

EECS 583 – Class 6

Dataflow Analysis II

University of Michigan

January 25, 2023

Announcements & Reading Material

- ❖ Today's class is virtual only due to weather
 - » Discussion is cancelled
- ❖ HW 2 is posted, Due Wed Feb 25, midnight
 - » Please start early, significantly harder than HW 1
 - » Take a look at the template code
 - » Aditya will discuss at the end of today's class
- ❖ Today's class
 - » *Compilers: Principles, Techniques, and Tools*,
A. Aho, R. Sethi, and J. Ullman, Addison-Wesley, 1988.
(Sections: 10.5, 10.6, 10.9, 10.10 Edition 1; 9.2, 9.3 Edition 2)
- ❖ Material for Monday
 - » "Practical Improvements to the Construction and Destruction of Static Single Assignment Form," P. Briggs, K. Cooper, T. Harvey, and L. Simpson, *Software--Practice and Experience*, 28(8), July 1998, pp. 859-891.

Recap: Liveness vs Reaching Defs

Liveness

$$\begin{aligned} \text{OUT} &= \text{Union}(\text{IN}(\text{succs})) \\ \text{IN} &= \text{GEN} + (\text{OUT} - \text{KILL}) \end{aligned}$$

Bottom-up dataflow

Any path

Keep track of variables/registers

Uses of variables $\rightarrow$ GEN

Defs of variables $\rightarrow$ KILL

Reaching Definitions/DU/UD

$$\begin{aligned} \text{IN} &= \text{Union}(\text{OUT}(\text{preds})) \\ \text{OUT} &= \text{GEN} + (\text{IN} - \text{KILL}) \end{aligned}$$

Top-down dataflow

Any path

Keep track of instruction IDs

Defs of variables $\rightarrow$ GEN

Defs of variables $\rightarrow$ KILL

Generalizing Dataflow Analysis

❖ Transfer function

- » How information is changed by “something” (BB)
- » $OUT = GEN + (IN - KILL)$ /* forward analysis, e.g., rdefs */
- » $IN = GEN + (OUT - KILL)$ /* backward analysis, e.g., liveness */

❖ Meet function

- » How information from multiple paths is combined
- » $IN = \text{Union}(OUT(\text{predecessors}))$ /* forward analysis */
- » $OUT = \text{Union}(IN(\text{successors}))$ /* backward analysis */

