
THEM'S FIGHTIN' WORDS, MISTER...UNLESS'N, O'COURSE, THEM'S JUST SEMANTICS.

#1

Today's Cunning Plan

- Review, Truth, and Provability
- Large-Step Opsem Commentary
- Small-Step Contextual Semantics
  - Reductions, Redexes, and Contexts
- Applications and Recent Research

Survey Results

++++++ lecture style

++++ good content (theory/practice)

++ research paper reading

++ fast pace of course

+ HW0 bonus problem

+ humor

+ revise fundamental math

+ trivia quiz

+ lectures are clear & easy

+ use of examples

+ no exams

+ materials posted early

--- Wes talks fast

--- old room was dark, new room is far

- want more implementation

- hit in eye by candy

- want more programming paradigms

- don't know what to focus on w/ papers

- imposter syndrome

- want more examples in class

- “ML is sort of a pain”

- trivia quiz focuses on Western culture

- want chocolate

Bookkeeping

- Hookkeeper (wire ring that holds a fly-fishing hook in place)
- Tattooee
- Bookkeeper
  - Subbookkeeper (!)
- Sweettooth

60 Second Summary - Semantics

- A **formal semantics** is a system for assigning meanings to programs.
- For now, programs are IMP commands and expressions
- In **operational semantics** the meaning of a program is "what it evaluates to"
- Any opsem system gives **rules of inference** that tell you how to evaluate programs

Summary - Judgments

- Rules of inference allow you to derive judgments ("something that is knowable") like

$$\langle e, \sigma \rangle \Downarrow n$$

  - In state σ, expression e evaluates to n

$$\langle c, \sigma \rangle \Downarrow \sigma'$$

  - After evaluating command c in state σ the new state will be σ'
- State σ maps variables to values ($\sigma : L \rightarrow Z$)
- Inferences equivalent up to variable renaming:

$$\langle c, \sigma \rangle \Downarrow \sigma' \quad === \quad \langle c', \sigma_7 \rangle \Downarrow \sigma_8$$

Notation: Rules of Inference

- We express the evaluation rules as rules of inference for our judgment
  - called the derivation rules for the judgment
  - also called the evaluation rules (for operational semantics)
- In general, we have one rule for each language construct:

$$\frac{\langle e_1, \sigma \rangle \Downarrow n_1 \qquad \langle e_2, \sigma \rangle \Downarrow n_2}{\langle e_1 + e_2, \sigma \rangle \Downarrow n_1 + n_2}$$

This is the only rule for $e_1 + e_2$

Evaluation By Inversion

- We must find n_1 and n_2 such that $e_1 \Downarrow n_1$ and $e_2 \Downarrow n_2$ are derivable
  - This is done recursively
- If there is exactly one rule for each kind of expression we say that the rules are syntax-directed
  - At each step at most one rule applies
  - This allows a simple evaluation procedure as above (recursive tree-walk)
  - True for our Aexp but not Bexp.

