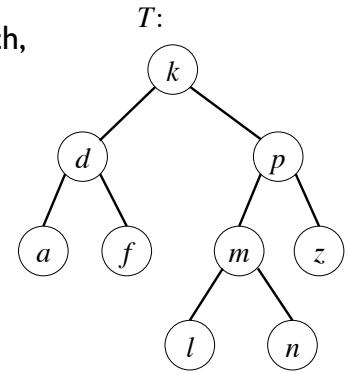
eecs 281 DATA STRUCTURES AND ALGORITHMS

Lecture 7: BST Range Search
k-d Trees
Binary Space Partitioning Trees
Nearest-Neighbor Search

BST Range Search

Instead of finding an exact match, find all items whose keys fall between a range of values, e.g., between *m* and *z*, inclusive

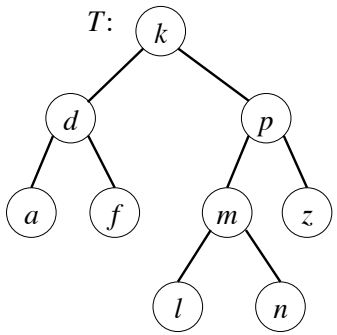
Example applications:



BST Range Search: Algorithm

```
void
rangesearch(Link root, Key searchrange[],
            Key subtreerange[], List results)
```

1. if root is in search range, add root to results
2. compute range of left subtree
3. if search range covers all or part of left subtree, search left
4. compute range of right subtree
5. if search range covers all or part of right subtree, search right
6. return results

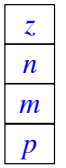


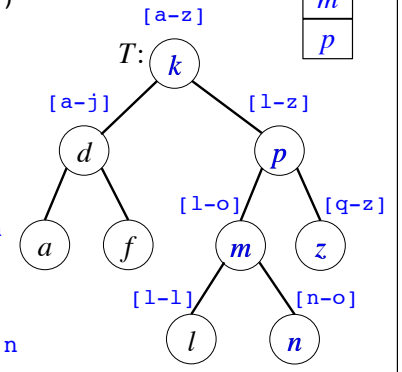
(Other traversal orders are also ok)

BST Range Search: Example

```
                searchrange, subtreerange
rangesearch(T, [m-z], [a-z], results)
(T's range is "whole universe")
is k in [m-z]?
does [a-j] overlap [m-z]?
does [l-z] overlap [m-z]?
 search p's subtree
 is p in [m-z]? results ← p
 does [l-o] overlap [m-z]?
   search m's subtree
   is m in [m-z]? results ← m
   does [l-l] overlap [m-z]?
   does [n-o] overlap [m-z]?
    search n's subtree
    is n in [m-z]? results ← n
 does [q-z] overlap [m-z]?
   search z's subtree
   is z in [m-z]? results ← z
```

results:





BST Range Search: Support Functions

1. if root is in search range, i.e.,
   `root->key <= searchrange[MAX]`, and
   `root->key >= searchrange[MIN]`
   add node to results

2. compute subtree's range: replace upper (lower) bound of left (right) subtree's range by `root->key-1 (+1)`

3. if search range covers all or part of subtree's range, search subtree
   - each subtree covers a range of key values
   - compute overlap between subtree's range and search range
   - no overlap if either
     `searchrange[MAX] < subtreerange[MIN]` or
     `searchrange[MIN] > subtreerange[MAX]`

BST Range Search: Other Details

How to express range when the keys are floats?
- be careful with numerical precision and floating point errors [one of this week's discussion topic]

How to support duplicate keys?
- be consistent about using $\leq$ or $<$
- if $\leq$, the range for the left subtree would be closed, e.g., $[-\infty, 0]$, and the range for the right subtree half open, e.g., $(0, +\infty]$

Multidimensional Search

Example applications:

A k-d tree can handle all these queries with $O(\log n)$ insert and search times (it can also handle partial, range, and approximate matches)

k-d Trees

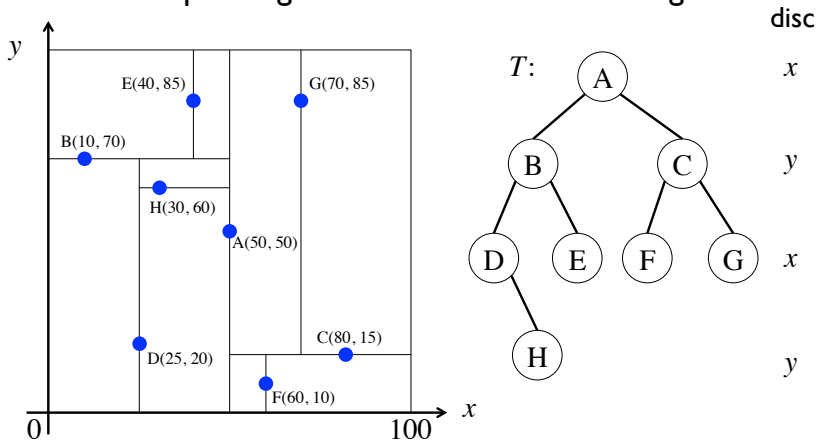
A k-d tree is a binary search tree (not covered in textbook, link to original 1975 paper on syllabus)

At each level of the k-d tree, keys from a different search dimension is used as the discriminator
- keys for any given level are all from the same dimension indicated by the discriminator
- nodes on the left subtree of a node have keys with value $<$ the node's key value along this dimension
- nodes on the right subtree have keys with value $>$ the node's key value along this dimension

We cycle through the dimensions as we go down the tree
- for example, given keys consisting of x- and y-coordinates, level 0 could discriminate by the x-coordinate, level 1 by the y-coordinate, level 2 again by the x-coordinate, etc.

k-d Trees: Example

Given points in a Cartesian plane on the left,
a corresponding k-d tree is shown on the right



k-d Tree Insert

```
void kdTree::
insert(Link &root, Item newitem, int disc)
{
  if (root == NULL) {
    root = new Node(newitem);
    return;
  }
  if (newitem.key[disc] < root->item.key[disc]) // or <=
    insert(root->left, newitem, (disc+1)%dim);
  else if (newitem.key[disc] > root->item.key[disc])
    insert(root->right, newitem, (disc+1)%dim);
}
```

If new item's key is smaller than root's along the dimension indicated by the discriminator, recursive call on left subtree

else recursive call on right subtree

In both cases, switch the discriminator before traversing the next level of the tree, cycling through the dimensions

k-d Tree Search

Search works similarly to insert, using a discriminator to cycle through the dimensions as one recurses down the levels: $O(\log n)$ time