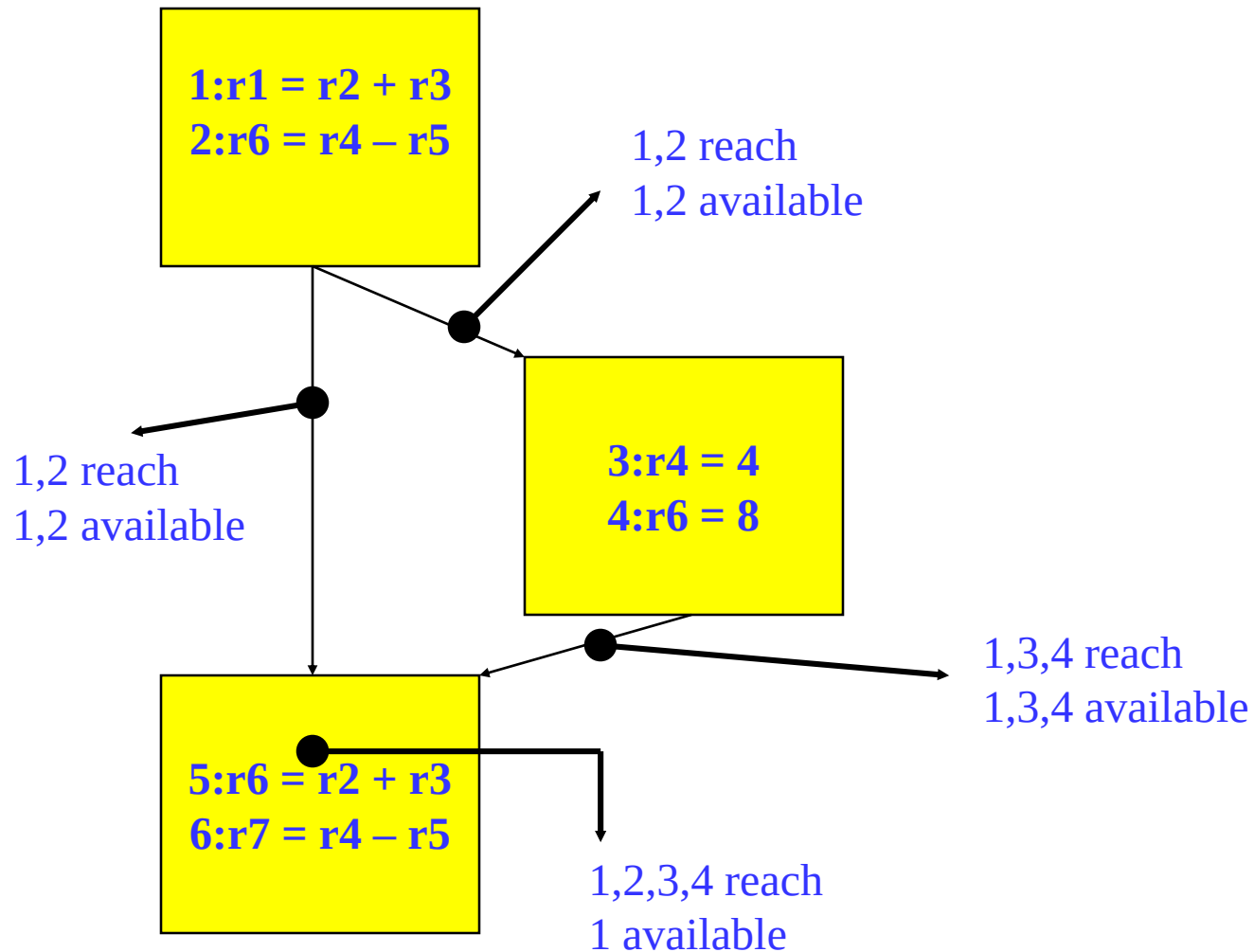
❖ Generalized dataflow algorithm

- » while (change)
 - change = false
 - for each BB
 - ◆ apply meet function
 - ◆ apply transfer functions
 - ◆ if any changes $\rightarrow$ change = true

What About All Path Problems?

- ❖ Up to this point
 - » Any path problems (maybe relations)
 - Definition reaches along some path
 - Some sequence of branches in which def reaches
 - Lots of defs of the same variable may reach a point
 - » Use of Union operator in meet function
- ❖ All-path: Definition guaranteed to reach
 - » Regardless of sequence of branches taken, def reaches
 - » Can always count on this
 - » Only 1 def can be guaranteed to reach
 - » Availability (as opposed to reaching)
 - Available definitions
 - Available expressions (could also have reaching expressions, but not that useful)

Reaching vs Available Definitions



Available Definition Analysis (Adefs)

- ❖ A definition d is available at a point p if along all paths from d to p , d is not killed
- ❖ Remember, a definition of a variable is killed between 2 points when there is another definition of that variable along the path
 - » $r1 = r2 + r3$ kills previous definitions of $r1$
- ❖ Algorithm
 - » Forward dataflow analysis as propagation occurs from defs downwards
 - » Use the Intersect function as the meet operator to guarantee the all-path requirement
 - » GEN/KILL/IN/OUT similar to reaching defs
 - Initialization of IN/OUT is the tricky part

Compute GEN/KILL Sets for each BB (Adefs)

