

# CS 6120/CS4120: Natural Language Processing

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# Parts of Speech

- Perhaps starting with Aristotle in the West (384–322 BCE), there was the idea of having parts of speech (POS)
  - a.k.a lexical categories, word classes, “tags”
- Lowest level of syntactic analysis

# English Parts of Speech (POS) Tagsets

- Original Brown corpus used a large set of 87 POS tags.
- Most common in NLP today is the Penn Treebank set of 45 tags.
  - Tagset used in the slides.
  - Reduced from the Brown set for use in the context of a parsed corpus (i.e. Penn Treebank).

# English Parts of Speech

- Noun (person, place or thing)
  - Singular (NN): dog, fork
  - Plural (NNS): dogs, forks
  - Proper (NNP, NNPS): John, Springfields
  - Personal pronoun (PRP): I, you, he, she, it
  - Wh-pronoun (WP): who, what
- Verb (actions and processes)
  - Base, infinitive (VB): eat
  - Past tense (VBD): ate
  - Gerund (VBG): eating
  - Past participle (VBN): eaten
  - Non 3<sup>rd</sup> person singular present tense (VBP): eat
  - 3<sup>rd</sup> person singular present tense: (VBZ): eats
  - Modal (MD): should, can
  - To (TO): to (to eat)

# English Parts of Speech (cont.)

- Adjective (modify nouns)
  - Basic (JJ): red, tall
  - Comparative (JJR): redder, taller
  - Superlative (JJS): reddest, tallest
- Adverb (modify verbs)
  - Basic (RB): quickly
  - Comparative (RBR): quicker
  - Superlative (RBS): quickest
- Preposition (IN): on, in, by, to, with
- Determiner:
  - Basic (DT) a, an, the
  - WH-determiner (WDT): which, that
- Coordinating Conjunction (CC): and, but, or,
- Particle (RP): off (took off), up (put up)

Open class (lexical) words

Nouns

Proper

*IBM*  
*Italy*

Common

*cat / cats*  
*snow*

Verbs

Main

*see*  
*registered*

Adjectives *old older oldest*

Adverbs *slowly*

Numbers

*122,312*  
*one*

*... more*

Closed class (functional)

Determiners *the some*

Conjunctions *and or*

Pronouns *he its*

Modals

*can*  
*had*

Prepositions *to with*

Particles *off up*

Interjections *Ow Eh*

*... more*

# Open vs. Closed classes

- Open vs. Closed classes
  - Closed:
    - determiners: *a, an, the*
    - pronouns: *she, he, I*
    - prepositions: *on, under, over, near, by, ...*
    - Why “closed”?
  - Open:
    - Nouns, Verbs, Adjectives, Adverbs.

# Ambiguity in POS Tagging

- “Like” can be a verb or a preposition
  - I like/VBP candy.
  - Time flies like/IN an arrow.
- “Around” can be a preposition, particle, or adverb
  - I bought it at the shop around/IN the corner.
  - I never got around/RP to getting a car.
  - A new Prius costs around/RB \$25K.



# POS Tagging

- The POS tagging problem is to determine the POS tag for a particular instance of a word.

# POS Tagging

NN\*: noun

VB\*: verb

UH: interjection

JJ: adjective

RB: adverb

IN: preposition/subordinating conjunction

- Input:            plays            well                            with others
- Ambiguity: NNS/VBZ UH/JJ/NN/RB    IN    NNS
- Output:        Plays/VBZ well/RB with/IN others/NNS
- Uses:
  - Text-to-speech (how do we pronounce “lead”?)
  - Can write regexps over the output for phrase extraction
    - Noun phrase: (Det) Adj\* N+
  - As input to or to speed up a full parser

# POS tagging performance

- How many tags are correct? (Tag accuracy)
  - About 97% currently
  - But baseline is already 90%
    - Baseline is performance of stupidest possible method
      - Take an annotated corpus (or a dictionary), tag every word with its most frequent tag
      - Tag unknown words as nouns
- Partly easy because
  - Many words are unambiguous
  - You get points for them (*the*, *a*, etc.) and for punctuation marks!

# How difficult is POS tagging?

- Word types: roughly speaking, unique words
- About 11% of the word types in the Brown corpus are ambiguous with regard to part of speech
- But they tend to be very common words. E.g., *that*
  - I know *that* he is honest = IN (preposition)
  - Yes, *that* play was nice = DT (determiner)
  - You can't go *that* far = RB (adverb)
- 40% of the word tokens are ambiguous

# Sources of information

- What are the main sources of information for POS tagging?
  - **Contextual:** Knowledge of neighboring words
    - Bill saw that man yesterday
    - NNP NN DT NN NN
    - VB VB(D) IN VB NN
  - **Local:** Knowledge of word probabilities
    - *man* is rarely used as a verb....
- The latter proves the most useful, but the former also helps
- Sometimes these preferences are in conflict:
  - *The trash can is in the garage*

# More and Better Features → Feature-based tagger

- Can do surprisingly well just looking at a word by itself:
  - Word                      the: the → DT
  - Lowercased word              Importantly: importantly → RB
  - Prefixes                      unfathomable: un- → JJ
  - Suffixes                      Importantly: -ly → RB
  - Capitalization              Meridian: CAP → NNP
  - Word shapes              35-year: d-x → JJ

# POS Tagging Approaches

- **Rule-Based**: Human crafted rules based on lexical and other linguistic knowledge.
- **Learning-Based**: Trained on human annotated corpora like the Penn Treebank.
  - **Statistical models**: Hidden Markov Model (HMM) – **this lecture!**, Maximum Entropy Markov Model (MEMM), Conditional Random Field (CRF)
  - **Rule learning**: Transformation Based Learning (TBL)
  - **Neural networks**: Recurrent networks like Long Short Term Memory (LSTMs)
- Generally, learning-based approaches have been found to be more effective overall, taking into account the total amount of human expertise and effort involved.

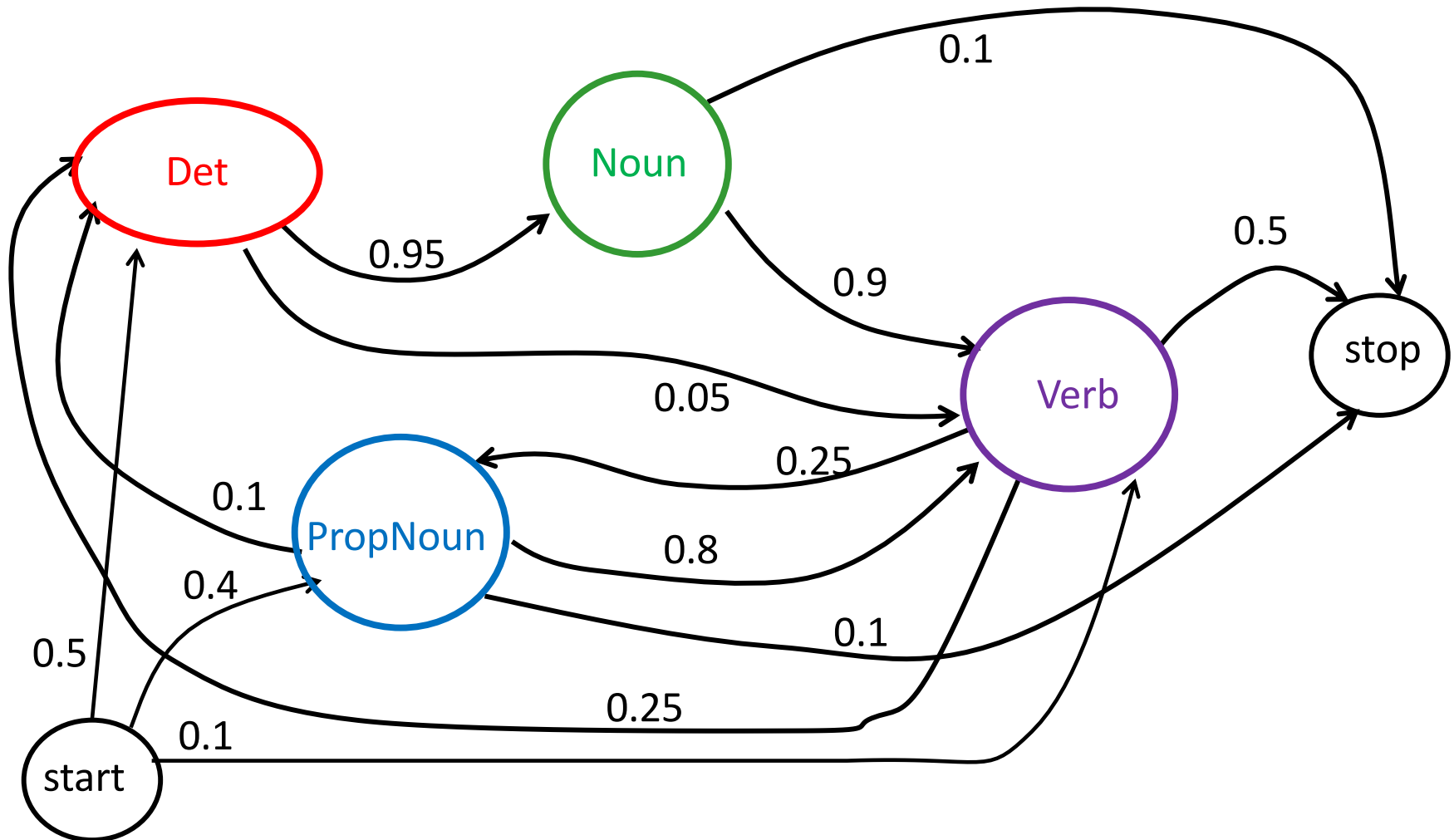
# Hidden Markov Model



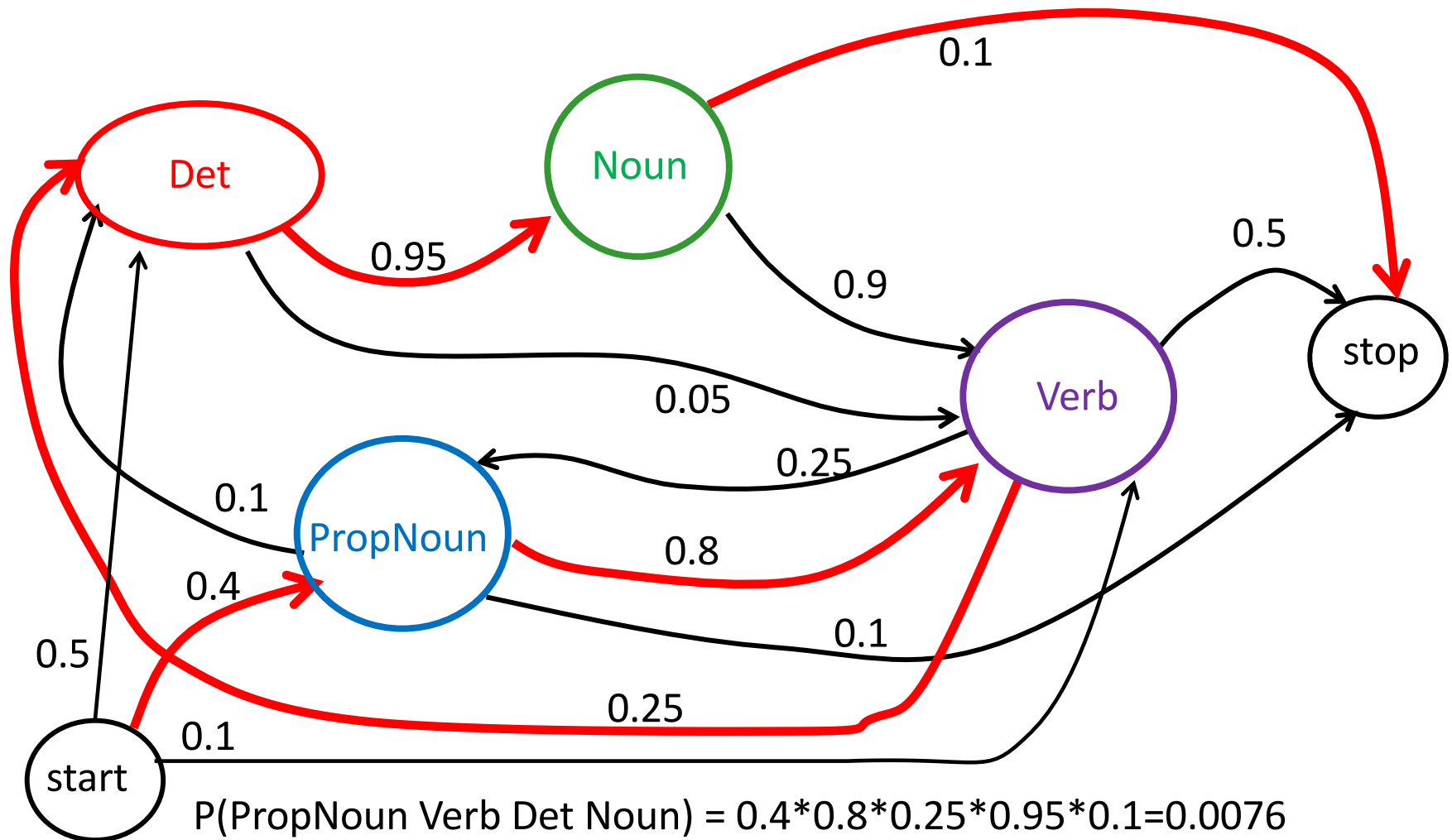
# Markov Model / Markov Chain

- A finite state machine with probabilistic state transitions.
- Makes Markov assumption that next state only depends on the current state and independent of previous history.

