Complex-valued Butterfly Transform for Efficient Hyperspectral Image Processing

Utkarsh Singhal

Stella X. Yu

Berkeley UNIVERSITY OF CALIFORNIA

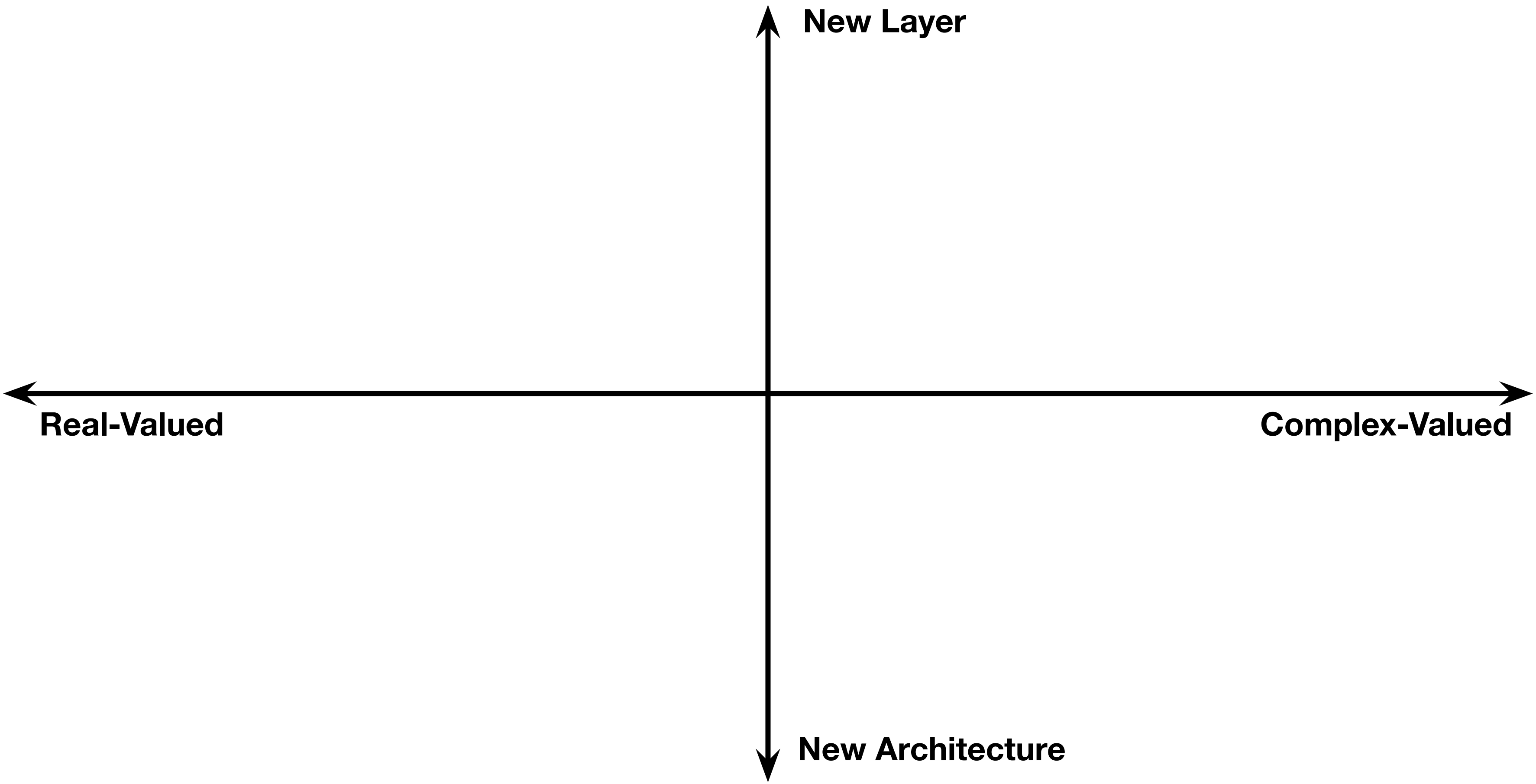
ICSI

IEEE WCCI 2022

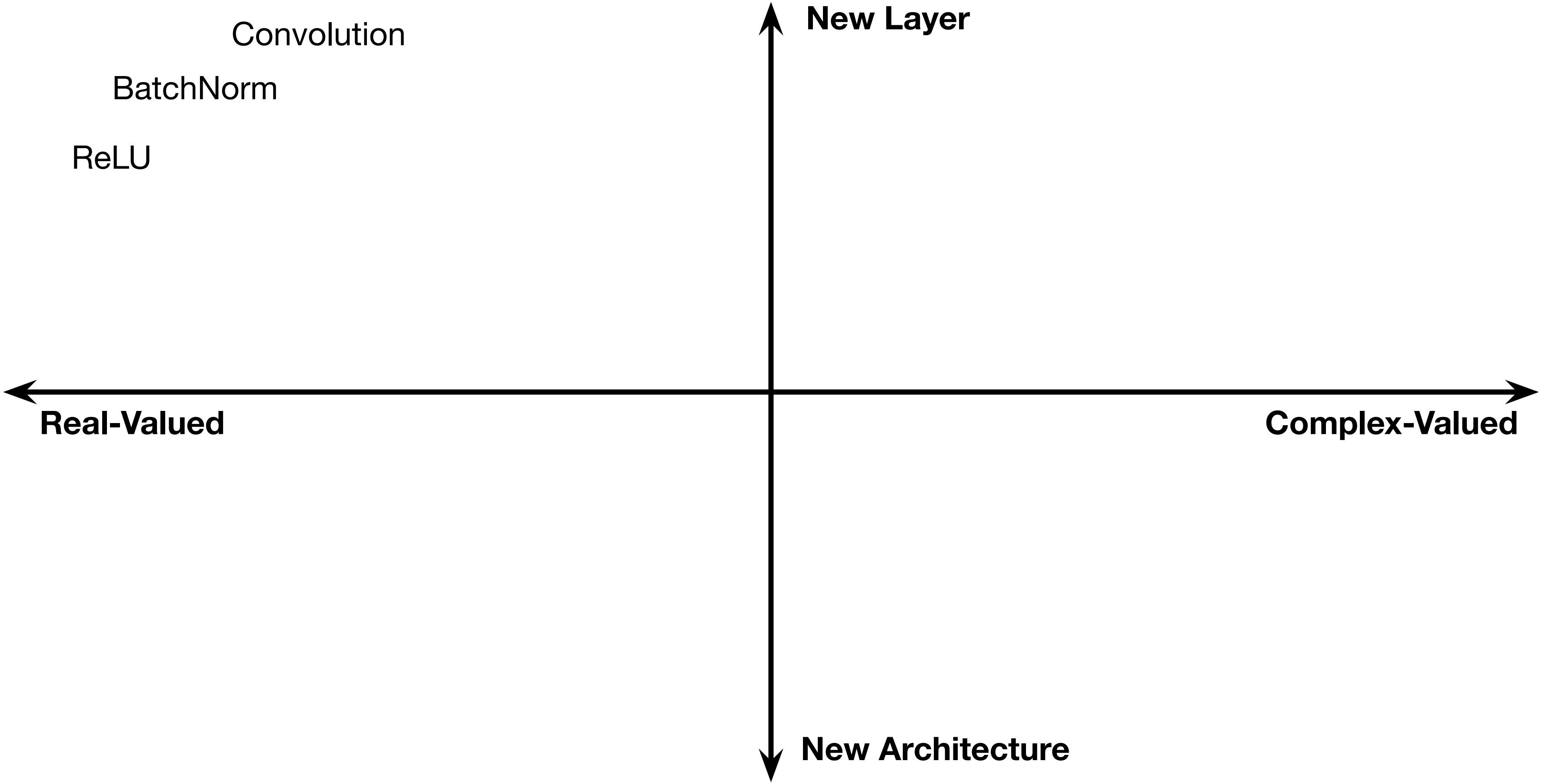
IEEE WORLD CONGRESS ON COMPUTATIONAL INTELLIGENCE

PADUA, ITALY • 18 - 23 July, 2022

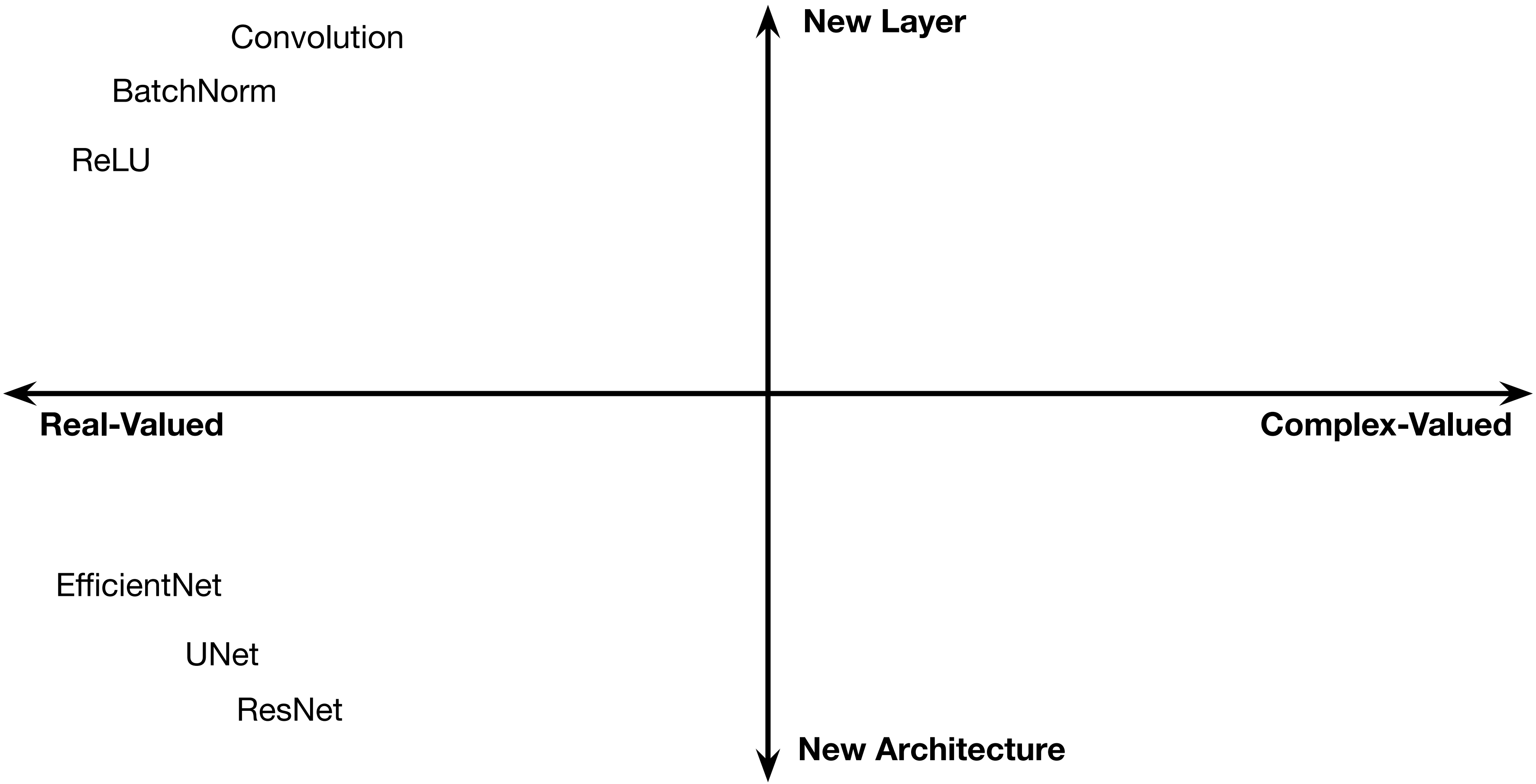
Landscape of Real/Complex-Valued Neural Net Research



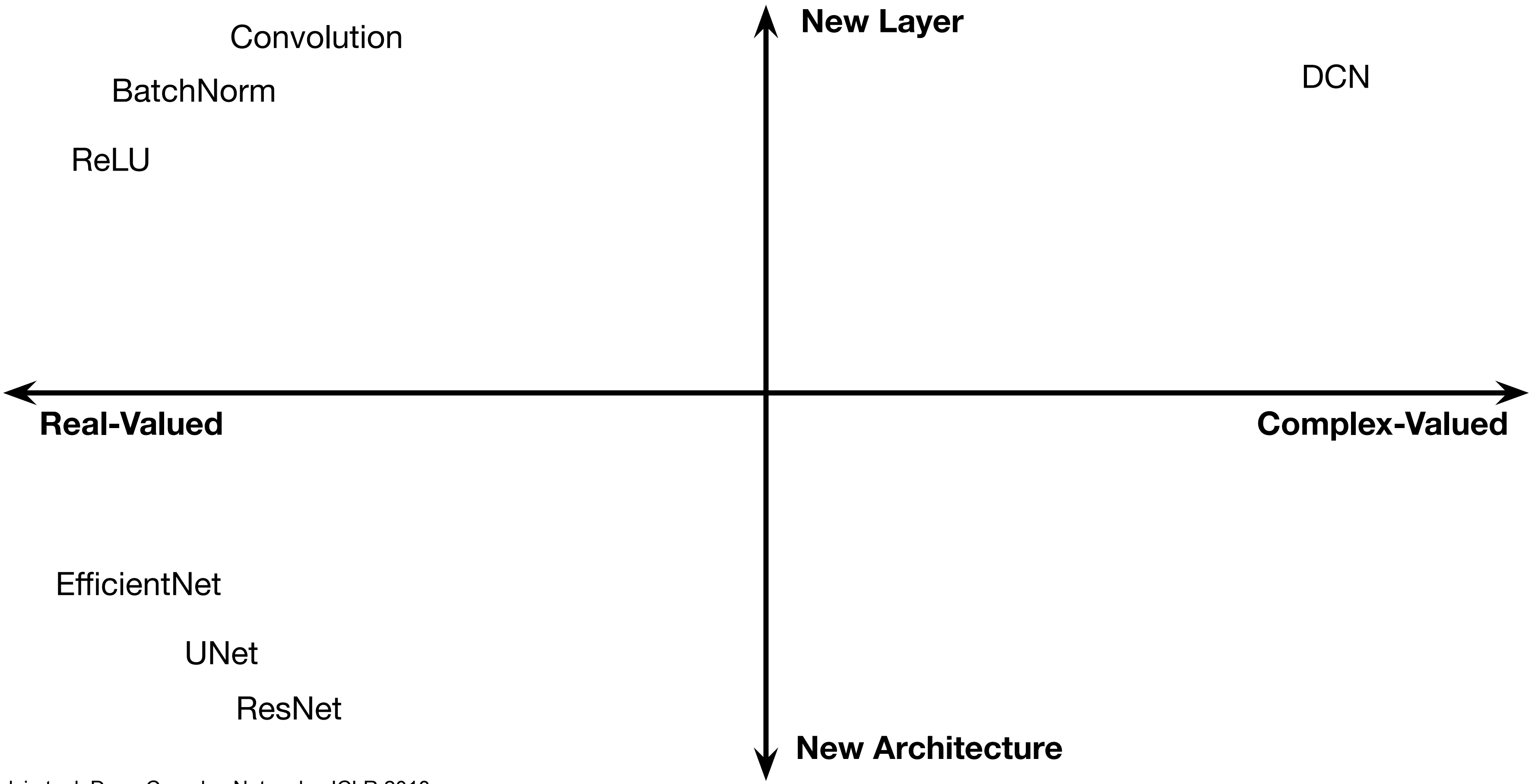
Landscape of Real/Complex-Valued Neural Net Research