The tree we built was nicely balanced

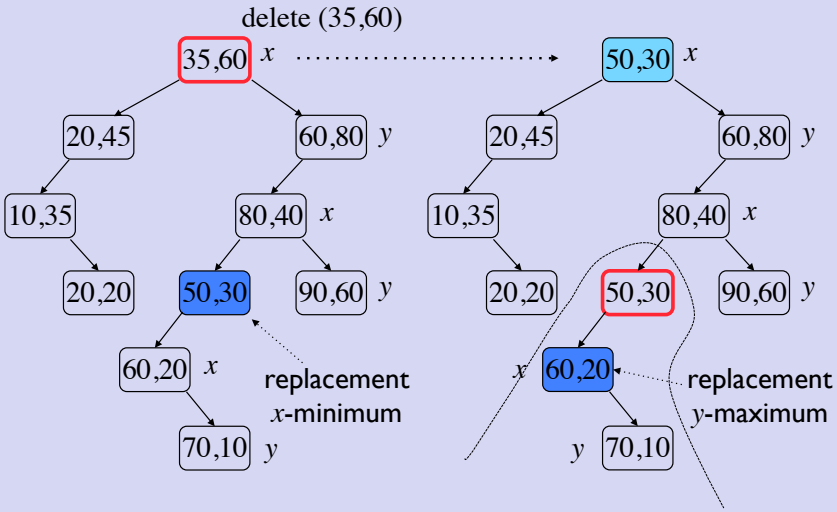
- if nodes are inserted randomly, on average we will get a balanced tree: $O(\log n)$ time
- if nodes inserted are sorted, recursively insert the median of the range to build a balanced tree: $O(n \log n)$ time

k-d Tree Remove

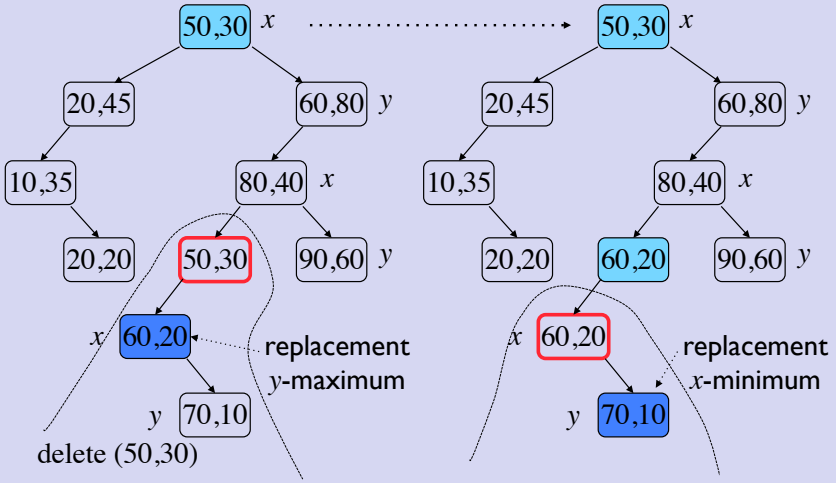
To remove a node on a level with discriminator along dimension j:

- if the node is a leaf, remove it
- else if node has right subtree, find the j-minimum node in the right subtree
- replace node with j-minimum node and repeat until you reach a leaf, then remove the leaf
- else find the j-maximum node in the left subtree, replace, repeat, remove
- j-minimum means minimum along the j dimension, analogously j-maximum
- $O(\log n)$ time

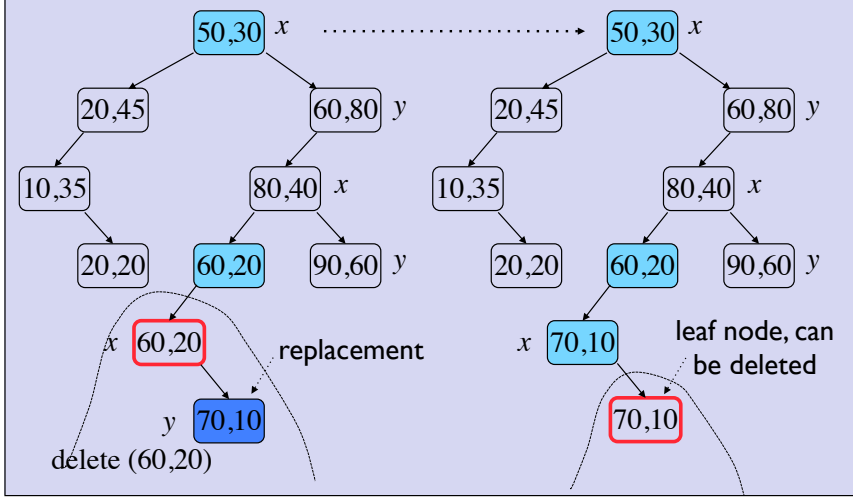
k-d Tree Remove Example



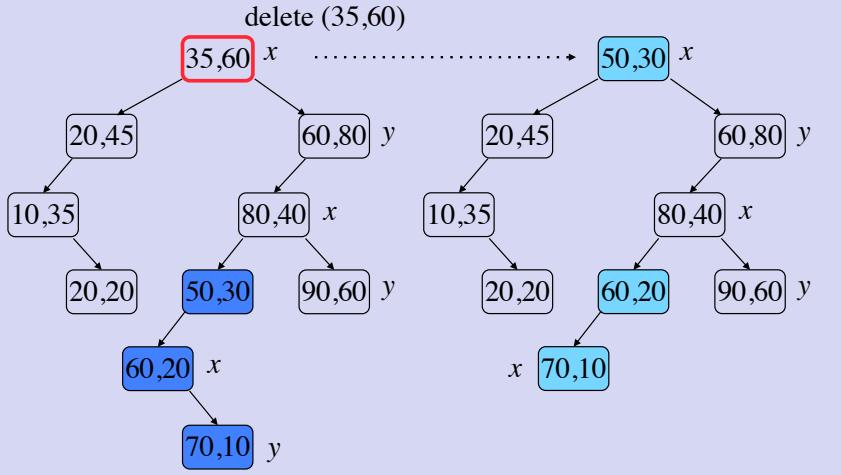
k-d Tree Remove Example



k-d Tree Remove Example



k-d Tree Remove Summary



Multidimensional Range Search

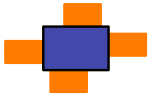
Example applications:

What would the `kdTree::rangesearch()` function signature be?
- What are its formal arguments?
- What would it return?

k-d Tree Range Search

```
void
kdTree::rangesearch(Link root,int disc,
        Key searchrange[],
        Key subtreerange[], List results)
```

- cycle through the dimensions as we traverse down the tree, same as with `kdTree::insert()`
- `searchrange[]` holds 2 values (min, max) per dimension
- subtree's ranges are defined for all dimensions
  - for 2D, both `searchrange[]` and `subtreerange[]` define a rectangle



  - for dimension j's range, `subtreerange[2*j]` holds the lower bound, `subtreerange[2*j+1]` holds the upper bound
    ⇒ these need to be updated as we go down the levels

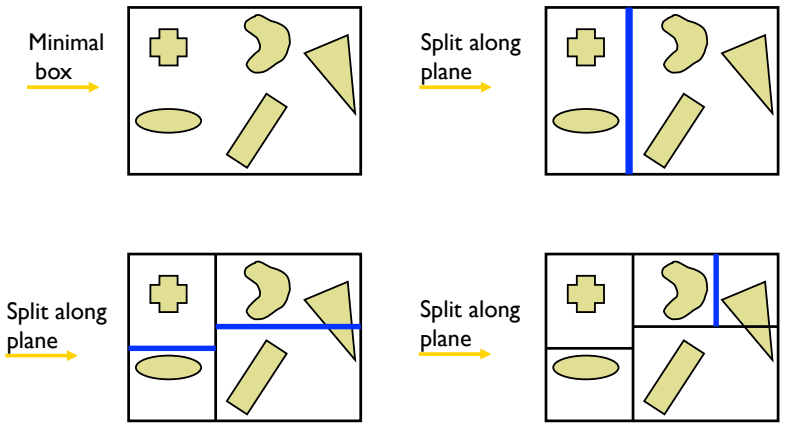
Example: Binary Space Partitioning (BSP) Trees

Axis-aligned BSP tree: a k-d tree used for spatial range search, with the following distinctions:
- items are stored in leaf nodes only
- other internal nodes store only the coordinates used to partition space
- an item that spans multiple partitions are stored in all corresponding leaf nodes

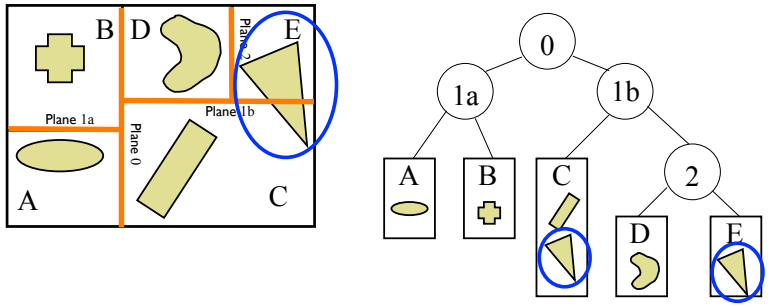
Example usages:
- occlusion culling and collision detection in computer graphics, computer games, robot motion planning

Axis-Aligned BSP Tree: Idea

Splitting plane aligned to x, y, or z axis



Axis-Aligned BSP Tree: Build



Each internal node holds a divider plane
Leaves hold geometry

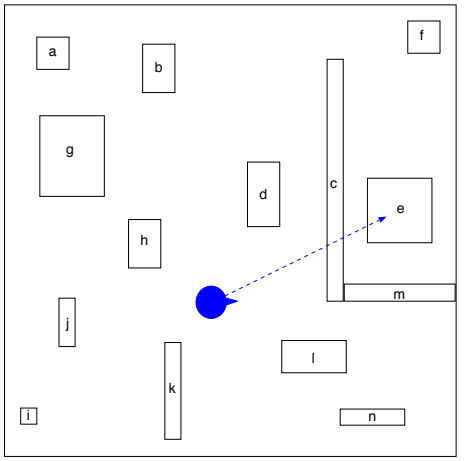
Example Use: Collision Detection

Blue wants to get to object *e*

Blue draws a straight line from itself to *e*

Now it needs to know which objects in the room could potentially obstruct it if it follows this straight line

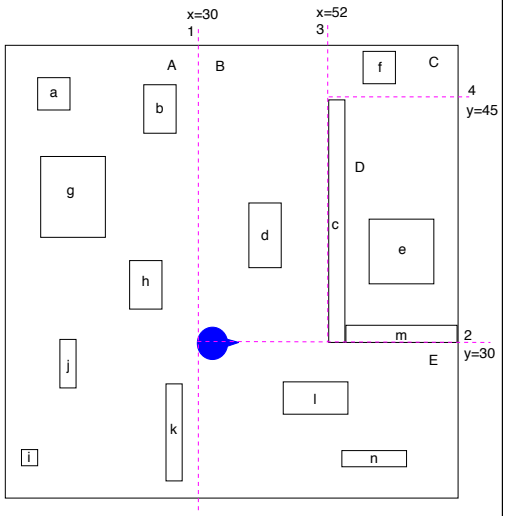
What is the brute force way of finding potentially obstructing objects?



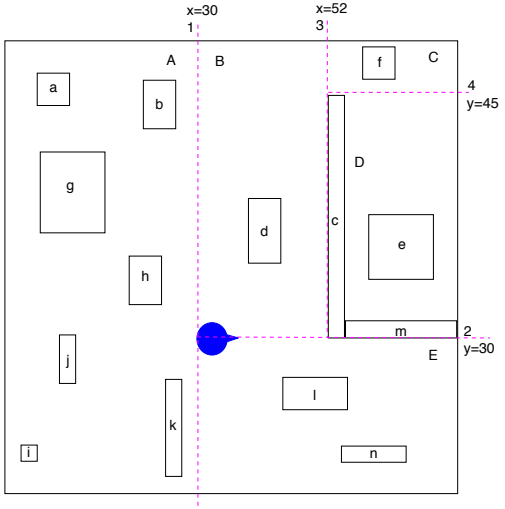
Collision Detection with BSP

Algorithm:

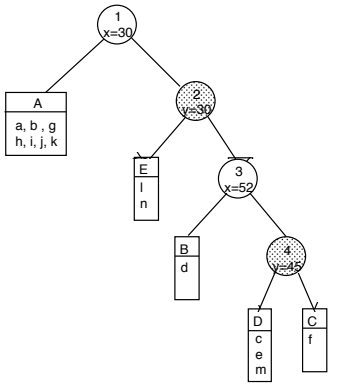
- use a plane to divide space into 2 parts
- objects whose *j*-coordinate is smaller than that of the partition go to the left subtree
- objects with *j*-coordinate larger than the partition's go to the right subtree
- repeat for the two partitions, cycling through the coordinates



