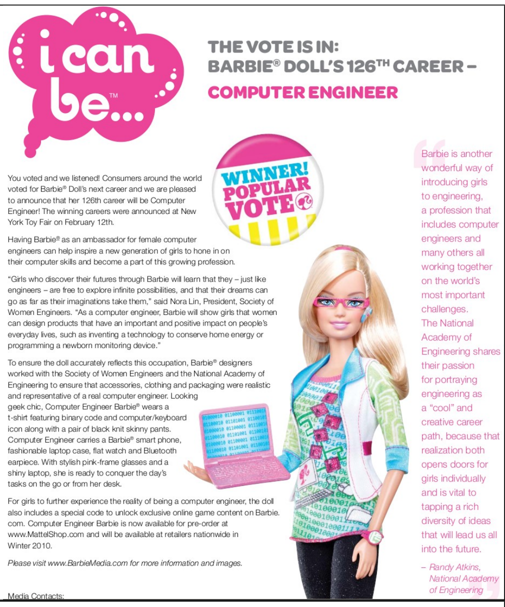
# Costs and Sneezewort and Growth

532. Achillea Ptarmica L.

*Sneezewort*

# Breaking News

- "Follow-up discussion on the CS education email list has focused on the clothes, the accessories, and whether members of the list or their students might have stuffed the ballot box electronically. (Might have happened!) One person did ask a good question: he asked that anyone with young kids who like Barbie to report back on how they react to CpE Barbie.
  - Tom Horton



i can be...™
WINNER! POPULAR VOTE

THE VOTE IS IN:
BARBIE® DOLL'S 126TH CAREER –
COMPUTER ENGINEER

You voted and we listened! Consumers around the world voted for Barbie® Doll's next career and we are pleased to announce that her 126th career will be Computer Engineer! The winning careers were announced at New York Toy Fair on February 12th.

Having Barbie® as an ambassador for female computer engineers can help inspire a new generation of girls to hone in on their computer skills and become a part of this growing profession.

"Girls who discover their futures through Barbie will learn that they – just like engineers – are free to explore infinite possibilities, and that their dreams can go as far as their imaginations take them," said Nora Lin, President, Society of Women Engineers. "As a computer engineer, Barbie will show girls that women can design products that have an important and positive impact on people's everyday lives, such as inventing a technology to conserve home energy or programming a newborn monitoring device."

To ensure the doll accurately reflects this occupation, Barbie® designers worked with the Society of Women Engineers and the National Academy of Engineering to ensure that accessories, clothing and packaging were realistic and representative of a real computer engineer. Looking geek chic, Computer Engineer Barbie® wears a t-shirt featuring binary code and computer/keyboard icon along with a pair of black knit skinny pants. Computer Engineer carries a Barbie® smart phone, fashionable laptop case, flat watch and Bluetooth earpiece. With stylish pink-frame glasses and a shiny laptop, she is ready to conquer the day's tasks on the go or from her desk.

For girls to further experience the reality of being a computer engineer, the doll also includes a special code to unlock exclusive online game content on Barbie.com. Computer Engineer Barbie is now available for pre-order at www.MattelShop.com and will be available at retailers nationwide in Winter 2010.

*Please visit www.BarbieMedia.com for more information and images.*

Media Contacts:

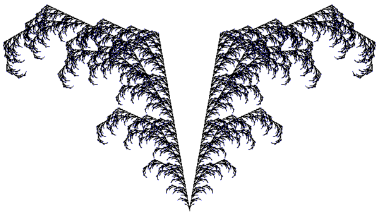
> "Barbie is another wonderful way of introducing girls to engineering, a profession that includes computer engineers and many others all working together on the world's most important challenges. The National Academy of Engineering shares their passion for portraying engineering as a "cool" and creative career path, because that realization both opens doors for girls individually and is vital to tapping a rich diversity of ideas that will lead us all into the future."
>
> – *Randy Atkins, National Academy of Engineering*

# One-Slide Summary

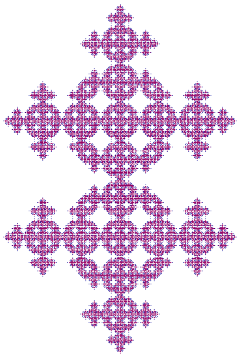
- The basic recursive computation of Fibonacci can take quite a while. **There are faster ways**.
- We can formally measure and evaluate the cost of a computer program. We abstract away details such as processor speed and instead measure how **the solving time increases as the input increases**.
- **g is in O(f)** iff there exist positive constants *c* and $n_0$ such that $g(n) \le cf(n)$ for all $n \ge n_0$.
- If g is in O(f) we say that f is an **upper bound** for g.

