eecs281 Data Structures and Algorithms

Discussion 4: Week of Sep 26, 2011

Outline

- BST Range Search
- K-D Trees and Multi-dimensional Search
- Nearest Neighbor Search

Binary Search Tree Review

- Invariants:
  - Left nodes are always < current node.
  - Right nodes are always > current node value.
- Therefore:
  - At each node, we can split the search space in half, enabling O(logn) search times.

Range Search

- Regular Search:

```
node * query(node * root, int target_val){
      if (target_val == root->value)
            return root;
      if (target_val < root->value)
            return query(root->left, target_val);
      if (target_val > root->value)
            return query(root->right, 
target_val);
}
```

Range Search

- In order to be able to query a range, what should we do?

Range Search

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  - What should we return?

Range Search

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  - We should return a list of matches
  - Recursive or Iterative?

Range Search

- In order to be able to query a range, what should we do?
  - We should return a list of matches
  - Recursive

Range Search – Recursive Definition

- Take 5 minutes to develop a recursive definition.
- On a node of value x:
  - 1. What if min < max < x?
  - 2. What if min <= x <= max
  - 3. What if x < min < max?

Range Search – Recursive Definition

- 1. What if min < max < x?
  - X is not in the set, search more to the left if possible.
- 2. What if min <= x <= max
  - X is in the set, add it, and then search both sub-trees if possible.
- 3. What if x < min < max?
  - X is not in the set, search more to the right if possible.

```
vector<node *> BST::range_query(int start_range, int end_range){
        vector<node *> answers;
        range_query_h(root_node, start_range, end_range, answers);
        return answers;
}

void BST::range_query_h(node * current, int start_range, int end_range,
                        vector<node *>& answers){
        if (end_range < current->val){
          range_query_h(current->left, start_range, end_range, answers);
          return;
        }

        if (start_range <= current->val && end_range >= current->val){
          range_query_h(current->left, start_range, end_range, answers);
          answers.push_back(this);
          range_query_h(current->right, start_range, end_range, answers);
          return;
        }

        if (start_range < current->val){
          range_query_h(current->right, start_range, end_range, answers);
          return;
        }
        assert(0); // something really bad happened
}
```

K-D Trees Review

- Like Binary Search Trees except:
  - Each node has multiple keys.
  - At each branch, you consider a different key to split on.
- Due to similarity, you can easily adapt the algorithms for querying BSTs for use in K-D trees.

Multi-dimensional Search

- BST version:
  - 1. if goal < current node value, return left
  - 2. if goal == current node value, return the node
  - 3. if goal > current node value, return right
- How to modify this for K-D trees?

Multi-dimensional Search

- K-D Tree version:
  - 1. if goal[dim] < current node value[dim], dim = (dim+1) % total_dim, return left
  - 2. if goal[dim] == current node value[dim], return the node
  - 3. if goal[dim] > current node value[dim], dim = (dim+1) % total_dim, return right
- How to modify this for K-D trees?
  - Keep track of which dimension you are searching.
  - As you keep searching down, be sure to keep track of which dimension the nodes are being split on.

Nearest Neighbor Search

- Goal:
  - Given a point in a K-dimensional space, find the closest point stored in a data structure.

Nearest Neighbor Search

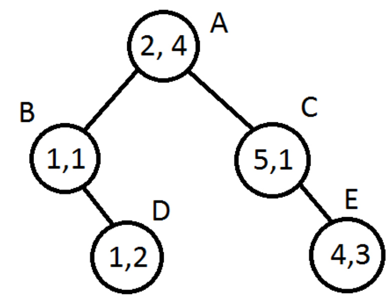
- Goal:
  - Given a point in a K-dimensional space, find the closest point stored in a data structure.
- Use a K-D Tree!

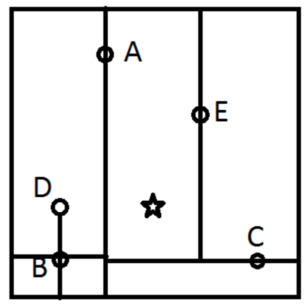
Nearest Neighbor Search

- Steps:
  - 1. Start with root node and use depth-first search to find where you would insert the node if you were inserting it. Save this as current best
  - 2. Go up one node. If it's better than closest best, it becomes closest best.
  - 3. Check whether there could be any points on the other side of the splitting plane by checking the distance between the target node and the splitting plane.
  - 4. Repeat steps 2-3 until you are at the root node.

Nearest Neighbor Search

- Given a K-D Tree with the points:
  - (1, 1) (1, 2) (5, 1) (4, 3) (2, 4)
  - Find the point closest to (3, 2) (manhatten space)





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- 1. Depth-first Search to E.
- 2. E is the best distance = 3.
- 3. Go back to C:
  - C is not closer (3 away). No nodes below C.
- 4. Go back to A.
  - A is not closer. Check if other side of A could be closer. Abs(Goal.x – A.x) = 1 < 3. Check other side.

Nearest Neighbor Search

- 1. Depth-first Search to E.
- 2. E is the best distance = 3.
- 3. Go back to C:
  - C is not closer (3 away). No nodes below C.
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- 5. DFS to D. D is closer than E (2 away)

Nearest Neighbor Search

- 3. Go back to C:
  - C is not closer (3 away). No nodes below C.
- 4. Go back to A.
  - A is not closer. Check if other side of A could be closer. Abs(Goal.x – A.x) = 1 < 3. Check other side.
- 5. DFS to D. D is closer than E (2 away)
- 6. Go back to B.
  - B is not closer. No nodes below B.

Nearest Neighbor Search

- 4. Go back to A.
  - A is not closer. Check if other side of A could be closer. Abs(Goal.x – A.x) = 1 < 3. Check other side.
- 5. DFS to D. D is closer than E (2 away)
- 6. Go back to B.
  - B is not closer. No nodes below B.
- 7. Go back to the A. Checked both sub-trees, therefore done.