Collision Detection with BSP



In this case, we use:
x=30, y=30, x=52, y=45
with the resulting BSP:



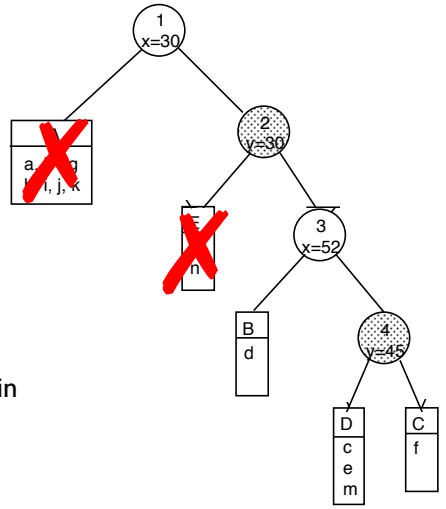
Collision Detection with BSP

Blue searches the BSP and found that at $x=30$, e is to its right, so it can ignore objects a, b, g, h, i, j, k

At $y=30$, e is to its left, so it can ignore objects l, n

It now knows that it would potentially have to navigate around d, f, c, and m

Of d, f, c, and m, c and m are in the same area as e, compute intersection with them first



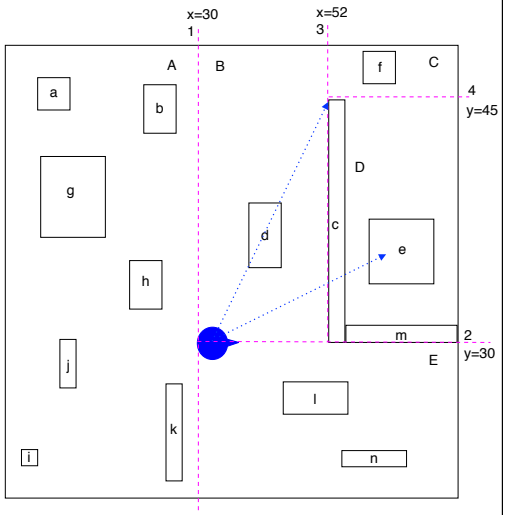
Collision Detection with BSP

Blue found out that c is in its way to e

So it needs to navigate around c

Assumes Blue can query object c for its dimension, it now knows that it needs to go to $(52, 45)$ to navigate around c

Next Blue needs to find out if any object is in its way to $(52, 45)$



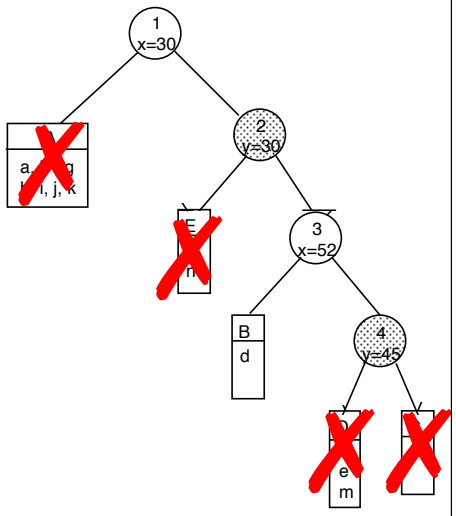
Collision Detection with BSP

Blue needs to find out if any object is in its way to $(52, 45)$

What objects does Blue need to compute intersection with?

Blue is in the left side of the $x=52$ partition, and only object d is on the same side

So Blue only needs to compute intersection with d

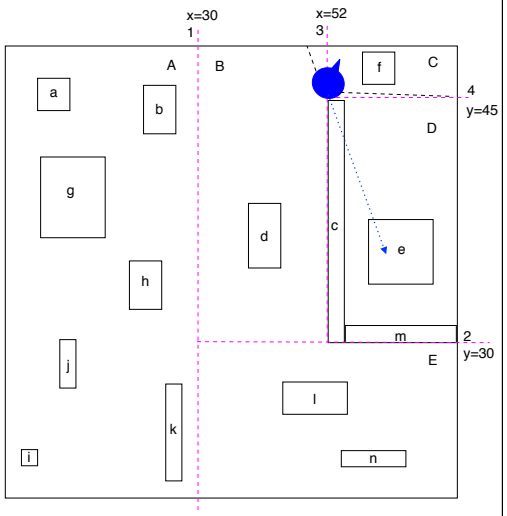


Collision Detection with BSP

Now Blue is at $(52, 45)$, it faces away from $y=45$, so it needs to turn around to go to e

To get to e, it needs to find if there are other objects in its way

Since it is now in the same region as e, it only needs to compute intersection with other objects in the region: c and m



Collision Detection with BSP

Compared to the brute force method, we only need to compute intersection with c, m, d, and c and m again: 5 intersection computations instead of $3*13$ intersections

Under normal k-d trees, we search both subtrees when the search range spans both subtrees

Under BSP trees however, items that span subtrees are split into two halves such that only one subtree needs to be searched

Fixed-Range Search

The range searches we have considered all have fixed ranges specified in world space, e.g.,

- of the whole alphabet space, search in range $[m–z]$
- of the whole Cartesian space, search in range $[(-\infty, 6), (-\infty, 8)]$
- find all (or k) departures within 2 hours of noon
  - relative to local point of reference
  - but can be transformed into a fixed- and absolute-range query in world-space, i.e., $[10\text{ am to }2\text{ pm}]$

Nearest-Neighbor Search

In nearest-neighbor search the range is not pre-specified and is in local space, e.g.,

- find the ATM machine nearest to me
- find the k restaurants nearest to me

⇐ range not specified!

