EECS 230
ENGINEERING ELECTROMAGNETICS
Leland Pierce

Wave Reflection and Transmission

University of Michigan - Fall 2024

Lecture Coverage

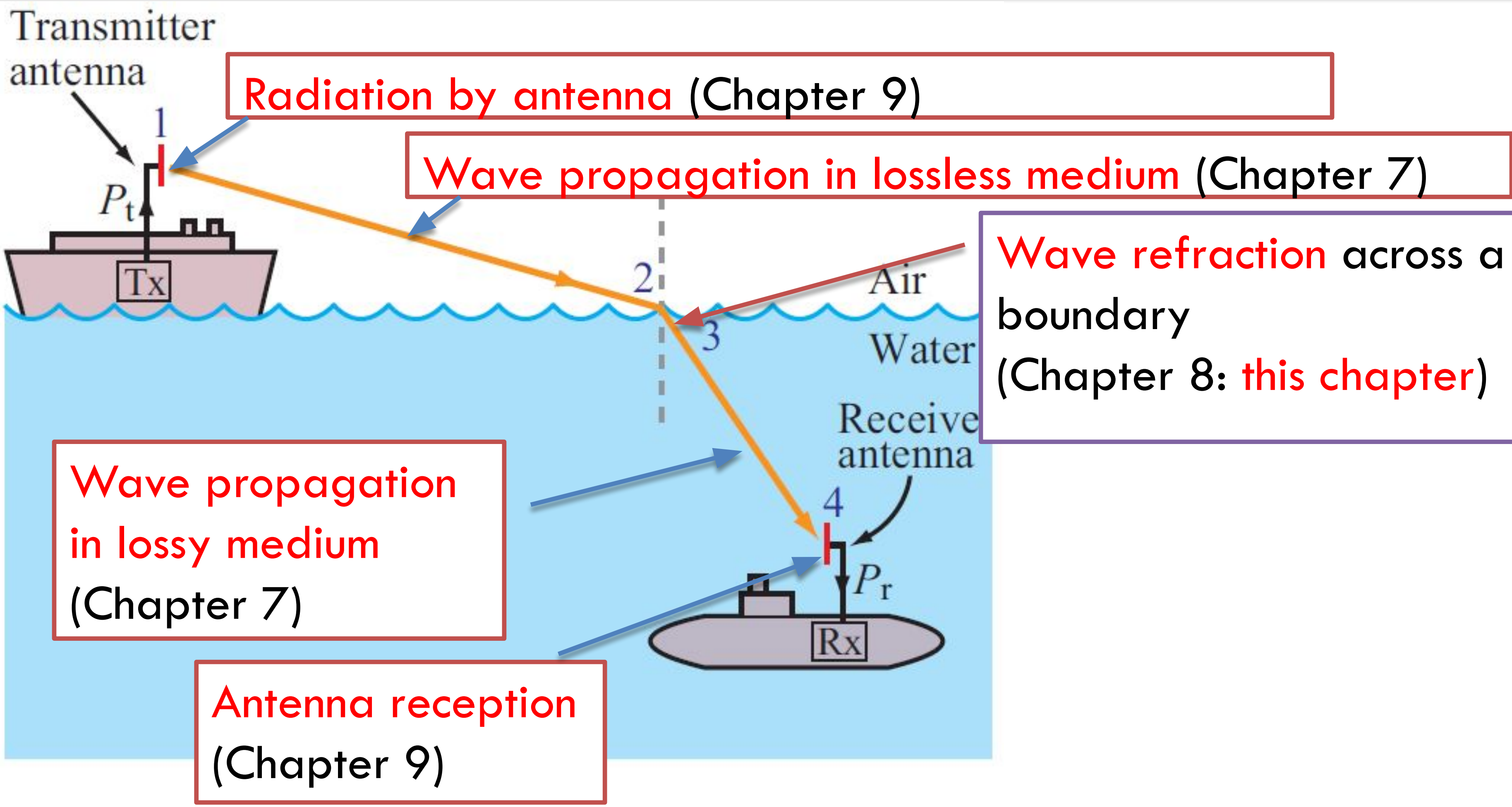
Today's lecture:

Sections 8-1 and 8-2 of the book:

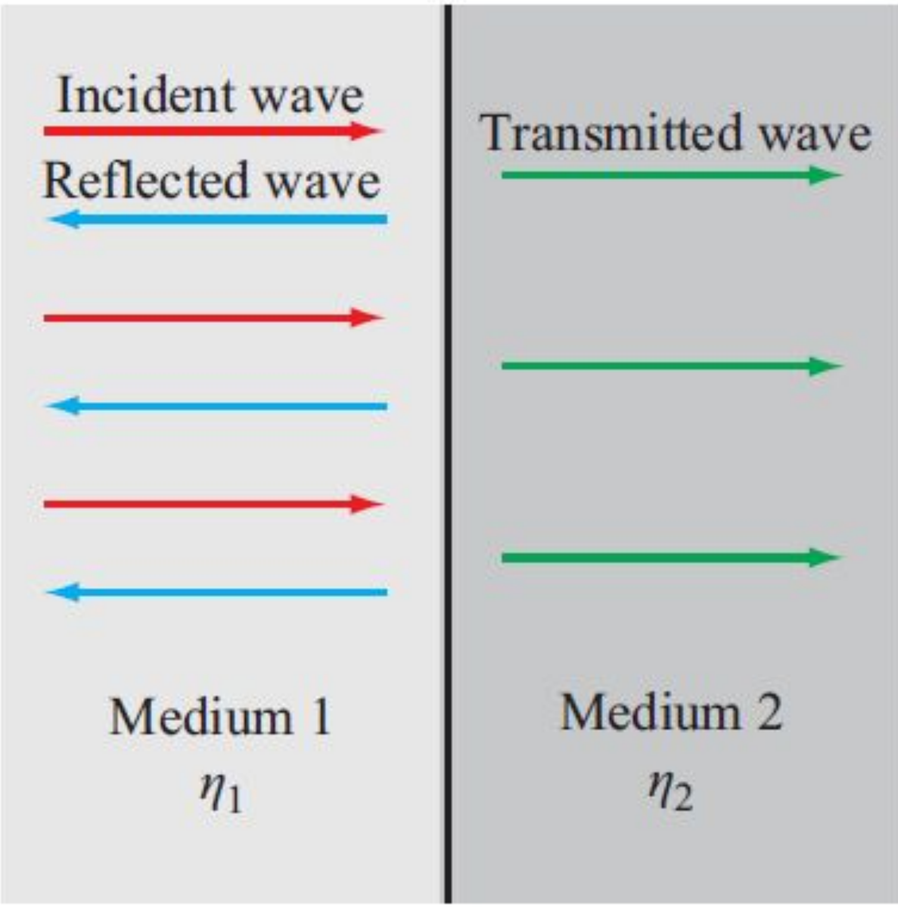
8-1: Wave Reflection and Transmission: Normal Incidence

