Banburismus and the Story So Far

One-Slide Summary

- British codebreakers used **cribs** (guesses), brute force, and **analysis** to break the Lorenz cipher. Guessed wheel settings were likely to be correct if they resulted in a message with the right linguistic properties for German.
- If you've guessed the right wheel settings, two adjacent letters are more likely to be the same than they are to be different letters: **Double Deltas**.
- We can tell if two messages were encrypted using the same wheel settings (= same key) because the output letters will match when the input letters match. So we can try to “line them up” using **Banburismus** to look for matches.
- Tree sorting is only efficient if the trees are **balanced**. If not, it's $\Theta(n^2)$. The best possible sorting is $\Theta(n \log n)$.

Outline

- WWII Codebreaking
- Double Deltas
- Machines
- Banburismus
- Tree Sorting
- Course Roadmap

Pick Up Graded Problem Sets Before Spring Break
Or Possibly Lose Points!

Breaking WWII Traffic

- Knew machine structure, but a different initial configuration was used for each message
- Need to determine wheel setting:
  - Initial position of each of the 12 wheels
  - 1271 possible starting positions
  - Needed to try them fast enough to decrypt message while it was still strategically valuable

This is what you did for PS4 (except with fewer wheels)

Recognizing a Good Guess

- Intercepted Message (divided into 5 channels for each Baudot code bit)

  $Z_c = z_0z_1z_2z_3z_4z_5z_6z_7\ldots$

  $z_{c,i} = m_{c,i} \oplus x_{c,i} \oplus s_{c,i}$

  Message Key (parts from S-wheels and rest)

- Look for statistical properties
  - How many of the $z_{c,i}$'s are 0? ½ (not useful)
  - How many of $(z_{c,i+1} \oplus z_{c,i})$ are 0? ½

Double Delta

$$\Delta Z_{c,i} = Z_{c,i} \oplus Z_{c,i+1}$$

Combine two channels:

$$\Delta Z_{1,i} \oplus \Delta Z_{2,i} = \Delta M_{1,i} \oplus \Delta M_{2,i} \quad > ½ \text{ Yippee!}$$

$$\oplus \Delta X_{1,i} \oplus \Delta X_{2,i} \quad = ½ \text{ (key)}$$

$$\oplus \Delta S_{1,i} \oplus \Delta S_{2,i} \quad > ½ \text{ Yippee!}$$

Why is $\Delta M_{1,i} \oplus \Delta M_{2,i} > ½$

Message is in German, more likely following letter is a repetition than random

Why is $\Delta S_{1,i} \oplus \Delta S_{2,i} > ½$

S-wheels only turn when M-wheel is 1

Actual Advantage

- Probability of repeating letters

  Prob[$\Delta M_{1,i} \oplus \Delta M_{2,i} = 0$] ~ 0.614

  3.3% of German digraphs are repeating
- Probability of repeating S-keys

  Prob[$\Delta S_{1,i} \oplus \Delta S_{2,i} = 0$] ~ 0.73

Prob[$\Delta Z_{1,i} \oplus \Delta Z_{2,i} \oplus \Delta X_{1,i} \oplus \Delta X_{2,i} = 0$]

$= 0.614 * 0.73 \quad + (1-0.614) * (1-0.73)$

Δ M and S are 0 $\quad \Delta$ M and S are 1

= 0.55 **if** the wheel settings guess is correct (0.5 otherwise)

Using the Advantage

- If the guess of **X** is correct, should see higher than ½ of the double deltas are 0
- Try guessing different configurations to find highest number of 0 double deltas
- **Problem**:

  # of double delta operations to try one config

  = length of Z * length of X

  = for 10,000 letter message = 12 M for each setting

  * 7 ⊕ per double delta

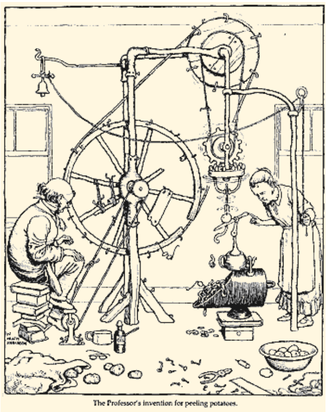
  = 89 M ⊕ operations

  (that's a lot!)

