

# The Use of the MPI Communication Library in the NAS Parallel Benchmarks

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*Abstract*—The statistical analysis of traces taken from the NAS Parallel Benchmarks can tell one much about the type of network traffic that can be expected from scientific applications run on distributed-memory parallel computers. For instance, such applications utilize a relatively few number of communication library functions, the length of their messages is widely varying, they use many more short messages than long ones, and within a single application the messages tend to follow relatively simple patterns. Information such as this can be used by hardware and software designers to optimize their systems for the highest possible performance.

*Keywords*—benchmarks, trace analysis, message-passing, distributed memory parallel computer, parallel computing

## I. INTRODUCTION

Parallel computing is a computer paradigm where multiple processors attempt to co-operate in the completion of a single task. Within the parallel computing paradigm, there are two memory models: shared-memory and distributed memory. The shared-memory model distinguishes itself by presenting the programmer with the illusion of a single memory space. The distributed-memory model, on the other hand, presents the programmer with a separate memory space for each processor. Processors, therefore, have to share information by sending messages to each other. To send these messages, usually applications call a standard communication library. The communication library is usually MPI (Message Passing Interface) [1] or PVM (Parallel Virtual Machine) [2], with MPI rapidly becoming the norm.

An important component in the performance of a distributed-memory parallel computing application is the performance of the communication library the application uses. Therefore, the hardware and software systems providing these communication functions must be tuned to the highest degree possible. An important class of information that would aid in the tuning of a communication library is an understanding of the communication patterns that occur within applications. This includes information such as the relative frequency with which the various functions within the communication library are called, the lengths of the messages involved, and the ordering of the messages.

Since it is not realistic to examine all the distributed-memory parallel applications in existence, one looks to find a small set of applications that reasonably represents the entire field. The representative set of applications that was chosen was the widely-used NAS Parallel Benchmarks

(NPB) [3]. The rest of this paper describes in further detail the NPB and the results obtained from analyzing the frequency and type of message calls which occur within the NPB.

Section II of the paper describes the NPB. Section III describes the instrumentation methodology used on the NPB. Following that is Section IV, which describes the assumptions made about the manner in which the MPI message-passing library was implemented. Section V gives a summary of the data gathered from the traces. Section VI provides an explanation for the patterns observed, in terms of the nature of the communication patterns of the NPB. Section VII provides some final conclusions.

## II. NAS PARALLEL BENCHMARKS DESCRIPTION

The NAS Parallel Benchmarks are a set of scientific benchmarks issued by the Numerical Aerodynamic Simulation (NAS) program located at the NASA Ames Research Center. The benchmarks have become widely accepted as a reliable indicator of supercomputer performance on scientific applications. As such, they have been extensively analyzed [4], [5], [6]. The benchmarks are largely derived from computational fluid dynamics code and are currently on version 2.2. The NAS Parallel Benchmarks 2.2 includes implementations of 7 of the 8 benchmarks in the NAS Parallel Benchmarks 1.0 suite. The eighth benchmark shall be implemented in a later version of the NAS Parallel Benchmarks. The benchmarks implemented are:

**BT**: a block tridiagonal matrix solver.

**EP**: Embarrassingly Parallel, an application where there is very minimal communication amongst the processes

**FT**: a 3-D FFT PDE solver benchmark.

**IS**: integer sort

**LU**: an LU solver.

**MG**: a multigrid benchmark.

**SP**: a pentadiagonal matrix solver.

The benchmark codes are written in Fortran with MPI function calls, except for the IS benchmark which is written in C with MPI function calls. The NAS Parallel Benchmarks can be compiled into three problem sizes known as classes A, B, and C. The class A benchmarks are tailored to run on moderately powerful workstations. Class B benchmarks are meant to run on high-end workstations or small parallel systems. Class C benchmarks are meant for high-end supercomputing.

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### III. TRACEGATHERING

#### A. Instrumenting the Benchmarks

The source code for the NPB was instrumented by pre-processing the source code with a filter written in Perl. Since all MPI function calls have the form:

```
call MPI_function_name(parameter1,parameter2,...)
```

the Perl preprocessor simply uses pattern matching to find the MPI function calls in the program and inserts code just before the MPI function call to print out the relevant parameters of the MPI function call. The static frequency of MPI function calls was determined by using the `grep` utility to search for “MPI\_” in all source files.

