# EECS 583 – Class 13 Software Pipelining

*University of Michigan*

*October 18, 2023*

# Announcements + Reading Material

- Project discussion meetings Oct 23 (M), Oct 25 (W), Oct 26 (Th) (slots 10am-noon each day)
	- » Each group meets 10 mins with Aditya, Tarun, and I on Zoom, use GSI office hour link
	- » Action items
		- Form or join a team (3-5 people per team)
		- Use piazza to recruit additional group members or express your availability
		- Project areas, start looking for research papers, think about the specifics
	- » Google calendar signup available see piazza post by Aditya Please just sign up once!
- $\div$  Project proposals
	- » Due Monday, Oct 30, midnight
	- » 1 paragraph summary of what you plan to work on
		- Topic, what are you going to do, what is the goal, 1-2 references
	- » Email to me & Aditya & Tarun, cc all your group members
- **❖** Today's class reading
	- » "Code Generation Schema for Modulo Scheduled Loops", B. Rau, M. Schlansker, and P. Tirumalai, MICRO-25, Dec. 1992.
- $\div$  Next class (Fri, Oct 20)
	- » "Register Allocation and Spilling Via Graph Coloring," G. Chaitin, Proc. 1982 SIGPLAN Symposium on Compiler Construction, 1982.

# Recap: Software Pipelining Terminology



Initiation Interval  $(II)$  = fixed delay between the start of successive iterations

Each iteration can be divided into stages consisting of II cycles each

Number of stages in 1 iteration is termed the stage count (SC)

Takes SC-1 cycles to fill/drain the pipe

## Recap: Resource Usage Legality

- **❖** Need to guarantee that
	- » No resource is used at 2 points in time that are separated by an interval which is a multiple of II
	- » I.E., within a single iteration, the same resource is never used more than 1x at the same time modulo II
	- » Known as modulo constraint, where the name modulo scheduling comes from
	- » Modulo reservation table solves this problem
		- To schedule an op at time T needing resource R
			- The entry for R at T mod II must be free
		- Mark busy at T mod II if schedule

$$
II=3
$$



# Dynamic Single Assignment (DSA) Form

Impossible to overlap iterations because each iteration writes to the same register. So, we'll have to remove the anti and output dependences.

Virtual rotating registers

- \* Each register is an infinite push down array (Expanded virtual reg or EVR)
- \* Write to top element, but can reference any element
- \* Remap operation slides everything down  $\rightarrow$  r[n] changes to r[n+1]

A program is in DSA form if the same virtual register (EVR element) is never assigned to more than 1x on any dynamic execution path



### Loop Dependence Example

 $r1 = \& A$  $r2 = \&B$ 

1:  $r3[-1] = load(r1[0])$ 2:  $r4[-1] = r3[-1] * 26$ 3: store (r2[0], r4[-1])  $4: r1[-1] = r1[0] + 4$ 5:  $r2[-1] = r2[0]+4$ 6:  $p1[-1] = \text{cmp}(r1[-1] < r9)$ remap r1, r2, r3, r4, p1 7: brct p1[-1] Loop Loop:

> In DSA form, there are no inter-iteration anti or output dependences!

Assume compiler can prove load and store are never dependent



<delay, distance>

#### Class Problem

Latencies:  $Id = 2$ ,  $st = 1$ ,  $add = 1$ ,  $cmp = 1$ ,  $br = 1$ 

```
1: r1[-1] = load(r2[0])2: r3[-1] = r1[1] - r1[2]3: store (r3[-1], r2[0])
4: r2[-1] = r2[0]+45: p1[-1] = \text{cmp}(r2[-1] < 100)remap r1, r2, r3
6: brct p1[-1] Loop
```
Draw the dependence graph showing both intra and inter iteration dependences



