A New Uniform Solution for Scattering by Thin Dielectric Strips: TM Wave Incidence

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I. Introduction

Scattering from thin dielectric sheets is encountered in certain practical situations. To simplify the problem a thin dielectric structure is usually modeled with a resistive sheet. Among the canonical problems scattering by a resistive strip has been intensively studied by a number of researchers. Many approximate solutions have been proposed for the problem, which all are based on the known exact solution for a resistive half plane. Uniform solutions valid at the transition regions between the shadow and reflection boundaries for the half-plane is needed to obtain higher order multiple interactions between two edges of a finite strip. One such method is the Extended Spectral Ray Method (ESRM) [1], which can be applied to a general multiple scattering problems with some analytical complexity. However as explained, all existing methods use the formulation for the half-plane, and hence the resulting solutions contain a transcendental function known as Maliuzhinets function.

Recently an approximate solution for a thin dielectric object with any size and shape was proposed [2]. The solution is represented in terms of a spectral integral whose integrand contains only elementary functions. Based on this formal solution, a uniform solution for bistatic scattering by a thin dielectric strip is formulated for a TM wave incidence. Through comparisons of results calculated by the uniform solution and a numerical method such as a method of moments (MoM), the new formulation is verified for several cases. One advantage of the new formulation is that it is expressed in terms of elementary functions, and thus it is much easer to understand scattering behavior.

II. Formulation

When a TM wave is incident on a thin dielectric strip as seen in Fig. 1, the scattered field in the far-field region is represented as $\vec{E}^s \sim \hat{y}\sqrt{\frac{2}{\pi k_0 \rho}} e^{i(k_0\rho - \pi/4)} P_e(\theta_i, \theta_s)$. Here $P_e(\cdot) = \frac{1}{\pi\eta} I$ is known as the far-field amplitude and an approximate solution for I is given by [2]

$$\begin{aligned} I &= \int_{-\infty}^{\infty} dk_x \frac{k_z}{k_z - \alpha k_0} \frac{\sin\left(\frac{k_x^i - k_x}{2} k_0 w\right) \sin\left(\frac{k_x^s - k_x}{2} k_0 w\right)}{(k_x^i - k_x)(k_x^s - k_x)} \\ &= -\frac{I_1}{2}\cos\left(\frac{k_x^i - k_x^s}{2} k_0 w\right) + \frac{I_2}{4} e^{-ik_0(k_x^i - k_x^s)w/2} + \frac{I_3}{4} e^{ik_0(k_x^i - k_x^s)w/2} \end{aligned} \quad (1)$$

where $k_z = \sqrt{1 - k_x^2}$, and $\alpha = \frac{i}{2} t(\varepsilon_r - 1)$. I_1, I_2, and I_3 are defined as

$$\begin{aligned} I_1 &= \int_{-\infty}^{\infty} dk_x \frac{k_z}{k_z - \alpha k_0} \frac{1}{(k_x^i - k_x)(k_x - k_x^s)}, \\ I_2 \text{ or } I_3 &= \int_{-\infty}^{\infty} dk_x \frac{k_z}{k_z - \alpha k_0} \frac{e^{\pm i k_0 k_x w}}{(k_x^i - k_x)(k_x - k_x^s)} \end{aligned}$$

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Here "+" is chosen for I_2, and "-" for I_3. Simply it can be shown that $I_2 = I_3$. After lengthy algebraic manipulation, I_1 can be evaluated analytically and given by

$$I_1 = -\frac{4i\eta}{\sqrt{1-\eta^2}} \tanh^{-1} \sqrt{\frac{1-\eta}{1+\eta}} + f_1 \tag{2}$$

f_1 is given by

$$f_1 = -\eta^2 \frac{f_2(k_x^i) - f_2(k_x^s)}{k_x^s - k_x^i}, \quad \text{where} \quad f_2(k_x^i) = \frac{g_1 k_x^i {k_z^i}^2 + \frac{2ik_z^i}{\eta} \sin^{-1} k_x^i}{1 - \eta^2 {k_z^i}^2}$$

where $g_1 = \frac{2\eta}{\sqrt{\eta^2-1}} \log \frac{1+\eta+\sqrt{\eta^2-1}}{1+\eta-\sqrt{\eta^2-1}} - 2i \tan^{-1} \sqrt{\frac{\eta^2-1}{i\eta+1}}$, $\alpha k_0 = -\frac{1}{\eta}$, and ${k_z^i}^2 = k_0^2 - {k_x^i}^2$.

