Basic Game Physics

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Based on *The Physics of the Game, Chapter 13 of Teach Yourself Game Programming in 21 Days,*

pp. 681-715

Why Physics?

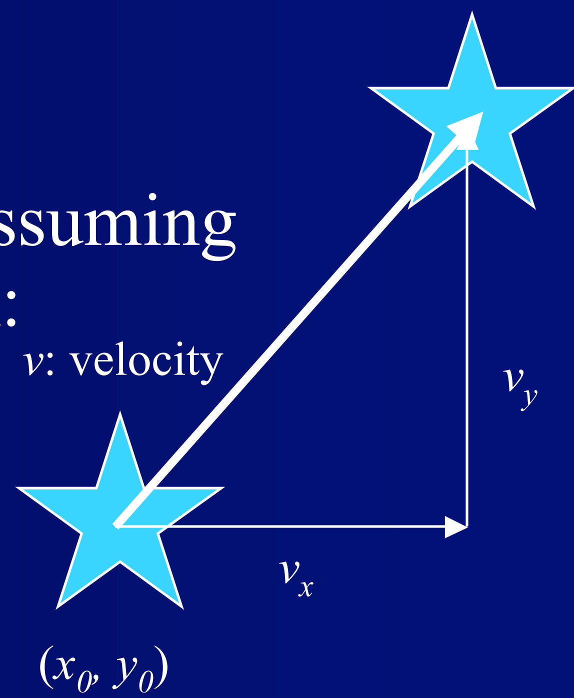
- Some games don't need any physics
- Games based on the real world should look realistic, meaning realistic action and reaction
  - More complex games need more physics:
    - sliding through a turn in a racecar, sports games, flight simulation, etc.
  - Running and jumping off the edge of a cliff
- Two types of physics:
  - Elastic, rigid-body physics, $F = ma$, e.g., pong
  - Non-elastic, physics with deformation: clothes, pony tails, a whip, chain, hair, volcanoes, liquid, boomerang
- Elastic physics is easier to get right

Game Physics

- Approximate real-world physics
- We don't want just the equations
- We want *efficient* ways to compute physical values
  - Assume fixed discrete simulation – constant time step
  - Must account for actual time passed for variable simulation
- Assumptions:
  - 2D physics, usually easy to generalize to 3D (add z)
  - Rigid bodies (no deformation)
  - Will just worry about center of mass
    - Not accurate for all physical effects
  - Constant time step

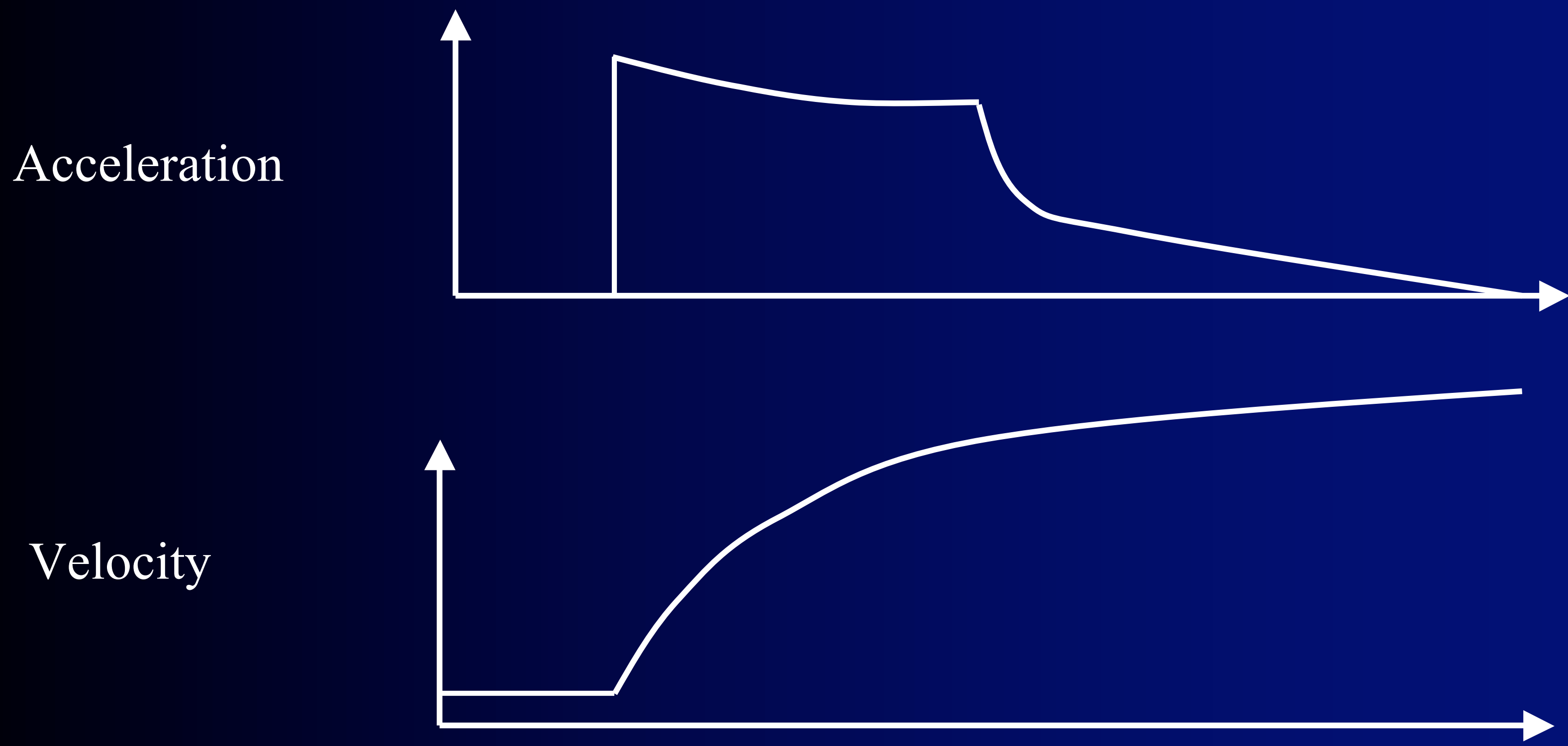
Position and Velocity

- Modeling the movement of objects with velocity
  - Where is an object at any time *t*?
  - Assume distance unit is in pixels
- Position at time *t* for an object moving at velocity *v*, from starting position x_0:
  - $x(t) = x_0 + v_x\, t$
  - $y(t) = y_0 + v_y\, t$
- Incremental computation per frame, assuming constant time step and no acceleration:
  - v_x and v_y constants, pre-compute
  - $x \mathrel{+}= v_x,\ y \mathrel{+}= v_y$



