EECS 583 – Class 13 Software Pipelining

University of Michigan

October 18, 2023

Announcements + Reading Material

- ❖ Project discussion meetings Oct 23 (M), Oct 25 (W), Oct 26 (Th) (slots 10am-noon each day)
 - » Each group meets 10 mins with Aditya, Tarun, and I on Zoom, use GSI office hour link
 - » Action items
 - Form or join a team (3-5 people per team)
 - Use piazza to recruit additional group members or express your availability
 - Project areas, start looking for research papers, think about the specifics
 - » Google calendar signup available see piazza post by Aditya Please just sign up once!

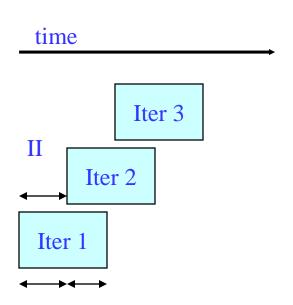
Project proposals

- » Due Monday, Oct 30, midnight
- » 1 paragraph summary of what you plan to work on
 - Topic, what are you going to do, what is the goal, 1-2 references
- » Email to me & Aditya & Tarun, cc all your group members

Today's class reading

- "Code Generation Schema for Modulo Scheduled Loops", B. Rau, M. Schlansker, and P. Tirumalai, MICRO-25, Dec. 1992.
- Next class (Fri, Oct 20)
 - » "Register Allocation and Spilling Via Graph Coloring," G. Chaitin, Proc. 1982 SIGPLAN Symposium on Compiler Construction, 1982.

Recap: Software Pipelining Terminology



<u>Initiation Interval</u> (II) = fixed delay between the start of successive iterations

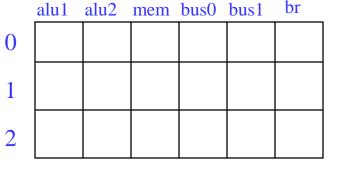
Each iteration can be divided into <u>stages</u> consisting of II cycles each

Number of stages in 1 iteration is termed the <u>stage count (SC)</u>

Takes SC-1 cycles to fill/drain the pipe

Recap: Resource Usage Legality

- Need to guarantee that
 - » No resource is used at 2 points in time that are separated by an interval which is a multiple of II
 - » I.E., within a single iteration, the same resource is never used more than 1x at the same time modulo II
 - » Known as modulo constraint, where the name modulo scheduling comes from
 - » Modulo reservation table solves this problem
 - To schedule an op at time T needing resource R
 - ◆ The entry for R at T mod II must be free
 - Mark busy at T mod II if schedule



II = 3

Dynamic Single Assignment (DSA) Form

Impossible to overlap iterations because each iteration writes to the same register. So, we'll have to remove the anti and output dependences.

Virtual rotating registers

- * Each register is an infinite push down array (<u>Expanded virtual reg or EVR</u>)
- * Write to top element, but can reference any element
- * Remap operation slides everything down \rightarrow r[n] changes to r[n+1]

A program is in DSA form if the same virtual register (EVR element) is never assigned to more than 1x on any dynamic execution path

```
1: r3 = load(r1)

2: r4 = r3 * 26

3: store (r2, r4)

4: r1 = r1 + 4

5: r2 = r2 + 4

6: p1 = cmpp (r1 < r9)

7: brct p1 Loop
```



DSA conversion

```
1: r3[-1] = load(r1[0])

2: r4[-1] = r3[-1] * 26

3: store (r2[0], r4[-1])

4: r1[-1] = r1[0] + 4

5: r2[-1] = r2[0] + 4

6: p1[-1] = cmpp (r1[-1] < r9)

remap r1, r2, r3, r4, p1

7: brct p1[-1] Loop
```

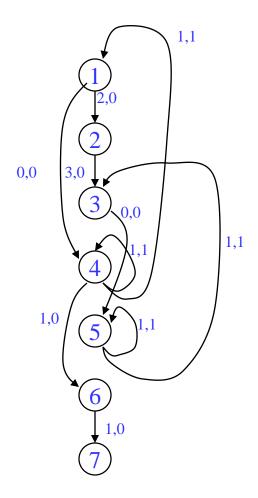
Loop Dependence Example

```
r1 = &Ar2 = &B
```

```
Loop: 1: r3[-1] = load(r1[0])
2: r4[-1] = r3[-1] * 26
3: store (r2[0], r4[-1])
4: r1[-1] = r1[0] + 4
5: r2[-1] = r2[0] + 4
6: p1[-1] = cmpp (r1[-1] < r9)
remap r1, r2, r3, r4, p1
7: brct p1[-1] Loop
```

In DSA form, there are no inter-iteration anti or output dependences!

