CS 6120/CS4120: Natural Language Processing

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Outline

- Vector Semantics
- Sparse representation
  - Pointwise Mutual Information (PMI)
- Dense representation
  - Singular Value Decomposition (SVD)
  - Neural Language Model (Word2Vec) (next lecture)

Why vector models of meaning? computing the similarity between words

"**fast**" is similar to "**rapid**"

"**tall**" is similar to "**height**"

Question answering:

*Q: "How **tall** is Mt. Everest?"*

*Candidate A: "The official **height** of Mount Everest is 29029 feet"*

Beyond Dead Parrots

- Automatically constricted clusters of semantically similar words (Charniak, 1997):

Friday Monday Thursday Wednesday Tuesday Saturday Sunday
People guys folks fellows CEOs commies blocks
water gas cola liquid acid carbon steam shale
that the theat
head body hands eyes voice arm seat eye hair mouth

Smoothing for statistical language models

- Two alternative guesses of speech recognizer:
  - *For breakfast, she ate durian.*
  - *For breakfast, she ate Dorian.*
- Our corpus contains neither *"ate durian"* nor *"ate Dorian"*
- But, our corpus contains *"ate orange"*, *"ate banana"*

Distributional models of meaning = vector-space models of meaning = vector semantics

Intuitions: Zellig Harris (1954):

- "oculist and eye-doctor … occur in almost the same environments"
- "If A and B have almost identical environments we say that they are synonyms."

Firth (1957):

- "You shall know a word by the company it keeps!"

Intuition of distributional word similarity

- Example:
  - What is ***tesgüino?***

Intuition of distributional word similarity

- Example:

```
A bottle of tesgüino is on the table
Everybody likes tesgüino
Tesgüino makes you drunk
We make tesgüino out of corn.
```

- From context words humans can guess ***tesgüino*** means
  - an alcoholic beverage like **beer**
- Intuition for algorithm:
  - Two words are similar if they have similar word contexts.

Four kinds of vector models

Sparse vector representations

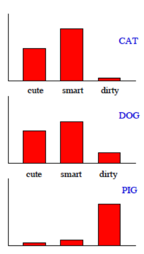
1. Mutual-information weighted word co-occurrence matrices

Dense vector representations:

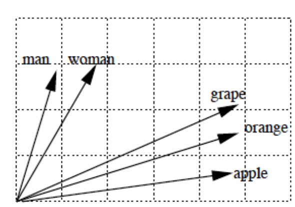
2. Singular value decomposition (and Latent Semantic Analysis)
3. Neural-network-inspired models (skip-grams, CBOW)
4. Brown clusters

Shared intuition

- Model the meaning of a word by "embedding" in a vector space.
- The meaning of a word is a vector of numbers
  - Vector models are also called "**embeddings**".



Sample Lexical Vector Space



Term-document matrix

- Each cell: count of term t in a document d: $tf_{t,d}$:
  - Each document is a count vector in $\mathbb{N}^v$: a column below

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	1	8	15
soldier	2	2	12	36
fool	37	58	1	5
clown	6	117	0	0

The words in a term-document matrix

- Each word is a count vector in $\mathbb{N}^D$: a row below

	As You Like It	Twelfth Night	Julius Caesar	Henry V
battle	1	1	8	15
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The words in a term-document matrix

- Two **words** are similar if their vectors are similar

	As You Like It	Twelfth Night	Julius Caesar	Henry V
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Term-context matrix for word similarity

- Two **words** are similar in meaning if their context vectors are similar

sugar, a sliced lemon, a tablespoonful of **apricot** preserve or jam, a pinch each of,

	aardvark	computer	data	pinch	result	sugar	…
apricot	0	0	0	1	0	1	
pineapple	0	0	0	1	0	1	
digital	0	2	1	0	1	0	
information	0	1	6	0	4	0	

The word-word matrix

- Instead of entire documents, use smaller contexts
  - Paragraph
  - Window of $\pm$ k (e.g. k=4) words
- A word is now defined by a vector over counts of context words
- Instead of each vector being of length D
- Each vector is now of length |V|
- The word-word matrix is |V|x|V|