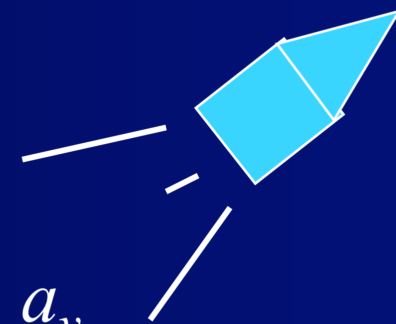
Acceleration

- Acceleration (a): change in velocity per unit time

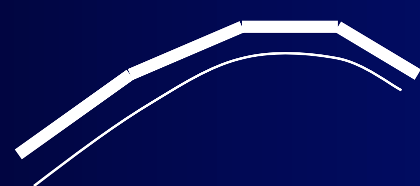


Approximate

Acceleration



- Constant acceleration: $v_x \mathrel{+}= a_x$, $v_y \mathrel{+}= a_y$
- Variable acceleration:
  - use table lookup based on other factors:
  - *acceleration* = acceleration_value(gear, speed, pedal_pressure)
    - Cheat a bit: *acceleration* = acceleration_value(gear, speed) * pedal_pressure
  - $a_x = cos\ (v) * acceleration$
  - $a_y = sin\ (v) * acceleration$
- Piece-wise linear approximation to continuous functions



Gravity

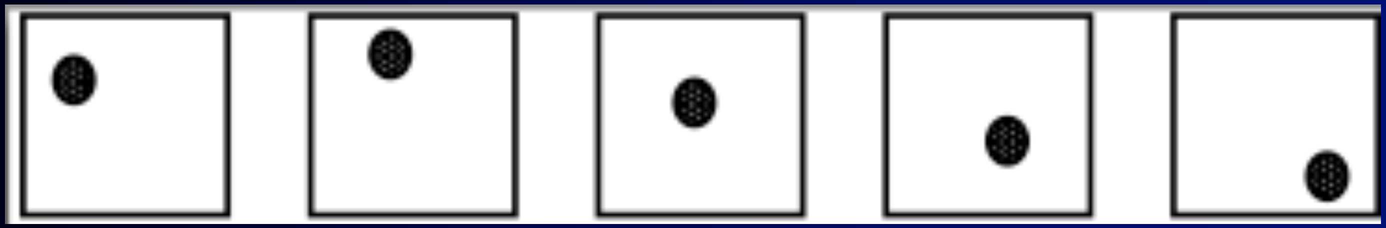
- Gravity is a force between two objects:
  - Force $F = G \ (m_1 m_2)/ D^2$
    - G = 6.67 x 10^{-11} Nm^2kg^{-2}
    - m_i: the mass of the two objects
    - D = distance between the two objects
  - So both objects have same force applied to them
    - $F=ma$ --> $a=F/m$
- On earth, assume mass of earth is so large it doesn't move, and D is constant
  - Assume uniform acceleration
  - Position of falling object at time t:
    - $x(t) = x_0$
    - $y(t) = y_0 + 1/2 * 9.8\ m/s^2 * t^2$
    - Incrementally, y += gravity (normalized to frame rate)

Space Game Physics

- Gravity
  - Influences both bodies
  - Can have two bodies orbit each other
  - Only significant for large mass objects
  - Consider *N*-body problem
- What happens after you apply a force to an object?
- What happens when you shoot a missile from a moving object?
- What types of controls do you expect to have on a space ship?
- What about a flying game?

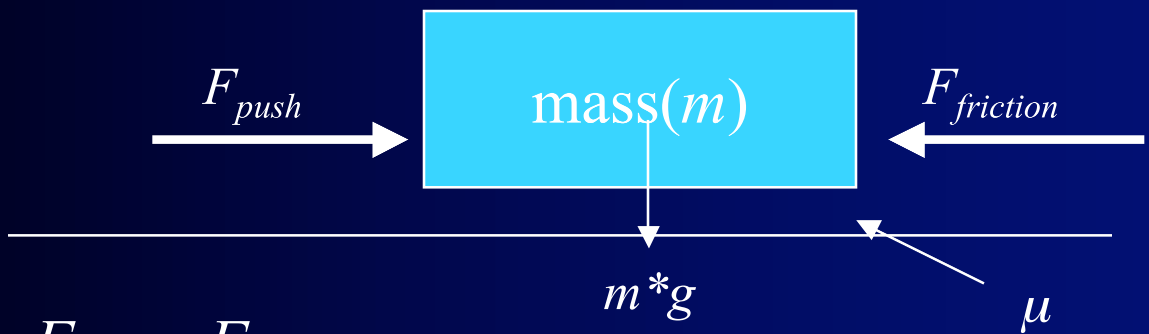
Mass

- Objects represented by their *center of mass*, not accurate for all physical effects
- Center of mass (x_c, y_c) for a polygon with n vertices:
  - Attach a mass to each vertex
  - $x_c = \Sigma x_i m_i / \Sigma m_i,\ i = 0 .. n$
  - $y_c = \Sigma y_i m_i / \Sigma m_i,\ i = 0 .. n$
- For sprites, put center of mass where pixels are densest
- For arcade games, model gravity in sprite frames:



Friction

- Conversion of kinetic energy into heat
- Frictional force $F_{friction} = m\ g\ \mu$
  - m = mass, $g = 9.8\ m/s^2$,
  - μ = frictional coefficient = amount of force to maintain a constant speed



- $F_{actual} = F_{push} - F_{friction}$
  - Careful that friction doesn't cause your object to move backward!
  - Consider inclined plane
- Usually two frictional forces
  - Static friction when at rest (velocity = 0). No movement unless overcome.
  - Kinetic friction when moving ($\mu_k < \mu_s$)