#### B. Running the Benchmarks

The benchmarks were compiled as class B benchmarks and traced on machines at the University of Michigan’s Center for Parallel Computing. When the dynamic frequencies of the MPI function calls were calculated from the traces, the `MPI_COMM_*` functions and the `MPI_WTIME` function were not included because the beforementioned functions are not inherently involved in the transfer of data between processors. Also, many of the benchmarks run an iteration of the code before the running and timing of the main loop. This is done to minimize variations in performance that would be caused by cache misses, TLB misses, and page faults. The portions of the traces that correspond to this extra iteration were not included in the dynamic results due to the fact that we wished the results to be as close as possible to those which would be found if the benchmarks were used in real-world situations.

The BT and SP benchmarks were run with 4, 9, and 16 processors. The FT, IS, LU, and MG benchmarks were run with 2, 4, 8, and 16 processors. As an example of the type of information gathered by the traces, for an `MPISEND` function call, the following information is output to stdout:

1. `MPISEND`
2. the ID of the sending process
3. number of data items being transferred
4. datatype of the transferred data (i.e. `INTEGER`, `DOUBLE`, or `DOUBLE_COMPLEX`)
5. the ID of the receiving process
6. the type tag for this interprocess communication

### IV. MPI IMPLEMENTATION MODEL

Given the fact that the low-level implementation of the MPI library is platform and vendor dependent, there are then widely varying answers to the question of how many messages it takes to implement any given MPI function. Therefore, for this paper, we created a limited MPI implementation model that we feel reasonably represents how the MPI library functions found within the NAS Parallel Benchmark 2.2 could be implemented. Then, for each function call, we determined how many messages the processor with pid equal to one would send. The processor with pid equal to one was chosen as the one to model over the more typical processor zero is because many times the processor

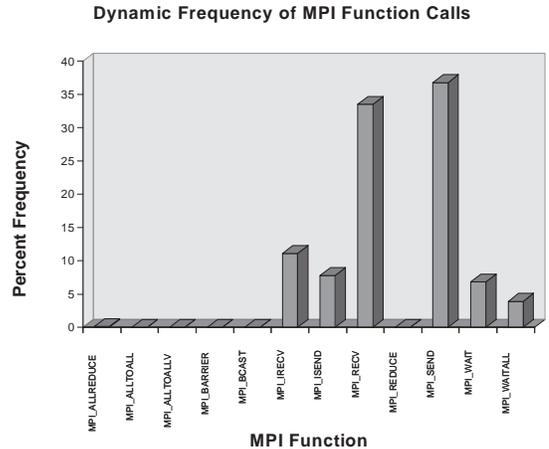


Fig. 1. Dynamic Frequency of MPI Function Calls in the NAS Benchmarks

with pid equal to zero is used to broadcast information, causing its traffic pattern to be unrepresentative of that of a typical node.

Our model is as follows:

**MPI\_ALLREDUCE** The `MPI_ALLREDUCE` function call is modeled as a reduction followed by a broadcast. The reduction and broadcast are each modeled as occurring via binary trees. In the binary tree model, one message travels over each edge. Also, in a binary tree of  $p$  vertices, there are  $p - 1$  edges. Therefore,  $(p - 1) \times 2$  total messages are sent globally for each `MPI_ALLREDUCE` function call. In this model, processor one sends two messages.

**MPI\_ALLTOALL** The straightforward method of having each node send out  $p - 1$  messages is the model used for the `MPI_ALLTOALL` function call. Globally, this implies that  $p(p - 1)$  messages are sent for each `MPI_ALLTOALL` function call. For processor one, the model implies  $p - 1$  messages.

**MPI\_ALLTOALLV** It is modeled in the same manner as the `MPI_ALLTOALL` function call is modeled.

**MPI\_BARRIER** It is treated as an `MPI_ALLREDUCE` where the message length is 4 bytes.

**MPI\_BCAST** It is modeled as occurring via a binary tree-based algorithm. Therefore,  $p - 1$  messages are sent globally. Processor one sends one message.