Summary - Rules

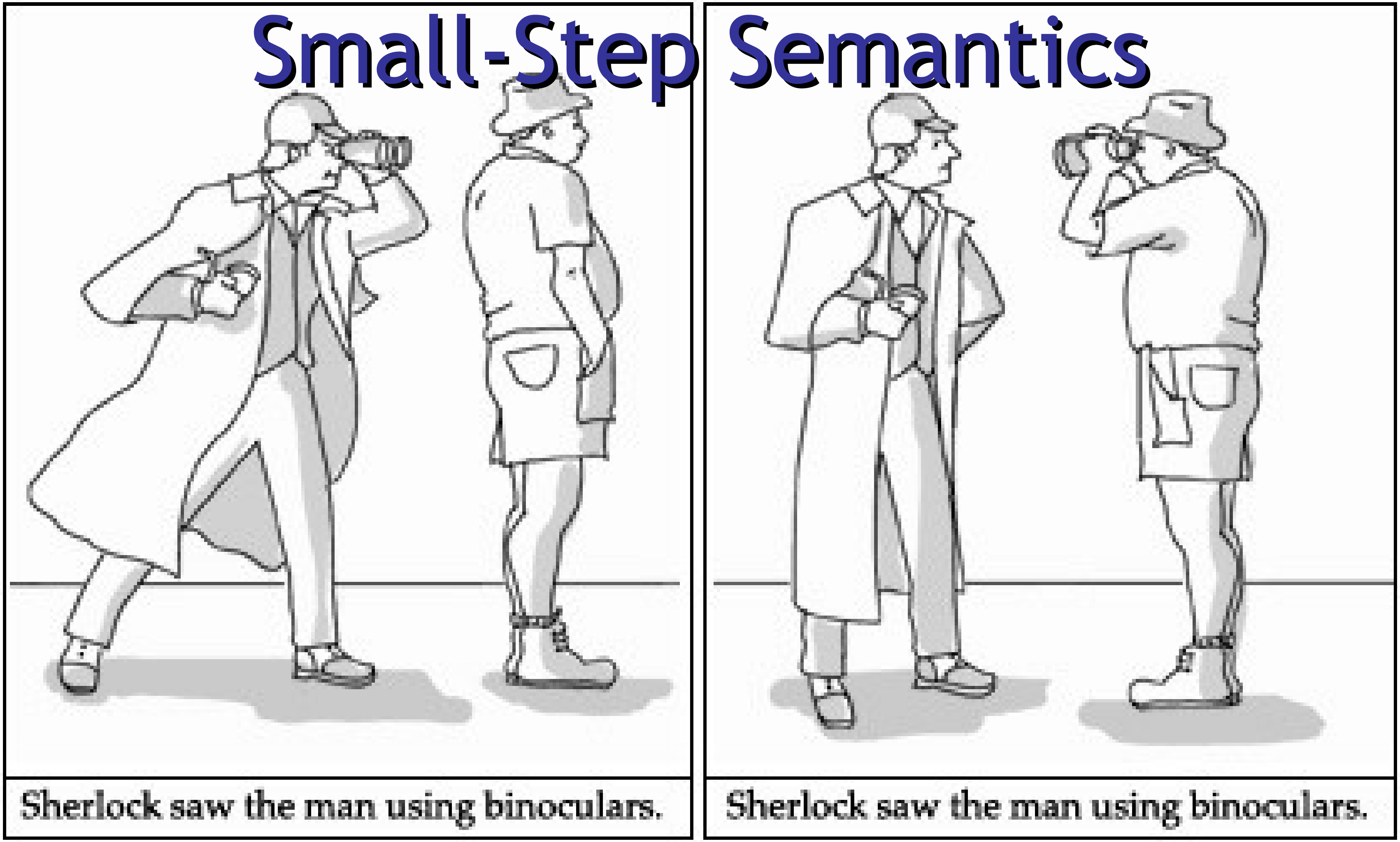
- Rules of inference list the hypotheses necessary to arrive at a conclusion

$$\frac{}{\langle x, \sigma\rangle \Downarrow \sigma(x)} \qquad \frac{\langle e_1, \sigma\rangle \Downarrow n_1 \quad \langle e_2, \sigma\rangle \Downarrow n_2}{\langle e_1 - e_2, \sigma\rangle \Downarrow n_1 \text{ minus } n_2}$$

- A derivation involves interlocking (well-formed) instances of rules of inference

$$\frac{\dfrac{\langle 4, \sigma_3\rangle \Downarrow 4 \quad \langle 2, \sigma_3\rangle \Downarrow 2}{\langle 4*2, \sigma_3\rangle \Downarrow 8} \qquad \langle 6, \sigma_3\rangle \Downarrow 6}{\langle (4*2) - 6, \sigma_3\rangle \Downarrow 2}$$

Operational Semantics
Small-Step Semantics



Provability

- Given an opsem system, $<e, \sigma> \Downarrow n$ is <u>**provable**</u> *if there exists* a well-formed derivation with $<e, \sigma> \Downarrow n$ as its conclusion
  - "well-formed" = "every step in the derivation is a valid instance of one of the rules of inference for this opsem system"
  - "$\vdash <e, \sigma> \Downarrow n$" = "it is provable that $<e, \sigma> \Downarrow n$"
- We would *like* truth and provability to be closely related

Truth?

