CO-SNE: Dimensionality Reduction & Visualization for Hyperbolic Data

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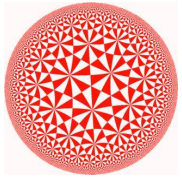
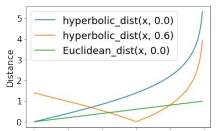
Hyperbolic Space





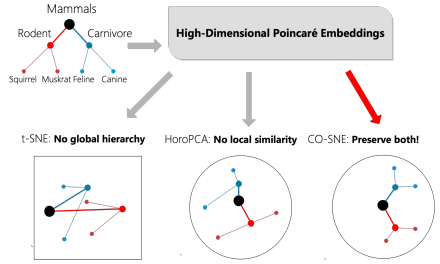
Hyperbolic Distance

$$d_{\mathbb{B}^n}(\boldsymbol{u},\boldsymbol{v}) = \operatorname{arcosh}\left(1 + 2\frac{\|\boldsymbol{u}-\boldsymbol{v}\|^2}{(1-\|\boldsymbol{u}\|^2)(1-\|\boldsymbol{v}\|^2)}\right)$$

- **Non-Euclidean space with constant negative curvature**
- **Can embed tree-like data continuously with low distortion**

Contributions

CO-SNE: A novel visualization method designed specifically for high-dimensional hyperbolic data



Losses in CO-SNE

Total Loss: $\mathcal{L} = \lambda_1 \mathcal{C} + \lambda_2 \mathcal{H}$

t-SNE Loss:

$$\mathcal{C} = KL(P||Q) = \sum_i \sum_j p_{ij} \log \frac{p_{ij}}{q_{ij}}$$

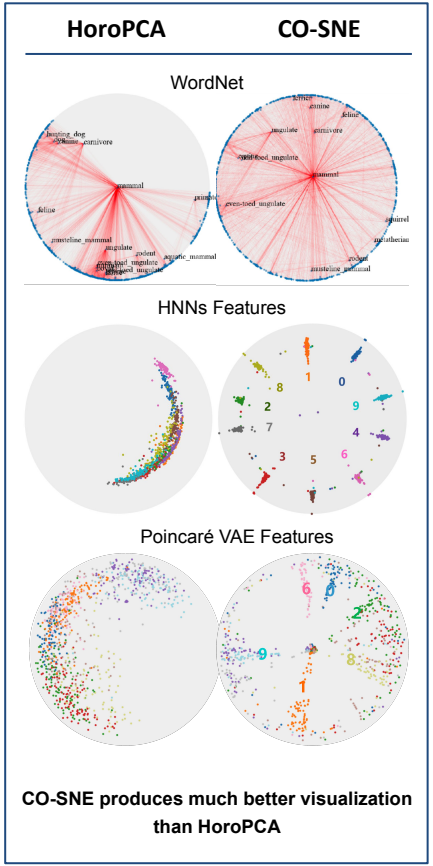
Maintaining local similarities

Distance Loss:

$$\mathcal{H} = \frac{1}{m}\sum_{i=1}^{m}(||\mathbf{x}_i||^2 - ||\mathbf{y}_i||^2)^2$$

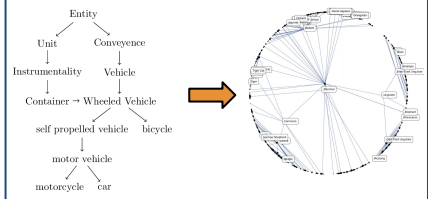
Maintaining global hierarchy

Visualizing Hyperbolic Features



CO-SNE produces much better visualization than HoroPCA

Visualizing Two-Dimensional Hyperbolic Space is Easy

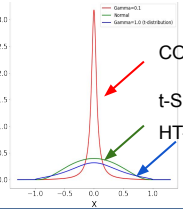


Embedding WordNet in a Two-dimensional Poincaré Ball

CO-SNE Uses Hyperbolic Cauchy Distribution

	Metric	low-dimensional Dist.	Losses
t-SNE	Euclidean	t-distribution	KL-div
HT-SNE	Hyperbolic	t-distribution	KL-div
CO-SNE	Hyperbolic	Cauchy	KL-div + Distance

CO-SNE uses a small γ



Hyperbolic Normal

$$\mathcal{N}_{\mathbb{B}^n}(x|\mu,\sigma^2) = \frac{1}{Z}\exp\left(-\frac{d_{\mathbb{B}^n}(\mu,x)^2}{2\sigma^2}\right)$$

Hyperbolic Cauchy

$$f(t;t_0,\gamma) = \frac{1}{\pi\gamma}\left[\frac{\gamma^2}{d_{\mathbb{B}^n}(t,t_0)^2+\gamma^2}\right]$$

CO-SNE Produces Stronger Repulsion Force

The gradients as a function of high-dimensional and low-dimensional hyperbolic distance