

Bitwidth Analysis with Application to Silicon Compilation

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Abstract

This paper introduces *Bitwise*, a compiler that minimizes the bitwidth — the number of bits used to represent each operand — for both integers and pointers in a program. By propagating static information both forward and backward in the program dataflow graph, *Bitwise* frees the programmer from declaring bitwidth invariants in cases where the compiler can determine bitwidths automatically. Because loop instructions comprise the bulk of dynamically executed instructions, *Bitwise* incorporates sophisticated loop analysis techniques for identifying bitwidths. We find a rich opportunity for bitwidth reduction in modern multimedia and streaming application workloads. For new architectures that support sub-word data-types, we expect that our bitwidth reductions will save power and increase processor performance.

This paper also applies our analysis to silicon compilation, the translation of programs into custom hardware, to realize the full benefits of bitwidth reduction. We describe our integration of *Bitwise* with the DeepC Silicon Compiler. By taking advantage of bitwidth information during architectural synthesis, we reduce silicon real estate by 15 – 86%, improve clock speed by 3 – 249%, and reduce power by 46 – 73%. The next era of general purpose and reconfigurable architectures should strive to capture a portion of these gains.

1 Introduction

The pioneers of the computing revolution described in Steven Levy's book *Hackers* competed to make the best use of every precious architectural resource. They hand-tuned each program statement and operand. In contrast, today's programmers pay little attention to small details such as the bitwidth (*e.g.*, 8, 16, 32) of data-types used in their programs. For instance, in the C programming language, it is common to use a 32-bit integer data-type to represent a single Boolean variable. We could dismiss this shift in emphasis as a consequence of abundant computing resources and expensive programmer time. However, there is another his-

torical reason: as processor architectures have evolved, the use of smaller operands eventually has provided no performance gains. Datapaths became wider, but the processor's entire data path was exercised regardless of operand size. In fact, the additional overhead of packing and unpacking words — now only to save space in memory — actually reduces performance.

1.1 A New Era: Software-Exposed Bits

Three new compilation targets for high-level languages are re-invigorating the need to conserve bits. Each of these architectures expose subword control. The first is the rejuvenation of SIMD architectures for multimedia workloads. These architectures include Intel's MultiMedia eXtension (MMX) and Motorola's AltiVec [20, 25]. For example, in AltiVec, data paths are used to operate on 8, 16, 32, or 64 bit quantities.

The second class of compilation targets consists of embedded systems which can effectively *turn off* bit slices [7]. The static information determined at compile time can be used to specify which portions of a datapath are on or off during program execution. Alternatively, for more traditional architectures this same information can be used to predict power consumption by determining which datapath bits will change over time.

The third class of compilation targets comprises fine-grain substrates such as gate and function-level reconfigurable architectures — including Field Programmable Gate Arrays (FPGAs) — and custom hardware, such as standard cell ASIC designs. In both cases, architectural synthesis is required to support high-level languages. There has been a recent surge of both industrial and academic interest in developing new reconfigurable architectures [18].

Unfortunately, there are no available commercial compilers that can effectively target any of these new architectures. Programmers have been forced to revert to writing low-level code. MMX libraries are written in assembly in order to expose the most sub-word parallelism. In the Verilog and VHDL hardware description languages, the burden of bitwidth specification lies on the programmer. To compete in the marketplace, designers must choose the minimum operand bitwidth for smaller, faster, and more energy-efficient circuits.

1.2 Benefits of Automating Bitwidth Specification

Automatic bitwidth analysis relieves the programmer of the burden of identifying and specifying derivable bitwidth in-

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formation. The programmer can work at a higher level of abstraction. In contrast, explicitly choosing the smallest data size for each operand is not only tedious, but also error prone. These programs are less malleable since a simple change may require hand propagation of bitwidth information across a large segment of the program. Furthermore, some of the bitwidth information may be dependent on a particular architecture or implementation technology, making programs less portable.

Even if the programmer explicitly specifies operand sizes in languages that allow it, bitwidth analysis can still be valuable. For example, bitwidth analysis can be used to verify that specified operand sizes do not violate program invariants – *e.g.*, array bounds.

1.3 The Bitwise Compiler

Bitwise minimizes the bitwidth required for each static operation and each static assignment of the program. The scope of Bitwise includes fixed-point arithmetic, bit manipulation, and Boolean operations. It uses additional sources of information such as type casts, array bounds, and loop iteration counts to refine variable bitwidths. We have implemented Bitwise within the SUIF compiler infrastructure [26].

In many cases, Bitwise is able to analyze the bitwidth information as accurately as the bitwidth information gathered from run-time profiles. On average we reduce the size of program scalars by 12 – 80% and program arrays by up to 93%.

1.4 Application to Silicon Compilation

In this paper we apply bitwidth analysis to the task of silicon compilation. In particular, we have integrated Bitwise with the *DeepC Silicon Compiler*. The compiler produces gate-level netlists from input programs written in C and FORTRAN. We compare end-to-end performance results for this system both with and without our bitwidth optimizations. The results demonstrate that the analysis techniques perform well in a real system. Our experiments show that Bitwise favorably impacts area, speed, and power of the resulting circuits.

1.5 Contributions

We summarize this paper’s contributions as follows:

- We formulate bitwidth analysis as a value range propagation problem.
- We introduce a suite of bitwidth extraction techniques that seamlessly perform bi-directional propagation.
- We formulate an algorithm to accurately find bitwidth information in the presence of loops by calculating closed-form solutions.
- We implement the compiler and demonstrate that the compile-time analysis can approach the accuracy of run-time profiling.
- We incorporate the analysis in a silicon compiler and demonstrate that bitwidth analysis impacts area, speed, and power consumption of a synthesized circuit.

1.6 Organization

The rest of the paper is organized as follows. Section 2 defines the bitwidth analysis problem. Bitwise’s implementation and our algorithms are described in Section 3. Section 4 provides empirical evidence of the success of Bitwise. Next, Section 5 describes the DeepC Silicon Compiler and Section 6 discusses the impact that bitwidth analysis has on silicon compilation. Finally, we present related work in Section 7 and conclude in Section 8.

2 Bitwidth Analysis

The goal of bitwidth analysis is to analyze each static instruction in a program to determine the narrowest return type that still retains program correctness. This information can in turn be used to find the minimum number of bits needed to represent each program operand.

Library calls, I/O routines, and loops make static bitwidth analysis challenging. In the presence of these constructs, we may have to make conservative assumptions about an operand’s bitwidth. Nevertheless, with careful static analysis, it is possible to infer bitwidth information.

Structures such as arrays and conditional statements provide us with valuable bitwidth information. For instance, we can use the bounds of an array to set an index variable’s maximum bitwidth. Other program constructs such as AND-masks, divides, right shifts, type promotions, and Boolean operations are also invaluable for reducing bitwidths.

```
(1) index += indexTable[delta];
(2) if ( index < 0 ) index = 0;
(3) if ( index > 88 ) index = 88;
(4) step = stepsizeTable[index];
(5)
(6) if ( bufferstep ) {
(7)     outputbuffer = (delta << 4) & 0xf0;
(8) } else {
(9)     *outp++ = (delta & 0x0f) |
(10)             (outputbuffer & 0xf0);
(11) }
(12) bufferstep = !bufferstep;
```

Figure 1: Sample C code used to illustrate the fundamentals of the analysis. This code fragment was taken from the loop of `adpcm_coder` in the `adpcm` multimedia benchmark.

The C code fragment in Figure 1 exhibits several such constructs. This code, which is an excerpt of the *adpcm* benchmark presented later in this paper, is typical of important multimedia applications. Each line of code in the figure is annotated with a line number to facilitate the following discussion.

Assume that we do not know the precise value of `delta`, referenced in lines (1), (7), and (9). Because it is used as an index variable in line (1), we know that its value is confined by the base and bounds of `indexTable`¹. Though we still do not know `delta`’s precise value, by restricting the range of values that it can assume, we effectively reduce the number of bits needed to represent it. In a similar fashion, the code

¹Our analysis assumes that the program being analyzed is error free. If the program exhibits bound violations, arithmetic underflow, or arithmetic overflow, changing operand bitwidths may alter its functionality.