Need a fast way to compute XOR!

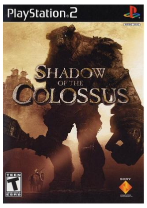
Heath Robinson Machine

- Dec 1942: Decide to build a machine to do these ⊕s quickly, due June 1943
- Apr 1943: first **"Heath Robinson" machine** is delivered!
  - Predecessor to Colossus
- Intercepted ciphertext on tape:
  - 2000 characters per second (12 miles per hour)
  - Needed to perform 7 ⊕ operations each ½ ms



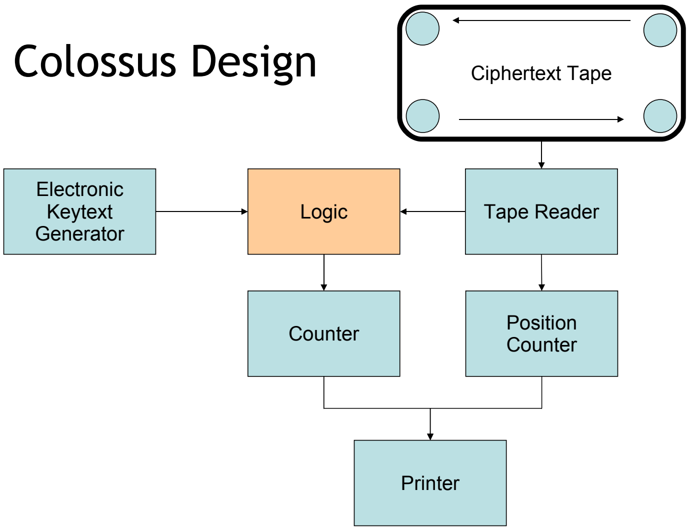
Heath Robinson, British Cartoonist (1872-1944)

Colossus



- Heath Robinson machines were too slow
- **Colossus** designed and first built in Jan 1944
- Replaced keytext tape loop with electronic keytext generator
- Speed up ciphertext tape:
  - 5,000 chars per second = 30 mph
  - Perform 5 double deltas simultaneously
  - Speedup = 2.5X for faster tape * 5X for parallelism

Colossus Design

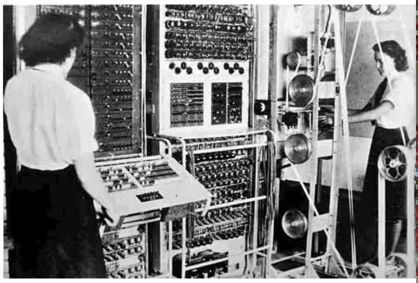


Impact on WWII

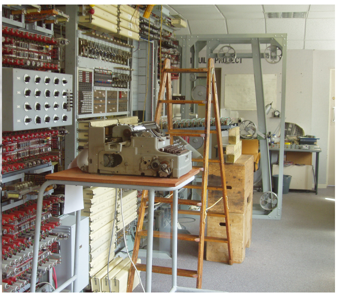
- 10 Colossus machines operated at Bletchley park
  - Various improvements in speed
- Decoded 63 million letters in Nazi command messages
- Learned German troop locations to plan D-Day (knew the deception was working)

