Introduction to 3D Graphics

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Basic Issues

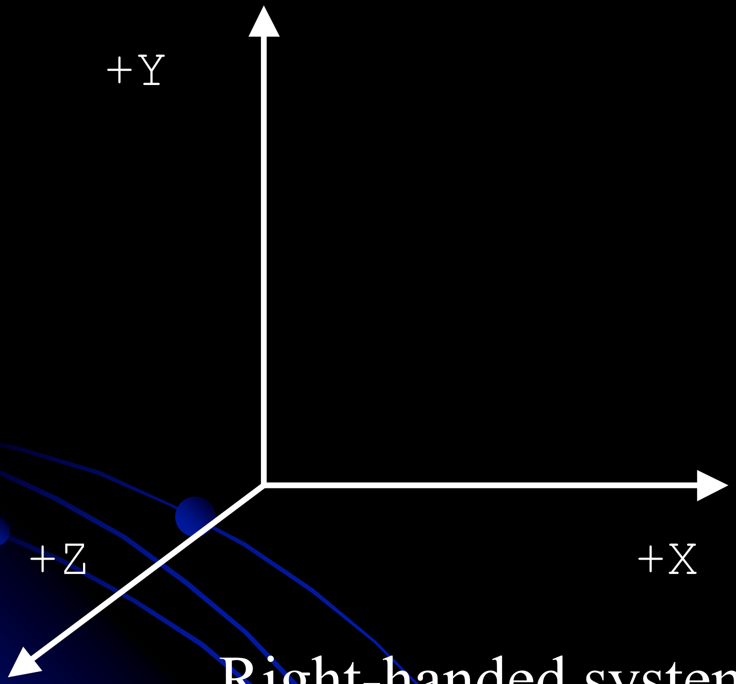
- Given an internal model of a 3D world, with textures and light sources how do you project it on the screen from any perspective *fast.*
  - Restrictions on geometry
  - Restrictions on viewing perspective
  - Lots of algorithms

- Questions
  - How do I draw polygons on the screens?
  - Which polygons should I draw?
  - How should I rasterize them (for lighting and texture)?

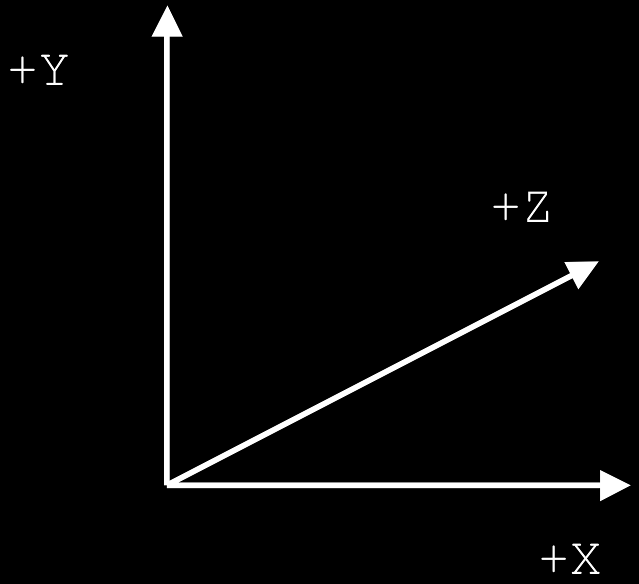
Overview: Simple 3D graphics

- 3D space
- Points, Lines, Polygons, and Objects in 3D
- Coordinate Systems
- Translation, Scaling, and Rotation in 3D
- Projections
- Solid Modeling
- Hidden-surface removal
- Z-Buffering