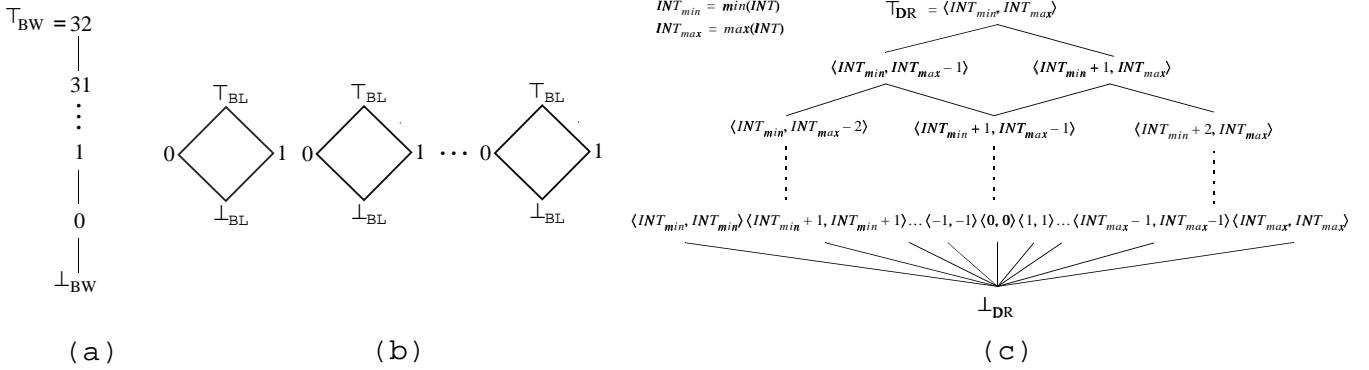


Figure 2: Three alternative data structures for bitwidth analysis. The lattice in (a) represents the number of bits needed to represent a variable. The lattice in (b) represents a vector of bits that can be assigned to a variable, and the lattice in (c) represents the range of values that can be assigned to a variable.

on lines (2) and (3) ensure that `index`'s value is restricted to be between 0 and 88.

The AND-mask on line (7) ensures that `outputbuffer`'s value is no greater than `0xf0`. Similarly, we can infer that the assignment to `*outp` on line (9) is no greater than `0xff` (`0x0f | 0xf0`).

Finally, we know that `bufferstep`'s value is either `true` or `false` after the assignment on line (12) because it is the result of the Boolean `not` (!) operation.

3 Bitwise Implementation

This section describes the infrastructure and algorithms of *Bitwise*, a compiler that performs bitwidth analysis. Bitwise uses SSA as its intermediate form. It performs a *numerical* data flow analysis. Because we are solving for absolute numerical bitwidths, the more complex symbolic analysis is not needed [23].

We continue by comparing the candidate data-flow lattices that were considered in our implementation.

3.1 Candidate Lattices

We considered three candidate data-structures for propagating the numerical information of our analysis. Figure 2 visually depicts the *lattice* that corresponds to each data-structure.

Propagating the bitwidth of each variable: Figure 2(a) is the most straightforward structure. While this representation permits an easy implementation, it does not yield accurate results on arithmetic operations. When applying the lattice's transfer function, incrementing an 8-bit number always produces a 9-bit resultant, even though it may likely need only 8-bits. In addition, only the most significant bits of a variable are candidates for bit-elimination.

Maintaining a bit vector for each variable: Figure 2(b) is a more complex representation, requiring the composition of several smaller bit-lattices [8, 21]. Although this lattice allows elimination of arbitrary bits from a variable's representation, it does not support precise arithmetic analysis. As an example of eliminating arbitrary bits, consider a particular variable that is assigned the values from the set $\{010_2, 100_2, 110_2\}$. After analysis, the variable's bit-vector will be $[TT0]$, indicating that we can eliminate the least

significant bit. Like the first data structure, the arithmetic is imprecise because the analysis must still conservatively assume that every addition results in a carry.

Propagating data-ranges: Figure 2(c) is the final lattice we considered. This lattice is also the implementation chosen in the compiler. A data-range is a single connected subrange of the integers from a lower bound to an upper bound (e.g., $[1..100]$ or $[-50..50]$). Thus a data-range keeps track of a variable's lower and upper bounds. Because only a single range is used to represent all possible values for a variable, this representation does not permit the elimination of low-order bits. However, it does allow us to operate on arithmetic expressions precisely. Technically, this representation maps bitwidth analysis to the more general *value range propagation problem*. Value range propagation is known to be useful in value prediction, branch prediction, constant propagation, procedure cloning, and program verification [19, 23].

For the *Bitwise* compiler we chose to propagate data-ranges, not only because of their generality, but also because most important applications use arithmetic and will benefit from their exact precision. Unlike a regular set union, we define the data-range union operation (\sqcup) to be the union over the single connected subrange of the integers where $\langle a_l, a_h \rangle \sqcup \langle b_l, b_h \rangle = \langle \min(a_l, b_l), \max(a_h, b_h) \rangle$. We also define the data-range intersection operation (\sqcap) to be the set of all integers in both subranges where $\langle a_l, a_h \rangle \sqcap \langle b_l, b_h \rangle = \langle \max(a_l, b_l), \min(a_h, b_h) \rangle$. The intersection of two non-overlapping data-ranges yields the value \perp_{DR} , which can be used to identify likely programmer errors (e.g., array bound violations). Additionally, note that the value \perp_{DR} , a part of the lattice, represents values that cannot be statically determined, or values that can potentially utilize the entire range of the integer type.

3.2 Data-Range Propagation

As concluded in the last section, our Bitwise implementation propagates data-ranges. These data-ranges can be propagated both *forward* and *backward* over a program's control flow graph. Figure 4 shows a subset of the transfer functions for propagation. The forward propagated values in the figure are subscripted with a down arrow (\Downarrow), and the backward propagated values with an up arrow (\Uparrow). In general the transfer functions take one or two data-ranges as input and return a single data-range.

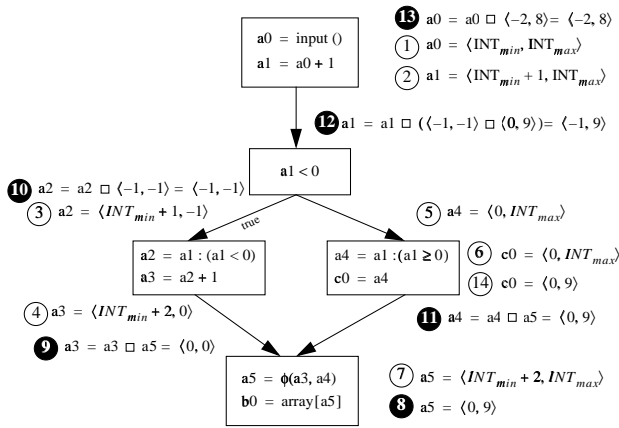


Figure 3: Forward and backward data-range propagation. The numbers denote the order of evaluation. Application of forward propagation rules are shown in white, while backward propagation rules are shown in black. We use `array`'s bounds information to tighten the ranges of some of the variables.

Initially, all of the variables in the SSA graph are initialized to the maximum range allowable for their type. Informally, forward propagation traverses the SSA graph in breadth-first order, applying the transfer functions for forward propagation. Because there is one unique assignment for each variable in SSA form, we can restrict a variable's data-range if the result of its assignment is less than the maximum data-range of its type.

To more accurately gather data-ranges, we extend standard SSA form to include the notion of range-refinement functions. For each node that is control dependent, a function is created which refines the range of control variables based on the outcome of the branch test. Consider the SSA graph shown in Figure 3. Range-refinement functions have been inserted in each of the nodes directly following the branch test. By taking control-dependent information into account, these functions facilitate a more accurate collection of data-ranges. Thus, if the branch in the figure is taken, we know that a_1 's value is less than zero. Similarly, a_1 's value has to be greater than or equal to zero if the branch is not taken.