#### Class Problem Answer

Latencies:  $Id = 2$ ,  $st = 1$ ,  $add = 1$ ,  $cmp = 1$ ,  $br = 1$ 

1:  $r1[-1] = load(r2[0])$ 2:  $r3[-1] = r1[1] - r1[2]$ 3: store (r3[-1], r2[0]) 4:  $r2[-1] = r2[0]+4$ 5:  $p1[-1] = \text{cmp}(r2[-1] < 100)$ remap r1, r2, r3 6: brct p1[-1] Loop

Draw the dependence graph showing both intra and inter iteration dependences



# Minimum Initiation Interval (MII)

- Remember,  $II =$  number of cycles between the start of successive iterations
- Modulo scheduling requires a candidate II be selected before scheduling is attempted
	- » Try candidate II, see if it works
	- » If not, increase by 1, try again repeating until successful
- MII is a lower bound on the II
	- $\rightarrow$  MII = Max(ResMII, RecMII)
	- $\rightarrow$  ResMII = resource constrained MII
		- Resource usage requirements of 1 iteration
	- $\rightarrow$  RecMII = recurrence constrained MII
		- Latency of the circuits in the dependence graph

# ResMII

Concept: If there were no dependences between the operations, what is the the shortest possible schedule?

Simple resource model

A processor has a set of resources R. For each resource r in R there is  $count(r)$  specifying the number of identical copies

 $ResMI = MAX$  (uses(r) / count(r)) for all r in R

 $uses(r) = number of times the resource is used in 1 iteration$ 

In reality its more complex than this because operations can have multiple alternatives (different choices for resources it could be assigned to), but we will ignore this for now

### ResMII Example

resources: 4 issue, 2 alu, 1 mem, 1 br latencies: add=1, mpy=3,  $ld = 2$ , st = 1, br = 1

1:  $r3 = load(r1)$ 2:  $r4 = r3 * 26$ 3: store (r2, r4) 4:  $r1 = r1 + 4$ 5:  $r2 = r2 + 4$ 6:  $p1 = \text{cmp}(r1 < r9)$ 7: brct p1 Loop

 $ResMI = MAX$  (uses(r) / count(r))  $uses(r) = number of times the resource is used$ in 1 iteration

ALU: used by 2, 4, 5, 6  $\rightarrow$  4 ops / 2 units = 2 Mem: used by 1, 3  $\rightarrow$  2 ops / 1 unit = 2 Br: used by 7  $\rightarrow$  1 op / 1 unit = 1

 $ResMI = MAX(2,2,1) = 2$ 

# RecMII

Approach: Enumerate all irredundant elementary circuits in the dependence graph

 $RecMI = MAX$  (delay(c) / distance(c)) for all c in C

 $delay(c) = total latency in dependence cycle c (sum of delays)$  $distance(c) = total iteration distance of cycle c (sum of distances)$ 



### RecMII Example



<delay, distance>

### Homework Problem

Latencies:  $Id = 2$ ,  $st = 1$ ,  $add = 1$ ,  $cmp = 1$ ,  $br = 1$ Resources: 1 ALU, 1 MEM, 1 BR

1:  $r1[-1] = load(r2[0])$  $2: r3[-1] = r1[1] - r1[2]$ 3: store (r3[-1], r2[0])  $4: r2[-1] = r2[0]+4$ 5:  $p1[-1] = \text{cmp}(r2[-1] < 100)$ remap r1, r2, r3 6: brct p1[-1] Loop

Calculate RecMII, ResMII, and MII



# Modulo Scheduling Process

- **↓** Use list scheduling but we need a few twists
	- » II is predetermined starts at MII, then is incremented
	- » Cyclic dependences complicate matters
		- Estart/Priority/etc.
		- Consumer scheduled before producer is considered
			- There is a window where something can be scheduled!
	- » Guarantee the repeating pattern
- $\div$  2 constraints enforced on the schedule
	- » Each iteration begin exactly II cycles after the previous one
	- » Each time an operation is scheduled in 1 iteration, it is tentatively scheduled in subsequent iterations at intervals of II
		- MRT used for this

# Priority Function

Height-based priority worked well for acyclic scheduling, makes sense that it will work for loops as well