# Outline

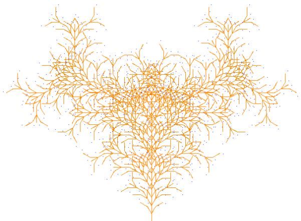
- Sneezewort and Fibonacci
- **Cost** of computing Fibonacci
- **Cost** of sorting
- Intro to Big-Oh Notation



*"V" shrubbery*
by Andrew Jesien, Becky Elstad



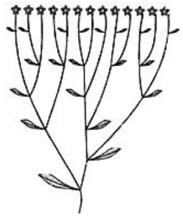
*After the Incident*
by Ben Morrison and Liz Peterson



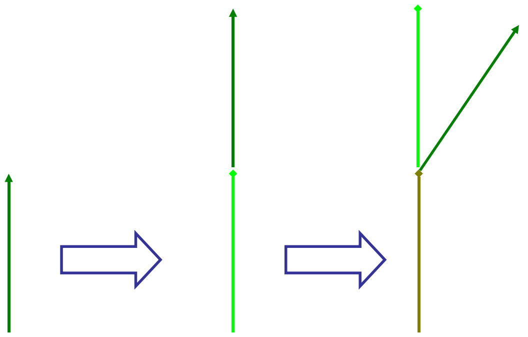
*Robot Cav Man*
by Jamie Jeon & Walter Borges

# Sneezewort

- Achillea ptarmica is real.
- It is "moste efficacious in the inflaming of the braine, and [is] therefore much used in Confusing and Befuddlement Draughts, where the wizard is desirous of producing hot-headedness and recklessness."
  - Order of the Phoenix, p.18
- Sneezewort's pattern of development displays the Fibonacci sequence.

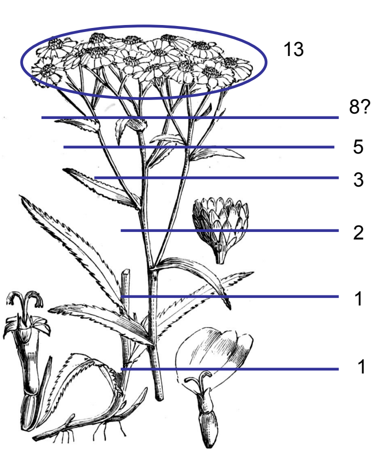


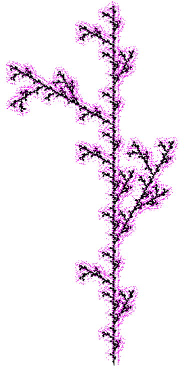
# Sneezewort Growth



Could we model Sneezewort with PS3 code?

# Sneezewort Numbers

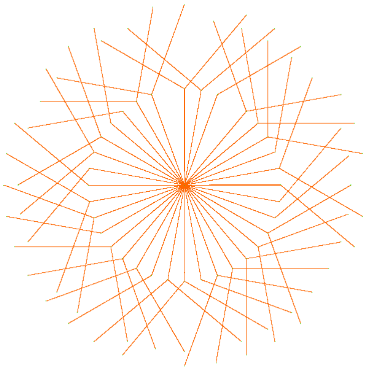




*pink*
by Jessica Geist, Ellen Clarke

# Fibo Results

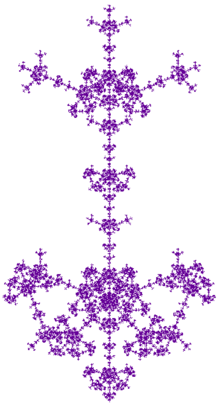
```
> (fibo 2)
1
> (fibo 3)
2
> (fibo 4)
3
> (fibo 10)
55
> (fibo 60)
Still working…
```



*At least we finished.*
by Dmitriy Semenov and Sara Alspaugh

# Tracing Fibo

```
> (require-library "trace.ss")
> (trace fibo)
(fibo)
> (fibo 3)
|(fibo 3)
|       (fibo 2)
|       1
|       (fibo 1)
|       1
|2
2
```