8-2: Snell's Laws

8.1 Signal Path



8.1 Normal and Oblique Incidence

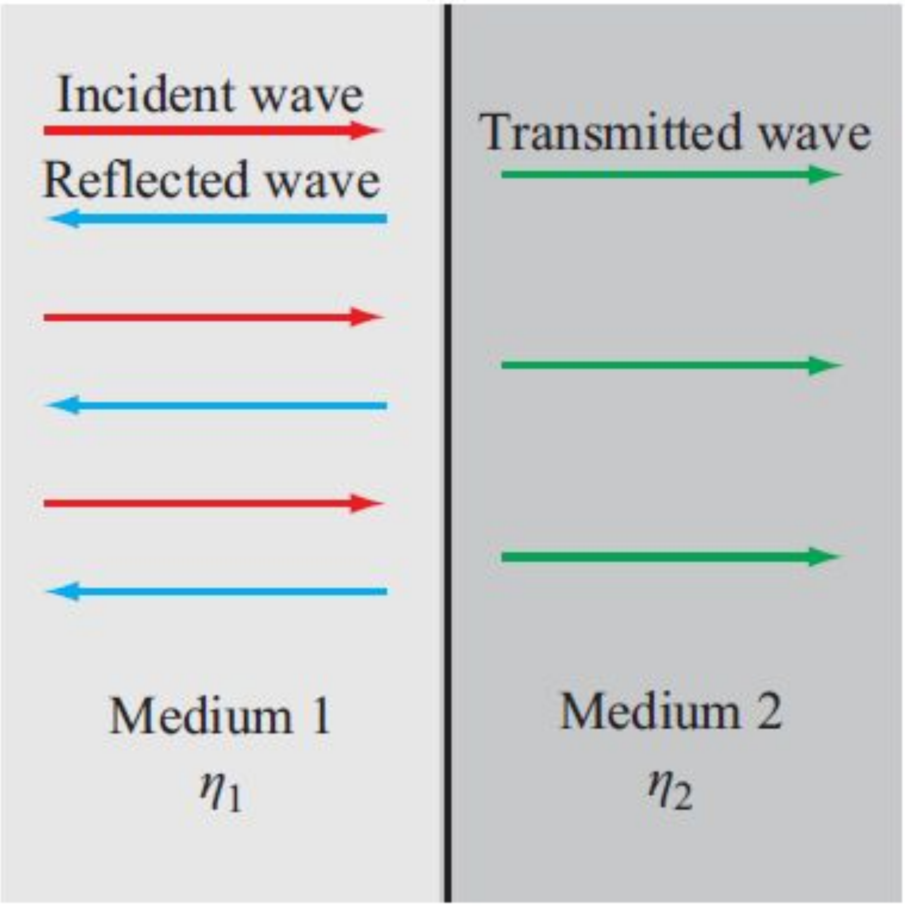


(a) Normal incidence

Lines represent direction of propagation of a plane wave.

This is called the "ray" view of a wave

8.1 Normal and Oblique Incidence



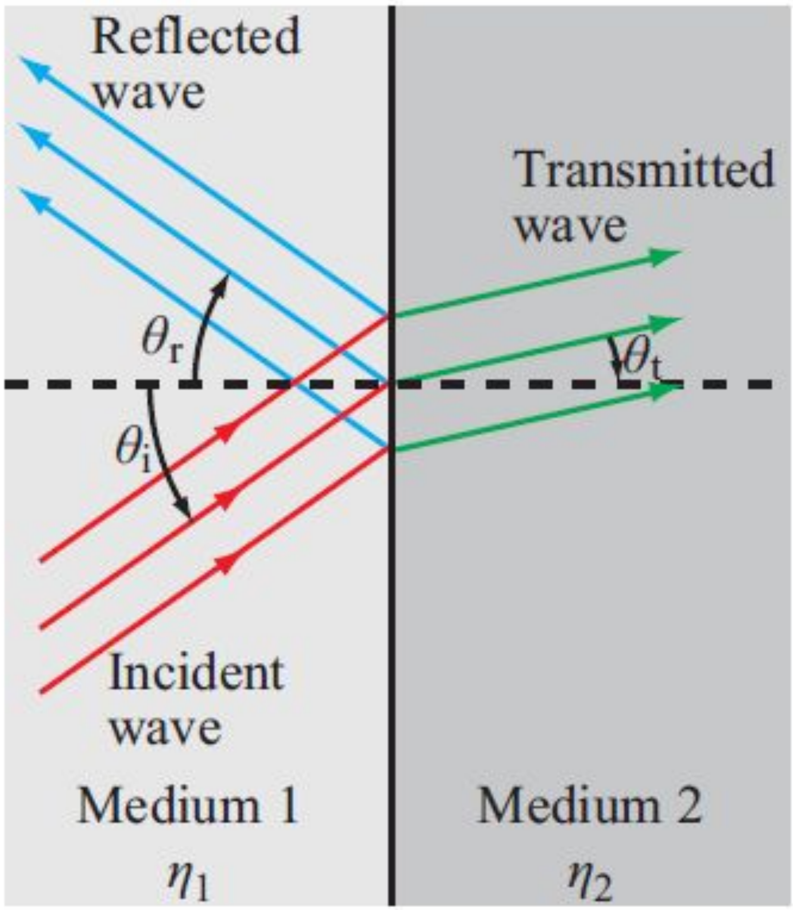
(a) Normal incidence

For a normally-incident wave:

Some is reflected
Some is transmitted

All are normal to the boundary.