[Known as the k-nearest neighbor (k-NN) problem]

Using k-d tree to support nearest-neighbor queries in 2D:

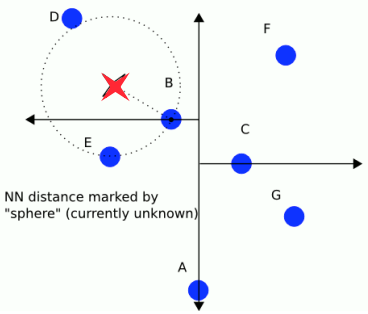
- not the most efficient solution in theory
- but everyone uses it in practice
- time complexity: $O(n^{1/2} + k)$

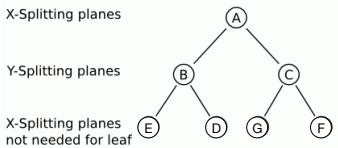
Nearest Neighbor Search: Algorithm

1. start with a reference point (✗) and an infinite search range
2. if root node matches query and the distance from reference point to root is smaller than current search range, reduce search range to this distance
3. determine whether the reference point is to the left or right of root (along the discriminating coordinate)
4. recursively search the branch of the tree where the reference point belongs
5. upon returning from the recursive search, the search range may have been further reduced, check to see if the current search range covers the other branch of the tree that has not been searched
6. if so, recursively search the other branch of the tree; otherwise prune the other branch
7. return results

Nearest Neighbor Search

```
void
nnsearch(Link root, int disc, Key reference,
         double *searchradius,
         Key subtreerange[], List results)
```

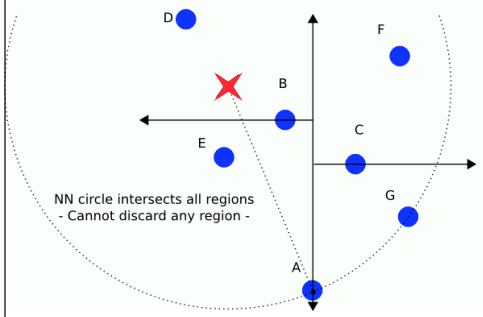


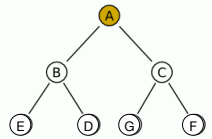


Assume all data points match query

[wikipedia]

Nearest Neighbor Search





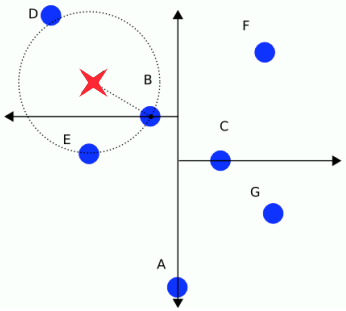
Start at A, then proceed in depth-first search (maintain a stack of parent-nodes if using a singly-linked tree). Set best estimate to A's distance Then examine left child node

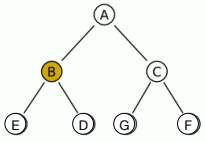
Reduce search range from initial infinite value to distance(reference, A) (A being the root of the k-d tree)

Current search range covers both children of A

[wikipedia]

Nearest Neighbor Search





Calculate B's distance and compare against best estimate - It is smaller distance, so update best estimate. Examine children (left then right)

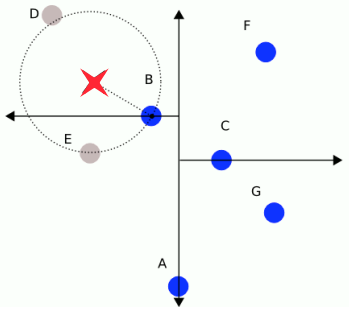
Reference is to the left of A, search left child of A first

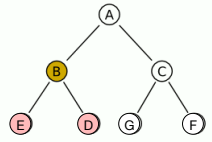
Found a node (B) closer to reference than A

Reduce search range to distance(reference, B)

[wikipedia]

Nearest Neighbor Search



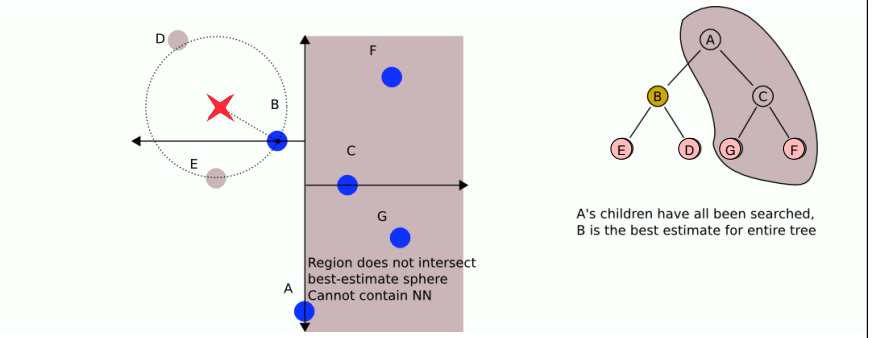


D & E Discarded as B (already visited) is closer. B is the best estimate for B's sub-branch Proceed back to parent node

Since B's children are not in the new search range, we're done with this branch

[wikipedia]

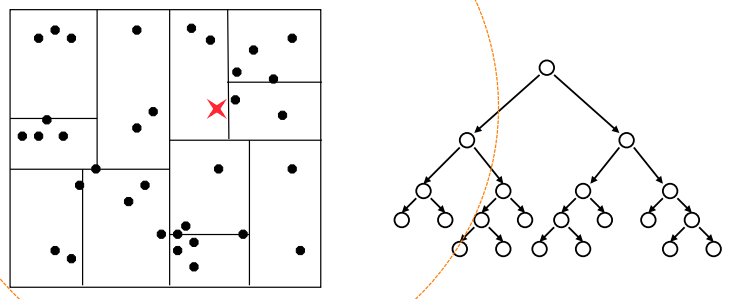
Nearest Neighbor Search



Returning back to A, current (new) search range no longer overlaps A's right child, prune the right child, return result

[wikipedia]

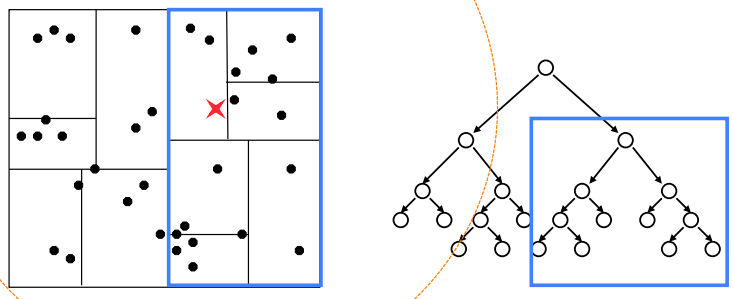
Nearest Neighbor Search



In this example, the k-d tree is formed by splitting planes, similar to an axis-aligned BSP, hence the internal nodes do not always hold data points

[Sellarès]