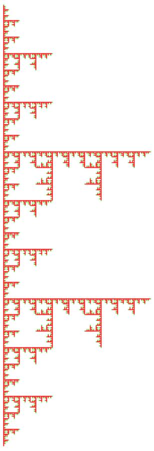


*Purple Arrow*
by Rachel Lathbury and Andrea Yoon

**See end of Class 5 lecture notes for information on "trace.ss"!**

```
> (fibo 5)
|(fibo 5)
| (fibo 4)
| |(fibo 3)
| | (fibo 2)
| | 1
| | (fibo 1)
| | 1
| |2
| |(fibo 2)
| |1
| 3
| (fibo 3)
| |(fibo 2)
| |1
| |(fibo 1)
| |1
| 2
|5
5
```

*A right-wing Christmas - awwwwww......*
by Andrew Baker & Emily Lam



To calculate (fibo 5) we calculated:

| | | |
|---|---|---|
| (fibo 4) | 1 time | |
| (fibo 3) | 2 times | |
| (fibo 2) | 3 times | 5 times total |
| (fibo 1) | 2 times | |

= 8 calls to fibo = (fibo 6)

How many calls to calculate (fibo 60)?

# Liberal Arts Trivia: History

- This 20th-century American inventor is credited with the phonograph, the carbon telephone transmitter, the practical electric light, and the phrase "Genius is one percent inspiration, ninety-nine percent perspiration." He fought against Nikola Tesla's alternating current in the so-called War of the Currents.

# Liberal Arts Trivia: Physics

- Count Alessandro Antonio Anastasio Volta was a 19th-century Italian physicist. Volta studied what we now call capacitance, developing separate means to study both electrical potential *V* and charge *Q*, and discovering that for a given object they are proportional. His experiments in “animal electricity”, in which two different metals were connected in series with frog's legs, eventually led to his most famous discovery. What was it?

# fast-fibo

```
(define (fast-fibo n)
  (define (fib-helper a b left)
    (if (<= left 0)
        b
        (fib-helper b (+ a b) (- left 1))))
  (fib-helper 1 1 (- n 2)))
```

# Fast-Fibo Results

```
> (fast-fibo 10)
55
> (time (fast-fibo 61))
cpu time: 0 real time: 0 gc time: 0
2504730781961
```

The original fibo would take at least 2.5 Trillion applications. A 2.5 GHz computer does 2.5 *Billion* simple operations per second, so 2.5 Trillion applications operations take ~1000 seconds.
Each application of fibo involves hundreds of simple operations…

```
  ;;; The Earth's mass is 6.0 x 10^24 kg
> (define mass-of-earth (* 6 (expt 10 24)))
  ;;; A typical rabbit's mass is 2.5 kilograms
> (define mass-of-rabbit 2.5)
> (/ (* mass-of-rabbit (fast-fibo 60)) mass-of-earth)
6.450036483e-013
> (/ (* mass-of-rabbit (fast-fibo 120)) mass-of-earth)
2.2326496895795693
```

According to Bonacci’s model, after less than 10 years, rabbits would out-weigh the Earth!

```
  ;;; The Earth's mass is 6.0 x 10^24 kg
> (define mass-of-earth (* 6 (expt 10 24)))
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> (/ (* mass-of-rabbit (fast-fibo 120)) mass-of-earth)
2.2326496895795693
```
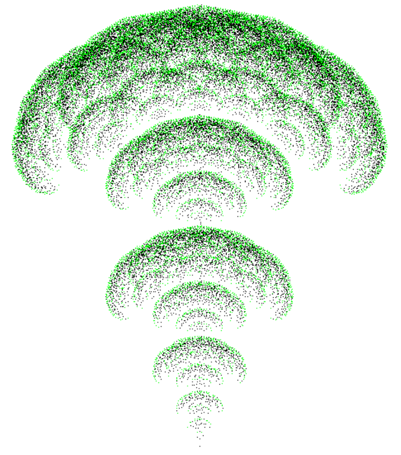

According to Bonacci’s model, after less than 10 years, rabbits would out-weigh the Earth!

Beware the Bunnies!!
Beware the Sneezewort!!