Exactly the same as reaching defs !!!

```
for each basic block in the procedure, X, do  
  GEN(X) = 0  
  KILL(X) = 0  
  for each operation in sequential order in X, op, do  
    for each destination operand of op, dest, do  
      G = op  
      K = {all ops which define dest – op}  
      GEN(X) = G + (GEN(X) – K)  
      KILL(X) = K + (KILL(X) – G)  
    endfor  
  endfor  
endwhile
```


Compute IN/OUT Sets for all BBs (Adefs)

U = universal set of all operations in the Procedure

IN(0) = 0

OUT(0) = GEN(0)

for each basic block in procedure, W, (W != 0), do

 IN(W) = 0

 OUT(W) = U – KILL(W)

change = 1

while (change) do

 change = 0

for each basic block in procedure, X, do

 old_OUT = OUT(X)

 IN(X) = **Intersect**(OUT(Y)) for all predecessors Y of X

 OUT(X) = GEN(X) + (IN(X) – KILL(X))

if (old_OUT != OUT(X)) then

 change = 1

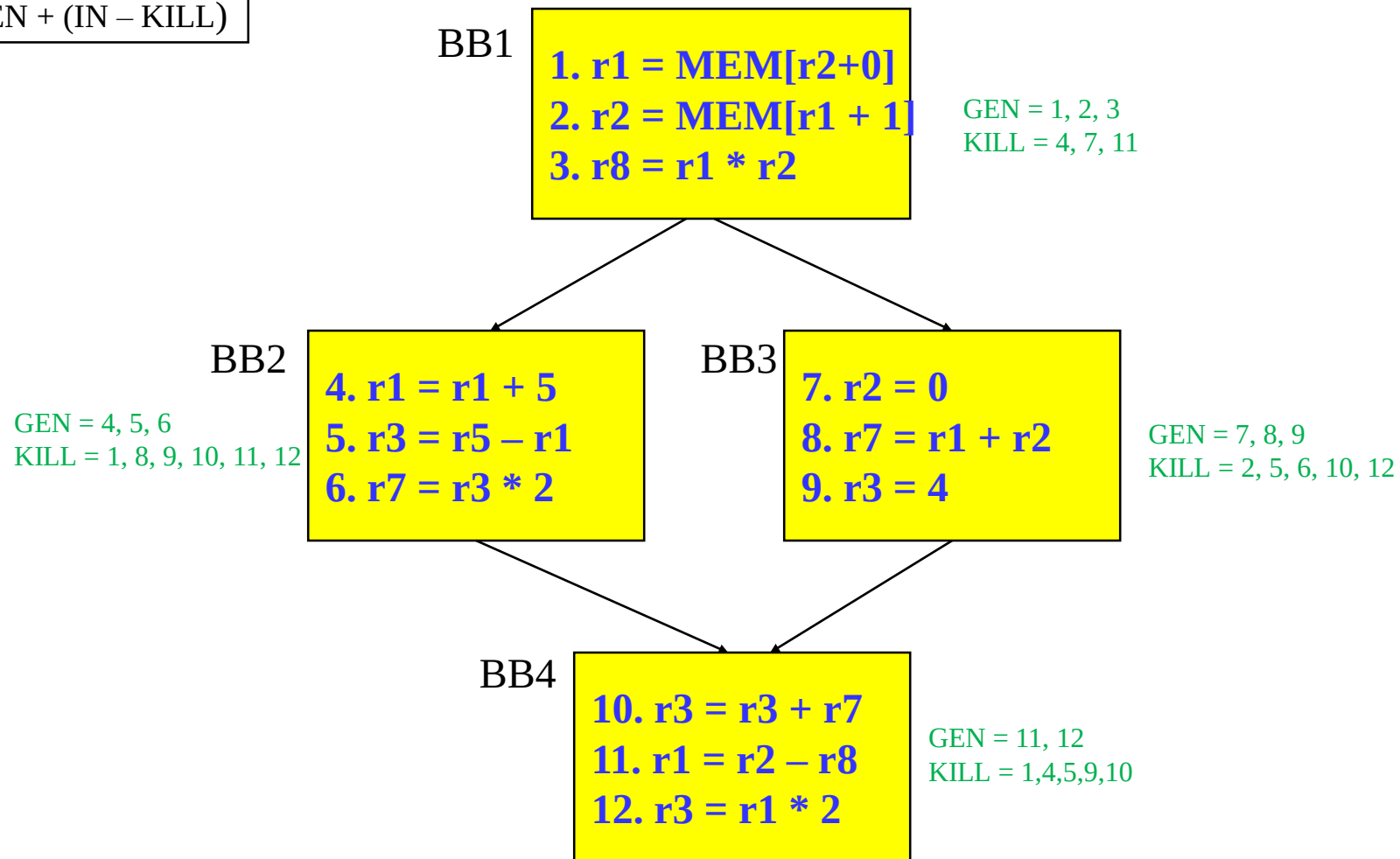
endif

endfor

endwhile

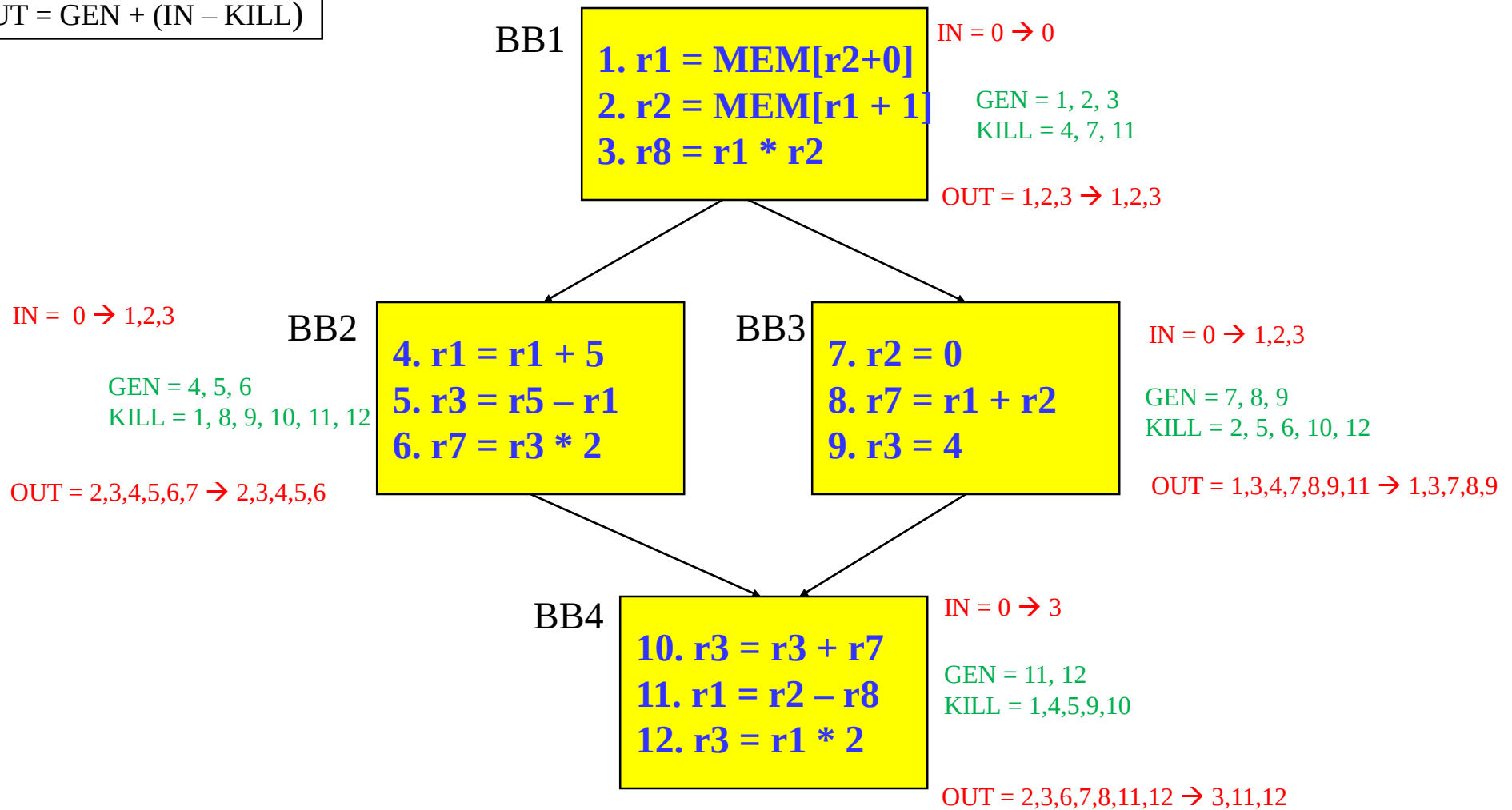
Example Adef Calculation

IN = Intersect(OUT(preds))
OUT = GEN + (IN - KILL)



Example Adef Calculation - Answer

IN = Intersect(OUT(preds))
OUT = GEN + (IN - KILL)



Available Expression Analysis (Aexprs)

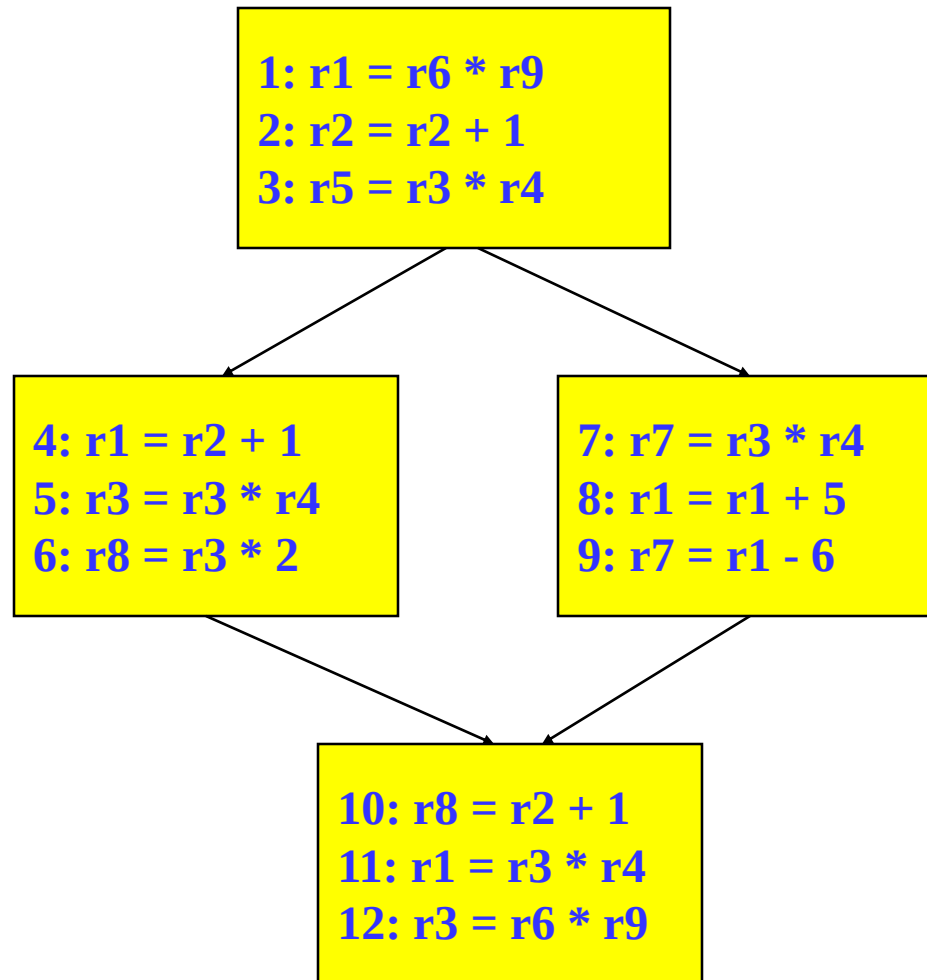
- ❖ An expression is a RHS of an operation
 - » $r2 = r3 + r4$, $r3+r4$ is an expression
- ❖ An expression e is available at a point p if along all paths from e to p , e is not killed
- ❖ An expression is killed between 2 points when one of its source operands are redefined
 - » $r1 = r2 + r3$ kills all expressions involving $r1$
- ❖ Algorithm
 - » Forward dataflow analysis as propagation occurs from defs downwards
 - » Use the Intersect function as the meet operator to guarantee the all-path requirement
 - » Looks exactly like adefs, except GEN/KILL/IN/OUT are the RHS's of operations rather than the LHS's

