## A Compact Ultra-Wideband Self-Complementary Antenna with Optimal Topology and Substrate

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- 1. Key Contributions: The focus in this paper is to design a compact ultra-wideband Self-Complementary Antenna (SCA) optimized for bandwidth constrained by size. The concept of frequency independent self-complementary geometries is applied to achieve the highest bandwidth possible, and a Genetic Algorithm (GA) optimization method is used to provide the optimal planar antenna topology and substrate design in an available finite space. A Finite Difference Time Domain (FDTD) technique is used to fully analyze the complex antenna structures. It is demonstrated that a GA-optimized SCA can provide a wide bandwidth ratio of about 1:5 which occupies an area as small as  $\lambda/4\times\lambda/4$  at the lowest frequency. An equally spaced alternate-leaves SCA on an optimized meta-substrate is also proposed that provides ultra-wideband bandwidth characteristic (bandwidth ratio 1:8) with as small dimension as  $\lambda/3\times\lambda/3$ .
- 2. Review of Self-Complementary Antennas: An antenna with an ideal self-complementary structure has a constant input impedance, independent of the source frequency and the antenna topology (geometry) [1], [2]. To briefly demonstrate the principal of SCA, let's consider the dual structures shown in Fig. 1. Maxwell's equations for EM fields in (a) with electric current source J=N<sub>+</sub> (in the right hand side of S) and J=N<sub>-</sub> (in the left hand side of S) and in (b) with magnetic current source M=N<sub>+</sub> (in the right hand side of S) and M=-N<sub>-</sub> (in the left hand side of S) are given as follow:

$$\nabla \times \mathbf{E}_{1} = -j\omega\mu \,\mathbf{H}_{1} , \nabla \times \mathbf{H}_{1} = j\omega\varepsilon \,\mathbf{E}_{1} + \mathbf{J} \qquad (1a)$$

 $\nabla \times \mathbf{E}_1 = -j\omega\mu \,\mathbf{H}_2 - \mathbf{M}$ ,  $\nabla \times \mathbf{H}_2 = j\omega\varepsilon \,\mathbf{E}_2$  (1b) where the boundary conditions are

$$\begin{cases} \hat{\mathbf{n}} \times \mathbf{E}_1 = 0 \text{ on } S_1 \\ \hat{\mathbf{n}} \times \mathbf{H}_1 = 0 \text{ on } S_2 \end{cases}, \begin{cases} \hat{\mathbf{n}} \times \mathbf{E}_2 = 0 \text{ on } S_2 \\ \hat{\mathbf{n}} \times \mathbf{H}_2 = 0 \text{ on } S_1 \end{cases}$$
(2)

It can be readily obtained that the structures in Figs. 1(a) and 1(b) are dual of each other with transformations  $-V_0E_2=\pm H_1$  and  $H_2=\pm Y_0E_1$ , where  $Y_0=\sqrt{\epsilon/\mu}$ . Here + and – signs refer, respectively, to the right- and left-hand-side of S. Moreover, for the symmetrical electric and anti-symmetrical magnetic

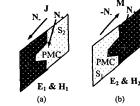


Fig. 1. Infinite planar dual sheets, (a) electric source, (b) magnetic source.

sources, as used in Fig.1, one can determine that the tangential component of the magnetic field is equal to zero over the entire domain  $S=S_1+S_2$ . This means that the PMC sheets on  $S_2$  in Fig. 1(a) and on  $S_1$  in Fig. 1(b) can be removed without any change in the original electromagnetic problem. Therefore, for a pair of arbitrary-shaped mutually-complementary PEC depicted in Fig. 2(a) with input impedances:

$$Z_{1} = \frac{\int_{b}^{d} \mathbf{E}_{1} \cdot d\mathbf{l}}{2 \int_{c}^{d} \mathbf{H}_{1} \cdot d\mathbf{l}}, Z_{2} = \frac{\int_{c}^{d} \mathbf{E}_{2} \cdot d\mathbf{l}}{2 \int_{c}^{d} \mathbf{H}_{2} \cdot d\mathbf{l}}, (3)$$

one can use the above duality transformations and obtain  $Z_1Z_2=(Z_0/2)^2$ . If the structure is itself self-complementary (Fig. 2(b)) then we have  $Z_1=Z_2$  and the antenna provides constant input impedance  $Z=60\pi$  independent of frequency.

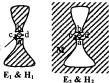




Fig. 2. (a) Mutually and (b) Self complementary structures.

- 3. Analytical Approach: To characterize the performance of SCA with complex geometries comprehensively, the finite difference time domain technique with Perfectly Matched Layered (PML) walls is used [3]. The Prony extrapolation scheme is also applied to expedite the computational time. For the purpose of obtaining the optimal design, the genetic algorithm optimization method is integrated with the FDTD. The GA is a very robust and effective method for searching for the global optima in a multidimensional, multi-modal, and highly nonlinear functional domain. To avoid the poor convergence performance of conventional GA the Micro Genetic Algorithm (MGA) is used [4]. It has a smaller population size and uses the population restart strategy to avoid the local optima. It also uses elitism to ensure the migration of best individual form one generation to the next.
- 4. Monopole-Slot SCA: Fig. 3 shows the geometry of a finite size monopole-slot self-complementary antenna [1]. The antenna is printed on a low dielectric material with  $\varepsilon_r$ =1.20 in order not to perturb the self-complementary condition. Since the structure is truncated and it has finite-size geometry, instead of observing a constant input impedance, a wideband performance is expected. The FDTD is applied and the input resistance and reactance of antenna are obtained and shown in Fig. 4. The monopole-slot SCA provides a wide bandwidth ratio of about 1:3. The size of antenna at the lowest frequency (1 GHz) is 0.36 $\lambda$ . The antenna radiation patterns in xz and yz planes at frequencies 1 and 2 GHz are plotted in Fig. 5. Antenna has a large cross polarization level, especially in higher frequency range, due to its asymmetry.

