_2} \quad \frac{\langle e_1, \sigma \rangle \Downarrow n_1 \quad \langle e_2, \sigma \rangle \Downarrow n_2}{\langle e_1 - e_2, \sigma \rangle \Downarrow n_1 - n_2}$$

$$\frac{\langle e_1, \sigma \rangle \Downarrow n_1 \quad \langle e_2, \sigma \rangle \Downarrow n_2}{\langle e_1 * e_2, \sigma \rangle \Downarrow n_1 * n_2}$$

- This is called structural operational semantics
 - rules defined based on the structure of the expression
- These rules do **not** impose an order of evaluation!

Evaluation Rules (for Bexp)

$$\frac{}{\langle \text{true}, \sigma \rangle \Downarrow \text{true}}$$

$$\frac{}{\langle \text{false}, \sigma \rangle \Downarrow \text{false}}$$

$$\frac{\langle e_1, \sigma \rangle \Downarrow n_1 \quad \langle e_2, \sigma \rangle \Downarrow n_2}{\langle e_1 \leq e_2, \sigma \rangle \Downarrow n_1 \leq n_2}$$

$$\frac{\langle e_1, \sigma \rangle \Downarrow n_1 \quad \langle e_2, \sigma \rangle \Downarrow n_2}{\langle e_1 = e_2, \sigma \rangle \Downarrow n_1 = n_2}$$

$$\frac{\langle b_1, \sigma \rangle \Downarrow \text{false}}{\langle b_1 \wedge b_2, \sigma \rangle \Downarrow \text{false}}$$

$$\frac{\langle b_2, \sigma \rangle \Downarrow \text{false}}{\langle b_1 \wedge b_2, \sigma \rangle \Downarrow \text{false}}$$

$$\frac{\langle b_1, \sigma \rangle \Downarrow \text{true} \quad \langle b_2, \sigma \rangle \Downarrow \text{true}}{\langle b_1 \wedge b_2, \sigma \rangle \Downarrow \text{true}}$$

(show: candidate \vee rule)

How to Read the Rules?

- Forward (top-down) = inference rules
 - if we know that the hypothesis judgments hold then we can **infer** that the conclusion judgment also holds
 - If we know that
$$\langle e_1, \sigma \rangle \Downarrow 5$$
 and
$$\langle e_2, \sigma \rangle \Downarrow 7,$$
 then we can infer that
$$\langle e_1 + e_2, \sigma \rangle \Downarrow 12$$

How to Read the Rules?

- Backward (bottom-up) = evaluation rules
 - Suppose we want to evaluate $e_1 + e_2$, i.e., find n s.t. $e_1 + e_2 \Downarrow n$ is derivable using the previous rules
 - By inspection of the rules we notice that the last step in the derivation of $e_1 + e_2 \Downarrow n$ **must be** the addition rule
 - the other rules have conclusions that would not match $e_1 + e_2 \Downarrow n$
 - this is called reasoning by **inversion** on the derivation rules

Evaluation By Inversion

- Thus we must find n_1 and n_2 such that $e_1 \Downarrow n_1$ and $e_2 \Downarrow n_2$ are derivable
 - This is done **recursively**
- If there is exactly one rule for each kind of expression we say that the rules are syntax-directed
 - At each step at most one rule applies
 - This allows a simple evaluation procedure as above (recursive tree-walk)
 - True for our Aexp but not Bexp. **Why?**

Evaluation of Commands

- The evaluation of a Com may have side effects but has **no direct result**
 - What is the result of evaluating a command ?
- The “result” of a Com is a **new state**:

$$\langle C, \sigma \rangle \Downarrow \sigma'$$

- But the evaluation of Com might not terminate! **Danger Will Robinson!** (huh?)



Com Evaluation Rules 1

$$\frac{}{\langle \text{skip}, \sigma \rangle \Downarrow \sigma} \quad \frac{\langle c_1, \sigma \rangle \Downarrow \sigma' \quad \langle c_2, \sigma' \rangle \Downarrow \sigma''}{\langle c_1 ; c_2, \sigma \rangle \Downarrow \sigma''}$$

$$\frac{\langle b, \sigma \rangle \Downarrow \text{true} \quad \langle c_1, \sigma \rangle \Downarrow \sigma'}{\langle \text{if } b \text{ then } c_1 \text{ else } c_2, \sigma \rangle \Downarrow \sigma'}$$

$$\frac{\langle b, \sigma \rangle \Downarrow \text{false} \quad \langle c_2, \sigma \rangle \Downarrow \sigma'}{\langle \text{if } b \text{ then } c_1 \text{ else } c_2, \sigma \rangle \Downarrow \sigma'}$$

Com Evaluation Rules 2

$$\frac{\langle e, \sigma \rangle \Downarrow n}{\langle x := e, \sigma \rangle \Downarrow \sigma[x := n]}$$

$$\begin{aligned} \text{Def: } \sigma[x := n](x) &= n \\ \sigma[x := n](y) &= \sigma(y) \end{aligned}$$

- Let's do **while** together



Com Evaluation Rules 3

$$\frac{\langle e, \sigma \rangle \Downarrow n}{\langle x := e, \sigma \rangle \Downarrow \sigma[x := n]}$$

Def: $\sigma[x := n](x) = n$
 $\sigma[x := n](y) = \sigma(y)$

$$\frac{\langle b, \sigma \rangle \Downarrow \text{false}}{\langle \text{while } b \text{ do } c, \sigma \rangle \Downarrow \sigma}$$

$$\frac{\langle b, \sigma \rangle \Downarrow \text{true} \quad \langle c; \text{while } b \text{ do } c, \sigma \rangle \Downarrow \sigma'}{\langle \text{while } b \text{ do } c, \sigma \rangle \Downarrow \sigma'}$$

Homework

- Homework 0 Peer Review
- Homework 1 Due Soon
- Reading!
 - If this wasn't intuitive, try some of the optional readings for more context.