Race Game Physics

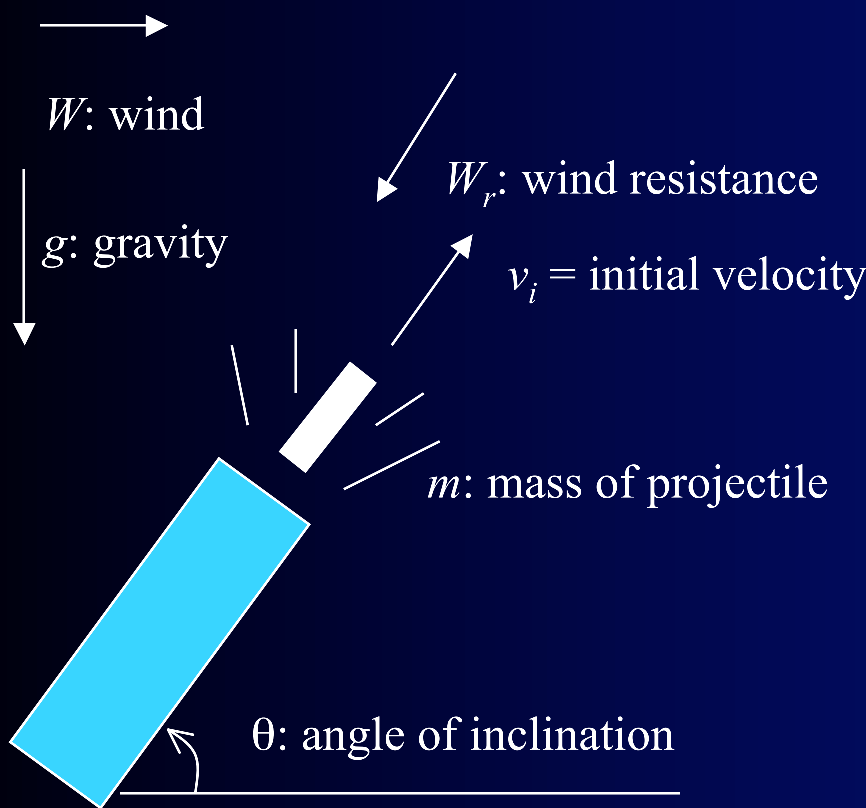
- Non-linear acceleration
- Resting friction > rolling friction
- Rolling friction < sliding friction
- Centripetal force?

- What controls do you expect to have for a racing game?
  - Turning requires forward motion!

- What about other types of racing games
  - Boat?
  - Hovercraft?

Projectile Motion

- Forces



$v_{i_x} = v_i\, cos(\theta)$

$v_{i_y} = v_i\, sin(\theta)$

Reaches apex at $t = v_i\, sin(\theta)/g$, hits ground at $x = v_{i_x} * v_{i_y}/g$

With wind:

$x \mathrel{+}= v_{i_x} + W$

$y \mathrel{+}= v_{i_y}$

With wind resistance and gravity:

$v_{i_x} \mathrel{+}= W_{r_x}$

$v_{i_y} \mathrel{+}= W_{r_y} + g$, g normalized

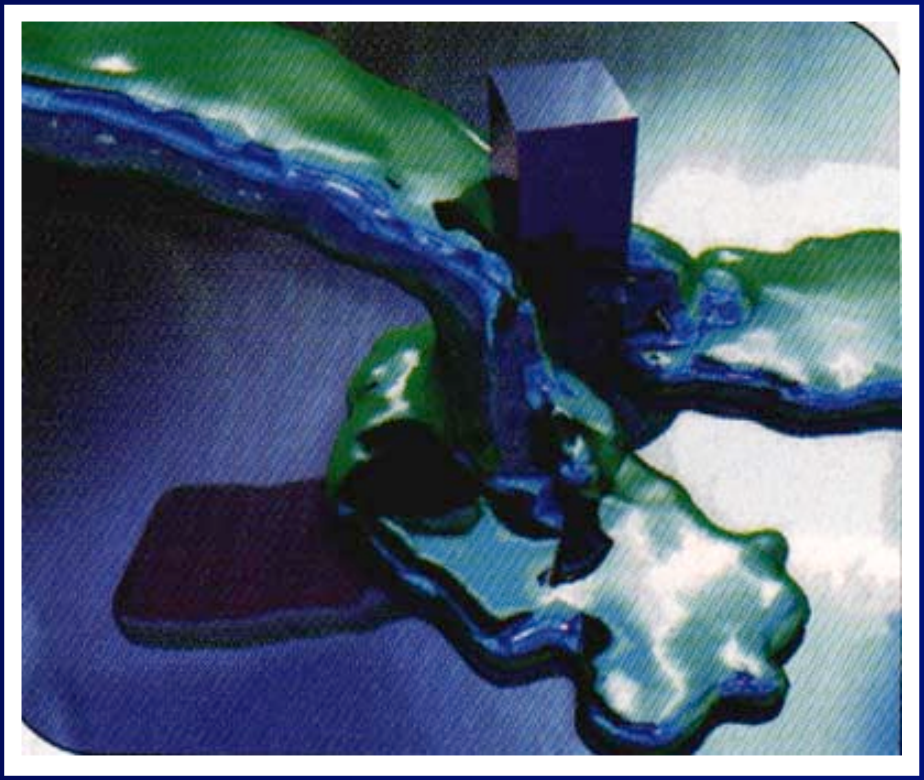
Particle System Explosions

- Start with lots of point objects (1-4 pixels)
- Initialize with random velocities based on velocity of object exploding
- Apply gravity
- Transform color intensity as a function of time
- Destroy objects upon collision or after fixed time

- Can add vapor trail (different color, lifetime, wind)