Assume compiler can prove load and store are never dependent



<delay, distance>

Class Problem

Latencies: 1d = 2, st = 1, add = 1, cmpp = 1, br = 1

```
1: r1[-1] = load(r2[0])

2: r3[-1] = r1[1] - r1[2]

3: store (r3[-1], r2[0])

4: r2[-1] = r2[0] + 4

5: p1[-1] = cmpp (r2[-1] < 100)

remap r1, r2, r3

6: brct p1[-1] Loop
```

Draw the dependence graph showing both intra and inter iteration dependences 1

2

3

4

(5)

6

Class Problem Answer

Latencies: 1d = 2, st = 1, add = 1, cmpp = 1, br = 1

```
1: r1[-1] = load(r2[0])

2: r3[-1] = r1[1] - r1[2]

3: store (r3[-1], r2[0])

4: r2[-1] = r2[0] + 4

5: p1[-1] = cmpp (r2[-1] < 100)

remap r1, r2, r3

6: brct p1[-1] Loop
```

Draw the dependence graph showing both intra and inter iteration dependences



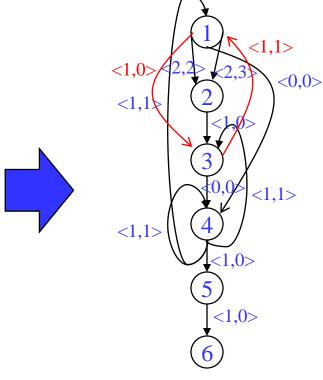












Red edges are memory anti and flow dependences

Instructions 1-5 have <0,0> control dependences to 6.

Minimum Initiation Interval (MII)

- Remember, II = number of cycles between the start of successive iterations
- Modulo scheduling requires a candidate II be selected before scheduling is attempted
 - » Try candidate II, see if it works
 - » If not, increase by 1, try again repeating until successful
- * MII is a lower bound on the II
 - » MII = Max(ResMII, RecMII)
 - » ResMII = resource constrained MII
 - Resource usage requirements of 1 iteration
 - » RecMII = recurrence constrained MII
 - Latency of the circuits in the dependence graph

ResMII

Concept: If there were no dependences between the operations, what is the shortest possible schedule?

Simple resource model

A processor has a set of resources R. For each resource r in R there is count(r) specifying the number of identical copies

```
ResMII = MAX (uses(r) / count(r))
for all r in R
```

uses(r) = number of times the resource is used in 1 iteration

In reality its more complex than this because operations can have multiple alternatives (different choices for resources it could be assigned to), but we will ignore this for now

ResMII Example

resources: 4 issue, 2 alu, 1 mem, 1 br latencies: add=1, mpy=3, ld = 2, st = 1, br = 1

```
1: r3 = load(r1)

2: r4 = r3 * 26

3: store (r2, r4)

4: r1 = r1 + 4

5: r2 = r2 + 4

6: p1 = cmpp (r1 < r9)

7: brct p1 Loop
```

```
ResMII = MAX (uses(r) / count(r))
```

uses(r) = number of times the resource is used in 1 iteration

ALU: used by 2, 4, 5, 6 \rightarrow 4 ops / 2 units = 2 Mem: used by 1, 3 \rightarrow 2 ops / 1 unit = 2 Br: used by 7