- "A Vorlon said understanding is a three-edged sword. Your side, their side and the truth."
  - Sheridan, Babylon 5, *Into The Fire*
- We will not formally define "truth" yet
- Instead we appeal to your intuition
  - $<2+2, \sigma> \Downarrow 4$ -- *should be* true
  - $<2+2, \sigma> \Downarrow 5$ -- *should be* false

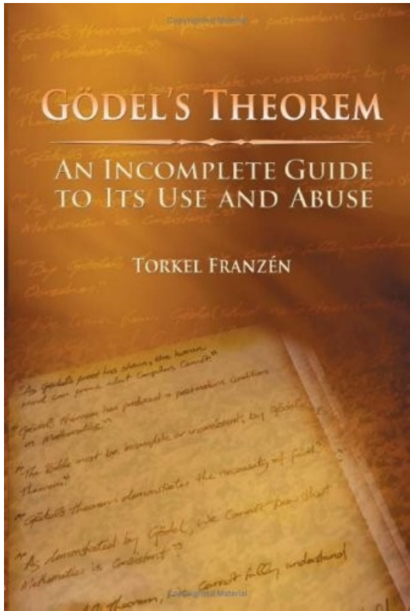
Completeness

- A proof system (like our operational semantics) is [complete](#) if every true judgment is provable.
- If we ***replaced*** the subtract rule with:

$$\frac{\langle e_1, \sigma\rangle \Downarrow n \qquad \langle e_2, \sigma\rangle \Downarrow 0}{\langle e_1 - e_2, \sigma\rangle \Downarrow n}$$

- Our opsem would be [incomplete](#):

$\langle 4\text{-}2, \sigma\rangle \Downarrow 2$ -- true but not provable



Consistency

- A proof system is **consistent** (or **sound**) if every provable judgment is true.
- If we ***replaced*** the subtract rule with:

$$\frac{\langle e_1, \sigma\rangle \Downarrow n_1 \qquad \langle e_2, \sigma\rangle \Downarrow n_2}{\langle e_1 - e_2, \sigma\rangle \Downarrow n_1 + 3}$$

- Our opsem would be **inconsistent** (or **unsound**):
  - $<6-1, \sigma> \Downarrow 9$ -- false but provable

"A foolish consistency is the hobgoblin of little minds, adored by little statesmen and philosophers and divines."
-- Ralph Waldo Emerson, *Essays. First Series. Self-Reliance*.

Desired Traits

- Typically a system (of operational semantics) is always complete (unless you forget a rule)
- If you are not careful, however, your system may be unsound
- Usually that is *very bad*
  - A paper with an unsound type system is usually rejected
  - Papers often prove (sketch) that a system is sound
  - Recent research (e.g., Engler, ESP) into useful but unsound systems exists, however
- In this class your work should be complete and consistent (e.g., on homework problems)

Dr. Peter Venkman: I'm a little fuzzy on the whole "good/bad" thing here. What do you mean, "bad"?
Dr. Egon Spengler: Try to imagine all life as you know it stopping instantaneously and every molecule in your body exploding at the speed of light.

