Unsupervised Deep Basis Pursuit:
Learning Reconstructions without Ground-Truth Data

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Declaration of Financial Interests or Relationships

Speaker Name: Jonathan I. Tamir

I have the following financial interest or relationship(s) to disclose with regard to the subject matter of this presentation:

- Grant/research support: GE Healthcare
- Employment: Subtle Medical
- Ownership Interest: Subtle Medical

Motivation

Deep learning + iterative optimization = Best of Both Worlds

$$\min_{\mathbf{x}} \frac{1}{2}||\mathbf{y} - \mathbf{A}\mathbf{x}||_2 + Q(\mathbf{x})$$

MoDL[1]:

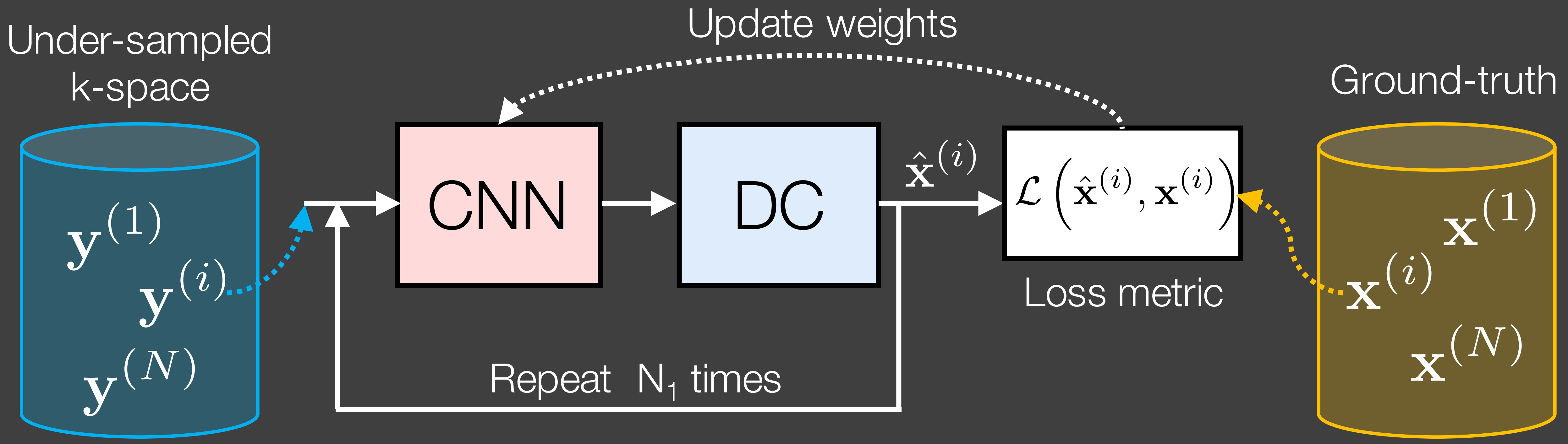
$$Q(\mathbf{x}) = \lambda||\mathbf{x} - \mathcal{R}_{\mathbf{w}}(\mathbf{x})||_2^2$$

CNN regularization with weights **w**

[1] S Diamond, arXiv, 2017. [2] J Schlemper, IPMI, 2017. [3] K Hammernik, MRM 2017. [4] J Zhang, CVPR 2018. [5] Aggarwal, IEEE TMI 2019.

Motivation

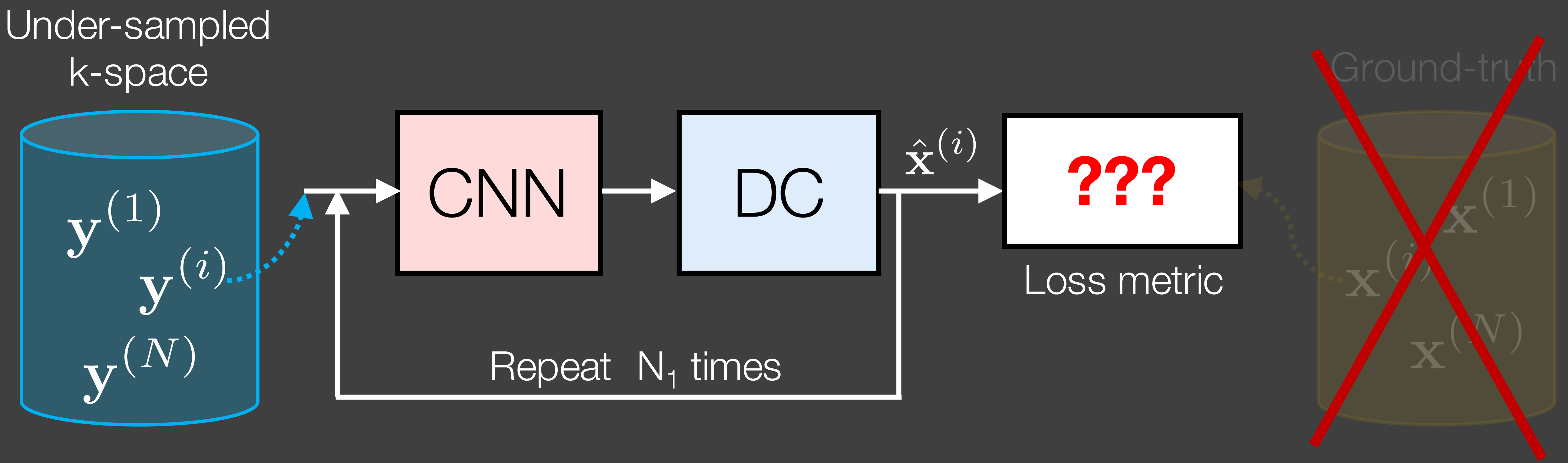
Deep learning + iterative optimization = Best of Both Worlds



[1] S Diamond, arXiv, 2017. [2] J Schlemper, IPMI, 2017. [3] K Hammernik, MRM 2017. [4] J Zhang, CVPR 2018. [5] HK Aggarwal, IEEE TMI 2019.

Challenge

What to do when there is no ground-truth data?



Challenge

What to do when there is no ground-truth data?

Noise2Noise[1]
CNN with SURE[2,3]
Deep Image Prior[4,5]
Convolutional sparse coding[6]
…
…

$$\min_{\mathbf{w}} ||\mathbf{y} - \mathbf{A}G_{\mathbf{w}}(\mathbf{z})||_2$$

Generator network
random vector $\mathbf{z}$

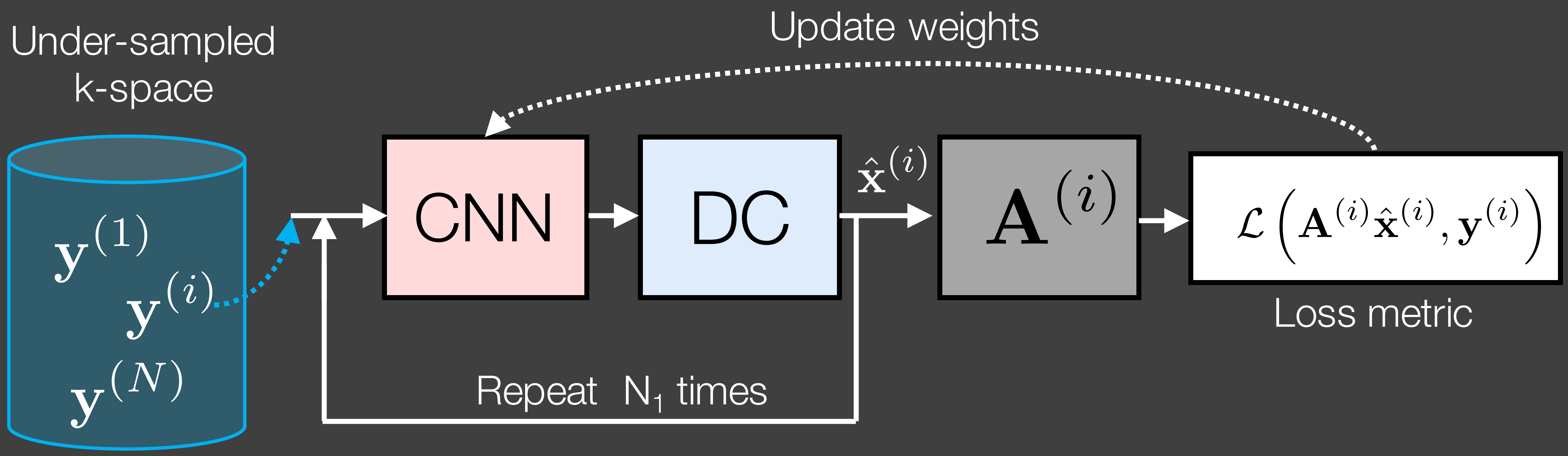
[1] J Lehtinen, ICML 2018. [2] S Soltanayev, NIPS 2018. [3] A Metzler, arXiv 2018. [4] D Ulyanov, CVPR 2018. [5] D Van Veen, arXiv 2018. [6] F Ong, ISMRM 2018