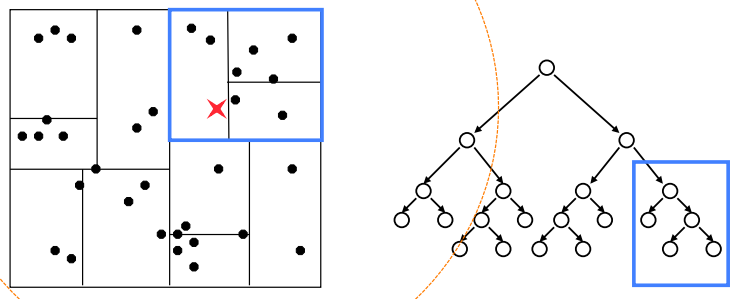
Nearest Neighbor Search



Since the root doesn't hold a data point, the search range remains at ∞ as we search the right side of the k-d tree where the reference point is

[Sellarès]

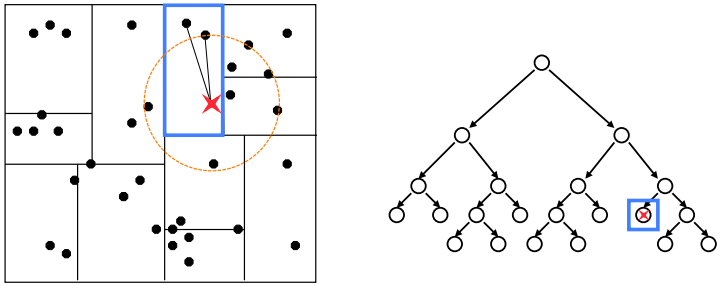
Nearest Neighbor Search



As we traverse to the second level of the k-d tree, we still cannot reduce the search range

[Sellarès]

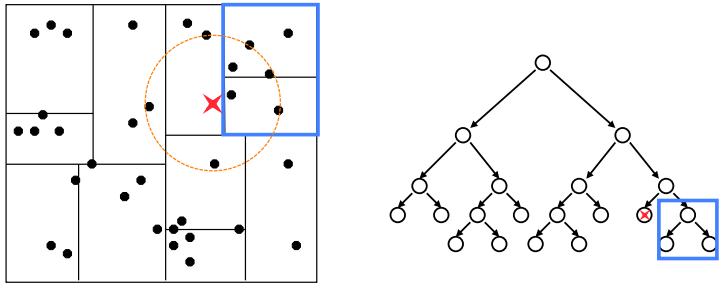
Nearest Neighbor Search



We've reached a leaf node and found two data points that match our query (assume all data points do), reduce the search range to the minimum distance between the reference point and the two data points

[Sellarès]

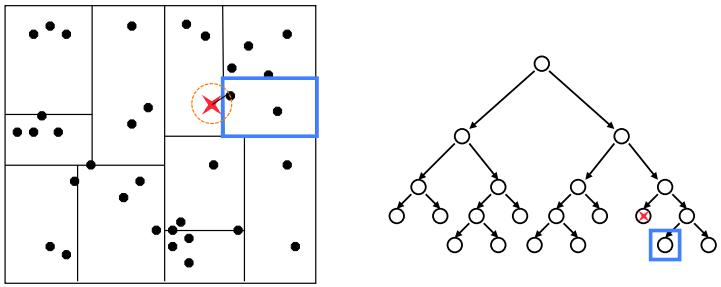
Nearest Neighbor Search



Returning back to the parent node, we found that the search range overlaps the range of the right subtree, search the right subtree

[Sellarès]

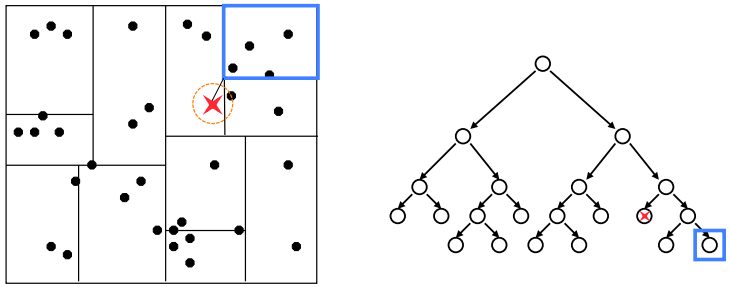
Nearest Neighbor Search



Found a closer neighbor in the left branch of the right subtree, further reduce the search range

[Sellarès]

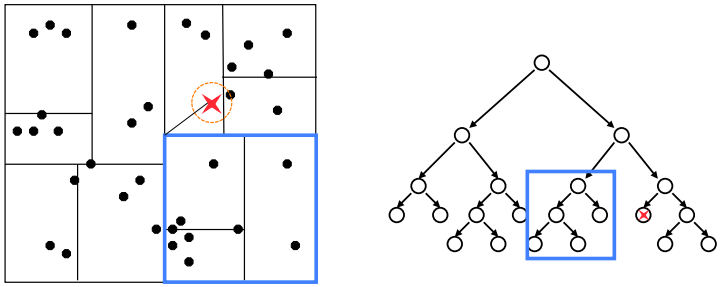
Nearest Neighbor Search



Using current search range and each subtree's range, prune parts of the tree that could NOT include the nearest neighbor

[Sellarès]

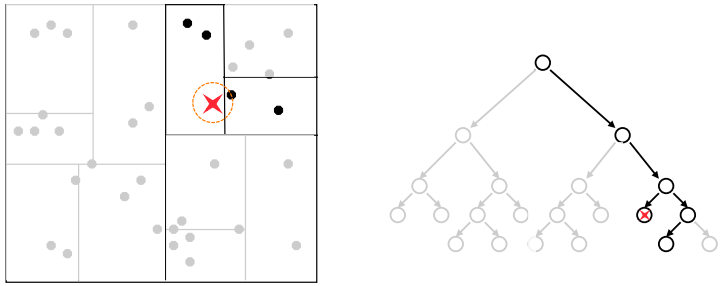
Nearest Neighbor Search



Using current search range and each subtree's range, prune parts of the tree that could NOT include the nearest neighbor

[Sellarès]

k Nearest Neighbors Search



To find k nearest neighbors, maintain k current best instead of just the best [different k from k-d tree]

Branches are only eliminated when they can't hold any neighbor closer than any of the k current best

[Sellarès]

Programming Assignment 2

Topic: Location-Based Services

> "Consumer and advertiser expenditure on location-based services (LBS) to approach $10B by 2016, with search advertising accounting for just over 50%." *LA Times*, 6/10/11