Word-Word matrix
Sample contexts $\pm$ 7 words

sugar, a sliced lemon, a tablespoonful of	**apricot**	preserve or jam, a pinch each of,
their enjoyment. Cautiously she sampled her first	**pineapple**	and another fruit whose taste she likened
well suited to programming on the digital	**computer**.	In finding the optimal R-stage policy from
for the purpose of gathering data and	**information**	necessary for the study authorized in the

Sample Word-Word matrix

	aardvark	computer	data	pinch	result	sugar	…
apricot	0	0	0	1	0	1	
pineapple	0	0	0	1	0	1	
digital	0	2	1	0	1	0	
information	0	1	6	0	4	0	
…	…						

Word-word matrix

- We showed only 4x6, but the real matrix is 50,000 x 50,000
  - So it's very **sparse**
    - Most values are 0.
  - That's OK, since there are lots of efficient algorithms for sparse matrices.

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- We showed only 4x6, but the real matrix is 50,000 x 50,000
  - So it's very **sparse**
    - Most values are 0.
  - That's OK, since there are lots of efficient algorithms for sparse matrices.
- The size of windows depends on your goals
  - The shorter the windows , the more **syntactic** the representation
    - ± 1-3 very syntacticy
    - You may see playing is similar to cooking or singing, played is similar to cooked or sang
  - The longer the windows, the more **semantic** the representation
    - ± 4-10 more semanticy

Positive Pointwise Mutual Information (PPMI)

Problem with raw counts

- Raw word frequency is not a great measure of association between words
  - It's very skewed
    - "the" and "of" are very frequent, but maybe not the most discriminative

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- Raw word frequency is not a great measure of association between words
  - It's very skewed
    - "the" and "of" are very frequent, but maybe not the most discriminative
- We'd rather have a measure that asks whether a context word is **particularly informative** about the target word.
  - Positive Pointwise Mutual Information (PPMI)

Pointwise Mutual Information

Pointwise mutual information:

Do events x and y co-occur more than if they were independent?

$$\text{PMI}(X,Y) = \log_2 \frac{P(x,y)}{P(x)P(y)}$$

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PMI between two words: (Church & Hanks 1989)

Do words x and y co-occur more than if they were independent?

$$\text{PMI}(word_1, word_2) = \log_2 \frac{P(word_1, word_2)}{P(word_1)P(word_2)}$$

Positive Pointwise Mutual Information

- PMI ranges from $-\infty$ to $+\infty$
- But the negative values are problematic
  - Things are co-occurring **less than** we expect by chance
  - Unreliable without enormous corpora
    - Imagine w1 and w2 whose probability is each 10^{-6}
    - Hard to be sure p(w1,w2) is significantly different than 10^{-12}
  - Plus it's not clear people are good at "unrelatedness"

Positive Pointwise Mutual Information

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    - Imagine w1 and w2 whose probability is each 10^{-6}
    - Hard to be sure p(w1,w2) is significantly different than 10^{-12}
  - Plus it's not clear people are good at "unrelatedness"
- So we just replace negative PMI values by 0
- Positive PMI (PPMI) between word1 and word2:

$$\text{PPMI}(word_1, word_2) = \max\left(\log_2 \frac{P(word_1, word_2)}{P(word_1)P(word_2)}, 0\right)$$

Computing PPMI on a term-context matrix

- Matrix F with W rows (words) and C columns (contexts, e.g. in the form of words)
- f_{ij} is number of times w_i occurs in context c_j

	aardvark	computer	data	pinch	result	sugar
apricot	0	0	0	1	0	1
pineapple	0	0	0	1	0	1
digital	0	2	1	0	1	0
information	0	1	6	0	4	0

$$p_{ij} = \frac{f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}} \qquad p_{i*} = \frac{\sum_{j=1}^{C} f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}} \qquad p_{*j} = \frac{\sum_{i=1}^{W} f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}}$$

$$pmi_{ij} = \log_2 \frac{p_{ij}}{p_{i*}p_{*j}} \qquad ppmi_{ij} = \begin{cases} pmi_{ij} & \text{if } pmi_{ij} > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$p_{ij} = \frac{f_{ij}}{\sum_{i=1}^{W}\sum_{j=1}^{C} f_{ij}}$$