**MPI\_RECV** Since a message is received, no messages are sent by processor one.

**MPI\_SEND** One message is sent by processor one.

**MPI\_RECV** Since a message is received, no messages are sent by processor one.

**MPI\_REDUCE** The model assumes a binary tree-based algorithm, implying  $p - 1$  total messages globally. One message is sent by processor one.

**MPI\_SEND** One message is sent by processor one.

### V. RESULTS

A plethora of conclusions can be drawn from the trace data:

MPI Function	Percent Frequency
MPLIRECV	14.4%
MPLSEND	10.6%
MPLISEND	10.2%
MPLBCAST	9.7%
MPLWAIT	9.7%
MPLALLREDUCE	7.2%
MPLBARRIER	7.2%
MPLABORT	4.7%
MPL_COMM_SIZE	4.2%
MPL_WAITALL	3.4%
MPL_FINALIZE	3.0%
MPL_COMM_RANK	2.5%
MPLINIT	2.5%
MPLREDUCE	2.5%
MPLALLTOALL	1.7%
MPL_COMM_DUP	1.7%
MPL_COMM_SPLIT	1.7%
MPLRECV	1.7%
MPLWTIME	0.8%
MPLALLTOALLV	0.4%

TABLE I

STATIC FREQUENCY OF MPI FUNCTION CALLS IN THE NAS PARALLEL BENCHMARKS 2.2

	Static	Dynamic
MPLALLREDUCE	17	268
MPLALLTOALL	4	128
MPLALLTOALLV	1	32
MPLBARRIER	17	4
MPLBCAST	23	86
MPLIRECV	34	45,604
MPLISEND	24	32,436
MPLRECV	4	139,500
MPLREDUCE	6	98
MPLSEND	25	152,628
MPLWAIT	23	27,568
MPL_WAITALL	8	16,206

TABLE II

DYNAMIC VERSUS STATIC MPI FUNCTION CALLS

MPI Function	IS Percent Frequency	non-IS Percent Frequency
MPLALLREDUCE	30.5%	0.1%
MPLALLTOALL	30.5%	0.0%
MPLALLTOALLV	24.4%	0.0%
MPLBARRIER	0.0%	0.0%
MPLBCAST	0.0%	0.0%
MPLIRECV	3.1%	11.0%
MPLISEND	0.0%	7.8%
MPLRECV	0.0%	33.7%
MPLREDUCE	6.1%	0.0%
MPLSEND	2.3%	36.8%
MPLWAIT	3.1%	6.7%
MPL_WAITALL	0.0%	3.9%

TABLE III

DYNAMIC FREQUENCY OF MPI FUNCTION CALLS IN THE IS BENCHMARK VS. THE OTHER BENCHMARKS

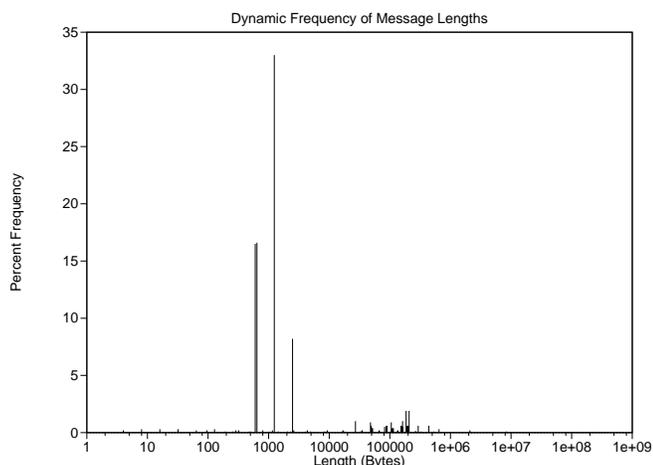


Fig. 2. Length of MPI Messages in the NAS Parallel Benchmarks

• One of the most interesting results reached by examining the traces was that relatively few of the functions in the MPI library are used by the NAS Parallel Benchmarks. Table I shows the static frequency in percent of MPI function calls in the NAS Parallel Benchmarks 2.2. Of the 125 functions in the MPI communication library, only 20 were actually used in the NAS Parallel Benchmarks. The functions used are:

1. MPLABORT (abort from MPI)
2. MPLALLREDUCE (reduction plus a broadcast)
3. MPLALLTOALL (all-to-all communication where all the messages are the same length)
4. MPLALLTOALLV (all-to-all communication where

the messages can be of different lengths)

5. MPLBARRIER (barrier synch)
6. MPLBCAST (broadcast)
7. MPL\_COMM\_DUP (duplicate a communication group pointer)
8. MPL\_COMM\_RANK (find processor id)
9. MPL\_COMM\_SIZE (find number of processors in group)
10. MPL\_COMM\_SPLIT (split current communication group into two groups)
11. MPL\_FINALIZE (shut MPI down cleanly)
12. MPLINIT (initialize MPI)
13. MPLIRECV (non-blocking receive)
14. MPLISEND (non-blocking send)
15. MPLRECV (blocking receive)
16. MPLREDUCE (reduction)
17. MPLSEND (blocking send)
18. MPLWAIT (wait for a non-blocking communication to complete)
19. MPL\_WAITALL (wait for a list of non-blocking com-

	IS	non-IS
MPIALLREDUCE	40	228
MPIALLTOALL	40	88
MPIALLTOALLV	32	0
MPIBARRIER	0	4
MPIBCAST	0	86
MPIIRECV	0	45,600
MPIISEND	0	32,436
MPIIRECV	0	139,500
MPIREDUCE	8	90
MPISEND	3	152,625
MPI_WAIT	4	27,564
MPI_WAITALL	0	16,206

TABLE IV

ABSOLUTE COUNT OF MPI FUNCTION CALLS IN THE IS BENCHMARK VS. THE OTHER BENCHMARKS

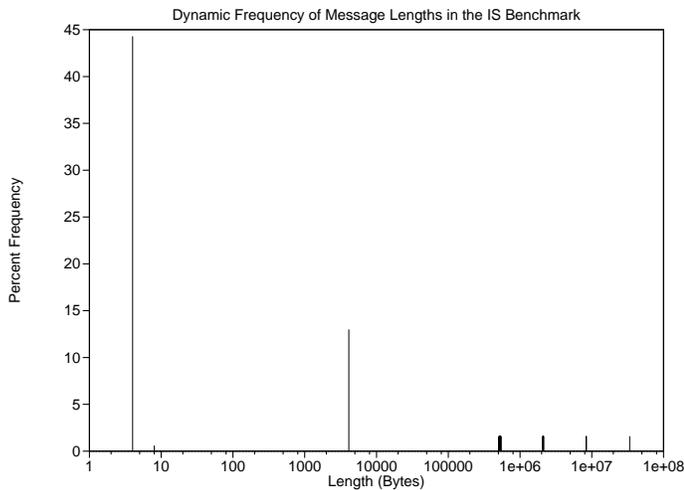


Fig. 3. Length of MPI Messages in the IS Benchmark

munications to complete)

#### 20. MPI\_WTIME (system time)

This phenomena is not unique to the NPB, as others have noted that relatively few MPI commands are needed for most programs. For example, in the tutorial by Gropp [7], he states “MPI is small (6 functions) — many parallel programs can be written with just 6 basic functions.”

- When looking at the dynamic frequency of the MPI functions in the NAS Parallel Benchmarks, as shown in Figure 1, another interesting result is noted. Fully 89% of the MPI functions calls are blocking or non-blocking sends and receives. None of the more complex MPI communication functions nor even more complex sends or receives such as buffered sends and buffered receives are used.

- Another interesting result is shown in Figure 2. It shows that 74.8% of the messages have a message length of 600, 640, 1240, or 2480 bytes, implying that short messages dominate NPB network traffic. However, the mean message length is 73,447 bytes, the median message length is 1240 bytes, and the standard deviation is a rather large 1,594,623 bytes. Thus there was wide variation in the message lengths. The largest message length

	IS Benchmark (bytes)	non-IS Benchmarks (bytes)
Mean	1,332,196	69,317
Median	4116	1240
Minimum	4	4
Maximum	33,716,496	134,217,728
Standard Deviation	4,568,524	1,574,036