Forward propagation allows us to identify a significant number of unused bits, sometimes achieving near optimal results. However, additional minimization can be achieved by integrating *backward propagation*². For example, when we find a data-range that has stepped outside of known array bounds, we can back-propagate this new reduced data-range to instructions that have already used its deprecated value to compute their results. Beginning at the node where the boundary violation is found, we propagate the reduced data-range in a reverse breadth-first order, using the transfer functions for backward propagation. This halts when either the graph's entry node is reached, or when a fixed point is reached. Forward propagation resumes from this point.

Forward and backward propagation steps have been annotated on the graph in Figure 3 to ease the following discussion. The numbers on the figure chronologically order each step. The step numbers in black represent the backward propagation of data-ranges. Without backward prop-

agation we arrive at the following data-ranges:

$$\begin{aligned}
 a_0 &= \langle INT_{min}, INT_{max} \rangle \\
 a_1 &= \langle INT_{min} + 1, INT_{max} \rangle \\
 a_2 &= \langle INT_{min} + 1, -1 \rangle \\
 a_3 &= \langle INT_{min} + 2, 0 \rangle \\
 a_4 &= \langle 0, INT_{max} \rangle \\
 a_5 &= \langle INT_{min} + 2, INT_{max} \rangle \\
 c_0 &= \langle 0, INT_{max} \rangle
 \end{aligned}$$

Let us assume we know that the length of the array, `array`, is 10 from its declaration. We can now substantially reduce the data-ranges of the variables in the graph with backward propagation. We use `array`'s bound information to clamp a_3 's data-range to $\langle 0, 9 \rangle$. We then propagate this value backward in reverse breadth-first order using the transfer functions for backward propagation. In our example, propagating a_3 's new value backward yields the following new data-ranges:

$$\begin{aligned}
 a_0 &= \langle -2, 8 \rangle \\
 a_1 &= \langle -1, 9 \rangle \\
 a_2 &= \langle -1, -1 \rangle \\
 a_3 &= \langle 0, 0 \rangle \\
 a_4 &= \langle 0, 9 \rangle \\
 a_5 &= \langle 0, 9 \rangle \\
 c_0 &= \langle 0, 9 \rangle
 \end{aligned}$$

Reverse propagation can halt after a_0 's range is determined (step 13). Because c_0 uses the results of a variable that has changed, we have to traverse the graph in the forward direction again. After we confine c_0 's data-range to $\langle 0, 9 \rangle$ we will have reached a fixed point and the analysis will be complete.

In this example we see that data-range propagation *subsumes* constant propagation; we can replace all occurrences of a_3 with the constant value 0.

3.3 Loops

Optimization of loop instructions is crucial — they usually comprise the bulk of dynamic instructions. Traditional data flow analysis techniques iterate over back edges in the graph until a fixed point is reached. However, this technique will saturate even the simplest loop-carried arithmetic expression. That is, because the method does not take into account any static knowledge of loop bounds, such an expression will eventually saturate at the maximum range of its type.

Because many important applications use loop-carried arithmetic expressions, a new approach is required. To this end, our implementation of the *Bitwise* compiler identifies loops and finds closed-form solutions. We ease loop identification in SSA form by converting all ϕ -functions that occur in loop headers to μ -functions [10]. These functions have exactly two operands; the first operand is defined outside the loop, and the second operand is loop carried. We take advantage of these properties when finding closed-form solutions.

3.3.1 Closed-Form Solutions

To find the closed-form solution to loop-carried expressions, we use the techniques introduced by Gerlek et al. [10]. These techniques allow us to identify and classify *sequences* in loops. A *sequence* is a mutually dependent group of instructions. In other words, a sequence is a strongly connected component (SCC) of the program's dependence graph. We

²SSA form is not an efficient form for performing backward propagation[13]. Bitwise currently reverts to standard data-flow analysis techniques *only* when analyzing in the reverse direction. If efficiency in the less common case of backward propagation is a concern, our form of SSA could readily be converted to SSI form, which was designed for bi-directional data-flow analyses[1].

(a)	$b_{\downarrow} = \langle b_l, b_h \rangle$ $c_{\downarrow} = \langle c_l, c_h \rangle$ $a_{\uparrow} = \langle a_l, a_h \rangle$	\downarrow $a = b + c$ \downarrow	$b_{\uparrow} = b_{\downarrow} \sqcap \langle a_l - c_h, a_h - c_l \rangle$ $c_{\uparrow} = c_{\downarrow} \sqcap \langle a_l - b_h, a_h - b_l \rangle$ $a_{\downarrow} = a_{\uparrow} \sqcap \langle b_l + c_l, b_h + c_h \rangle$
(b)	$b_{\downarrow} = \langle b_l, b_h \rangle$ $c_{\downarrow} = \langle c_l, c_h \rangle$ $a_{\uparrow} = \langle a_l, a_h \rangle$	\downarrow $a = b - c$ \downarrow	$b_{\uparrow} = b_{\downarrow} \sqcap \langle a_l + c_l, a_h + c_h \rangle$ $c_{\uparrow} = c_{\downarrow} \sqcap \langle a_l + b_l, a_h + b_h \rangle$ $a_{\downarrow} = a_{\uparrow} \sqcap \langle b_l - c_h, b_h - c_l \rangle$
(c)	$b_{\downarrow} = \langle b_l, b_h \rangle$ $c_{\downarrow} = \langle c_l, c_h \rangle$ $a_{\uparrow} = \langle a_l, a_h \rangle$	\downarrow $a = b \ \& \ c$ \downarrow	$b_{\uparrow} = b_{\downarrow}$ $c_{\uparrow} = c_{\downarrow}$ $a_{\downarrow} = a_{\uparrow} \sqcap \langle -2^{n-1}, 2^{n-1} - 1 \rangle$, <i>where</i> $n = \min(\text{bitwidth}(b_{\downarrow}), \text{bitwidth}(c_{\downarrow}))$
(d)	$b = \langle b_l, b_h \rangle$ $a = \langle a_l, a_h \rangle$	\downarrow $a = b$ \downarrow	$b = b \sqcap a$ $a = a \sqcap b$
(e)	$x = \langle a_l, a_h \rangle$	\downarrow $\{x_l \leq x \leq x_h\}$ \downarrow	$x = x \sqcap \langle x_l, x_h \rangle$
(f)	$x^b_{\downarrow} = \langle b_l, b_h \rangle$ $x^c_{\downarrow} = \langle c_l, c_h \rangle$ $x^a_{\uparrow} = \langle a_l, a_h \rangle$	x^b x^c $\swarrow \quad \nwarrow$ $x^a = \phi(x^b, x^c)$ \downarrow	$x^b_{\uparrow} = x^b_{\downarrow} \sqcap x^a_{\uparrow}$ $x^c_{\uparrow} = x^c_{\downarrow} \sqcap x^a_{\uparrow}$ $x^a_{\downarrow} = x^a_{\uparrow} \sqcap (x^b_{\downarrow} \sqcup x^c_{\downarrow})$
(g)	$x^a_{\downarrow} = \langle a_l, a_h \rangle$ $y_{\downarrow} = \langle y_l, y_h \rangle$ $x^b_{\uparrow} = \langle b_l, b_h \rangle$ $x^c_{\uparrow} = \langle c_l, c_h \rangle$	x^a \downarrow $x^a < y$ $\swarrow \quad \searrow$ $x^b \quad x^c$	$x^a_{\uparrow} = x^a_{\downarrow} \sqcap (x^b_{\uparrow} \sqcup x^c_{\uparrow})$ $x^b_{\downarrow} = x^a_{\downarrow} \sqcap x^b_{\uparrow} \sqcap \langle a_l, y_h - 1 \rangle$ $x^c_{\downarrow} = x^a_{\downarrow} \sqcap x^c_{\uparrow} \sqcap \langle y_l, a_h \rangle$