Computation of Aexpr GEN/KILL Sets

We can also formulate the GEN/KILL slightly differently so you do not need to break up instructions like “ $r2 = r2 + 1$ ”.

```
for each basic block in the procedure, X, do
  GEN(X) = 0
  KILL(X) = 0
  for each operation in sequential order in X, op, do
    K = 0
    for each destination operand of op, dest, do
      K += {all ops which use dest}
    endfor
    if (op not in K)
      G = op
    else
      G = 0
    GEN(X) = G + (GEN(X) – K)
    KILL(X) = K + (KILL(X) – G)
  endfor
endfor
```

Example Aexpr Calculation



Example Aexpr Calculation - Answer

IN/OUT sets

$A \rightarrow B$

A = initial state

B = after first iteration

IN = - $\rightarrow$ 1,3

GEN = 4, 6
KILL = 3, 5, 7, 8, 9, 11

OUT = 1,2,4,6,10,12 $\rightarrow$ 1,4,6

1: $r1 = r6 * r9$
2: $r2 = r2 + 1$
3: $r5 = r3 * r4$

IN = - $\rightarrow$ -

GEN = 1,3 (remember {1, 3} means {"r6*r9", "r3*r4"})
KILL = 2, 4, 8, 9, 10

OUT = 1,3,5,6,7,11,12 $\rightarrow$ 1,3

4: $r1 = r2 + 1$
5: $r3 = r3 * r4$
6: $r8 = r3 * 2$

7: $r7 = r3 * r4$
8: $r1 = r1 + 5$
9: $r7 = r1 - 6$

IN = - $\rightarrow$ 1,3

GEN = 7, 9
KILL = 8

OUT = 1,2,3,4,5,6,7,9,10,11,12 $\rightarrow$ 1,3,7,9

10: $r8 = r2 + 1$
11: $r1 = r3 * r4$
12: $r3 = r6 * r9$

IN = - $\rightarrow$ 1

GEN = 10, 12
KILL = 3, 5, 6, 7, 8, 9, 11

OUT = 1,2,4,10,12 $\rightarrow$ 1,10,12

Dataflow Summary

Liveness

OUT = Union(IN(succs))
IN = GEN + (OUT - KILL)

Bottom-up dataflow

Any path

Keep track of variables/registers

Uses of variables → GEN

Defs of variables → KILL

Reaching Definitions/DU/UD

IN = Union(OUT(preds))
OUT = GEN + (IN - KILL)

Top-down dataflow

Any path

Keep track of instruction IDs

Defs of variables → GEN

Defs of variables → KILL

Available Expressions

IN = Intersect(OUT(preds))
OUT = GEN + (IN - KILL)

Top-down dataflow

All path

Keep track of instruction IDs

Expressions of variables → GEN

Defs of variables → KILL

Available Definitions

IN = Intersect(OUT(preds))
OUT = GEN + (IN - KILL)

Top-down dataflow

All path

Keep track of instruction IDs

Defs of variables → GEN

Defs of variables → KILL