CS 6120/CS 4120: Natural Language Processing

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Outline

- Maximum Entropy
- Feedforward Neural Networks
- Recurrent Neural Networks

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Maximum Entropy (MaxEnt)

- Or logistic regression

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Features

- In these slides and most MaxEnt work: *features (or feature functions)* f are elementary pieces of evidence that link aspects of what we observe d with a category c that we want to predict
- A feature is a function with a **bounded** real value: $f: C \times D \rightarrow \mathbb{R}$

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Example Task: Named Entity Type

LOCATION *in Arcadia* LOCATION *in Québec* DRUG *taking Zantac* PERSON *saw Sue*

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Example features

- $f_1(c, d) \equiv [c = \text{LOCATION} \wedge w_{-1} = \text{“in”} \wedge \text{isCapitalized}(w)]$
- $f_2(c, d) \equiv [c = \text{LOCATION} \wedge \text{hasAccentedLatinChar}(w)]$
- $f_3(c, d) \equiv [c = \text{DRUG} \wedge \text{ends}(w, \text{“c”})]$

LOCATION *in Arcadia* LOCATION *in Québec* DRUG *taking Zantac* PERSON *saw Sue*

- Models will assign to each feature a *weight:*
  - A positive weight votes that this configuration is likely correct
  - A negative weight votes that this configuration is likely incorrect

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Example features

- $f_1(c, d) \equiv [c = \text{LOCATION} \wedge w_{-1} = \text{“in”} \wedge \text{isCapitalized}(w)]$ -> weight 1.8
- $f_2(c, d) \equiv [c = \text{LOCATION} \wedge \text{hasAccentedLatinChar}(w)]$ -> weight -0.6
- $f_3(c, d) \equiv [c = \text{DRUG} \wedge \text{ends}(w, \text{“c”})]$ -> weight 0.3

- Weights will be learned by training on a labeled dataset

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More about feature functions:

an indicator function – a yes/no boolean matching function – of properties of the input and a particular class

$$f_i(c, d) \equiv [\Phi(d) \wedge c = c_j] \quad \text{[Value is 0 or 1]}$$

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Feature-Based Models

- The decision about a data point is based only on the features active at that point.

Data	Data	Data
BUSINESS: Stocks hit a yearly low …	… to restructure bank:MONEY debt.	DT JJ NN … The previous fall …
Label: BUSINESS Features {…, stocks, hit, a, yearly, low, …}	Label: MONEY Features {…, w_{-1}=restructure, w_{+1}=debt, L=12, …}	Label: NN Features {w=fall, t_{-1}=JJ w_{-1}=previous}
Text Classification	Word Sense Disambiguation	POS Tagging

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Feature-Based Linear Classifiers

- Linear classifiers at classification time:
  - Linear function from feature sets $\{f_i\}$ to classes $\{c\}$.
  - Assign a weight λ_i to each feature f_i.
  - We consider each class for sample d
  - For a pair (c,d), features vote with their weights:
    - vote(c) = $\sum_i \lambda_i f_i(c,d)$

PERSON	LOCATION	DRUG
in Québec	*in Québec*	*in Québec*

  - Choose the class c which maximizes $\sum \lambda_i f_i(c,d)$

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- Maximum Entropy:
  - Make a probabilistic model from the linear combination $\sum \lambda_i f_i(c,d)$

$$P(c \mid d, \lambda) = \frac{\exp \sum_i \lambda_i f_i(c,d)}{\sum_{c'} \exp \sum_i \lambda_i f_i(c',d)}$$

Makes votes positive

Normalizes votes

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Feature-Based Linear Classifiers

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Makes votes positive

Normalizes votes

- P(LOCATION|*in Québec*) = $e^{1.8}e^{-0.6}/(e^{1.8}e^{-0.6} + e^{0.3} + e^{0}) = 0.586$
- P(DRUG|*in Québec*) = $e^{0.3}/(e^{1.8}e^{-0.6} + e^{0.3} + e^{0}) = 0.238$
- P(PERSON|*in Québec*) = $e^{0}/(e^{1.8}e^{-0.6} + e^{0.3} + e^{0}) = 0.176$
- The weights are the parameters of the probability model, combined via a “soft max” function