3D Space



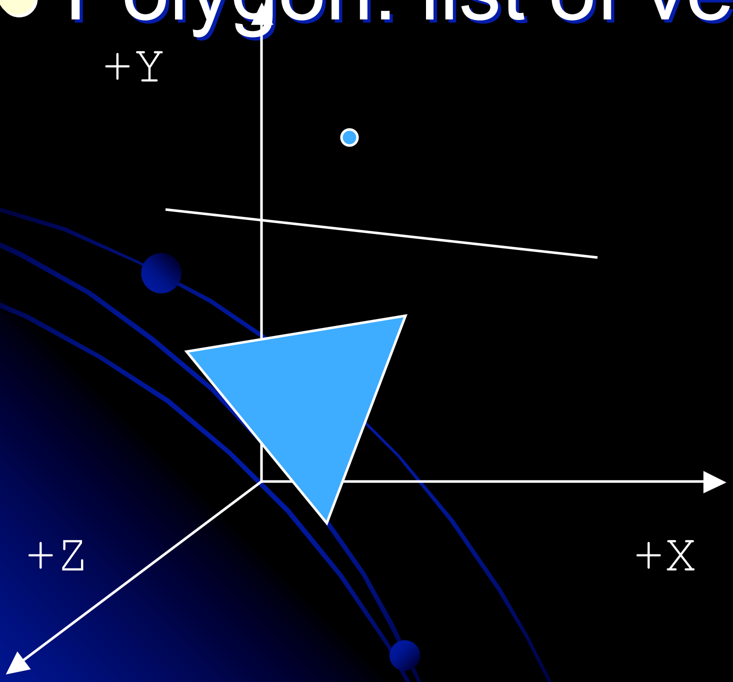
Right-handed system



Left-handed system

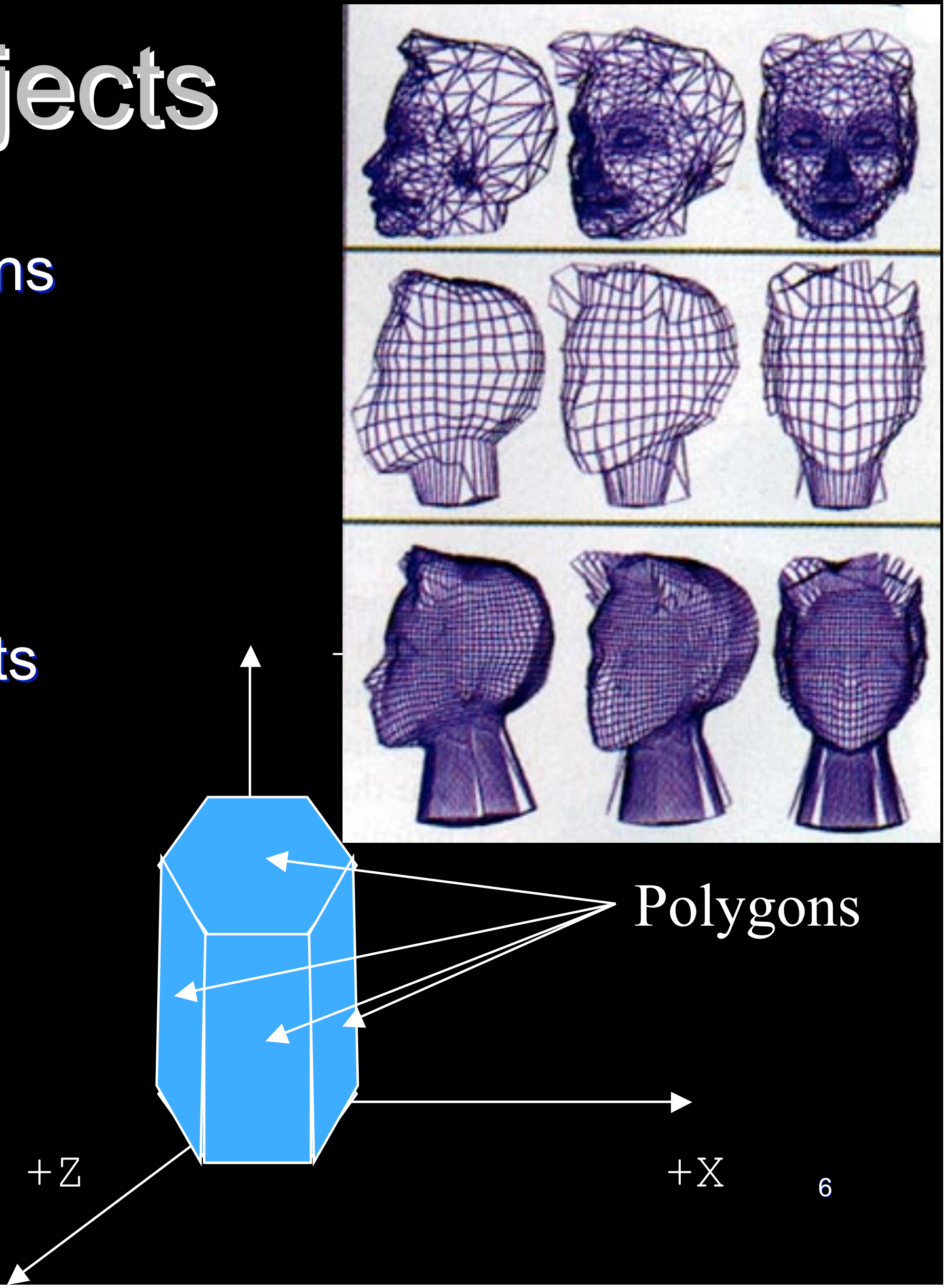
Points, Lines, Polygons

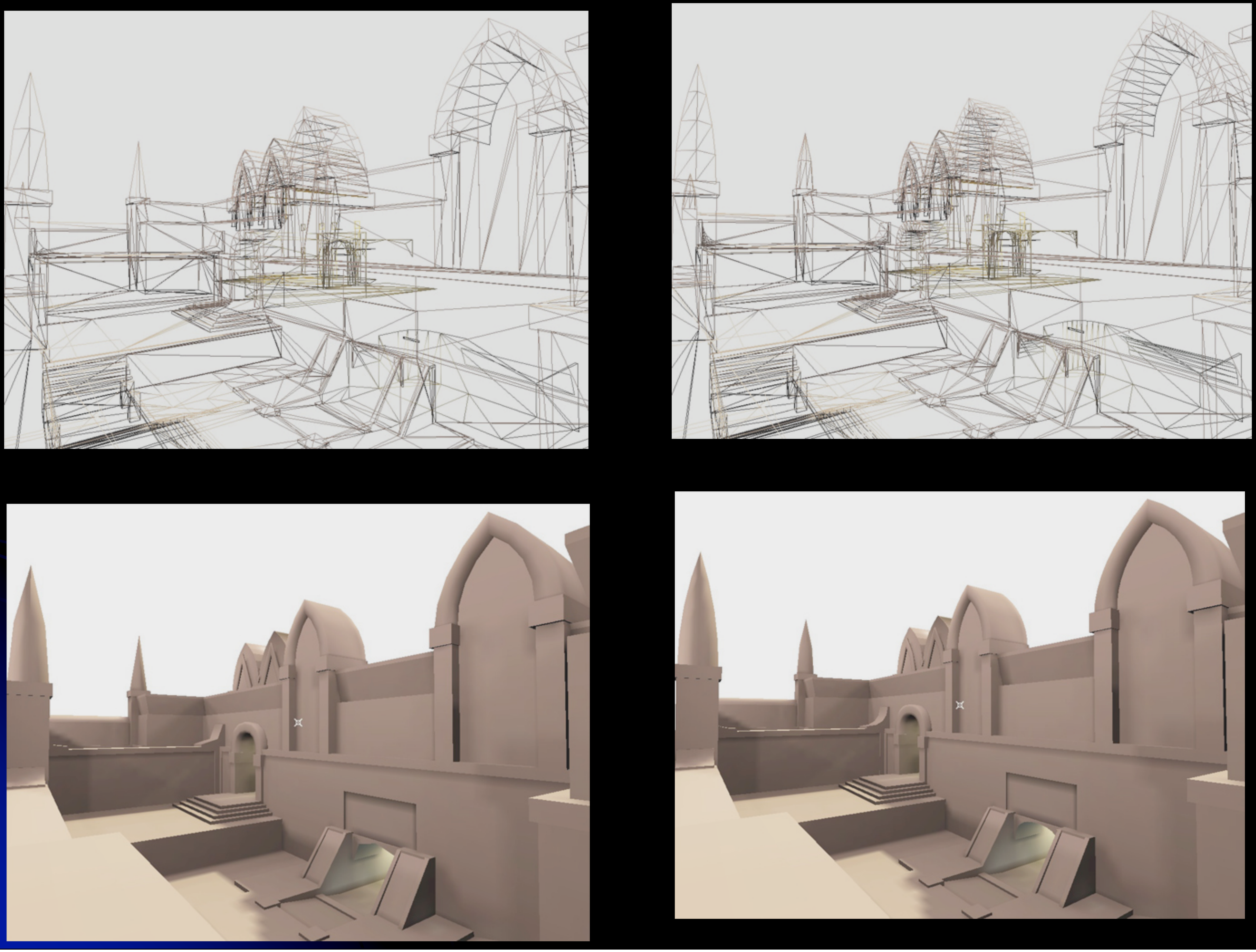
- Points: x, y, z
- Line: two points
- Polygon: list of vertices, color/texture



Objects

- Made up of sets of polygons
  - Which are made up of lines
  - Which are made of *points*
  - No curved surfaces
- Just a “shell”
  - Not a solid object
- Everything is a set of points
  - *In local coordinate system*





Object Transformations

- Since all objects are just sets of points, we just need to translate, scale, rotate the points.
- To manipulate a 3D point, use matrix multiplication.
- Translation:

$$[x' \quad y' \quad z' \quad 1] = [x \quad y \quad z \quad 1] \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ dx & dy & dz & 1 \end{vmatrix}$$

Scaling

- Constant Axis Scaling

$$[x' \; y' \; z' \; 1] = [x \; y \; z \; 1] \left| \begin{matrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \right|$$

- Variable Axis Scaling

$$[x' \; y' \; z' \; 1] = [x \; y \; z \; 1] \left| \begin{matrix} sx & 0 & 0 & 0 \\ 0 & sy & 0 & 0 \\ 0 & 0 & sz & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \right|$$

Rotation

- Parallel to x-axis

```
[x' y' z' 1] = [x y z 1] |  1       0       0      0 |
                         |  0   cos r   sin r      0 |
                         |  0  -sin r   cos r      0 |
                         |  0       0       0      1 |
```