Figure 4: A selected subset of transfer functions for bi-directional data-range propagation. Intermediate results on the left are inputs to the transfer functions on the right. The variables in the figure are subscripted with the direction in which they are computed. The transfer function in (a) adds two data-ranges, and (b) subtracts two data-ranges. Both of these functions assume saturating semantics which will confine the resulting range to be within the bounds of the type on which they operate. The AND-masking operation for signed data-types in (c) returns a data-range corresponding to the smallest of its two inputs. It makes use of the *bitwidth* function which returns the number of bits needed to represent the data-range. The type-casting operation shown in (d) confines the resulting range to be within the range of the smaller data-type. Because variables are initialized to the largest range that can be represented by their types, ranges are propagated seamlessly, even in the case of type conversion. The function in (e) is applied when we know that a value must be within a specified range. For instance, this rule is applied to limit the data-range of a variable that is indexing into a static array. Note that rules (d) and (e) are not directionally dependent. Rule (f) is applied at merge points, and rule (g) is applied at locations where control-flow splits. In rule (g), we see that x^b corresponds to an occurrence of x^a such that $x^a < y$. We can use this information to refine the range of x^b based on the outcome of the branch test, $x^a < y$.

```

LSS: InstList List  $\times$  Int Cur  $\times$  Range Trip
       $\times$  SSAVar Sentinel  $\rightarrow$  Range  $\times$  Int
Range R  $\leftarrow$   $\langle 0, 0 \rangle$ 
Int i  $\leftarrow$  Cur
while i < |List| do
  if List[i] has form  $\langle a_k = \mu(a_l, a_m)$  with tripcount tc  $\rangle$  then
     $a_k \leftarrow a_l$ 
     $\langle R, i \rangle \leftarrow$  LSS(List, i + 1, tc  $\times_{DR}$  Trip,  $a_m$ )
  else if List[i] has form  $\langle a_k = a_l$  linop C  $\rangle$  then
     $a_k \leftarrow a_l$  linop C  $\times_{DR}$  Trip
  else if List[i] has form  $\langle a_k = \phi(a_l, a_m) \rangle$  then
     $a_k \leftarrow a_l \sqcup a_m$ 
  if  $a_k =$  Sentinel then
    return  $\langle a_k, i \rangle$ 
  i  $\leftarrow$  i + 1
return  $\langle R, i \rangle$ 

```

```

CLASSIFYSEQUENCE: InstList List  $\rightarrow$  Void
Range Val
if |List| = 1 then
  EVALUATEINST(List[0])
else
  SeqType  $\leftarrow$  SEQUENCETYPE(List)
  if SeqType = Linear then
     $\langle Val, x \rangle \leftarrow$  LSS(List, 0,  $\langle 1, 1 \rangle$ , NIL)
    foreach Inst I  $\in$  List do
       $a_k \leftarrow$  Val where  $a_k$  is destination of I
  else if ...
  :
  else if SeqType =  $\perp_{sequence}$ 
    FIX(List, MaxIters)

```

Figure 5: Pseudocode for the algorithm that classifies and solves closed-form solutions of commonly occurring sequences. The SEQUENCETYPE function identifies the type of sequence we are considering. Based on the sequence type, we can invoke the appropriate solver. We provide pseudocode for the linear sequence solver (LSS). The FIX function attempts to find a fixed-point solution for unidentifiable sequences.

can examine the instructions of the sequence to try and find a closed-form solution to the sequence.

Thus, the algorithm begins by finding all the sequences in the loop. We then order them according to dependences between the sequences. At this point we can classify each sequence in turn. The algorithm for classifying sequences is shown in Figure 5.

A sequence’s type is identified by examining its composition of instructions. This functionality corresponds to the SEQUENCETYPE procedure called in Figure 5. We provide a sketch of our approach in Section 3.3.2.

Once we have determined the type of sequence the component represents, the algorithm invokes a *solver* to compute the sequence’s closed-form solution. For each type of sequence, a different method is needed to compute the closed-form solution. If no sequence is identified, the algorithm resorts to fixed point iteration up to a user defined maximum.

3.3.2 Sequence Identification

We sketch our sequence identification algorithm as follows. First, we create a partial order on the types of expressions we wish to identify. We employ the *Expression* lattice (Figure 6) to order various expressions according to set containment. For example, *linear* sequences are the composition of an induction variable and loop invariants, and polynomial sequences are the composition of loop invariants and linear sequences. The top of the lattice ($\top_{sequence}$) represents

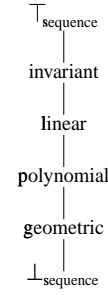


Figure 6: A lattice that orders sequences according to set containment.

an undetermined expression, and the bottom of the lattice ($\perp_{sequence}$) represents all possible expressions.

Next, we create transfer functions for each instruction type in the source language. A transfer function, which operates on the lattice, is implemented as a table that is indexed by the expression types of its source operands. The destination operand is then tagged with the expression type dictated by the transfer function.

We proceed by classifying the sequence based on the types of its expressions and its composition of ϕ - and μ -functions. For instance, a linear sequence can contain any number of loads, stores, additions, or subtractions of invariant values. In addition, linear sequences must have at least one μ -function³. Remember that μ -functions define loop headers, and thus denote the start of all non-trivial sequences. Trivial sequences contain exactly one instruction, and thus, the sequence itself represents the closed-form solution.

3.3.3 Sequence Example

Figure 7 is an example loop and Figure 8 is its corresponding SSA graph. In this example all μ -functions are annotated with the loop’s tripcount ($\langle 0, 64 \rangle$). While we can restrict the range of the loop’s induction variable without the annotations, knowing the tripcount allows us to analyze other unrelated sequences.

The next step is to find all of the strongly connected components in the loop’s body and create the sequence dependence graph. The sequence dependence graph for the loop in Figure 7 is shown in Figure 9.

We then analyze each of the sequences according to the dependence graph. The algorithm classifies the sequence based on the types of its constituent expressions. The component below, from the example, is determined to be a linear sequence because it contains a μ -function and a linear-type expression:

<i>Sequence</i>	<i>Sum</i>
$\text{addr1} = \mu(\text{addr0}, \text{addr2})$	$\langle 0, 0 \rangle$
$\text{addr2} = \text{addr1} + 4$	$\langle 4, 4 \rangle \times \langle 0, 64 \rangle = \langle 0, 256 \rangle$

Based on the tripcount of the μ -function ($\langle 0, 64 \rangle$) and addr0 ’s range ($\langle 0, 0 \rangle$), the function LSS in Figure 5 finds the maximum range that any of the variables in the linear sequence can possibly assume. The function steps through the sequence summing up all of the invariants. This sum is

³Gerlek et al. process inner-loops first and provide mechanisms to propagate closed-form solutions to enclosing loop nests. We consider all loops simultaneously.

```

addr = 0;
even = 0;
line = 0;
for (word = 0; word < 64; word++) {
    addr = addr + 4;
    even = !even;
    line = addr & 0x1c;
}

```

Figure 7: Example loop.

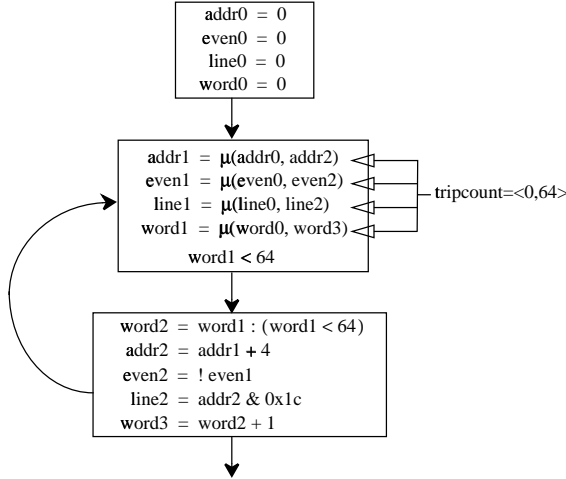


Figure 8: SSA graph corresponding to example loop.

multiplied by the total number of times the loop in question will be executed. For this example, the function determines the maximum range to be $\langle 0, 256 \rangle$. At this point we set all of the destination variables in the sequence to this range, and the sequence is solved.