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Feature-Based Linear Classifiers

- Given this model form, we will choose parameters $\{\lambda_i\}$ that *maximize the conditional likelihood* of the data according to this model.
- Parameter learning is omitted and not required for this course, but is often discussed in a machine learning class.
  - E.g. gradient descent for parameter learning

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Outline

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- Recurrent Neural Networks

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Neural Network Learning

- Learning approach based on modeling adaptation in biological neural systems.
- Perceptron: Initial algorithm for learning simple neural networks (single layer) developed in the 1950’s.
- Backpropagation: More complex algorithm for learning multi-layer neural networks developed in the 1980’s. (not required for this class)

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ARTIFICIAL NEURON

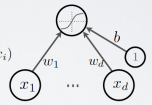
Topics: connection weights, bias, activation function

- Neuron pre-activation (or input activation):

$$a(\mathbf{x}) = b + \sum_i w_i x_i = b + \mathbf{w}^\top \mathbf{x}$$

- Neuron (output) activation

$$h(\mathbf{x}) = g(a(\mathbf{x})) = g(b + \sum_i w_i x_i)$$



- $\mathbf{w}$ are the connection weights
- b is the neuron bias
- $g(\cdot)$ is called the activation function

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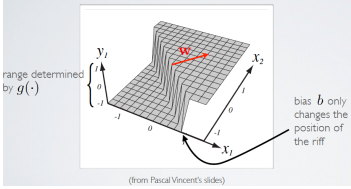
ARTIFICIAL NEURON

Topics: connection weights, bias, activation function



(from Pascal Vincent's slides)

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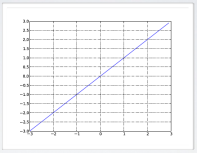
ACTIVATION FUNCTION

Topics: linear activation function

- Performs no input squashing
- Not very interesting...



$$g(a) = a$$

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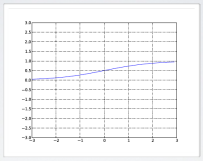
ACTIVATION FUNCTION

Topics: sigmoid activation function

- Squashes the neuron's pre-activation between 0 and 1
- Always positive
- Bounded
- Strictly increasing



$$g(a) = \text{sigm}(a) = \frac{1}{1+\exp(-a)}$$

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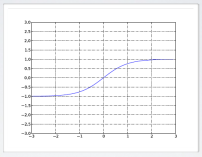
ACTIVATION FUNCTION

Topics: hyperbolic tangent ("tanh") activation function

- Squashes the neuron's pre-activation between -1 and 1
- Can be positive or negative
- Bounded
- Strictly increasing



$$g(a) = \tanh(a) = \frac{\exp(a)-\exp(-a)}{\exp(a)+\exp(-a)} = \frac{\exp(2a)-1}{\exp(2a)+1}$$

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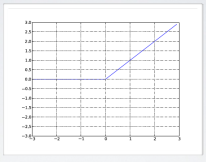
ACTIVATION FUNCTION

Topics: rectified linear activation function

- Bounded below by 0 (always non-negative)
- Not upper bounded
- Strictly increasing
- Tends to give neurons with sparse activities