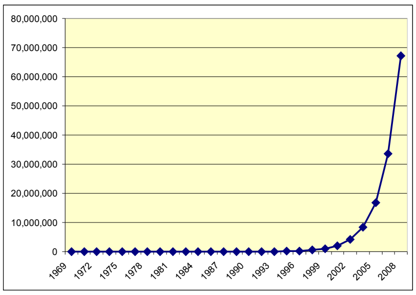


*Broccoli Fallout* by Paul DiOrio, Rachel Phillips

# Evaluation Cost

Actual running times vary according to:

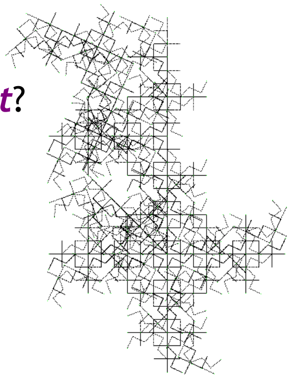
- How fast a processor you have
- How much memory you have
- Where data is located in memory
- How hot it is
- What else is running
- etc...



Moore's "Law" – computing power doubles every 18 months

# Measuring Cost

- How does the cost scale with the ***size of the input***?
- If the input size increases by one, how much longer will it take?
- If the input size *doubles*, how much longer will it take?



*Untitled*
Nokomis McCaskill
Chris Hooe

# Cost of Fibonacci Procedures

```
(define (fibo n)
        (if (or (= n 1) (= n 2))
    1 ;;; base case
    (+ (fibo (- n 1))
       (fibo (- n 2)))))
```

```
(define (fast-fibo n)
  (define (fib-helper a b left)
    (if (= left 0)
        b
        (fib-helper b (+ a b) (- left 1))))
  (fib-helper 1 1 (- n 2)))
```

| Input | fibo | fast-fibo |
|---|---|---|
| $m$ | $q$ | $mk$ |
| $m+1$ | | $(m+1)k$ |
| $m+2$ | at least $q^2$ | $(m+2)k$ |

# Cost of Fibonacci Procedures

```
(define (fibo n)
        (if (or (= n 1) (= n 2))
    1 ;;; base case
    (+ (fibo (- n 1))
       (fibo (- n 2)))))
```

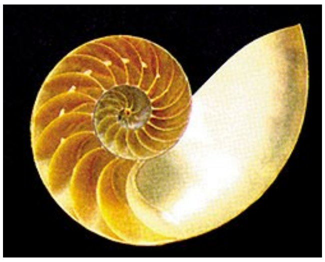
```
(define (fast-fibo n)
  (define (fib-helper a b left)
    (if (= left 0)
        b
        (fib-helper b (+ a b) (- left 1))))
  (fib-helper 1 1 (- n 2)))
```

| Input | fibo | fast-fibo |
|---|---|---|
| $m$ | $q$ | $mk$ |
| $m+1$ | $q*\Phi$ | $(m+1)k$ |
| $m+2$ | at least $q^2$ | $(m+2)k$ |

Φ = (/ (+ 1 (sqrt 5)) 2) = "The Golden Ratio" ~ 1.618033988749895...
~ (/ (fast-fibo 61) (fast-fibo 60)) = 1.618033988749895

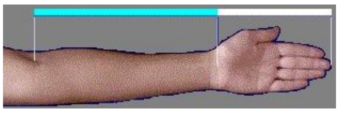
# The Golden Ratio

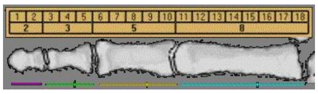
Parthenon



Nautilus Shell

# More Golden Ratios





http://www.fenkefeng.org/essaysm18004.html
by Oleksiy Stakhov

# PS2 Question

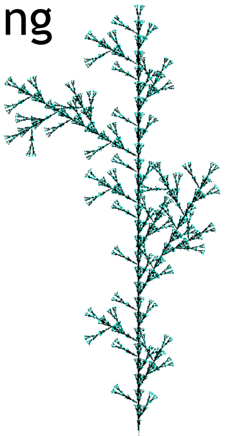
```
(define (find-best-hand hands)
  (car (sort hands higher-hand?)))
```

```
(define (find-best lst cf)
  (if (= 1 (length lst)) (car lst)
      (pick-better cf (car lst) (find-best (cdr lst) cf))))
(define (pick-better cf num1 num2)
  (if (cf num1 num2) num1 num2))
(define (find-best-hand hands)
   (find-best hands higher-hand?))
```

Which is better and by how much?