Obtaining this conservative result is simpler than finding the precise range for each variable in the sequence. Because there is typically little variation between ranges of destination variables in the same sequence, this method works well in practice.

Unlike linear sequences, not all sequences are readily identifiable. In such cases we iterate over the sequence until a fixed point is reached. For example, the sequence labeled (2) in Figure 9, will reach a fixed point after only two iterations. Not surprisingly, sequences that contain Boolean operations, AND-masks, left-shifts, or divides – all common in multimedia kernels – can quickly reach a fixed-point. The following section addresses the cases when a fixed-point is not reached quickly.

3.3.4 Termination

For cases in which we cannot find a closed-form solution, lattice height could lead to seemingly boundless iteration. For example, by traversing back-edges in the control flow graph, it could take nearly 2^{32} iterations to reach a fixed point for typical 32-bit integers.

In order to solve this problem, we consider what happens to a data-range after applying a transfer function to a static assignment. The data-range either:

- reaches a fixed point, or
- monotonically decreases.

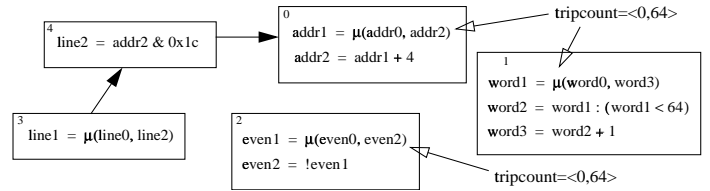


Figure 9: A dependence graph of sequences corresponding to the code in Figure 7. The sequences labeled (3) and (4) are trivial sequences. In other words, the sequences are themselves the closed-form solution. Using the tripcount of the loop, we can calculate the final ranges for the linear sequences labeled (0) and (1). Though we do not identify Boolean sequences such as the one marked (2), they quickly reach a fixed point.

Thus it is possible to add a user-defined limit to the number of iterations. When iteration is limited, the resulting data-range will be an improved but potentially sub-optimal solution.

3.4 Arrays and Pointers

In traditional SSA form, arrays and pointers are not renamed. Special extensions to SSA form have been proposed that provide element-level data flow information for arrays [15]. While such extensions to SSA form can potentially provide more accurate data-range information, for bitwidth analysis it is more convenient to conservatively treat arrays as scalars. The following sections describe further implementation details related to arrays and pointers.

3.4.1 Arrays

When treating an array as a scalar, if an array is modified we must insert a new ϕ -function to merge the array's old data-range with the new data-range. A side-effect of this approach is that a uniform data-range must be used for every element in the array. Another drawback of this method is that a ϕ -function is required for every array assignment, increasing the size of the code. However, def-use chains are still inherent in the intermediate representation, simplifying the analysis. Furthermore, when compiling to silicon this analysis determines the size of embedded RAMs.

3.4.2 Pointers

Pointers complicate the analysis of memory instructions, potentially creating aliases and ambiguities that can obscure data-range discovery. To handle pointers, we use the SPAN pointer analysis package developed by Radu Rugina and Martin Rinard [22]. SPAN can determine the sets of variables – commonly referred to as *location sets* – that a pointer *may* or *must* reference. We distinguish between *reference location sets* and *modify location sets*. A reference location set is a location set annotation that occurs on the right hand side of an expression, whereas a modify location set occurs on the left hand side of an expression.

As an example, consider the following C memory instruction, assuming that p_0 is a pointer that can point to variable a_0 or b_0 , and that q_0 is a pointer that can only point to variable b_0 :

```
*p0 = *q0 + 1
```

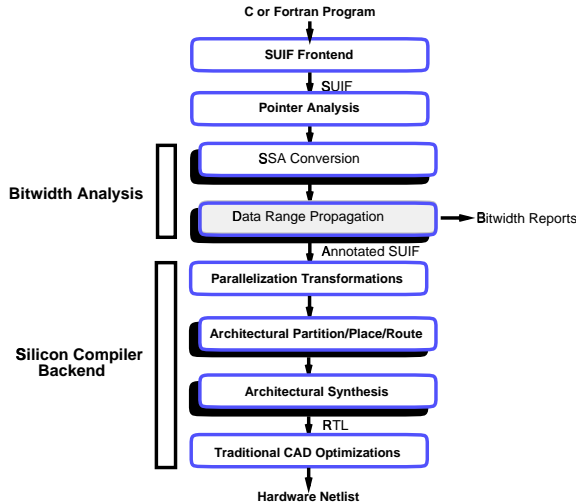


Figure 10: Compiler flow: includes general SUIF, Bitwise, silicon, and CAD processing steps. The raised steps are new *Bitwise* or *DeepC* passes, and the remaining steps are re-used from previous SUIF compiler passes.

The location set that the instruction may modify is $\{a0, b0\}$, and the location set that the instruction must reference is $\{b0\}$. Since $b0$ is the only variable in the instruction’s reference location set, the instruction *must* reference it. Also, because there are two variables in the modify location set, either $a0$ or $b0$ *may* be modified.

Keeping the SSA guarantee that there is a unique assignment associated with each variable, we have to rename $a0$ and $b0$ in the instruction’s modify location set. Furthermore, since it is not certain that either variable will be modified, a ϕ -function has to be inserted for each variable in the modify location set to merge the previous version of the variable with the renamed version:

$$\begin{aligned} \{a1, b1\} &= \{b0\} + 1 \\ a2 &= \phi(a0, a1) \\ b2 &= \phi(b0, b1) \end{aligned}$$

If the modify location set has only one element, the element *must* be modified, and a ϕ -function does not need to be inserted. This extension to SSA form allows us to treat de-referenced pointers in exactly the same manner as scalars.

4 Bitwise Results

This section presents results from a stand-alone *Bitwise Compiler*. The compiler is composed of the first four steps shown in Figure 10. Further results, after processing with the silicon compiler backend (the last four steps in the flowchart), are presented in Section 6.

The frontend of the compiler takes as input a program written in C or FORTRAN and produces a bitwidth-annotated SUIF file. After parsing the input program into SUIF, the compiler performs traditional optimizations and then pointer analysis [22]. The next two passes, labeled “Bitwidth Analysis”, are the realization of the algorithms discussed in this paper. Here, the SUIF intermediate representation is converted to SSA form, including the extensions

Benchmark	Type	Source	Lines	Description
softfloat	Emulation	Berkeley	1815	Floating Point
adpcm	Multimedia	UTdsp	195	Audio Compress
bubblesort	Scientific	Raw	62	Bubble Sort
life	Automata	Raw	150	Game of Life
intmatmul	Scientific	Raw	78	Int. Matrix Mult.
jacobi	Scientific	Raw	84	Jacobi Relation
median	Multimedia	UTdsp	86	Median Filter
mpegcorr	Multimedia	MIT	144	From MPEG Kernel
sha	Encryption	MIT	638	Secure Hash
bilinterp	Multimedia	MMX	110	Bilinear Interp.
convolve	Multimedia	MIT	74	Convolution
histogram	Multimedia	UTdsp	115	Histogram
intfir	Multimedia	UTdsp	64	Integer FIR
newlife	Automata	MIT	119	New Game of Life
parity	Multimedia	MIT	54	Parity Function
pmatch	Multimedia	MIT	63	Pattern Matching
sor	Scientific	MIT	60	5-point Stencil

Table 1: Benchmark characteristics

discussed in Section 3. Finally, the data-range propagation pass is invoked to produce bitwidth-annotated SUIF along with the appropriate bitwidth reports. In total, they comprise roughly 12,000 lines of C++ code. We first discuss the bitwidth reports that are generated after these passes.

4.1 Experiments

The prototype compiler does not currently support recursion. Although this restriction limits the complexity of the benchmarks we can analyze, it provides adequate support of programs written for high-level silicon synthesis.

Table 1 lists the benchmarks presented in this section. The source code for the benchmarks can be found at [6]. We include several contemporary multimedia applications as well as standard applications that contain predominantly bit-level or byte-level operations, such as *life* and *softfloat*.