With That In Mind

- We now return to opsem for IMP

$$\frac{\langle e, \sigma \rangle \Downarrow n}{\langle x := e, \sigma \rangle \Downarrow \sigma[x := n]}$$

Def: $\sigma[x := n](x) = n$
$\sigma[x := n](y) = \sigma(y)$

$$\frac{\langle b, \sigma \rangle \Downarrow \text{false}}{\langle \text{while } b \text{ do } c, \sigma \rangle \Downarrow \sigma}$$

$$\frac{\langle b, \sigma \rangle \Downarrow \text{true} \quad \langle c; \text{while } b \text{ do } c, \sigma \rangle \Downarrow \sigma'}{\langle \text{while } b \text{ do } c, \sigma \rangle \Downarrow \sigma'}$$

Command Evaluation Notes

- The order of evaluation is important
  - c_1 is evaluated before c_2 in c_1; c_2
  - c_2 is not evaluated in "if true then c_1 else c_2"
  - c is not evaluated in "while false do c"
  - b is evaluated first in "if b then c_1 else c_2"
  - this is explicit in the evaluation rules
- Conditional constructs (e.g., $b_1 \vee b_2$) have multiple evaluation rules
  - but only one can be applied at one time

Command Evaluation Trials

- The evaluation rules are not syntax-directed
  - See the rules for while, $\wedge$
  - The evaluation might not terminate
- Recall: the evaluation rules suggest an interpreter
- Natural-style semantics has two big disadvantages (continued …)

Disadvantages of Natural-Style Operational Semantics

- It is hard to talk about commands whose evaluation does not terminate
  - i.e., when there is no σ' such that $<c, \sigma> \Downarrow \sigma'$
  - But that is true also of ill-formed or erroneous commands (in a richer language)!
- It does not give us a way to talk about intermediate states
  - Thus we cannot say that on a parallel machine the execution of two commands is interleaved (= no modeling threads)

Semantics Solution

- Small-step semantics addresses these problems
  - Execution is modeled as a (possible infinite) sequence of states
- Not quite as easy as large-step natural semantics, though
- Contextual semantics is a small-step semantics where the atomic execution step is a rewrite of the program

Contextual Semantics

- We will define a relation $<c, \sigma> \rightarrow <c', \sigma'>$
  - c' is obtained from c via an atomic rewrite step
  - Evaluation terminates when the program has been rewritten to a terminal program
    - one from which we cannot make further progress
  - For IMP the terminal command is "skip"
  - As long as the command is not "skip" we can make further progress
    - some commands *never* reduce to skip (e.g., "while true do skip")

Contextual Derivations

- In small-step contextual semantics, derivations are not tree-structured
- A contextual semantics derivation is a sequence (or list) of atomic rewrites:

$$\langle x+(7-3),\sigma\rangle \rightarrow \langle x+(4),\sigma\rangle \rightarrow \langle 5+4,\sigma\rangle \rightarrow \langle 9,\sigma\rangle$$

$\sigma(x)=5$

What is an Atomic Reduction?

- What is an atomic reduction step?
  - Granularity is a choice of the semantics designer
- How to select the next reduction step, when several are possible?
  - This is the order of evaluation issue

Q: Music

- This Hong Kong singer is one of the original four cantopop Heavenly Kings (四大天王), and possesses a rich baritone/tenor. He is sometimes called the God of Songs (歌神). His most famous work is perhaps Goodbye Kiss (吻别) – one of the best-selling albums of all time, with over 3 million copies sold in 1993 alone. Give the English or Romanized name of this singer.

Correcting English Prose

4. Lizzy drank in the sight of him like a thirst craven man consumes water.

421. "I go here, silly," said Kimi with a proud expression. "And how I might ask? Your scores were not legible for this school."

312. Every member of the Thespians, or anyone who has ever acted in one of our school plays was a pre-Madonna, mellow-dramatic; over-actor and I didn't want to be one of them.

198. Nobody goes into Donovan's Layer, For they sence evil. But Livvy doesn't she see's something no one else does.

Q: Events (615 / 842)

- This Egyptian-born United Nations Secretary-General served from 1992 to 1996. He was criticized for, among other things, failing to act during the 1994 Rwandan genocides and during the continuing Angolan civil war.