Landscape of Real/Complex-Valued Neural Net Research

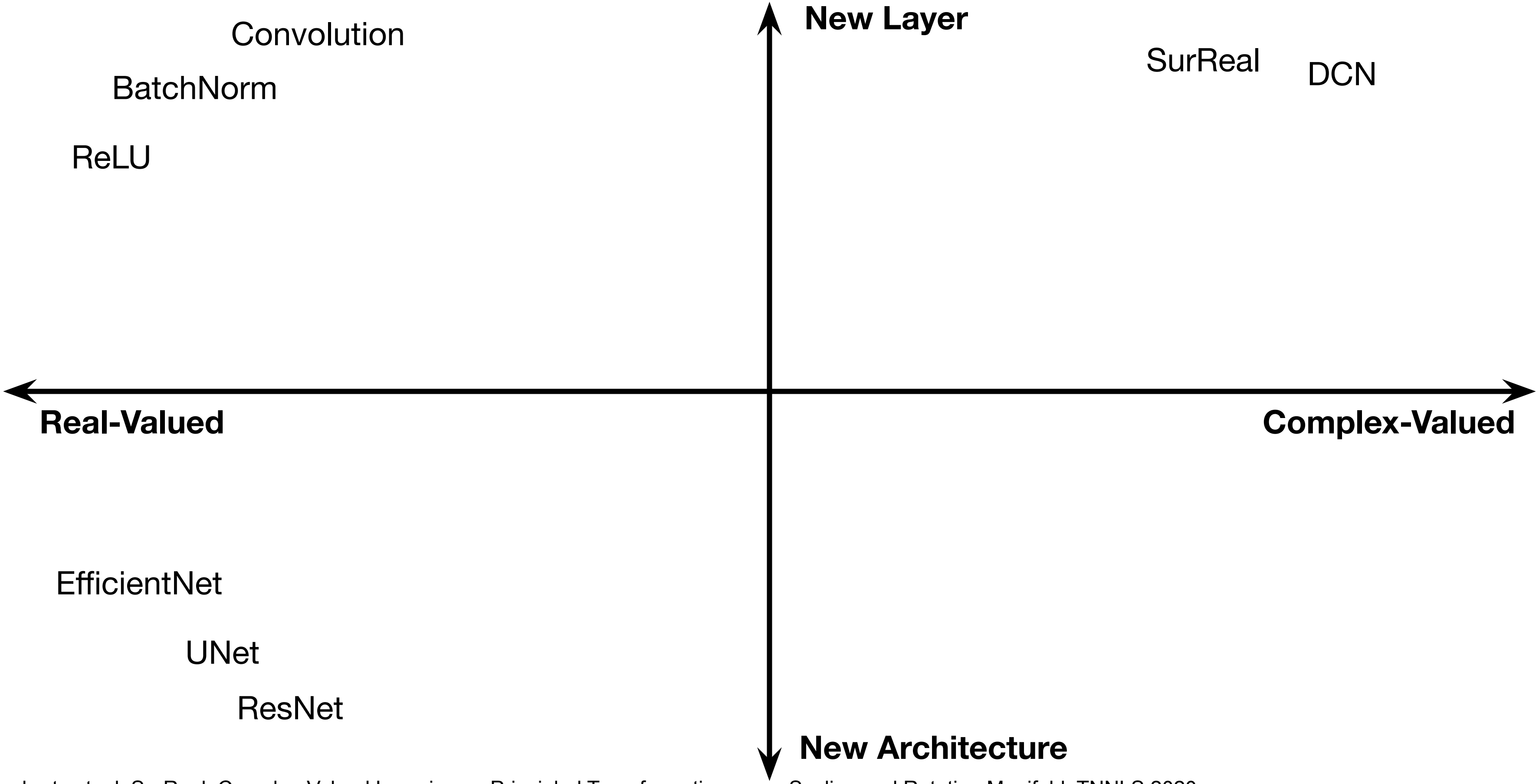


Landscape of Real/Complex-Valued Neural Net Research



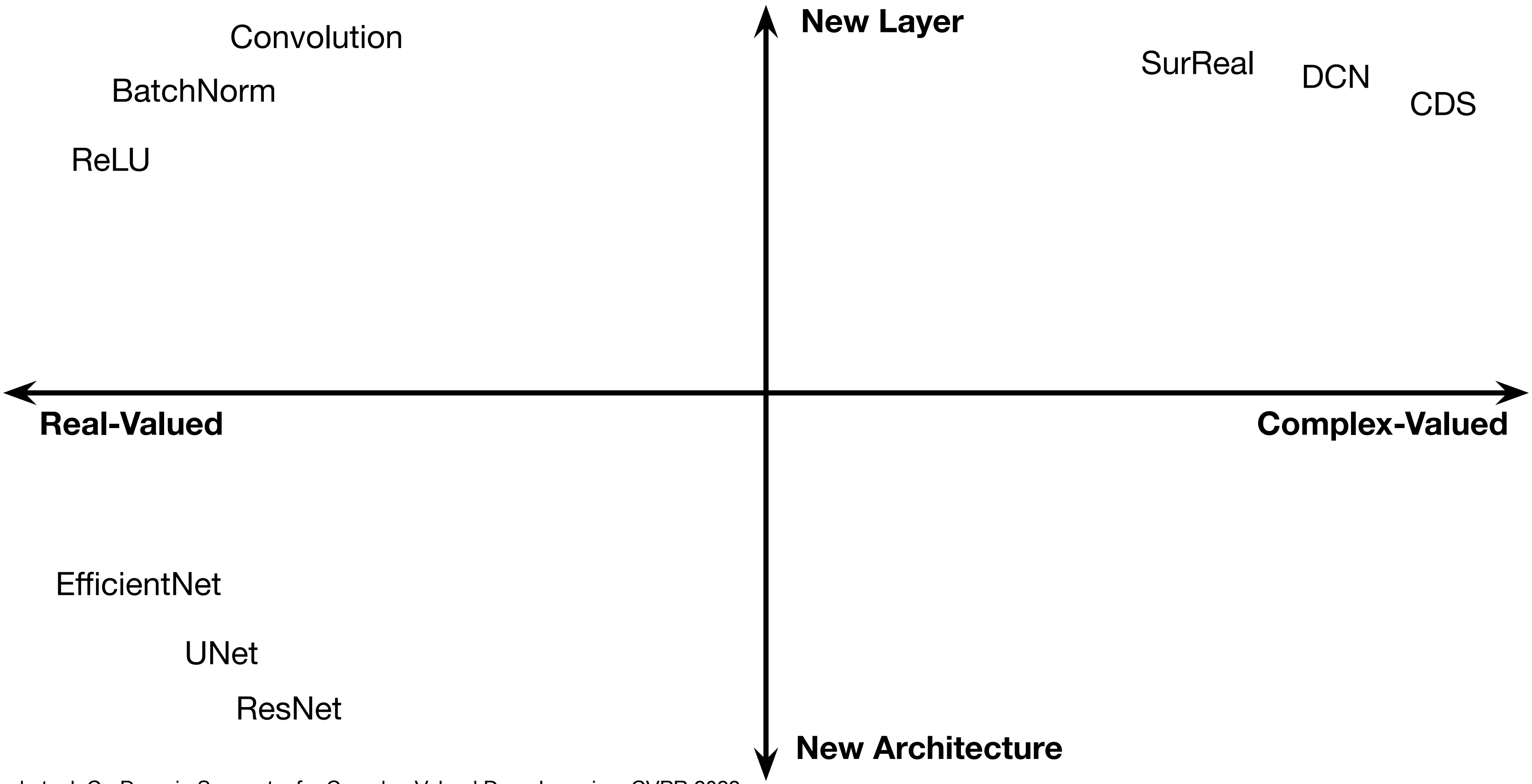
Trabelsi et. al, Deep Complex Networks, ICLR 2018

Landscape of Real/Complex-Valued Neural Net Research



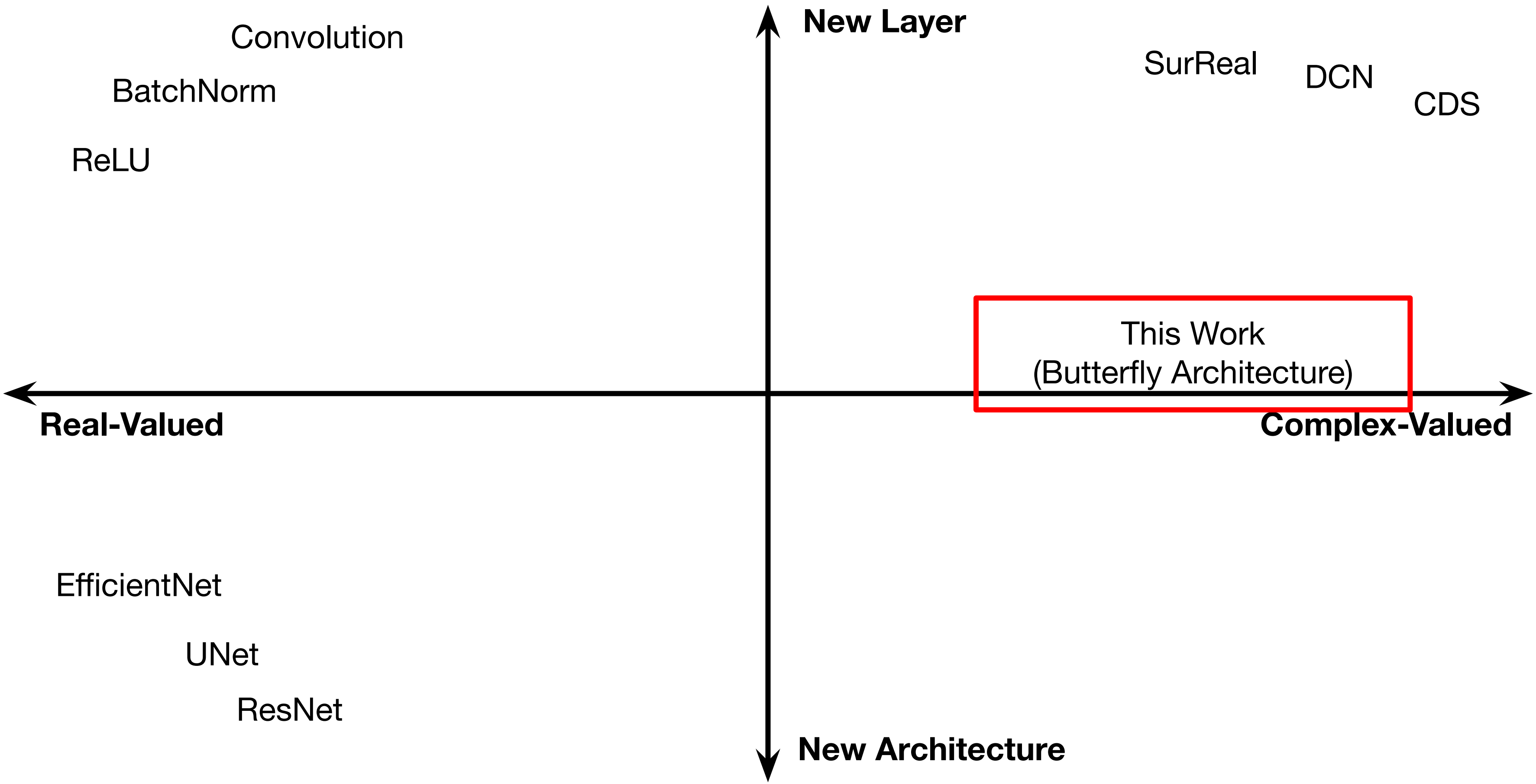
Chakraborty et. al, SurReal: Complex-Valued Learning as Principled Transformations on a Scaling and Rotation Manifold, TNNLS 2020