 \rightarrow 1 op / 1 unit = 1

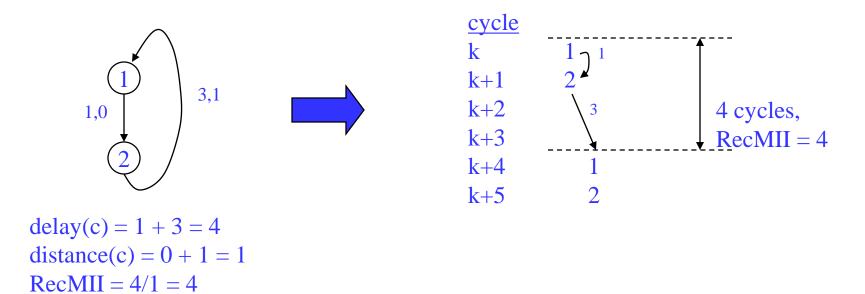
ResMII =
$$MAX(2,2,1) = 2$$

RecMII

Approach: Enumerate all irredundant elementary circuits in the dependence graph

RecMII = MAX
$$(delay(c) / distance(c))$$

delay(c) = total latency in dependence cycle c (sum of delays) distance(c) = total iteration distance of cycle c (sum of distances)



RecMII Example

```
1: r3 = load(r1)

2: r4 = r3 * 26

3: store (r2, r4)

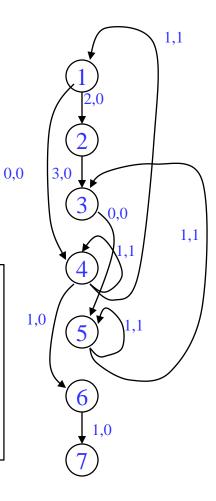
4: r1 = r1 + 4

5: r2 = r2 + 4

6: p1 = cmpp (r1 < r9)

7: bret p1 Loop
```

RecMII = MAX(delay(c) / distance(c))



 $4 \rightarrow 4: 1/1 = 1$

 $5 \rightarrow 5: 1/1 = 1$

 $4 \rightarrow 1 \rightarrow 4$: 1 / 1 = 1

 $5 \rightarrow 3 \rightarrow 5: 1/1 = 1$

RecMII = MAX(1,1,1,1) = 1

Then,

MII = MAX(ResMII, RecMII)

MII = MAX(2,1) = 2

<delay, distance>

Homework Problem

Latencies: 1d = 2, st = 1, add = 1, cmpp = 1, br = 1Resources: 1 ALU, 1 MEM, 1 BR

```
1: r1[-1] = load(r2[0])

2: r3[-1] = r1[1] - r1[2]

3: store (r3[-1], r2[0])

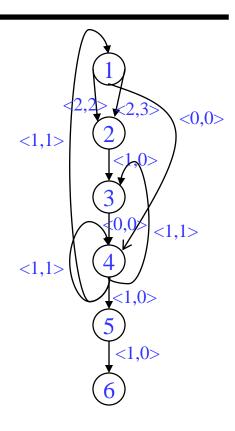
4: r2[-1] = r2[0] + 4

5: p1[-1] = cmpp (r2[-1] < 100)

remap r1, r2, r3

6: brct p1[-1] Loop
```

Calculate RecMII, ResMII, and MII



Modulo Scheduling Process

- Use list scheduling but we need a few twists
 - » II is predetermined starts at MII, then is incremented
 - » Cyclic dependences complicate matters
 - Estart/Priority/etc.
 - Consumer scheduled before producer is considered
 - ◆ There is a window where something can be scheduled!
 - Guarantee the repeating pattern
- * 2 constraints enforced on the schedule
 - » Each iteration begin exactly II cycles after the previous one
 - Each time an operation is scheduled in 1 iteration, it is tentatively scheduled in subsequent iterations at intervals of II
 - MRT used for this

Priority Function

Height-based priority worked well for acyclic scheduling, makes sense that it will work for loops as well

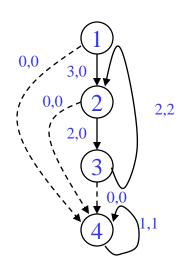
$$\frac{A cyclic:}{Height(X) =} \begin{tabular}{c} 0, if X has no successors \\ MAX & ((Height(Y) + Delay(X,Y)), otherwise \\ for all Y = succ(X) \\ \end{tabular}$$

$$EffDelay(X,Y) = Delay(X,Y) - II*Distance(X,Y)$$

Calculating Height

- 1. Insert pseudo edges from all nodes to branch with latency = 0, distance = 0 (dotted edges)
- 2. Compute II, For this example assume II = 2
- 3. HeightR(4) =
- 4. HeightR(3) =