Redexes

- A **redex** is a syntactic expression or command that can be reduced (transformed) in one atomic step
- Redexes are defined via a grammar:

r ::= x **($x \in L$)**
| $n_1 + n_2$
| x := n
| skip; c
| if true then c_1 else c_2
| if false then c_1 else c_2
| while b do c

- For brevity, we mix exp and command redexes
- Note that (1 + 3) + 2 is not a redex, but 1 + 3 is

Local Reduction Rules for IMP

- One for each redex: **$\langle r, \sigma\rangle \rightarrow \langle e, \sigma'\rangle$**
  - means that in state σ, the redex r can be *replaced in one step* with the expression e

$\langle x, \sigma\rangle \rightarrow \langle \sigma(x), \sigma\rangle$

$\langle n_1 + n_2, \sigma\rangle \rightarrow \langle n, \sigma\rangle$ where $n = n_1$ plus n_2

$\langle n_1 = n_2, \sigma\rangle \rightarrow \langle \text{true}, \sigma\rangle$ if $n_1 = n_2$

$\langle x := n, \sigma\rangle \rightarrow \langle \text{skip}, \sigma[x := n]\rangle$

$\langle \text{skip}; c, \sigma\rangle \rightarrow \langle c, \sigma\rangle$

$\langle \text{if true then } c_1 \text{ else } c_2, \sigma\rangle \rightarrow \langle c_1, \sigma\rangle$

$\langle \text{if false then } c_1 \text{ else } c_2, \sigma\rangle \rightarrow \langle c_2, \sigma\rangle$

$\langle \text{while } b \text{ do } c, \sigma\rangle \rightarrow$

$\langle \text{if } b \text{ then } c; \text{while } b \text{ do } c \text{ else skip}, \sigma\rangle$

Not happy? I'll explain with pictures soon!

The Global Reduction Rule

- General idea of contextual semantics
  - Decompose the current expression into the redex-to-reduce-next and the remaining program
    - The remaining program is called a **context**
  - Reduce the redex "r" to some other expression "e"
  - The resulting (reduced) expression consists of "e" with the original context

As A Picture (1)

(Context)

…

x := 2+2 ;

print x

Step 1: Find The Redex

As A Picture (2)

(Context)

…

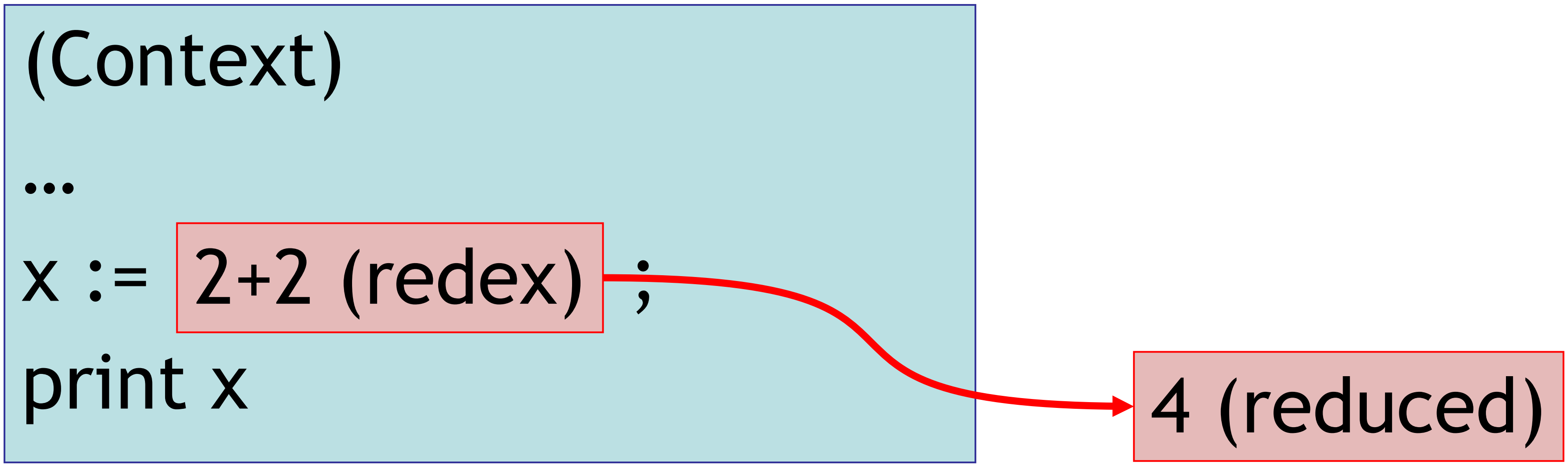
x := $\boxed{\text{2+2 (redex)}}$;