- Parallel to y-axis

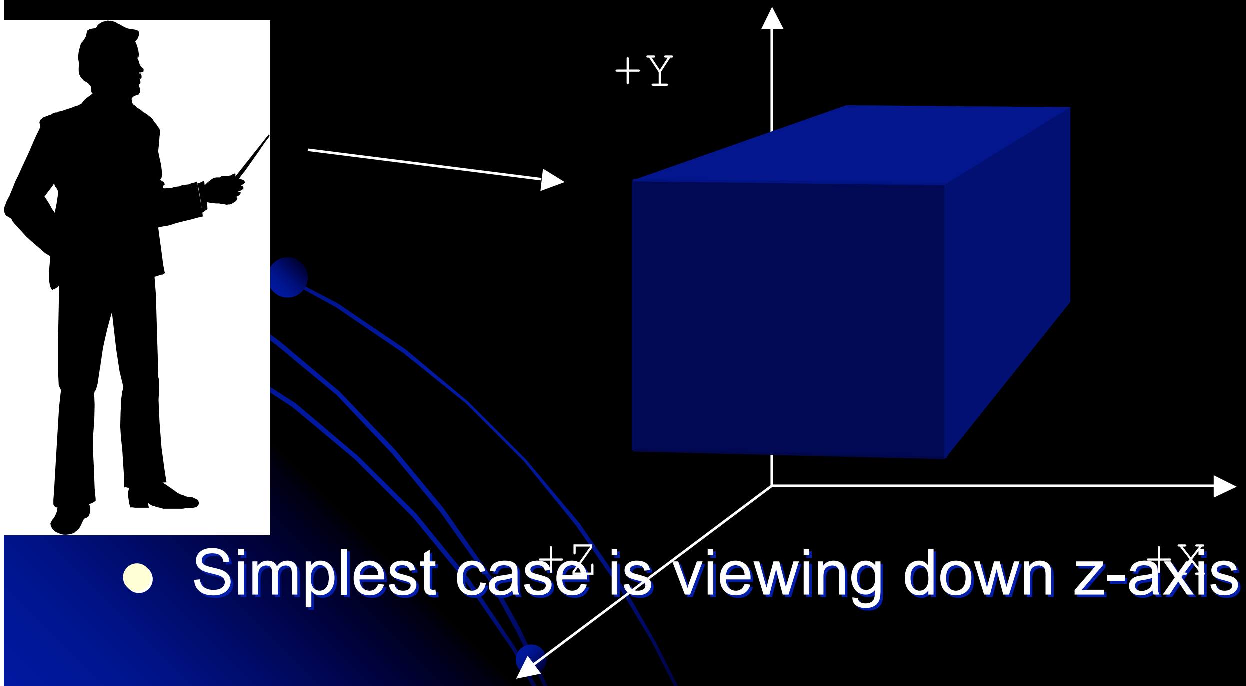
```
[x' y' z' 1] = [x y z 1] | cos r      0 -sin r      0 |
                         |     0      1      0      0 |
                         | sin r      0  cos r      0 |
                         |     0      0      0      1 |
```

- Parallel to z-axis

```
[x' y' z' 1] = [x y z 1] | cos r sin r      0      0 |
                         |-sin r cos r      0      0 |
                         |     0     0      1      0 |
                         |     0     0      0      1 |
```

Three Coordinate Systems

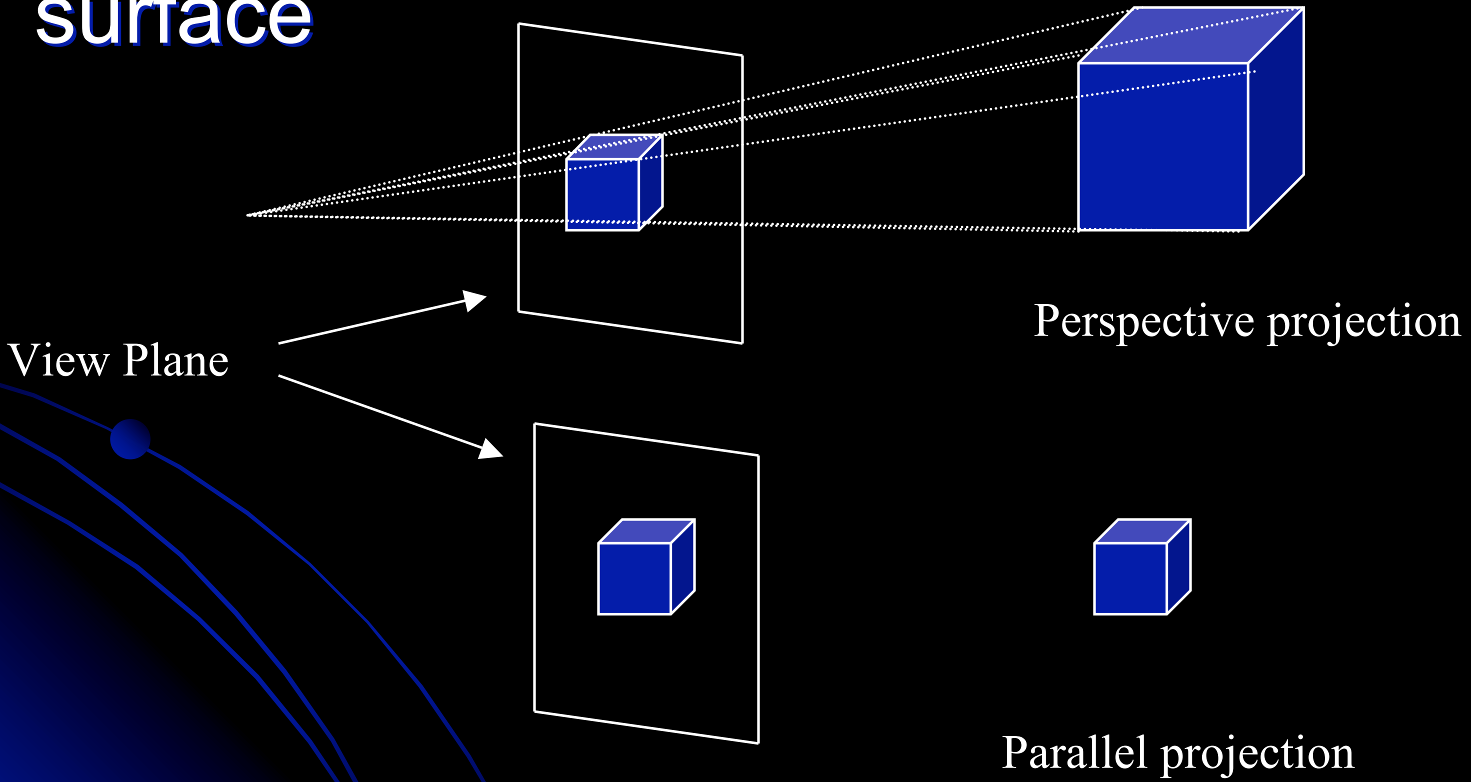
- World-centered: Where objects are in the world
- Object-centered: Relative to position of object
- View-centered: Relative to the position of viewer