4.2 Register Bit Elimination

Figure 11 shows the percentage of the original register bits remaining in the program after *Bitwise* has run. Register bits are used to store scalar program variables. The lower bound — which was obtained by profiling the code — is included for reference. For the particular data sets supplied to the benchmark, this lower bound represents the fewest number of bits needed to retain program correctness. That is, it forms a lower bound on the minimum bitwidth that can be determined by static analysis, which must be correct over all input data-sets. The graph assumes that each variable is assigned to its own register. However, downstream architectural synthesis passes include a register allocator. If variables with differing bitwidths share the same physical register, the final hardware may not capture all of the gains of our analysis. Our metric is a useful overall gauge because register bitwidths affect functional unit size, data path bitwidths, and circuit switching activity.

Our analysis dramatically reduces the total number of register bits needed. In most cases, the analysis is near optimal, which is especially exciting for applications that perform abundant multi-granular computations. For instance, *Bitwise* nearly matches the lower bound for *life* and *mpegcorr*, which are bit-level and byte-level applications respectively.

The only applications in the figure with substantially sub-optimal performance compared to the dynamic profile

are median and softfloat. In the case of median, the analyzer was unable to determine the bitwidth of the input data, thus variables that were dependent on the input data assumed the maximum possible bitwidths. Although dynamic profiling of softfloat shows plenty of opportunities for bitwidth reduction, these opportunities are specific to particular control flow paths and were not discovered during our static analysis of the whole program.

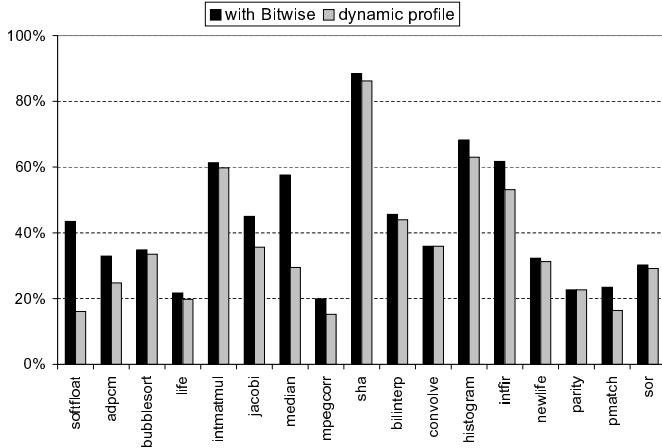


Figure 11: Percentage of total register bits remaining: post-bitwidth analysis versus dynamic profile-based lower bound.

4.3 Memory Bit Elimination

Figure 12 shows the percentage of the original memory bits remaining in the program. Here memory bits are defined as data allocated for static arrays and dynamically allocated variables. This is an especially useful metric when compiling to non-conventional devices such as an FPGA, where memories may be segmented into many small chunks. In addition, because memory systems are one of the primary consumers of power in modern processors, this is a useful metric for estimating power consumption [14].

In almost all cases, the analyzer is able to determine near-optimal bitwidths for the memories. There are a couple of contributing factors for Bitwise’s success in reducing array bitwidths. First, many multimedia applications initialize static constant tables which represent a large portion of the memory savings shown in the figure. Second, Bitwise capitalizes on arrays of Boolean variables.

4.4 Bitwidth Distribution

It is interesting to categorize variable bitwidths according to grain size. The stacked bar chart in Figure 13 shows the distribution of variable bitwidths both before and after bitwidth analysis. We call this distribution a *Bitspectrum*. To make the graph more coherent, bitwidths are rounded up to the nearest typical machine data-type size. In most cases, the number of 32-bit variables is substantially reduced to 16, 8, and 1-bit values.

For silicon compilation, this figure estimates the overall register bits that can be saved. As we will see in the next sections, reducing register bits results in smaller datapaths and subsequently smaller, faster, and more efficient circuits.

Compilers for multimedia extensions can utilize bitwidth information to extract higher degrees of parallelism [16]. In this context, the spectrum shows which applications will

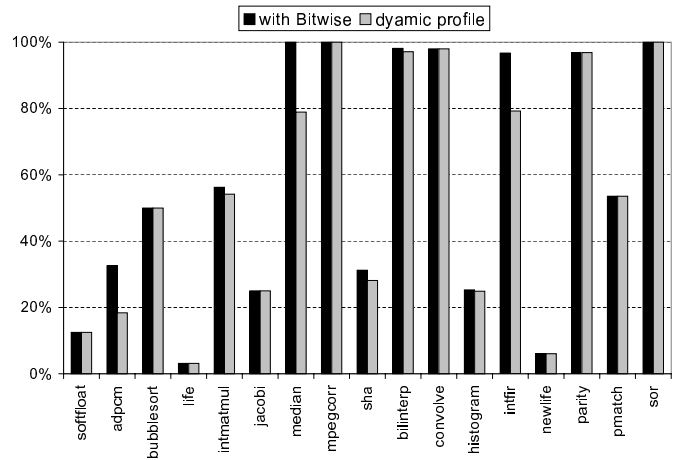


Figure 12: Percentage of total memory remaining: post-bitwidth analysis versus dynamic profile-based lower bound.

have the best prospect for packing values into sub-word instructions.

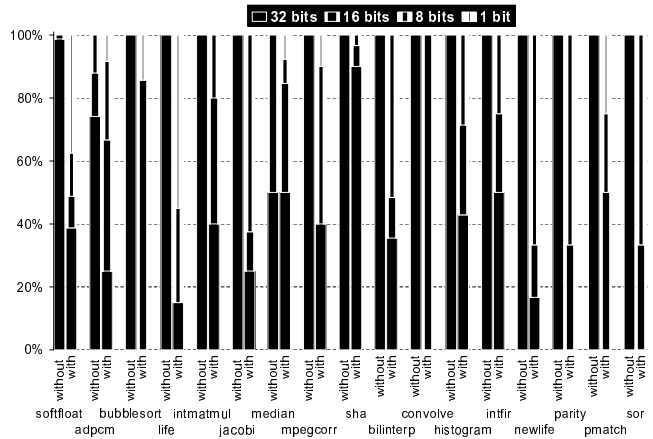


Figure 13: Bitspectrum. This graph is a stacked bar chart that shows the distribution of register bitwidths for each benchmark. Without bitwidth analysis, almost all bitwidths are 32-bits. With Bitwise, many widths are reduced to 16, 8, and 1 bit machine types, as denoted by the narrower 16, 8, and 1 bit machine types.

5 DeepC Silicon Compiler

Thus far we have shown that bitwidth analysis is a generally effective optimization and that our *Bitwise Compiler* is capable of performing this task well. We now turn to a concrete application. We have applied bitwidth analysis to the difficult problem of silicon compilation. For lack of space, this section gives brief treatment to the design of a high-level silicon compiler. The following section discusses the impact of bitwidth analysis in this context.

5.1 Overview

We have integrated *Bitwise* with the *DeepC Silicon Compiler* [4]. DeepC is a research compiler under development that is capable of translating sequential applications, written in either C or FORTRAN, directly into a hardware netlist. The compiler automatically generates a specialized parallel

architecture for every application. To make this translation feasible, the compilation system incorporates the latest code optimization and parallelization techniques as well as modern hardware synthesis technology. Figure 10 shows the details of integrating *Bitwise* into *DeepC*'s overall compiler flow. After reading in the program and performing traditional compiler optimizations and pointer analysis, the bitwidth analysis steps are then invoked. These steps were described in detail in Section 3. The additional steps of the silicon compiler backend are as follows. First, loop-level parallelizations are applied, followed by an architectural-level partition, place, and route. At this point the program has been formed into an array of communicating threads. Then an architectural synthesis step translates these threads into custom hardware. Finally, the compiler applies traditional computer-aided-design (CAD) optimizations to generate the final hardware netlist.