# Sample Markov Model for POS (a finite state machine)



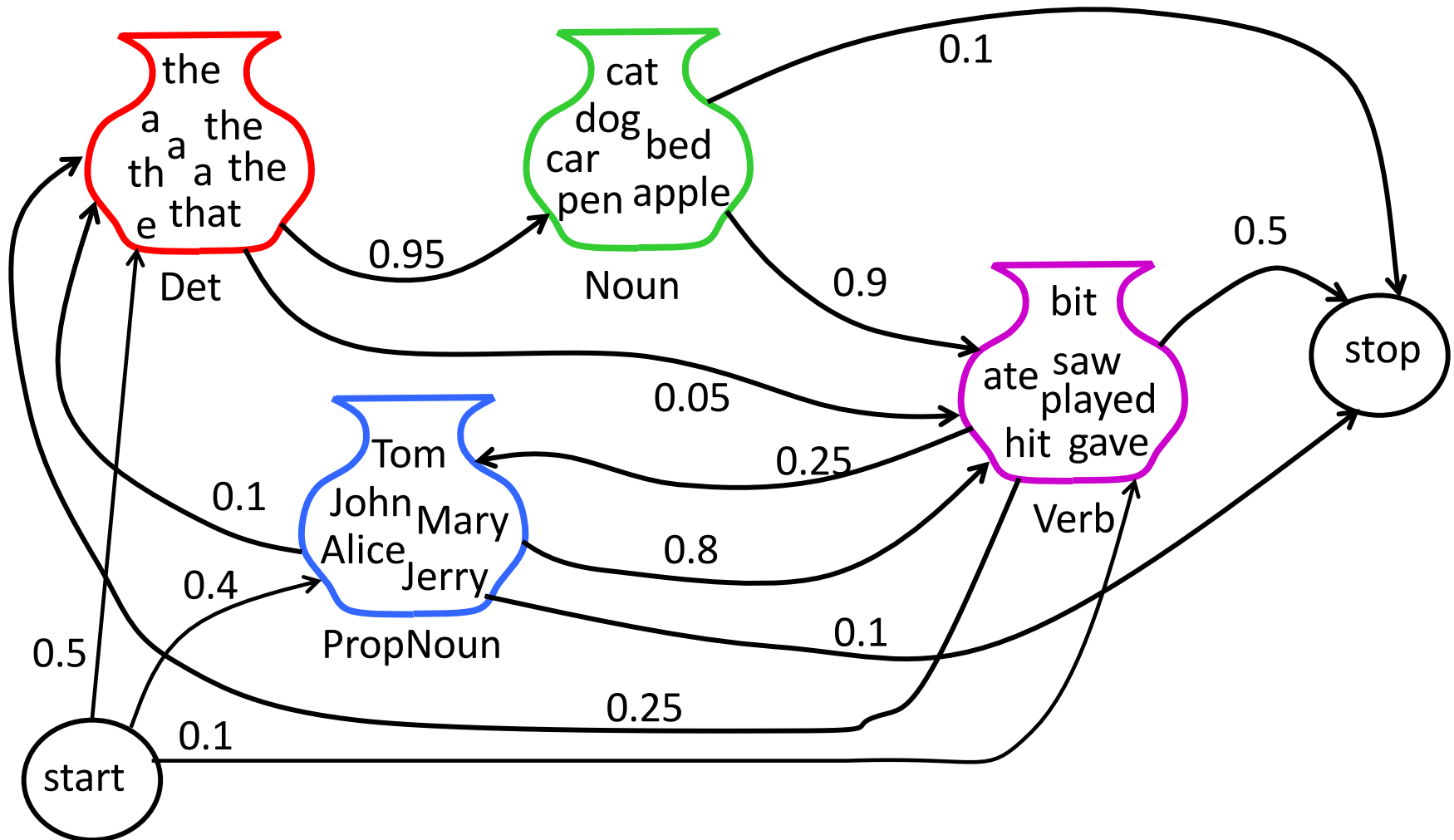
# Sample Markov Model for POS



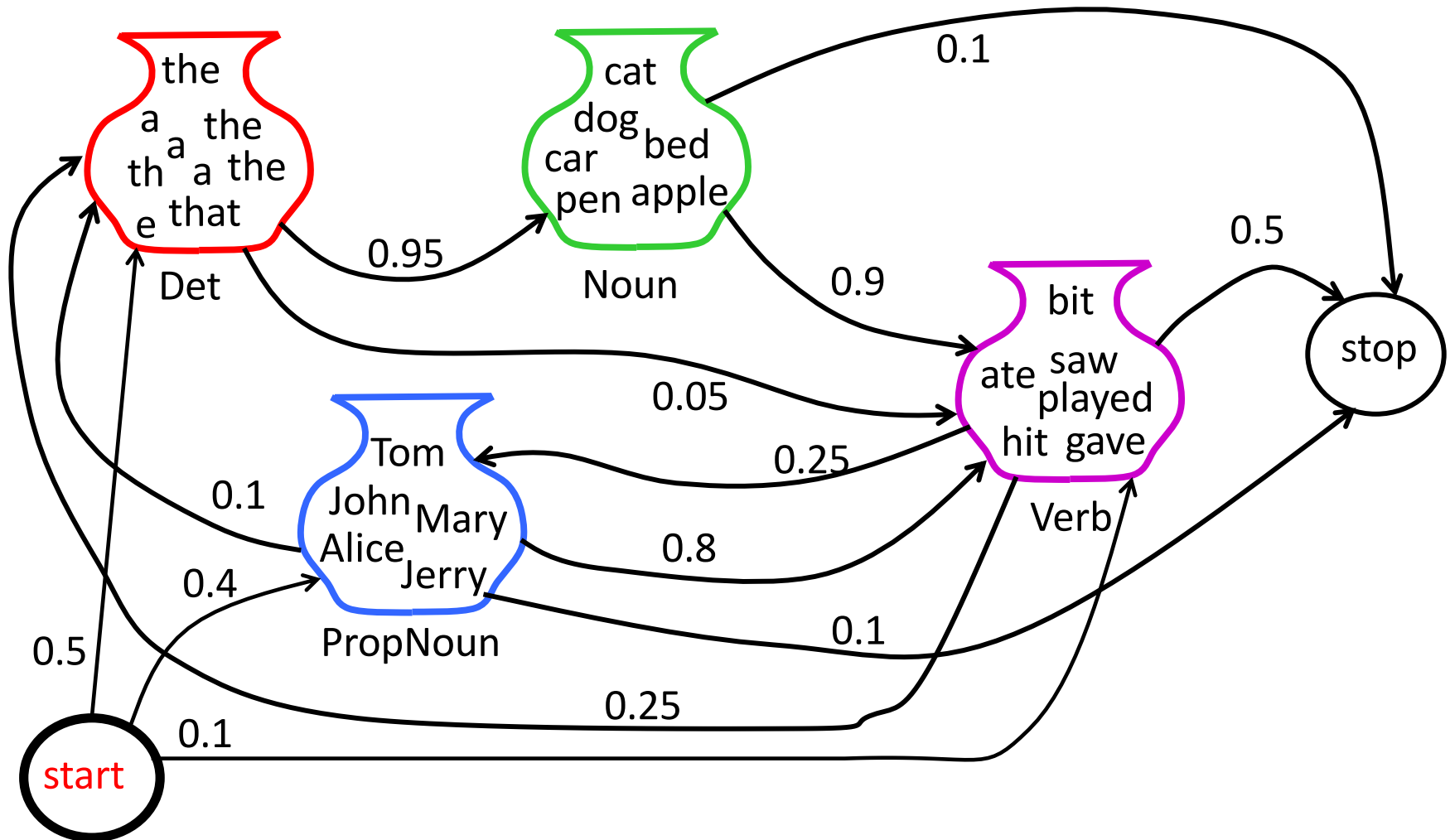
# Hidden Markov Model

- Probabilistic generative model for sequences.
- Assume an underlying set of *hidden* (unobserved) states in which the model can be (e.g. part-of-speech).
- Assume probabilistic transitions between states over time (e.g. transition from POS to another POS as sequence is generated).
- Assume a *probabilistic* generation of tokens from states (e.g. words generated for each POS).