$$g(a) = \text{reclin}(a) = \max(0, a)$$

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```python
class Neuron(object):
  # ...
  def forward(inputs):
    """ assume inputs and weights are 1-D numpy arrays and bias is a number """
    cell_body_sum = np.sum(inputs * self.weights) + self.bias
    firing_rate = 1.0 / (1.0 + math.exp(-cell_body_sum)) # sigmoid activation function
    return firing_rate
```

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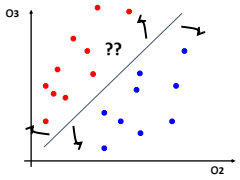
Linear Separator

- Since one-layer neuron (aka perceptron) uses linear threshold function, it is searching for a linear separator that discriminates the classes.



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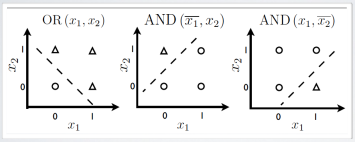
ARTIFICIAL NEURON

Topics: capacity of single neuron

- Can solve linearly separable problems



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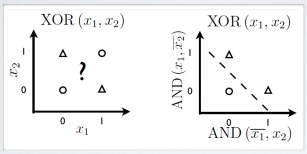
ARTIFICIAL NEURON

Topics: capacity of single neuron

- Can't solve non linearly separable problems...



- ... unless the input is transformed in a better representation

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NEURAL NETWORK

Topics: single hidden layer neural network

- Hidden layer pre-activation:

$$\mathbf{a}(\mathbf{x}) = \mathbf{b}^{(1)} + \mathbf{W}^{(1)}\mathbf{x}$$

$$\left(a(\mathbf{x})_i = b_i^{(1)} + \textstyle\sum_j W_{i,j}^{(1)} x_j\right)$$

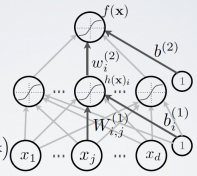
- Hidden layer activation:

$$\mathbf{h}(\mathbf{x}) = \mathbf{g}(\mathbf{a}(\mathbf{x}))$$

- Output layer activation:

$$f(\mathbf{x}) = o\left(b^{(2)} + \mathbf{w}^{(2)\top}\mathbf{h}^{(1)}\mathbf{x}\right)$$

output activation function



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NEURAL NETWORK

Topics: softmax activation function

- For multi-class classification:
  - we need multiple outputs (1 output per class)
  - we would like to estimate the conditional probability $p(y = c|\mathbf{x})$
- We use the softmax activation function at the output:

$$\mathbf{o}(\mathbf{a}) = \text{softmax}(\mathbf{a}) = \left[\frac{\exp(a_1)}{\sum_c \exp(a_c)} \cdots \frac{\exp(a_C)}{\sum_c \exp(a_c)}\right]^\top$$

  - strictly positive
  - sums to one
- Predicted class is the one with highest estimated probability

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NEURAL NETWORK

Topics: multilayer neural network

- Could have L hidden layers:
  - layer pre-activation for $k>0$ $(\mathbf{h}^{(0)}(\mathbf{x}) = \mathbf{x})$

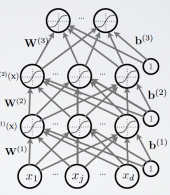
$$\mathbf{a}^{(k)}(\mathbf{x}) = \mathbf{b}^{(k)} + \mathbf{W}^{(k)}\mathbf{h}^{(k-1)}(\mathbf{x})$$

  - hidden layer activation (k from 1 to L):

$$\mathbf{h}^{(k)}(\mathbf{x}) = \mathbf{g}(\mathbf{a}^{(k)}(\mathbf{x}))$$

  - output layer activation ($k=L+1$):

$$\mathbf{h}^{(L+1)}(\mathbf{x}) = \mathbf{o}(\mathbf{a}^{(L+1)}(\mathbf{x})) = \mathbf{f}(\mathbf{x})$$



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```
# forward-pass of a 3-layer neural network:
f = lambda x: 1.0/(1.0 + np.exp(-x)) # activation function (use sigmoid)
x = np.random.randn(3, 1) # random input vector of three numbers (3x1)
h1 = f(np.dot(W1, x) + b1) # calculate first hidden layer activations (4x1)
h2 = f(np.dot(W2, h1) + b2) # calculate second hidden layer activations (4x1)
out = np.dot(W3, h2) + b3 # output neuron (1x1)
```

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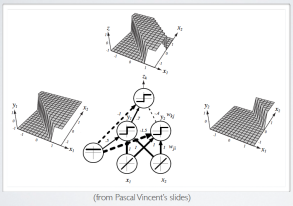
CAPACITY OF NEURAL NETWORK

Topics: single hidden layer neural network



(from Pascal Vincent's slides)