 $EffDelay(X, Y) = Delay(X, Y) - II^*Distance(X, Y)$ 

# Calculating Height

- 1. Insert pseudo edges from all nodes to branch with latency = 0, distance =  $0$  (dotted edges)
- 2. Compute II, For this example assume  $II = 2$
- 3. HeightR(4) =
- 4. Height $R(3) =$

5. HeightR $(2)$  =

6. HeightR(1)



### Calculating Height Solution

- 1. Insert pseudo edges from all nodes to branch with latency  $= 0$ , distance  $= 0$  (dotted edges)
- 2. Compute II, For this example assume  $II = 2$
- 3. HeightR(4) = H(4) + (1  $II^*1$ ) (Assume H(4) is 0 since not calculated yet

 $0 + 1-2 = -1 \rightarrow 0$  (Always MAX answer with 0)

4. HeightR(3) =  $MAX(H(4) + 0 - II^*0 = 0 + 0 - 2^*0 = 0,$  $H(2) + 2 - H^*2 = 0 + 2 - 2^*2 = -2$ Assume H(2) is 0 since not calculated yet  $= 0$ 0,0

5. HeightR(2) = MAX(H(4) + 0 – II\*0 = 0 + 0 – 2\*0 = 0, H(3) + 2 – II\*0 = 0 + 2 – 2\*0 = 2) = 2

- 6. HeightR(1) = MAX(H(4) + 0 II\*0 = 0 + 0 2\*0 = 0,  $H(2) + 3 - H^*0 = 2 + 3 - 2^*0 = 5$  $= 5$
- 7. Now recalculate the heights to see if anything changes since HeightR(3) assumed wrong value for node 2  $HeightR(3) = MAX(H(4) + 0 - II^{*}0 = 0 + 0 - 2^{*}0 = 0,$  $H(2) + 2 - H^*2 = 2 + 2 - 2^*2 = 0$  $= 0 \rightarrow$  Unchanged, so no need to compute any other heights again

1

2

3,0

0,0

2,0

3

0,0

4

1,1

2,2

# The Scheduling Window

With cyclic scheduling, not all the predecessors may be scheduled, so a more flexible earliest schedule time is:

$$
E(Y) = \underset{\text{for all } X = \text{pred}(Y)}{\text{MAX}}
$$
 0, if X is not scheduled  
MAX (0, SchedTime(X) + EffDelay(X,Y)),  
otherwise  
otherwise

where  $EffDelay(X, Y) = Delay(X, Y) - II^*Distance(X, Y)$ 

Every II cycles a new loop iteration will be initialized, thus every II cycles the pattern will repeat. Thus, you only have to look in a window of size II, if the operation cannot be scheduled there, then it cannot be scheduled.

Latest schedule time(Y) =  $L(Y) = E(Y) + II - 1$ 

# Loop Prolog and Epilog



Only the kernel involves executing full width of operations

Prolog and epilog execute a subset (ramp-up and ramp-down)

# Removing Prolog/Epilog



Execute loop kernel on every iteration, but for prolog and epilog selectively disable the appropriate operations to fill/drain the pipeline

# Kernel-only Code Using Rotating Predicates



# Modulo Scheduling Architectural Support

- Loop requiring N iterations
	- Will take  $N + (S 1)$  where S is the number of stages
- $\div$  2 special registers created
	- » LC: loop counter (holds N)
	- » ESC: epilog stage counter (holds S)
- ❖ Software pipeline branch operations
	- $\rightarrow$  Initialize LC = N, ESC = S in loop preheader
	- » All rotating predicates are cleared
	- » SWP-BR (BRF)
		- While  $LC > 0$ , decrement LC and RRB,  $P[0] = 1$ , branch to top of loop
			- This occurs for prolog and kernel
		- If  $LC = 0$ , then while  $\text{ESC} > 0$ , decrement RRB and write a 0 into P[0], and branch to the top of the loop
			- $\bullet$  This occurs for the epilog