However, I_2 cannot be carried out analytically, and so an asymptotic technique such as steepest descent method (SDM) can be used to obtain an analytical formulation. Since the integrand in I_2 has four poles at $k_x = k_x^i$, k_x^s, and $\pm\sqrt{1-\alpha}k_0$, first, the integrand can be transformed into a more conventional form applying the SDM approximation:

$$I_2 = \frac{\eta}{k_0} \int_{-\infty}^{\infty} dk_x \left[\frac{1}{k_z} - \frac{1}{k_z - \alpha k_0} \right] e^{ik_0 k_x w} + \frac{k_0}{k_x^s - k_x^i} \int_{-\infty}^{\infty} dk_x \left[\frac{1}{k_z} - \frac{1}{k_z - \alpha k_0} \right] \left[\frac{{k_z^i}^2}{k_x - k_x^i} - \frac{{k_z^s}^2}{k_x - k_x^s} \right] e^{ik_0 k_x w}$$

Since these integrands have poles, the pole contribution should be carefully taken account into to obtain uniform solution. Following the standard SDM procedure [3], a uniform solution is obtained and given by

$$I_2 \sim \eta \left[I_4 + \frac{e^{ik_0 w}}{k_x^i - k_x^s} \left\{ f_3(k_x^i) - f_3(k_x^s) \right\} \right] \tag{3}$$

I_4 is given by

$$I_4 = \pi H_0^{(1)}(k_0 w) + e^{ik_0 w} \left[T_0 \sqrt{\frac{\pi}{k_0 w}} + i\pi b w(\sqrt{k_0 w} s_1) \right]$$

where $T_0 = b/s_1$, and $s_1 = \sqrt{i(1 - \sqrt{1 - 1/\eta^2})}$, $b = \frac{1}{\eta^2 - 1}$, and $w(z) = \frac{i}{\pi} \int_{-\infty}^{\infty} \frac{e^{-t^2}}{z-t} dt = e^{-z^2} erfc(-iz)$ [3]. Here $erfc(\cdot)$ is the complementary error function. And f_3 is given by

$$f_3(k_x^i) = \frac{\eta {k_z^i}^2}{1 - \eta^2 {k_z^i}^2} \left[\eta {k_z^i}^2 \left\{ T_1^i \sqrt{\frac{\pi}{k_0 w}} + i\pi b^i w(\sqrt{k_0 w} s_1) \right\} + P_i - T_2^i \sqrt{\frac{\pi}{k_0 w}} + i2\pi k_z^i w(\sqrt{k_0 w} s_2^i) \right]$$

where $b^i = -1 - \frac{\eta k_x^i}{\sqrt{\eta^2-1}}$, $T_1^i = \eta(1-i)(1+k_x^i) + b^i/s_1$, $T_2^i = \sqrt{2}(1+k_x^i) - 2e^{-i\pi/4}\sqrt{1+k_x^i}$, $P_i = i\pi k_z^i(\eta k_z^i - 1) e^{ik_0 k_x^i w}$, and $s_2^i = \sqrt{i(1-k_x^i)}$.