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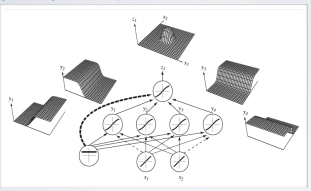
CAPACITY OF NEURAL NETWORK

Topics: single hidden layer neural network



(from Pascal Vincent's slides)

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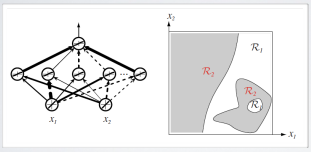
CAPACITY OF NEURAL NETWORK

Topics: single hidden layer neural network



(from Pascal Vincent's slides)

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CAPACITY OF NEURAL NETWORK

Topics: universal approximation

- Universal approximation theorem (Hornik, 1991):
  - "a single hidden layer neural network with a linear output unit can approximate any continuous function arbitrarily well, given enough hidden units"
- The result applies for sigmoid, tanh and many other hidden layer activation functions
- This is a good result, but it doesn't mean there is a learning algorithm that can find the necessary parameter values!

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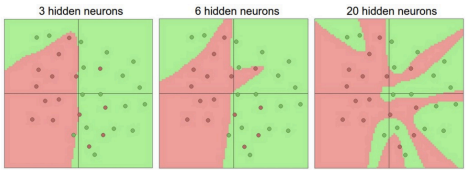

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How to train a neural network? (Not covered in this course, only for reference)

Topics: multilayer neural network

- Could have L hidden layers:
  - layer input activation for $k>0$ $(\mathbf{h}^{(0)}(\mathbf{x}) = \mathbf{x})$
    $$\mathbf{a}^{(k)}(\mathbf{x}) = \mathbf{b}^{(k)} + \mathbf{W}^{(k)}\mathbf{h}^{(k-1)}(\mathbf{x})$$
  - hidden layer activation (k from 1 to L):
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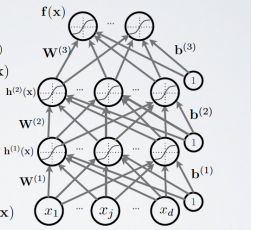

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Empirical Risk Minimization

Topics: empirical risk minimization, regularization

- Empirical risk minimization
  - framework to design learning algorithms

$$\arg\min_{\boldsymbol{\theta}} \frac{1}{T}\sum_t l(f(\mathbf{x}^{(t)};\boldsymbol{\theta}), y^{(t)}) + \lambda\Omega(\boldsymbol{\theta})$$

  - $l(f(\mathbf{x}^{(t)};\boldsymbol{\theta}), y^{(t)})$ is a loss function
  - $\Omega(\boldsymbol{\theta})$ is a regularizer (penalizes certain values of $\boldsymbol{\theta}$)
- Learning is cast as optimization
  - ideally, we'd optimize classification error, but it's not smooth
  - loss function is a surrogate for what we truly should optimize (e.g. upper bound)

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LOSS FUNCTION

Topics: loss function for classification

- Neural network estimates $f(\mathbf{x})_c = p(y = c|\mathbf{x})$
  - we could maximize the probabilities of $\mathbf{y}^{(t)}$ given $\mathbf{x}^{(t)}$ in the training set
- To frame as minimization, we minimize the negative log-likelihood

natural log (ln)

$$l(\mathbf{f}(\mathbf{x}), y) = -\sum_c 1_{(y=c)} \log f(\mathbf{x})_c = -\log f(\mathbf{x})_y$$

  - we take the log to simplify for numerical stability and math simplicity
  - sometimes referred to as cross-entropy

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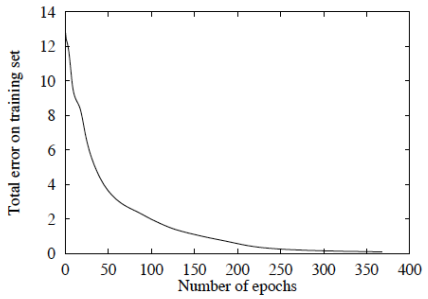

[figure from Greg Mori's slides]