Due date: Thu, 10/13, 10:00 pm

To be done individually (no group or team)

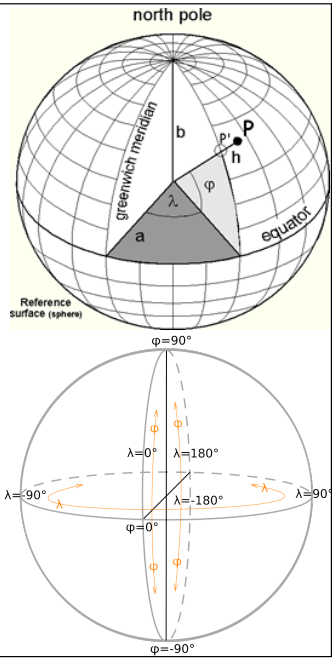
No STL (`iostream` and `string` are allowed, they are part of the C++ standard library, not part of STL)

Geographic Coordinate System

A point on earth is given by its latitude and longitude

Latitude 0° is at the Equator
Latitude 90° is at the North Pole
Latitude –90° is at the South Pole

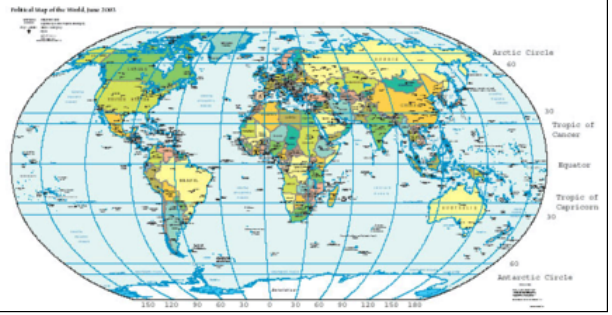
Longitude 0° is the Prime Meridian
Longitude ±180° is a the International Dateline
(more or less)



Standard Coordinate Frame

The World Geodetic System 84 represents the earth as an ellipsoid with the origin located at the Earth's center of mass (± 2 cm)

WGS84 is the current standard coordinate frame used by GPS systems



Lat/Lon of Some Cities on Earth

City	Latitude	Longitude
Kinshasa	–4.325	15.322222
Istanbul	41.01224	28.976018
Mumbai	18.975	72.825833
Jakarta	–6.2	106.8
Shanghai	31.2	121.5
Queenstown	–45.031111	168.6625
Honolulu	21.308889	–157.826111
San Francisco	37.7793	–122.4192
Ann Arbor	42.281389	–83.748333
Rio de Janeiro	–22.908333	–43.196389
Reykjavik	64.133333	–21.933333
Casablanca	33.533333	–7.583333

Problem Specification

Read in a "database" of location data
Each entry consists of:

```
lat lon name tag
```

where the `lat` and `lon` are floats and `name` and `tag` are single words

To simplify the assignment, we limit acceptable latitude to between 0° and 90° and we limit acceptable longitude to between 0° and -180° (covering North America)

Sample Location Database

In one column:

42.2893 –83.7391 NorthsideGrill restaurant
42.3036 –83.7090 UMCU bank
42.2831 –83.7485 TheBrokenEgg restaurant
42.2982 –83.7200 GreatPlainsBurger fastfood
42.3033 –83.7053 Evergreen restaurant
42.2785 –83.7413 CometCoffee cafe
42.2845 –83.7463 Yamato restaurant
42.2806 –83.7497 GrizzlyPeak pub
42.2810 –83.7486 Vinology restaurant
42.3030 –83.7066 TCF bank
42.2803 –83.7479 ArborBrewingCompany pub
42.2780 –83.7449 CreditUnion bank
42.2804 –83.7497 Sweetwaters cafe
42.2795 –83.7438 TCF bank
42.2984 –83.7195 Qdoba fastfood
42.2780 –83.7409 Ashley's pub
42.2984 –83.7195 Panera bakery
42.2846 –83.7451 Zingerman's restaurant
42.2797 –83.7496 WestendGrill restaurant
42.2808 –83.7486 KaiGarden restaurant
42.2909 –83.7178 UMCU bank
42.2806 –83.7493 CafeZola restaurant
42.3047 –83.7090 Kroger supermarket
42.2830 –83.7467 NoThai fastfood
42.3048 –83.7083 AABank bank
42.2827 –83.7470 CafeVerde cafe
42.2780 –83.7449 UMCU bank
42.2828 –83.7485 Heidelberg pub
42.2792 –83.7409 PotBelly fastfood

Problem Specification

You can assume there will be no duplicate records (all four fields being the same) in the database

The lat/lon may be duplicated, but as long as the name and/or the tag is different, records with the same lat/lon are not considered duplicates

We limit the lat/lon precision to 4 decimal places ($\pm 0.0001°$), which translates to about 11 m longitudinal distance at the equator and about 5.56 m longitudinal distance at latitude 60°

Problem Specification

The database is followed by a single blank line and one or more queries

There are three types of queries (not ordered):
1. exact-match query, led by an '@' sign:

```
@ 42.2982 –83.7200
@ 42.2984 –83.7195
@ 42.2980 –83.7109
```

2. range-match query, led by an '`r`':

```
r 42.2806 –83.7493 0.0004
r 42.2812 –83.7521 0.002
```

Problem Specification

3. nearest neighbor query, led by an 'n' sign:

```
n 42.2785 –83.7461 bank
n 42.2785 –83.7461 bookstore
n 42.3033 –83.7078 bank
n 42.3034 –83.7078 bank
```

Problem Specification

The two floats following the '@' and '`r`' signs should be obvious

The third float following the '`r`' sign specifies the search range from the provided location, in degree

We limit the range to be at most 0.5 degree from the provided location, so in total the search range is limited to 1 degree (enough to cover the New York Metropolitan Area or the Tokyo/Yokohama megalopolis, the two largest urban conglomerations in the world, by area)

Problem Specification

Given the acceptable lat/lon and the limit on search range, the acceptable latitude on a range search is limited to 0.5° to 89.5° and the acceptable longitude is limited to -0.5° to -179.5°

No need to translate degree to linear units!