Landscape of Real/Complex-Valued Neural Net Research

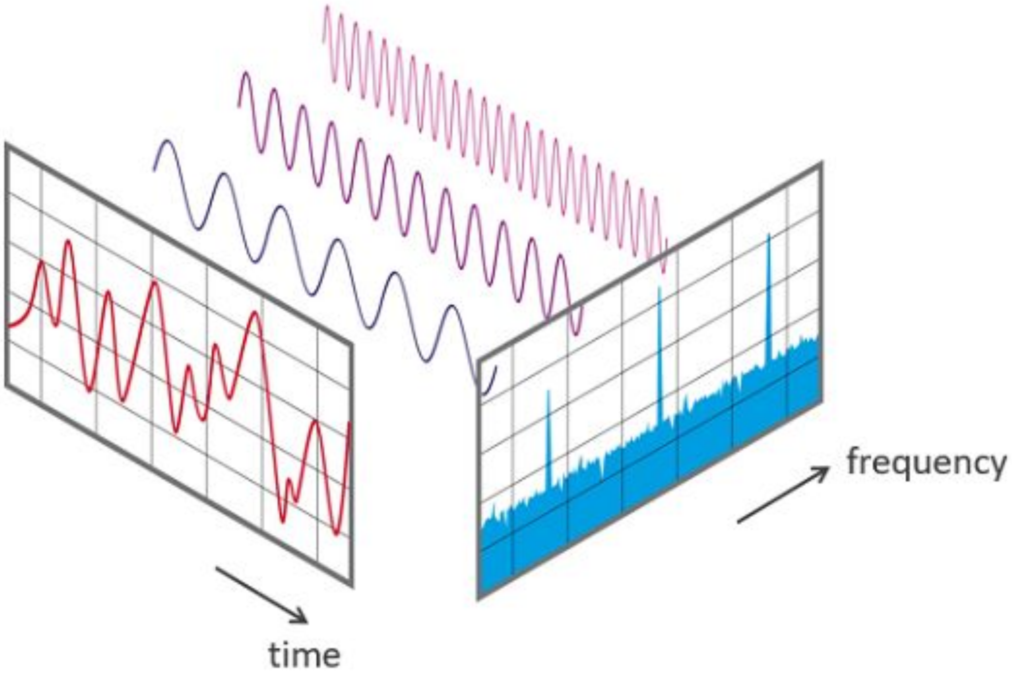


Singhal et. al, Co-Domain Symmetry for Complex-Valued Deep Learning, CVPR 2022

Landscape of Real/Complex-Valued Neural Net Research



Butterfly Architecture and FFT

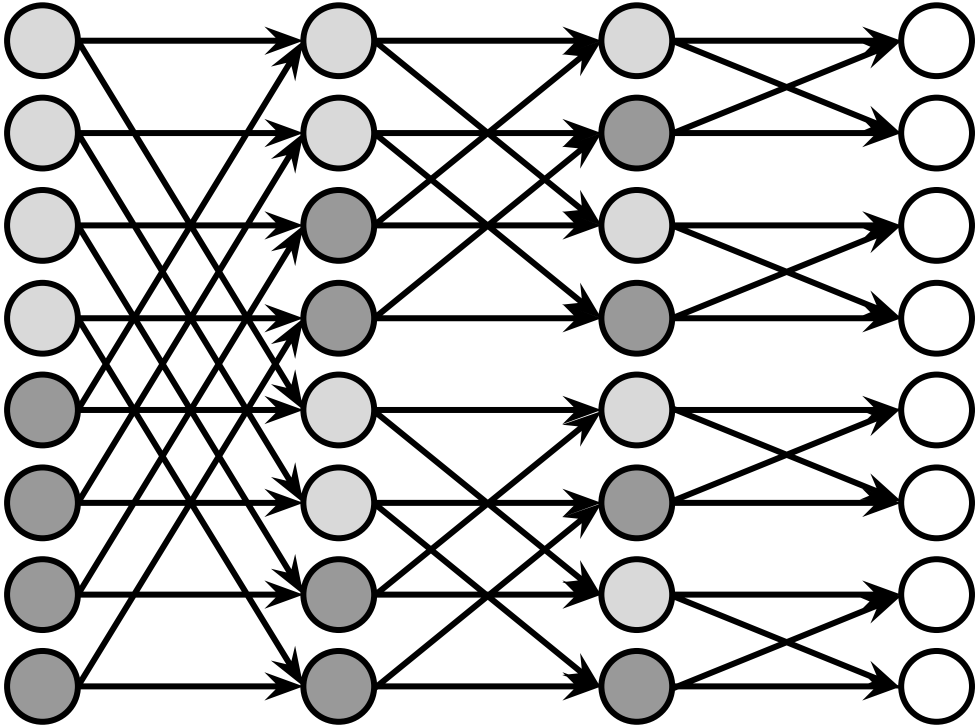


$$x[k] = \sum_{n=0}^{N-1} x[n] e^{\frac{-j2\pi kn}{N}}$$

Discrete Fourier Transform

https://www.nti-audio.com/en/support/know-how/fast-fourier-transform-fft
https://towardsdatascience.com/fast-fourier-transform-937926e591cb

Butterfly Architecture and FFT

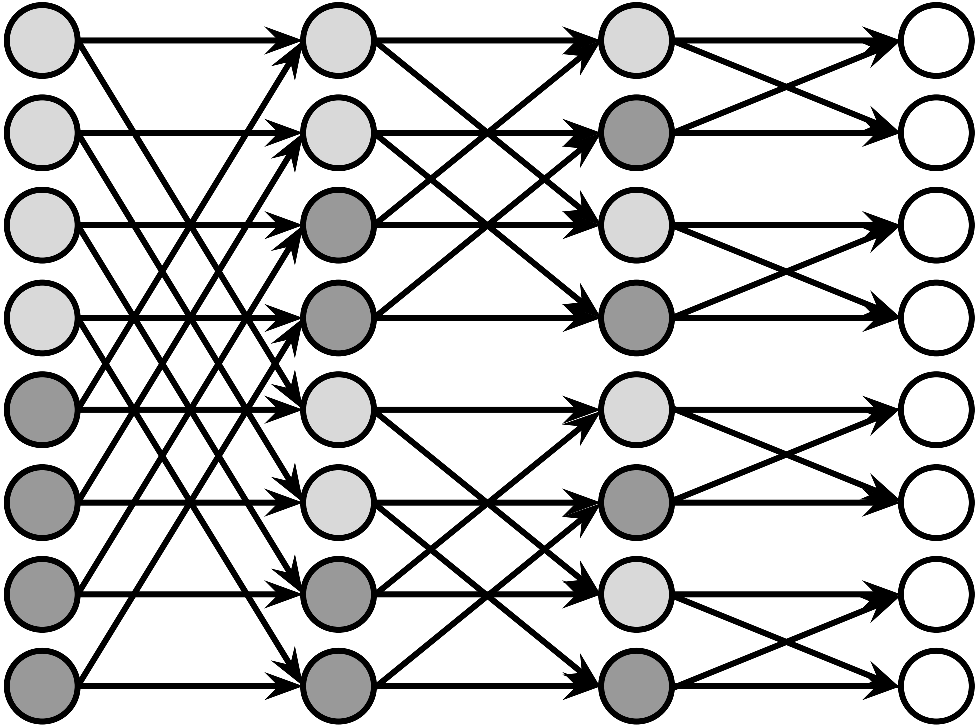


Butterfly Diagram

[1]: Cipra et. al, SIAM: https://archive.siam.org/pdf/news/637.pdf

Butterfly Architecture and FFT

- **Efficient Architecture:** Recursive and Sparse

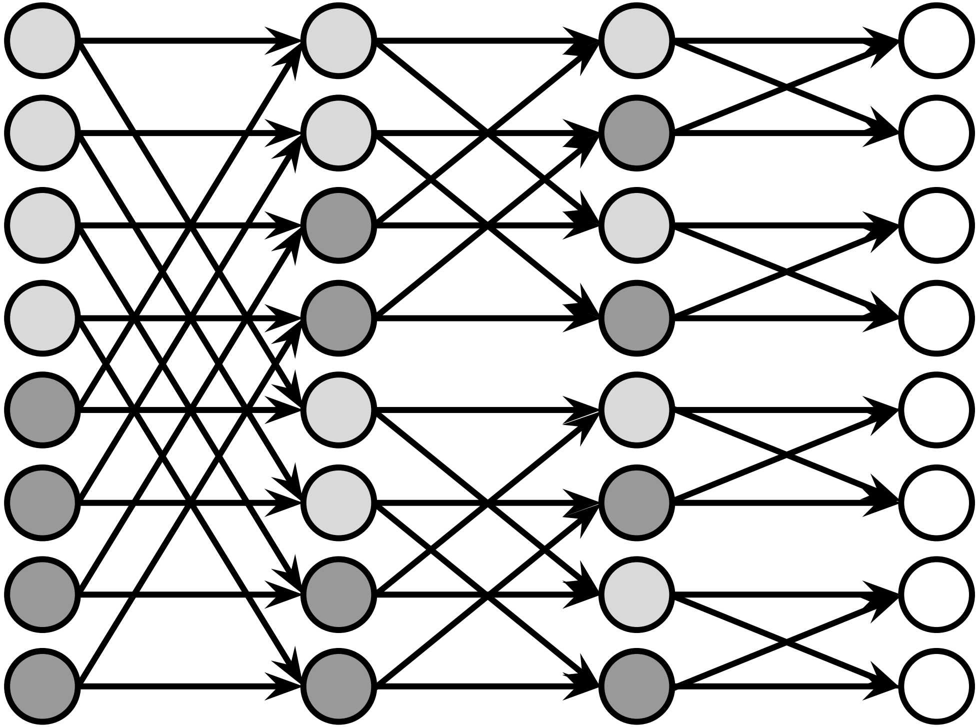


Butterfly Diagram

[1]: Cipra et. al, SIAM: https://archive.siam.org/pdf/news/637.pdf

Butterfly Architecture and FFT

- **Efficient Architecture:** Recursive and Sparse
- **Multiscale:** Extracts correlations at multiple scales in a single pass

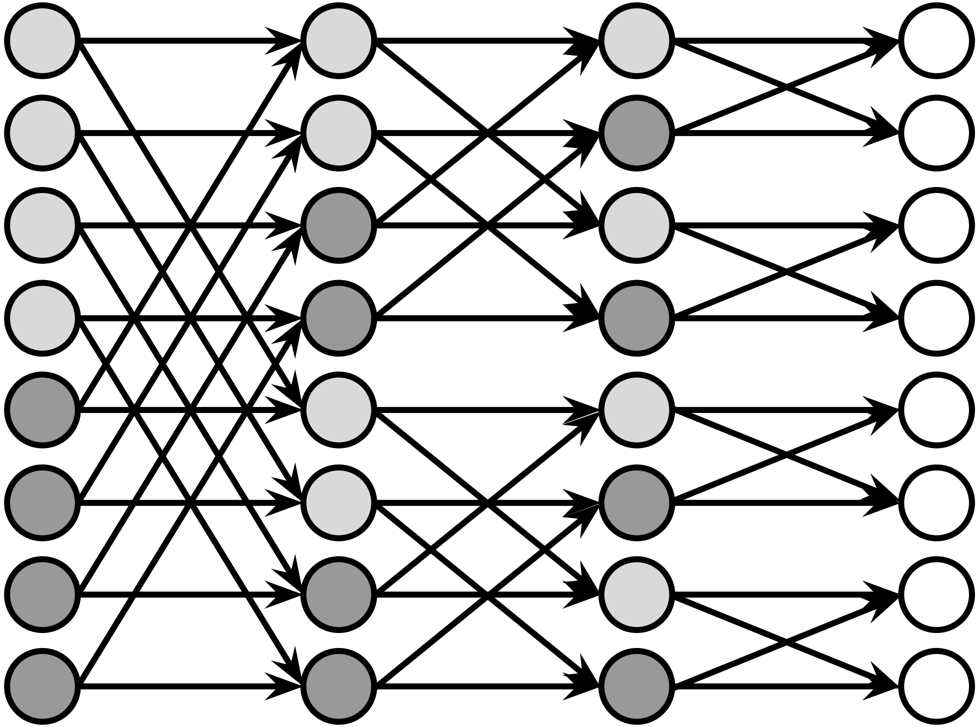


Butterfly Diagram

[1]: Cipra et. al, SIAM: https://archive.siam.org/pdf/news/637.pdf

Butterfly Architecture and FFT

- **Efficient Architecture:** Recursive and Sparse
- **Multiscale:** Extracts correlations at multiple scales in a single pass
- **Among the most important algorithms of 20th century[1]**



Butterfly Diagram

[1]: Cipra et. al, SIAM: https://archive.siam.org/pdf/news/637.pdf

Butterfly Architectures in Machine Learning

Learnable weights?				
Complex-valued?				
Correlated Input data?				

Butterfly Architectures in Machine Learning

	FFT			
Learnable weights?	✗			
Complex-valued?	✓			
Correlated Input data?	✓			

Butterfly Architectures in Machine Learning

	FFT	Butterfly Transform [1]		
Learnable weights?	✗	✓		
Complex-valued?	✓	✗		
Correlated Input data?	✓	✗		

[1] Vahid et al, *Butterfly Transform: An Efficient FFT Based Neural Architecture Design, 2019*
[2] Dao et al, *Learning Fast Algorithms for Linear Transforms Using Butterfly Factorizations, 2020*
[3] Dao et al, *Monarch: Expressive Structured Matrices for Efficient and Accurate Training, 2022*

Butterfly Architectures in Machine Learning

	FFT	Butterfly Transform [1]	Monarch [2,3]	
Learnable weights?	✗	✓	✓	
Complex-valued?	✓	✗	✓	
Correlated Input data?	✓	✗	✗	

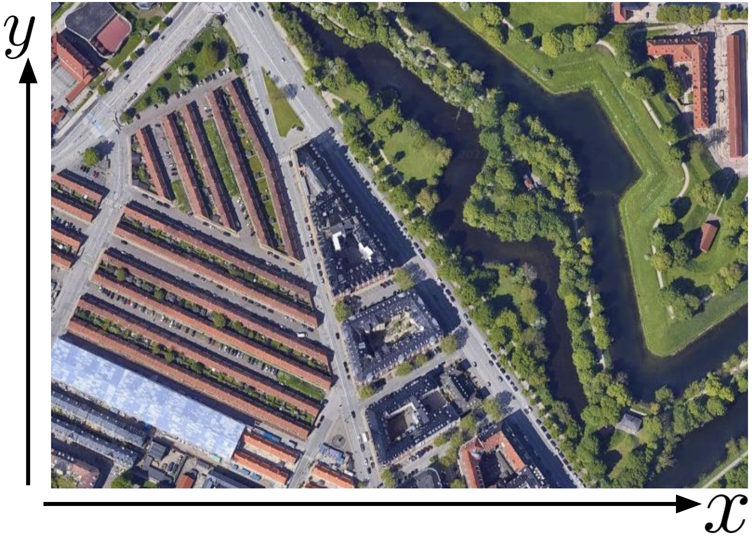
[1] Vahid et al, *Butterfly Transform: An Efficient FFT Based Neural Architecture Design, 2019*
[2] Dao et al, *Learning Fast Algorithms for Linear Transforms Using Butterfly Factorizations, 2020*
[3] Dao et al, *Monarch: Expressive Structured Matrices for Efficient and Accurate Training, 2022*