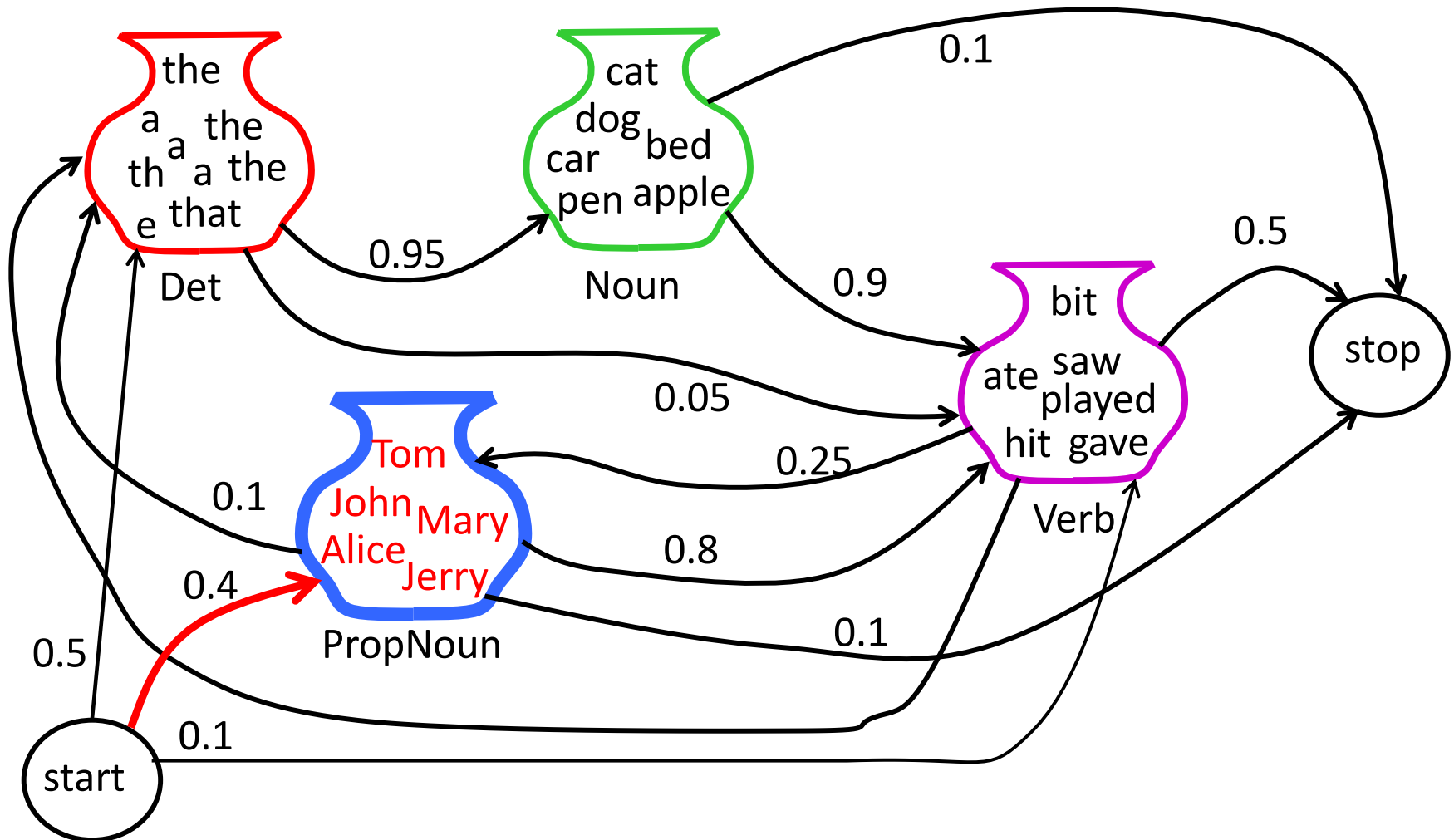
# Sample HMM for POS



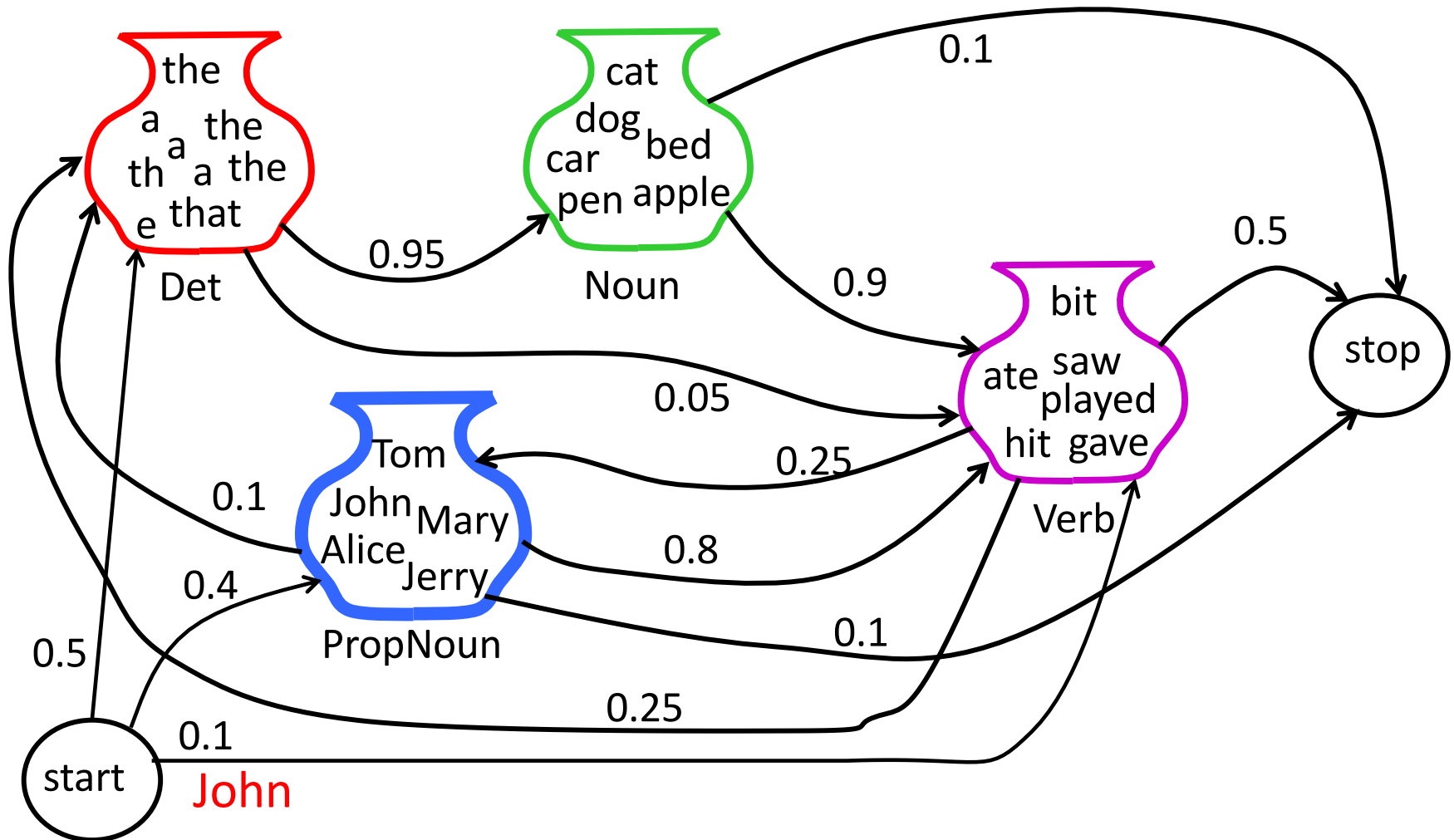
# Sample HMM Generation



# Sample HMM Generation

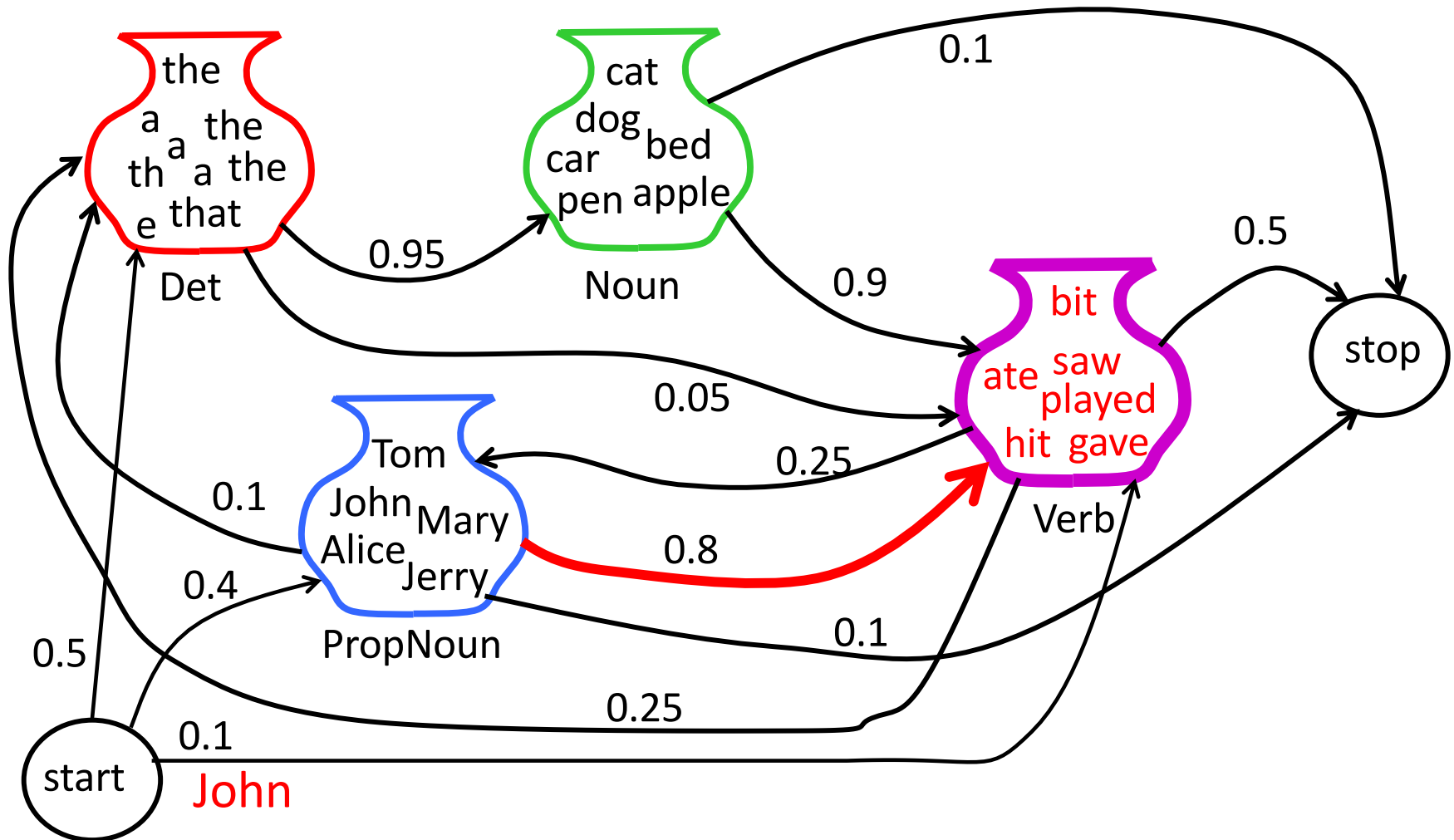


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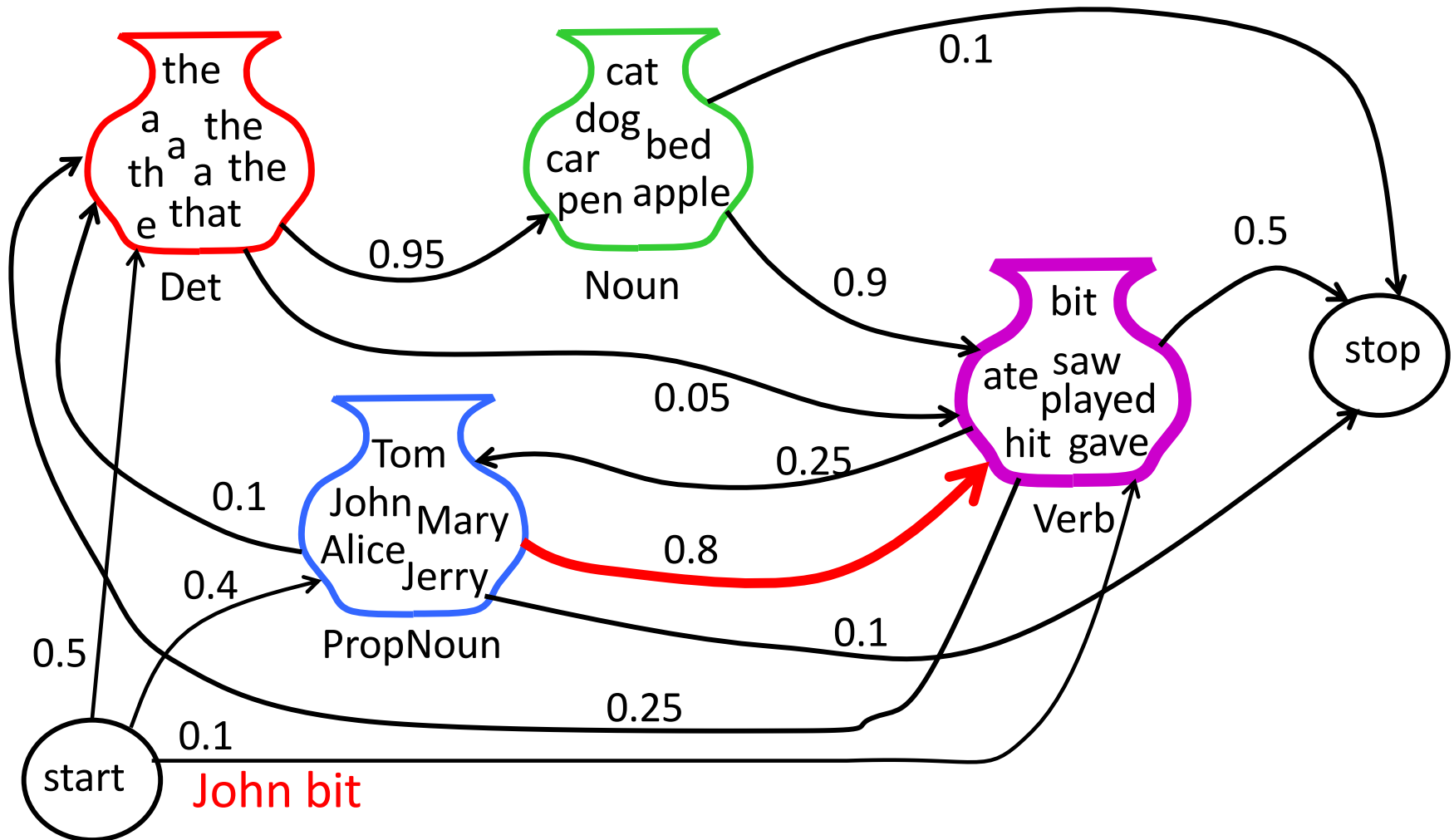