- Simplest case is viewing down z-axis

Projections

- Mapping a 3D object onto a 2D viewing surface



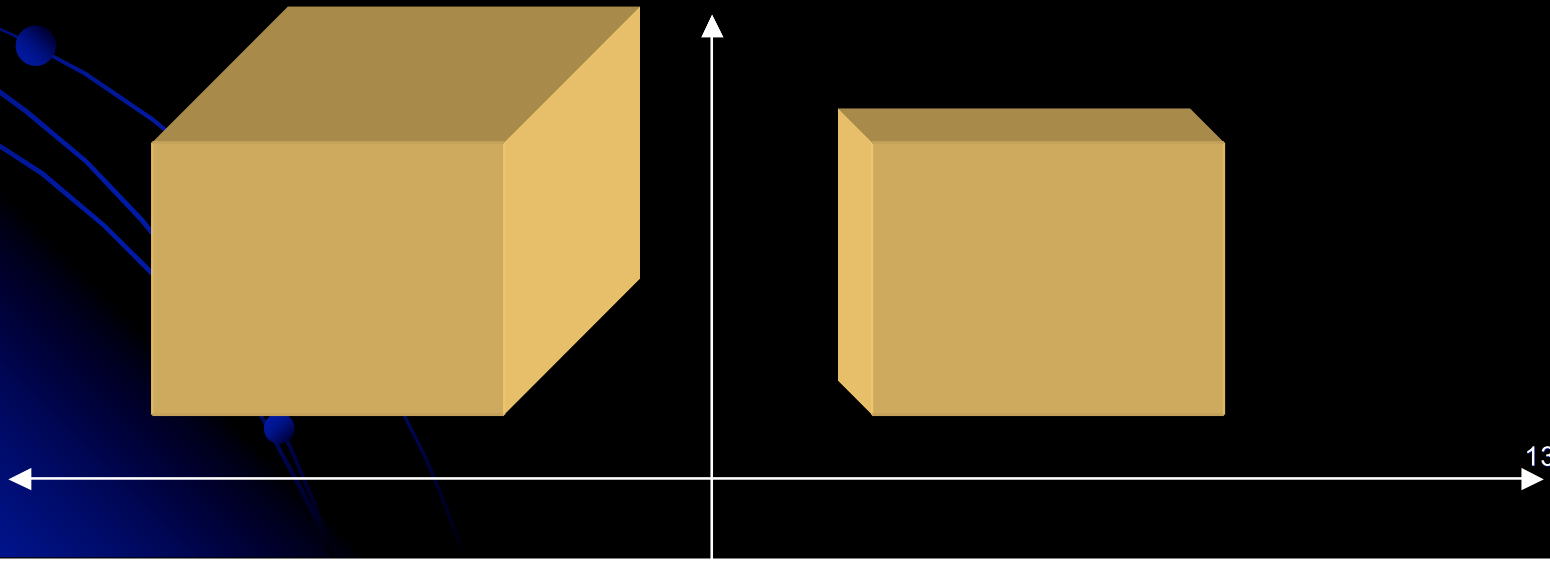
View Plane

Perspective projection

Parallel projection

Projections

- Parallel
  - If viewing down z-axis, just discard z component
- Perspective
  - If viewing down z-axis, scale points based on distance.
  - x_screen = x / z
  - y_screen = y / z



Projections

- Usually not viewing down center of z axis.
- Usually x = 0 and y = 0 at bottom left
- Correct by adding 1/2 screen size
  - x_screen = x/z + 1/2 screen width
  - y_screen = y/z + 1/2 screen height

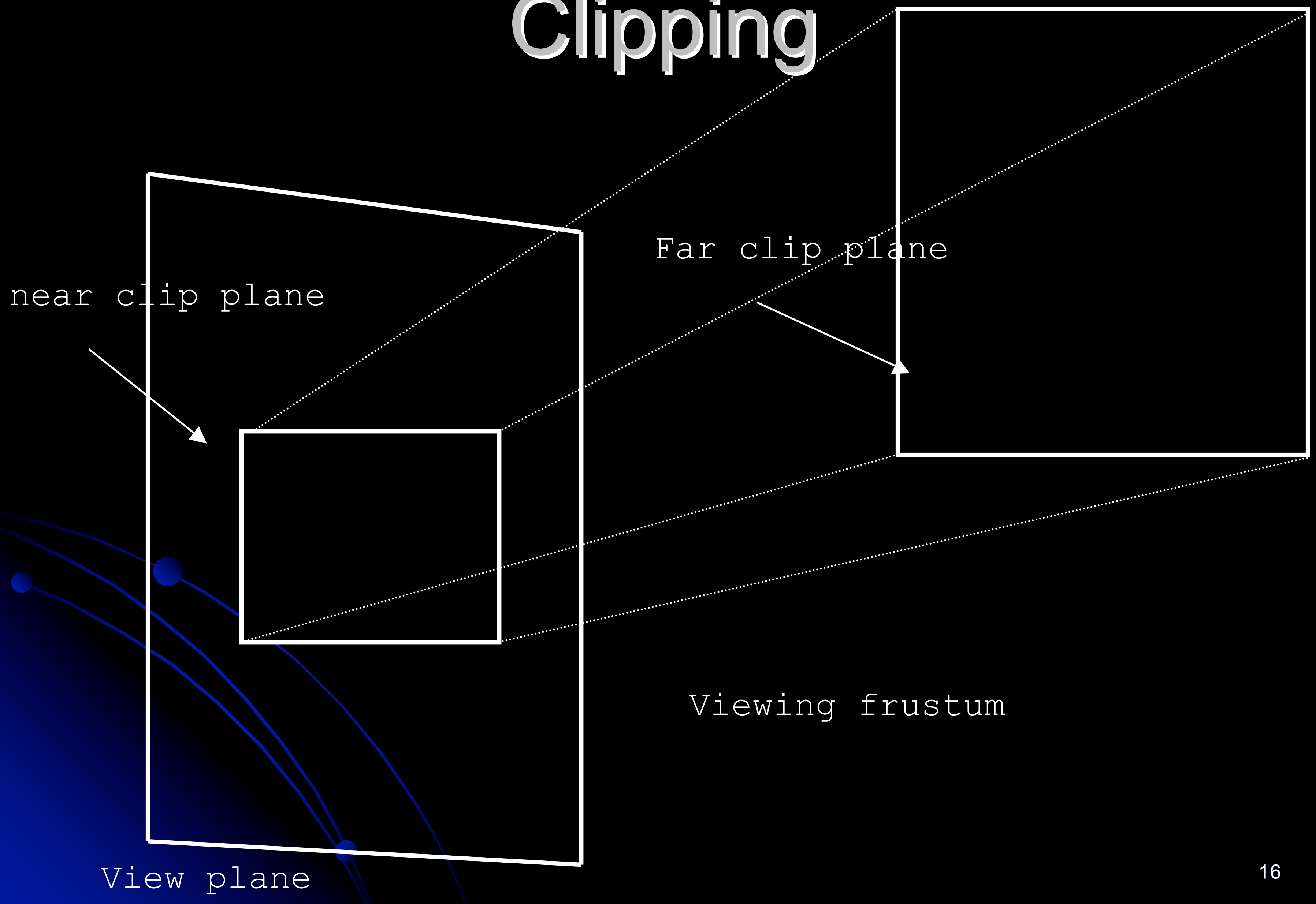
- To get perspective right, need to know field of view, distance to screen, aspect ratio.
  - Often add scaling factor to get it to look right
  - x_screen = x*scale /z + 1/2 screen width