8.1 Normal and Oblique Incidence



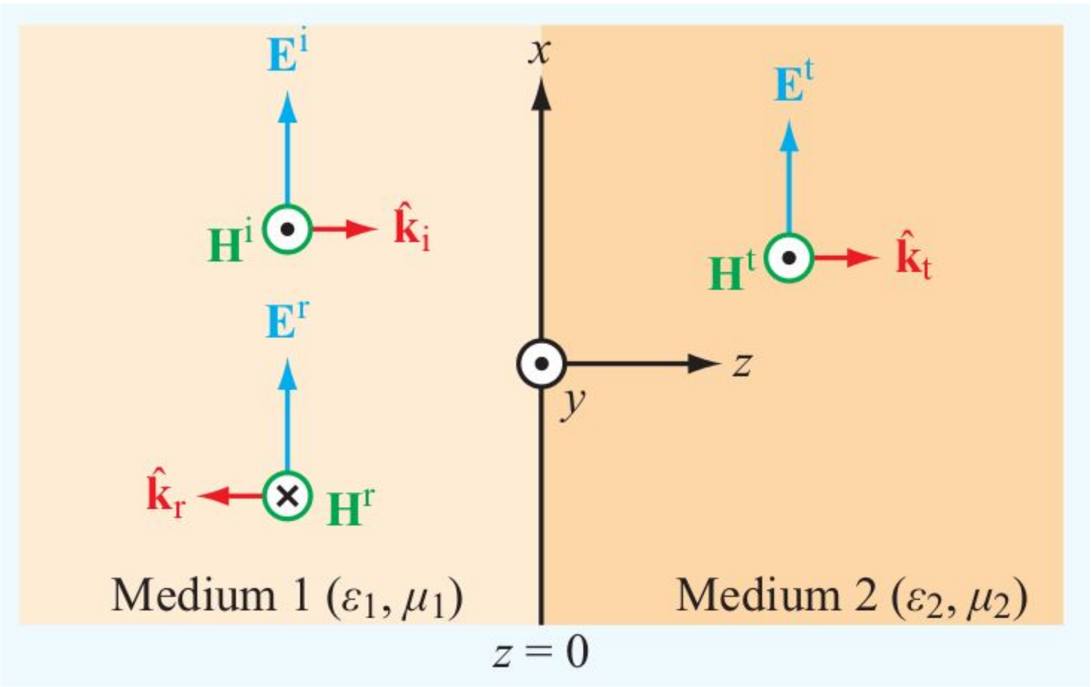
(b) Ray representation of oblique incidence

For an obliquely-incident wave:

Some is reflected
Some is transmitted

All are generally at different directions.

8.1 Normal Incidence: Lossless Media



Incident plane wave direction:

$$\hat{\mathbf{k}}_i = \hat{\mathbf{z}}$$

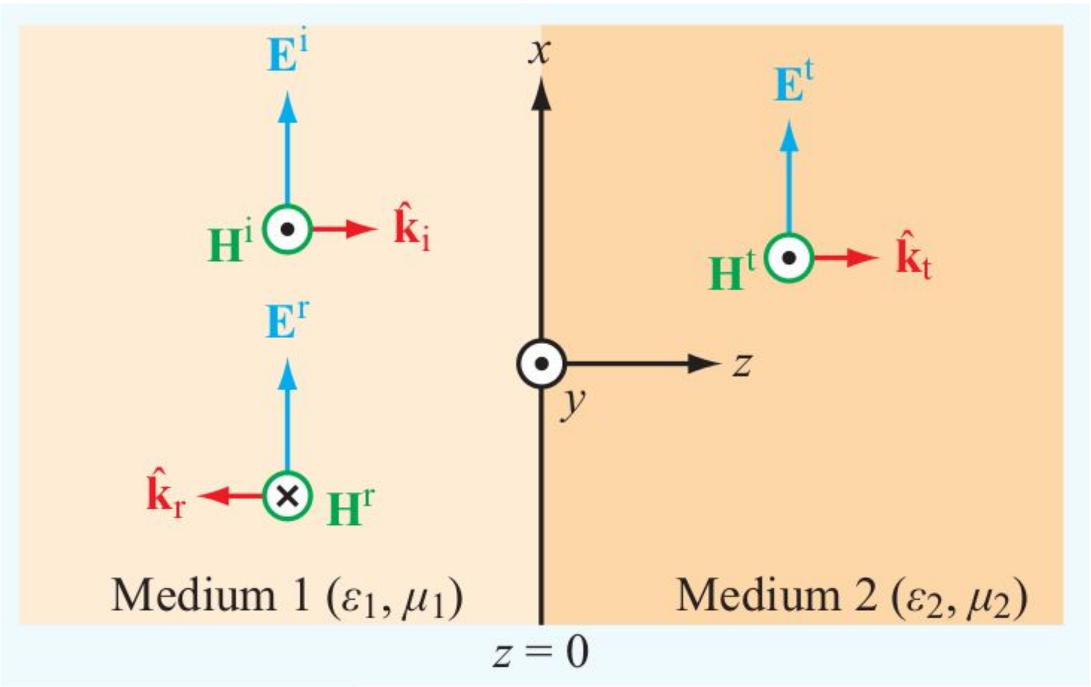
Reflected:

$$\hat{\mathbf{k}}_r = -\hat{\mathbf{z}}$$

Transmitted:

$$\hat{\mathbf{k}}_t = \hat{\mathbf{z}}$$

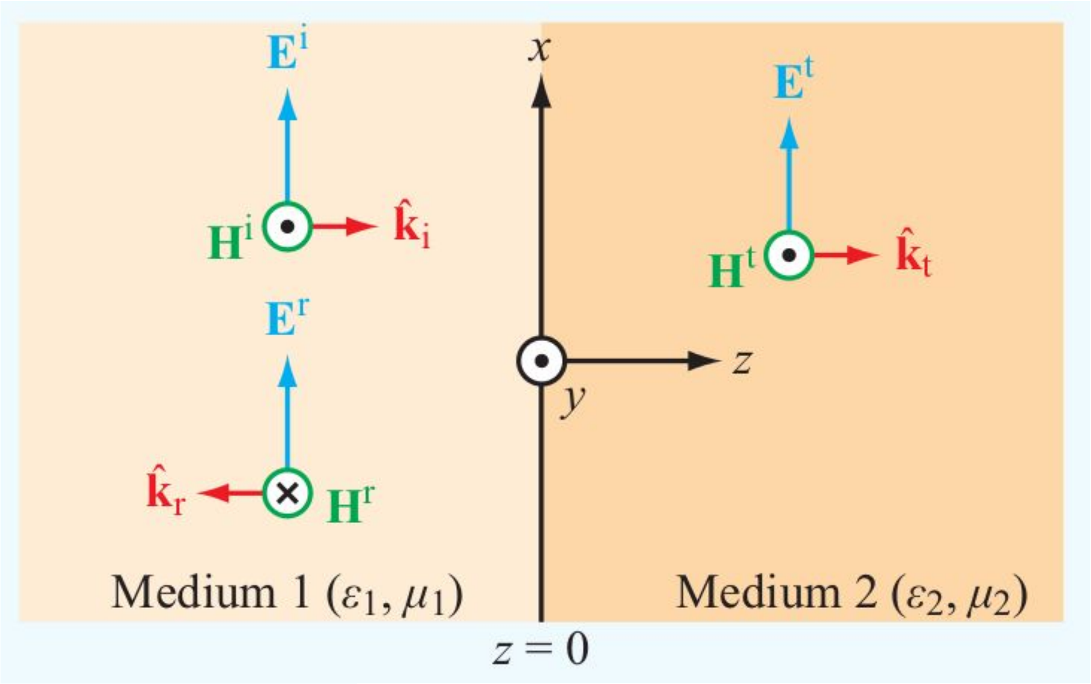
8.1 Normal Incidence: Lossless Media



Incident Field in medium 1

$$\widetilde{\mathbf{E}}^i(z) = \hat{\mathbf{x}} E_0^i e^{-jk_1 z},$$

8.1 Normal Incidence: Lossless Media

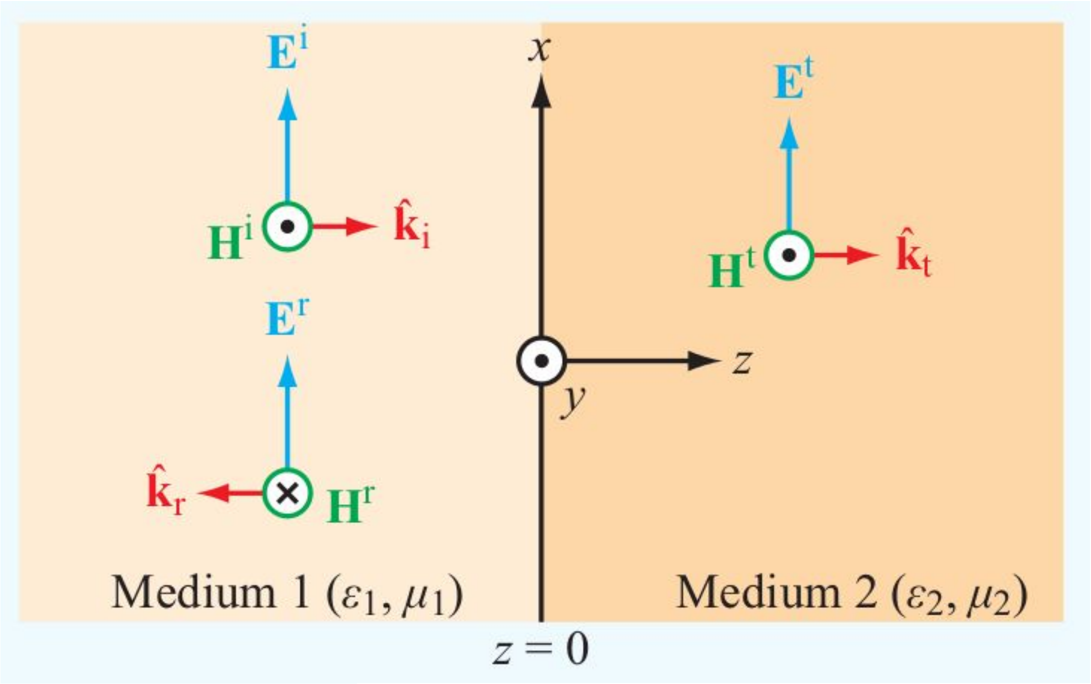


wavenumber for the incident wave in medium 1:

$$k_1 = \omega\sqrt{\mu_1 \varepsilon_1}$$

$$\widetilde{\mathbf{E}}^i(z) = \hat{\mathbf{x}} E_0^i e^{-jk_1 z},$$

8.1 Normal Incidence: Lossless Media



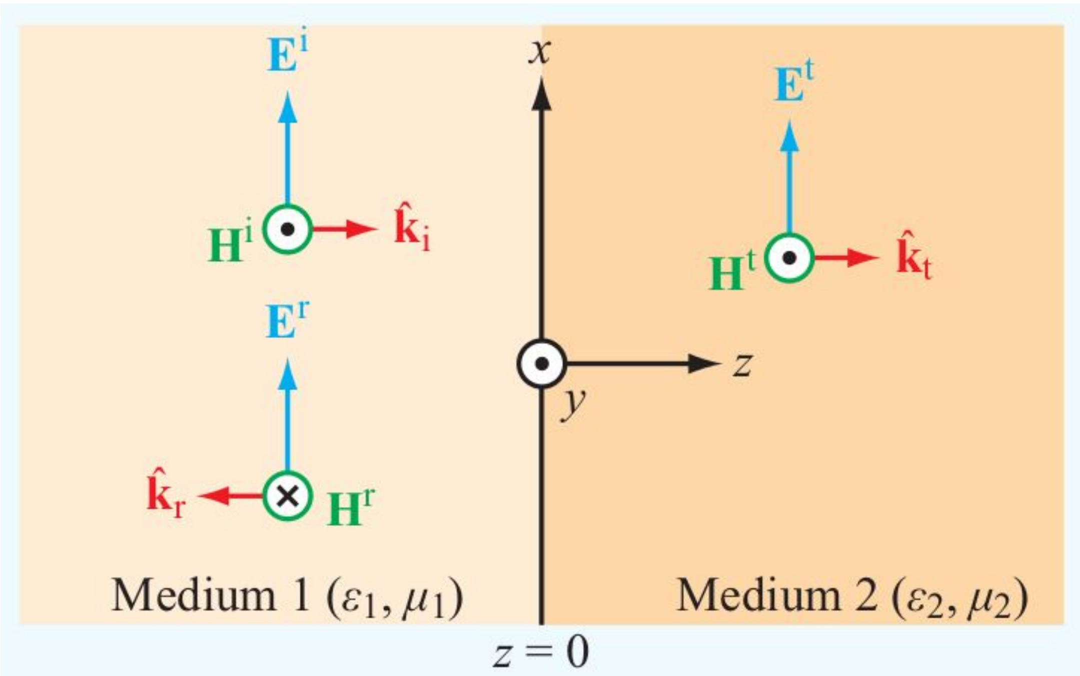
Intrinsic impedance of medium 1:

$$\eta_1 = \sqrt{\mu_1/\varepsilon_1}$$

$$\widetilde{\mathbf{E}}^i(z) = \hat{\mathbf{x}} E_0^i e^{-jk_1 z},$$

$$\widetilde{\mathbf{H}}^i(z) = \hat{\mathbf{z}} \times \frac{\widetilde{\mathbf{E}}^i(z)}{\eta_1} = \hat{\mathbf{y}} \frac{E_0^i}{\eta_1} e^{-jk_1 z}.$$

