An Approximate Solution for Scattering by Thin Dielectric Objects

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I. Introduction

Scattering from thin dielectric objects is a classic research topic in electromagnetics which has found a number of useful applications. A traditional approach is to model thin dielectrics with resistive sheets, and for several canonical geometries of resistive sheet bodies, exact solutions are found [1]. However, for many important geometries such as disks, exact solutions are not known and approximate solutions may be applicable to limited cases of interest. In this paper a new approximate solution is formulated based on a volumetric integral equation using Fourier transform, and it is shown that the solution is uniformly valid from low to high frequencies at all incidence angles including edge-on incidence. Validity of the solution is demonstrated through a comparison with canonical objects such as an infinite dielectric slab, and a number of 2-D and 3-D dielectric scatterers. For 2-D and 3-D scatterers, the approximate solution is compared with a Method of Moment solution.

II. Formulation

Figure 1 shows the problem geometry in which a very thin dielectric scatterer is located in x-y plane of a cartesian coordinate system. The scatterer is assumed homogeneous with a relative permittivity ε_r and arbitrary shape having a constant thickness $t \ll \lambda$. Using the notation of polarization current, the scatterer can be replaced with a volumetric current distribution of $\vec{J}(x,y,z) = ik_0Y_0(\varepsilon_r - 1)\vec{E}^t$ embedded in the host medium with permittivity ε_0 as seen in Fig. 2. Here $\vec{E}^t = \vec{E}^i + \vec{E}^s$ is the total electric field inside the scatterer. It can easily be shown that $\vec{J}$ must satisfy the following Fredholm integral equation.

$$\vec{J}(\vec{r}) - k_0^2(\varepsilon_r - 1)\int_{V} \bar{\bar{G}}(\vec{r},\vec{r}') \cdot \vec{J}(\vec{r}')dv' = -ik_0Y_0(\varepsilon_r - 1)\vec{E}^i(\vec{r}) \tag{1}$$

where $\vec{E}^i$ is the incident wave, and $\bar{\bar{G}}$ is the free-space dyadic Green's function [2]. By evaluating the delta function term in the dyadic Green's function analytically, the volumetric integral equation can be expressed in a more compact form given by

$$\vec{J} - \bar{\bar{A}}k_0^2(\varepsilon_r - 1)\int_{V} \bar{\bar{G}}_r \cdot \vec{J}dv' = \vec{J}_{RG} \tag{2}$$

where $\vec{J}_{RG}$ is the well-known Rayleigh-Gans current. By invoking the thin dielectric approximation, the polarization current can be assumed a constant with respect to z-axis, i.e., $\vec{J}(x,y,z) \approx \vec{J}(x,y,0)$. Hence the integral in (2) with respect to z can be evaluated using the mid-point approximation and according to thin dielectric assumption, $\frac{\partial}{\partial z}J_x = \frac{\partial}{\partial z}J_y = 0$, we have

$$\vec{I} \approx \frac{t}{8\pi^2}\iint_{-\infty}^{\infty} d^2k \frac{\bar{\bar{A}} \cdot \bar{\bar{L}}}{k_z}\int_{s'} ds' \vec{J}(x',y')e^{i[k_x(x-x')+k_y(y-y')]}$$

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where $\bar{\bar{L}}$ is given by

$$\bar{\bar{L}} = \frac{1}{k_0^2}\begin{pmatrix} k_0^2 - k_x^2 & -k_x k_y & 0 \\ -k_x k_y & k_0^2 - k_y^2 & 0 \\ 0 & 0 & k_0^2 - k_z^2 \end{pmatrix}$$

Now the original integral equation given by (2) can be written as

$$\vec{J} - \iota \frac{k_0^2}{8\pi^2}(\varepsilon_r - 1)\iint_{-\infty}^{\infty} d^2k \frac{\bar{\bar{A}}\cdot\bar{\bar{L}}}{k_z}\int_{s'} ds' \vec{J}(x', y')e^{i[k_x(x-x')+k_y(y-y')]} = \vec{J}_{RG} \tag{3}$$