Goal

Learn network parameters **directly from under-sampled data**

Approach

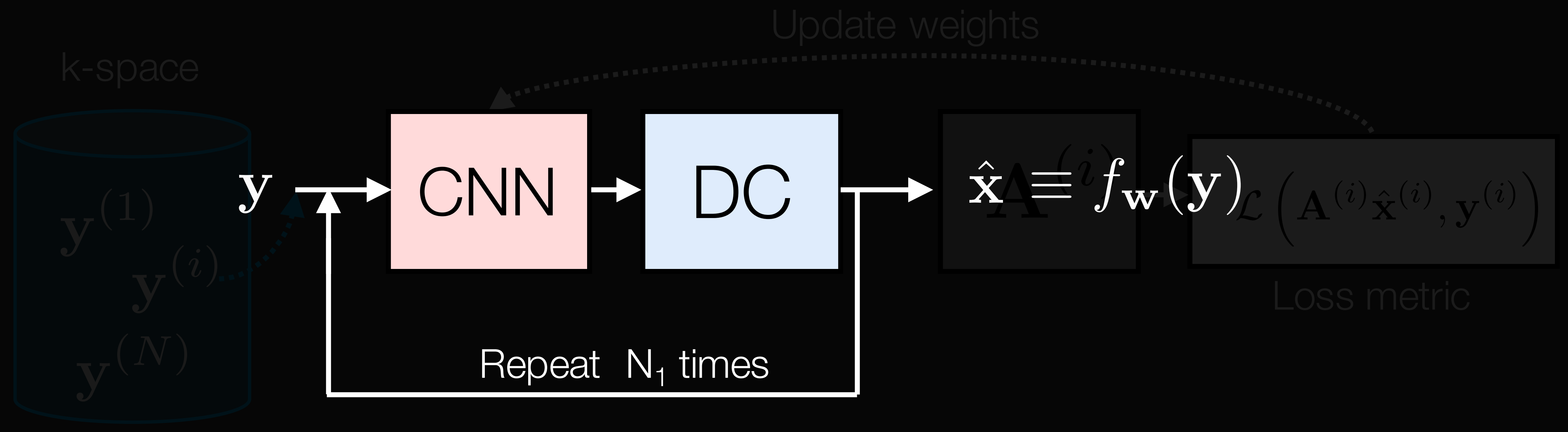
Impose loss directly on measured data[1,2]



[1] D Van Veen, arXiv 2018. [2] F Ong, ISMRM 2018

Approach

Impose loss directly on measured data[1,2]

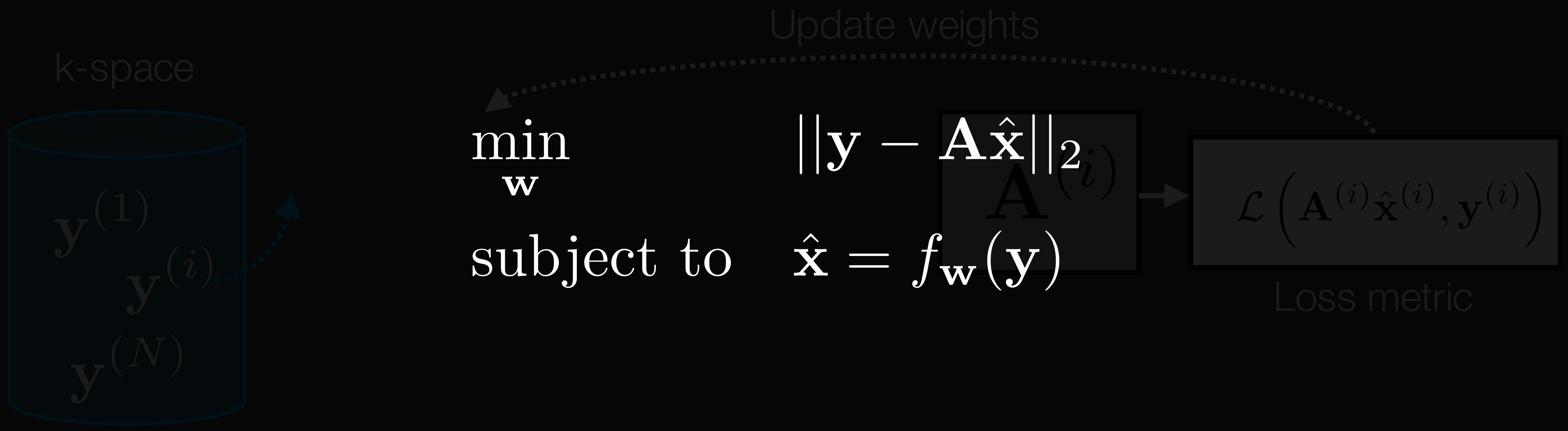


[1] D Van Veen, arXiv 2018. [2] F Ong, ISMRM 2018

Approach

Impose loss directly on measured data[1,2]

$$\min_{\mathbf{w}} \quad ||\mathbf{y} - \mathbf{A}\hat{\mathbf{x}}||_2$$
$$\text{subject to} \quad \hat{\mathbf{x}} = f_{\mathbf{w}}(\mathbf{y})$$



[1] D Van Veen, arXiv 2018. [2] F Ong, ISMRM 2018

Approach

Training on N data sets

$$\min_{\mathbf{w}} \sum_{i=1}^{N} \|\mathbf{y}^{(i)} - \mathbf{A}^{(i)}\hat{\mathbf{x}}^{(i)}\|_2$$

$$\text{subject to} \quad \boxed{\hat{\mathbf{x}}^{(i)} = f_{\mathbf{w}}\left(\mathbf{y}^{(i)}\right)}$$

How to design *f*

Update weights

k-space

$\mathbf{y}^{(1)}$ $\mathbf{y}^{(i)}$ $\mathbf{y}^{(N)}$

$\mathbf{A}^{(i)}$

$\mathcal{L}\left(\mathbf{A}^{(i)}\hat{\mathbf{x}}^{(i)}, \mathbf{y}^{(i)}\right)$

Loss metric

[1] D Van Veen, arXiv 2018. [2] F Ong, ISMRM 2018

Deep Basis Pursuit (DBP)

- Combine MoDL[1] regularization with basis pursuit denoising[2]

$$\mathcal{N}_{\mathbf{w}}(\mathbf{x}) = \mathbf{x} - \mathcal{R}_{\mathbf{w}}(\mathbf{x})$$

Estimates noise and aliasing

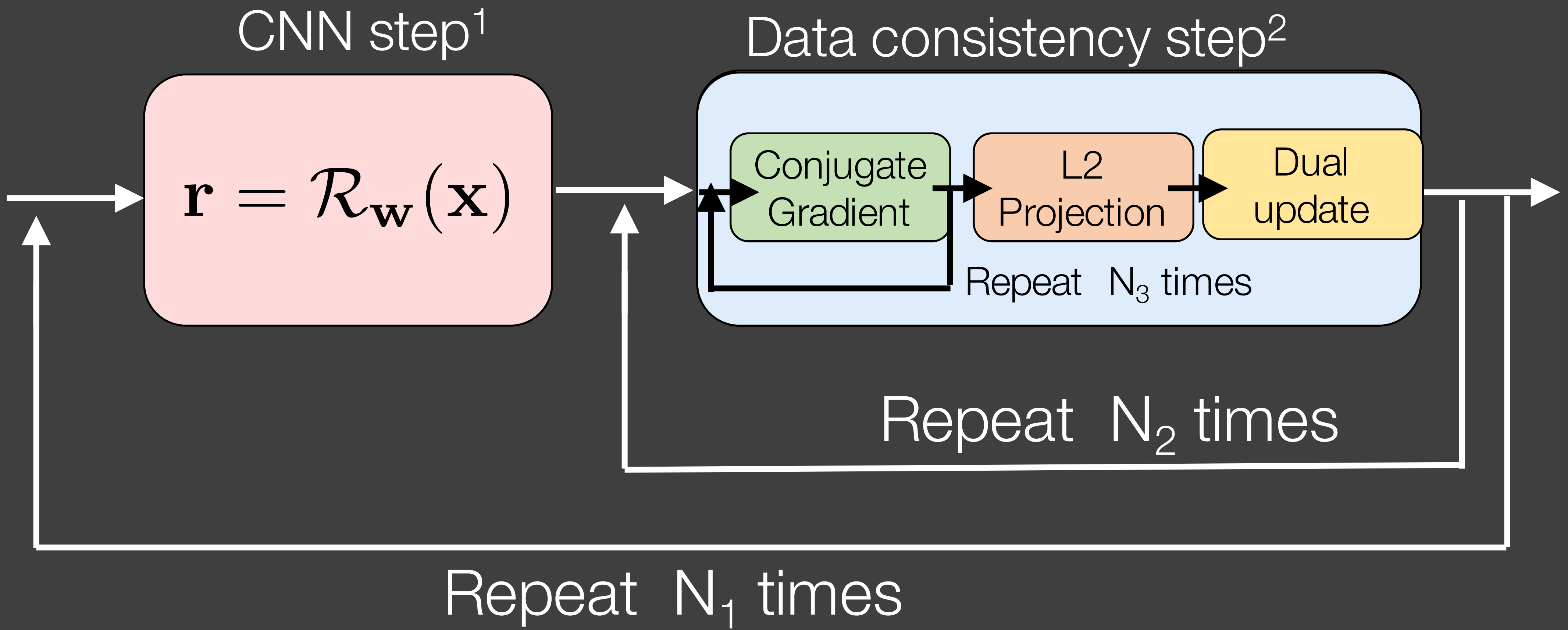
$$\underbrace{\begin{aligned} \min_{\mathbf{x}} \quad & \frac{1}{2}\|\boxed{\mathcal{N}_{\mathbf{w}}(\mathbf{x})}\|_2^2 \\ \text{subject to} \quad & \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2 \leq \epsilon \end{aligned}}_{f_{\mathbf{w}}(\mathbf{y})}$$

Known noise level

[1] HK Aggarwal, IEEE TMI 2019. [2] S Chen, Stanford technical report.