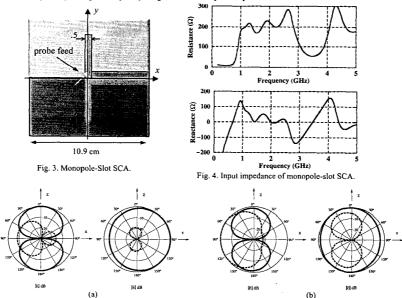


Fig. 5. Radiation patterns of monopole-slot SCA at (a) 1 GHz, and (b) 2 GHz (— co-pol, .... x-pol).

5. Alternate-Leaves SCA: It was demonstrated in the previous section that due to the truncation of self-complementary structure, the impedance bandwidth of finite-size SCA is not the same as one predicted theoretically (constant impedance  $Z=60\pi$ ). However, one can reduce the effect of truncation by a proper topology of self-complementary geometry. To this end, Mushiake proposed an equally spaced alternate leaves SCA (see Fig. 6) in 1977, and measured its

performance [1]. He experimentally demonstrated the frequency independent property of this antenna at frequencies higher than fL at which the longest leaf of the antenna is a quarter wavelength. Here, we apply the FDTD to present a more detailed/analytical study of this antenna. The input impedance is determined in Fig. 7 and the very wide bandwidth ratio 1:4 is illustrated. The antenna radiation patterns at frequencies 1, 2, and 3 GHz are plotted in Fig. 8. It is observed that the antenna has almost the same radiation patterns over its wide band impedance characteristics. Note that in this case the cross polarization level is much smaller than the one obtained for monopole-slot antenna. The maximum dimension of antenna is about  $0.37\lambda$  at  $f_{\rm L}$ .

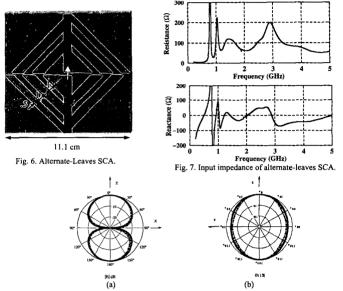
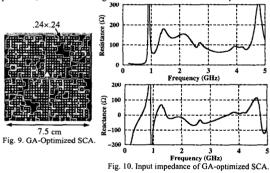


Fig. 8. Radiation patterns of alternate-leaves SCA in (a) xz and (b) yz planes ( - 1, .... 2, and - · · 3 (GHz))

Compact GA-Optimized SCA: Our goal is to maximize the bandwidth of an SCA for given antenna dimension. To accomplish this, the MGA is integrated with FDTD to obtain the optimal

antenna geometry. In other words, the MGA investigates the best complementary topology such that the adverse effect of truncation on the antenna current distribution is minimized. GA-optimized structure is shown in Fig. 9. The input impedance of antenna is presented in Fig. 10. It is demonstrated that the antenna occupies an area as small as  $\lambda/4 \times \lambda/4$  while it provides



very wide bandwidth ratio 1:5. The radiation patterns at 1, 2, and 3 GHz are obtained in Fig. 11. Note that for the higher frequency range, above 3 GHz, the antenna cross polarization is increased.

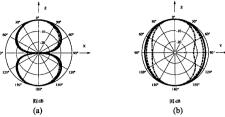


Fig. 11. Radiation patterns of GA-optimized SCA in (a) xz and (b) yz planes ( — 1, .... 2, and — · · 3 (GHz))

7. SCA on Meta-Substrate: In this section we investigate the performance of equally spaced alternate leaves SCA (shown in Fig. 6) printed on a novel GA-optimized metasubstrate. The MGA uses two materials to construct an inhomogeneous substrate. One is dielectric material with  $\varepsilon_r$ =2.2 and the second one magnetic material with  $\mu_r$ =5.0. Fig. 12 shows the optimal substrate pattern. The alternate leaves antenna printed on the designed meta-substrate provides the much higher bandwidth ratio of about 1:8 as illustrated in Fig. 13. The radiation patterns at 1, 2, 3, and 5 GHz are determined in Fig. 14. The maximum dimension of structure is around  $\lambda/3$ . It should be mentioned that the magnetic materials in GHz range are unachievable. However, the frequency can be scaled to UHF-VHF band for designing extremely wideband compact antennas.



Fig. 12. Engineered meta-substrate.

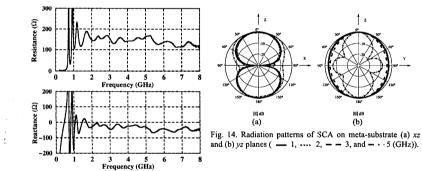


Fig. 13. Input Impedance of alternate leaves SCA printed on meta-substrate.

## References

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