TABLE V

MESSAGE LENGTH STATISTICS FOR THE IS BENCHMARK VS. THE REST OF THE NAS PARALLEL BENCHMARKS 2.2

is 128 MB when two nodes exchange 128 MB messages using MPIALLTOALL in the 2-processor version of the FT benchmark. The shortest message length is 4 bytes which is mainly used by MPIBCAST synchronization messages. Overall, the conclusion that can be drawn is that inter-processor communication hardware and software must be optimized for both short and long messages. It is the norm now for parallel machines to realize their full bandwidth for extremely long messages. These traces show that a large percentage of messages are not extremely long. These short messages should also be experiencing the full bandwidth of the interconnect. If this occurred, then many applications would notice increased performance from their distributed environment. Note, however, that “short” is not a few bytes, but hundreds of bytes. Another study has been done characterizing the amount and type of communication in distributed memory code [8]. This study also obtained results indicating that there is a wide variation in the amount and the size of messages.

- The 80/80 rule states that 80% of the messages on a network are 256 bytes or less and that 80% of the data on a network is sent by messages of 8K bytes or greater [9]. For the NPB, one finds that 80% of the messages are 27,040 bytes or less and that 80% of the data is from messages of length 184,960 bytes or greater. Therefore, the traffic from the NAS Parallel Benchmarks roughly demonstrates the characteristics predicted by an 80/80 rule.

- Some more interesting results occur when one contrasts the statistics for the IS benchmark, which performs integer computation, against the other benchmarks, which perform floating-point computations. Table III indicates that the IS code is dominated by reductions and all-to-all communication as opposed to the floating-point code which is dominated by sends and receives. Figure 3 shows the distribution of message lengths in the IS benchmark and Table V compares the message length statistics of the IS benchmark versus the rest of the NAS Parallel Benchmarks. It shows that the mean of the IS benchmark is two orders of magnitude larger than the mean of the rest of the NPB, and the standard deviation of the IS benchmark is three times larger than the standard deviation of the rest of the NPB. Thus the IS benchmark has a wider variation in message lengths than the rest of the NPB, a fact which may be indicative of some differences between data-intensive and

computation-intensive applications. However, it should not be taken to be indicative of general differences between codes dominated by integer operations versus floating-point operations.

- Just from comparing the total number of dynamic versus static MPI function calls, as shown in Table II, one can immediately conclude that some of the function calls in the NAS application codes are being visited a large number of times. This would lead one to believe that the communication patterns of most distributed memory applications are dominated by their behavior within loops. Section VI explores this issue further.

## VI. NAS PARALLEL BENCHMARK COMMUNICATION KERNELS

Within the NAS Parallel Benchmarks there is a very simple structure to the communication that occurs between nodes. The Appendix contains pseudocode delineating the communication structure of the NPB codes. As the Appendix shows, the communication is dominated by a loop-based repetition of a few simple communication calls.

Tables VI and VII are another view of this simple communication structure. It contains equations that describe the relationship between the frequency of an MPI function being called and the number of processors a benchmark is run on for the various NAS Parallel Benchmarks. The boxes labeled N/A indicate that the MPI function is not called within the benchmark.

The loop-based patterns mean that the communication structure is static for long periods of time, leading one to believe that simple strategies can detect the pattern. Most likely, there are hardware and software optimizations that can be used to take advantage of these communication patterns. This is an ongoing area of research for us.

## VII. CONCLUSION

The fact that only the knowledge of a few MPI functions are really necessary in order to create applications is emphasized to programmers even as early as the first MPI tutorial [10]. This paper, however, goes one step further and quantitatively describes which MPI functions are important. We considered both their static frequency, i.e., how often they were written, and their dynamic frequency, i.e., how often they were executed.

Our statistical analysis of traces taken from the NAS Parallel Benchmarks can tell one much about the type of network traffic to be expected from parallel distributed-memory scientific applications. For instance, these applications will utilize a relatively few number of communication library functions, and the length of these messages will be widely varying. It is also true that they obey an 80/80 rule, in that most of the messages are shorter, but most of the traffic volume is from long messages. Further, for an application, the majority of the messages are issued within loops having a simple communication pattern.