DBP: Alternating minimization

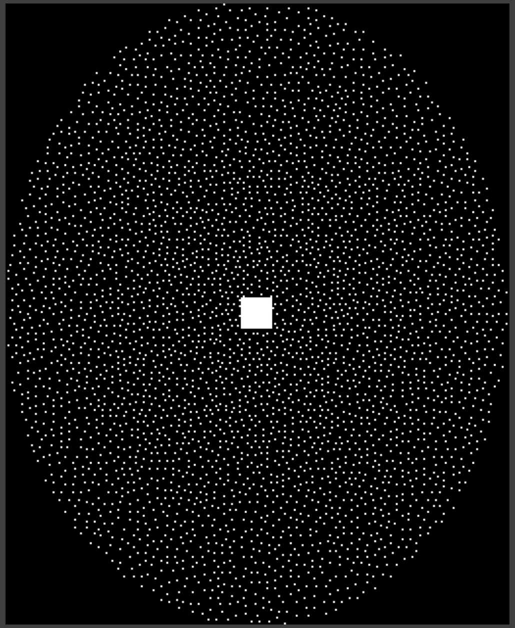
$$\begin{aligned} \min_{\mathbf{x}} \quad & \frac{1}{2}||\mathbf{x} - \mathcal{R}_{\mathbf{w}}(\mathbf{x})||_2^2 \\ \text{subject to} \quad & ||\mathbf{y} - \mathbf{A}\mathbf{x}||_2 \leq \epsilon \end{aligned}$$



[1] HK Aggarwall, IEEE TMI 2019. [2] Boyd, FTML 2011.

Methods

Sampling pattern



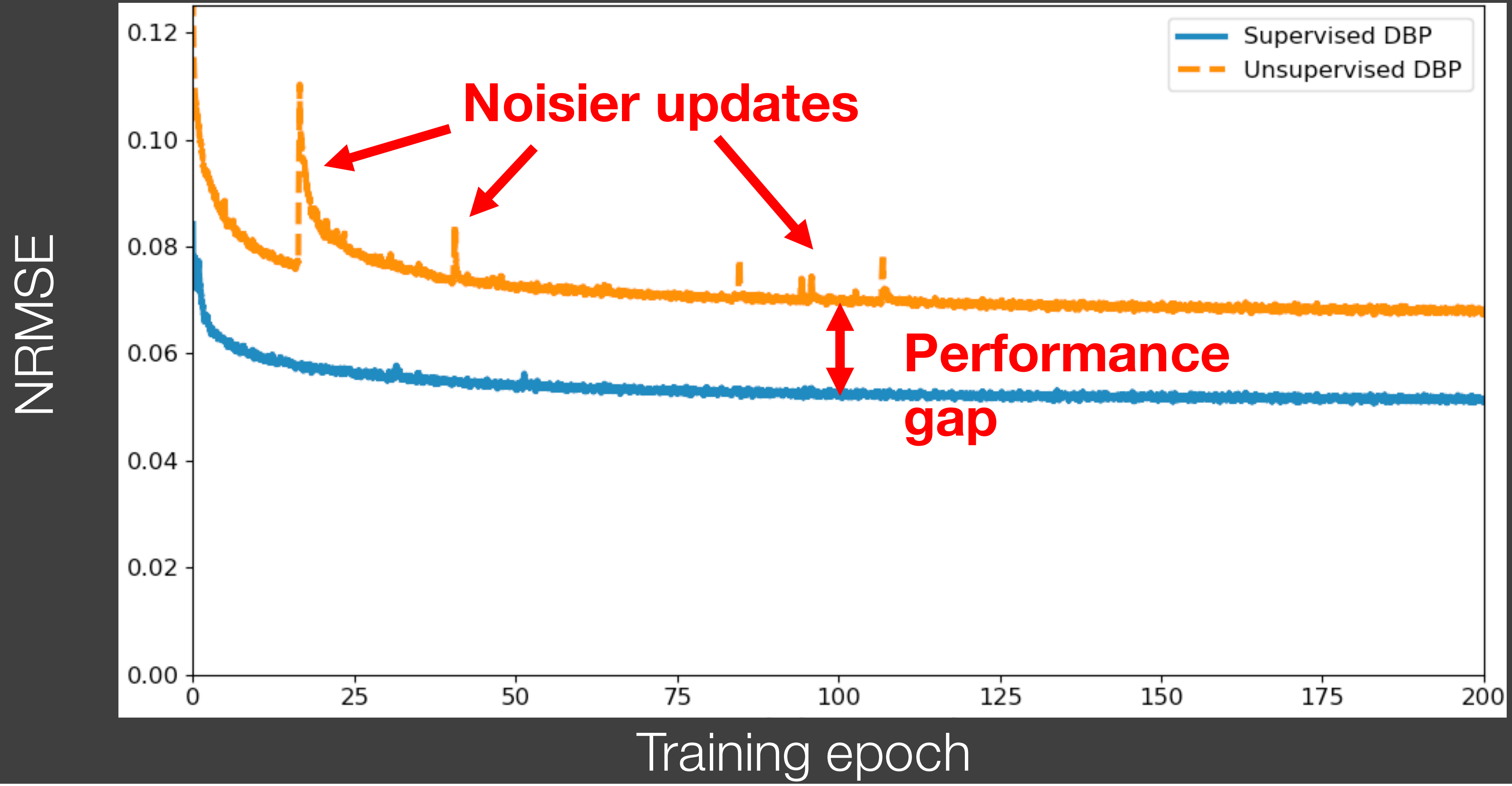
Data:
- Train on 4,384 2D slices from 16 3D FSE knee scans (mridata.org)
- Validation: 2 separate scans. Test: 2 separate scans (548 slices each)
- Average 7 slices to act as noise-free ground-truth
- Apply retrospective Poisson disc[1] sampling and add noise

Network:
- 5 unrolls, 2 ADMM iterations, 6 Conjugate Gradient iterations
- Denoising CNN with UNet[2] (2ch real/imaginary)
- Train with Adam optimizer

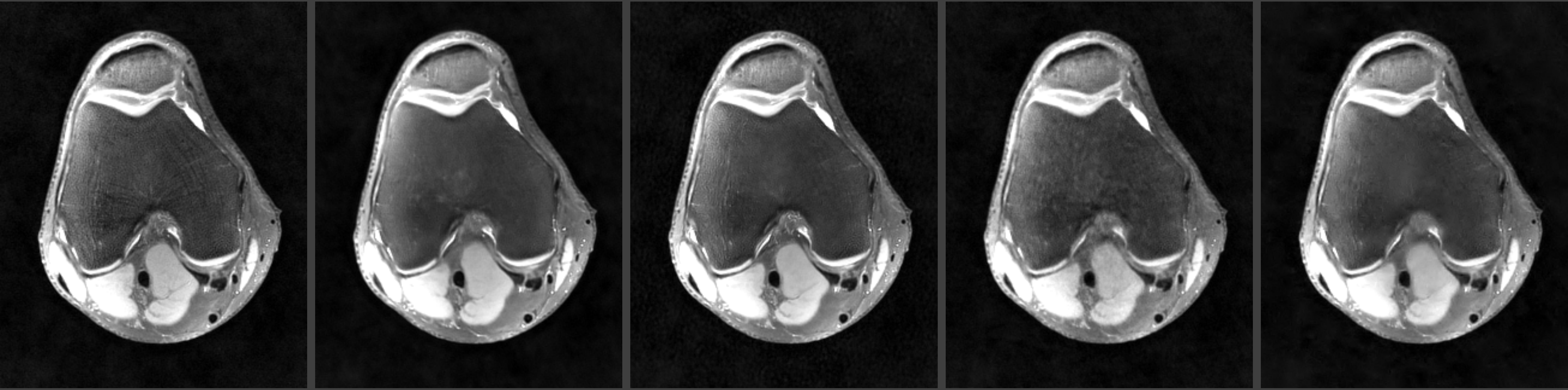
Evaluation:
- DBP with and without ground-truth, MoDL with ground-truth, PICS[3] (l1-wavelet)

[1] M Lustig, MRM 2007. [2] O Ronneberger, MICCAI 2015. [3] M Uecker, BART v0.4.04.

NRMSE vs. training epoch



Test slice



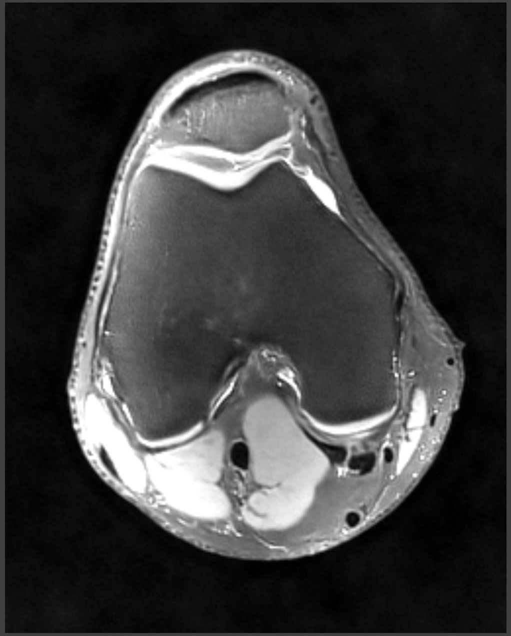
Ground truth	MoDL	Supervised DBP	Unsupervised DBP	PICS
NRMSE:	0.091369	0.067340	0.090781	0.076315