# Execution History With LC/ESC



Clear all rotating predicates

 $P[0] = 1$ 

A if P[0]; B if P[1]; C if P[2]; D if P[3]; P[0] = BRF;



4 iterations, 4 stages,  $II = 1$ , Note  $4 + 4 - 1$  iterations of kernel executed

### Modulo Scheduling Example

resources: 4 issue, 2 alu, 1 mem, 1 br latencies: add=1, mpy=3, ld = 2, st = 1, br = 1

for  $(j=0; j<100; j++)$  $b[j] = a[j] * 26$ 

Step1: Compute to loop into form that uses LC

 $LC = 99$ 

![](_page_24_Figure_5.jpeg)

resources: 4 issue, 2 alu, 1 mem, 1 br latencies: add=1, mpy=3, ld = 2, st = 1, br = 1

![](_page_25_Figure_2.jpeg)

![](_page_25_Figure_3.jpeg)

resources: 4 issue, 2 alu, 1 mem, 1 br latencies: add=1, mpy=3,  $ld = 2$ , st = 1, br = 1

Step3: Draw dependence graph Calculate MII

![](_page_26_Figure_3.jpeg)

![](_page_27_Figure_1.jpeg)

Step 4 – Calculate priorities (MAX height to pseudo stop node)

![](_page_27_Picture_125.jpeg)

resources: 4 issue, 2 alu, 1 mem, 1 br latencies:  $add=1$ , mpy=3,  $ld = 2$ ,  $st = 1$ ,  $br = 1$  Schedule brlc at time II - 1

![](_page_28_Figure_3.jpeg)

Step6: Schedule the highest priority op

Op1:  $E = 0, L = 1$ Place at time  $0 (0 % 2)$ 

Loop:

![](_page_29_Figure_4.jpeg)

Unrolled

Step7: Schedule the highest priority op

Op4:  $E = 0, L = 1$ Place at time  $0 (0 % 2)$ 

Loop:

![](_page_30_Figure_4.jpeg)

Unrolled

Step8: Schedule the highest priority op

Op2:  $E = 2$ ,  $L = 3$ Place at time 2 (2 % 2)

 $LC = 99$ 

1:  $r3[-1] = load(r1[0])$ 2:  $r4[-1] = r3[-1] * 26$ 3: store (r2[0], r4[-1])  $4: r1[-1] = r1[0] + 4$ 5:  $r2[-1] = r2[0]+4$ remap r1, r2, r3, r4 7: brlc Loop Loop:

![](_page_31_Figure_5.jpeg)

Step9: Schedule the highest priority op

Op3:  $E = 5$ ,  $L = 6$ Place at time 5 (5 % 2)

Loop:

![](_page_32_Figure_4.jpeg)

Unrolled

Step10: Schedule the highest priority op

Op5:  $E = 5$ ,  $L = 6$ Place at time 5 (5 % 2)

Loop:

![](_page_33_Figure_4.jpeg)

Unrolled

Step11: calculate ESC,  $SC = ceiling(max$  unrolled sched length / ii) unrolled sched time of branch = rolled sched time of br  $+$  (ii\*esc)

![](_page_34_Figure_2.jpeg)

Finishing touches - Sort ops, initialize ESC, insert BRF and staging predicate, initialize staging predicate outside loop

 $LC = 99$  $\text{ESC} = 2$  $p1[0] = 1$ 

Loop:

1:  $r3[-1] = load(r1[0])$  if  $p1[0]$ 2:  $r4[-1] = r3[-1] * 26 \text{ if } p1[1]$ 4:  $r1[-1] = r1[0] + 4$  if  $p1[0]$ 3: store (r2[0], r4[-1]) if p1[2] 5:  $r2[-1] = r2[0] + 4$  if  $p1[2]$ 7: brlc Loop if p1[2]

Staging predicate, each successive stage increment the index of the staging predicate by 1, stage 1 gets px[0]

> Unrolled Schedule

![](_page_35_Figure_7.jpeg)

# Example – Dynamic Execution of the Code

![](_page_36_Picture_102.jpeg)