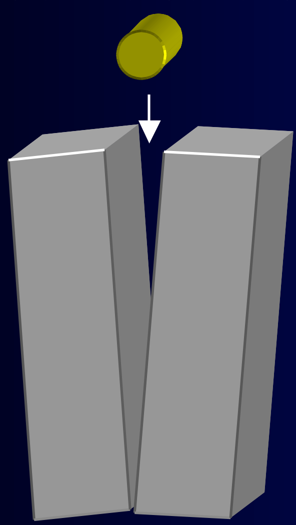
Advanced Physics

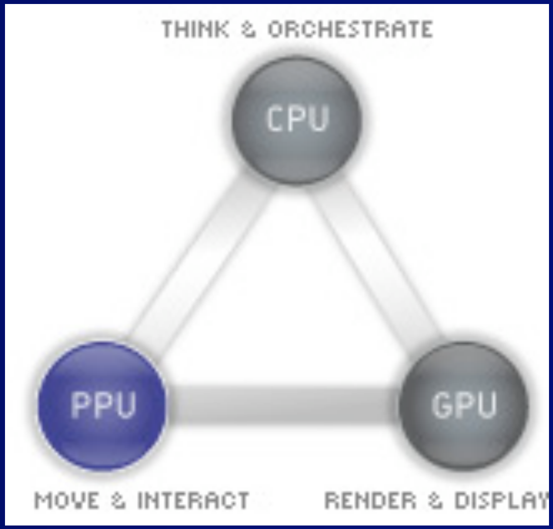
- Modeling liquid (*Shrek, Finding Nemo*)
- Movement of clothing
- Movement of hair (*Monster Inc.*)
- Fire/Explosion effects
- Reverse Kinematics



Physics Engines

- Havok, AGEIA PhysX, Tokamak, etc.
- Strengths
  - Do all of the physics for you as a package
- Weaknesses
  - Can be slow when there are many objects (use PPU?)
  - May have trouble with small vs. big object interactions
  - Have trouble with boundary cases

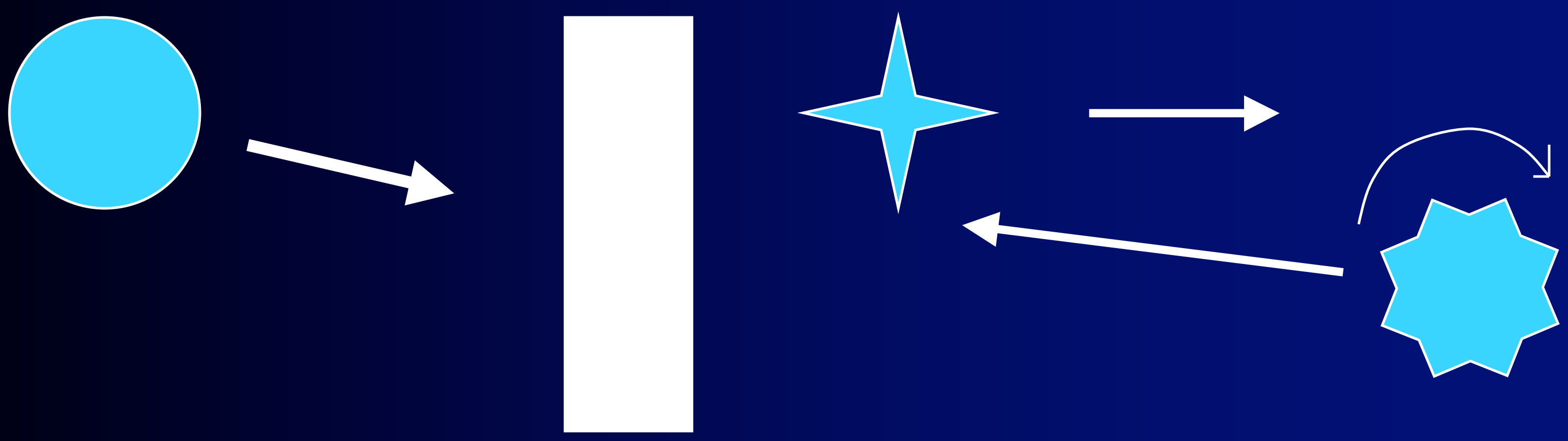




Source: AGEIA

Back to Collisions

- Steps of analysis for different types of collisions
  - Circle/sphere against a fixed, flat object
  - Two circles/spheres
  - Rigid bodies
  - Deformable
- Model the simplest - don’t build a general engine

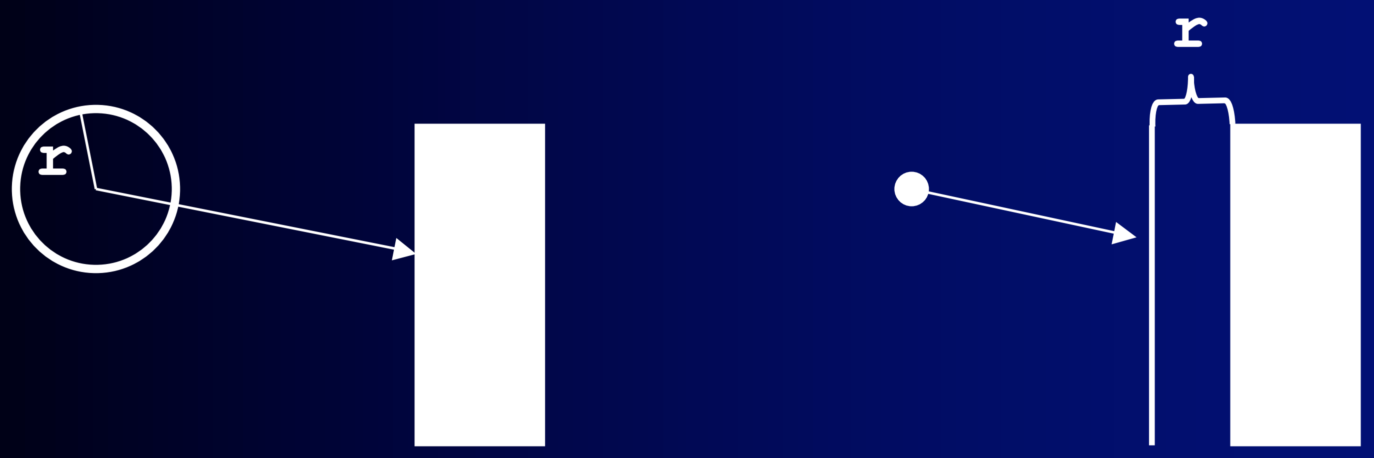


Collisions: Steps of Analysis

- Detect that a collision has occurred
- Determine the time of the collision
  - So can back up to point of collision
- Determine where the objects were at time of collision
- Determine the collision angle off the collision normal
- Determine the velocity vectors after collision
- Determine changes in rotation