Since $\vec{J}$ and $\vec{J}_{RG}$ are non-zero only inside the scatterer, the embedded integral in (3) can be interpreted as a convolution integral. Taking the Fourier transform of both sides and assuming $(\varepsilon_r - 1)$ is a constant function of position in the entire domain, (3) can be solved analytically as

$$\tilde{J} = \bar{\bar{B}}^{-1}\tilde{J}_{RG} \tag{4}$$

where $\tilde{J}$ and $\tilde{J}_{RG}$ are the Fourier transform of $\vec{J}$ and $\vec{J}_{RG}$, respectively. Also $\bar{\bar{B}}^{-1} = \left[\bar{\bar{I}} - \alpha k_0^2 \frac{A\cdot L}{k_z}\right]^{-1}$, $\bar{\bar{I}}$ is an unit dyadic, and $\alpha = \frac{i}{2}\iota(\varepsilon_r - 1)$. Explicitly $\bar{\bar{B}}^{-1}$ can be expressed as

$$\bar{\bar{B}}^{-1} = \begin{pmatrix} \frac{1}{D}\left[k_z - \alpha(k_0^2 - k_y^2)\right] & -\frac{\alpha}{D}k_x k_y & 0 \\ -\frac{\alpha}{D}k_x k_y & \frac{1}{D}\left[k_z - \alpha(k_0^2 - k_x^2)\right] & 0 \\ 0 & 0 & \frac{\varepsilon_r k_z}{\varepsilon_r k_z - \alpha k_\rho^2} \end{pmatrix}$$

where $D = k_z(1 + \alpha^2 k_0^2) - \alpha(k_0^2 + k_z^2)$, and $k_\rho^2 = k_x^2 + k_y^2$. By taking the inverse Fourier transform, a closed-form expression of $\vec{J}$ can be obtained and is given by

$$\vec{J}(x,y) \approx \frac{1}{4\pi^2}\iint_{-\infty}^{\infty} d^2k \bar{\bar{B}}^{-1} e^{i(k_x x + k_y y)} \int_{s'} ds' \vec{J}_{RG} e^{-i(k_x x' + k_y y')} \tag{5}$$

This formulation is valid for a situation that the integral contribution in (3) is smaller than the current itself.

The scattered field in the far-field region can also be obtained easily from (5). Considering a plane wave incidence, $\vec{E}^i = \vec{e}_i e^{i(k_x^i x + k_y^i y + k_z^i z)}$, the electric field in far-field region can be calculated as

$$\vec{E} \sim i(\varepsilon_r - 1)\bar{\bar{A}}\cdot\vec{e}_i \frac{k_0^3 Y_0}{16\pi^3}\frac{e^{k_0 r}}{r}\hat{r}\times\hat{r}\times\iint_{\infty}^{\infty} d^2k \bar{\bar{B}}^{-1}\int_{s'} ds' e^{i[(k_x^i - k_x)x' + (k_y^i - k_y)y']}\int_{s''} ds'' e^{-i[(k_x^s - k_x)x'' + (k_y^s - k_y)y'']}$$

Here $\hat{k}^s = \frac{1}{k_0}(k_x^s\hat{x} + k_y^s\hat{y} + k_z^s\hat{z})$ is a unit vector along the direction of observation point.

III. Dielectric Strip and Half-plane

An example considered here is the problem of scattering from a thin dielectric strip whose geometry is shown in Fig. 3. For a plane wave incidence, the polarization current (5) can be reduced to a single integral given by

$$\vec{J} \approx -ik_0 Y_0(\varepsilon_r - 1)\frac{w}{2\pi}\int_{-\infty}^{\infty} dk_x \bar{\bar{B}}_1^{-1}\bar{\bar{A}}\cdot\vec{e}_i \operatorname{sinc}\left[\frac{(k_x^i - k_x)}{2}w\right] e^{i(k_x x + k_y^i y)} \tag{6}$$