5.2 Implementation Details

The *DeepC Compiler* is implemented as a set of over 50 SUIF passes followed by commercial RTL synthesis. The current implementation uses the latest version of Synopsys Design Compiler and FPGA compiler for synthesis. A large set of the SUIF passes are taken directly from MIT's Raw compiler [17], whose backend is built on Harvard's Mach-SUIF compiler [24]. The backend Verilog generator is implemented on top of Stanford's VeriSUIF [9] data structures. Despite the large number of SUIF passes, CAD synthesis tools consume the majority of the compiler's run-time.

5.3 Verilog Bitwidth Rule

Because our compiler generates RTL Verilog for commercial tools, bitwidth information must be totally communicated via register and wire widths. We expect conformation to Verilog's operation bitwidth rule: the bitwidth of each operation is set to the maximum bitwidth of the containing assignment expression's input and output variables. For example, the bitwidth of the expression $A = B + C$ is the maximum bitwidth of A , B , and C .

5.4 Usage

There has been a limited release of the compiler and it is in use by researchers at MIT, Princeton, and the University of Massachusetts. We are studying both reconfigurable computing and embedded system-on-a-chip design. When used for reconfigurable computing, the compiler is coupled with further silicon compilation tools, such as the VirtualLogic [12] emulation system from IKOS, or software and drivers for Annapolis System's Wild-one PCI card [2]. For use in embedded chip design, downstream CAD tools accepting a logic netlist then perform physical place and route, mapping the design onto a specific silicon substrate.

6 Impact on Silicon Compilation

As described in the previous section, the *DeepC Silicon Compiler* has the opportunity to specialize memory, register, and datapath widths to match application characteristics. We expect bitwidth analysis to have a large impact in this domain. However, because backend CAD tools already implicitly perform some bitwidth calculation during optimizations (such as dead logic elimination), accurate measurements require end-to-end compilation. A fair comparison

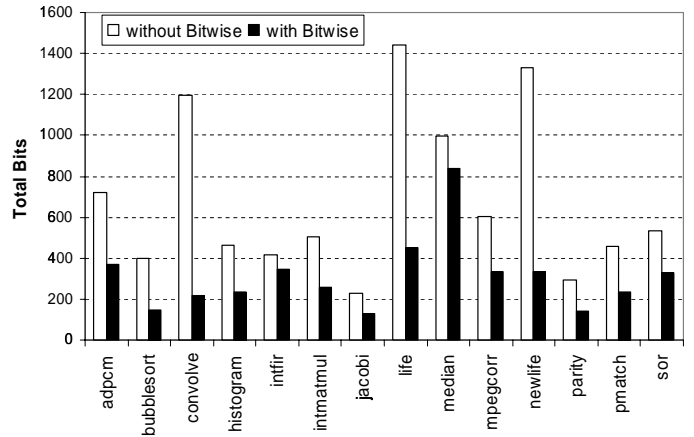


Figure 14: Register bits after bitwise optimization. In every case *Bitwise* saves substantial register resources in the final silicon implementation.

is to measure final silicon both with and without bitwidth analysis.

We introduce our benchmarks in the next section, then describe the dramatic area, latency, and power savings that bitwidth analysis enables⁴.

6.1 Experiments

We present experimental results for an initial set of applications that we have compiled to hardware. For each application, our compilation system produces an architecture description in RTL Verilog. We further synthesize this architecture to logic gates with a commercial CAD tool (Synopsys). In this paper we report area and speed results for Xilinx 4000 series FPGAs, and power results for IBM's SA27E process – a 0.15 micron, 6-layer copper, standard-cell process.

The benchmarks used for silicon compilation are included in Table 1. These applications are generally short benchmarks, but include many multimedia kernels. It is important to note that the relatively small size of the benchmarks is dictated by the current synthesis time of our compilation approach and not *Bitwise*. Also note that there are slight variations from the benchmarks presented in Section 4.

6.2 Registers Saved in Final Silicon

We first compiled each benchmark into a netlist capable of being accepted by either Xilinx or IBM CAD tools to produce “final silicon.” The memory savings reported in Section 4 translate directly into silicon memory savings when we allow a separate small memory for each program variable. This small memory partitioning process is further described in earlier work [4].

Register savings, on the other hand, vary as additional compiler and CAD optimizations transform the program's variables. Variable renaming and register allocation also distort the final result by placing some scalars in more than one register and others in a shared register. Figure 14 shows the total FPGA bits saved by bitwidth optimization. For Xilinx FPGA compilation, the fixed allocation of registers to combinational logic will distort the exact translation of this savings to chip area, as some registers may go unused.

⁴Note that we also found considerable synthesis compile time savings which are not reported here.

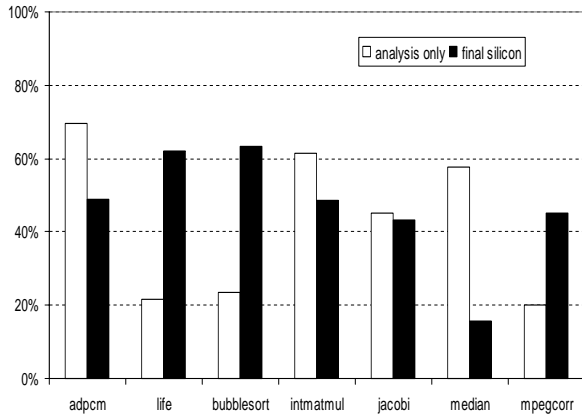


Figure 15: Register bit reduction, after high-level analysis versus final silicon. The fluctuation in bitwidth savings between final silicon and high-level analysis is due to factors such as variable renaming and register allocation.

Our findings are very positive — the earlier bitwidth savings translate into dramatic savings in final silicon, despite the possibilities for loss of this information or potential overlap with other optimizations. However, because there is not a one-to-one mapping from program scalars to hardware registers, the exact savings do not match. Examining Figure 15, we see that the percentage of bits saved by high-level analysis are sometimes greater and sometimes less than those bits saved in final silicon. We explain these differences as follows. First, there are many compiler and CAD passes between high-level analysis and final silicon generation. If in any of these passes the bitwidth information is “lost”, for example when a new variable is cloned, then the full complement of saved bits will not be realized. On the other hand, the backend passes, especially the CAD tools, are also attempting to save bits through logic optimizations. Thus these passes may find savings that the current high-level pass is not finding. Finally, variable renaming and register sharing also change the percentages.

6.3 Area

Register bits saved translate directly into area saved. Area savings also result from the reduction of associated datapaths. Figure 16 shows the total area savings with Bitwise optimizations versus without. We save from 15% to 86% in overall silicon area, nearly an 8× savings in the best case.

Note that in the DeepC Compilation system pointers do not require the full complement of 32-bits. Using the MAPS [5] compiler developed for Raw, arrays have been assigned to a set of *equivalence classes*. By definition, a given pointer can only point to one equivalence class, and thus needs to be no wider than $\log \sum_a S_a$, where S_a is the size of each memory array specified in the equivalence class. This technique is further described in [3].

6.4 Clock Speed

We also expect bitwidth optimization to reduce the latency along the critical paths of the circuit and increase maximum system clock speed. If circuit structures are linear, such as a ripple carry adder, then we expect a linear increase. However, common structures such as carry-save adders, multiplexors, and barrel shifters are typically implemented with logarithmic latency. Thus, bitwidth reduction translates

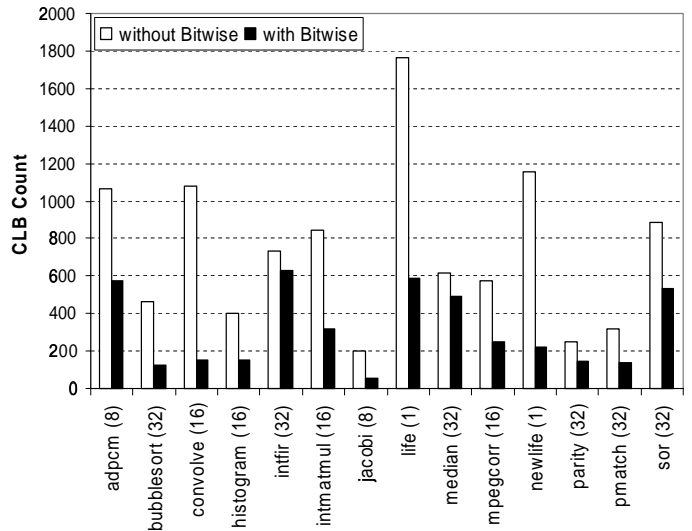


Figure 16: FPGA area after Bitwise optimization. Register savings translate directly into area savings for FPGAs. In the figure, post-synthesis CLB count measures the number of internal combinational logic blocks required to implement the benchmark when compiled to FPGAs. Combinational logic blocks (CLBs) each include 2 four input lookup tables and 2 flip-flop registers. Wasted CLBs due to routing difficulties during vendor place and route are not included in this result, but should reduce proportionally. The number in parenthesis by each benchmark is the resulting bitwidth of the main datapath.

into a less-than-linear yet significant speedup. Figure 17 shows the results for a few of our benchmarks. The largest speedup is for convolve, in which the reduction of constant multiplications increased clock speeds by nearly 3×. On the other hand, the MPEG correlation kernel did not speed up because the original bitwidths were already close to optimal.