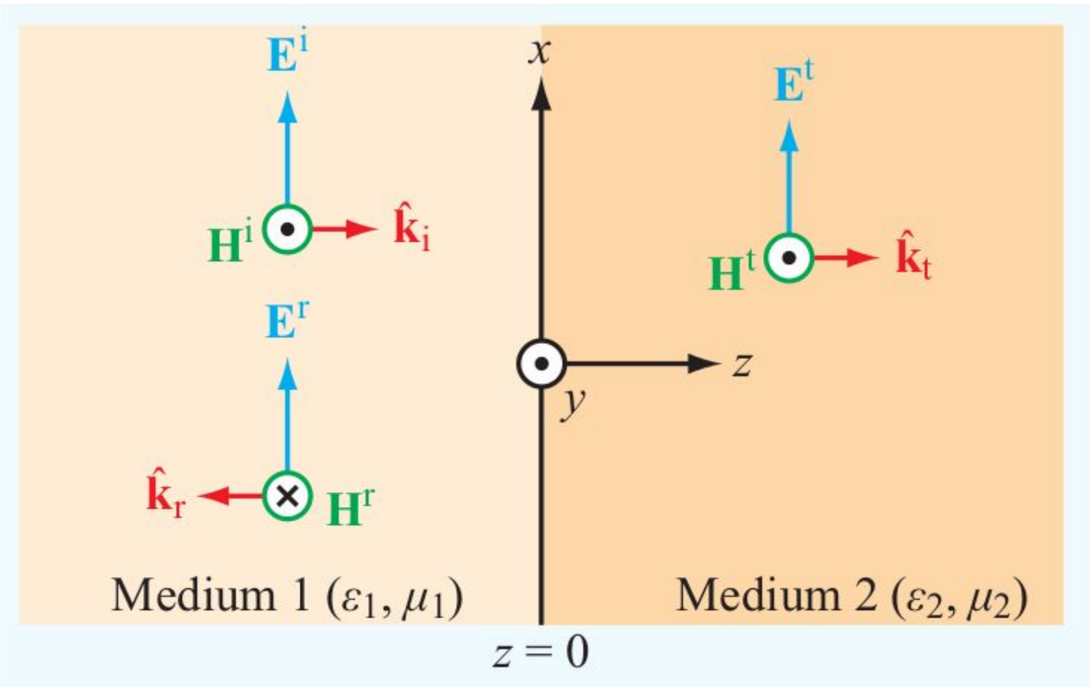
8.1 Normal Incidence: Lossless Media



Reflected Field in medium 1

$$\widetilde{\mathbf{E}}^r(z) = \hat{\mathbf{x}} E_0^r e^{jk_1 z},$$

8.1 Normal Incidence: Lossless Media



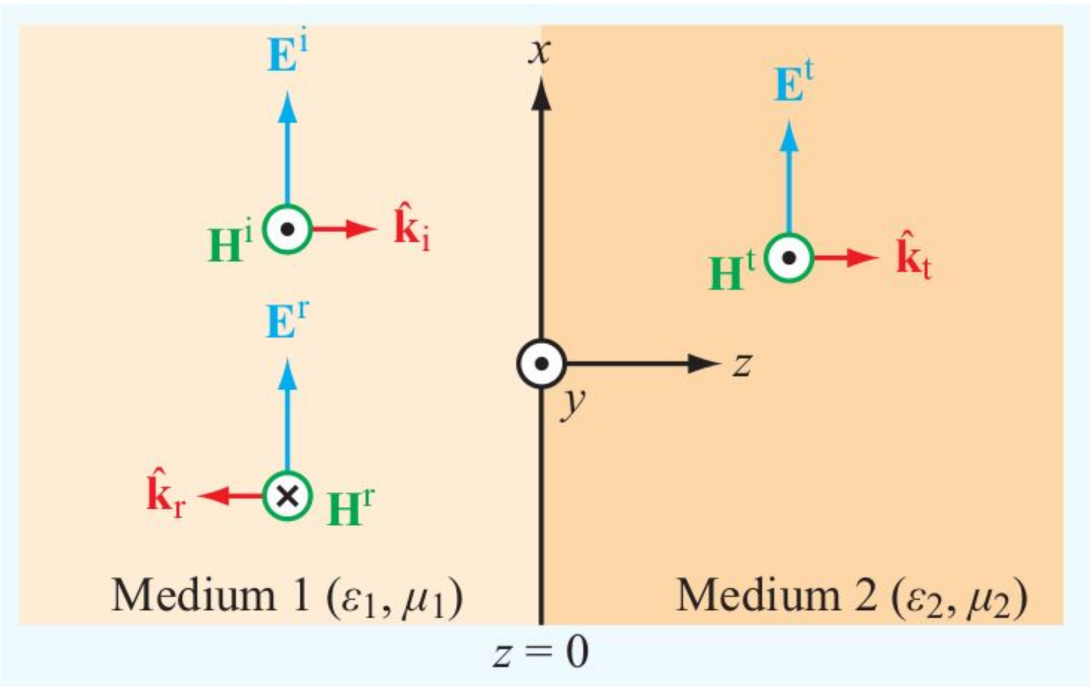
$$\widetilde{\mathbf{E}}^{\mathrm{r}}(z) = \hat{\mathbf{x}} E_0^{\mathrm{r}} e^{jk_1 z},$$

wavenumber for the reflected field in medium 1:

$$k_1 = \omega\sqrt{\mu_1 \varepsilon_1}$$

Note different sign for exponential: opposite propagation direction

8.1 Normal Incidence: Lossless Media



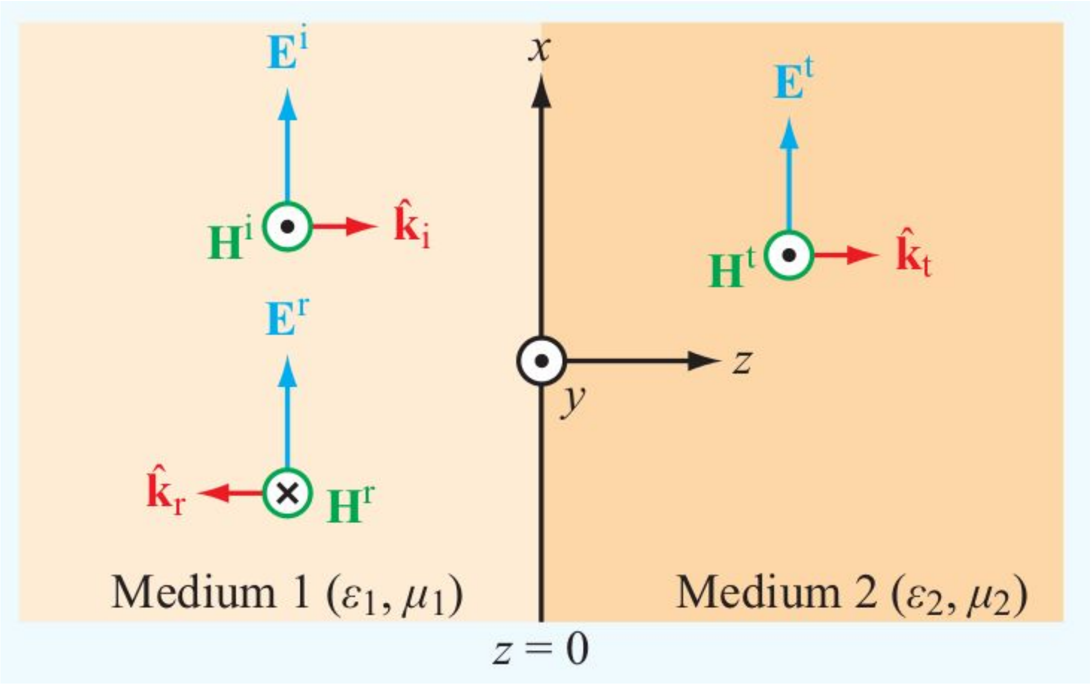
Propagation direction for the *reflected* field is opposite that of the *incident* field:

$$\hat{\mathbf{k}}_r = -\hat{\mathbf{z}}$$

$$\widetilde{\mathbf{E}}^r(z) = \hat{\mathbf{x}} E_0^r e^{jk_1 z},$$

$$\widetilde{\mathbf{H}}^r(z) = (-\hat{\mathbf{z}}) \times \frac{\widetilde{\mathbf{E}}^r(z)}{\eta_1} = -\hat{\mathbf{y}} \frac{E_0^r}{\eta_1} e^{jk_1 z}.$$