Colossus History

Kept secret after the war, all machines destroyed

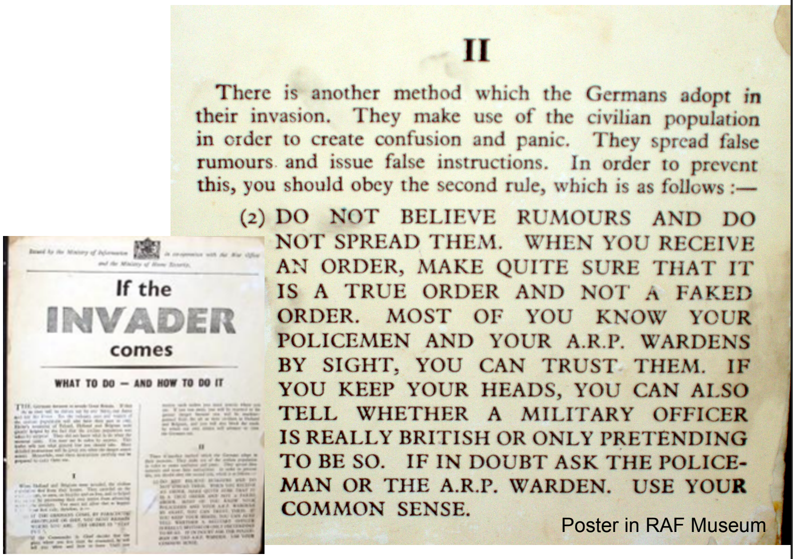


During WWII



Rebuild, Bletchley Park, Summer 2004

How could the folks at Bletchley Park solve a problem ~ 1 quintillion times harder than ps4?



Issued by the Ministry of Information in co-operation with the War Office and the Ministry of Home Security.

If the INVADER comes

WHAT TO DO — AND HOW TO DO IT

II

There is another method which the Germans adopt in their invasion. They make use of the civilian population in order to create confusion and panic. They spread false rumours and issue false instructions. In order to prevent this, you should obey the second rule, which is as follows :—

(2) DO NOT BELIEVE RUMOURS AND DO NOT SPREAD THEM. WHEN YOU RECEIVE AN ORDER, MAKE QUITE SURE THAT IT IS A TRUE ORDER AND NOT A FAKED ORDER. MOST OF YOU KNOW YOUR POLICEMEN AND YOUR A.R.P. WARDENS BY SIGHT, YOU CAN TRUST THEM. IF YOU KEEP YOUR HEADS, YOU CAN ALSO TELL WHETHER A MILITARY OFFICER IS REALLY BRITISH OR ONLY PRETENDING TO BE SO. IF IN DOUBT ASK THE POLICEMAN OR THE A.R.P. WARDEN. USE YOUR COMMON SENSE.

Poster in RAF Museum

Motivation Helps…

Confronted with the prospect of defeat, the Allied cryptanalysts had worked night and day to penetrate German ciphers. It would appear that fear was the main driving force, and that adversity is one of the foundations of successful codebreaking.

Simon Singh, *The Code Book*

Liberal Arts Trivia: Maritime Law

- A letter of marque is an official government document authorizing an agent to search, seize, or destroy specified assets or personnel belonging to a foreign party beyond the borders of the nation ("marque" or frontier). They are usually used to authorize private parties to raid and capture merchant shipping of an enemy nation. In the past, a ship operating under a letter of marque and reprisal was privately owned and was called a "private man-of-war" or ... what?

Liberal Arts Trivia: Geography

- This capital city of Uttar Pradesh, the most populous state of India, is popularly known as the The City of Nawabs. It is also known as the Golden City of the East, Shiraz-i-Hind and The Constantinople of India. It is a center of Hindi and Urdu literature, and the birthplace of Kathak, a classic Indian dance form. The city was besieged during the Indian Rebellion of 1857.

Banburismus

Given two Enigma-encrypted messages, how can we determine if they were encrypted starting with the same wheel settings?