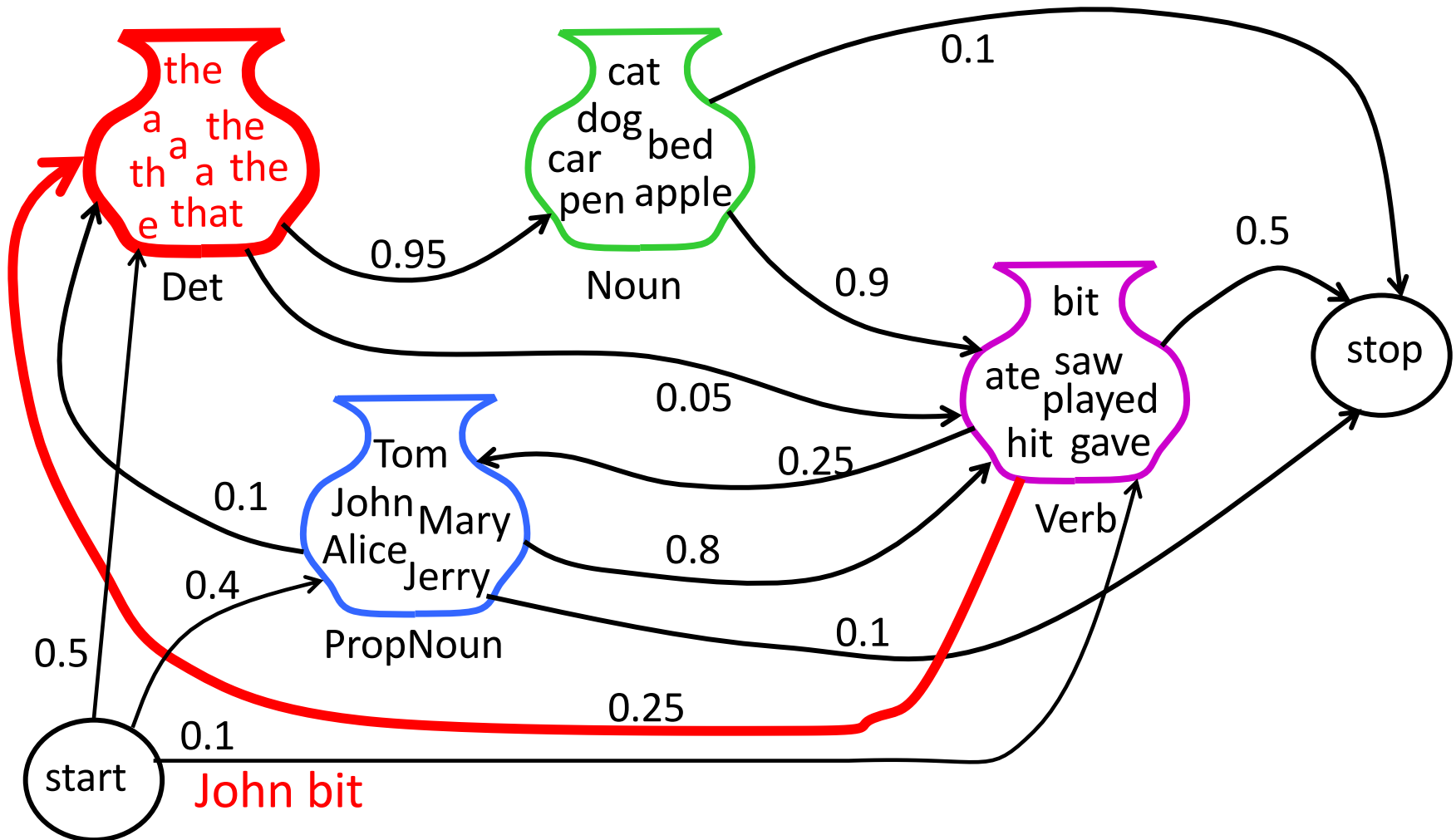
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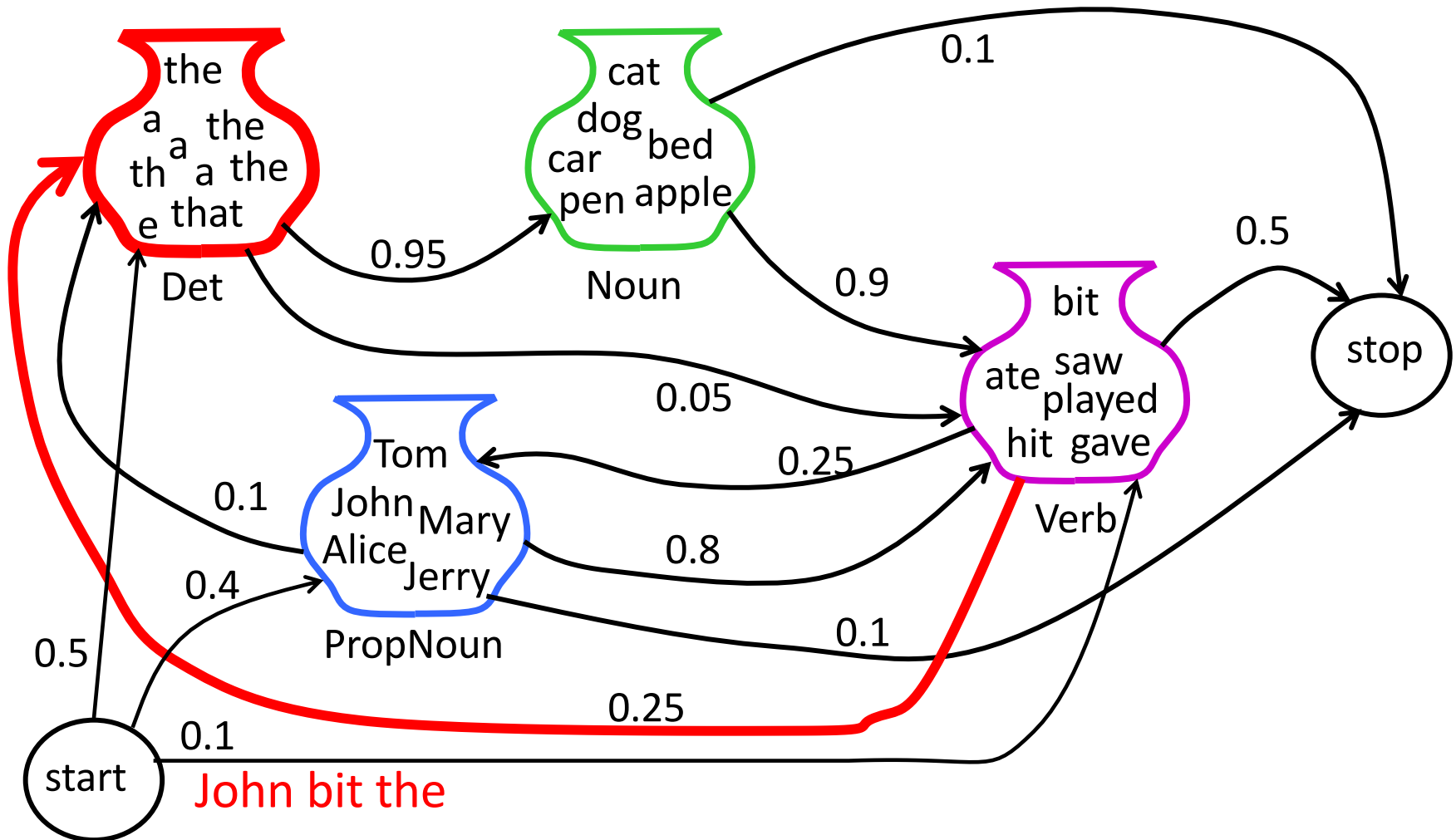
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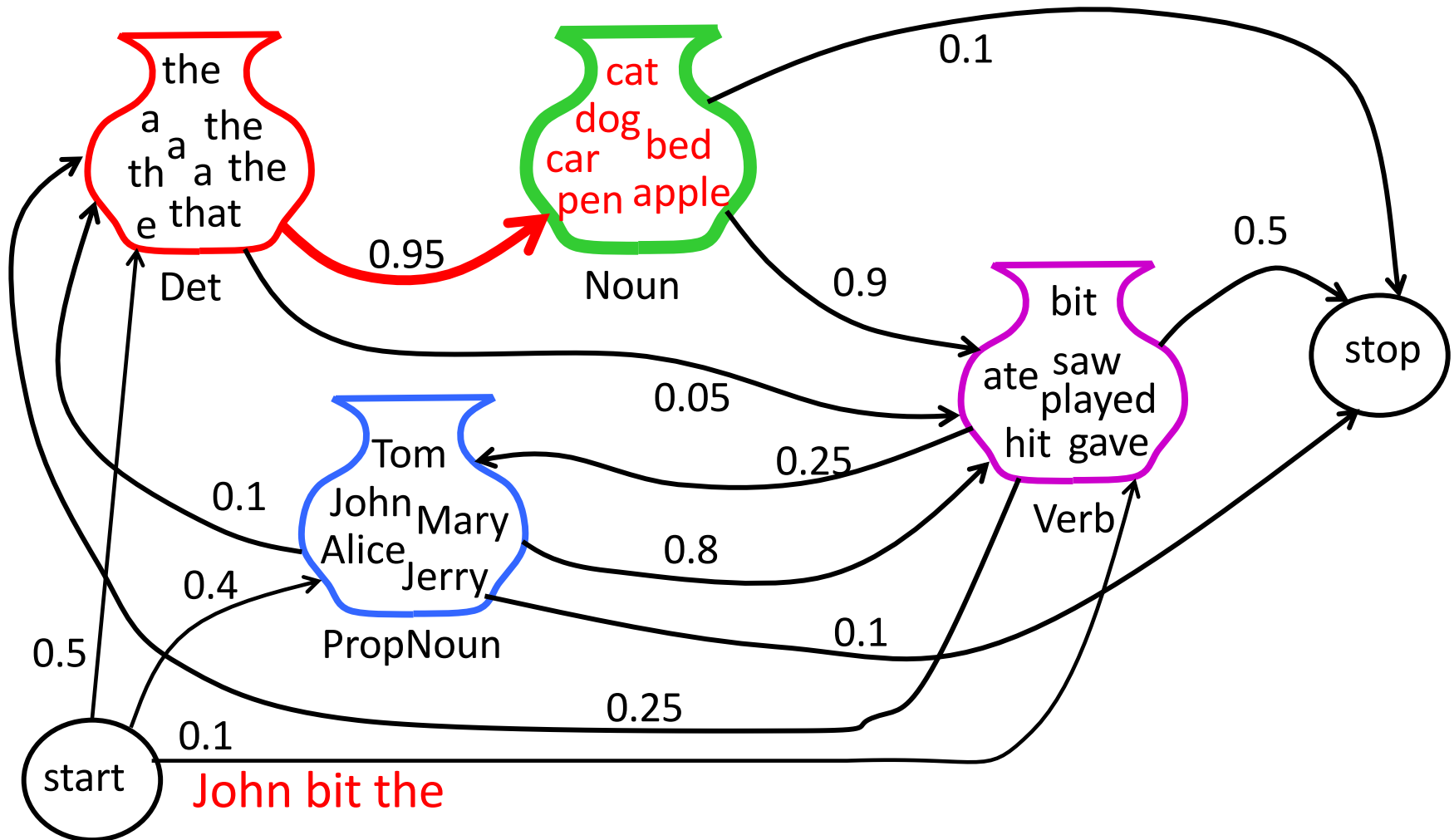
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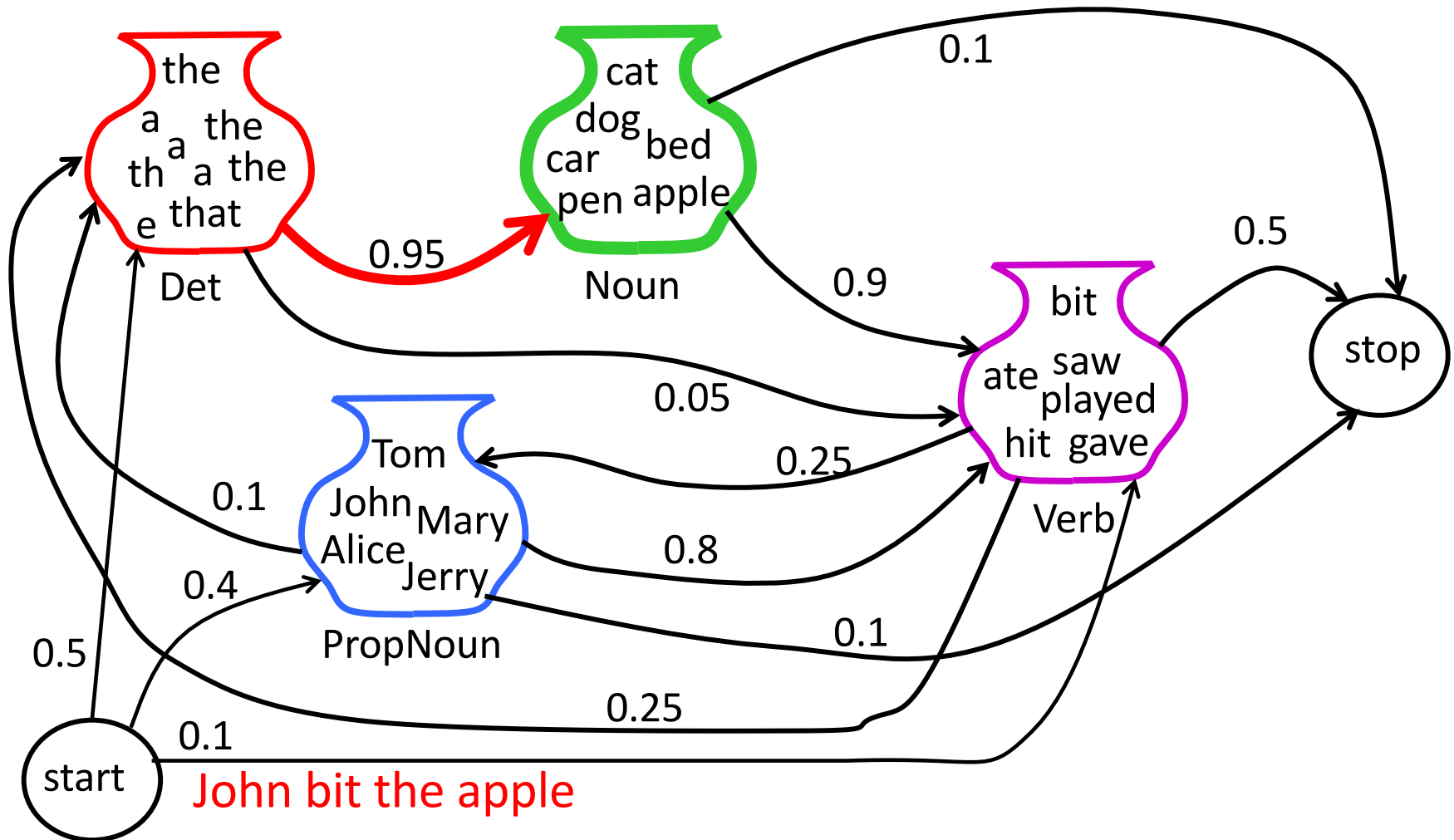
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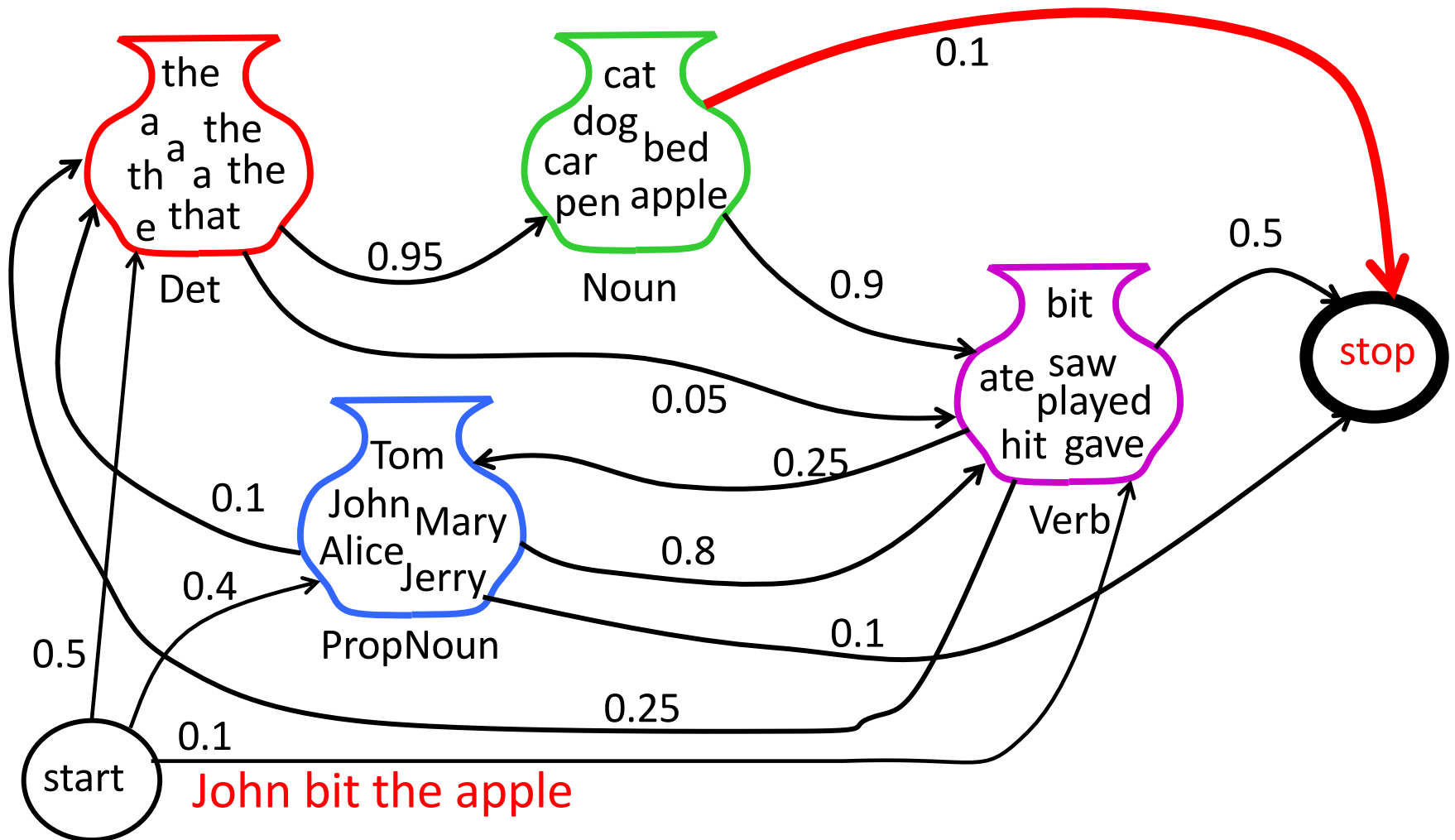
# Sample HMM Generation