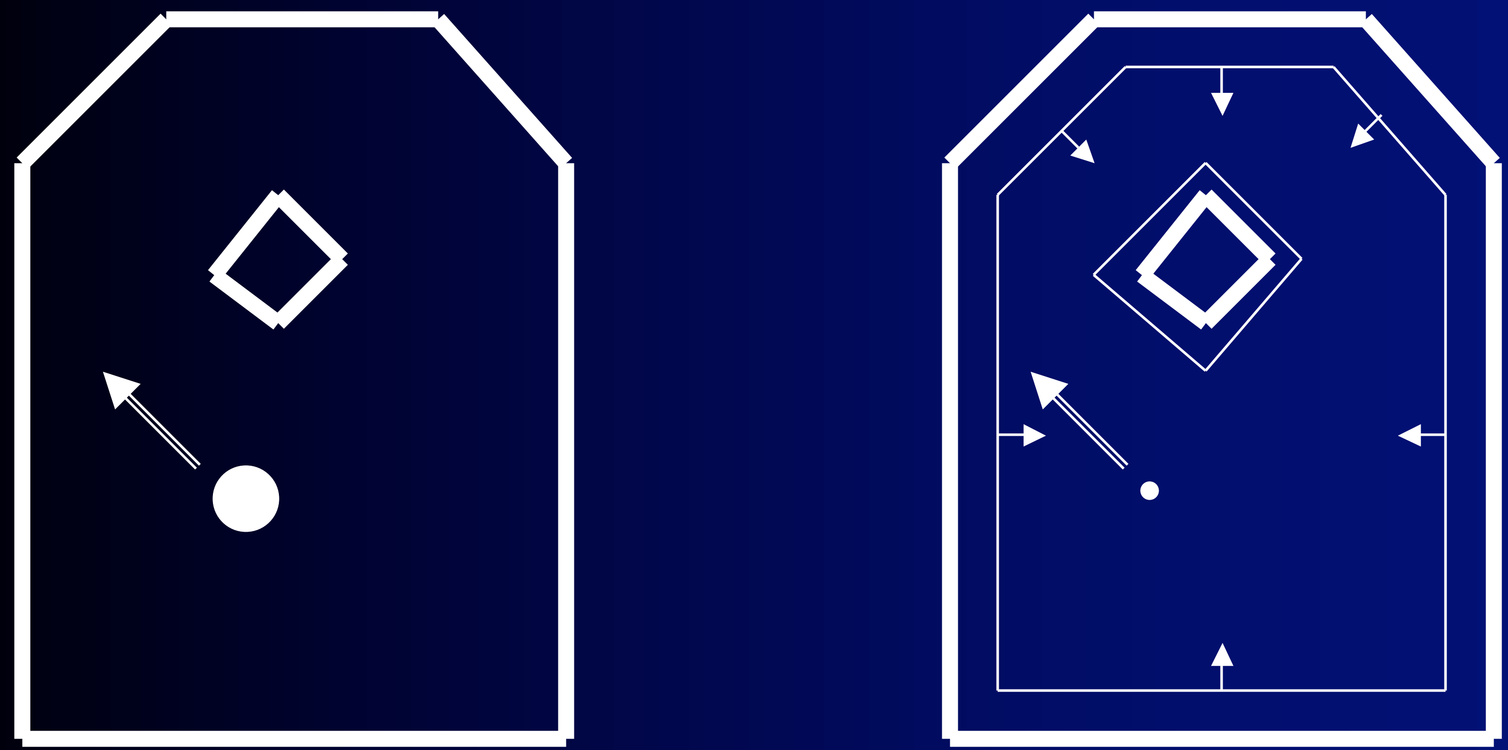
Circles and Lines

- Simplest case
  - Good step for your games - pinball
  - Assume circle hitting an *immovable* barrier
- Detect that a collision occurred
  - If the distance from the circle to the line < circle radius
  - Reformulate as a point about to hit a bigger wall
  - If vertical and horizontal walls, simple test of x, y

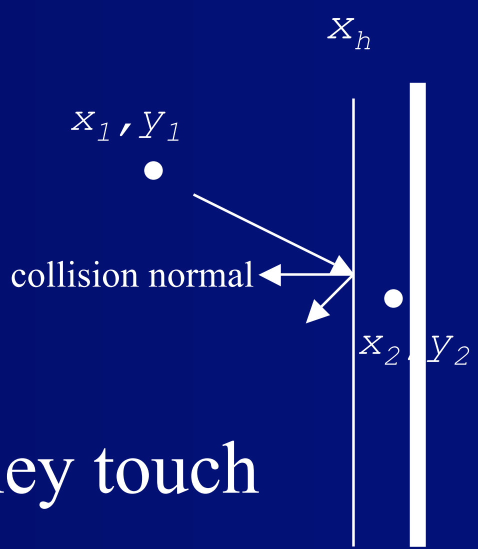


Circles and Angled Lines

- What if more complex background: pinball?
  - For complex surfaces, pre-compute and fill an array with collision points (and surface normals)



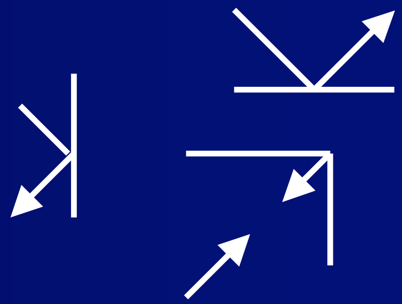
Circle on Wall Collision Response



- Determine the time of collision (t_c):
  - $t_c = t_i + (x_h - x_1)/(x_2 - x_1) * \Delta t$
  - t_i = initial time
  - Δt = time increment
- Determine where the objects are when they touch
  - $y_c = y_1 - (y_1 - y_2) * (t_c - t_i)/\Delta t$
- Determine the collision angle against collision normal
  - Collision normal is the surface normal of the wall in this case
  - Compute angle of line using $(x_1 - x_h)$ and $(y_1 - y_c)$