Enigma in Use,
10 December 1943

Enigma

Enigma machine at Bletchley Park

- Invented commercially, 1923
- German Navy, Army, Air Force
- About 50,000 in use (many were captured by Allies)
- Modified throughout WWII, Germans believed perfectly secure
- Kahn's *Codebreakers* (1967) didn't know it was broken
- Turing's 1940 Treatise on Enigma declassified in 1996

Reverse Engineering Enigma

"This fictional movie about a fictional U.S. submarine mission is followed by a mention in the end credits of those actual British missions. Oh, the British deciphered the Enigma code, too. Come to think of it, they pretty much did everything in real life that the Americans do in this movie."

Roger Ebert's review of **U-571**
(2000 Academy Award Winner)

Simple Substitution Ciphers

ABCDEFGHIJKLMNOPQRSTUVWXYZ

encrypt ↓ decrypt ↑

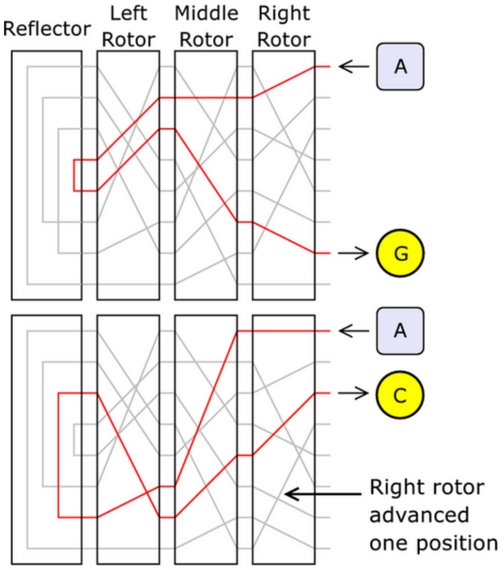
JIDKQACRSHLGWNFEXUZVTPMYOB

HELLO ⇒ RQGGF

Rotor Wheels

Simple substitution

Latch turns next rotor once per rotation



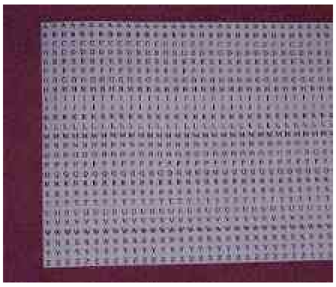
Right rotor advanced one position

Image from
http://en.wikipedia.org/wiki/Image:Enigma-action.png

Language is Non-Random

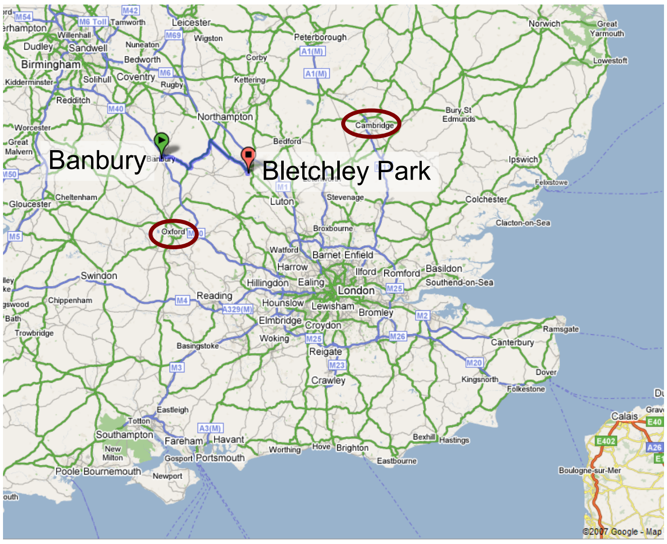
- Random strings: the probability of two letters in the two messages matching is 1/26 (number of letters in alphabet)
- Same-encrypted strings: the output letters will match when the input letters match
  - This happens much more frequently because some letters (e.g., "e" is ~13% of all letters) are more common