Butterfly Architectures in Machine Learning

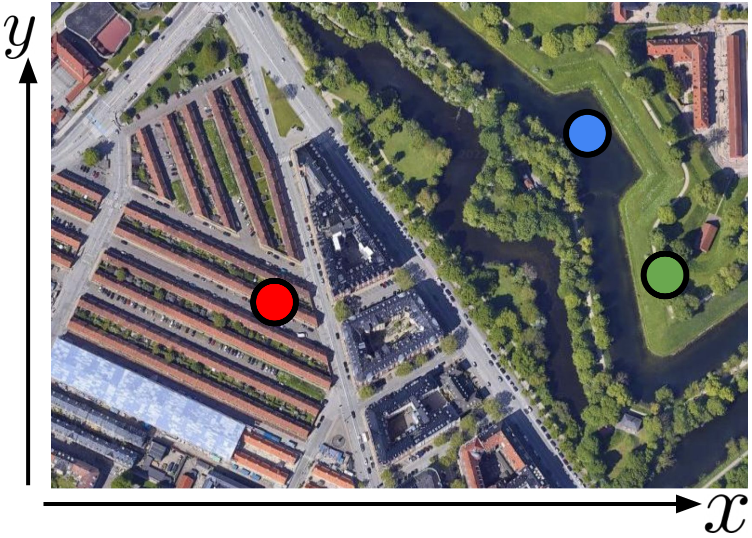
	FFT	Butterfly Transform [1]	Monarch [2,3]	Ours
Learnable weights?	✗	✓	✓	✓
Complex-valued?	✓	✗	✓	✓
Correlated Input data?	✓	✗	✗	✓

[1] Vahid et al, *Butterfly Transform: An Efficient FFT Based Neural Architecture Design, 2019*
[2] Dao et al, *Learning Fast Algorithms for Linear Transforms Using Butterfly Factorizations, 2020*
[3] Dao et al, *Monarch: Expressive Structured Matrices for Efficient and Accurate Training, 2022*

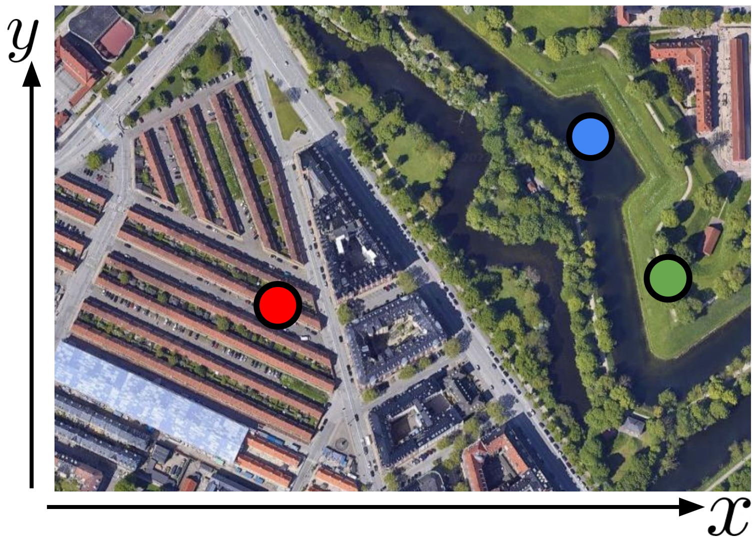
Hyperspectral Imaging

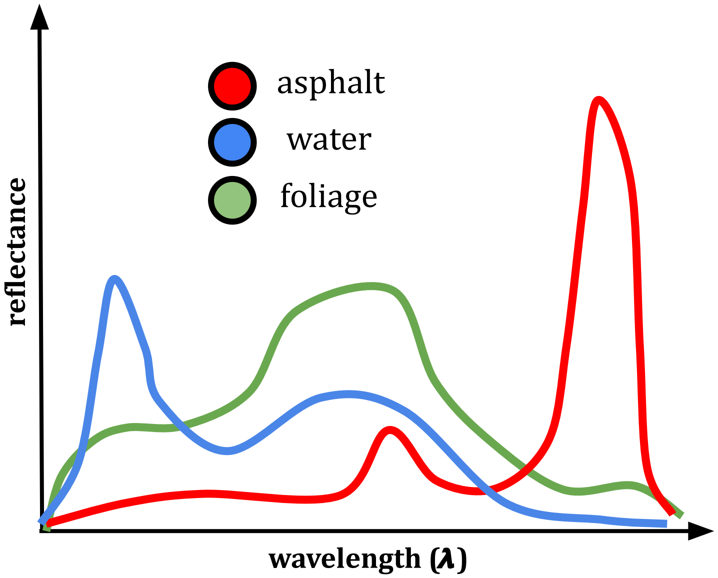


Hyperspectral Imaging

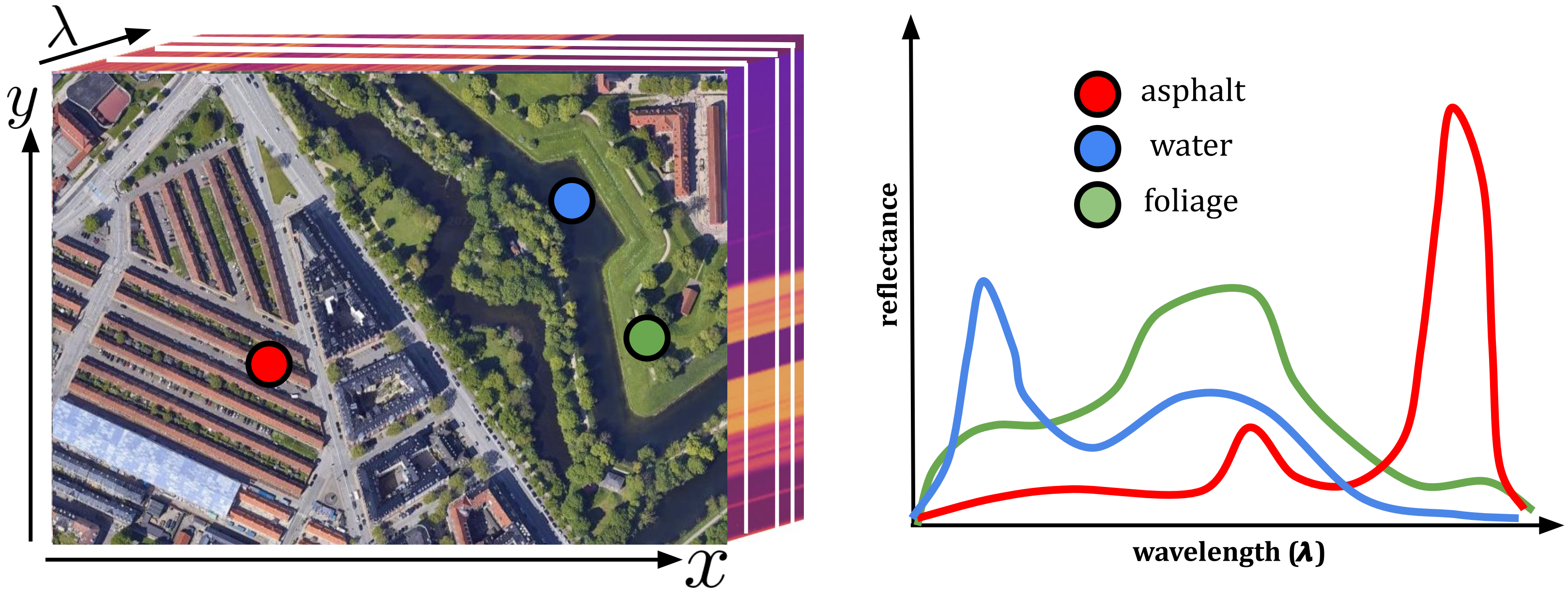


Hyperspectral Imaging

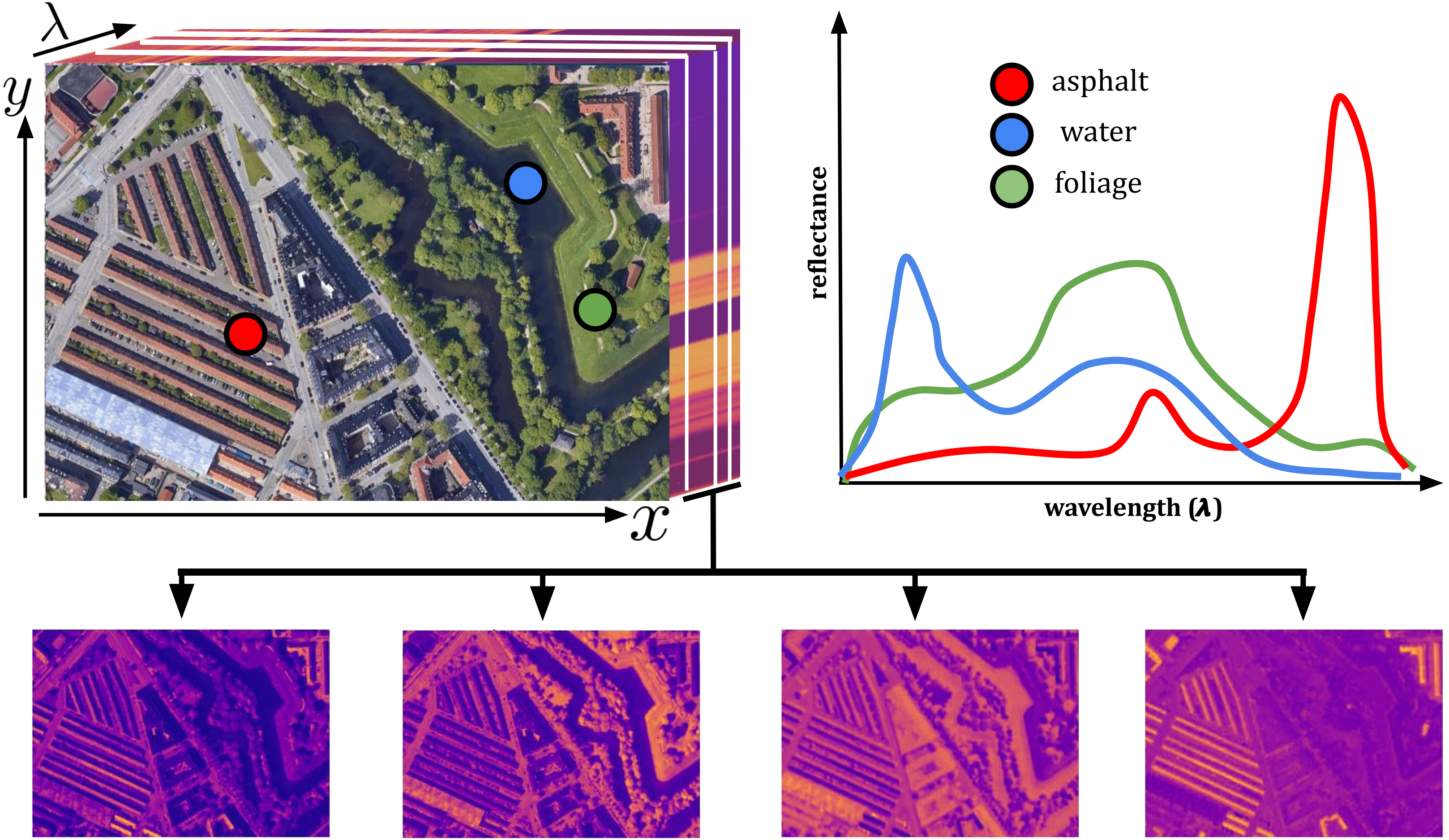




Hyperspectral Imaging



Hyperspectral Imaging

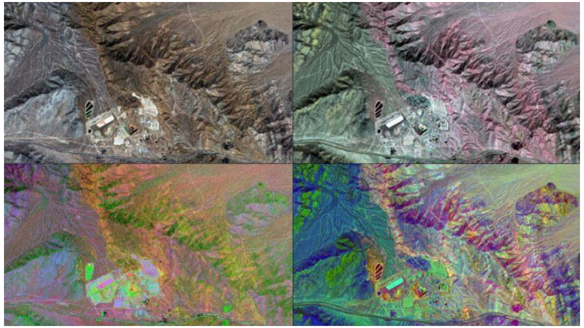


Hyperspectral Imaging is Invaluable for Remote Sensing Applications

Agriculture

Environmental Monitoring

Urban Planning



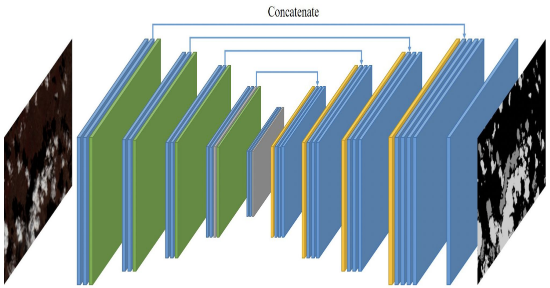
Mineral Exploration

HSI classification: Challenges and Current Approaches

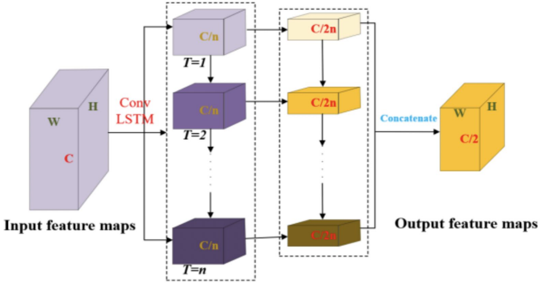
Main Challenge: Exploiting correlations across spectral channels and spatial pixel locations

HSI classification: Challenges and Current Approaches

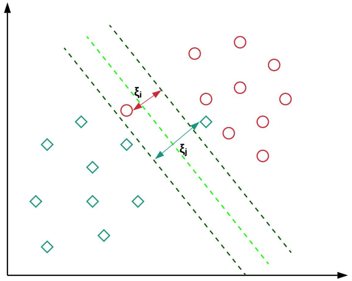
Main Challenge: Exploiting correlations across spectral channels and spatial pixel locations



CNN-Based [1,2]



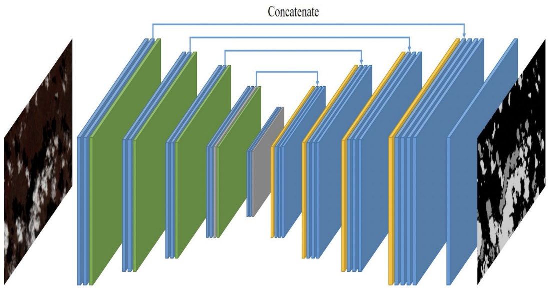
LSTM-Based [3,4]