where $\bar{\bar{B}}_1^{-1}$ is the same as $\bar{\bar{B}}^{-1}$ with $k_y = k_y^i$. The corresponding electric field in the far-field region can be easily formulated as

$$\vec{E} \sim i\iota(\varepsilon_r - 1)\frac{(k_0 w)^2}{8\pi}\sqrt{\frac{2}{\pi k_0 \rho}}e^{i(k_0\rho - \pi/4)}\cdot\hat{\rho}\times\hat{\rho}\times\int_{-\infty}^{\infty} dk_x \bar{\bar{B}}_1^{-1}\bar{\bar{A}}\cdot\vec{e}_i \operatorname{sinc}\left[\frac{k_x^i - k_x}{2}w\right]\operatorname{sinc}\left[\frac{k_x - k_x^s}{2}w\right] \tag{7}$$

Unfortunately the integral can't be evaluated analytically and must be calculated numerically. To examine the validity of (7), examples are chosen from [3]. Figure 4 shows comparisons of normalized radar echo width of a strip with thickness $t = 0.025\lambda_0$ and $\epsilon_r = 4 + i0.4$ as a function of w/λ, which are calculated from (7) and MoM in forward and backward directions for an edge-on TM polarized incident wave.

Next, backscattering from a thin dielectric half-plane along the negative x-axis is considered. For this structure, exact and approximate solutions are known [1]. For this geometry, (5) is reduced to

$$\vec{J} = \frac{1}{2}\vec{J}_{inf} - ik_0Y_0(\epsilon_r - 1)\frac{i}{2\pi}\int_{-\infty}^{\infty} dk_x \bar{\bar{B}}_1^{-1}\bar{\bar{A}} \cdot \vec{e}_i \frac{e^{i(k_x x + k_y^i y)}}{k_x - k_x^i} \quad (8)$$

where $\vec{J}_{inf}$ is the current inside an infinite slab (known as physical optics (PO) current). If we focus on just edge-on incidence case, this PO current always becomes zero. Considering a TM polarized wave having an electric field along $\hat{y}$ and $k_y^i = 0$, in the far-field region, the electric field can be written as $\vec{E} \sim \hat{y}\sqrt{\frac{2}{\pi k_0 \rho}}P_e(\theta_i, \theta_s)$, where $P_e(\theta_i, \theta_s)$ is known as the far-field amplitude. $P_e(\theta_i, \theta_s)$ can be computed for the backscattering direction from

$$P_e(\frac{\pi}{2}, \frac{\pi}{2}) = \frac{k_0}{4\pi\eta}\int_{-\infty}^{\infty} dk_x \frac{1}{k_z(k_z - \alpha k_0^2)}$$

where $\alpha k_0 = -\frac{1}{\eta}$, and $\eta = \frac{2R}{Z_0}$ is the normalized surface impedance, and $R = \frac{iZ_0}{k_0 t(\epsilon_r - 1)}$. The integral in the above equation can be evaluated analytically as

$$P_e(\frac{\pi}{2}, \frac{\pi}{2}) = -\frac{i}{\pi\sqrt{1-\eta^2}}\tanh^{-1}\sqrt{\frac{1-\eta}{1+\eta}} \quad (9)$$

Figure 5, and 6 are plots of P_e as a function of the imaginary part of η and a fixed real part of 0.1. As seen from this figure the new formulation agrees very well with the asymptotic results. Hence (9) can provide a very good approximation for the Maliuzhinets half-plane function for moderate to large value of $|\eta|$.