Alan Turing's Solution

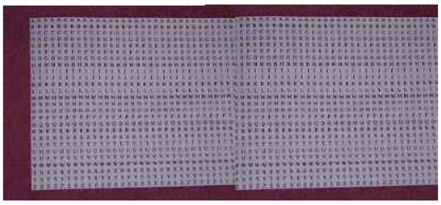


M1: GXCYBGDSLVWBDJLKWIPEHVYGQZWDTHRQXIKEESQS

M2: YNSCFCCPVIPEMSGIZWFLHESCIYSPVRXMCFQAXVXDVU



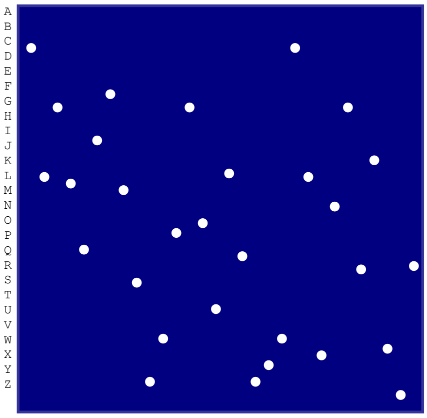
Banburismus



M1: GXCYBGDSLVWBDJLKWIPEHVYGQZWDTHRQXIKEESQS

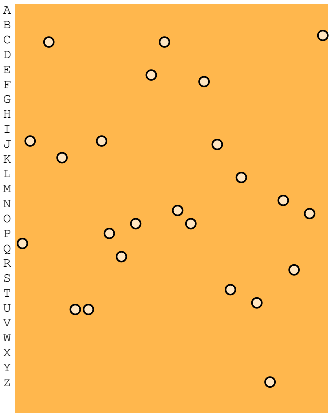
M2: YNSCFCCPVIPEMSGIZWFLHESCIYSPVRXMCFQAXVXDVU

Intercepted Message 1



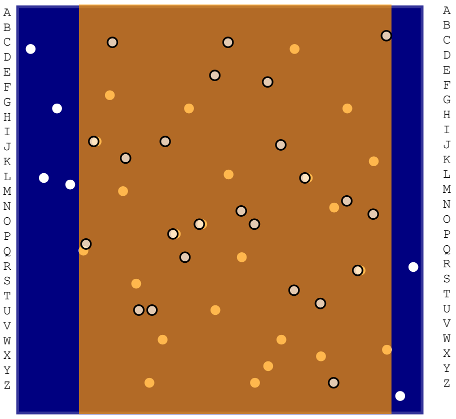
CKGLPIFLR...

Intercepted Message 2



PICJTTIOQN...

Intercepted Message 1


A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Trying Possible Alignments

```
GXCYBGDSLVWBDJLKWIPEHVYGQZWDTHRQXIKEESQS
YNSCFCCPVIPEMSGIZWFLHESCIYSPVRXMCFQAXVXDVU
 YNSCFCCPVIPEMSGIZWFLHESCIYSPVRXMCFQAXVXDVU
  YNSCFCCPVIPEMSGIZWFLHESCIYSPVRXMCFQAXVXDVU

       …

         YNSCFCCPVIPEMSGIZWFLHESCIYSPVRXMCFQAX..
```

Trying Possible Alignments

```
GXCYBGDSLVWBDJLKWIPEHVYGQZWDTHRQXIKEESQS
YNSCFCCPVIPEMSGIZWFLHESCIYSPVRXMCFQAXVXDVU
 YNSCFCCPVIPEMSGIZWFLHESCIYSPVRXMCFQAXVXDVU
  YNSCFCCPVIPEMSGIZWFLHESCIYSPVRXMCFQAXVXDVU

       …

         YNSCFCCPVIPEMSGIZWFLHESCIYSPVRXMCFQAX..
```

Trying Possible Alignments

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GXCYBGDSLVWBDJLKWIPEHVYGQZWDTHRQXIKEESQS
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 YNSCFCCPVIPEMSGIZWFLHESCIYSPVRXMCFQAXVXDVU
  YNSCFCCPVIPEMSGIZWFLHESCIYSPVRXMCFQAXVXDVU