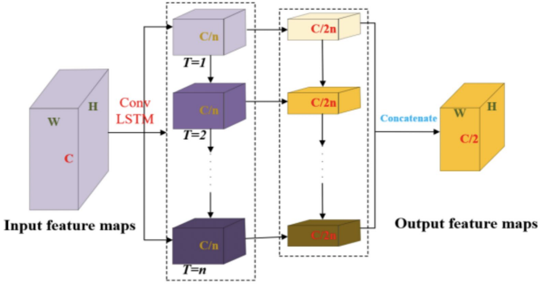
SVM-based [5,6]

[1] Zheng et al, *FPGA: Fast Patch-Free Global Learning Framework for Fully End-to-End Hyperspectral Image Classification*
[2] Lie et al, *A semi-supervised convolutional neural network for hyperspectral image classification*
[3] Seydgar et al, *3-D convolution-recurrent networks for spectral-spatial classification of hyperspectral images*
[4] Zhu et al, *A Spectral-Spatial Dependent Global Learning Framework for Insufficient and Imbalanced Hyperspectral Image Classification*
[5] Fauvel et al, *Spectral and spatial classification of hyperspectral data using SVMs and morphological profiles*
[6] Li et al, *Spectral–spatial hyperspectral image segmentation using subspace multinomial logistic regression and Markov random fields,*

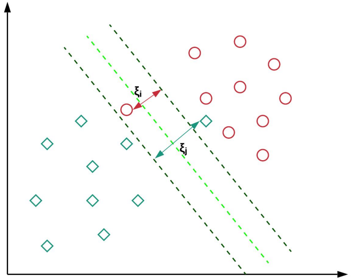
Exploiting correlations across spectral channels and spatial pixel locations



CNN-Based [1,2]



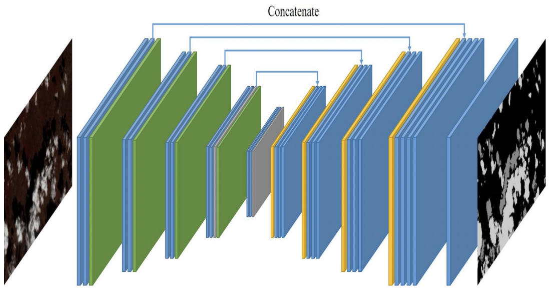
LSTM-Based [3,4]



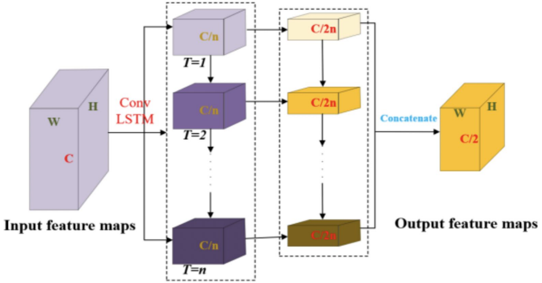
SVM-based [5,6]

[1] Zheng et al, *FPGA: Fast Patch-Free Global Learning Framework for Fully End-to-End Hyperspectral Image Classification*
[2] Lie et al, *A semi-supervised convolutional neural network for hyperspectral image classification*
[3] Seydgar et al, *3-D convolution-recurrent networks for spectral-spatial classification of hyperspectral images*
[4] Zhu et al, *A Spectral-Spatial Dependent Global Learning Framework for Insufficient and Imbalanced Hyperspectral Image Classification*
[5] Fauvel et al, *Spectral and spatial classification of hyperspectral data using SVMs and morphological profiles*
[6] Li et al, *Spectral–spatial hyperspectral image segmentation using subspace multinomial logistic regression and Markov random fields,*

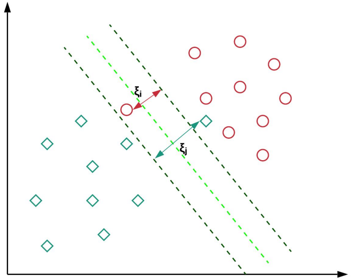
Exploiting correlations across spectral channels and spatial pixel locations



CNN-Based [1,2]



LSTM-Based [3,4]

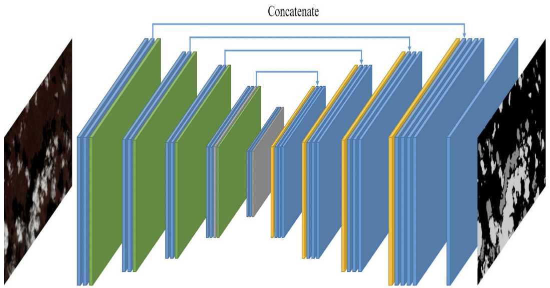


SVM-based [5,6]

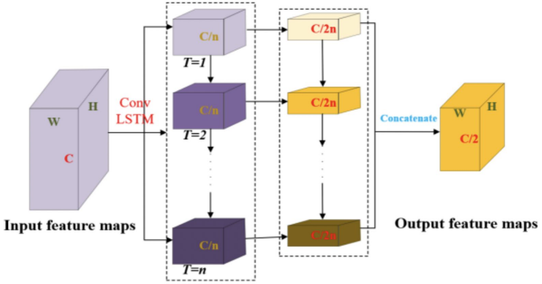
+ Simple
- Treat Channels Independently
- O(N^2) Parameters

[1] Zheng et al, *FPGA: Fast Patch-Free Global Learning Framework for Fully End-to-End Hyperspectral Image Classification*
[2] Lie et al, *A semi-supervised convolutional neural network for hyperspectral image classification*
[3] Seydgar et al, *3-D convolution-recurrent networks for spectral-spatial classification of hyperspectral images*
[4] Zhu et al, *A Spectral-Spatial Dependent Global Learning Framework for Insufficient and Imbalanced Hyperspectral Image Classification*
[5] Fauvel et al, *Spectral and spatial classification of hyperspectral data using SVMs and morphological profiles*
[6] Li et al, *Spectral–spatial hyperspectral image segmentation using subspace multinomial logistic regression and Markov random fields,*

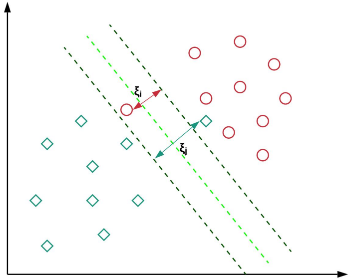
Exploiting correlations across spectral channels and spatial pixel locations



CNN-Based [1,2]



LSTM-Based [3,4]



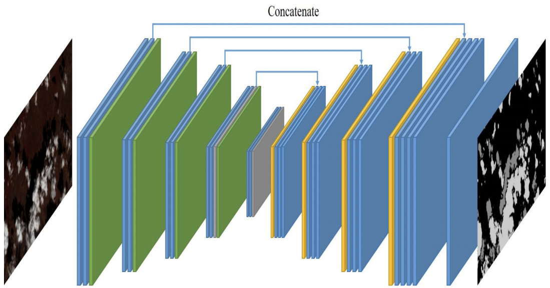
SVM-based [5,6]

+ **Simple**
- **Treat Channels Independently**
- **O(N^2) Parameters**

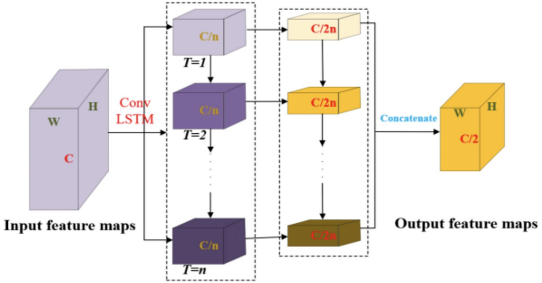
+ **Channels Processed Jointly**
- **Difficult to train**
- **Treat Input as Generic 1D sequence**

[1] Zheng et al, *FPGA: Fast Patch-Free Global Learning Framework for Fully End-to-End Hyperspectral Image Classification*
[2] Lie et al, *A semi-supervised convolutional neural network for hyperspectral image classification*
[3] Seydgar et al, *3-D convolution-recurrent networks for spectral-spatial classification of hyperspectral images*
[4] Zhu et al, *A Spectral-Spatial Dependent Global Learning Framework for Insufficient and Imbalanced Hyperspectral Image Classification*
[5] Fauvel et al, *Spectral and spatial classification of hyperspectral data using SVMs and morphological profiles*
[6] Li et al, *Spectral–spatial hyperspectral image segmentation using subspace multinomial logistic regression and Markov random fields,*

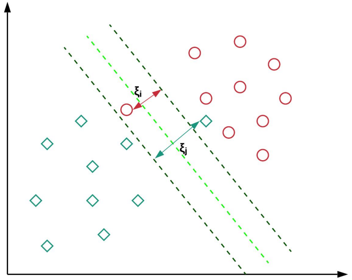
Exploiting correlations across spectral channels and spatial pixel locations



CNN-Based [1,2]



LSTM-Based [3,4]



SVM-based [5,6]

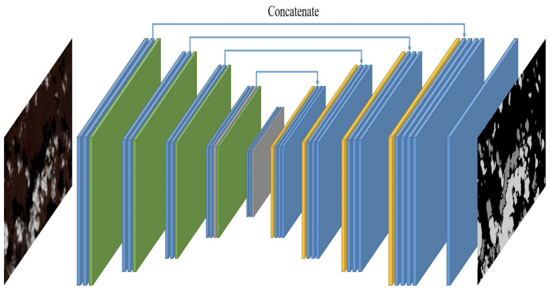
+ Simple
- Treat Channels Independently
- $O(N^2)$ Parameters

+ Channels Processed Jointly
- Difficult to train
- Treat Input as Generic 1D sequence

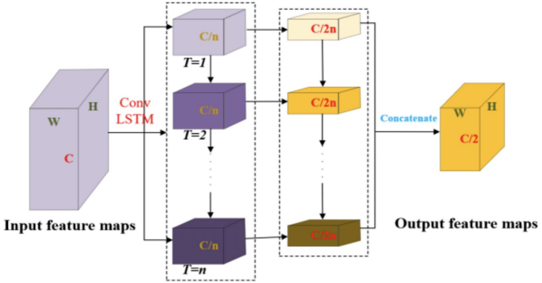
+ Few Parameters
+ Simple
- Low Accuracy

[1] Zheng et al, *FPGA: Fast Patch-Free Global Learning Framework for Fully End-to-End Hyperspectral Image Classification*
[2] Lie et al, *A semi-supervised convolutional neural network for hyperspectral image classification*
[3] Seydgar et al, *3-D convolution-recurrent networks for spectral-spatial classification of hyperspectral images*
[4] Zhu et al, *A Spectral-Spatial Dependent Global Learning Framework for Insufficient and Imbalanced Hyperspectral Image Classification*
[5] Fauvel et al, *Spectral and spatial classification of hyperspectral data using SVMs and morphological profiles*
[6] Li et al, *Spectral–spatial hyperspectral image segmentation using subspace multinomial logistic regression and Markov random fields,*

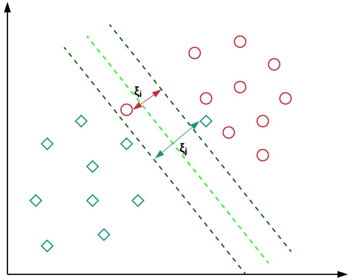
Exploiting correlations across spectral channels and spatial pixel locations



CNN-Based [1,2]



LSTM-Based [3,4]



SVM-based [5,6]

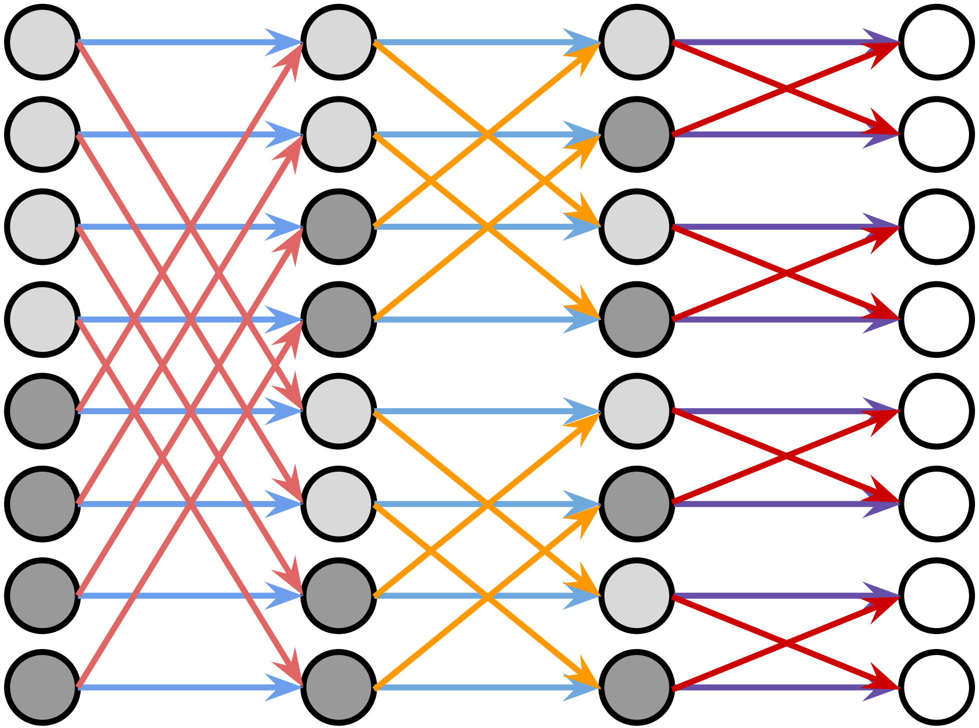
+ Simple
- Treat Channels Independently
- O(N^2) Parameters

+ Channels Processed Jointly
- Difficult to train
- Treat Input as Generic 1D sequence