8.1 Normal Incidence: Lossless Media



Intrinsic impedance of medium 1:

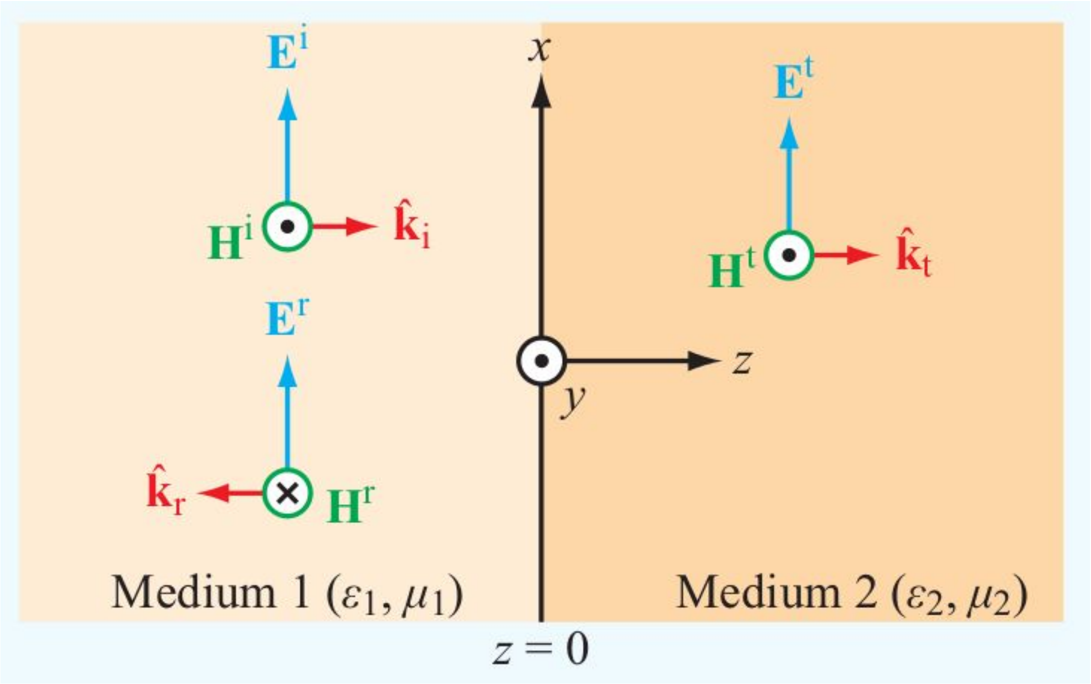
$$\eta_1 = \sqrt{\mu_1/\varepsilon_1}$$

Reflected **H** is oppositely-directed compared to incident **H**

$$\widetilde{\mathbf{E}}^r(z) = \hat{\mathbf{x}} E_0^r e^{jk_1 z},$$

$$\widetilde{\mathbf{H}}^r(z) = (-\hat{\mathbf{z}}) \times \frac{\widetilde{\mathbf{E}}^r(z)}{\eta_1} = -\hat{\mathbf{y}} \frac{E_0^r}{\eta_1} e^{jk_1 z}.$$