Count(w,context)

	computer	data	pinch	result	sugar
apricot	0	0	1	0	1
pineapple	0	0	1	0	1
digital	2	1	0	1	0
information	1	6	0	4	0

p(w=information,c=data) = 6/19 = .32

p(w=information) = 11/19 = .58

p(c=data) = 7/19 = .37

$$p(w_i) = \frac{\sum_{j=1}^{C} f_{ij}}{N} \qquad p(c_j) = \frac{\sum_{i=1}^{W} f_{ij}}{N}$$

	p(w,context)					**p(w)**
	computer	data	pinch	result	sugar	
apricot	0.00	0.00	0.05	0.00	0.05	0.11
pineapple	0.00	0.00	0.05	0.00	0.05	0.11
digital	0.11	0.05	0.00	0.05	0.00	0.21
information	0.05	0.32	0.00	0.21	0.00	0.58
p(context)	0.16	0.37	0.11	0.26	0.11	

$$pmi_{ij} = \log_2 \frac{p_{ij}}{p_{i*}p_{*j}}$$

p(w,context)

	computer	data	pinch	result	sugar	p(w)
apricot	0.00	0.00	0.05	0.00	0.05	0.11
pineapple	0.00	0.00	0.05	0.00	0.05	0.11
digital	0.11	0.05	0.00	0.05	0.00	0.21
information	0.05	0.32	0.00	0.21	0.00	0.58
p(context)	0.16	0.37	0.11	0.26	0.11	

- pmi(information,data) = $\log_2$ (.32 / (.37*.58)) = .57

PPMI(w,context)

	computer	data	pinch	result	sugar
apricot	-	-	2.25	-	2.25
pineapple	-	-	2.25	-	2.25
digital	1.66	0.00	-	0.00	-
information	0.00	0.57	-	0.47	-

Weighting PMI

- PMI is biased toward infrequent events
  - Very rare words have very high PMI values
- Two solutions:
  - Give rare words slightly higher probabilities
  - Use add-one (or k) smoothing (which has a similar effect)

Weighting PMI: Giving rare context words slightly higher probability

- Raise the context probabilities to $\alpha = 0.75$:

$$\text{PPMI}_\alpha(w,c) = \max(\log_2 \frac{P(w,c)}{P(w)P_\alpha(c)},0)$$

$$P_\alpha(c) = \frac{count(c)^\alpha}{\sum_c count(c)^\alpha}$$

- This helps because $P_\alpha(c) > P(c)$ for rare c
- Consider two events, P(a) = .99 and P(b)=.01 (here we use probability to show the effect)
- $P_\alpha(a) = \frac{.99^{.75}}{.99^{.75}+.01^{.75}} = .97 \quad P_\alpha(b) = \frac{.01^{.75}}{.99^{.75}+.01^{.75}} = .03$

Add-k smoothing

Add-2 Smoothed Count

	computer	data	pinch	result	sugar
apricot	2	2	3	2	3
pineapple	2	2	3	2	3
digital	4	3	2	3	2
information	3	8	2	6	2

p(w,context) [add-2]

	computer	data	pinch	result	sugar	p(w)
apricot	0.03	0.03	0.05	0.03	0.05	0.20
pineapple	0.03	0.03	0.05	0.03	0.05	0.20
digital	0.07	0.05	0.03	0.05	0.03	0.24
information	0.05	0.14	0.03	0.10	0.03	0.36
p(context)	0.19	0.25	0.17	0.22	0.17	

PPMI versus add-2 smoothed PPMI

PPMI(w,context)

	computer	data	pinch	result	sugar
apricot	-	-	2.25	-	2.25
pineapple	-	-	2.25	-	2.25
digital	1.66	0.00	-	0.00	-
information	0.00	0.57	-	0.47	-

PPMI(w,context) [add-2]

	computer	data	pinch	result	sugar
apricot	0.00	0.00	0.56	0.00	0.56
pineapple	0.00	0.00	0.56	0.00	0.56
digital	0.62	0.00	0.00	0.00	0.00
information	0.00	0.58	0.00	0.37	0.00

Measuring similarity

- Given 2 target words *v* and *w*
- We'll need a way to measure their similarity.
- Most measure of vectors similarity are based on the:
- **Dot product** or **inner product** from linear algebra (raw counts)

$$\text{dot-product}(\vec{v},\vec{w}) = \vec{v}\cdot\vec{w} = \sum_{i=1}^{N} v_i w_i = v_1 w_1 + v_2 w_2 + \ldots + v_N w_N$$

  - High when two vectors have large values in same dimensions.
  - Low (in fact 0) for **orthogonal vectors** with zeros in complementary distribution