# Sample HMM Generation



# Sample HMM Generation



# Formally, Markov Sequences

- ▶ Consider a sequence of random variables  $X_1, X_2, \dots, X_m$  where  $m$  is the length of the sequence
- ▶ Each variable  $X_i$  can take any value in  $\{1, 2, \dots, k\}$
- ▶ How do we model the joint distribution

$$P(X_1 = x_1, X_2 = x_2, \dots, X_m = x_m)$$



# The Markov Assumption

$$\begin{aligned} & P(X_1 = x_1, X_2 = x_2, \dots, X_m = x_m) \\ = & P(X_1 = x_1) \prod_{j=2}^m P(X_j = x_j | X_1 = x_1, \dots, X_{j-1} = x_{j-1}) \\ = & P(X_1 = x_1) \prod_{j=2}^m P(X_j = x_j | X_{j-1} = x_{j-1}) \end{aligned}$$

- ▶ The first equality is exact (by the chain rule).
- ▶ The second equality follows from *the Markov assumption*: for all  $j = 2 \dots m$ ,

$$P(X_j = x_j | X_1 = x_1, \dots, X_{j-1} = x_{j-1}) = P(X_j = x_j | X_{j-1} = x_{j-1})$$

# Homogeneous Markov Chains

- ▶ In a *homogeneous* Markov chain, we make an additional assumption, that for  $j = 2 \dots m$ ,

$$P(X_j = x_j | X_{j-1} = x_{j-1}) = q(x_j | x_{j-1})$$

where  $q(x'|x)$  is some function

- ▶ Idea behind this assumption: the transition probabilities do not depend on the position in the Markov chain (do not depend on the index  $j$ )

# Markov Models

- ▶ Our model is then as follows:

$$p(x_1, x_2, \dots, x_m; \underline{\theta}) = q(x_1) \prod_{j=2}^m q(x_j | x_{j-1})$$

- ▶ Parameters in the model:

- ▶  $q(x)$  for  $x = \{1, 2, \dots, k\}$   
Constraints:  $q(x) \geq 0$  and  $\sum_{x=1}^k q(x) = 1$
- ▶  $q(x' | x)$  for  $x = \{1, 2, \dots, k\}$  and  $x' = \{1, 2, \dots, k\}$   
Constraints:  $q(x' | x) \geq 0$  and  $\sum_{x'=1}^k q(x' | x) = 1$

# Probabilistic Models for Sequence Pairs

- ▶ We have two sequences of random variables:  
 $X_1, X_2, \dots, X_m$  and  $S_1, S_2, \dots, S_m$
- ▶ Intuitively, each  $X_i$  corresponds to an “observation” and each  $S_i$  corresponds to an underlying “state” that generated the observation. Assume that each  $S_i$  is in  $\{1, 2, \dots, k\}$ , and each  $X_i$  is in  $\{1, 2, \dots, o\}$
- ▶ How do we model the joint distribution

$$P(X_1 = x_1, \dots, X_m = x_m, S_1 = s_1, \dots, S_m = s_m)$$

# Probabilistic Models for Sequence Pairs

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**Words**

**Part-of-Speech tags**

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$$P(X_1 = x_1, \dots, X_m = x_m, S_1 = s_1, \dots, S_m = s_m)$$

Firstly, why would we want to model the joint distribution?

$$P(X_1 = x_1, \dots, X_m = x_m, S_1 = s_1, \dots, S_m = s_m)$$

**Words**                      **Part-of-Speech tags**

# Supervised Learning Problems

- ▶ We have training examples  $x^{(i)}, y^{(i)}$  for  $i = 1 \dots m$ . Each  $x^{(i)}$  is an input, each  $y^{(i)}$  is a label.
- ▶ Task is to learn a function  $f$  mapping inputs  $x$  to labels  $f(x)$

# Generative Models

- ▶ We have training examples  $x^{(i)}, y^{(i)}$  for  $i = 1 \dots m$ . Task is to learn a function  $f$  mapping inputs  $x$  to labels  $f(x)$ .
- ▶ Generative models:
  - ▶ Learn a distribution  $p(x, y)$  from training examples
  - ▶ Often we have  $p(x, y) = p(y)p(x|y)$
- ▶ Note: we then have

$$p(y|x) = \frac{p(y)p(x|y)}{p(x)}$$

where  $p(x) = \sum_y p(y)p(x|y)$



# Prediction with Generative Models

- ▶ We have training examples  $x^{(i)}, y^{(i)}$  for  $i = 1 \dots m$ . Task is to learn a function  $f$  mapping inputs  $x$  to labels  $f(x)$ .
- ▶ Generative models:
  - ▶ Learn a distribution  $p(x, y)$  from training examples
  - ▶ Often we have  $p(x, y) = p(y)p(x|y)$
- ▶ Output from the model:

$$\begin{aligned} f(x) &= \arg \max_y p(y|x) \\ &= \arg \max_y \frac{p(y)p(x|y)}{p(x)} \\ &= \arg \max_y p(y)p(x|y) \end{aligned}$$

# Probabilistic Models for Sequence Pairs

- ▶ We have two sequences of random variables:

$X_1, X_2, \dots, X_m$  and  $S_1, S_2, \dots, S_m$

**Words**

**Part-of-Speech tags**

- ▶ Intuitively, each  $X_i$  corresponds to an “observation” and each  $S_i$  corresponds to an underlying “state” that generated the observation. Assume that each  $S_i$  is in  $\{1, 2, \dots, k\}$ , and each  $X_i$  is in  $\{1, 2, \dots, o\}$
- ▶ How do we model the joint distribution

$$P(X_1 = x_1, \dots, X_m = x_m, S_1 = s_1, \dots, S_m = s_m)$$

# Hidden Markov Models (HMMs)

- ▶ In HMMs, we assume that:

**Words**

**Part-of-Speech tags**

$$\begin{aligned} &P(X_1 = x_1, \dots, X_m = x_m, S_1 = s_1, \dots, S_m = s_m) \\ &= P(S_1 = s_1) \prod_{j=2}^m P(S_j = s_j | S_{j-1} = s_{j-1}) \prod_{j=1}^m P(X_j = x_j | S_j = s_j) \end{aligned}$$

# Independence Assumptions in HMMs

- ▶ By the chain rule, the following equality is exact:

$$\begin{aligned} & P(X_1 = x_1, \dots, X_m = x_m, S_1 = s_1, \dots, S_m = s_m) \\ &= P(S_1 = s_1, \dots, S_m = s_m) \times \\ & \quad P(X_1 = x_1, \dots, X_m = x_m | S_1 = s_1, \dots, S_m = s_m) \end{aligned}$$

- ▶ Assumption 1: the state sequence forms a Markov chain

**e.g. Part-of-Speech tags**

$$P(S_1 = s_1, \dots, S_m = s_m) = P(S_1 = s_1) \prod_{j=2}^m P(S_j = s_j | S_{j-1} = s_{j-1})$$

- ▶ By the chain rule, the following equality is exact:

$$\begin{aligned} & P(X_1 = x_1, \dots, X_m = x_m | S_1 = s_1, \dots, S_m = s_m) \\ = & \prod_{j=1}^m P(X_j = x_j | S_1 = s_1, \dots, S_m = s_m, X_1 = x_1, \dots, X_{j-1} = x_j) \end{aligned}$$

- ▶ Assumption 2: each observation depends only on the underlying state

$$\begin{aligned} & P(X_j = x_j | S_1 = s_1, \dots, S_m = s_m, X_1 = x_1, \dots, X_{j-1} = x_j) \\ = & P(X_j = x_j | S_j = s_j) \end{aligned}$$

# Formally

- ▶ The model takes the following form:

$$p(x_1 \dots x_m, s_1 \dots s_m; \underline{\theta}) = t(s_1) \prod_{j=2}^m t(s_j | s_{j-1}) \prod_{j=1}^m e(x_j | s_j)$$

- ▶ Parameters in the model:

1. Initial state parameters  $t(s)$  for  $s \in \{1, 2, \dots, k\}$
2. Transition parameters  $t(s' | s)$  for  $s, s' \in \{1, 2, \dots, k\}$
3. Emission parameters  $e(x | s)$  for  $s \in \{1, 2, \dots, k\}$  and  $x \in \{1, 2, \dots, o\}$

# HMM

- Parameter estimation
  - Learning the probabilities from training data
  - $P(\text{verb} | \text{noun})?$ ,  $P(\text{apple} | \text{noun})?$
- Inference: Viterbi algorithm (dynamic programming)
  - Given a new sentence, what are the POS tags for the words?

# HMM

- Parameter estimation
- Inference: Viterbi algorithm (dynamic programming)



# Parameter Estimation with Fully Observed Data

- ▶ We'll now discuss parameter estimates in the case of *fully observed data*: for  $i = 1 \dots n$ , we have pairs of sequences  $x_{i,j}$  for  $j = 1 \dots m$  and  $s_{i,j}$  for  $j = 1 \dots m$ . (i.e., we have  $n$  training examples, each of length  $m$ .)

# Parameter Estimation: Transition Parameters

- $P(\text{verb} | \text{noun})?$

- ▶ Assume we have fully observed data: for  $i = 1 \dots n$ , we have pairs of sequences  $x_{i,j}$  for  $j = 1 \dots m$  and  $s_{i,j}$  for  $j = 1 \dots m$
- ▶ Define  $\text{count}(i, s \rightarrow s')$  to be the number of times state  $s'$  follows state  $s$  in the  $i$ 'th training example. More formally:

$$\text{count}(i, s \rightarrow s') = \sum_{j=1}^{m-1} [[s_{i,j} = s \wedge s_{i,j+1} = s']]$$

(We define  $[[\pi]]$  to be 1 if  $\pi$  is true, 0 otherwise.)

- ▶ The maximum-likelihood estimates of transition probabilities are then

$$t(s'|s) = \frac{\sum_{i=1}^n \text{count}(i, s \rightarrow s')}{\sum_{i=1}^n \sum_{s'} \text{count}(i, s \rightarrow s')}$$

# Parameter Estimation: Emission Parameters

- $P(\text{apple} | \text{noun})?$

- ▶ Assume we have fully observed data: for  $i = 1 \dots n$ , we have pairs of sequences  $x_{i,j}$  for  $j = 1 \dots m$  and  $s_{i,j}$  for  $j = 1 \dots m$
- ▶ Define  $\text{count}(i, s \rightsquigarrow x)$  to be the number of times state  $s$  is paired with emission  $x$ . More formally:

$$\text{count}(i, s \rightsquigarrow x) = \sum_{j=1}^m [[s_{i,j} = s \wedge x_{i,j} = x]]$$

- ▶ The maximum-likelihood estimates of emission probabilities are then

$$e(x|s) = \frac{\sum_{i=1}^n \text{count}(i, s \rightsquigarrow x)}{\sum_{i=1}^n \sum_x \text{count}(i, s \rightsquigarrow x)}$$

# Parameter Estimation: Initial State Parameters

- ▶ Assume we have fully observed data: for  $i = 1 \dots n$ , we have pairs of sequences  $x_{i,j}$  for  $j = 1 \dots m$  and  $s_{i,j}$  for  $j = 1 \dots m$
- ▶ Define  $\text{count}(i, s)$  to be 1 if state  $s$  is the initial state in the sequence, and 0 otherwise:

$$\text{count}(i, s) = [[s_{i,1} = s]]$$

- ▶ The maximum-likelihood estimates of initial state probabilities are:

$$t(s) = \frac{\sum_{i=1}^n \text{count}(i, s)}{n}$$

# HMM

- Parameter estimation
- Inference: Viterbi algorithm (dynamic programming)

# The Viterbi Algorithm

- ▶ Goal: for a given input sequence  $x_1, \dots, x_m$ , find

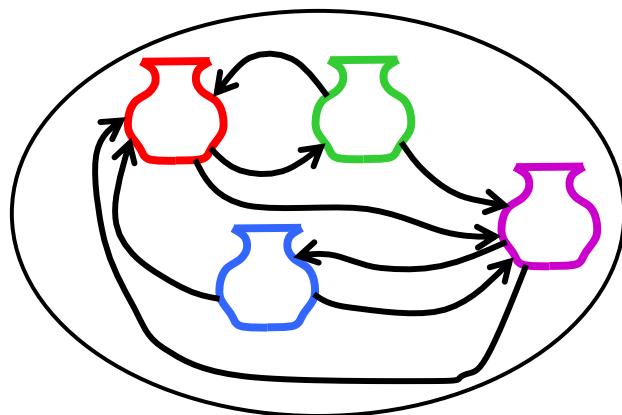
$$\arg \max_{s_1, \dots, s_m} p(x_1 \dots x_m, s_1 \dots s_m; \underline{\theta})$$

- ▶ This is the most likely state sequence  $s_1 \dots s_m$  for the given input sequence  $x_1 \dots x_m$



# Most Likely State Sequence

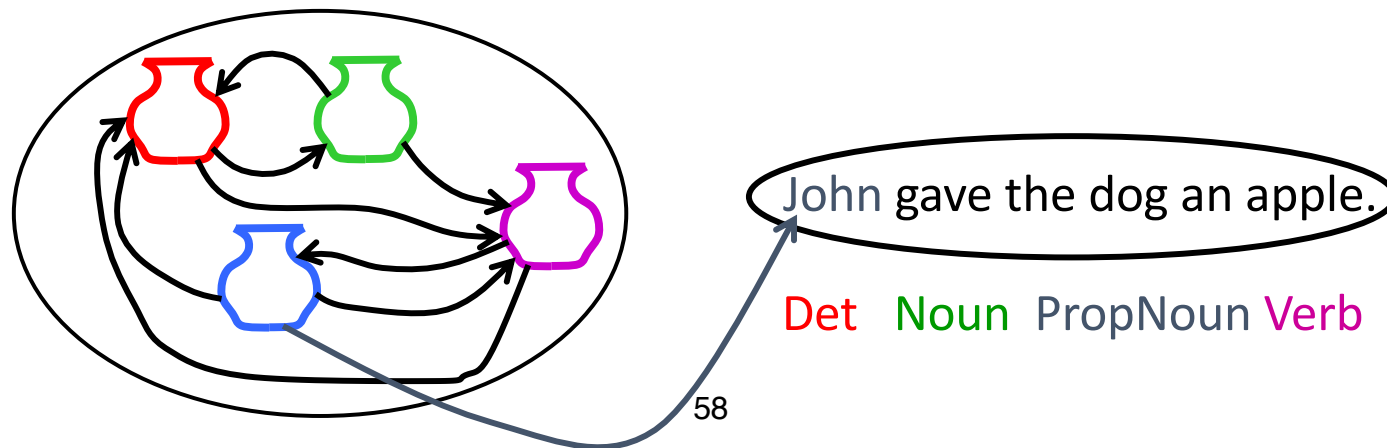
- Given an observation sequence,  $X$ , and a model, what is the most likely state sequence,  $S=s_1, s_2, \dots, s_m$ , that generated this sequence from this model?
- Used for sequence labeling, assuming each state corresponds to a tag, it determines the globally best assignment of tags to all tokens in a sequence using a principled approach grounded in probability theory.



John gave the dog an apple.