Since communications is a critical component of distributed-memory parallel computing, it is important

that it be carefully optimized. Studies such as those in this paper can be used by hardware and software designers to tune their communications systems to increase their performance on real applications. This in turn should enable users to achieve higher performance and increased scalability of their codes.

## ACKNOWLEDGMENTS

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<b>MPI Function</b>	<b>BT</b>	<b>FT</b>	<b>IS</b>
MPI_ALLREDUCE	2	N/A	10
MPI_ALLTOALL	N/A	22	10
MPI_ALLTOALLV	N/A	N/A	if $procs = 2, 5$ if $procs = 4, 8$ if $procs = 8, 9$ if $procs = 16, 10$
MPI_BARRIER	N/A	N/A	N/A
MPI_BCAST	3	2	N/A
MPI_RECV	$1200\sqrt{procs} + 6$	N/A	N/A
MPI_SEND	$1200\sqrt{procs} + 6$	N/A	N/A
MPI_RECV	N/A	N/A	N/A
MPI_REDUCE	1	20	2
MPI_SEND	N/A	N/A	if $procs = 2, 0$ if $procs > 2, 1$
MPI_WAIT	$2400\sqrt{procs} - 2400$	N/A	N/A
MPI_WAITALL	201	N/A	N/A

TABLE VI

MPI FUNCTION CALL FREQUENCY IN THE NAS PARALLEL BENCHMARKS 2.2 AS A FUNCTION OF THE NUMBER OF PROCESSORS, PART 1

<b>MPI Function</b>	<b>LU</b>	<b>MG</b>	<b>SP</b>
MPI_ALLREDUCE	8	46	2
MPI_ALLTOALL	N/A	N/A	N/A
MPI_ALLTOALLV	N/A	N/A	N/A
MPI_BARRIER	N/A	1	N/A
MPI_BCAST	9	6	3
MPI_RECV	if $procs = 2$ , 252 if $procs = 4$ , 506 if $procs = 8$ , 759 if $procs = 16$ , 759	if $procs < 16$ , 2772 if $procs = 16$ , 2572	$2400\sqrt{procs} + 6$
MPI_SEND	N/A	N/A	$2400\sqrt{procs} + 6$
MPI_RECV	if $procs < 16$ , $15500 \log_2(procs)$ if $procs = 16$ , 46500	N/A	N/A
MPI_REDUCE	N/A	1	1
MPI_SEND	if $procs = 2$ , 15755 if $procs = 4$ , 31506 if $procs = 8$ , 47258 if $procs = 16$ , 47258	if $procs < 16$ , 2772 if $procs = 16$ , 2532	N/A
MPI_WAIT	if $procs = 2$ , 252 if $procs = 4$ , 506 if $procs = 8$ , 759 if $procs = 16$ , 759	if $procs < 16$ , 2772 if $procs = 16$ , 2572	N/A
MPI_WAITALL	N/A	N/A	$2400\sqrt{procs} - 1999$

TABLE VII

MPI FUNCTION CALL FREQUENCY IN THE NAS PARALLEL BENCHMARKS 2.2 AS A FUNCTION OF THE NUMBER OF PROCESSORS, PART 2

## APPENDIX

This appendix contains the pseudocode which exposes the structure of interprocessor communication within the various NAS Parallel Benchmarks. The pseudocode was formulated from the gathered communication traces and is, therefore, from the viewpoint of the processor with its id equal to 1 running a class B compilation of the benchmark. The pseudocode contains the variable **number\_of\_processors** which represents the total number of processors utilized during an execution of the application. Within the pseudocode, the variable **pid** is a number between 0 and **number\_of\_processors**-1. Each processor's **pid** variable is assigned a unique value within the beforementioned range. Furthermore, all reduction operations have the processor with pid equal to 0 as the root.

The MPI function calls within the pseudocode have been abbreviated for simplicity. A short description of the meaning of the fields within the pseudocode MPI function calls is, therefore, necessary:

1. **mpi\_allreduce**(*number of items, data type of items, reduction operation*)
2. **mpi\_alltoall**(*number of items, data type of items*)
3. **mpi\_bcast**(*number of items, data type of items*)
4. **mpi\_irecv**(*number of items, data type of items, destination*)
5. **mpi\_reduce**(*number of items, data type of items, reduction operation*)
6. **mpi\_send**(*number of items, data type of items, destination*)

For function calls where the source, destination, or message length cannot be succinctly mathematically expressed, the function call does not contain any fields.