       …

         YNSCFCCPVIPEMSGIZWFLHESCIYSPVRXMCFQAX..
```

Trying Possible Alignments

```
GXCYBGDSLVWBDJLKWIPEHVYGQZWDTHRQXIKEESQS
YNSCFCCPVIPEMSGIZWFLHESCIYSPVRXMCFQAXVXDVU
 YNSCFCCPVIPEMSGIZWFLHESCIYSPVRXMCFQAXVXDVU
  YNSCFCCPVIPEMSGIZWFLHESCIYSPVRXMCFQAXVXDVU

       …

         YNSCFCCPVIPEMSGIZWFLHESCIYSPVRXMCFQAX..
```

Turing's Hut 8 at Bletchley Park

Don't complain about your working space.
You can do good computer science anywhere.
But find a quiet, undisturbed place to work on the exam.

Liberal Arts Trivia: Geology

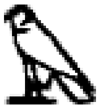
- A stratovolcano or composite volcano is a tall, conical volcano made of many layers of lava, tephra and volcanic ash: they are characterized by steep sides and periodic eruptions. They are common in subduction zones where the ocean crust is drawn under the continental crust. Mount St. Helens and Mount Fuji are both stratovolcanos: name the country containing each one.

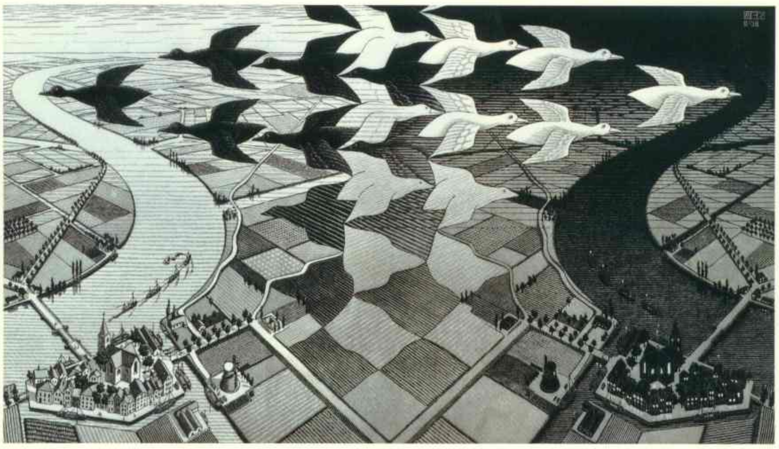
Liberal Arts Trivia: Philosophy

- This 18th century Prussian philosopher wrote on epistemology, as well as religion, law, and history. One of his most prominent works is the *Critique of Pure Reason*, an investigation into the limitations and structure of reason itself, encompassing an attack on traditional metaphysics and epistemology. He sought to create a compromise between empiricists and rationalists, claiming that the grand questions of speculative metaphysics cannot be answered by the human mind, but that the sciences are firmly grounded in laws of the mind.

Liberal Arts Trivia: Mythology

- In Egyptian mythology, this falcon-headed son of Isis and Osiris fought with Seth for the throne of Egypt. In the battle his eye was wounded and later healed by Isis; this became an important symbol for renewal. He united Egypt and bestowed divinity on the pharaohs (who were viewed as his living incarnations). Name this sun, sky and war god, shown here in hieroglyphs:





The Story So Far

insert-one-tree

```
(define (insert-one-tree cf el tree)
  (if (null? tree)
      (make-tree null el null)
      (if (cf el (get-element tree))
          (make-tree
           (insertel-tree cf el (get-left tree))
           (get-element tree) (get-right tree))
          (make-tree (get-left tree)
           (get-element tree)
           (insertel-tree cf el (get-right tree))))))
```

Each time we call insert-one-tree, the size of the tree approximately **halves** (if it is well balanced).

Each application is constant time.

The running time of insert-one-tree is in $\Theta(\log n)$ where n is the number of elements in the input tree, which must be well-balanced.

insert-sort-helper

```
(define (insert-sort-helper cf lst)
   (if (null? lst) null
       (insert-one-tree
        cf (car lst)
        (insert-sort-helper cf (cdr lst)))))
```

No change (other than using insert-one-tree)…but evaluates to a tree not a list!

((((() 1 ()) 2 ()) 5 (() 8 ()))

extract-elements

We need to make a list of all the tree elements, from left to right.