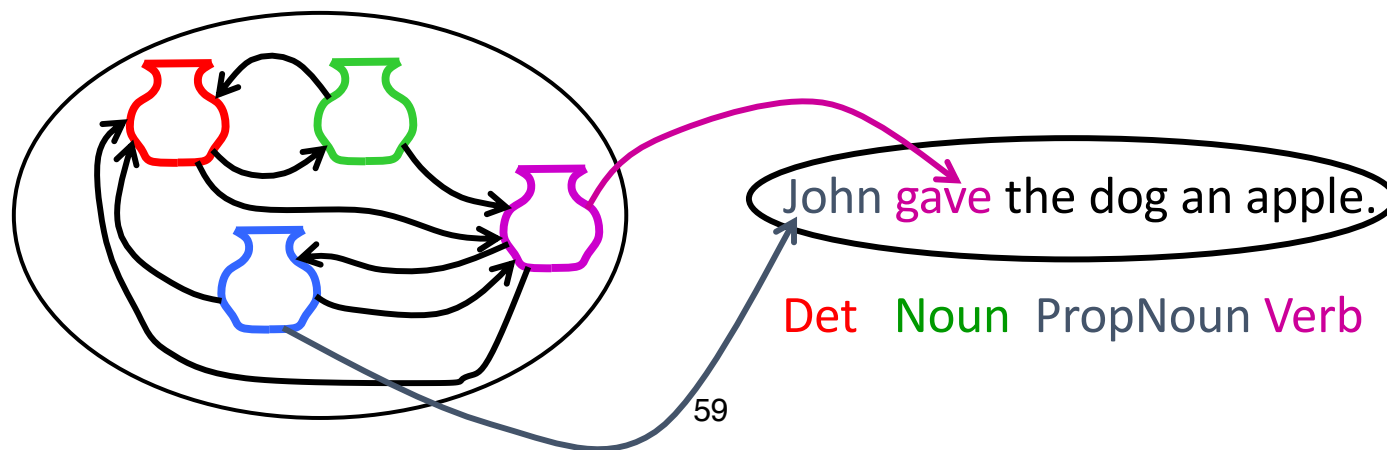
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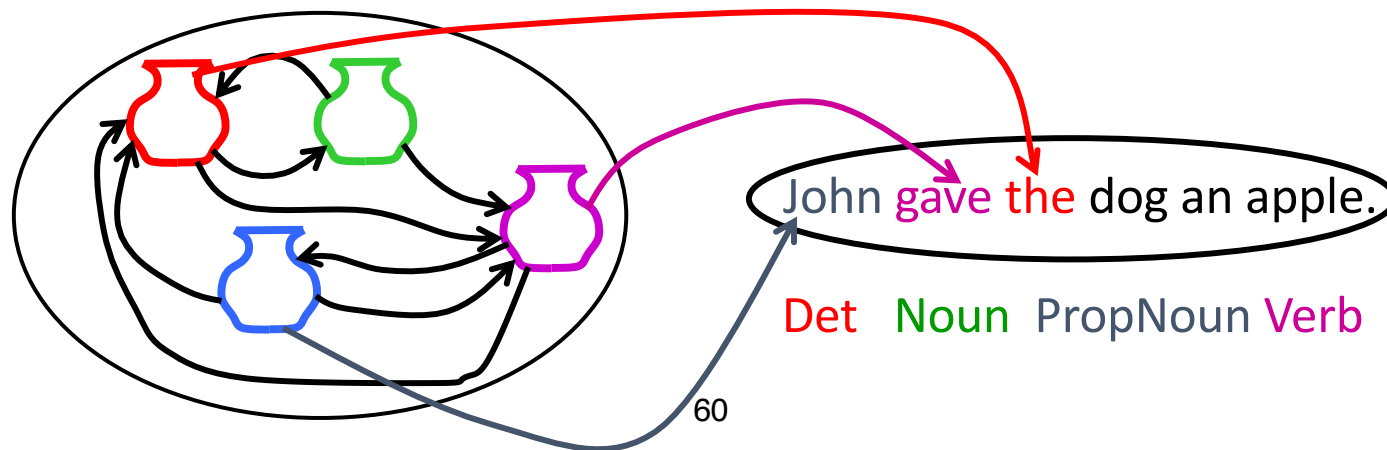
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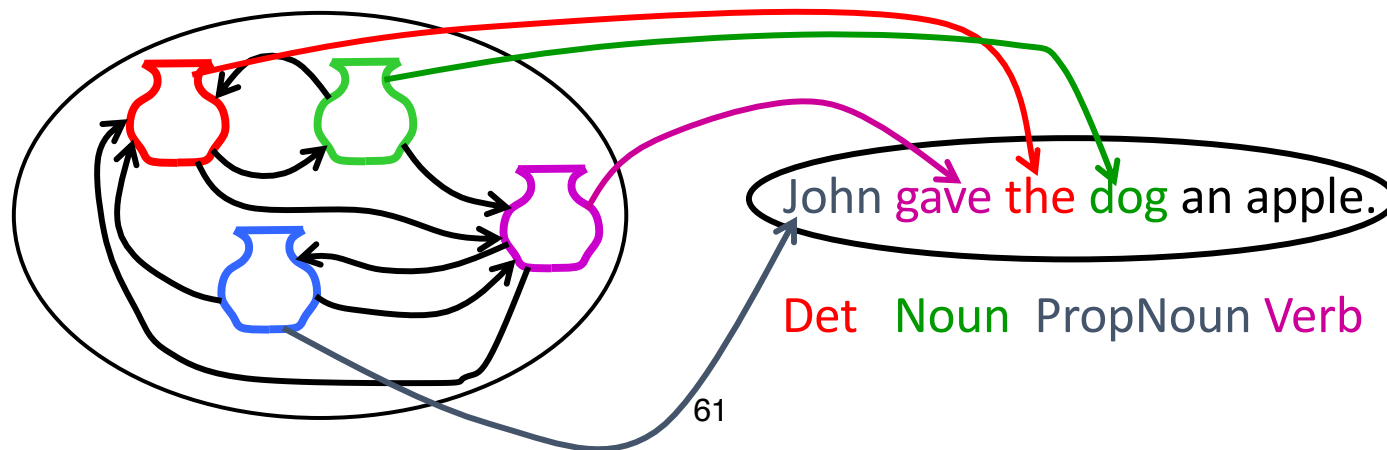
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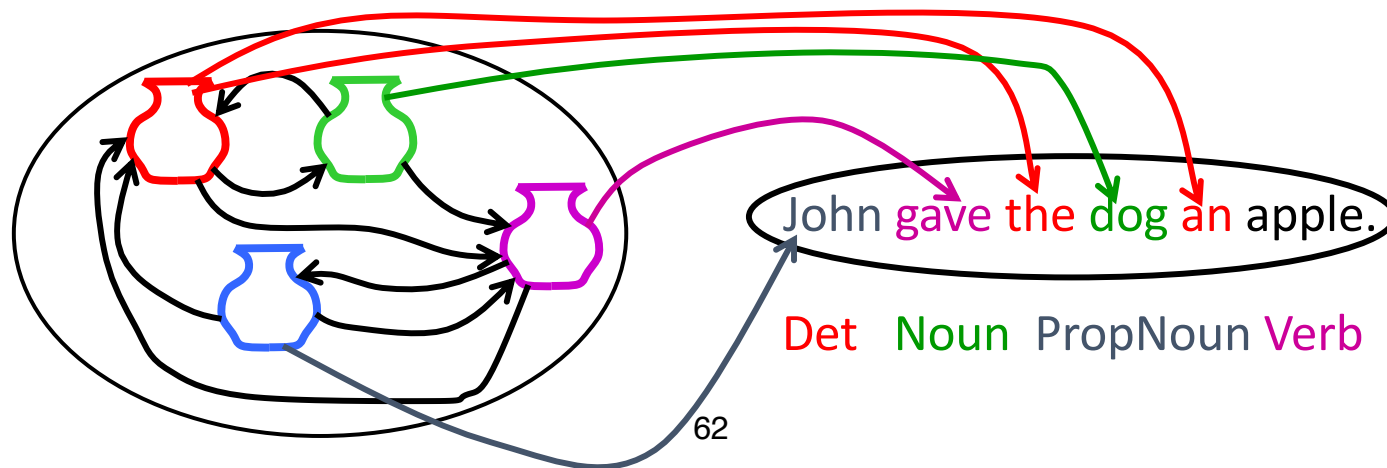
# Most Likely State Sequence

- Given an observation sequence,  $X$ , and a model, what is the most likely state sequence,  $S=s_1, s_2, \dots, s_m$ , that generated this sequence from this model?
- Used for sequence labeling, assuming each state corresponds to a tag, it determines the globally best assignment of tags to all tokens in a sequence using a principled approach grounded in probability theory.



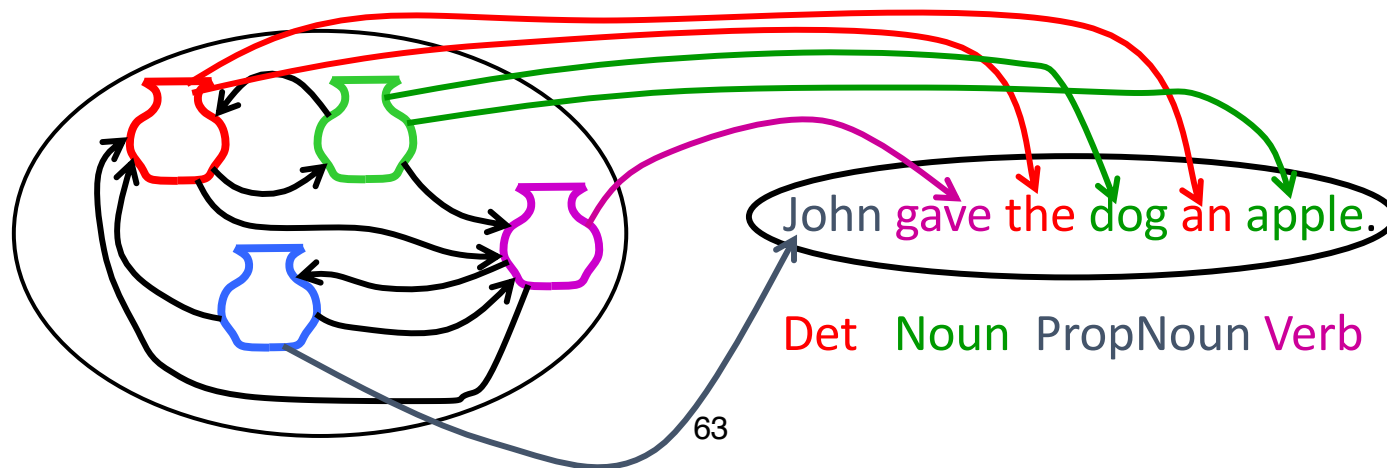
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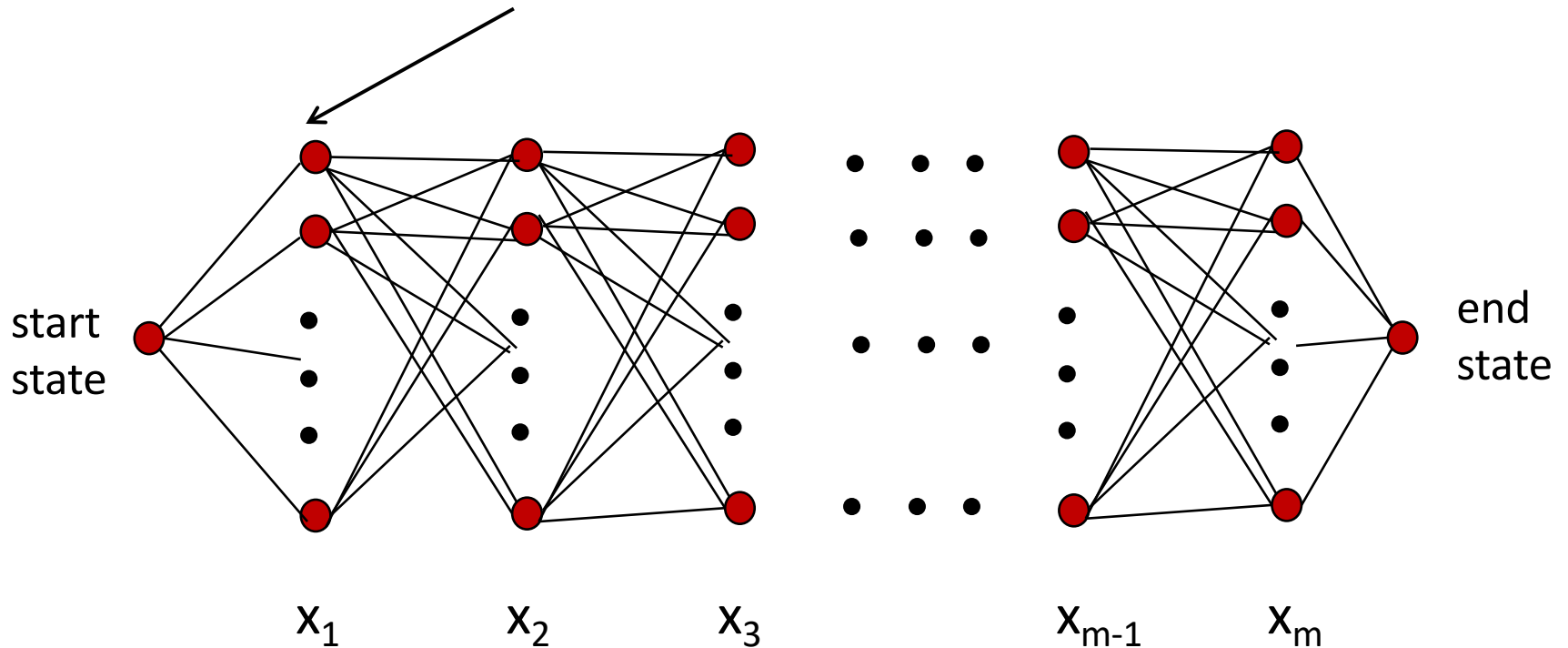


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All possible states (e.g., pos tags)



- Continue forward in time until reaching final time point.
- **The goal:** find a path with highest probability