Circle on Wall Collision Response

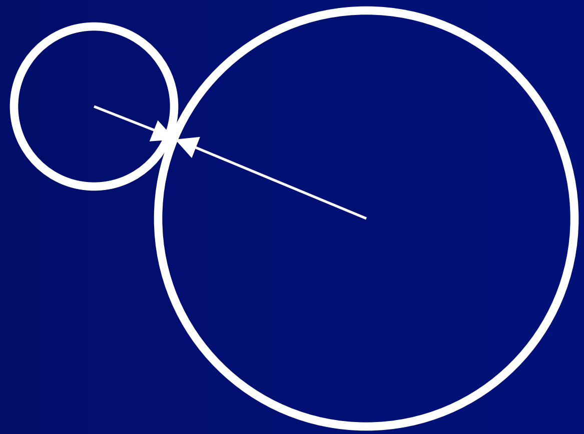
- Determine the velocity vectors after collision
  - Angle of reflectant = angle of incidence; reflect object at an angle equal and opposite off the surface normal
  - If surface is co-linear with the *x*- or *y*-axes:
    - Vertical - change sign of *x* velocity
    - Horizontal - change sign of *y* velocity
    - Corner - change sign of both



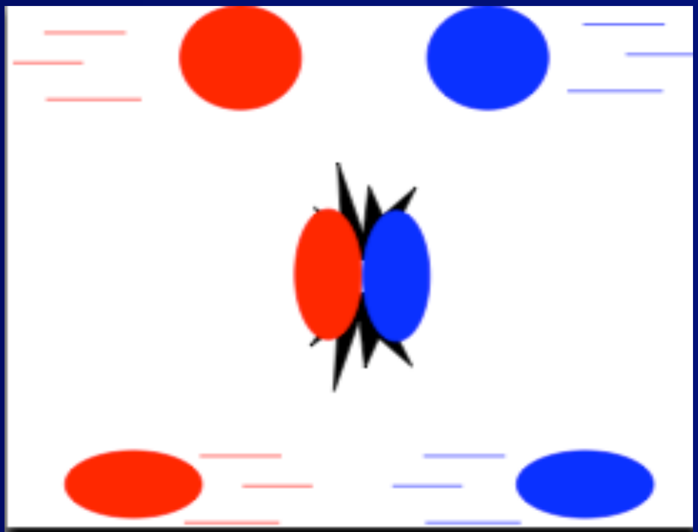
- Compute new position
  - Use Δt - t_c to calculate new position from collision point
- Determine changes in rotation
  - None!

- Is this worth it? Depends on speed of simulation, …

Circle-circle Collision

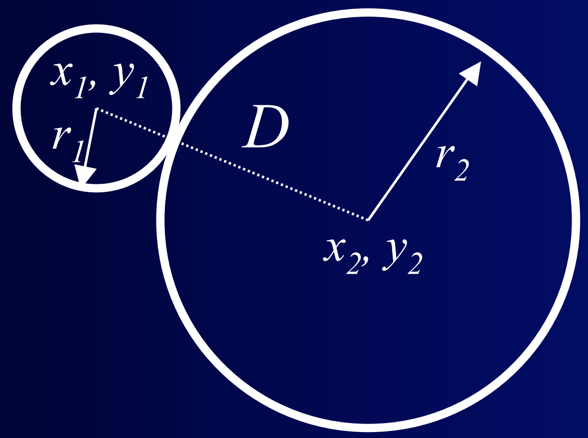


- Another important special case
  - Good step for your games
  - Many techniques developed here can be used for other object types
- Assume elastic collisions:
  - Conservation of momentum
  - Conservation of kinetic energy
- Non-elastic collision converts kinetic energy into heat and/or mechanical deformations



Detect that a collision occurred

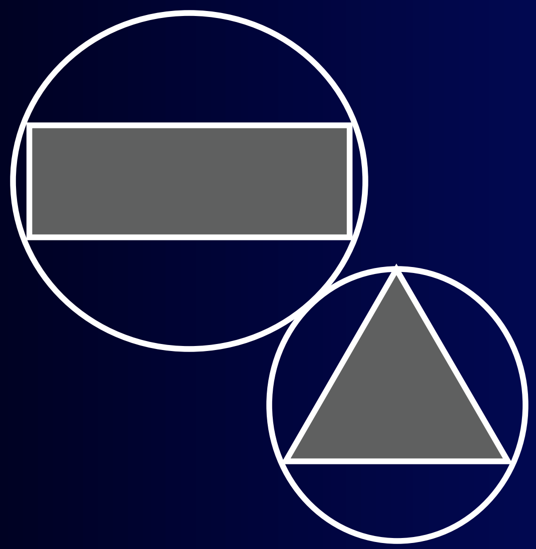
- If the distance between two circles is less than the sum of their radii
  - Trick: avoid square root in computing distance!
  - Instead of checking $(r_1+r_2) > D$, where $D = sqrt((x_1-x_2)^2 + (y_1-y_2)^2)$
  - Check $(r_1 + r_2)^2 > ((x_1-x_2)^2 + (y_1-y_2)^2)$

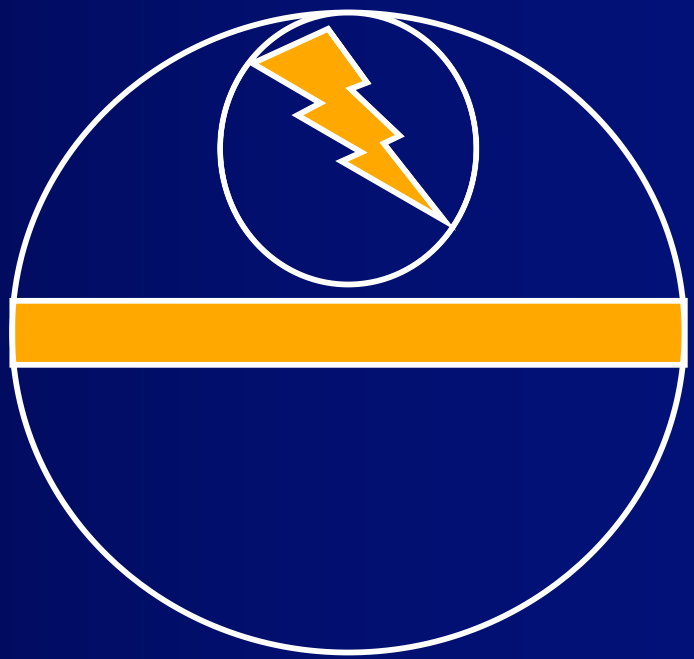


- Unfortunately, this is still $O(N^2)$ comparisons, N number of objects

Detect that a collision occurred

- With non-circles, gets more complex and more expensive for each pair-wise comparison
- Use bounding circles/spheres and check for overlap
  - Pretty cheap
  - Not great for thin objects