print x

Step 1: Find The Redex

Step 2: Reduce The Redex

As A Picture (3)



Step 1: Find The Redex

Step 2: Reduce The Redex

As A Picture (4)

(Context)

…

x := $\boxed{4}$;

print x

Step 1: Find The Redex

Step 2: Reduce The Redex

Step 3: Replace It In The Context

Contextual Analysis

- We use H to range over contexts
- We write H[r] for the expression obtained by placing redex r in context H
- Now we can define a small step

If $<r, \sigma> \to <e, \sigma'>$

then $<H[r], \sigma> \to <H[e], \sigma'>$

Contexts

- A **context** is like an expression (or command) with a marker • in the place where the redex goes
- Examples:
  - To evaluate "(1 + 3) + 2" we use the redex 1 + 3 and the context "• + 2"
  - To evaluate "if x > 2 then c_1 else c_2" we use the redex x and the context "if • > 2 then c_1 else c_2"

Context Terminology

- A context is also called an "expression with a hole"
- The marker • is sometimes called a hole
- H[r] is the expression obtained from H by replacing • with the redex r

"Avoid context and specifics; generalize and keep repeating the generalization."
-- Jack Schwartz

Contextual Semantics Example

- x := 1 ; x := x + 1 with initial state [x:=0]

<Comm, State>	**Redex •**	**Context**
<x := 1; x := x+1, [x := 0]>	x := 1	•; x := x+1
<skip; x := x+1, [x := 1]>	skip; x := x+1	•
<x := x+1, [x := 1]>	x	x := • + 1
What happens next?		

Contextual Semantics Example

- x := 1 ; x := x + 1 with initial state [x:=0]

<Comm, State>	**Redex •**	**Context**
<x := 1; x := x+1, [x := 0]>	x := 1	•; x := x+1
<skip; x := x+1, [x := 1]>	skip; x := x+1	•
<x := x+1, [x := 1]>	x	x := • + 1
<x := 1 + 1, [x := 1]>	1 + 1	x := •
<x := 2, [x := 1]>	x := 2	•
<skip, [x := 2]>		

More On Contexts

- Contexts are defined by a grammar:

$$
\begin{aligned}
\mathbf{H} ::= \bullet \ & | \ \mathbf{n + H} \\
& | \ \mathbf{H + e} \\
& | \ \mathbf{x := H} \\
& | \ \mathbf{if\ H\ then\ c_1\ else\ c_2} \\
& | \ \mathbf{H;\ c}
\end{aligned}
$$

- A context has exactly one • marker
- A redex is never a value

What's In A Context?

- Contexts specify precisely how to find the next redex
  - Consider $e_1 + e_2$ and its decomposition as H[r]
  - If e_1 is n_1 and e_2 is n_2 then $H = \bullet$ and $r = n_1 + n_2$
  - If e_1 is n_1 and <u>e_2 is not n_2</u> then $H = n_1 + H_2$ and $e_2 = H_2[r]$
  - If <u>e_1 is not n_1</u> then $H = H_1 + e_2$ and $e_1 = H_1[r]$
  - In the last two cases the decomposition is done recursively
  - Check that in each case the solution is unique