5 unrolls at training
5 unrolls at inference

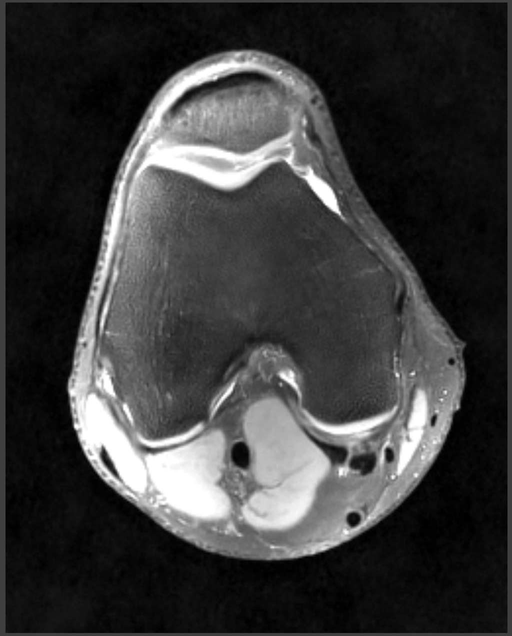
Test slice

Ground truth

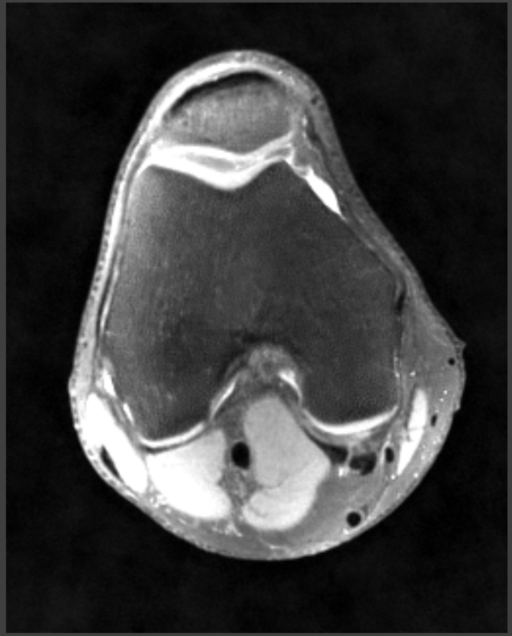
MoDL



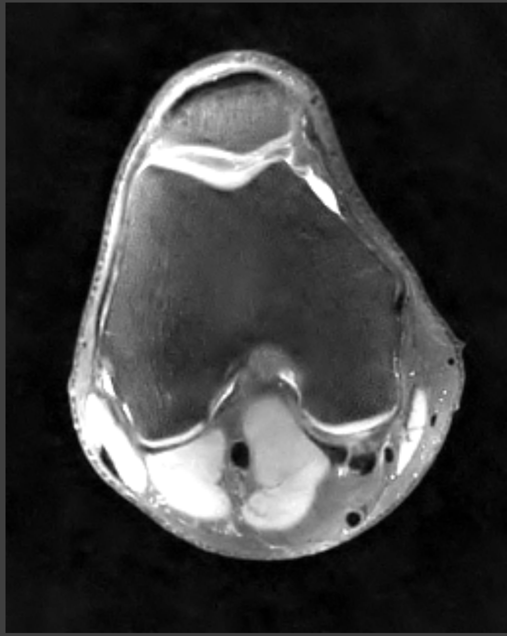
Supervised DBP



Unsupervised DBP



PICS

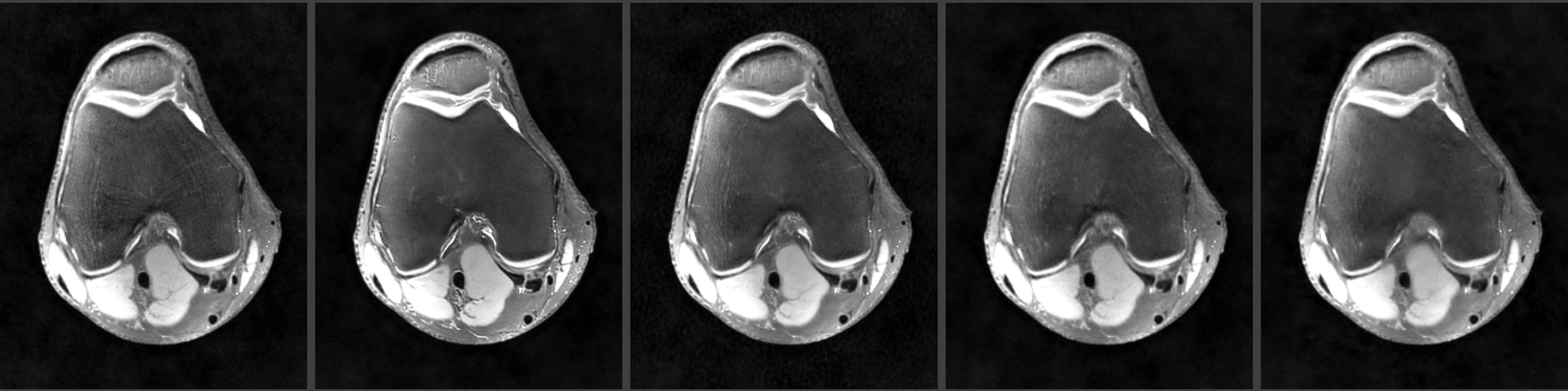


NRMSE:	MoDL	Supervised DBP	Unsupervised DBP	PICS
	0.095818 ↑	0.060486 ↓	0.072552 ↓	0.076315

