A New Scattering Formulation for Broad Leaves

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I. Introduction

Scattering from thin dielectric objects is a classic research topic in electromagnetics. A traditional approach is to model thin dielectrics with resistive sheets, and for several canonical geometries of resistive sheet bodies, exact solutions are found in [1]. However, for many important geometries such as disks, exact solutions are not known and only approximate solutions can be applicable to limited cases of interest. In this paper a new approximate solution is formulated based on a volumetric integral equation using Fourier transform, and it is shown that the solution is uniformly valid from low to high frequencies at all incidence angles including edge-on incidence. Validity of the solution is demonstrated through a series of comparisons with known exact solutions and a numerical solution, Method of Moment, for canonical objects such as an infinite dielectric half-plane, strip, and a circular disk for 2-D and 3-D dielectric scatterers, respectively.

II. Formulation

Figure 1 shows the problem geometry in which a very thin dielectric scatterer is located in x-y plane of a cartesian coordinate system. The scatterer is assumed homogeneous with a relative permittivity ε_r and arbitrary shape having a constant thickness $t \ll \lambda$. Using the concept of polarization current, the scatterer can be replaced with a volumetric current distribution of $\vec{J}(x,y,z) = ik_0Y_0(\varepsilon_r - 1)\vec{E}^t$ embedded in the host medium with permittivity ε_0 as seen in Fig. 1. Here $\vec{E}^t = \vec{E}^i + \vec{E}^s$ is the total electric field inside the scatterer. It can easily be shown that $\vec{J}$ must satisfy the following Fredholm integral equation.

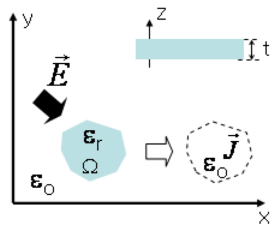

Fig. 1. Original and equivalent problems.

$$\vec{J}(\vec{r}) - k_0^2(\varepsilon_r - 1)\int_{v'} \bar{\bar{G}}(\vec{r},\vec{r}')\cdot\vec{J}(\vec{r}')dv' = -ik_0Y_0(\varepsilon_r - 1)\vec{E}^i(\vec{r}) \quad (1)$$

where $\vec{E}^i$ is the incident wave, and $\bar{\bar{G}}$ is the free-space dyadic Green's function [2]. By evaluating the delta function term in the dyadic Green's function analytically, the volumetric integral equation can be expressed in a more compact form given by

$$\vec{J} - \bar{\bar{A}}k_0^2(\varepsilon_r - 1)\int_{v'} \bar{\bar{G}}_r\cdot\vec{J}dv' = \vec{J}_{RG} \quad (2)$$

where $\vec{J}_{RG}$ is the well-known Rayleigh-Gans current. By invoking the thin dielectric approximation, the polarization current can be assumed a constant with respect to z-axis, i.e., $\vec{J}(x,y,z) \approx \vec{J}(x,y,0)$. Hence the integral in (2) with respect to z can be evaluated using the mid-point approximation and according to thin dielectric assumption, $\frac{\partial}{\partial z}J_x = \frac{\partial}{\partial z}J_y = 0$, we have

$$\vec{I} \approx \frac{t}{8\pi^2}\iint_{-\infty}^{\infty} d^2k \frac{\bar{\bar{A}}\cdot\bar{\bar{L}}}{k_z}\int_{s'} ds'\vec{J}(x',y')\cdot e^{i[k_x(x-x')+k_y(y-y')]}$$

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where $\bar{\bar{L}}$ is given by

$$\bar{\bar{L}} = \frac{1}{k_0^2}\begin{pmatrix} k_0^2-k_x^2 & -k_xk_y & 0 \\ -k_xk_y & k_0^2-k_y^2 & 0 \\ 0 & 0 & k_0^2-k_z^2 \end{pmatrix}$$

Now the original integral equation given by (2) can be written as

$$\vec{J} - k_0^2(\varepsilon_r-1)\bar{\bar{A}}\cdot\vec{I} = \vec{J}_{RG} \qquad (3)$$