Unique Next Redex: Proof By Handwaving Examples

- e.g. c = "c_1; c_2" – either
  - c_1 = skip and then $c = H[\text{skip}; c_2]$ with $H = \bullet$
  - or $c_1 \neq$ skip and then $c_1 = H[r]$; so $c = H'[r]$ with $H' = H; c_2$
- e.g. c = "if b then c_1 else c_2"
  - either b = true or b = false and then $c = H[r]$ with $H = \bullet$
  - or b is not a value and $b = H[r]$; so $c = H'[r]$ with H' = if H then c_1 else c_2

Context Decomposition

- Decomposition theorem:

  If c is not "skip" then there exist unique H and r such that c is H[r]

  – "Exist" means progress
  – "Unique" means determinism

Short-Circuit Evaluation

- What if we want to express short-circuit evaluation of $\wedge$?
  - Define the following contexts, redexes and local reduction rules

$$H ::= \ldots \mid H \wedge b_2$$

$$r ::= \ldots \mid \text{true} \wedge b \mid \text{false} \wedge b$$

$$<\text{true} \wedge b, \sigma> \rightarrow <b, \sigma>$$

$$<\text{false} \wedge b, \sigma> \rightarrow <\text{false}, \sigma>$$

  - the local reduction kicks in before b_2 is evaluated

Contextual Semantics Summary

- Can view • as representing the program counter
- The advancement rules for • are non-trivial
  - At each step the entire command is decomposed
  - This makes contextual semantics inefficient to implement directly

- The major advantage of contextual semantics: it allows a *mix* of local and global reduction rules
  - For IMP we have only local reduction rules: only the redex is reduced
  - Sometimes it is useful to work on the context too
  - We'll do that when we study memory allocation, etc.

Reading Real-World Examples

- Cobbe and Felleisen, POPL 2005
- Small-step contextual opsem for Java
- Their rule for object field access:

$$P \vdash \langle \mathsf{E}[obj.fd], \mathcal{S} \rangle \hookrightarrow \langle \mathsf{E}[\mathcal{F}(fd)], \mathcal{S} \rangle$$
$$\text{where } \mathcal{F} = fields(\mathcal{S}(obj)) \text{ and } fd \in \mathrm{dom}(\mathcal{F})$$

$P \vdash$ <E[obj.fd],S> $\to$ <E[F(fd)],S>

- where F=fields(S(obj)) and fd $\in$ dom(F)

- They use "E" for context, we use "H"
- They use "S" for state, we use "σ"