Finally, the pseudocode for the all benchmarks is appropriate for an arbitrary number of processors except the pseudocode for the MG benchmark. Within the MG benchmark, multiple grids of varying resolution are used to find the solution to an equation. The resolution of these fields depends on a variety of factors such as the number of processors, number of grids, and problem size. As the number of processors increases, then, the communication structure of the benchmark changes. To reduce the complexity of the pseudocode, we have only given a version appropriate for sixteen or fewer processors.



## II. FT BENCHMARK CODE

```
count=33554432/(number_of_processors*number_of_processors)

mpi_bcast(3,MPI_INTEGER)
mpi_bcast(1,MPI_INTEGER)

mpi_alltoall(count,MPI_DOUBLE_COMPLEX)

do i=1,20
  mpi_alltoall(sendbuf,count,MPI_DOUBLE_COMPLEX)
  mpi_reduce(1,MPI_DOUBLE_COMPLEX,MPI_SUM)
enddo
```

## III. IS BENCHMARK CODE

```
do i=1,10
  loop()
end do

mpi_reduce(1,MPI_DOUBLE_PRECISION,MPI_MAX)

if (pid > 0)
  mpi_irecv(1,MPI_INTEGER,pid-1)
end if

if (pid < number_of_processors-1)
  mpi_send(1,MPI_INTEGER,pid+1);
end if

mpi_wait()

mpi_reduce(1,MPI_INTEGER,MPI_SUM)

-----

loop()
{
  mpi_allreduce(1029,MPI_INTEGER,MPI_SUM)

  mpi_alltoall(1,MPI_INTEGER)

  mpi_alltoallv()
}
```

## IV. LU BENCHMARK CODE

```

mpi_bcast(1,MPI_INTEGER)
mpi_bcast(1,MPI_INTEGER)
mpi_bcast(1,MPI_INTEGER)
mpi_bcast(1,MPI_DOUBLE_PRECISION)
mpi_bcast(1,MPI_DOUBLE_PRECISION)
mpi_bcast(5,MPI_DOUBLE_PRECISION)
mpi_bcast(1,MPI_DOUBLE_PRECISION)
mpi_bcast(1,MPI_DOUBLE_PRECISION)
mpi_bcast(1,MPI_DOUBLE_PRECISION)

do i=1,2
  mpi_irecv()
  if (number_of_processors > 4)
    mpi_send()
  end if
  mpi_wait()

  if (number_of_processors > 4)
    mpi_irecv()
  end if

  mpi_send()

  if (number_of_processors > 4)
    mpi_wait()
  end if

  if (number_of_processors > 2)
    mpi_send()
    mpi_irecv()
    mpi_wait()
  end if
end do

mpi_allreduce(5,MPI_DOUBLE_PRECISION,MPI_SUM)

mpi_barrier()

do i=1,249
  j_loop()
  k_loop()
  mpi_irecv()
  if (number_of_processors > 4)
    mpi_send()
  end if
  mpi_wait()
  if (number_of_processors > 4)
    mpi_irecv()
  end if
  mpi_send()
  if (number_of_processors > 4)
    mpi_wait()
  end if
  if (number_of_processors > 2)
    mpi_send()

```

```

  mpi_irecv()
  mpi_wait()
end if
end do

j_loop()
k_loop()

mpi_allreduce(5,MPI_DOUBLE_PRECISION,MPI_SUM)
mpi_irecv()

if (number_of_processors > 4)
  mpi_send()
end if

mpi_wait()
mpi_send()

if (number_of_processors > 4)
  mpi_wait()
end if

if (number_of_processors > 2)
  mpi_send()
  mpi_irecv()
  mpi_wait()
end if

mpi_allreduce(5,MPI_DOUBLE_PRECISION,MPI_SUM)
mpi_allreduce(1,MPI_DOUBLE_PRECISION,MPI_MAX)
mpi_allreduce(5,MPI_DOUBLE_PRECISION,MPI_SUM)

if (number_of_processors > 2)
  mpi_irecv()
  mpi_wait()
end if

if (number_of_processors > 4)
  mpi_irecv()
  mpi_wait()
end if

mpi_send()
mpi_allreduce(1,MPI_DOUBLE_PRECISION,MPI_SUM)

if (number_of_processors == 4)
  mpi_irecv()
  mpi_wait()
end if