Since $\vec{J}$ and $\vec{J}_{RG}$ are non-zero only inside the scatterer, the embedded integral in (3) can be interpreted as a convolution integral. Taking the Fourier transform of both sides and assuming (ε_r-1) is a constant function of position in the entire domain, (3) can be solved analytically as

$$\tilde{J} = \bar{\bar{B}}^{-1}\tilde{J}_{RG} \qquad (4)$$

where $\tilde{J}$ and $\tilde{J}_{RG}$ are the Fourier transform of $\vec{J}$ and $\vec{J}_{RG}$, respectively. Also $\bar{\bar{B}}^{-1} = \left[\bar{\bar{I}} - \alpha k_0^2\frac{A\cdot L}{k_z}\right]^{-1}$, $\bar{\bar{I}}$ is an unit dyadic, and $\alpha = \frac{i}{2}t(\varepsilon_r-1)$. Explicitly $\bar{\bar{B}}^{-1}$ can be evaluated as

$$\bar{\bar{B}}^{-1} = \begin{pmatrix} \frac{B_{11}}{D} & -\frac{\alpha}{D}k_xk_y & 0 \\ -\frac{\alpha}{D}k_xk_y & \frac{B_{22}}{D} & 0 \\ 0 & 0 & \frac{\varepsilon_r k_z}{\varepsilon_r k_z - \alpha k_\rho^2} \end{pmatrix}$$

where $B_{11} = k_z - \alpha(k_0^2-k_y^2)$, $B_{22} = k_z - \alpha(k_0^2-k_x^2)$, $D = k_z(1+\alpha^2k_0^2) - \alpha(k_0^2+k_z^2)$, and $k_\rho^2 = k_x^2+k_y^2$. By taking the inverse Fourier transform, a closed-form expression of $\vec{J}$ can be obtained and is given by

$$\vec{J}(x,y) \approx \frac{1}{4\pi^2}\iint_{-\infty}^{\infty} d^2k\bar{\bar{B}}^{-1}e^{i(k_xx+k_yy)}\cdot \int_{s'} ds'\vec{J}_{RG}e^{-i(k_xx'+k_yy')} \qquad (5)$$

The scattered field in the far-field region can also be obtained easily from (5). Considering a plane wave incidence, $\vec{E}^i = \vec{e}_i e^{i(k_x^ix+k_y^iy+k_z^iz)}$, the electric field in far-field region can be calculated as

$$\vec{E} \sim i(\varepsilon_r-1)\bar{\bar{A}}\cdot\vec{e}_i\frac{k_0^3Y_0}{16\pi^3}\frac{e^{k_0r}}{r}\hat{r}\times\hat{r}\times\iint_{\infty}^{\infty} d^2k\bar{\bar{B}}^{-1}\cdot \int_{s'} ds' e^{i[(k_x^i-k_x)x'+(k_y^i-k_y)y']}\cdot \int_{s''} ds'' e^{-i[(k_x^s-k_x)x''+(k_y^s-k_y)y'']}$$

Here $\hat{k}^s = \frac{1}{k_0}(k_x^s\hat{x}+k_y^s\hat{y}+k_z^s\hat{z})$ is a unit vector along the direction of observation point.

III. Dielectric Strip and Half-plane

An example considered here is the problem of scattering from a thin dielectric strip whose geometry is shown in Fig. 2. For a plane wave incidence, the polarization current (5) can be reduced to a single integral given by

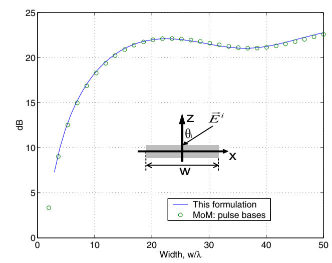

Fig. 2. Forward scattering from a thin dielectric strip.

$$\vec{J} \approx -ik_0Y_0(\varepsilon_r-1)\frac{w}{2\pi}\int_{-\infty}^{\infty} dk_x\bar{\bar{B}}_1^{-1}\bar{\bar{A}}\cdot\vec{e}_i\cdot \operatorname{sinc}\left[\frac{(k_x^i-k_x)}{2}w\right]e^{i(k_xx+k_y^iy)} \qquad (6)$$

where $\bar{\bar{B}}_1^{-1}$ is the same as $\bar{\bar{B}}^{-1}$ with $k_y = k_y^i$. Unfortunately the integral can't be evaluated analytically and must be calculated numerically. To examine the validity of the obtained formulation, examples are chosen from [3]. Figure 2 shows a comparison of normalized radar echo width of a strip with thickness $t = 0.025\lambda_0$ and $\varepsilon_r = 4+i0.4$ as a function of w/λ, which are calculated in forward direction for an edge-on TM polarized incident wave.