+ Few Parameters
+ Simple
- Low Accuracy

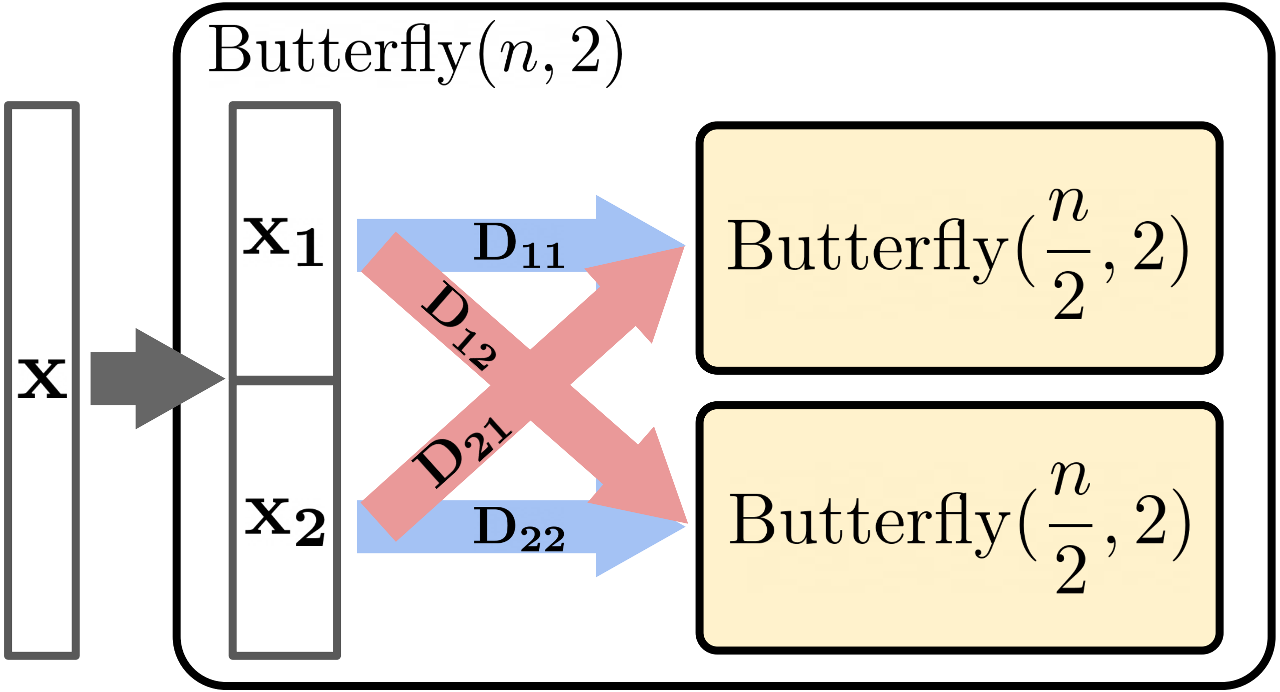
[1] Zheng et al, *FPGA: Fast Patch-Free Global Learning Framework for Fully End-to-End Hyperspectral Image Classification*
[2] Lie et al, *A semi-supervised convolutional neural network for hyperspectral image classification*
[3] Seydgar et al, *3-D convolution-recurrent networks for spectral-spatial classification of hyperspectral images*
[4] Zhu et al, *A Spectral-Spatial Dependent Global Learning Framework for Insufficient and Imbalanced Hyperspectral Image Classification*
[5] Fauvel et al, *Spectral and spatial classification of hyperspectral data using SVMs and morphological profiles*
[6] Li et al, *Spectral–spatial hyperspectral image segmentation using subspace multinomial logistic regression and Markov random fields,*

Proposed Method: Learnable Complex-Valued Butterfly



- \+ O(N log(N)) parameters
- \+ Extracts features at multiple scales
- \+ Utilizes redundant structure in input signal

Proposed Method: Learned Complex-Valued Butterfly



Complex-Valued Butterfly: Recursive Definition

Proposed Method: Learned Complex-Valued Butterfly

$$\mathbf{B}^{(n,k)} = \begin{bmatrix} \mathbf{B}_1^{(\frac{n}{k},k)}\mathbf{D}_{11} & \dots & \mathbf{B}_1^{(\frac{n}{k},k)}\mathbf{D}_{1k} \\ \vdots & \ddots & \vdots \\ \mathbf{B}_k^{(\frac{n}{k},k)}\mathbf{D}_{k1} & \dots & \mathbf{B}_k^{(\frac{n}{k},k)}\mathbf{D}_{kk} \end{bmatrix}$$

Recursive Matrix Definition

Proposed Method: Learned Complex-Valued Butterfly

$$\mathbf{B}^{(n,k)} = \begin{bmatrix} \mathbf{B}_1^{(\frac{n}{k},k)}\mathbf{D}_{11} & \dots & \mathbf{B}_1^{(\frac{n}{k},k)}\mathbf{D}_{1k} \\ \vdots & \ddots & \vdots \\ \mathbf{B}_k^{(\frac{n}{k},k)}\mathbf{D}_{k1} & \dots & \mathbf{B}_k^{(\frac{n}{k},k)}\mathbf{D}_{kk} \end{bmatrix}$$

Recursive Matrix Definition

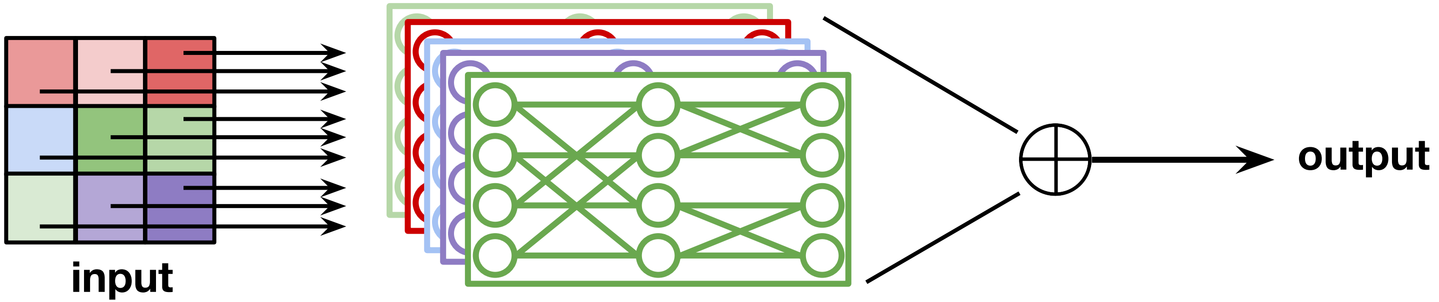
$$\mathbf{B}^{(n,k)}\mathbf{x} = \begin{bmatrix} \mathbf{B}_1^{(\frac{n}{k},k)} \sum_{j=1}^{k} \mathbf{D}_{1j}\mathbf{x}_j \\ \vdots \\ \mathbf{B}_k^{(\frac{n}{k},k)} \sum_{j=1}^{k} \mathbf{D}_{kj}\mathbf{x}_j \end{bmatrix}$$

Efficient Block-wise Computation

Proposed Method: Generalized Spatial-Spectral Complex-Valued Butterfly

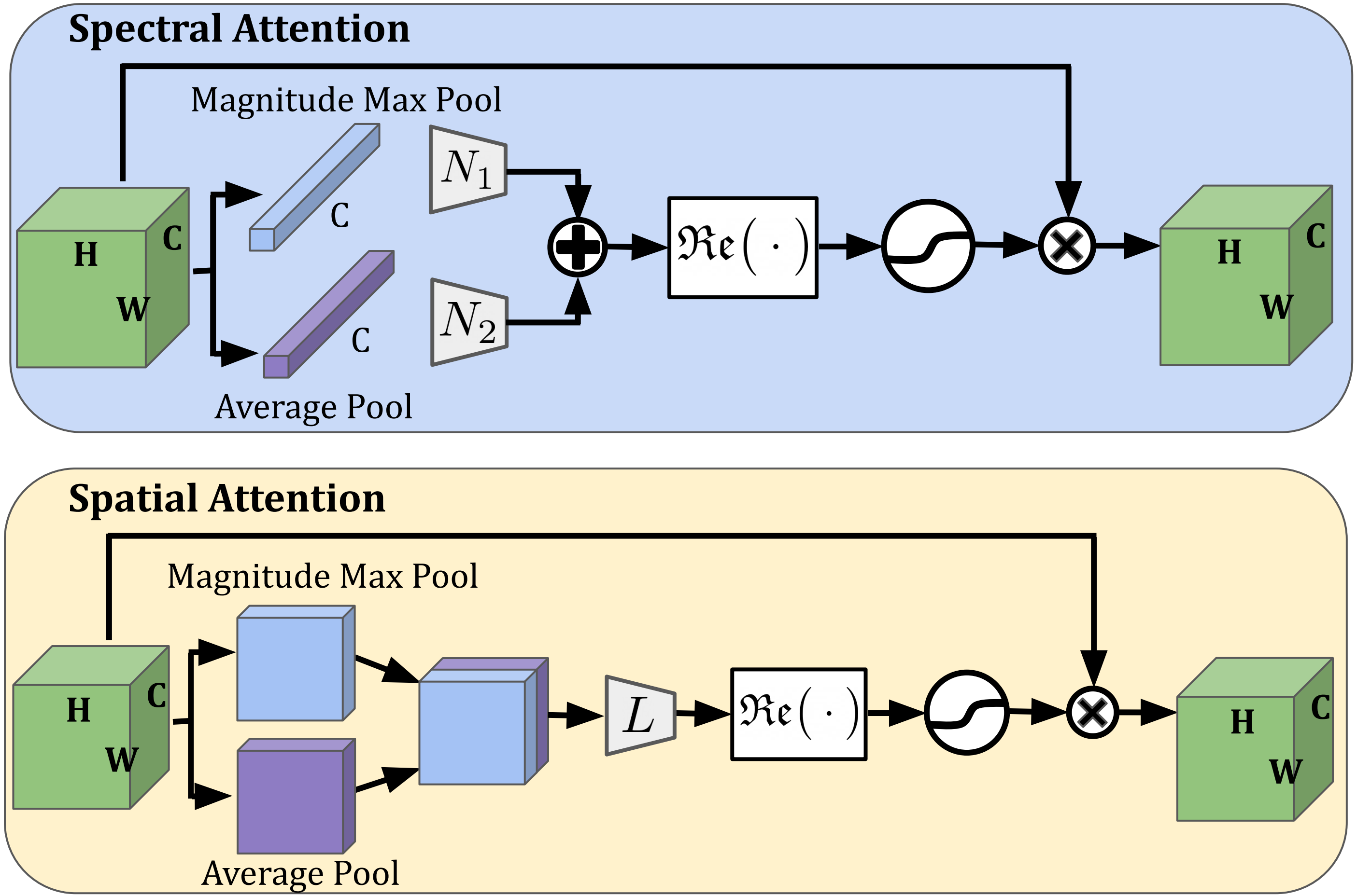
$$\mathbf{B}^{(n,k)}\mathbf{x} = \begin{bmatrix} \mathbf{B}_1^{(\frac{n}{k},k)} \sum_{j=1}^{k} \sum_{p=1}^{m\times m} \mathbf{D}_{1jp}\mathbf{x}_j^{(p)} \\ \vdots \\ \mathbf{B}_k^{(\frac{n}{k},k)} \sum_{j=1}^{k} \sum_{p=1}^{m\times m} \mathbf{D}_{kjp}\mathbf{x}_j^{(p)} \end{bmatrix}$$

Generalization: Spatial + Spectral Butterfly

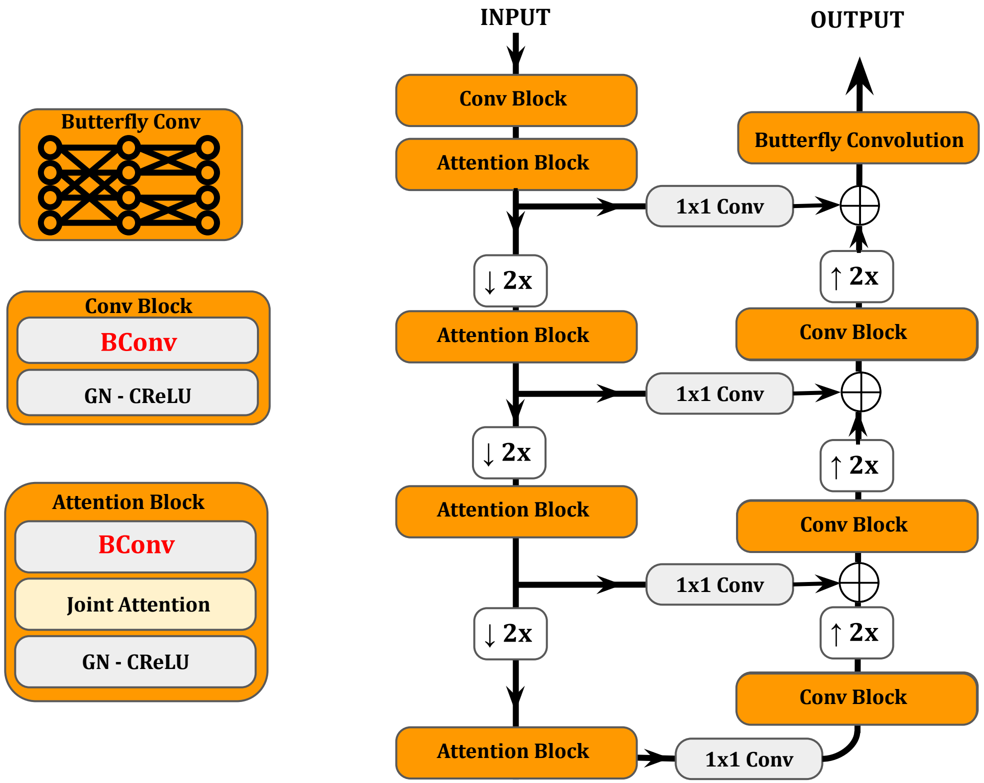


Aggregating over local pixel context

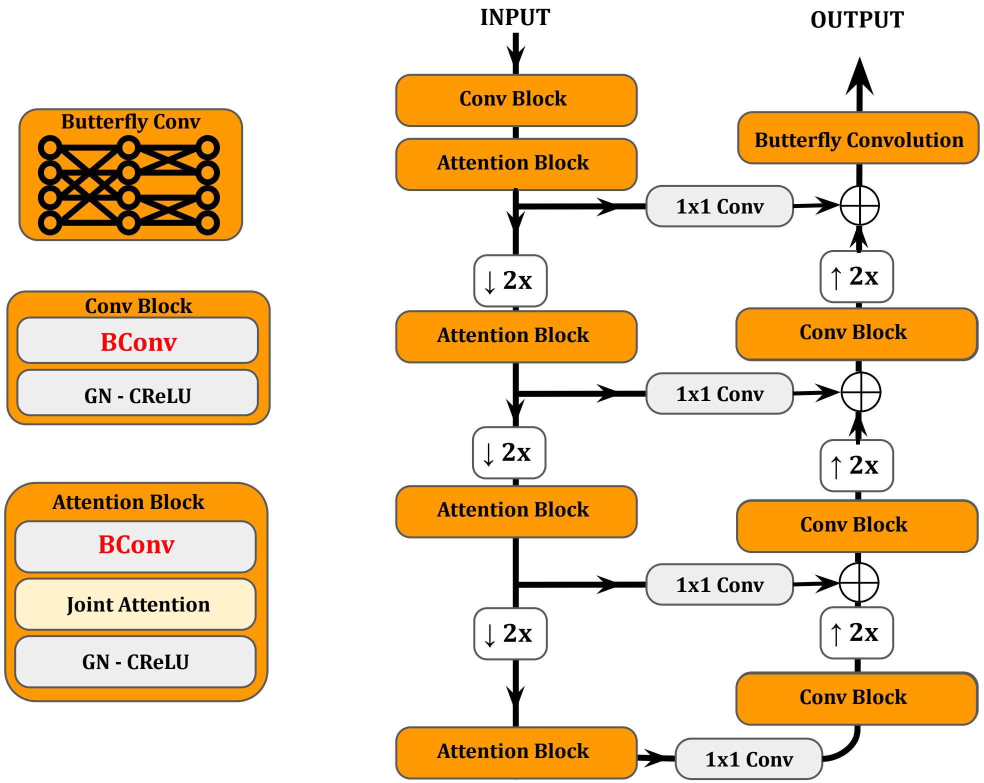
Proposed Method: Complex-Valued Spatial & Spectral Attention