Field of View

- To simulate human vision:
  - 110-120 degrees horizontally
  - < 90 vertically

- Think of the viewing pyramid or frustum

Clipping

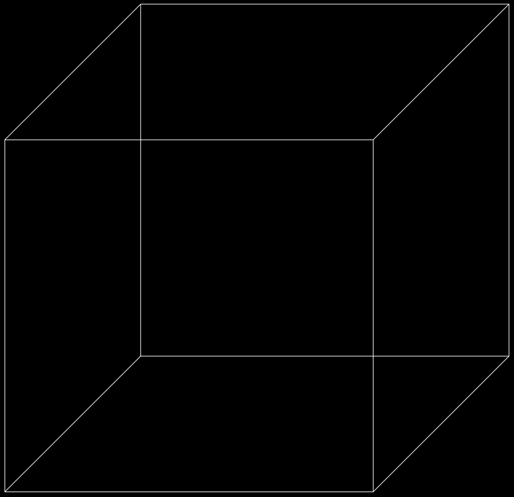


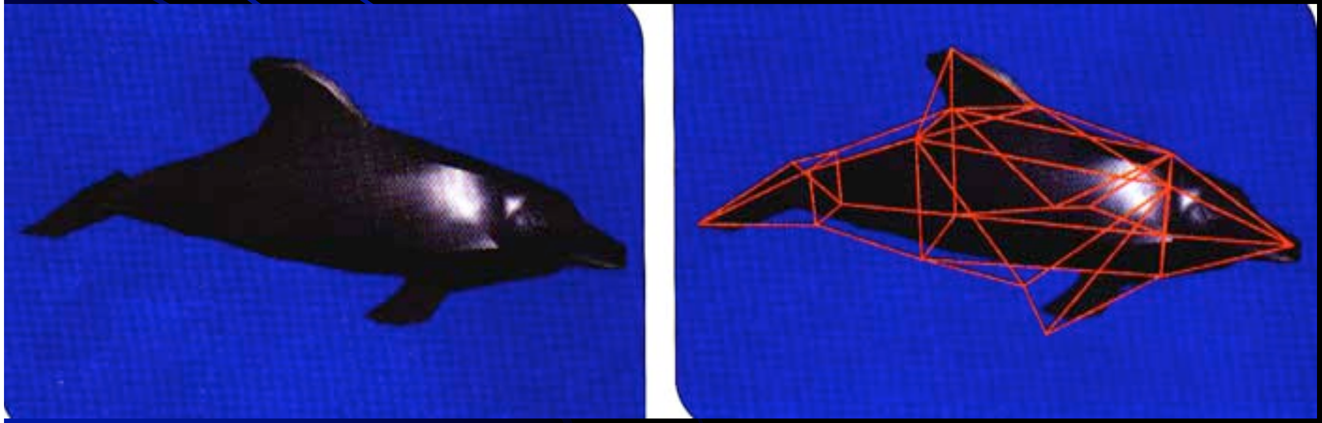
Drawing the Surface

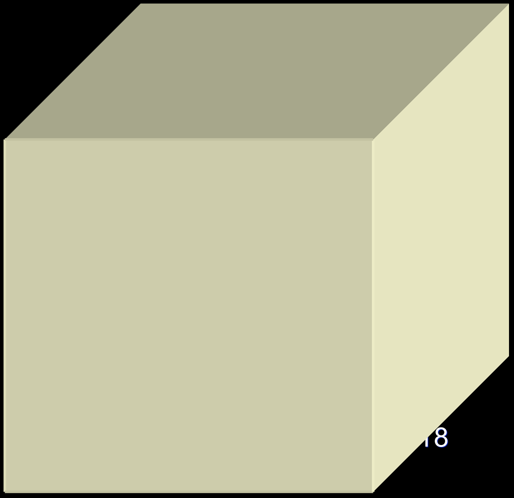
- Split triangles and fill in as described earlier

Solid Modeling

- Which surfaces should be drawn?
  - Object space methods
    - Hidden Surface Removal
    - Painters Algorithm
    - BSP Trees
  - Image space methods
    - Z-Buffering
    - Ray Casting