Next, backscattering from a thin dielectric half-plane along the negative x-axis is considered. For this structure, exact and approximate solutions are known [1]. For this geometry, (5) is reduced to

$$\vec{J} = \frac{1}{2}\vec{J}_{inf} - ik_0Y_0(\varepsilon_r-1)\frac{i}{2\pi}\int_{-\infty}^{\infty} dk_x\bar{\bar{B}}_1^{-1}\bar{\bar{A}}\cdot\vec{e}_i\frac{e^{i(k_xx+k_y^iy)}}{k_x-k_x^i} \qquad (7)$$

where $\vec{J}_{inf}$ is the current inside an infinite slab (known as physical optics (PO) current). If we focus on just edge-on incidence case, this PO current always becomes zero. Considering a TM polarized wave having an electric field along $\hat{y}$ and $k_y^i = 0$, in the far-field region, the electric field can be written as $\vec{E} \sim \hat{y}\sqrt{\frac{2}{\pi k_0\rho}}P_e(\theta_i,\theta_s)$, where $P_e(\theta_i, \theta_s)$ is known as the far-field amplitude. $P_e(\theta_i, \theta_s)$ can be evaluated analytically for the backscattering direction as

$$P_e(\frac{\pi}{2}, \frac{\pi}{2}) = -\frac{i}{\pi\sqrt{1-\eta^2}} \tanh^{-1}\sqrt{\frac{1-\eta}{1+\eta}} \quad (8)$$

where $\alpha k_0 = -\frac{1}{\eta}$, and $\eta = \frac{2R}{Z_0}$ is the normalized surface impedance, and $R = \frac{iZ_0}{k_0 t(\varepsilon_r - 1)}$. Figure 3 shows a comparison of exact result and result calculated by (8) as a function of real η. Therefore (8) can provide a very good approximation for the Maliuzhinets half-plane function for moderate to large value of $|\eta|$.

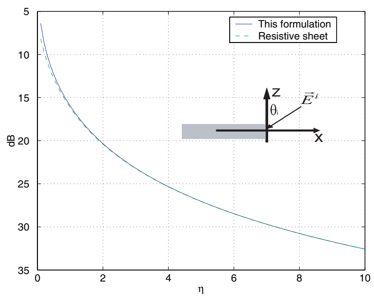

Fig. 3. Backscattering from a dielectric half-plane.

IV. Dielectric Disks

In this section scattering by a very thin dielectric disk is examined. Scattering from an elliptical disk with major and minor axes of a and b is investigated. Choosing a coordinate system with x along the major axis and y along the minor axis of the ellipse, the surface integral of (5) can be evaluated in a closed-form and is given by

$$\iint_{s'} e^{i[(k_x-k_x^i)x'+(k_y-k_y^i)y']}ds' = \frac{2AJ_1\left(\sqrt{a^2(k_x-k_x^i)^2+b^2(k_y-k_y^i)^2}\right)}{\sqrt{a^2(k_x-k_x^i)^2+b^2(k_y-k_y^i)^2}}$$

Forward scattering from a circular disk with a diameter of $3cm$, and a thickness of $0.2mm$, and $\varepsilon_r = 26.6 + i11.56$ is considered. Figures 4 shows a plot of the scattering of the dielectric disk in forward direction as a function of incidence angles. In this figure two approximate formulations such as VIPO (high frequencies) and Rayleigh-Gans (low frequencies) as well as MoM results are shown. It is shown that the new formulation can reproduce the MoM results very accurately over the entire comparison region where low and high frequency results are only valid at high and low incidence angles. Next examination is on the bistatic scattering from a square disk. For this shape,

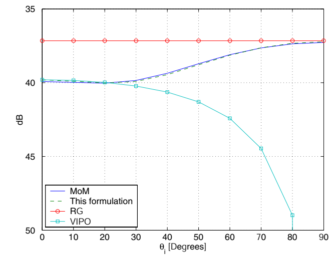

Fig. 4. Forward scattering from a thin circular disk.

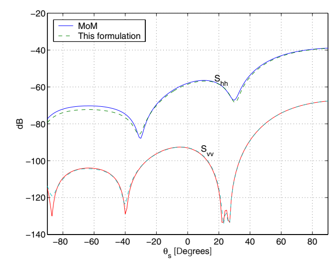

Fig. 5. Bistatic scattering from a thin square disk.

$$\iint_{s'} e^{i[(k_x-k_x^i)x'+(k_y-k_y^i)y']}ds' = w_x w_y \text{sinc}\left[\frac{k_x^i - k_x}{2}w_x\right] \text{sinc}\left[\frac{k_y^i - k_y}{2}w_y\right]$$

where w_x and w_y are the width and length of the rectangular disk, respectively. Figure 5 shows the bistatic scattering radar cross section from the square disk which is calculated for an incident wave propagating along the negative x-axis (edge-on incidence). The observation point (θ_s) is varied from $-90°$ to $90°$ along the diagonal direction of the disk ($\phi_s = 45°$). Some small discrepancy is observed in the backward direction.

References

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[3] J. H. Richmond, "Scattering by Thin Dielectric Strips," *IEEE Trans. Antennas Propagat.*, vol. AP-33, no. 1, pp. 64–68, January 1985.