```
(define (extract-elements tree)
  (if (null? tree) null
      (append (extract-elements (get-left tree))
              (cons
                 (get-element tree)
                 (extract-elements (get-right tree))))))
```

Running time of insert-sort-tree

```
(define (insert-one-tree cf el tree)
  (if (null? tree)
     (make-tree null el null)
     (if (cf el (get-element tree))
        (make-tree (insert-one-tree cf el (get-left tree))
                        (get-element tree)
                        (get-right tree))
        (make-tree (get-left tree)
                        (get-element tree)
                        (insert-one-tree cf el (get-right tree))))))
```

n = number of elements in tree

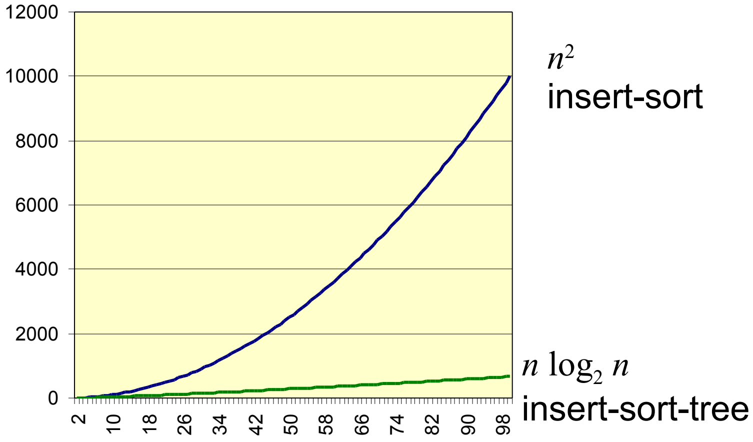
$\Theta(\log n)$