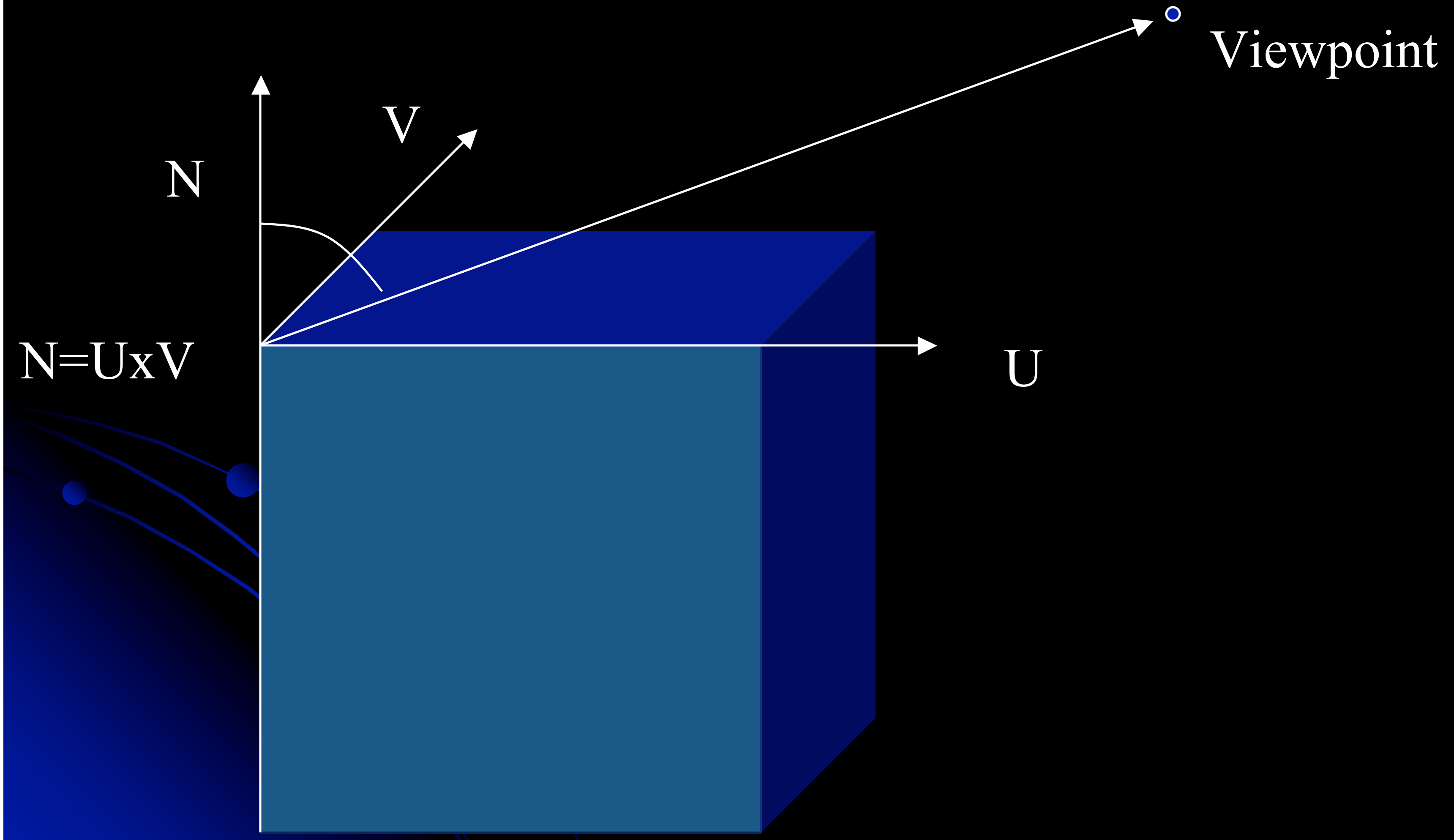




Hidden Surface Removal

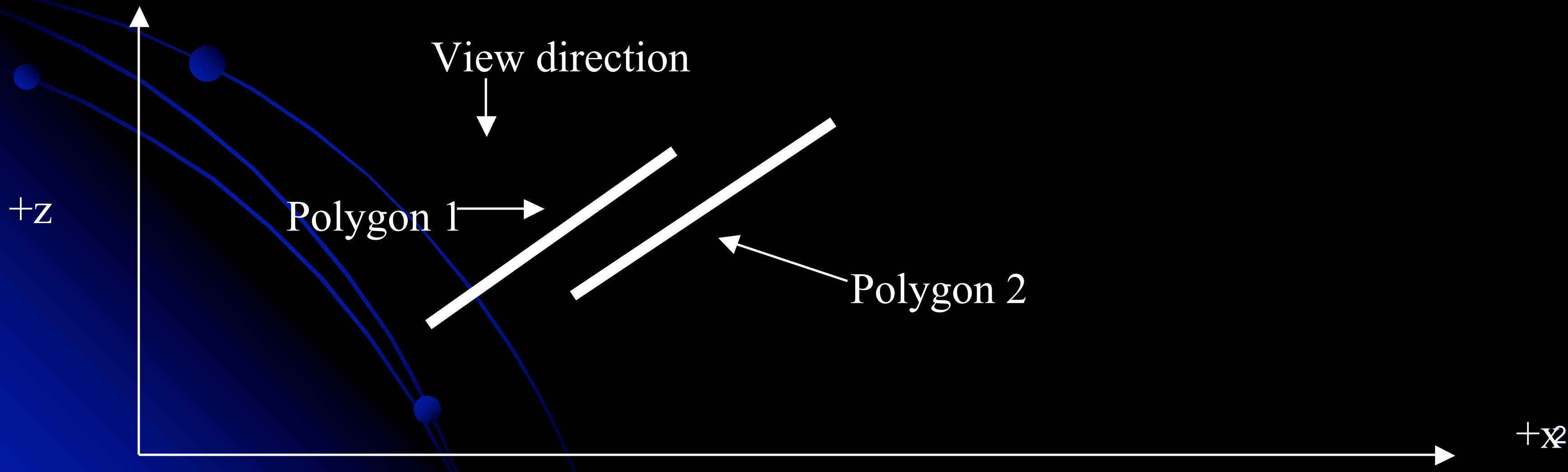
- Step 1:
  - Remove all polygons outside of viewing frustum
- Step 2:
  - Remove all polygons that are facing away from the viewer
  - If the dot product of the view vector and the surface normal is >= 90 degrees, it is facing away.
    - Surface normal = cross product of two co-planar edges.
    - View vector from normal point to viewpoint
- Step 3:
  - Draw the visible faces in an order so the object looks right.

Testing if Surface is Visible



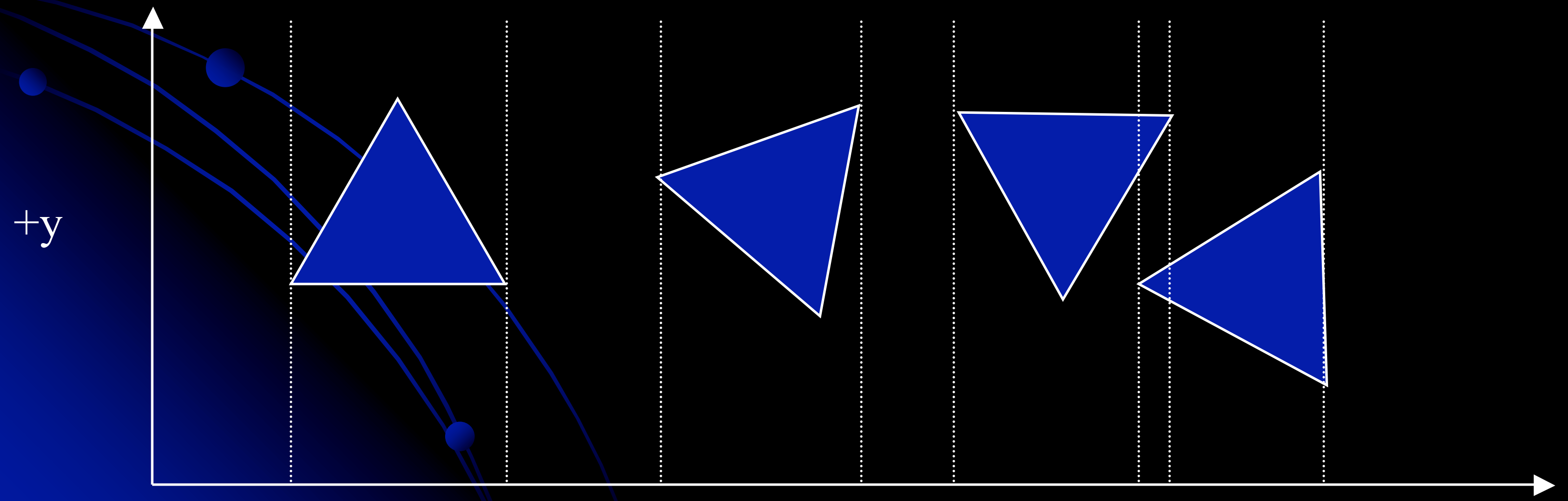
Painter's Algorithm

- Basic idea
  - Sort surfaces and then draw so looks right.
  - If all surface are parallel to view plane, sort based on distance to viewer, and draw from back to front.
  - Worst-case is O(n^2)
  - Otherwise, have five tests applied to each pair to sort.
  - Order tests by cheap to expensive
- Why can't come up with order (max, min, mean)?



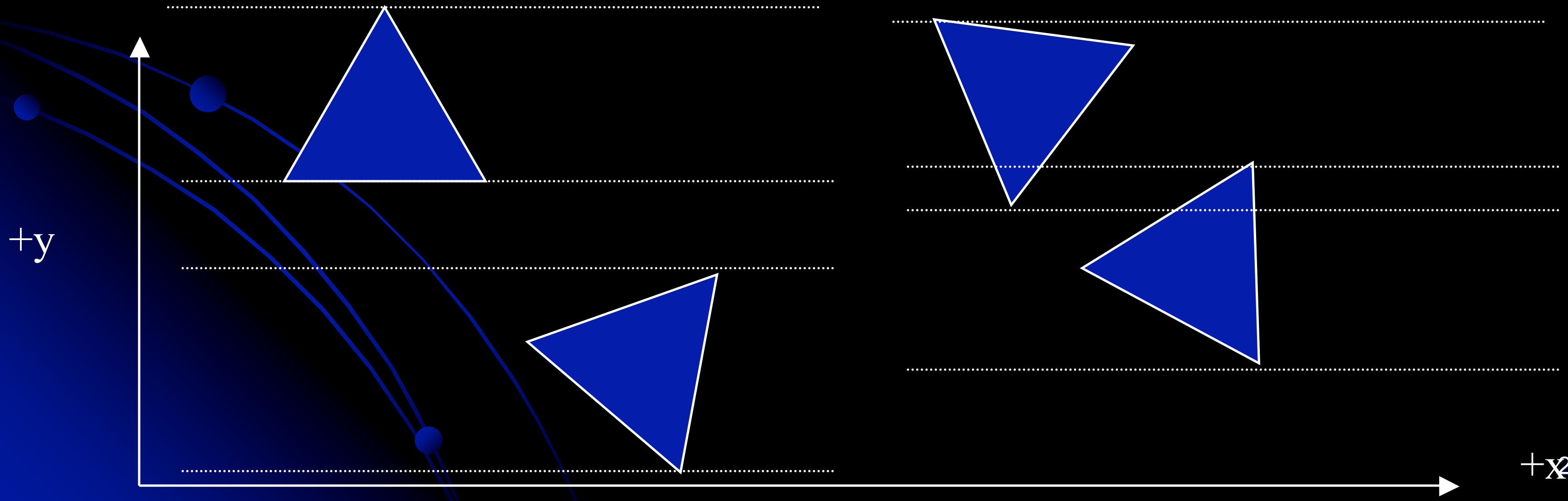
Test 1: X overlap

- If the x extents of two polygons do not overlap, then order doesn't matter and go to next pair.
- If x extents overlap, goto test 2.