```

```
mpi_allreduce(1,MPI_DOUBLE_PRECISION,MPI_SUM)
```

```
-----
```

```
j_loop()
{
  do j=1,62
    mpi_recv()
    if (number_of_processors > 4)
      mpi_send()
    end if
    if (number_of_processors > 2)
      mpi_send()
    end if
  end do
}
```

```
-----
```

```
k_loop()
{
  do k=1,62
    if (number_of_processors > 4)
      mpi_recv()
    end if
    if (number_of_processors > 2)
      mpi_recv()
    end if
    mpi_send()
  end do
}
```

## V. MG BENCHMARK CODE

```

if (number_of_processors < 16)
{
  mpi_bcast(1,MPI_INTEGER)
  mpi_bcast(1,MPI_INTEGER)
  mpi_bcast(1,MPI_INTEGER)
  mpi_bcast(1,MPI_INTEGER)
  mpi_bcast(1,MPI_INTEGER)
  mpi_bcast(8,MPI_INTEGER)
  mpi_barrier()

  do i=1,10
    loop3()
  end do

  mpi_barrier()

  do i=1,2
    loop1()
  end do

  do i=1,23
    loop2()
  end do

  mpi_barrier()

  do i=1,10
    loop3()
  end do

  mpi_barrier()

  loop2()

  mpi_barrier()

  loop1()

  do i=1,459
    loop2()
  end do

  loop1()

  mpi_reduce(1,MPI_DOUBLE_PRECISION,MPI_MAX)
}
else
{
  mpi_bcast(1,MPI_INTEGER)
  mpi_bcast(1,MPI_INTEGER)
  mpi_bcast(1,MPI_INTEGER)
  mpi_bcast(1,MPI_INTEGER)
  mpi_bcast(1,MPI_INTEGER)
  mpi_bcast(8,MPI_INTEGER)
  mpi_barrier()

  do i=1,10
    loop3()
  end do

  mpi_barrier()

  do i=1,2
    loop1()
  end do

  do i=1,6
    loop2()
  end do

  mpi_irecv()
  mpi_irecv()
  mpi_wait()
  mpi_wait()

  do i=1,5
    loop2()
  end do

  mpi_barrier()

  do i=1,10
    loop3()
  end do

  mpi_barrier()

  loop2()

  mpi_barrier()

  loop1()

  do i=1,6
    loop2()
  end do

  do i=1,19
    mpi_irecv()
    mpi_irecv()
    mpi_wait()
    mpi_wait()
    do j=1,21
      loop2()
    end do
  end do

  mpi_irecv()
  mpi_irecv()
  mpi_wait()
  mpi_wait()
}

```

```

do j=1,14
  loop2()
end do

mpi_reduce(1,MPI_DOUBLE_PRECISION,MPI_MAX)
}

```

-----

```

loop1()
{
  loop2()
  mpi_allreduce(1,MPI_DOUBLE_PRECISION,MPI_MAX)
  mpi_allreduce(1,MPI_DOUBLE_PRECISION,MPI_SUM)
}

```

-----

```

loop2()
{
  mpi_irecv()
  mpi_irecv()
  mpi_send()
  mpi_send()
  mpi_wait()
  mpi_wait()
  mpi_irecv()
  mpi_irecv()
  mpi_send()
  mpi_send()
  mpi_wait()
  mpi_wait()
  mpi_irecv()
  mpi_irecv()
  mpi_send()
  mpi_send()
  mpi_wait()
  mpi_wait()
}

```

-----

```

loop3()
{
  mpi_allreduce(1,MPI_DOUBLE_PRECISION,MPI_MAX)
  mpi_allreduce(1,MPI_DOUBLE_PRECISION,MPI_MAX)
  mpi_allreduce(1,MPI_DOUBLE_PRECISION,MPI_MIN)
  mpi_allreduce(4,MPI_INTEGER,MPI_MAX)
}

```