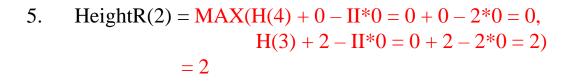
5. HeightR(2) =

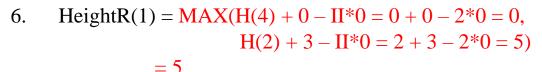


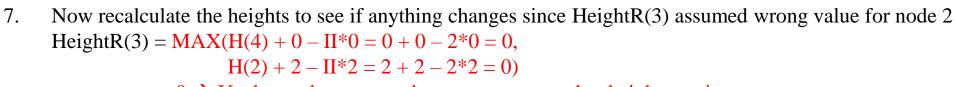
6. HeightR(1)

Calculating Height Solution

- 1. Insert pseudo edges from all nodes to branch with latency = 0, distance = 0 (dotted edges)
- 2. Compute II, For this example assume II = 2
- 3. HeightR(4) = H(4) + (1 II*1) (Assume H(4) is 0 since not calculated yet $0 + 1 2 = -1 \rightarrow 0$ (Always MAX answer with 0)
- 4. HeightR(3) = MAX(H(4) + 0 II*0 = 0 + 0 2*0 = 0, H(2) + 2 - II*2 = 0 + 2 - 2*2 = -2) Assume H(2) is 0 since not calculated yet = 0







2,2



The Scheduling Window

With cyclic scheduling, not all the predecessors may be scheduled, so a more flexible <u>earliest schedule time</u> is:

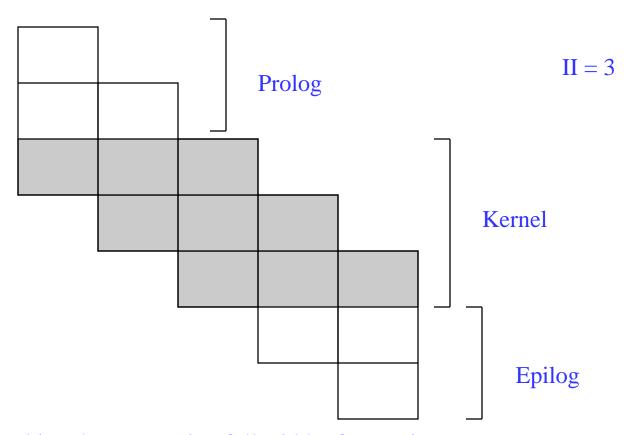
$$E(Y) = \underset{\text{for all } X = pred(Y)}{\text{MAX}} \\ \text{MAX (0, SchedTime(X) + EffDelay(X,Y)),} \\ \text{otherwise}$$

where EffDelay(X,Y) = Delay(X,Y) - II*Distance(X,Y)

Every II cycles a new loop iteration will be initialized, thus every II cycles the pattern will repeat. Thus, you only have to look in a window of size II, if the operation cannot be scheduled there, then it cannot be scheduled.