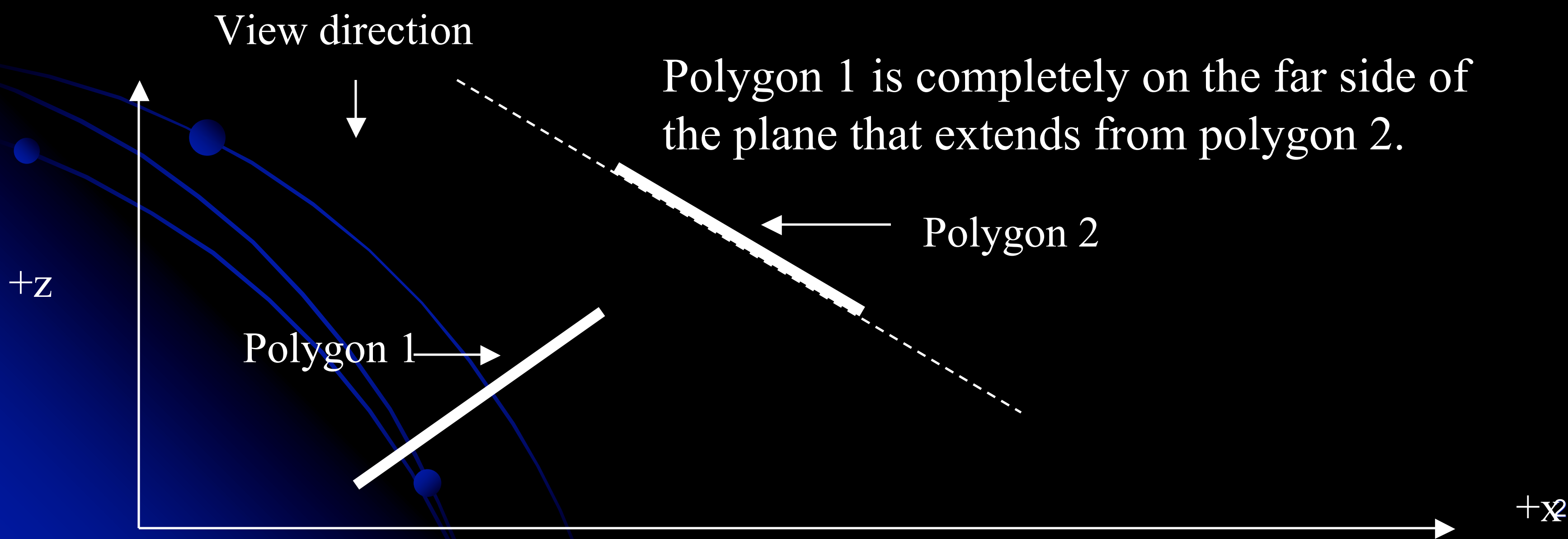
Test 2: Y Overlap

- If the y extents of two polygons do not overlap, then order doesn't matter.
- If y extents overlap, goto test 3.



Tests 3 & 4

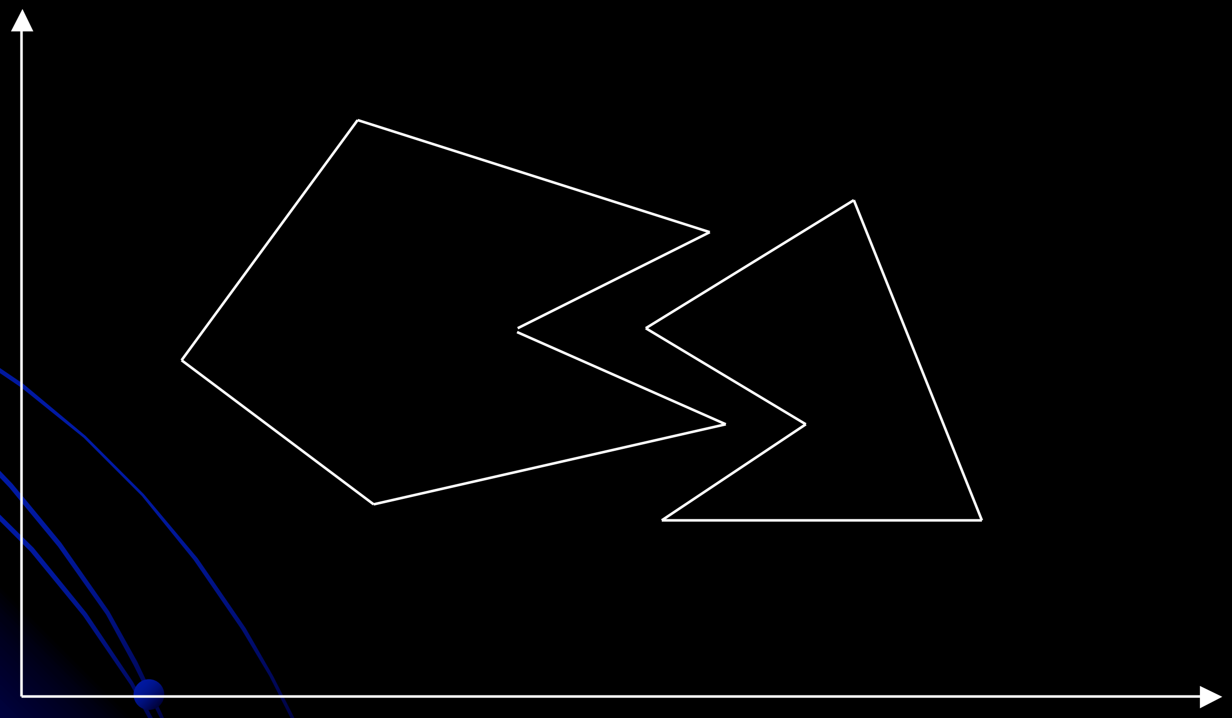
- Extend polygons to be a cutting plane
  - If a polygon can be contained within the cutting plane of the other, that polygon should be drawn first.
  - If neither can be contained, go to step 5.



Polygon 1 is completely on the far side of the plane that extends from polygon 2.

Test 5

- Only needs to be consider if have concave polygons.



How to make it easier

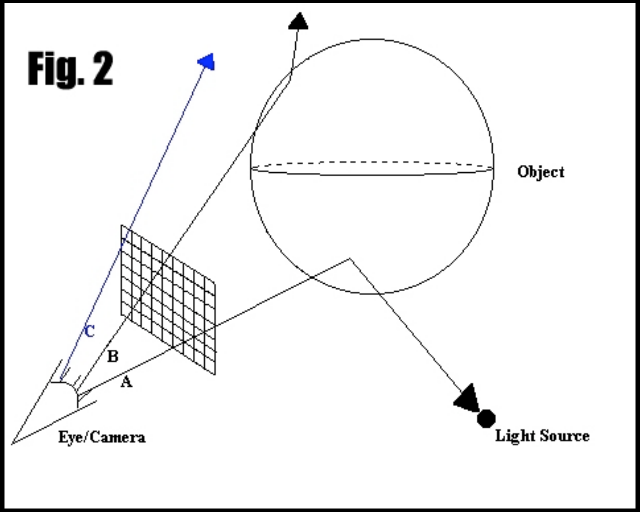
- Use convex objects
- Avoid long objects (like walls) that can overlap each other in multiple dimensions
- Avoid intersecting objects and polygons

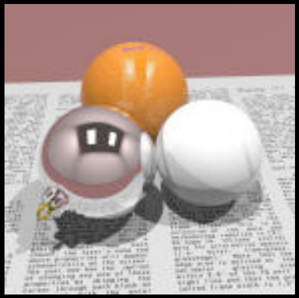
Z-buffer

- Z-buffer holds the z-coordinate of ever pixel
  - Usually 16 or 32-bits/pixel
- Initialize all values to maximum depth
- Compute the z value of every point of every non-back facing polygon
  - Not too hard if all polygons are triangles or rectangles
  - Do this during the filling of the triangles
- If z of point < z in Z-buffer, save the color of the current point and update Z-buffer
  - otherwise throw away point and move on
- In all 3D hardware now