Problem with dot product

$$\text{dot-product}(\vec{v},\vec{w}) = \vec{v}\cdot\vec{w} = \sum_{i=1}^{N} v_i w_i = v_1 w_1 + v_2 w_2 + \ldots + v_N w_N$$

- Dot product is longer if the vector is longer. Vector length:

$$|\vec{v}| = \sqrt{\sum_{i=1}^{N} v_i^2}$$

- Vectors are longer if they have higher values in each dimension
- That means more frequent words will have higher dot products
- That's bad: we don't want a similarity metric to be sensitive to word frequency

Solution: cosine

- Just divide the dot product by the length of the two vectors!

$$\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

- This turns out to be the cosine of the angle between them!

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \cos\theta$$

Cosine for computing similarity

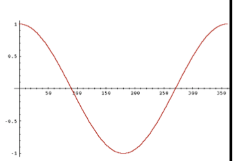
$$\cos(\vec{v},\vec{w}) = \underbrace{\frac{\vec{v}\bullet\vec{w}}{|\vec{v}||\vec{w}|}}_{\text{Dot product}} = \underbrace{\frac{\vec{v}}{|\vec{v}|}\bullet\frac{\vec{w}}{|\vec{w}|}}_{\text{Unit vectors}} = \frac{\sum_{i=1}^{N} v_i w_i}{\sqrt{\sum_{i=1}^{N} v_i^2}\sqrt{\sum_{i=1}^{N} w_i^2}}$$

v_i is the PPMI value for word *v* in context *i*
w_i is the PPMI value for word *w* in context *i*.

Cos($\vec{v}$,$\vec{w}$) is the cosine similarity of $\vec{v}$ and $\vec{w}$

Cosine as a similarity metric

- -1: vectors point in opposite directions
- +1: vectors point in same directions
- 0: vectors are orthogonal



- Raw frequency or PPMI are non-negative, so cosine range 0-1

$$\cos(\vec{v},\vec{w}) = \frac{\vec{v}\bullet\vec{w}}{|\vec{v}||\vec{w}|} = \frac{\vec{v}}{|\vec{v}|}\bullet\frac{\vec{w}}{|\vec{w}|} = \frac{\sum_{i=1}^{N} v_i w_i}{\sqrt{\sum_{i=1}^{N} v_i^2}\sqrt{\sum_{i=1}^{N} w_i^2}}$$

	large	data	computer
apricot	2	0	0
digital	0	1	2
information	1	6	1

Which pair of words is more similar?

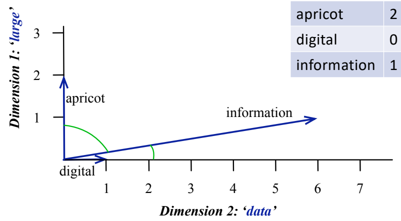
cosine(apricot,information) = $\frac{2+0+0}{\sqrt{2+0+0}\ \sqrt{1+36+1}} = \frac{2}{\sqrt{2}\sqrt{38}} = .23$

cosine(digital,information) = $\frac{0+6+2}{\sqrt{0+1+4}\ \sqrt{1+36+1}} = \frac{8}{\sqrt{38}\sqrt{5}} = .58$

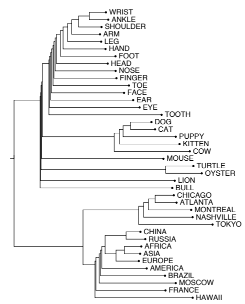
cosine(apricot,digital) = $\frac{0+0+0}{\sqrt{1+0+0}\ \sqrt{0+1+4}} = 0$

Visualizing vectors and angles

	large	data
apricot	2	0
digital	0	1
information	1	6



Clustering vectors to visualize similarity in co-occurrence matrices



Rohde et al. (2006)

Other possible similarity measures

$$\text{sim}_{\text{cosine}}(\vec{v},\vec{w}) = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|} = \frac{\sum_{i=1}^{N} v_i \times w_i}{\sqrt{\sum_{i=1}^{N} v_i^2}\sqrt{\sum_{i=1}^{N} w_i^2}}$$

$$\text{sim}_{\text{Jaccard}}(\vec{v},\vec{w}) = \frac{\sum_{i=1}^{N} \min(v_i,w_i)}{\sum_{i=1}^{N} \max(v_i,w_i)}$$

$$\text{sim}_{\text{Dice}}(\vec{v},\vec{w}) = \frac{2 \times \sum_{i=1}^{N} \min(v_i,w_i)}{\sum_{i=1}^{N} (v_i+w_i)}$$