5 unrolls at training

8 unrolls at inference

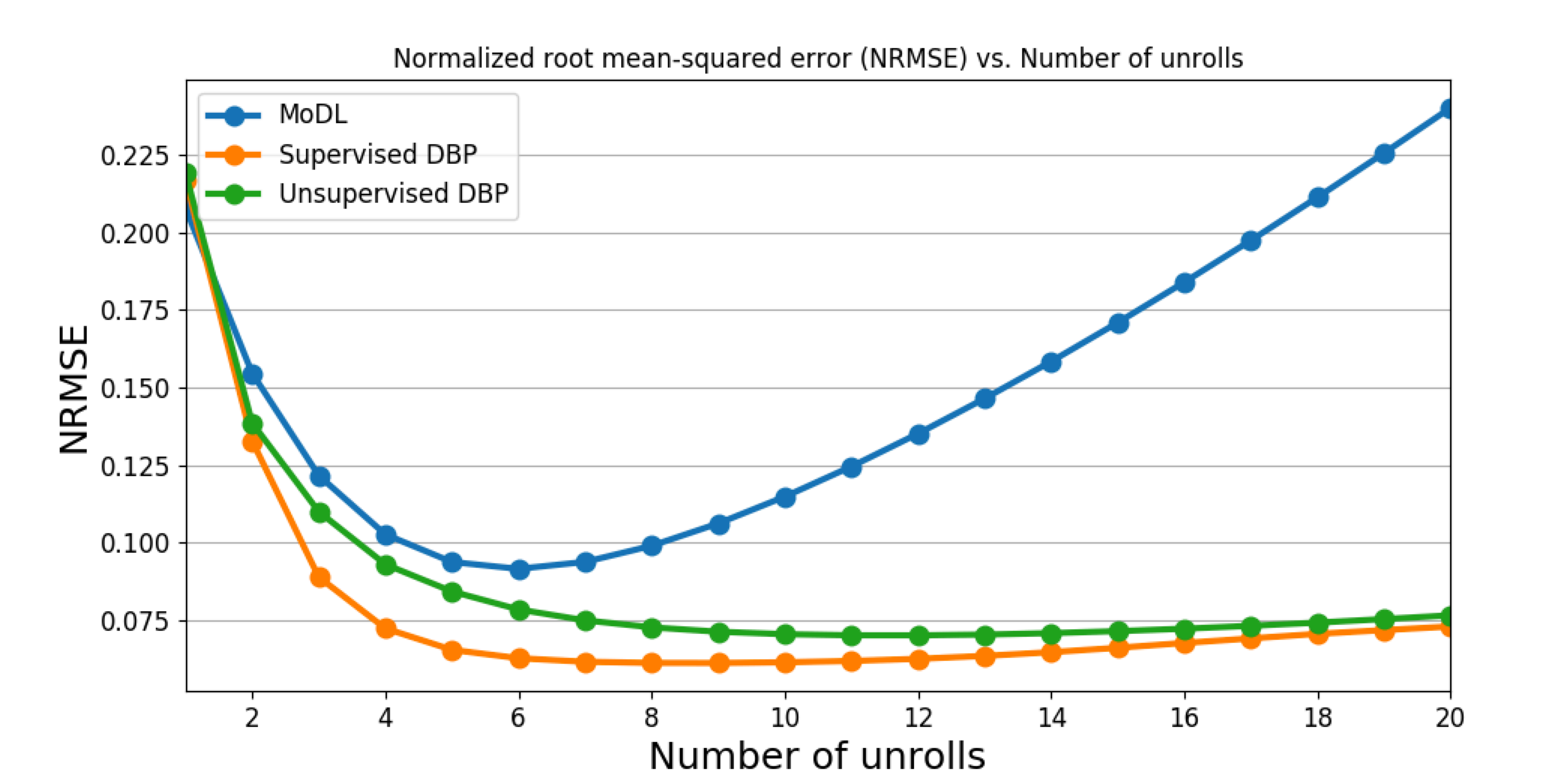
Test slice



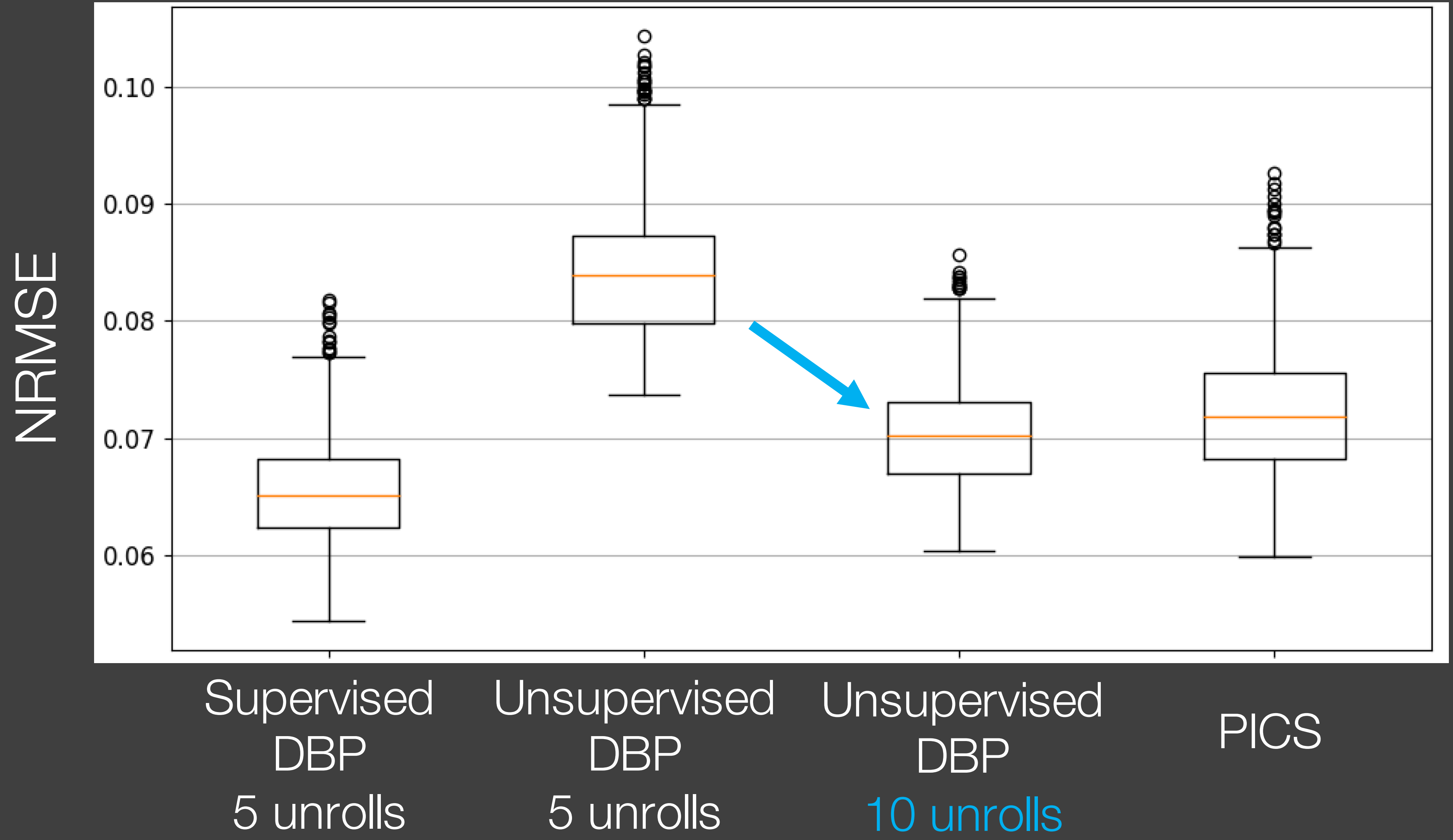
	Ground truth	MoDL	Supervised DBP	Unsupervised DBP	PICS
NRMSE:		0.113256	0.067340	0.068922	0.076315

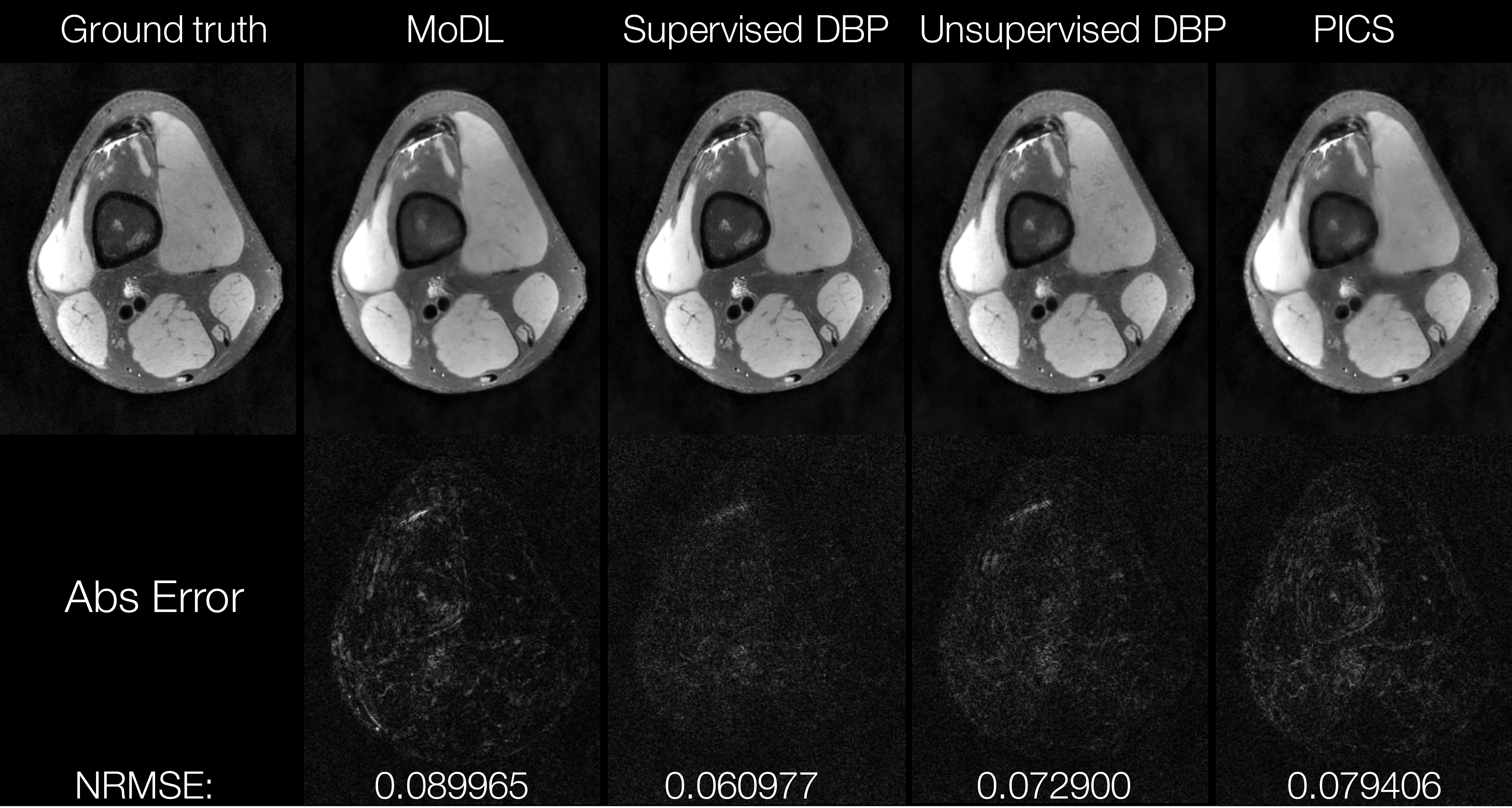
5 unrolls at training
10 unrolls at inference

Test set error



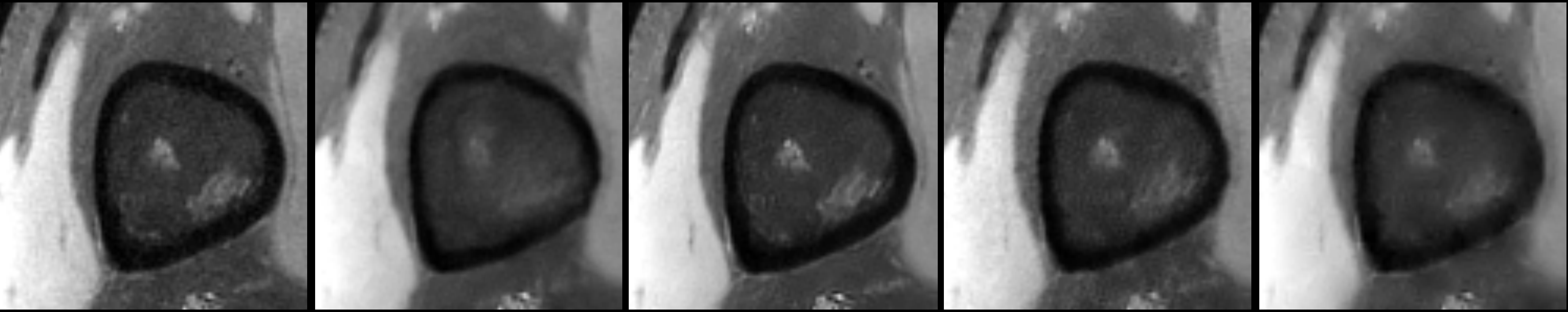
NRMSE on test set




Ground truth
MoDL
Supervised DBP
Unsupervised DBP
PICS
Abs Error
NRMSE:
0.089965
0.060977
0.072900
0.079406