Ray Tracing

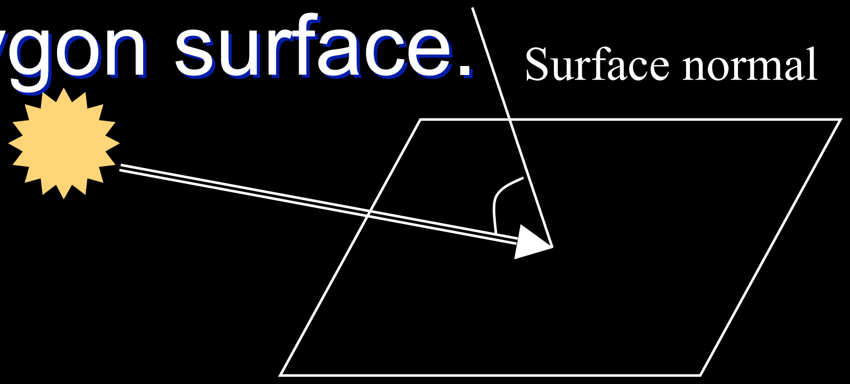
- Technique that mimics physical processes of light
- Extremely computationally intensive, but beautiful
  - Hidden surface removal
  - Transparency
  - Reflections
  - Refraction
  - Ambient lighting
  - Point source lighting
  - Shadows





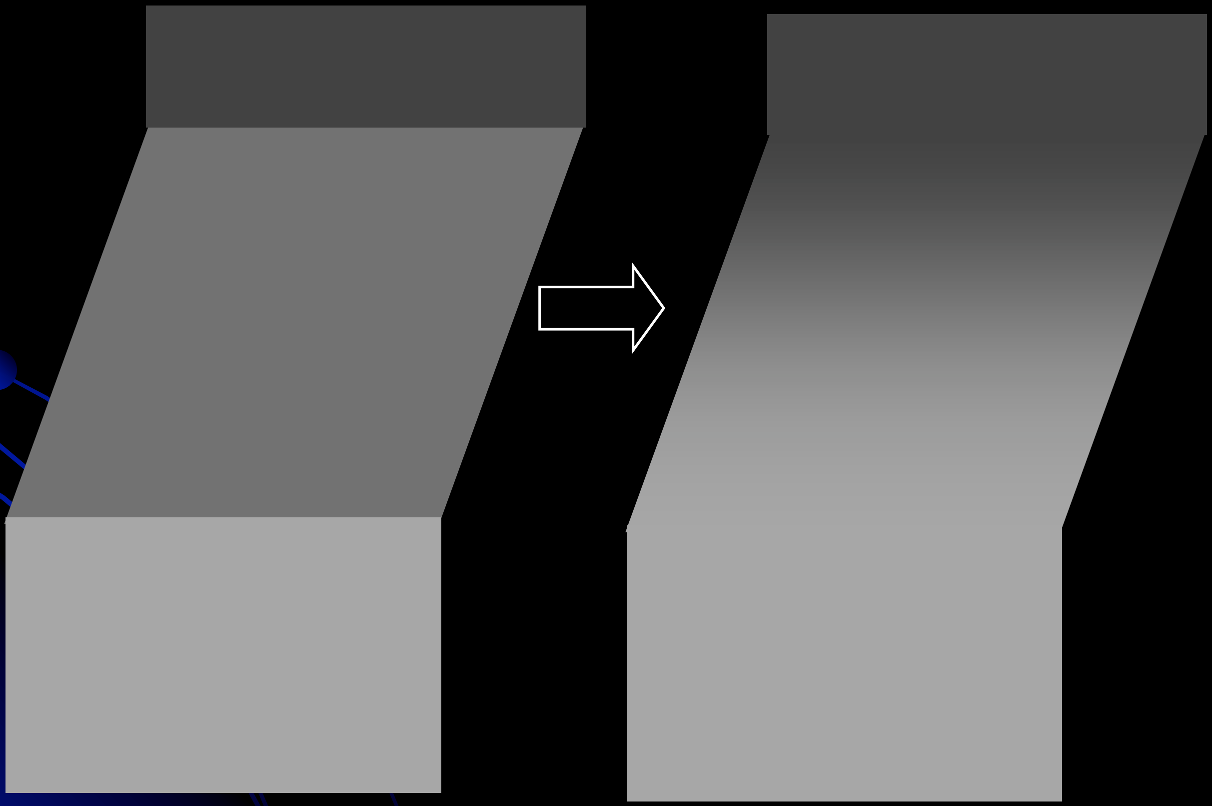
Shading

- Compute lighting based on angle of light on polygon surface.



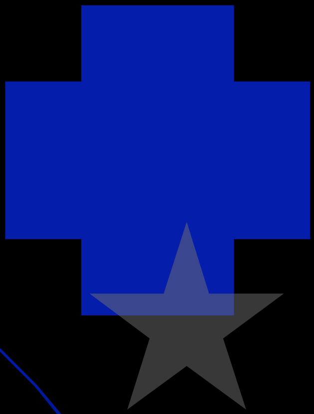
Gouraud Shading

- Compute shading for each pixel by averaging shading based on distance and shading of vertices.



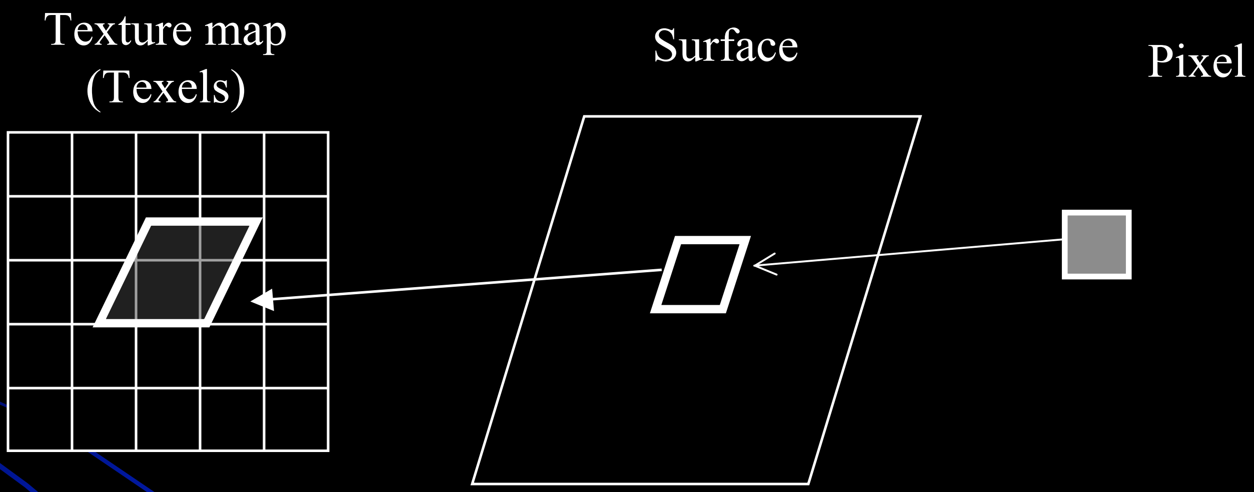
Transparency

- Use an extra set of bits to determine transparency
  - Alpha
  - Blend present value of the color buffer with new values.



Texture Mapping

- Apply stored bit map to a surface



- Average texels covered by pixel image

3D Collision Detection

- Can’t be done in image space
- Usually use hierarchical approach
  - First find objects in same 3D cells
  - Second test for overlaps in bounding sphere or box
  - Third
    - Good enough!
    - Check for polygon collisions
- Accurate 3D collision detection is very expensive