Using syntax to define a word's context

- Zellig Harris (1968)
  "The meaning of entities, and the meaning of grammatical relations among them, is related to the restriction of combinations of these entities relative to other entities"
- **Two words are similar if they have similar syntactic contexts**

Duty and **responsibility** have similar syntactic distribution:

Modified by adjectives	additional, administrative, assumed, collective, congressional, constitutional …
Objects of verbs	assert, assign, assume, attend to, avoid, become, breach..

Co-occurrence vectors based on syntactic dependencies

Dekang Lin, 1998 "Automatic Retrieval and Clustering of Similar Words"

- Each dimension: a context word in one of R grammatical relations
  - Subject-of- "absorb"
- Instead of a vector of $|V|$ features, a vector of $R|V|$
- Example: counts for the word *cell* :

	subj-of, absorb	subj-of, adapt	subj-of, behave	...	pobj-of, inside	pobj-of, into	...	nmod-of, abnormality	nmod-of, anemia	nmod-of, architecture	...	obj-of, attack	obj-of, call	obj-of, come from	obj-of, decorate	...	nmod, bacteria	nmod, body	nmod, bone marrow
cell	1	1	1		16	30		3	8	1		6	11	3	2		3	2	2

Co-occurrence ı syntactic dependencies

De ... etrieval and Clustering of Similar Words"

- Each dimension: a contex ... ımmatical relations
  - Subject-of- "absorb"
- Instead of a vector of $|V|$ ... $R|V|$
- Example: counts for the w

cell	
1	subj-of, absorb
1	subj-of, adapt
1	subj-of, behave
	⋮
16	pobj-of, inside
30	pobj-of, into
	⋮
3	nmod-of, abnormality
8	nmod-of, anemia
1	nmod-of, architecture
	⋮
6	obj-of, attack
11	obj-of, call

Syntactic dependencies for dimensions

- Alternative (Padó and Lapata 2007):
  - Instead of having a |V| x R|V| matrix
  - Have a |V| x |V| matrix
  - But the co-occurrence counts aren't just counts of words in a window
  - But counts of words that occur in one of R dependencies (subject, object, etc).
  - So M("cell","absorb") = count(subj(cell,absorb)) + count(obj(cell,absorb)) + count(pobj(cell,absorb)), etc.

PMI applied to dependency relations

Hindle, Don. 1990. Noun Classification from Predicate-Argument Structure. ACL

Object of "drink"	Count	PMI
tea	2	11.8
liquid	2	10.5
wine	2	9.3
anything	3	5.2
it	3	1.3

- "`Drink it`" more common than "`drink wine`"
- But "`wine`" is a better "drinkable" thing than "`it`"

Alternative to PPMI for measuring association

- The combination of two factors
  - **Term frequency** (Luhn 1957): frequency of the word (can be logged)
  - **Inverse document frequency** (IDF) (Spark Jones 1972)
    - N is the total number of documents
    - df_i = "document frequency of word i"
    - = number of documents with word i
  - w_{ij} : for word i in document j

$$w_{ij} = tf_{ij} \cdot idf_i$$

$$\mathrm{idf}_i = \log\left(\frac{N}{df_i}\right)$$

tf-idf not generally used for word-word similarity

- But is by far the most common weighting when we are considering the relationship of words to documents

Evaluating similarity (Revisit)

- Extrinsic (task-based, end-to-end) Evaluation:
  - Question Answering
  - Spell Checking
  - Essay grading
- Intrinsic Evaluation:
  - Correlation between algorithm and human word similarity ratings
    - Wordsim353: 353 noun pairs rated 0-10. *sim(plane,car)=5.77*
  - Taking TOEFL multiple-choice vocabulary tests
    - `Levied is closest in meaning to:`
      `imposed, believed, requested, correlated`

Summary and next step

- Distributional (vector) models of meaning
  - **Sparse** (PPMI-weighted word-word co-occurrence matrices)
  - **Dense**:
    - Word-word SVD (50-2000 dimensions)
    - Skip-grams and CBOW (100-1000 dimensions)