# Simple Sorting

- Can we use find-best to implement sort?
  - Yes!
- Use (find-best lst) to find the best
- Remove it from the list
  - Adding it to the answer
- Repeat until the list is empty



*crazy blue tree*
by Victor Malaret, Folami Williams

# Simple Sort

```
;; cf = comparison function
(define (sort lst cf)   ;; simple sort
  (if (null? lst) lst
      (let ((best (find-best lst cf)))
        (cons
          best
          (sort (delete lst best) cf)))))
;; delete lst x = (filter ... (not (eq? x ...
```

# Sorting Hands

```
(define (sort lst cf)
  (if (null? lst) lst
      (let ((best (find-best lst cf)))
        (cons
          best
          (sort (delete lst best) cf)))))

  (define (sort-hands lst)
    (sort lst higher-hand?))
```

# Sorting

```
(define (sort lst cf)
  (if (null? lst) lst
      (let ((best (find-best lst cf)))
        (cons best (sort (delete lst best) cf)))))
```

```
(define (find-best lst cf)
   (if (= 1 (length lst)) (car lst)
       (pick-better cf (car lst) (find-best (cdr lst) cf))))
(define (pick-better cf num1 num2)
   (if (cf num1 num2) num1 num2))
```

**How much work is sort?**

# Sorting Cost

- What grows?
  - $n$ = the number of elements in lst
- How much work are the pieces?

find-best:

delete:

# Sorting Cost

- What grows?
  - $n$ = the number of elements in lst
- How much work are the pieces?

  find-best: work scales as $n$ (increases by one)

  delete: work scales as $n$ (increases by one)
- How many times does sort evaluate find-best and delete?

#31

# Sorting Cost

- What grows?
  - $n$ = the number of elements in lst
- How much work are the pieces?

  find-best: work scales as $n$ (increases by one)

  delete: work scales as $n$ (increases by one)
- How many times does sort evaluate find-best and delete? $n$
- Total cost: scales as

#32

# Sorting Cost

- What grows?
  - $n$ = the number of elements in lst
- How much work are the pieces?

  find-best: work scales as $n$ (increases by one)

  delete: work scales as $n$ (increases by one)
- How many times does sort evaluate find-best and delete? $n$
- Total cost: scales as $n^2$

#33

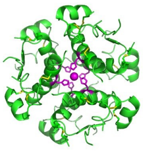
# Sorting Cost

```
(define (sort lst cf)
  (if (null? lst) lst
      (let ((best (find-best lst cf)))
        (cons best (sort (delete lst best) cf)))))
(define (find-best lst cf)
   (if (= 1 (length lst)) (car lst)
       (pick-better cf (car lst) (find-best (cdr lst) cf))))
```

If we double the length of the list, the amount of work *approximately* quadruples: there are twice as many applications of find-best, and each one takes twice as long

#34

# Liberal Arts Trivia: Medicine



- Nicolae Paulescu was a 20th century physiologist and professor of medicine. He is considered the true discoverer of hormone that causes most of the body's cells to take up glucose from the blood. His first experiments involved an aqueous pancreatic extract which, when injected into a diabetic dog, proved to have a normalizing effect on blood sugar levels. Name the hormone.

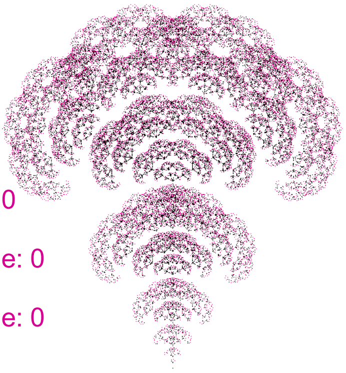
#35

# Liberal Arts Trivia: Sailing

- Name the collection of apparatus through which the force of the wind is transferred to the ship in order to propel it forward - this includes the masts, yardarms, sails, spars and cordage.