Latest schedule time(Y) =
$$L(Y) = E(Y) + II - 1$$

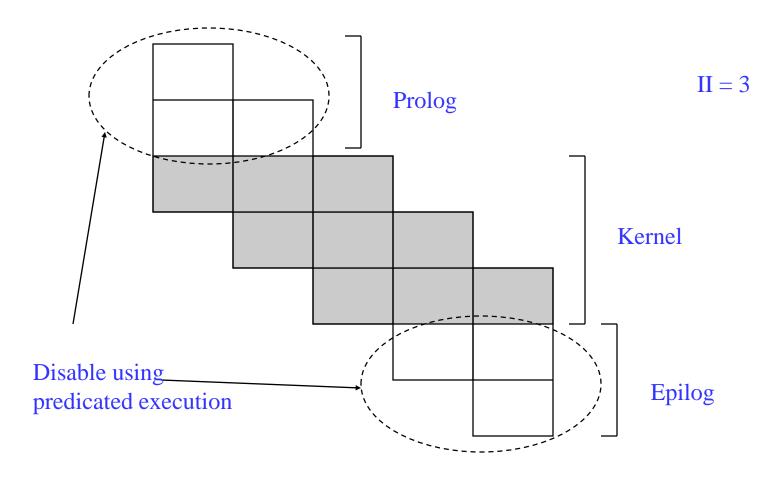
Loop Prolog and Epilog



Only the kernel involves executing full width of operations

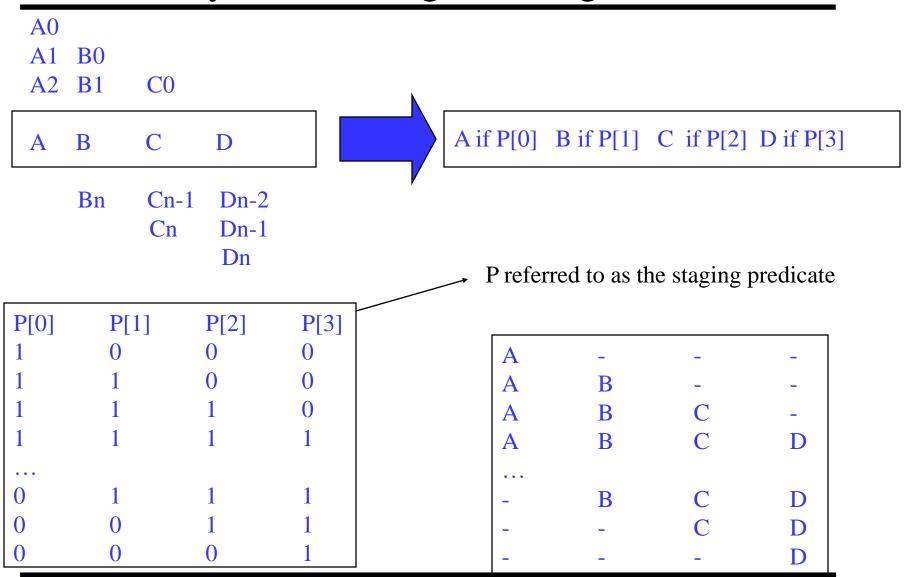
Prolog and epilog execute a subset (ramp-up and ramp-down)

Removing Prolog/Epilog



Execute loop kernel on every iteration, but for prolog and epilog selectively disable the appropriate operations to fill/drain the pipeline

Kernel-only Code Using Rotating Predicates



Modulo Scheduling Architectural Support

- Loop requiring N iterations
 - » Will take N + (S 1) where S is the number of stages
- 2 special registers created
 - » LC: loop counter (holds N)
 - » ESC: epilog stage counter (holds S)
- Software pipeline branch operations
 - \rightarrow Initialize LC = N, ESC = S in loop preheader
 - » All rotating predicates are cleared
 - » SWP-BR (BRF)
 - While LC > 0, decrement LC and RRB, P[0] = 1, branch to top of loop
 - This occurs for prolog and kernel
 - If LC = 0, then while ESC > 0, decrement RRB and write a 0 into P[0], and branch to the top of the loop
 - This occurs for the epilog

Execution History With LC/ESC

```
LC = 3, ESC = 3 /* Remember 0 relative!! */
Clear all rotating predicates
P[0] = 1

A if P[0]; B if P[1]; C if P[2]; D if P[3]; P[0] = BRF;
```

LC	ESC	P[0]	P[1]	P[2]	P[3]				
3	3	1	0	0	0	A			
			1	0	0	A	В		
	3	1	1	1	0	A	В	\mathbf{C}	
0	3	1	1	1	1	A	В	\mathbf{C}	D
0	2	0	1	1	1	_	В	\mathbf{C}	D
0	1	0	0	1	1	_	-	C	D
0	0	0	0	0	1	_	_	-	D

4 iterations, 4 stages, II = 1, Note 4 + 4 - 1 iterations of kernel executed

Modulo Scheduling Example

resources: 4 issue, 2 alu, 1 mem, 1 br

latencies: add=1, mpy=3, ld = 2, st = 1, br = 1

```
for (j=0; j<100; j++)

b[j] = a[j] * 26
```

Step1: Compute to loop into form that uses LC

$$LC = 99$$

Loop:

1: r3 = load(r1)

2: r4 = r3 * 26

3: store (r2, r4)

4: r1 = r1 + 4

 $5: r^2 = r^2 + 4$

6: p1 = cmpp (r1 < r9)

7: brct p1 Loop

Loop:



1: r3 = load(r1)

2: r4 = r3 * 26

3: store (r2, r4)