Test slice

Ground truth	MoDL	Supervised DBP	Unsupervised DBP	PICS



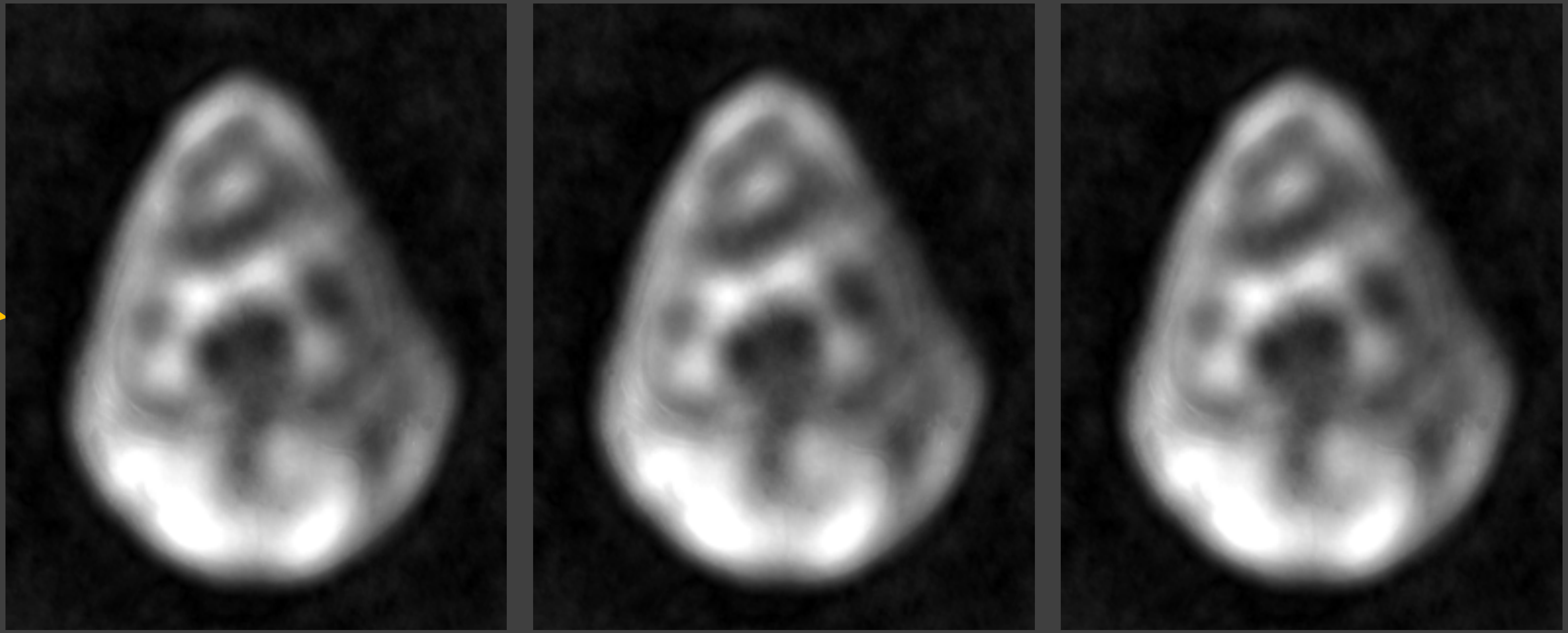
NRMSE:	0.089965	0.060977	0.072900	0.079406