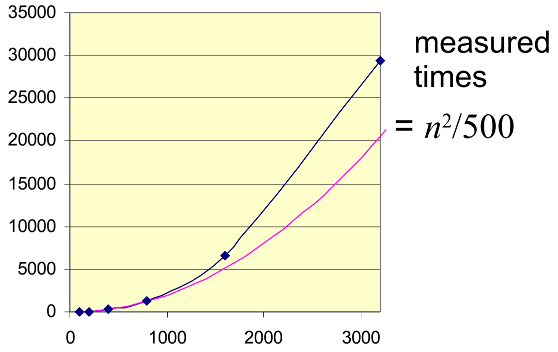
## Timing Sort

```
> (time (sort < (revintsto 100)))
cpu time: 20 real time: 20 gc time: 0
> (time (sort < (revintsto 200)))
cpu time: 80 real time: 80 gc time: 0
> (time (sort < (revintsto 400)))
cpu time: 311 real time: 311 gc time: 0
> (time (sort < (revintsto 800)))
cpu time: 1362 real time: 1362 gc time: 0
> (time (sort < (revintsto 1600)))
cpu time: 6650 real time: 6650 gc time: 0
```



*Cherry Blossom*
by Ji Hyun Lee, Wei Wang

## Timing Sort



## Growth Notations

- $g \in O(f)$ ("Big-Oh")
  $g$ grows **no faster than** $f$ ($f$ is upper bound)
- $g \in \Theta(f)$ ("Theta")
  $g$ grows **as fast as** $f$ ($f$ is tight bound)
- $g \in \Omega(f)$ ("Omega")
  $g$ grows **no slower than** $f$ ($f$ is lower bound)

Which one would we most like to know?

## Meaning of $O$ ("big Oh")

**$g$ is in $O(f)$ iff:**

There are positive constants $c$ and $n_0$ such that

$$g(n) \le cf(n)$$

for all $n \ge n_0$.

O
THE OPRAH MAGAZINE
LIVE YOUR BEST LIFE
LOVE AT LAST
JUST WHEN YOU THINK YOU'VE MISSED YOUR MOMENT...
EXERCISE: THE LEAST YOU CAN DO
(WOULD YOU BELIEVE 10 MINUTES A DAY?)
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DR. OZ'S GENIUS 2-WEEK PLAN
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OPRAH.COM

## $O$ Examples

$g$ is in $O(f)$ iff there are positive constants $c$ and $n_0$ such that $g(n) \le cf(n)$ for all $n \ge n_0$.

Is $n$ in $O(n^2)$?

Is $10n$ in $O(n)$?

Is $n^2$ in $O(n)$?

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Get unlimited nationwide calling from your home phone with T-Mobile @Home service.

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## $O$ Examples

$g$ is in $O(f)$ iff there are positive constants $c$ and $n_0$ such that $g(n) \le cf(n)$ for all $n \ge n_0$.

Is $n$ in $O(n^2)$? Yes, $c = 1$ and $n_0=1$ works.

Is $10n$ in $O(n)$? Yes, $c = 1/10$ and $n_0=1$ works.

Is $n^2$ in $O(n)$? No, no matter what $c$ we pick, $cn^2 > n$ for big enough $n$ ($n > c$)

# Revenge of $O$ Examples

$g$ is in $O(f)$ iff there are positive constants $c$ and $n_0$ such that $g(n) \leq cf(n)$ for all $n \geq n_0$.

Is $n+5$ in $O(n^2)$?

Is $n^2$-100 in $O(n)$?

Is $n^2$ in $O(n^3)$?

# Revenge of $O$ Examples

$g$ is in $O(f)$ iff there are positive constants $c$ and $n_0$ such that $g(n) \leq cf(n)$ for all $n \geq n_0$.

Is $n+5$ in $O(n^2)$? Yes, $c = 1$ and $n_0$=3 works.

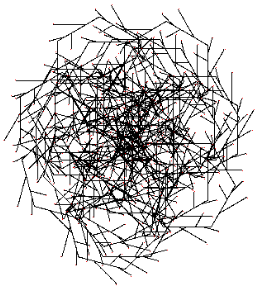
Is $n^2$-100 in $O(n)$? No, no matter what c we pick, $cn^2\text{-}100 > n$ for big enough n.

Is $n^2$ in $O(n^3)$?

Yes, $c = 1$ and $n_0$=1 works.
Yes, $c = 2$ and $n_0$=77 works.
Yes, $c = 55$ and $n_0$=102 works.

# Homework

- Read Course Book Chapter 7 before Wednesday
  - Has a formal notation for this kind of analysis!
- Problem Set 3 due!



*The Mask*
by Zachary Pruckowski,
Kristen Henderson