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REGULARIZATION

Topics: L2 regularization

$$\Omega(\boldsymbol{\theta}) = \sum_k \sum_i \sum_j \left(W_{i,j}^{(k)}\right)^2 = \sum_k ||\mathbf{W}^{(k)}||_F^2$$

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Empirical Risk Minimization

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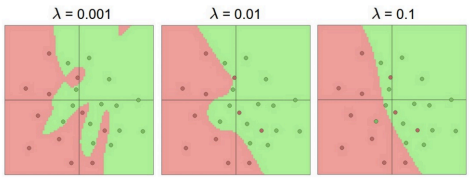

[http://cs231n.github.io/neural-networks-1/]

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INITIALIZATION

Topics: initialization

- For biases
  - initialize all to 0
- For weights
  - Can't initialize weights to 0 with tanh activation
    - we can show that all gradients would then be 0 (saddle point)
  - Can't initialize all weights to the same value
    - we can show that all hidden units in a layer will always behave the same
    - need to break symmetry
  - Recipe: sample $\mathbf{W}_{ij}^{(k)}$ from $U[-b, b]$ where $b = \frac{\sqrt{6}}{\sqrt{H_k + H_{k-1}}}$ (size of $\mathbf{h}^{(k)}(\mathbf{x})$)
    - the idea is to sample around 0 but break symmetry
    - other values of b could work well (not an exact science) (see Glorot & Bengio, 2010)

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Model Learning

- Backpropagation (BP) algorithm (not required for this course)
- Further reading on BP:
  - https://towardsdatascience.com/understanding-backpropagation-algorithm-7bb3aa2f95fd
  - https://mattmazur.com/2015/03/17/a-step-by-step-backpropagation-example/

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Outline

- Maximum Entropy
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- Recurrent Neural Networks

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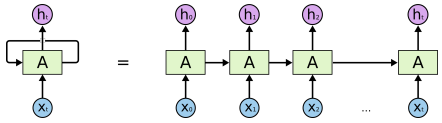
Long Distance Dependencies

- It is very difficult to train NNs to retain information over many time steps
- This makes it very difficult to handle long-distance dependencies, such as subject-verb agreement.
- E.g. Jane walked into the room. John walked in too. It was late in the day. Jane said hi to _?_



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Recurrent Neural Networks

Feed-forward NN	Recurrent NN
$\mathbf{h} = g(\mathbf{V}\mathbf{x} + \mathbf{c})$	$\mathbf{h}_t = g(\mathbf{V}\mathbf{x}_t + \mathbf{U}\mathbf{h}_{t-1} + \mathbf{c})$
$\hat{\mathbf{y}} = \mathbf{W}\mathbf{h} + \mathbf{b}$	$\hat{\mathbf{y}}_t = \mathbf{W}\mathbf{h}_t + \mathbf{b}$

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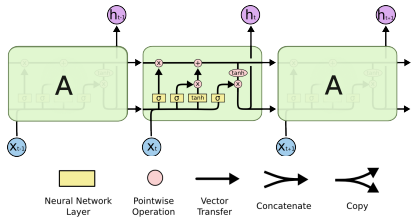
Long-Short Term Memory Networks (LSTMs)



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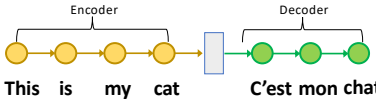
Sequence to Sequence

- Encoder/Decoder framework maps one sequence to a "deep vector" then another LSTM maps this vector to an output sequence.



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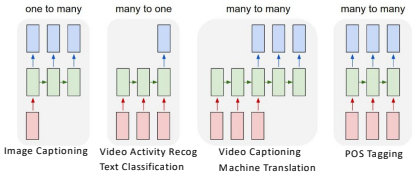
Summary of LSTM Application Architectures



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Successful Applications of LSTMs

- Speech recognition: Language and acoustic modeling
- Sequence labeling
  - POS Tagging
  - NER
  - Phrase Chunking
- Neural syntactic and semantic parsing
- Image captioning
- Sequence to Sequence
  - Machine Translation (Sustkever, Vinyals, & Le, 2014)
  - Summarization
  - Video Captioning (input sequence of CNN frame outputs)

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