IV. Dielectric Disk

In this section scattering by a very thin dielectric disk is examined. Scattering from an elliptical disk with major and minor axes of a and b is investigated. Choosing a coordinate system with x along the major axis and y along the minor axis of the ellipse, the surface integral of (5) can be evaluated in a closed-form and is given by

$$\iint_{s'} e^{i[(k_x - k_x^i)x' + (k_y - k_y^i)y']} ds' = \frac{2AJ_1\left(\sqrt{a^2(k_x - k_x^i)^2 + b^2(k_y - k_y^i)^2}\right)}{\sqrt{a^2(k_x - k_x^i)^2 + b^2(k_y - k_y^i)^2}}$$

Forward scattering from a circular disk with a diameter of $3cm$, and a thickness of $0.2mm$, and $\epsilon_r = 26.6 + i11.56$ is considered. Figures 7 and 8 show plots of the scattering of the dielectric disk in forward direction as a function of incidence angles. In this figure two approximate formulations such as VIPO (high frequencies) and Rayleigh-Gans (low frequencies) as well as MoM results are shown. It is shown that the new formulation can reproduce the MoM results very accurately over the entire comparison region where low and high frequency results are only valid at high and low incidence angles.

References

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[2] L.Tsang J.Kong and R.T.Shin, *Theory of Microwave Remote Sensing*, Wiley, New York, 1985.

[3] J. H. Richmond, "Scattering by Thin Dielectric Strips," *IEEE Trans. Antennas Propagat.*, vol. AP-33, no. 1, pp. 64–68, January 1985.

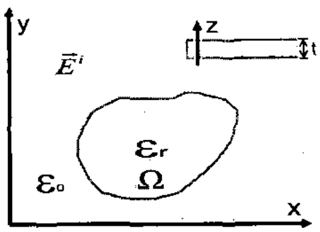

Fig. 1. Original problem geometry.

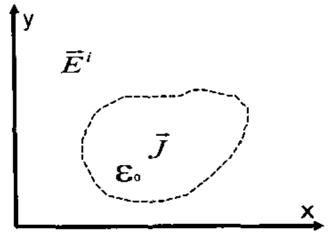

Fig. 2. Equivalent problem.

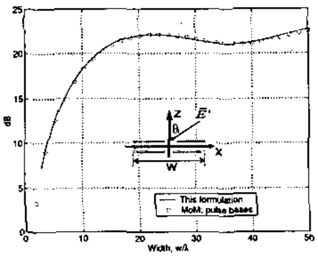

Fig. 3. Forward scattering from a thin dielectric strip.

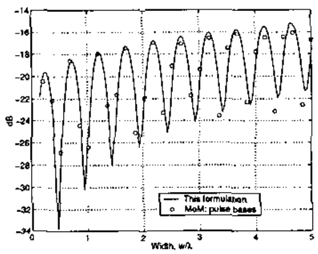

Fig. 4. Backscattering from a thin dielectric strip.

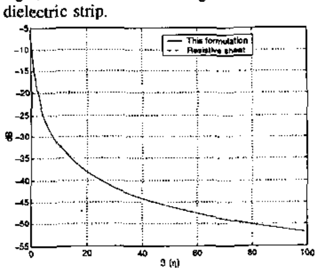

Fig. 5. Backscattering from a thin dielectric half-plane: Magnitude.

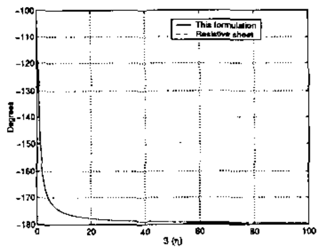

Fig. 6. Backscattering from a thin dielectric half-plane: Phase.

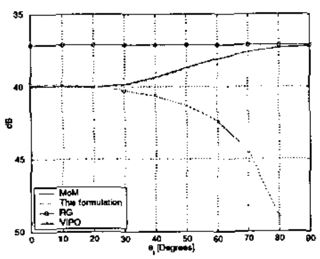

Fig. 7. Forward scattering from a thin dielectric disk: Magnitude.

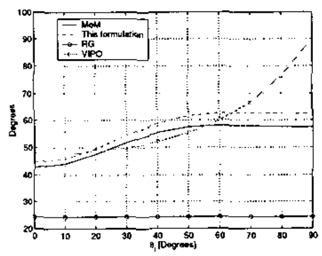

Fig. 8. Forward scattering from a thin dielectric disk: Phase.