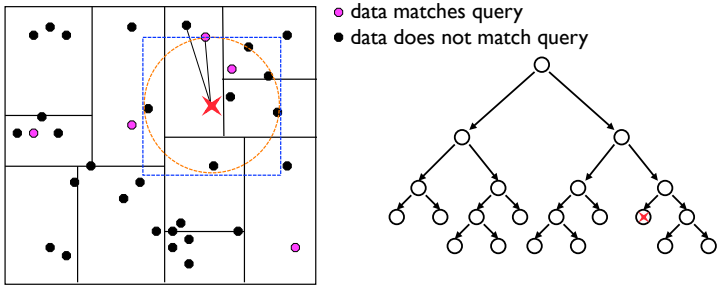
Problem Specification

The tag following the lat/lon in nearest location search is the queried tag, i.e., a record matches the query only if the record contains the same tag as the queried tag

To simplify implementation, we will use a "bounding box" covering the search radius to perform nearest neighbor search

```
void
rangesearch(Link root, int disc, Key reference,
            double *searchradius, Key bbox[],
            Key subtreerange[], List results)
```

Nearest Neighbor Bounding Box

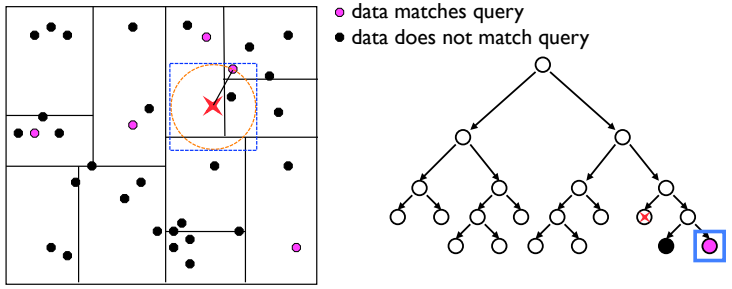


Bounding box search range covers the circle whose radius is the distance between the reference point and the nearest neighbor

$$\text{distance}(A, B) = \sqrt{(B_x - A_x)^2 + (B_y - A_y)^2}$$

[Sellarès]

Nearest Neighbor Bounding Box



Bounding box is shrunk as a nearer neighbor is found

A data point is considered a "neighbor" only if its tag matches that of the queried tag

[Sellarès]

Output

Given the above database and queries, the output of your program should be:

@ 42.2982 –83.7200	42.2982 -83.7200 GreatPlainsBurger fastfood
@ 42.2984 –83.7195	42.2984 -83.7195 Panera bakery
	42.2984 -83.7195 Qdoba fastfood
@ 42.2980 –83.7109	No record found
r 42.2806 –83.7493 0.0004	42.2806 -83.7493 CafeZola restaurant
	42.2806 -83.7497 GrizzlyPeak pub
	42.2804 -83.7497 Sweetwaters cafe
r 42.2812 –83.7521 0.002	No record found
n 42.2785 –83.7461 bank	42.2780 -83.7449 CreditUnion bank
n 42.2785 –83.7461 bookstore	No record found
n 42.3033 –83.7078 bank	42.3030 -83.7066 TCF bank
n 42.3034 –83.7078 bank	42.3036 -83.7090 UMCU bank

Location-Based Search

Finding exact matches:

- must be implemented using hashing
- figure out: what hash function to use
- figure out: how to resolve collisions
- assume your program will be used all over the acceptable region

Finding range and nearest neighbor matches:

- must be implemented using *k*-d tree
- search range forms a rectangle/bounding box
- no need to implement node removal
- may assume database is not sorted

k-d Tree

```
level.right(disc) lat lon name tag
0.-1(0) 42.2893 -83.7391 NorthsideGrill restaurant
 1.0(1) 42.2831 -83.7485 TheBrokenEgg restaurant
  2.0(0) 42.2806 -83.7497 GrizzlyPeak pub
   3.0(1) 42.2804 -83.7497 Sweetwaters cafe
    4.1(0) 42.2797 -83.7496 WestendGrill restaurant
     5.1(1) 42.2806 -83.7493 CafeZola restaurant
   3.1(1) 42.2810 -83.7486 Vinology restaurant
  4.0(0) 42.2808 -83.7486 KaiGarden restaurant
  4.1(0) 42.2828 -83.7485 Heidelberg pub
 2.1(0) 42.2785 -83.7413 CometCoffee cafe
  3.0(1) 42.2780 -83.7449 CreditUnion bank
  4.0(0) 42.2780 -83.7449 UMCU bank
  4.1(0) 42.2780 -83.7409 Ashley's pub
  3.1(1) 42.2845 -83.7463 Yamato restaurant
   4.0(0) 42.2803 -83.7479 ArborBrewingCompany pub
    5.1(1) 42.2830 -83.7467 NoThai fastfood
     6.0(0) 42.2827 -83.7470 CafeVerde cafe
   4.1(0) 42.2795 -83.7438 TCF bank
    5.0(1) 42.2792 -83.7409 PotBelly fastfood
    5.1(1) 42.2846 -83.7451 Zingerman's restaurant
1.1(1) 42.3036 -83.7090 UMCU bank
 2.0(0) 42.2982 -83.7200 GreatPlainsBurger fastfood
  3.0(1) 42.2909 -83.7178 UMCU bank
  3.1(1) 42.2984 -83.7195 Qdoba fastfood
  4.0(0) 42.2984 -83.7195 Panera bakery
  4.1(0) 42.3047 -83.7090 Kroger supermarket
 2.1(0) 42.3033 -83.7053 Evergreen restaurant
  3.0(1) 42.3030 -83.7066 TCF bank
  3.1(1) 42.3048 -83.7083 AABank bank
```

PA2 Grading Criteria

Working, efficient solution (75%):

- autograder will use –O3 compile flag for timing, so make sure your `Makefile` also uses the –O3 flag

Test cases (20%)

Code is readable, well-documented (5%):

- pay attention to PA1 grade report and avoid being penalized for the same stylistic issues

Files Organization

How would you organize your code into files?

Alternative 1: `main.cpp` (NOT)

Alternative 2: `main.cpp, hash.h, hash.cpp, kdtree.h, kdtree.cpp, array.h, array.cpp, linkedlist.h linkedlist.cpp, location.h, location.cpp` (NOT)

Alternative 3: `lbs281.cpp, adts.h, hash.h, kdtree.h, location.h, location.cpp`

Your choice would be different, but try not to split it up into too many files!

Time Requirements

How long does it take to do PA2?

Task	Lines of Code	% Total Time
design (and writing spec)		n/a (2 days)
parse input (incl. unit test)	42	4
hashing (incl. unit test)	132	27
kdtree insert	88	7
kdtree range search	142	23
kdtree nearest neighbor	130	38
whole globe (excl. nn)	102	+28