4: r1 = r1 + 4

5: r2 = r2 + 4

7: brlc Loop

resources: 4 issue, 2 alu, 1 mem, 1 br

latencies: add=1, mpy=3, ld = 2, st = 1, br = 1

Step 2: DSA convert

LC = 99LC = 991: r3 = load(r1)Loop: 1: r3[-1] = load(r1[0])Loop: 2: r4 = r3 * 262: r4[-1] = r3[-1] * 263: store (r2, r4) 3: store (r2[0], r4[-1]) 4: r1 = r1 + 44: r1[-1] = r1[0] + 45: r2[-1] = r2[0] + 45: r2 = r2 + 4remap r1, r2, r3, r4 7: brlc Loop 7: brlc Loop

resources: 4 issue, 2 alu, 1 mem, 1 br

latencies: add=1, mpy=3, ld = 2, st = 1, br = 1

Step3: Draw dependence graph Calculate MII

LC = 99

Loop:

```
1: r3[-1] = load(r1[0])

2: r4[-1] = r3[-1] * 26

3: store (r2[0], r4[-1])

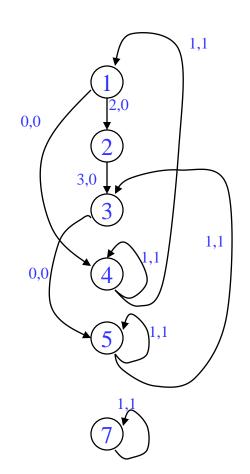
4: r1[-1] = r1[0] + 4

5: r2[-1] = r2[0] + 4

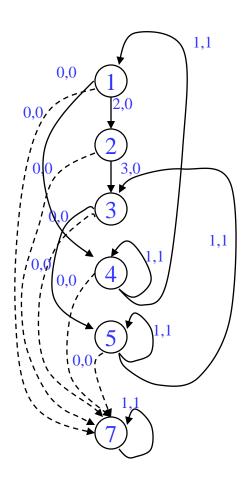
remap r1, r2, r3, r4

7: brlc Loop
```

No memory dependences since load and store refer to distinct arrays



 $\begin{aligned} & RecMII = 1 \\ & RESMII = 2 \\ & MII = 2 \end{aligned}$



Step 4 – Calculate priorities (MAX height to pseudo stop node)

<u>Iter1</u>	Iter2			
1: $H = 5$	1: $H = 5$			
2: H = 3	2: H = 3			
3: H = 0	3: H = 0			
4: H = 0	4: H = 4			
5: H = 0	5: H = 0			
7: H = 0	7: H = 0			

resources: 4 issue, 2 alu, 1 mem, 1 br

latencies: add=1, mpy=3, ld = 2, st = 1, br = 1

Schedule brlc at time II - 1

LC = 99

Loop:

1: r3[-1] = load(r1[0]) 2: r4[-1] = r3[-1] * 26 3: store (r2[0], r4[-1]) 4: r1[-1] = r1[0] + 4 5: r2[-1] = r2[0] + 4 remap r1, r2, r3, r4 7: brlc Loop

Unrolled Schedule Rolled Schedule 0 () 5 6 mem br alu0 alu1 0 **MRT** X

Step6: Schedule the highest priority op

Op1:
$$E = 0$$
, $L = 1$
Place at time 0 (0 % 2)

$$LC = 99$$

```
1: r3[-1] = load(r1[0])

2: r4[-1] = r3[-1] * 26

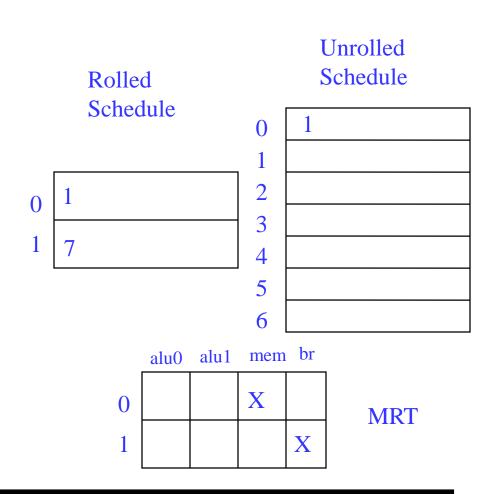
3: store (r2[0], r4[-1])

4: r1[-1] = r1[0] + 4

5: r2[-1] = r2[0] + 4

remap r1, r2, r3, r4

7: brlc Loop
```