# The Viterbi Algorithm

- ▶ Goal: for a given input sequence  $x_1, \dots, x_m$ , find

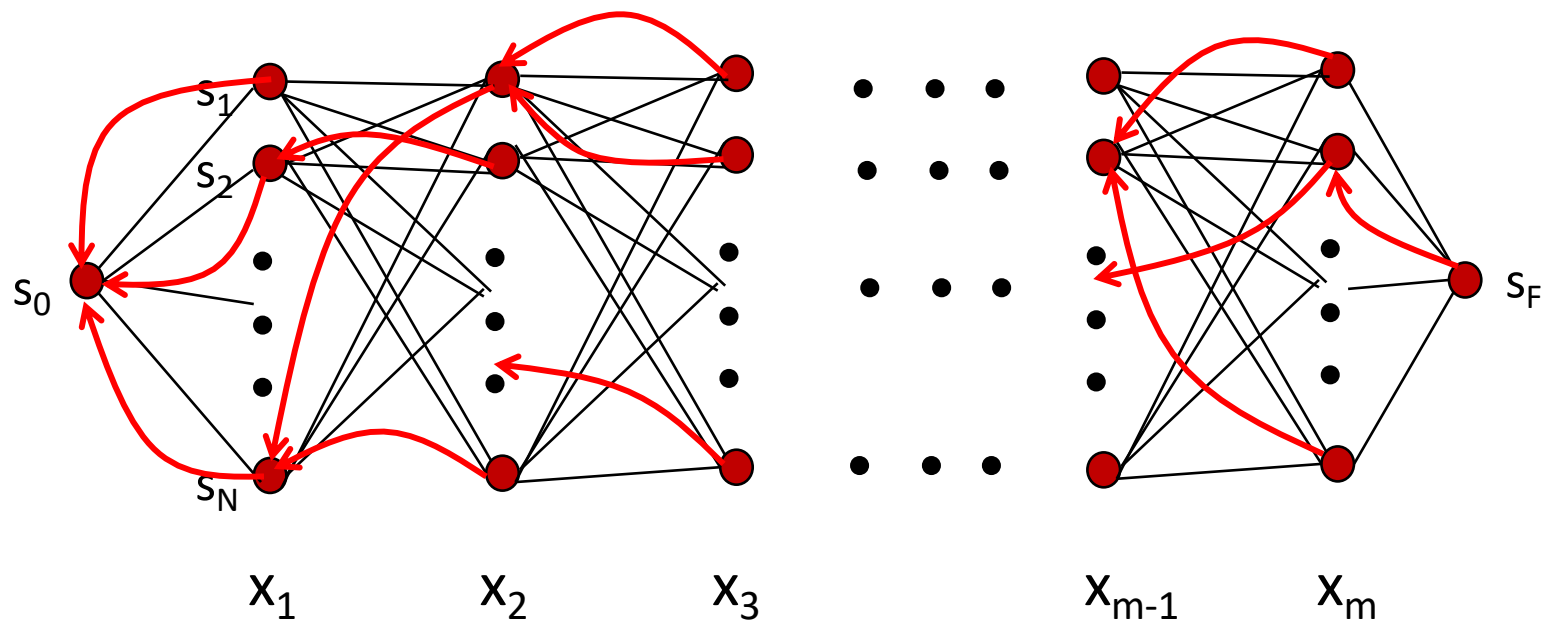
$$\arg \max_{s_1, \dots, s_m} p(x_1 \dots x_m, s_1 \dots s_m; \underline{\theta})$$

- ▶ The *Viterbi algorithm* is a dynamic programming algorithm.  
Basic data structure:

$$\pi[j, s]$$

will be a table entry that stores the maximum probability for any state sequence ending in state  $s$  at position  $j$ . More formally:  $\pi[1, s] = t(s)e(x_1|s)$ , and for  $j > 1$ ,

$$\pi[j, s] = \max_{s_1 \dots s_{j-1}} \left[ t(s_1)e(x_1|s_1) \left( \prod_{k=2}^{j-1} \underset{\substack{\text{Transition} \\ \downarrow}}{t(s_k|s_{k-1})} \underset{\substack{\text{Emission} \\ \downarrow}}{e(x_k|s_k)}} \right) \underset{\substack{\text{State } s \\ \downarrow}}{t(s|s_{j-1})} \underset{\substack{\text{Emission} \\ \text{from} \\ \text{state } s \\ \downarrow}}{e(x_j|s)} \right]$$



# The Viterbi Algorithm

- ▶ Initialization: for  $s = 1 \dots k$

$$\pi[1, s] = t(s)e(x_1|s)$$

- ▶ For  $j = 2 \dots m$ ,  $s = 1 \dots k$ :

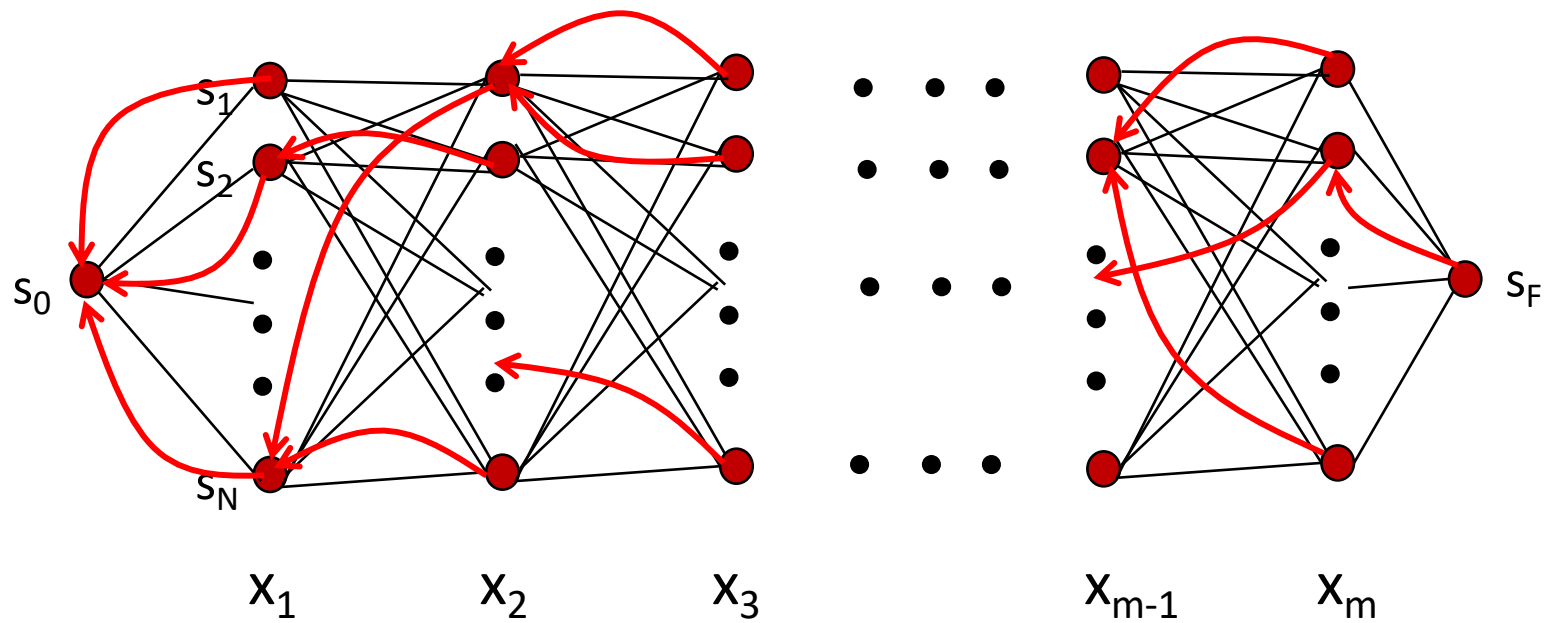
$$\pi[j, s] = \max_{s' \in \{1 \dots k\}} [\pi[j - 1, s'] \times t(s|s') \times e(x_j|s)]$$

- ▶ We then have

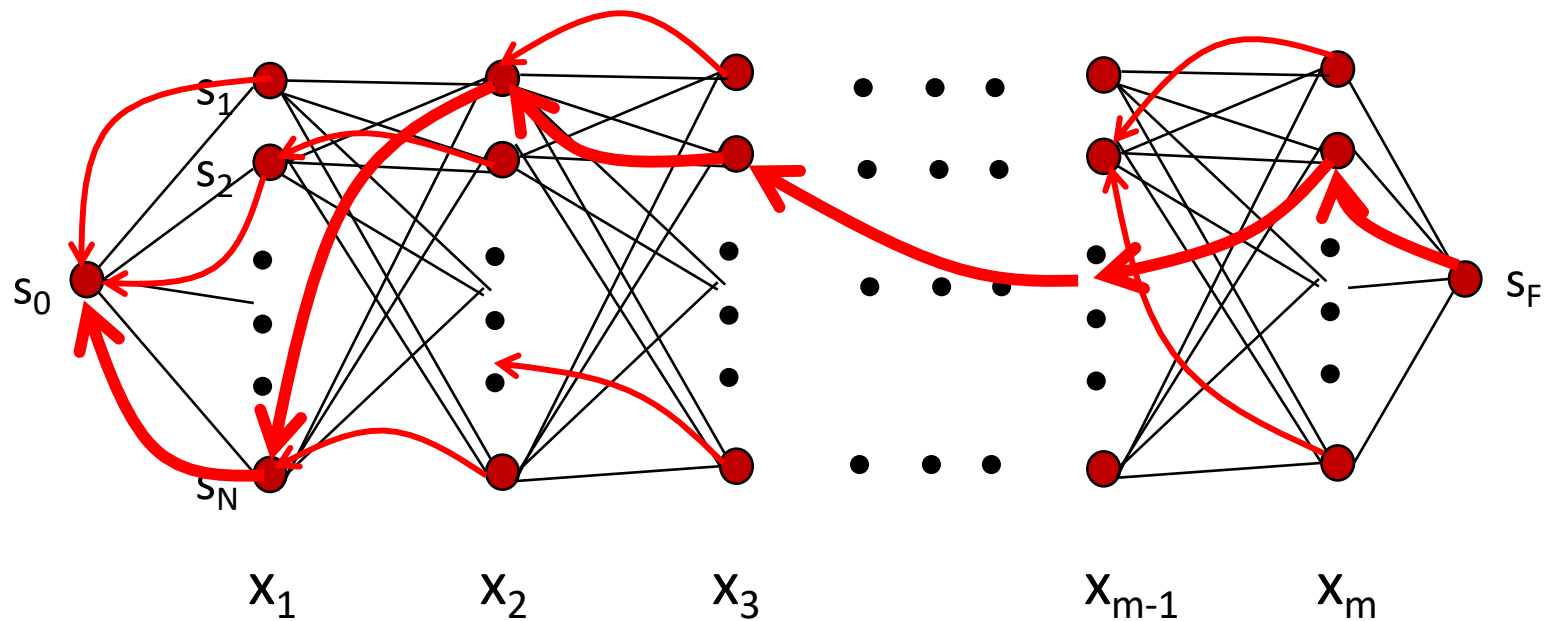
$$\max_{s_1 \dots s_m} p(x_1 \dots x_m, s_1 \dots s_m; \underline{\theta}) = \max_s \pi[m, s]$$

- ▶ The algorithm runs in  $O(mk^2)$  time

# Viterbi Backpointers



# Viterbi Backtrace



Most likely Sequence:  $s_0 s_N s_1 s_2 \dots s_2 s_F$

# The Viterbi Algorithm: Backpointers

- ▶ Initialization: for  $s = 1 \dots k$

$$\pi[1, s] = t(s)e(x_1|s)$$

- ▶ For  $j = 2 \dots m$ ,  $s = 1 \dots k$ :

$$\pi[j, s] = \max_{s' \in \{1 \dots k\}} [\pi[j - 1, s'] \times t(s|s') \times e(x_j|s)]$$

and

$$bp[j, s] = \arg \max_{s' \in \{1 \dots k\}} [\pi[j - 1, s'] \times t(s|s') \times e(x_j|s)]$$

- ▶ The  $bp$  entries are backpointers that will allow us to recover the identity of the highest probability state sequence

- ▶ Highest probability for any sequence of states is

$$\max_s \pi[m, s]$$

- ▶ To recover identity of highest-probability sequence:

$$s_m = \arg \max_s \pi[m, s]$$

and for  $j = m \dots 2$ ,

$$s_{j-1} = bp[j, s_j]$$

- ▶ The sequence of states  $s_1 \dots s_m$  is then

$$\arg \max_{s_1, \dots, s_m} p(x_1 \dots x_m, s_1 \dots s_m; \underline{\theta})$$

# Homework

- Reading J&M ch5&6
- Reading ch6 at <https://web.stanford.edu/~jurafsky/slp3/6.pdf>
- HMM notes
  - <http://www.cs.columbia.edu/~mcollins/hmms-spring2013.pdf>
- Assignment 1 is out. Due Feb 6.
- Start thinking about course project and find a team.