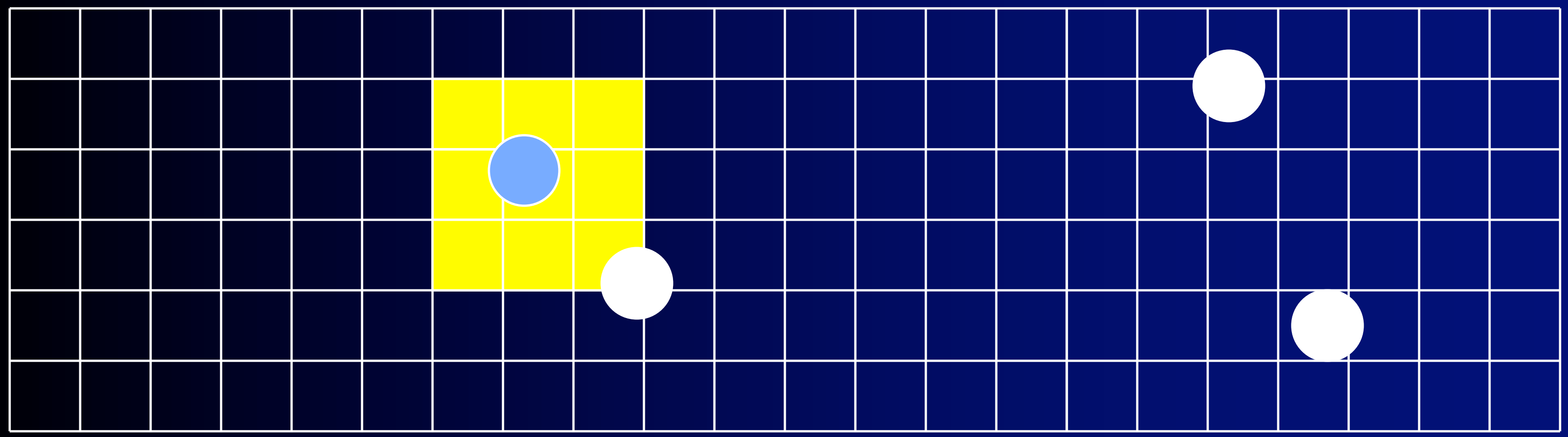


Avoiding Collision Detection

- General approach:
  - Observations: collisions are rare
    - Most of the time, objects are not colliding
  - Use various filters to remove as many objects as possible from the comparison set

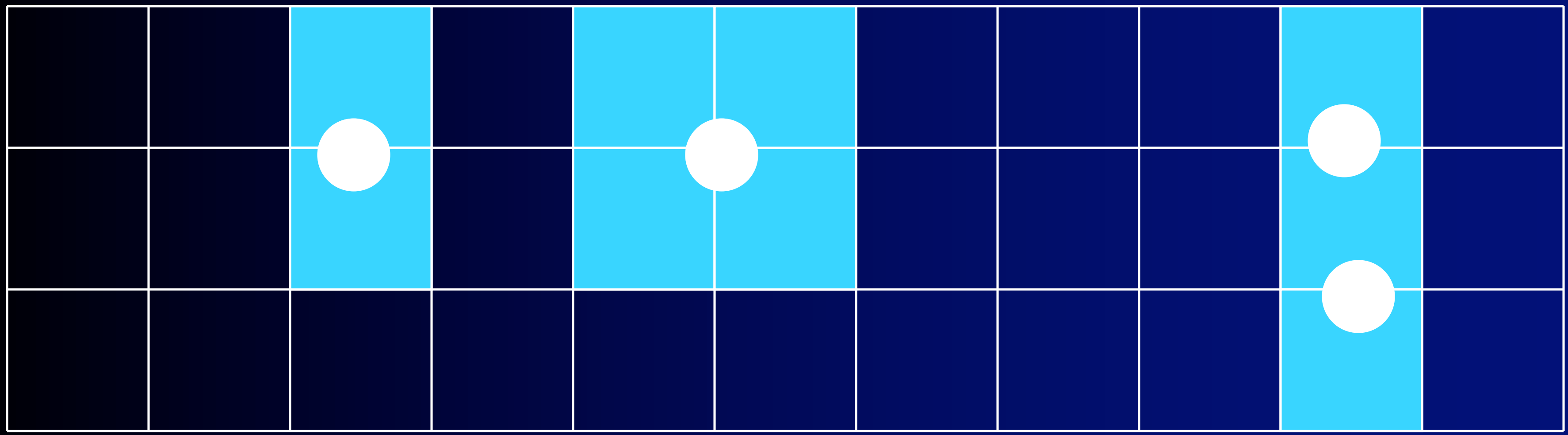
Area of Interest

- Avoid most of the calculations by using a grid:
  - Size of cell = diameter of biggest object
- Test objects in cells adjacent to object’s center
  - Can be computed using mod’s of objects coordinates:
    - bin sort
  - Linear in number of objects