Iteration progress

CNN → DC → CNN → DC → CNN → DC → CNN → DC $\cdots$

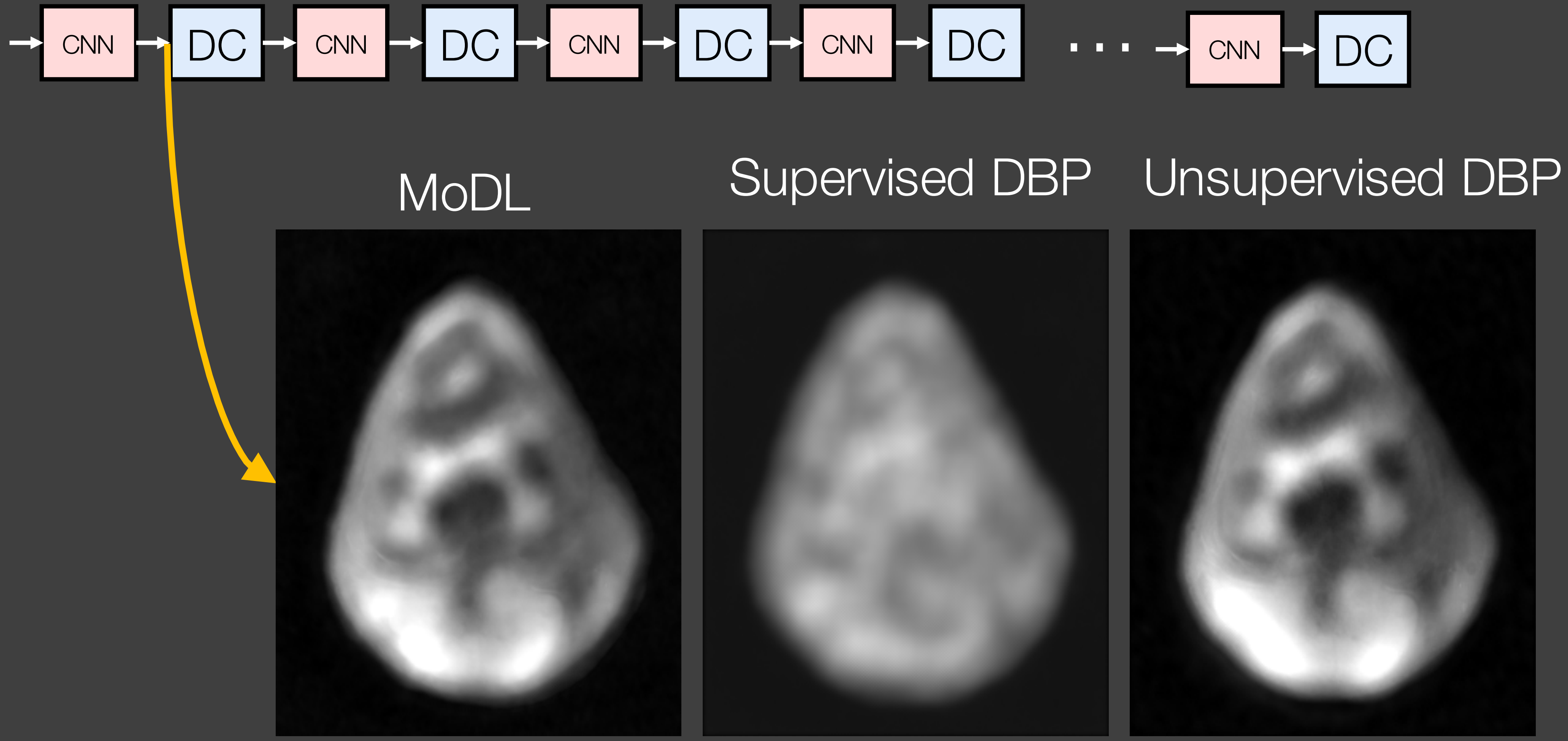
$$\mathbf{A}^*\mathbf{y}$$

MoDL | Supervised DBP | Unsupervised DBP



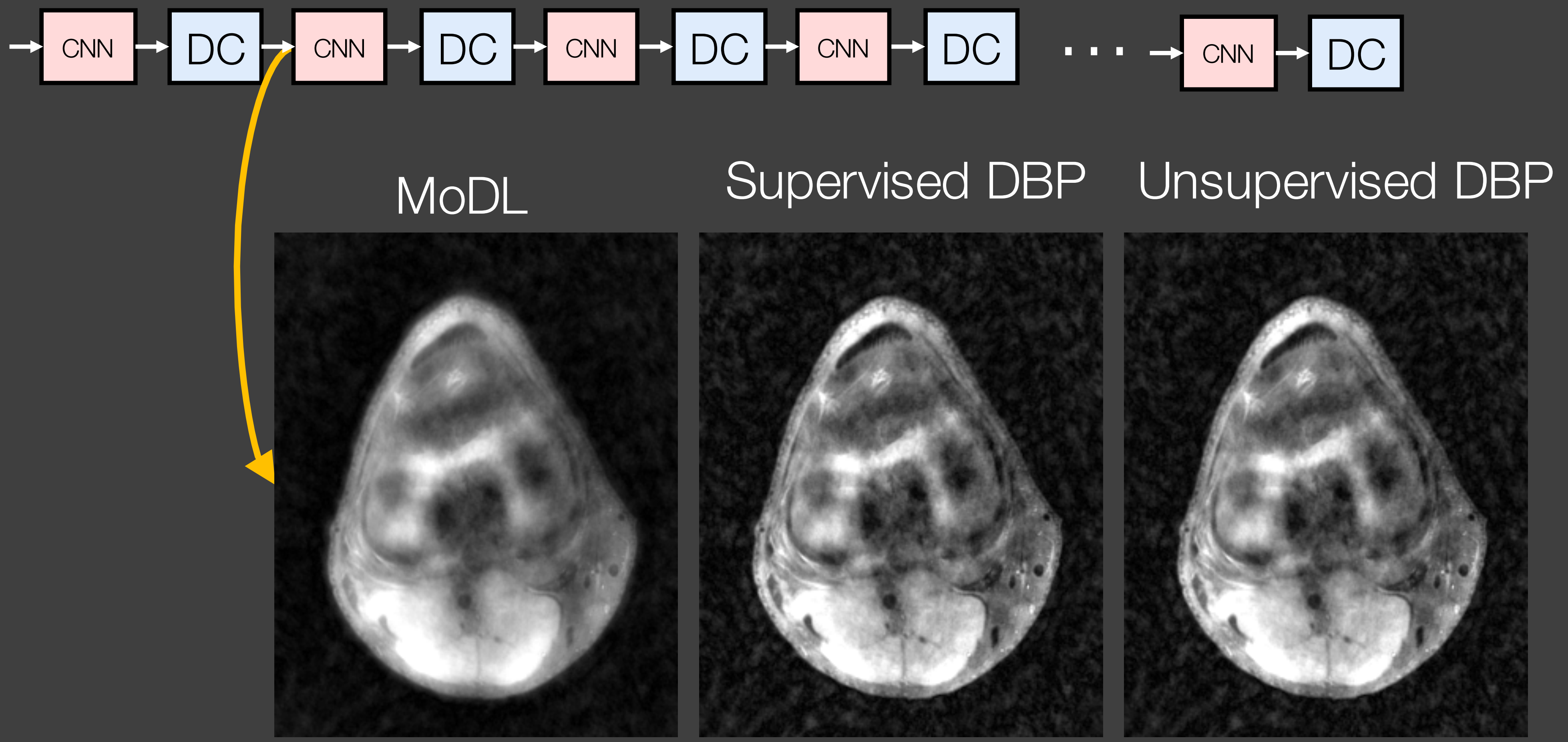
Iteration progress

CNN 1



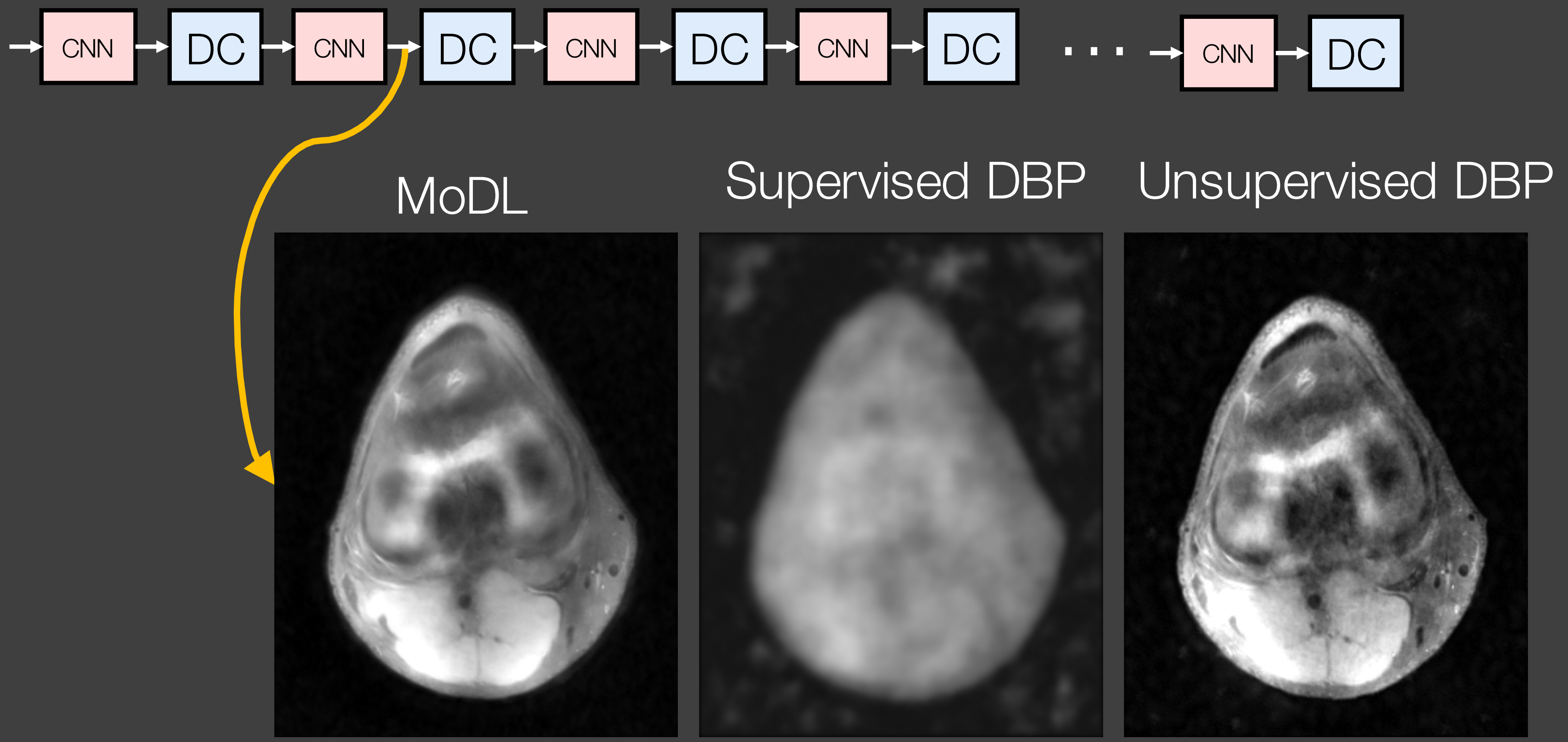
Iteration progress

DC 1



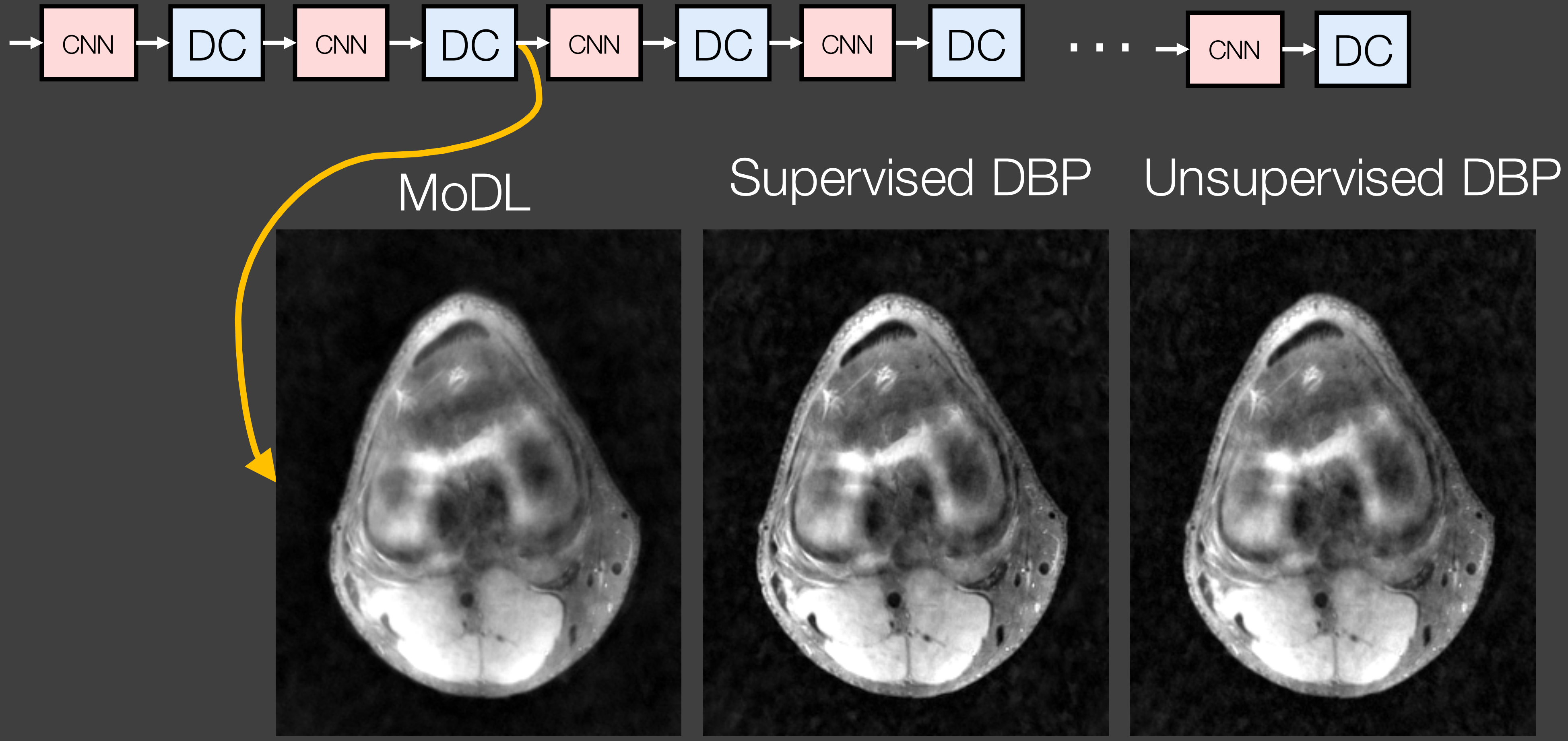
Iteration progress

CNN 2

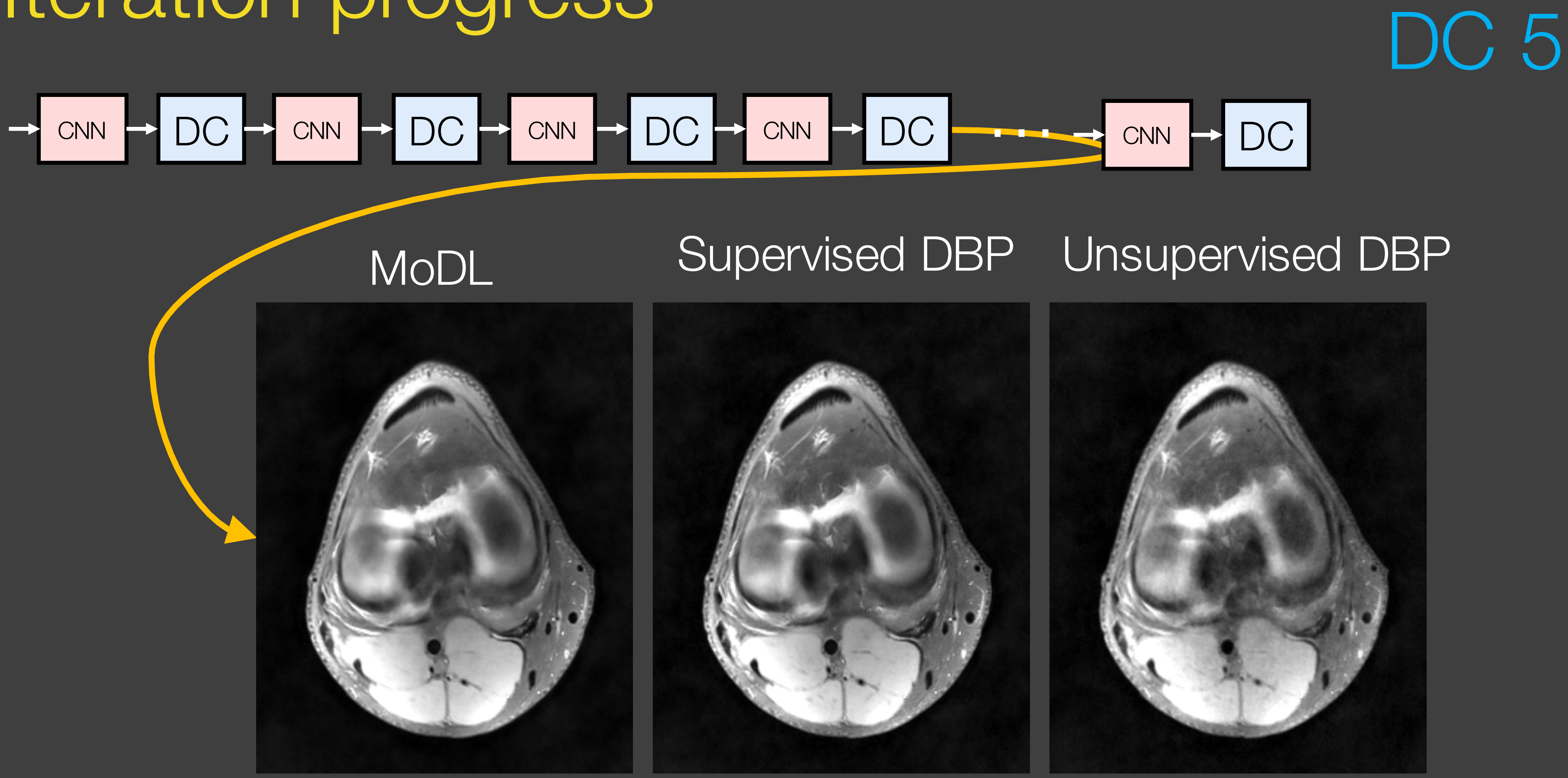


Iteration progress

DC 2