Step7: Schedule the highest priority op

Op4:
$$E = 0$$
, $L = 1$
Place at time 0 (0 % 2)

$$LC = 99$$

```
1: r3[-1] = load(r1[0])

2: r4[-1] = r3[-1] * 26

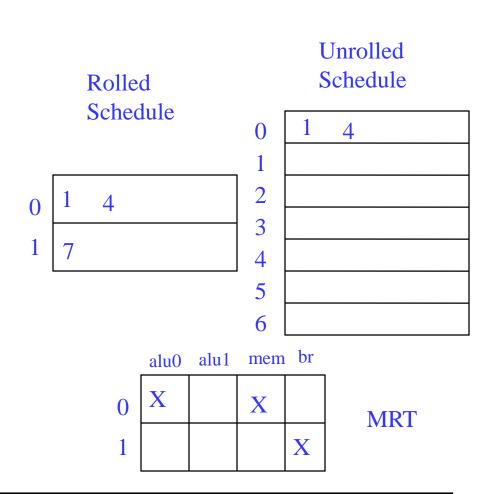
3: store (r2[0], r4[-1])

4: r1[-1] = r1[0] + 4

5: r2[-1] = r2[0] + 4

remap r1, r2, r3, r4

7: brlc Loop
```



Step8: Schedule the highest priority op

Op2:
$$E = 2$$
, $L = 3$
Place at time 2 (2 % 2)

$$LC = 99$$

```
1: r3[-1] = load(r1[0])

2: r4[-1] = r3[-1] * 26

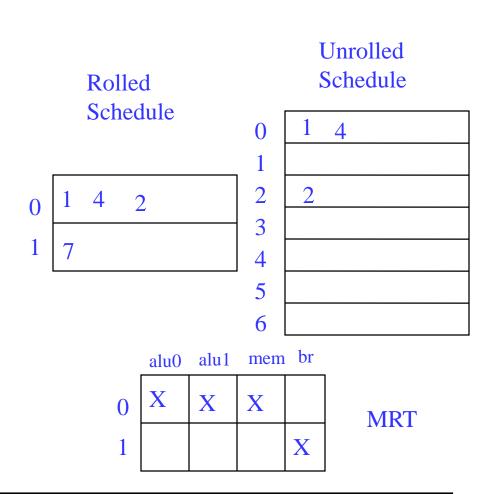
3: store (r2[0], r4[-1])

4: r1[-1] = r1[0] + 4

5: r2[-1] = r2[0] + 4

remap r1, r2, r3, r4

7: brlc Loop
```



Step9: Schedule the highest priority op

Op3:
$$E = 5$$
, $L = 6$
Place at time 5 (5 % 2)

$$LC = 99$$

```
1: r3[-1] = load(r1[0])

2: r4[-1] = r3[-1] * 26

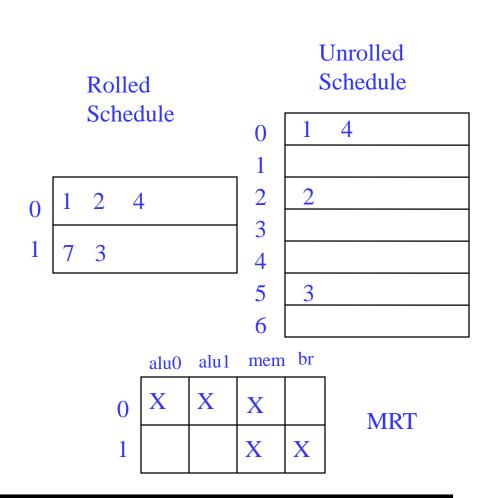
3: store (r2[0], r4[-1])

4: r1[-1] = r1[0] + 4

5: r2[-1] = r2[0] + 4

remap r1, r2, r3, r4

7: brlc Loop
```