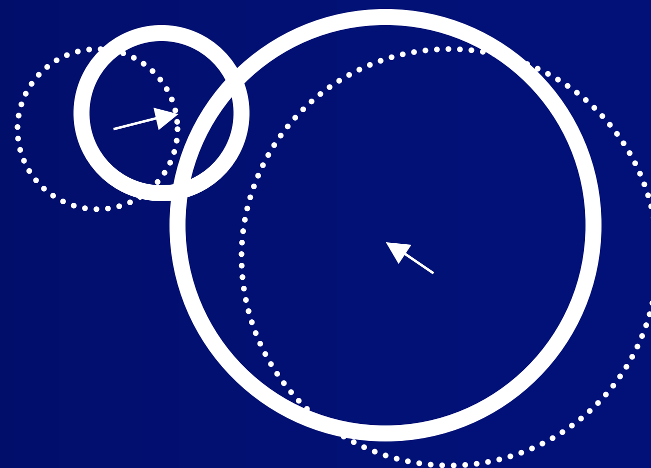
Detect that a collision occurred

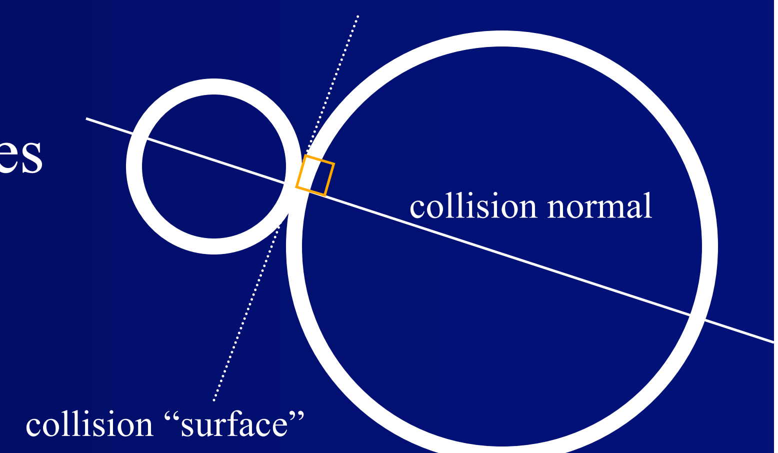
- Alternative if many different sizes
  - Cell size can be arbitrary
  - E.g., twice size of average object
- Test objects in cells touched by object
  - Must determine all the cells the object touches
  - Works for non-circles also



Circle-circle Collision Response

- Determine the time of the collision
  - Interpolate based on old and new positions of objects
- Determine where objects are when they touch
  - Backup positions to point of collision
- Determine the collision normal
  - Bisects the centers of the two circles through the colliding intersection





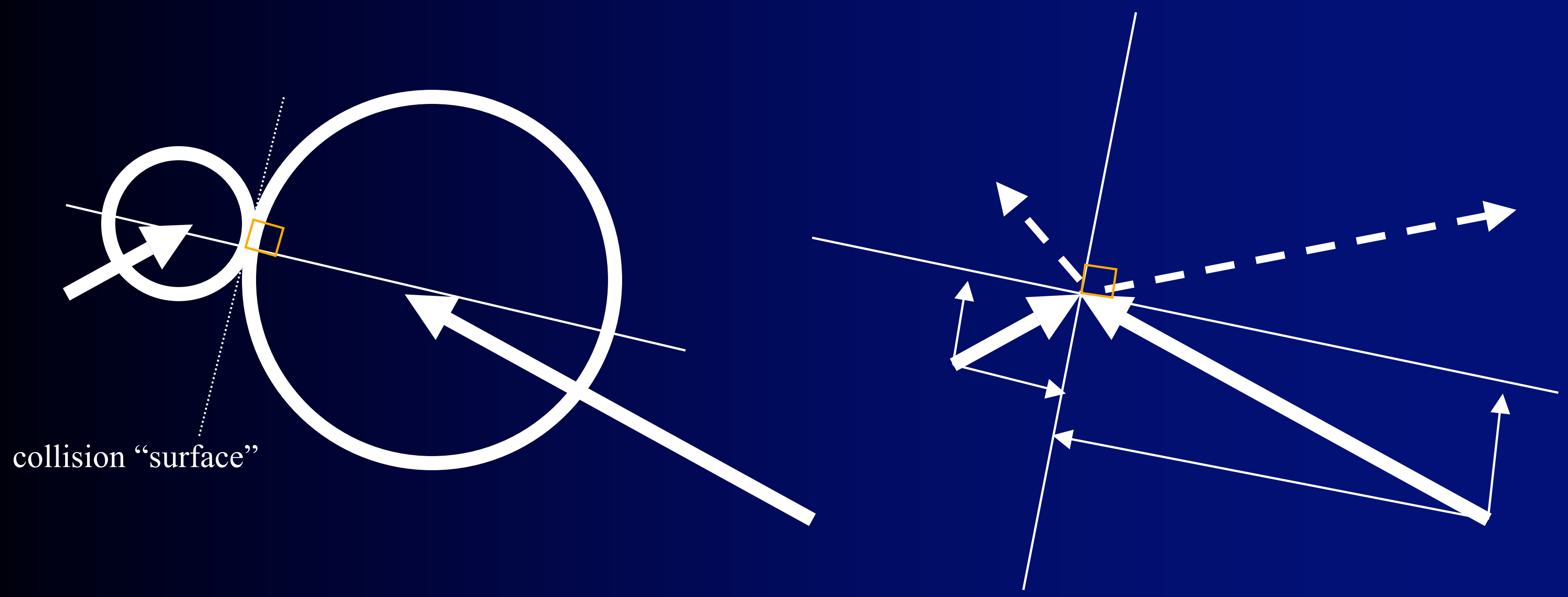
Circle-circle Collision Response

- Determine the velocity: assume elastic, no friction, head on collision
- Conservation of Momentum (mass * velocity):
  - $m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2$
- Conservation of Energy (Kinetic Energy):
  - $m_1 v_1^2 + m_2 v_2^2 = m_1 v'^2_1 + m_2 v'^2_2$
- Final Velocities
  - $v'_1 = (2m_2 v_2 + v_1(m_1 - m_2))/(m_1 + m_2)$
  - $v'_2 = (2m_1 v_1 + v_2(m_1 - m_2))/(m_1 + m_2)$
    - What if equal mass, $m_1 = m_2$
    - What if m_2 is infinite mass?

Circle-circle Collision Response

For non-head on collision, but still no friction:

- Velocity change:
  - Maintain conservation of momentum
  - Change of velocity reflect against the collision normal



Must be careful

- Round-off error in floating point arithmetic can throw off computation
  - Careful with divides
- Especially with objects of very different masses

Avoiding Physics in Collisions

- For simple collisions, don't do the math
  - Two identical balls swap velocities
- For collisions between dissimilar objects
  - Create a collision matrix