For forward scattering direction ($k_x^i = k_x^s$), the expressions for I_1 and I_2 have a removable pole. Therefore for this case we need to take limit of I_1 and I_2 as $k_x^i \to k_x^s$. For I_1, only f_1 should be modified as

$$\begin{aligned} f_1 &= -\frac{\eta^2}{1 - \eta^2 {k_z^i}^2} \left[f_1' + g_1 (2{k_x^i}^2 - {k_z^i}^2) + \frac{2i}{\eta} \left\{ \frac{k_x^i}{k_z^i} \sin^{-1} k_x^i - 1 \right\} \right] \\ f_1' &= \frac{2k_x^i \eta^2}{1 - \eta^2 {k_z^i}^2} \left(g_1 k_x^i {k_z^i}^2 + \frac{2ik_z^i}{\eta} \sin^{-1} k_x^i \right) \end{aligned}$$

I_2 is more complicated, and using L'Hospital rule, the final result is given by

$$I_2 = \frac{\eta}{1-\eta^2 k_z^{i\,2}} \left[I_4 - i\pi e^{ik_0 k_x^i w} \left\{ k_x^i (2\eta - \frac{1}{k_z^i}) - ik_0 k_z^i w (k_z^i \eta - 1) \right\} - e^{ik_0 w} \left\{ e^{-i\pi/4} \left(\sqrt{2} - \frac{1}{\sqrt{1+k_x^i}} \right) \cdot \right.\right.$$
$$\left.\left. \sqrt{\frac{\pi}{k_0 w}} + 2\pi i \left(\frac{k_x^i}{k_z^i} w(\sqrt{k_0 w} s_2^i) + e^{i\pi/4} \sqrt{k_0 w} \sqrt{1+k_x^i} (-\sqrt{k_0 w} s_2^i\, w(\sqrt{k_0 w} s_2^i) + \frac{i}{\sqrt{\pi}}) \right) \right\} \right] -$$
$$\frac{2\eta^3 k_x^i e^{ik_0 w}}{(1-\eta^2 k_z^{i\,2})^2} \left[\eta k_z^{i\,2} \left\{ T^i \sqrt{\frac{\pi}{k_0 w}} + i\pi b^i w(\sqrt{k_0 w} s_1) \right\} + P_i' - (\sqrt{2}(1+k_x^i) - 2e^{-i\pi/4} \sqrt{1+k_x^i}) \cdot \right.$$
$$\left. \sqrt{\frac{\pi}{k_0 w}} + i2\pi k_z^i w(\sqrt{k_0 w} s_2^i) \right] - \frac{2\eta^2 k_x^i e^{ik_0 w}}{1-\eta^2 k_z^{i\,2}} \left[T^i \sqrt{\frac{\pi}{k_0 w}} + i\pi b^i w(\sqrt{k_0 w} s_1) \right]$$

where $P_i' = i\pi e^{ik_0 k_x^i w}(\eta k_z^i - 1)$. For the case of $k_x^i = k_x^s$ and $|k_x^i| = 1$ (forward scattering when edge-on incidence), the above expression contains several divergent terms such as $\frac{1}{\sqrt{1+k_x^i}}$, but these terms cancel each other. Therefore by rearranging the divergent terms and using the first-order asymptotic expansion of $w(\cdot)$ function, (1) is modified slightly into

$$I \sim -\frac{I_1}{2} + \frac{I_2'}{4} e^{-ik_0 k_x^i w} + \frac{I_3'}{4} e^{ik_0 k_x^i w} + 4\eta \left[e^{i\pi/4} \sqrt{\frac{\pi k_0 w}{2}} - i \right] \tag{4}$$

In I_1, f_1 is simplified again into $f_1 = -2\eta^2 (g_1 - \frac{i}{\eta})$. I_2' and I_3' are given by

$$I_2' \text{ or } I_3' = I_4 - i2\pi\eta e^{ik_0 k_x^i w} - \pi H_0^{(1)}(k_0 w) + (1 \pm k_x^i) e^{ik_0 w} \left(\frac{1}{2\sqrt{2}} + \sqrt{\pi k_0 w}(1+i) \right)$$

Here "+" is chosen for I_2' and "-" for I_3'.