Step10: Schedule the highest priority op

Op5:
$$E = 5$$
, $L = 6$
Place at time 5 (5 % 2)

$$LC = 99$$

```
1: r3[-1] = load(r1[0])

2: r4[-1] = r3[-1] * 26

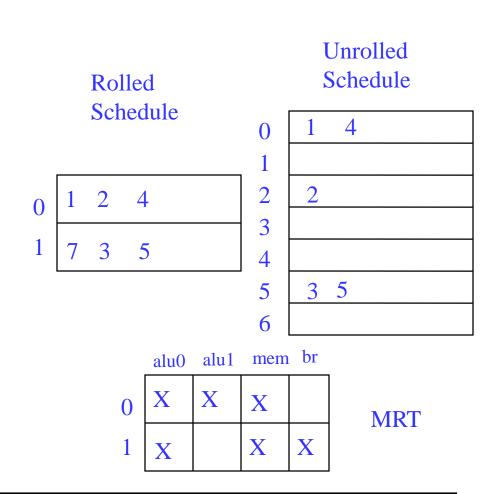
3: store (r2[0], r4[-1])

4: r1[-1] = r1[0] + 4

5: r2[-1] = r2[0] + 4

remap r1, r2, r3, r4

7: brlc Loop
```



Step11: calculate ESC, SC = ceiling(max unrolled sched length / ii) unrolled sched time of branch = rolled sched time of br + (ii*esc)

$$SC = 6 / 2 = 3$$
, $ESC = SC - 1$
time of $br = 1 + 2*2 = 5$
 $LC = 99$

```
1: r3[-1] = load(r1[0])

2: r4[-1] = r3[-1] * 26

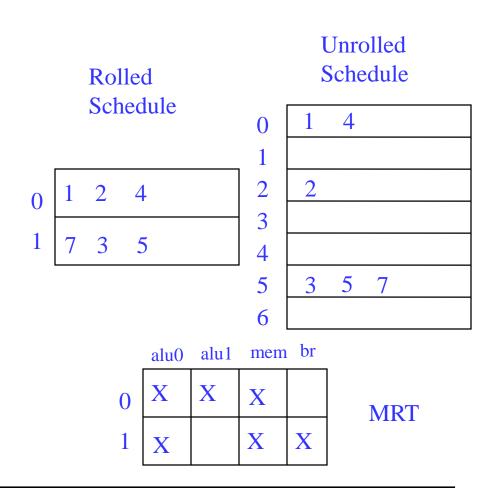
3: store (r2[0], r4[-1])

4: r1[-1] = r1[0] + 4

5: r2[-1] = r2[0] + 4

remap r1, r2, r3, r4

7: brlc Loop
```



Finishing touches - Sort ops, initialize ESC, insert BRF and staging predicate, initialize staging predicate outside loop

$$LC = 99$$

 $ESC = 2$
 $p1[0] = 1$

Loop:

```
1: r3[-1] = load(r1[0]) if p1[0]

2: r4[-1] = r3[-1] * 26 if p1[1]

4: r1[-1] = r1[0] + 4 if p1[0]

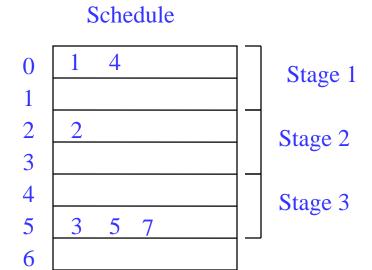
3: store (r2[0], r4[-1]) if p1[2]

5: r2[-1] = r2[0] + 4 if p1[2]

7: brlc Loop if p1[2]
```

Staging predicate, each successive stage increment the index of the staging predicate by 1, stage 1 gets px[0]

Unrolled



Example – Dynamic Execution of the Code

```
time: ops executed
         LC = 99
         ESC = 2
                                                                 0:1,4
         p1[0] = 1
                                                                 2: 1,2,4
                                                             3:
  Loop: 1: r3[-1] = load(r1[0]) if p1[0]
        2: r4[-1] = r3[-1] * 26 if p1[1]
                                                                 4: 1,2,4
        4: r1[-1] = r1[0] + 4 if p1[0]
                                                             5: 3,5,7
        3: store (r2[0], r4[-1]) if p1[2]
                                                                 6: 1,2,4
        5: r2[-1] = r2[0] + 4 \text{ if } p1[2]
                                                             7: 3,5,7
        7: brlc Loop if p1[2]
                                                                 198: 1,2,4
                                                             199: 3,5,7
Total time = II(num\_iteration + num\_stages - 1)
                                                                 200: 2
           = 2(100 + 3 - 1) = 204 cycles
                                                             201: 3,5,7
                                                                 202: -
                                                                 203 3,5,7
```