```
(define (insert-sort-tree cf lst)
   (define (insert-sort-helper cf lst)
      (if (null? lst) null
          (insert-one-tree
           cf (car lst)
           (insert-sort-helper cf (cdr lst)))))
   (extract-elements (insert-sort-helper cf lst)))
```

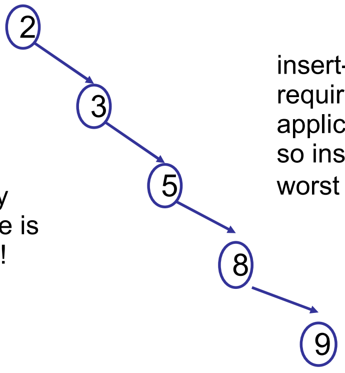
n = number of elements in lst

$\Theta(n \log n)$

Growth of time to sort random list



What if tree is not well-balanced?



A pathologically unbalanced tree is as bad as a list!

insert-one worst case requires n recursive applications, so insert-sort-tree worst case is in $\Theta(n^2)$

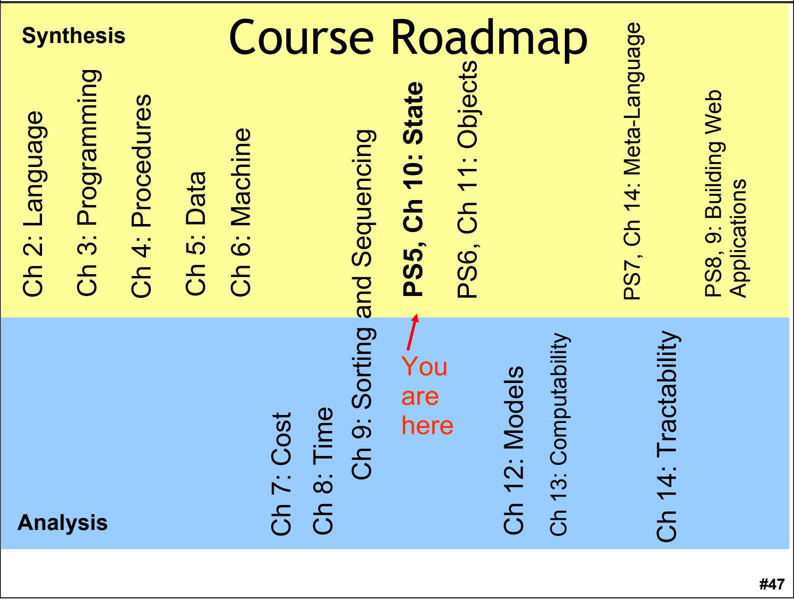
Can we do better?

- Making all those trees is a lot of work
- Can we divide the problem in two halves, without making trees?

This is the famous "**Quicksort**" algorithm invented by Sir Tony Hoare. See Course Book.

There are lots of ways to do a little bit better, but **no** way to do asymptotically better. **All** possible sort procedure have running times in $\Omega(n \log n)$. (We'll explain why later in the course...)

Course Roadmap



Computer Science 1120 so far

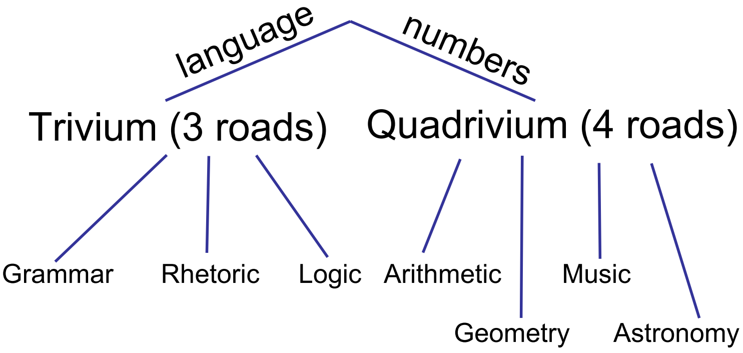
- How to describe **information processes** by defining **procedures**
  - Programming with procedures, lists, **recursion**
  - Chapters 3, 4, 5
- How to **predict properties** about information processes
  - Predicting running time, Θ, *O*, Ω
- How to elegantly and **efficiently implement** information processes
  - Chapter 3 (rules of evaluation)
  - Chapter 6 (machines)

CS1120 upcoming

- How to describe information processes by defining procedures
  - Programming with state, objects, networks
- How to predict properties about information processes
  - What is the fastest process that can solve a given problem?
  - Are there problems which can't be solved by algorithms?
- How to elegantly and efficiently implement information processes
  - How to implement a Scheme interpreter

From Lecture 1:

The Liberal Arts



language

numbers

Trivium (3 roads)

Quadrivium (4 roads)

Grammar

Rhetoric

Logic

Arithmetic

Geometry

Music

Astronomy

Liberal Arts Checkup

Trivium

- Grammar: study of meaning in written expression

BNF replacement rules for describing languages, rules of evaluation for meaning

- Rhetoric: comprehension of verbal and written discourse

Not much yet… interfaces between components (PS6-9), program and user (PS8-9)

- Logic: argumentative discourse for discovering truth

Rules of evaluation, if, recursive definitions

Quadrivium

- Arithmetic: understanding numbers

Not much yet… wait until April

- Geometry: quantification of space

Curves as procedures, fractals (PS3)

- Music: number in time

Yes, listen to "Hey Jude!"

- Astronomy

Read Neil deGrasse Tyson's essay

Homework

- **Exam 1 Due Wednesday**
  - **Out Today**