III. Numerical Results

The first example is a simulation of backscattering and scattering in forward direction from a thin dielectric strip with thickness $0.025\lambda_0$ and $\varepsilon_r = 4 + i0.4$. Figures 2 and 3 show comparisons of normalized radar echo width of the strip as a function of w/λ_0, which are calculated by the proposed solution given in Section II and MoM. Excellent agreement is observed for the two cases. For the rest of calculations presented here the width of the strip is fixed to be $5\lambda_0$. Figure 4 is a plot of backscattering as a function of incidence angle. Except at some angles around $\theta_i = 75°$, the new uniform solution provides very accurate results. Figures 5 and 6 are plots of bistatic scattering by the strip with the same dielectric constant and thickness as the previous case, for $\theta_i = 30°$ and $\theta_i = 90°$ (edge-on incidence), respectively. As shown in these figures, the asymptotic solution produces very accurate results, but some discrepancy is observed at angles around 75° for edge-on incidence. The final example is an investigation of effect of dielectric constant. Figure 7 shows a comparison of echo width as a function of the real part of dielectric constant with a fixed imaginary part of 0.4. The next figure is the echo width as a function of the imaginary part of the dielectric constant with a fixed real part of 20. For these simulations the incidence angle (θ_i) is fixed to be 30° and the observation point (θ_s) $-30°$ (forward direction). The asymptotic solution is again in excellent agreement with MoM.

References

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[2] I. Koh and K. Sarabandi, "A New Approximate Solution for Scattering by Thin Dielectric Disks of Arbitrary Size and Shape," *Submitted for publication in IEEE Trans. Antennas Propagat.*

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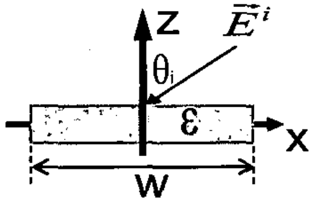

Fig. 1. Problem geometry.

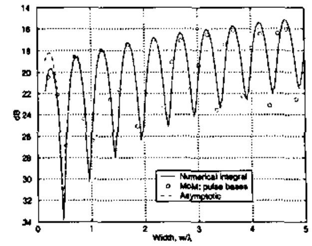

Fig. 2. Backscattering for edge-on incidence.

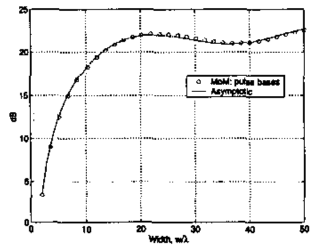

Fig. 3. Forward scattering for edge-on incidence.

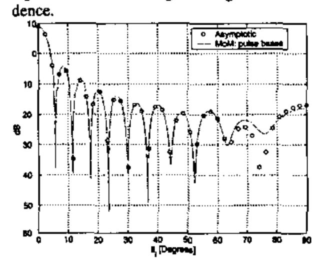

Fig. 4. Backscattering as a function of incidence angles.

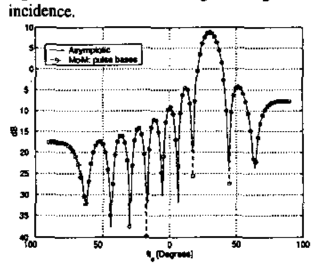

Fig. 5. Bistatic scattering as a function of incidence angles for $\theta_i = 30°$.

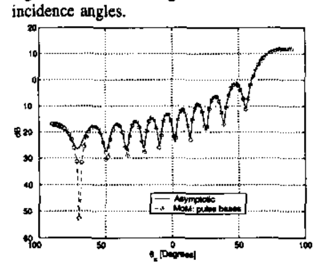

Fig. 6. Bistatic scattering as a function of incidence angles for $\theta_i = 90°$.

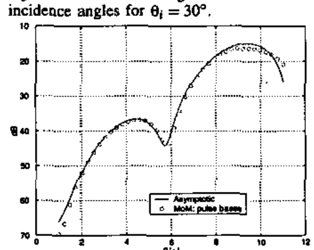

Fig. 7. Scattering as a function of real part of dielectric constant.

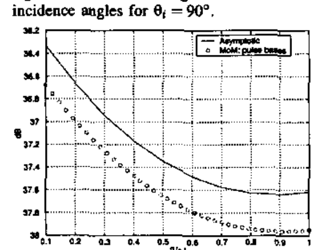

Fig. 8. Scattering as a function of imaginary part of dielectric constant.