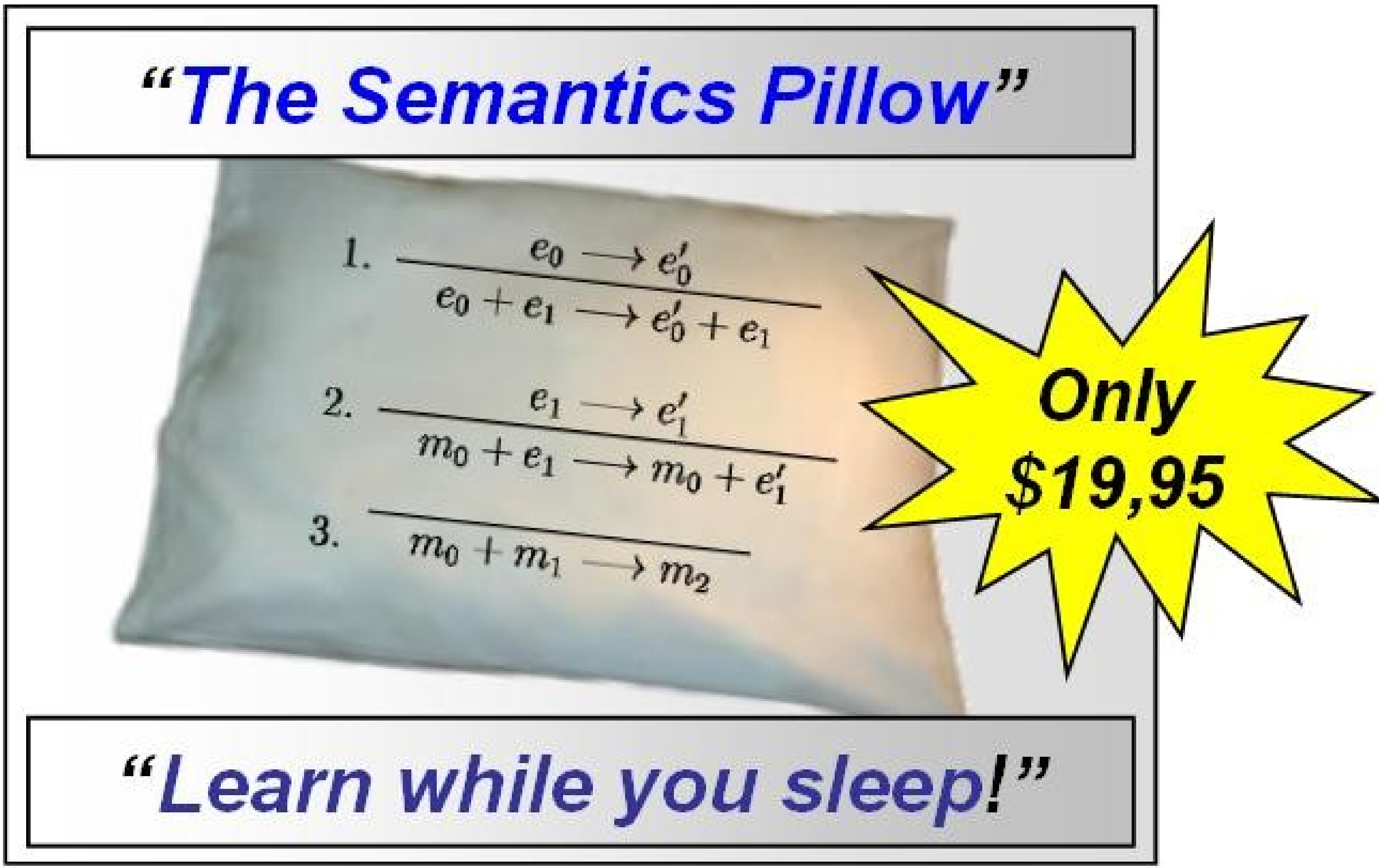
Lost In Translation

- $P \vdash \langle H[obj.fd],\sigma\rangle \rightarrow \langle H[F(fd)],\sigma\rangle$
  - Where F=fields(σ(obj)) and $fd \in dom(F)$
- They have "$P \vdash$", but that just means "it can be proved in our system given P"
- $\langle H[obj.fd],\sigma\rangle \rightarrow \langle H[F(fd)],\sigma\rangle$
  - Where F=fields(σ(obj)) and $fd \in dom(F)$

Lost In Translation 2

- $<H[obj.fd],\sigma> \rightarrow <H[F(fd)],\sigma>$
  - Where $F=fields(\sigma(obj))$ and $fd \in dom(F)$
- They model objects (like obj), but we do not (yet) – let's just make fd a variable:
- $<H[fd],\sigma> \rightarrow <H[F(fd)],\sigma>$
  - Where $F=\sigma$ and $fd \in L$
- Which is just our variable-lookup rule:
- $<H[fd],\sigma> \rightarrow <H[\sigma(fd)],\sigma>$ (when $fd \in L$)

“Sleep On It”


“The Semantics Pillow”
1. $\dfrac{e_0 \longrightarrow e_0'}{e_0 + e_1 \longrightarrow e_0' + e_1}$
2. $\dfrac{e_1 \longrightarrow e_1'}{m_0 + e_1 \longrightarrow m_0 + e_1'}$
3. $\dfrac{}{m_0 + m_1 \longrightarrow m_2}$
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Homework

- Homework 1 Due Thursday
- Read Hooimeijer & Weimer paper