Proposed Method: Architecture Details



Proposed Method: Architecture Details



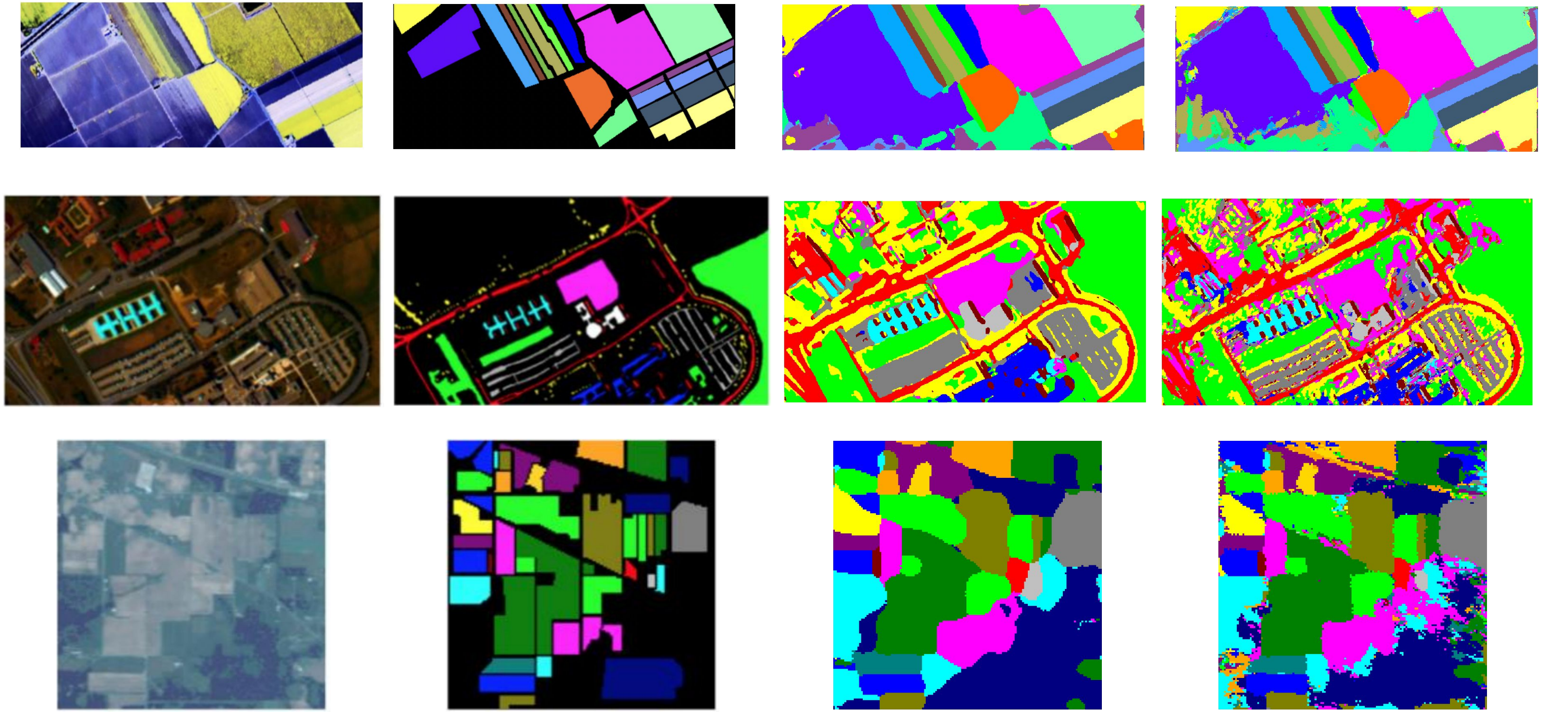
Experiments: HSI Pixel-wise Classification