6.5 Power

As expected, the area saved by bitwidth reduction translates directly into power savings. Our first hypothesis was that these savings might be lessened by the fact that inactive registers and datapaths would not consume power. Our experiments show otherwise. The muxes and control logic leading to these registers still consume power. Figure 18 shows the reduction in power achieved for a subset of our benchmarks. In order to make these power measurements, we first ran a Verilog simulation of the design to gather switching activity. This switching activity records when each register toggles in the design. This information is then used by logic synthesis, along with an internal zero delay simulation, to determine how often each wire changes state. The synthesizer then reports average dynamic power consumption in milliWatts, which we report here. Note that we do not include the power consumption of on-chip memories. Furthermore, we do not attempt to decrease the total cycle count with bitwidth reduction, giving a total energy reduction in proportion to total power savings.

We measured power for bubblesort, histogram, jacobi, pmatch, and newlife. Newlife had the largest power savings, reduced from 14 mW to 4 mW, while the other four benchmarks had more modest power savings. We expect that at least a portion of these savings can be translated to the processor regime, in which power consumption is typically hundreds of times higher.

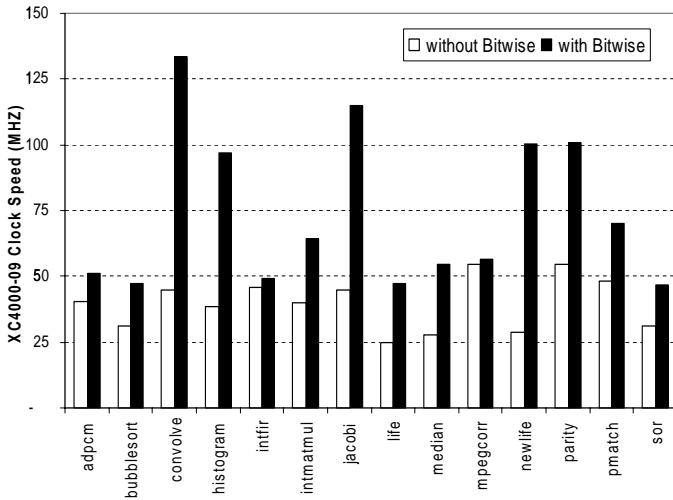


Figure 17: FPGA clock speed after Bitwise optimization. Benchmarks are universally faster after bitwidth analysis when compiled to Xilinx XC4000 FPGAs (-09 speed grade) with Synopsys. Clock speed is determined by the worst case delay reported during synthesis and does not account for skew, etc. The actual number of CLBs on the critical paths, ranging from 15-38 before bitwidth optimization and 7-16 afterwards, is the key factor in determining clock speed.

6.6 Discussion

For reconfigurable computing applications, bitwidth savings can be a “make or break” difference when comparing computational density – performance per area – to that of conventional processors. Because FPGAs provide an additional layer of abstraction (emulated logic), it is important to compile-through as many higher levels of abstraction as possible. Statically taking advantage of bitwidth information is a form of partial evaluation. It can help to make FPGAs competitive with more traditional, but less adaptive, computing solutions. Thus, bitwidth analysis is a key technology *enabler* for FPGA computing.

For ASIC implementations, bitwidth savings will directly translate into reduced silicon costs. Of course, many of these cost savings could be captured by manually specifying more precise variable bitwidths. However, manual optimization comes at the cost of manual labor. Additionally, reducing the probability of errors is invaluable in an ASIC environment, where companies who miss with first silicon often miss entire market windows. As we approach the billion-transistor era, raising the level of abstraction for ASIC designers will be a requirement, not a luxury.

7 Related Work

Brooks et al., dynamically recognize operands with narrow bitwidths to exploit sub-word parallelism [7]. Their research confirms our claim that a wide range of applications, particularly multimedia applications, exhibit narrow bitwidth computations. Using their techniques, they are able to detect and exploit bitwidth information that is not statically known. However, because they are detecting bitwidths dynamically, their research cannot be applied to applications that require a priori bitwidth information.

Scott Ananian also recognized the importance of static bitwidth information [1]. He uses bitwidth analysis in the context of a Java to silicon compiler. Because bitwidth anal-

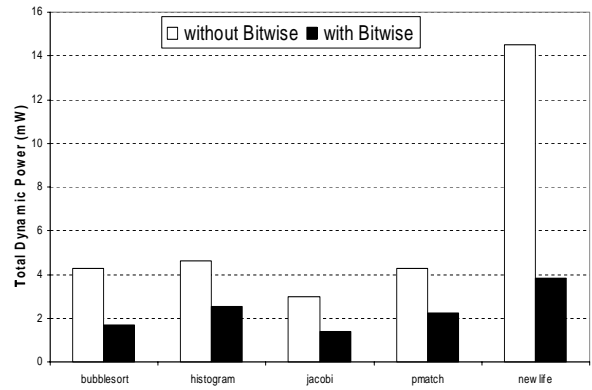


Figure 18: ASIC power after Bitwise optimization. Here we assume a 200MHz clock for the .15 micron IBM SA27E process. The total cycle count (number of clocks ticks to complete each benchmark) is not affected by bitwidth, and thus total energy will scale proportionally. These numbers do not include power consumed by RAM.

ysis is not the main thrust of his research, he uses a simple data flow technique that propagates bitwidth information. Our method of propagating data-ranges is a more precise method for discovering bitwidths.

Rahul Razdan developed techniques to successfully analyze bitwidths [21]. His “function width” analysis is a combination of forward and backward analyses on a vector of bits. In this sense, his analysis is similar to traditional CAD dead-bit elimination algorithms. Furthermore, with the exception of the loop induction variables, his analysis does not handle loop-carried expressions well. Razdan’s function width results for his PRISC architecture achieve modest speedups when used in combination with other logic-level optimizations.

Budiu et al. [8] also perform bitwidth analysis. They use methods similar to Razdan’s to improve performance in a reconfigurable device.

The data-range propagation techniques presented by Jason Patterson [19] and William Harrison [11] are similar to those presented in this paper. While their work proved to be effective, they did not consider backward propagation and their techniques for discovering loop-carried sequences do not include the general methods discussed in this paper.

8 Conclusion

Accurate bitwidth analysis of high-level programs requires sophisticated compiler techniques. Prior to this work, only simple or ad-hoc approaches to automatic bitwidth analysis have been applied. In this work we have *formalized* bitwidth analysis as a value range propagation problem. We have described algorithms for bi-directional data-range propagation and for finding closed-form solutions of loop-carried expressions. We have presented an initial implementation which works well: our compile-time analysis approaches the accuracy of run-time profile-based analyses. When incorporated into a silicon compiler, bitwidth analysis dramatically reduced the logic area by 15 – 86%, improved the clock speed by 3 – 249%, and reduced the power by 46 – 73% of the resulting circuits. Anticipated future uses of this technique include compilation for SIMD and low power architectures.

9 Acknowledgments

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