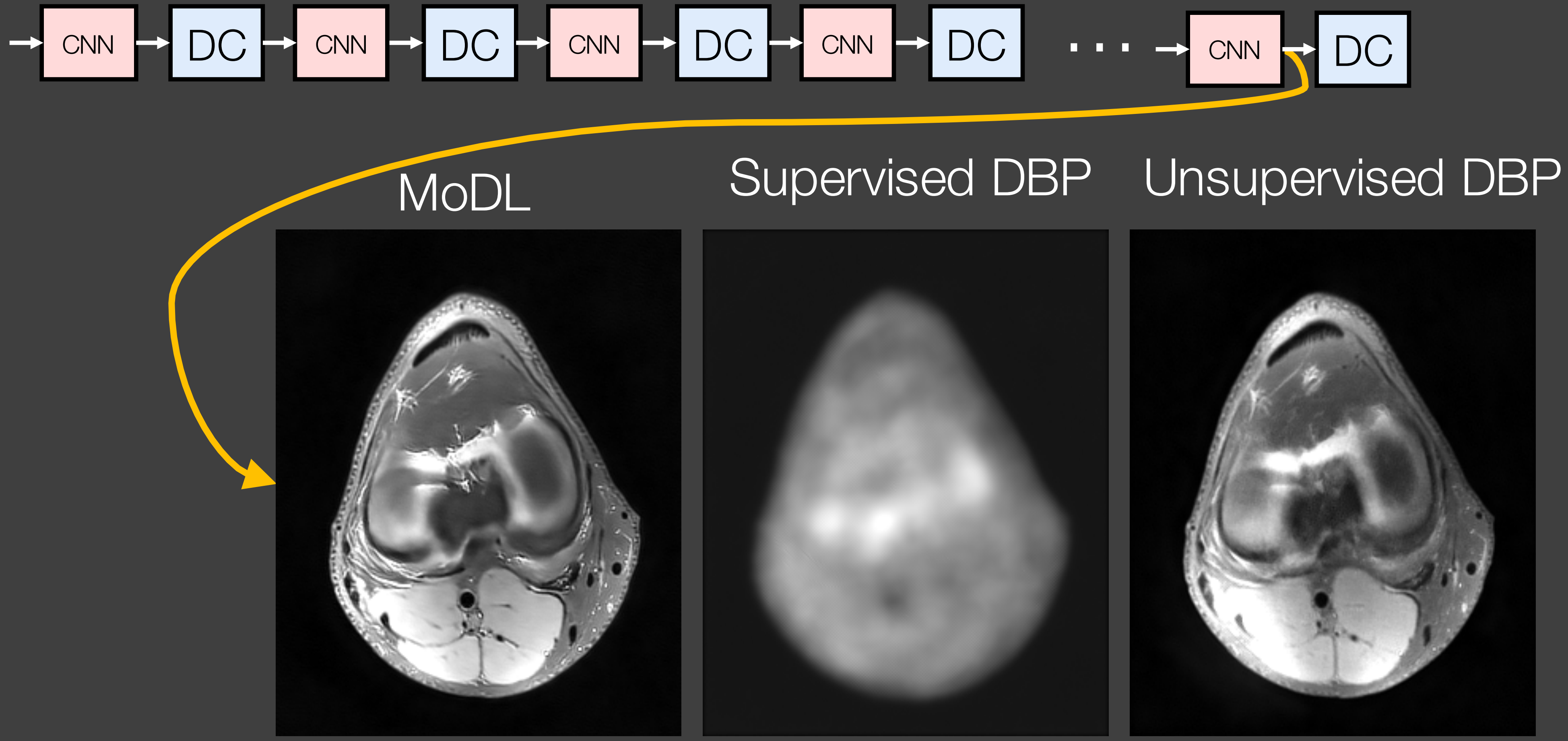


Iteration progress

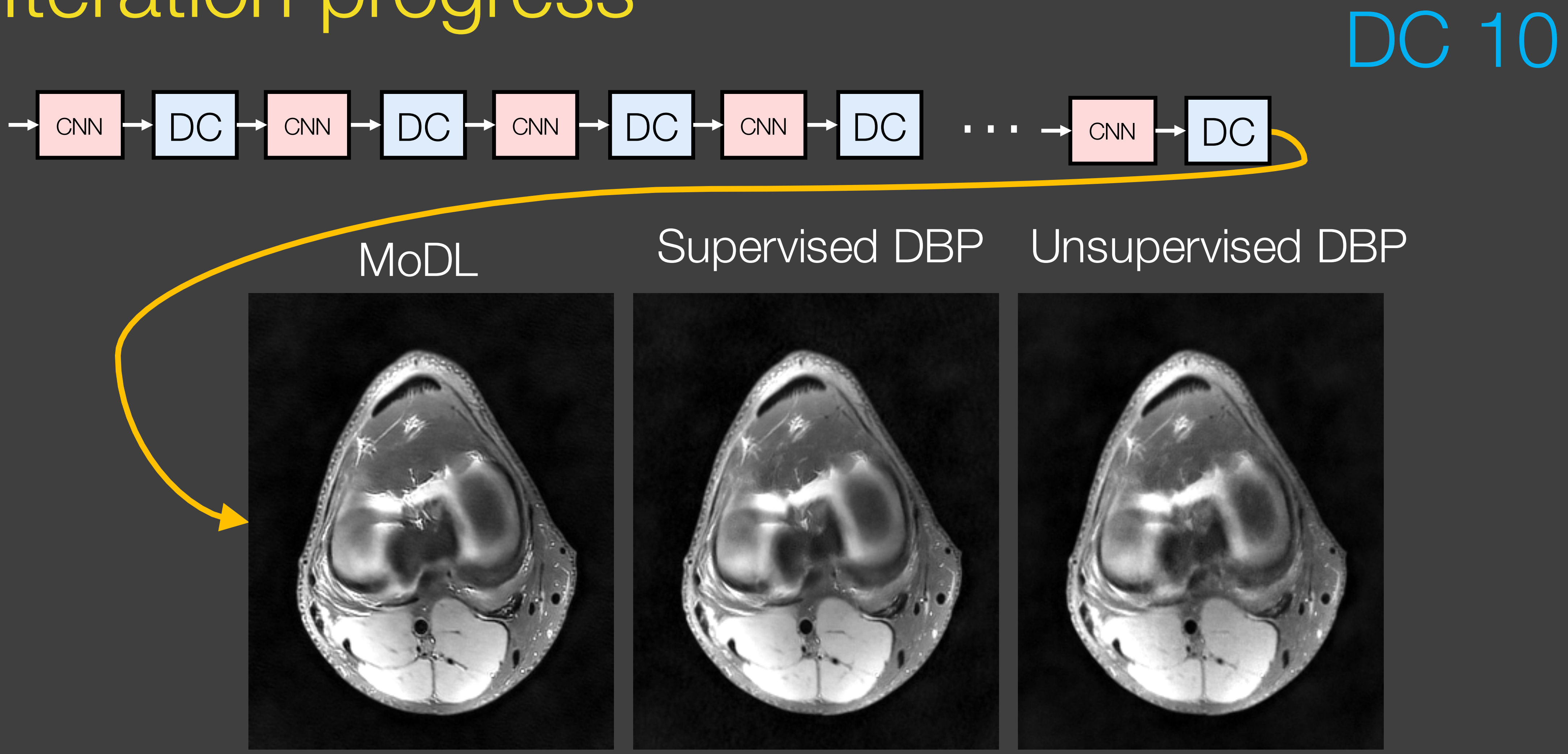


Iteration progress

CNN 9



Iteration progress



Conclusion

Unsupervised training across under-sampled data:

✓ Promising alternative when no ground-truth available

✕ Performance cost relative to supervised approach

Deep Basis Pursuit formulation:

- Takes advantage of known noise statistics
- Robust to number of unrolls

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