Input Image | **Ground Truth** | **SSDGL** | **Ours**

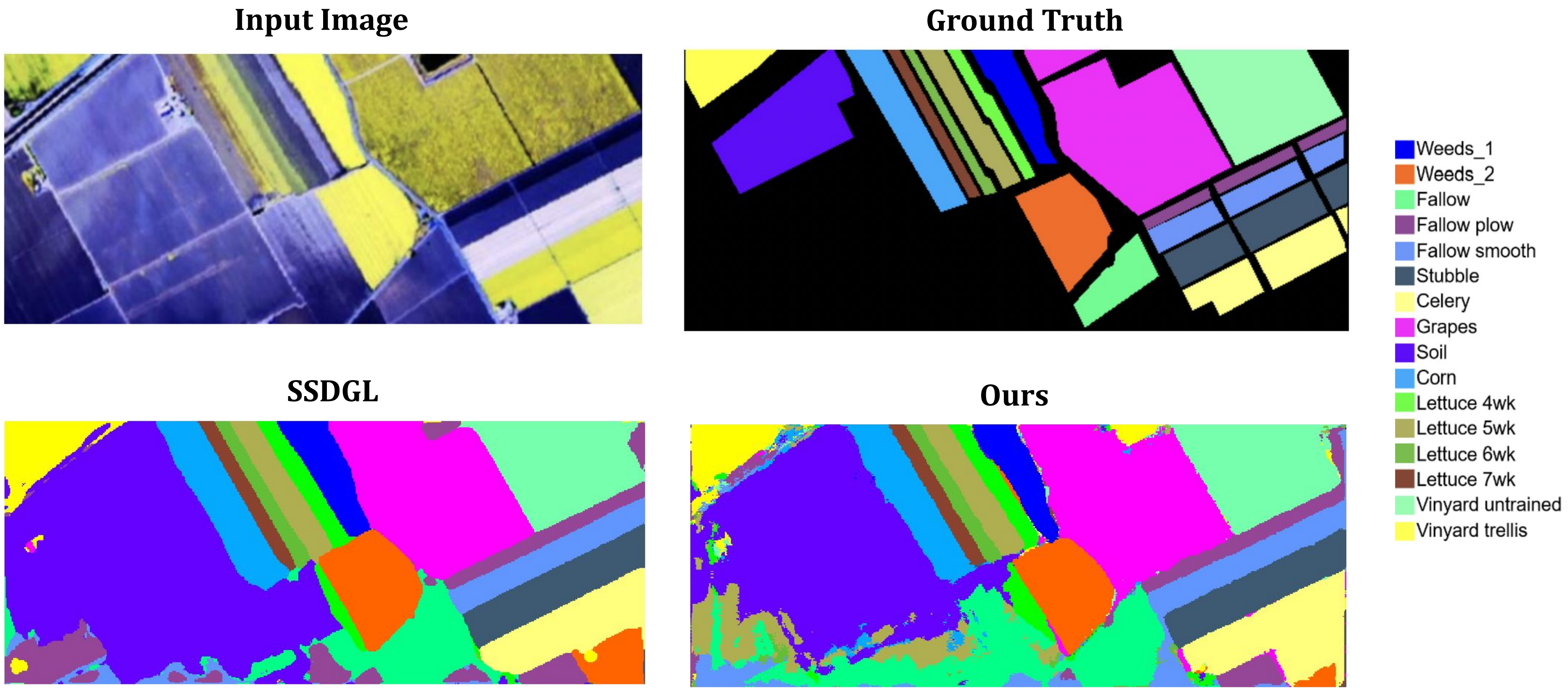
Salinas

Pavia University

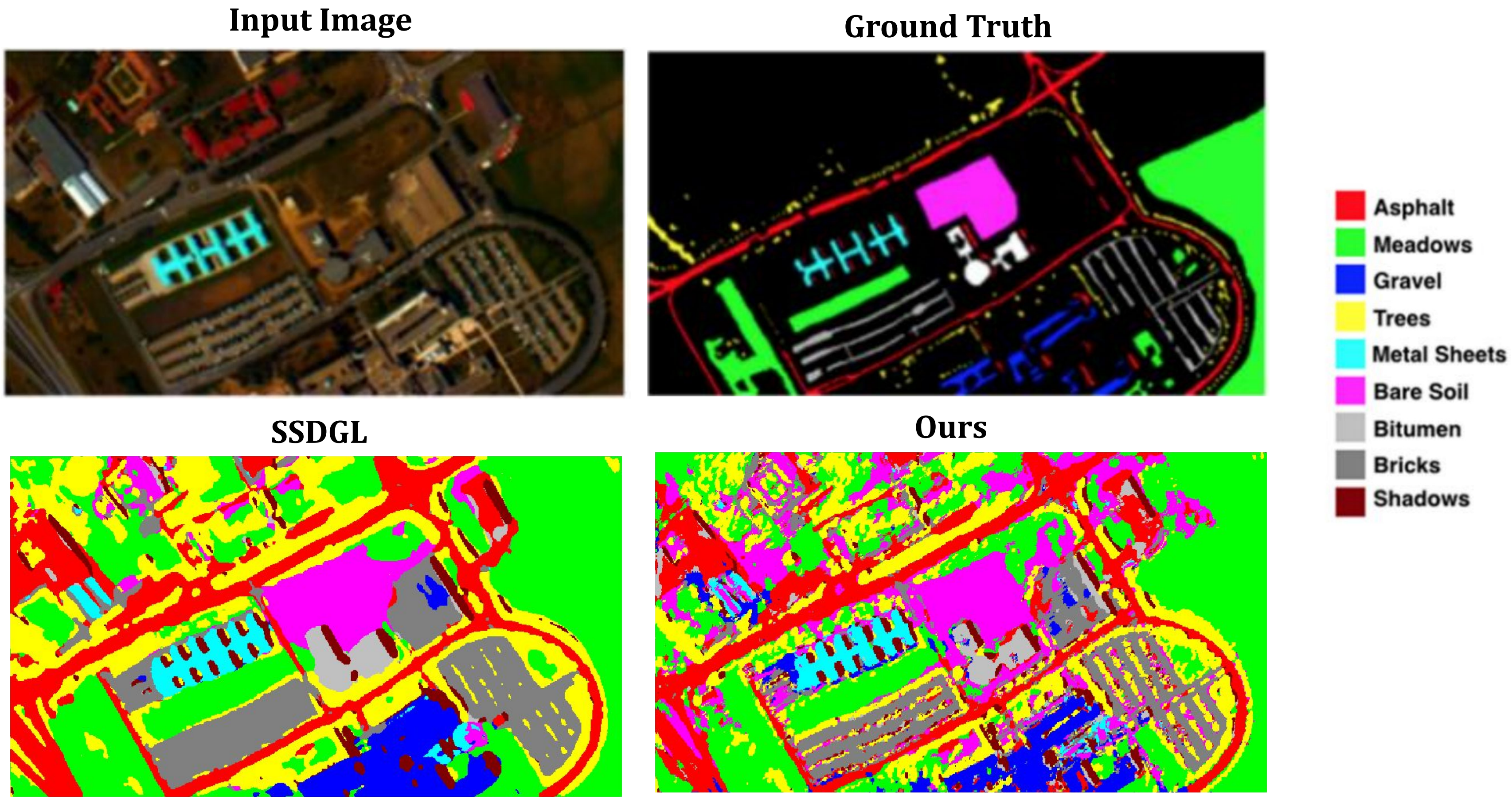
Indian Pines



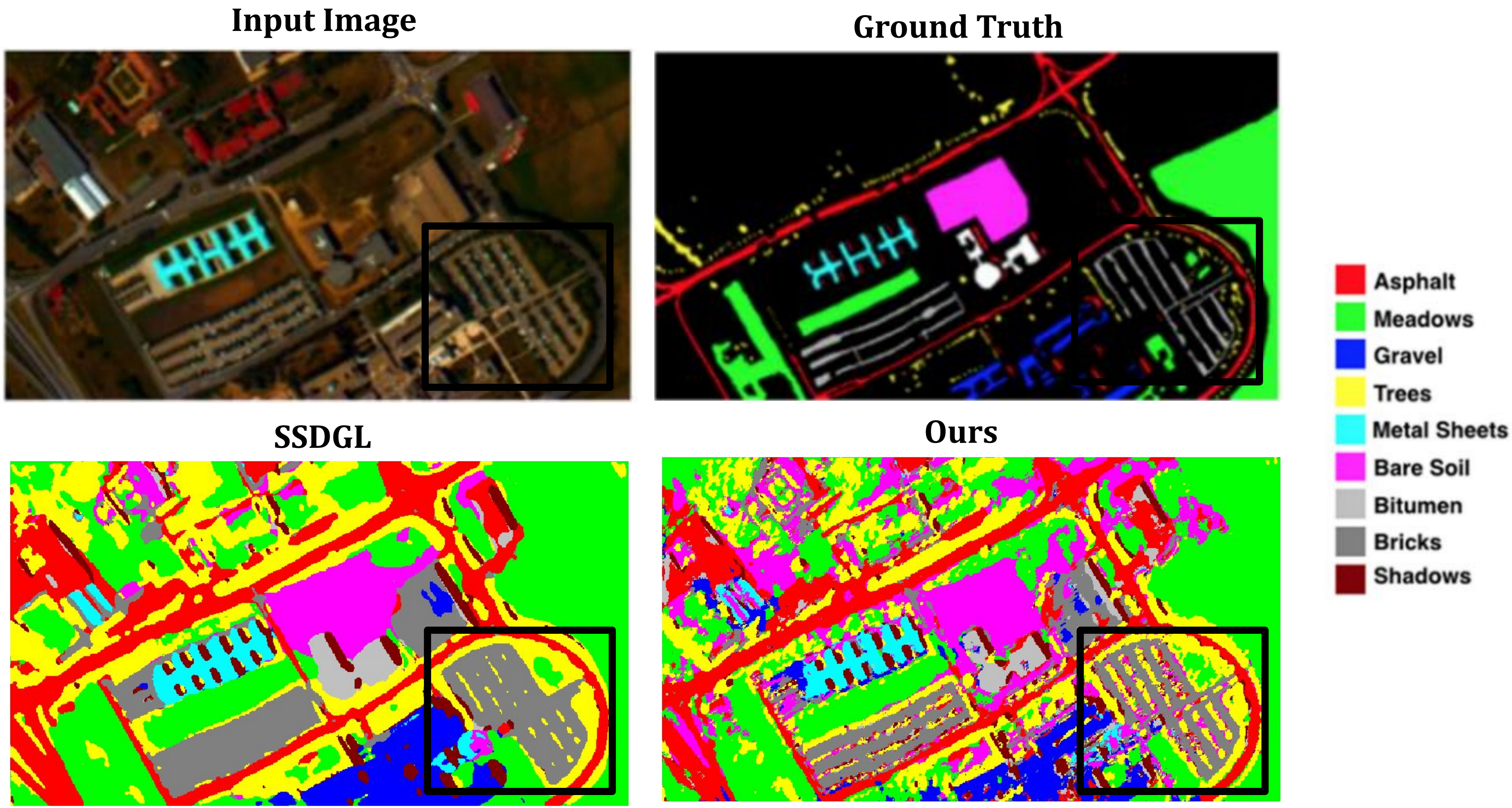
Salinas Classification Map



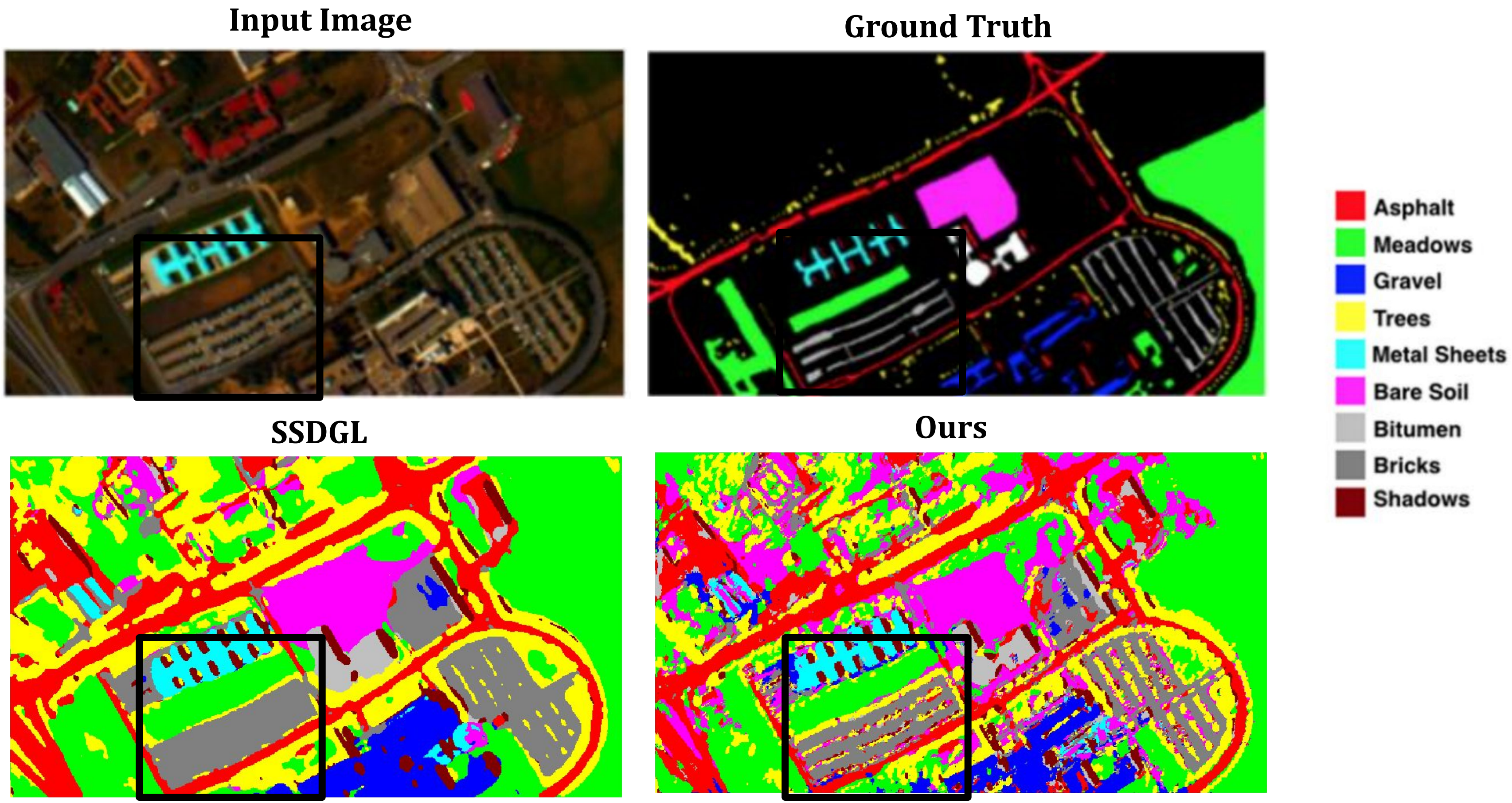
Pavia University Classification Map



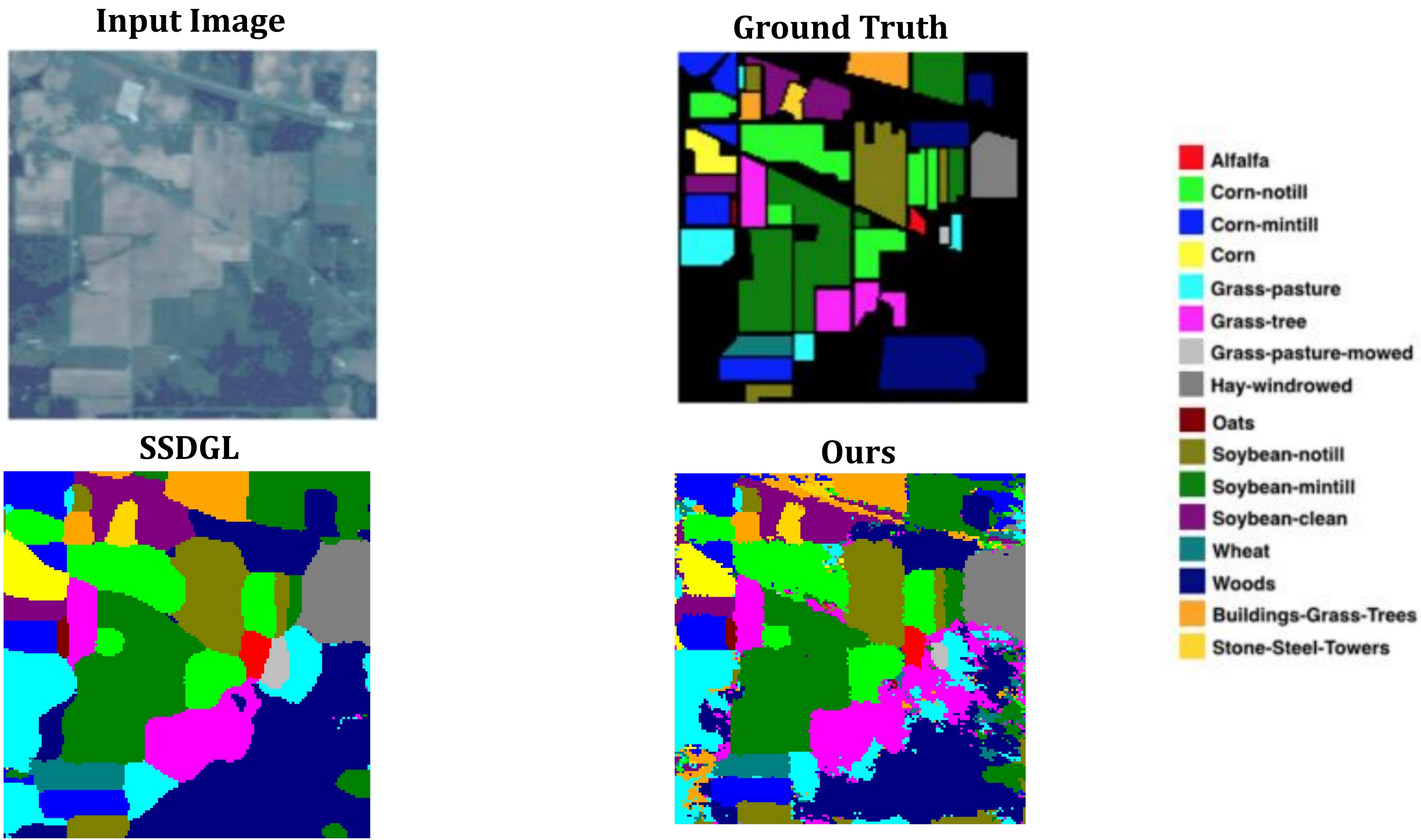
Pavia University Classification Map



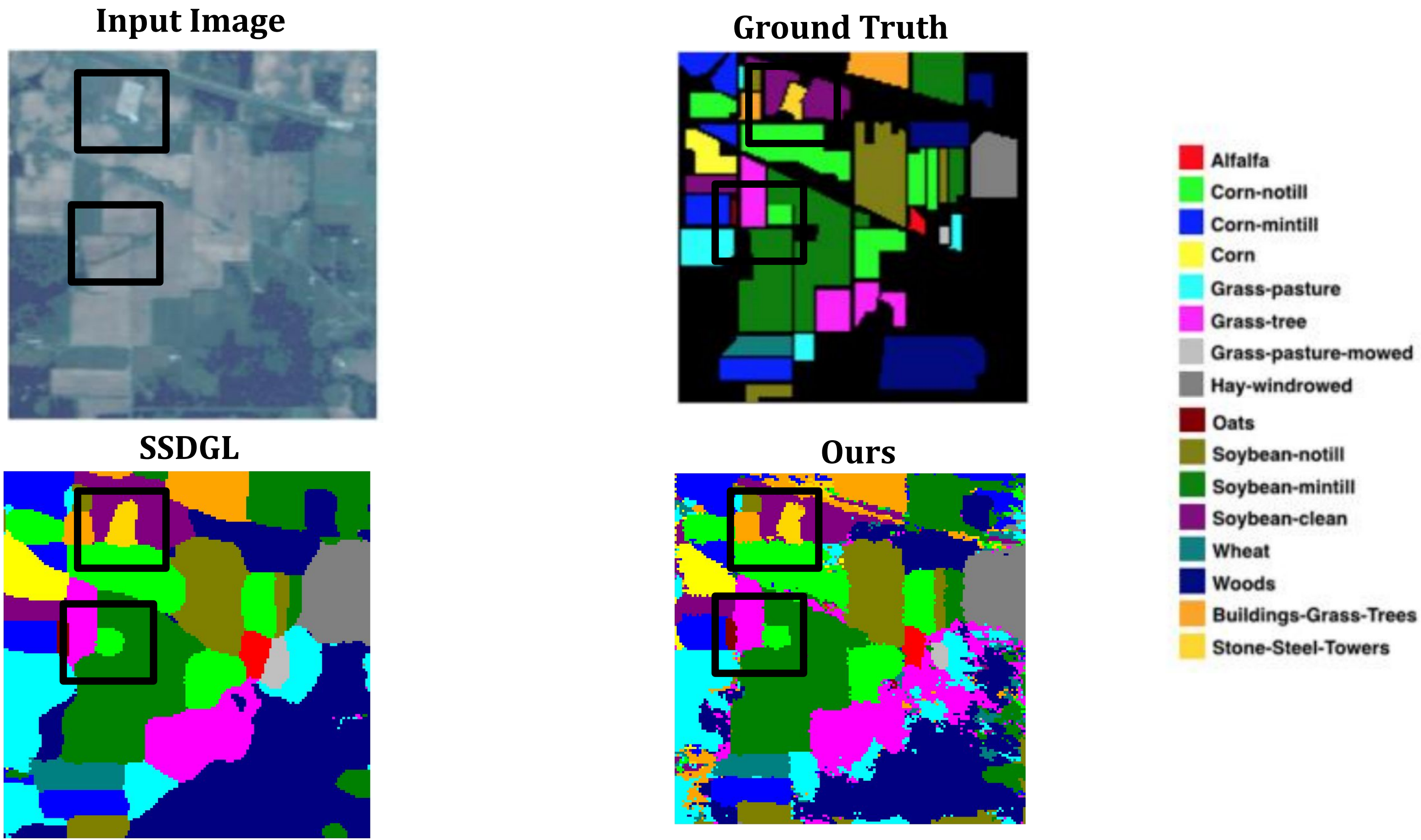
Pavia University Classification Map



Indian Pines Classification Map



Indian Pines Classification Map



Results: HSI Classification

Model	Indian Pines Dataset			Pavia University Dataset			Salinas Dataset		
	OA (%)	**AA (%)**	**Kappa**	**OA (%)**	**AA (%)**	**Kappa**	**OA (%)**	**AA (%)**	**Kappa**
SVM	75.3	71.1	0.717	86.5	73.5	0.819	90.9	96.2	0.899
U-Net	93.2	92.1	0.922	96.1	93.5	0.948	-	-	-
SSDGL	99.6	99.8	0.996	**99.9**	**99.9**	**0.999**	**100**	**100**	**1.0**
Ours	**99.9**	**99.9**	**0.998**	**99.9**	**99.9**	**0.999**	**99.9**	**100**	**99.9**

7x fewer parameters with higher/similar accuracy!

Summary

- We propose a class of Complex-Valued Learned Butterfly Transforms
- We discuss the challenges of applying butterfly transforms in neural networks
- We propose using HSI data as a source of input with redundant correlated channels
- On popular HSI classification datasets, our method shows similar accuracy while using 7x fewer parameters than the baseline.