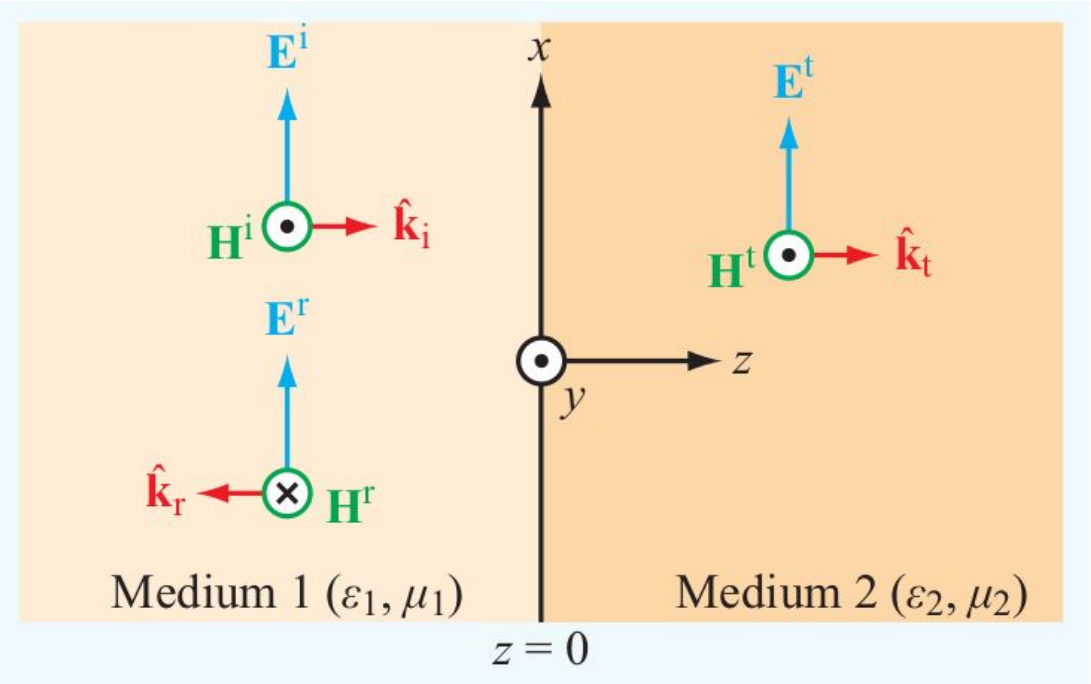
8.1 Normal Incidence: Lossless Media



Transmitted Field in medium 2

$$\widetilde{\mathbf{E}}^t(z) = \hat{\mathbf{x}} E_0^t e^{-jk_2 z},$$

8.1 Normal Incidence: Lossless Media

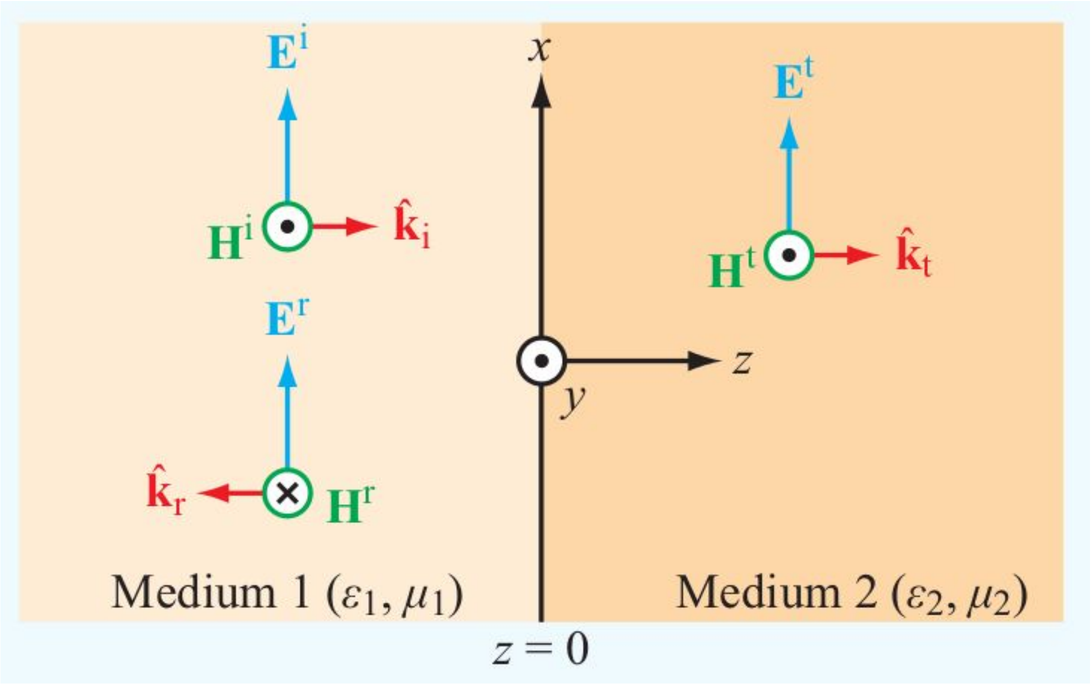


the value of the wavenumber for medium 2:

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$\widetilde{\mathbf{E}}^t(z) = \hat{\mathbf{x}}E_0^t e^{-jk_2z},$$

8.1 Normal Incidence: Lossless Media



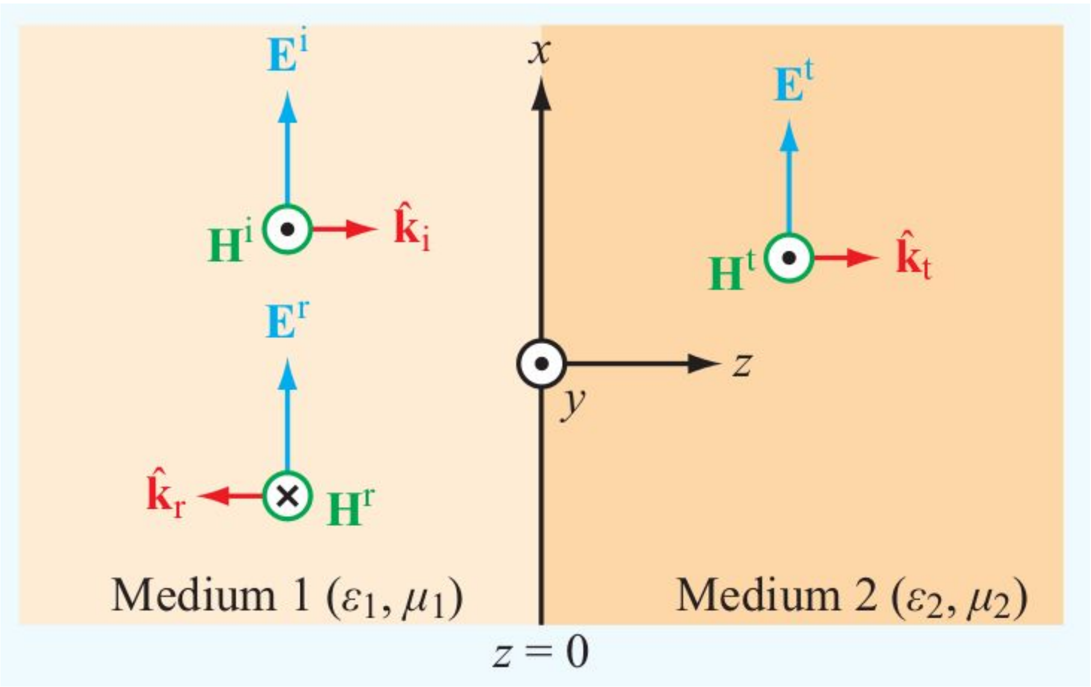
Intrinsic impedance of medium 2:

$$\eta_2 = \sqrt{\mu_2/\varepsilon_2}\,.$$

$$\widetilde{\mathbf{E}}^{\mathrm{t}}(z) = \hat{\mathbf{x}} E_0^{\mathrm{t}} e^{-jk_2 z},$$

$$\widetilde{\mathbf{H}}^{\mathrm{t}}(z) = \hat{\mathbf{z}} \times \frac{\widetilde{\mathbf{E}}^{\mathrm{t}}(z)}{\eta_2} = \hat{\mathbf{y}}\,\frac{E_0^{\mathrm{t}}}{\eta_2}\,e^{-jk_2 z}.$$

8.1 Normal Incidence: Lossless Media

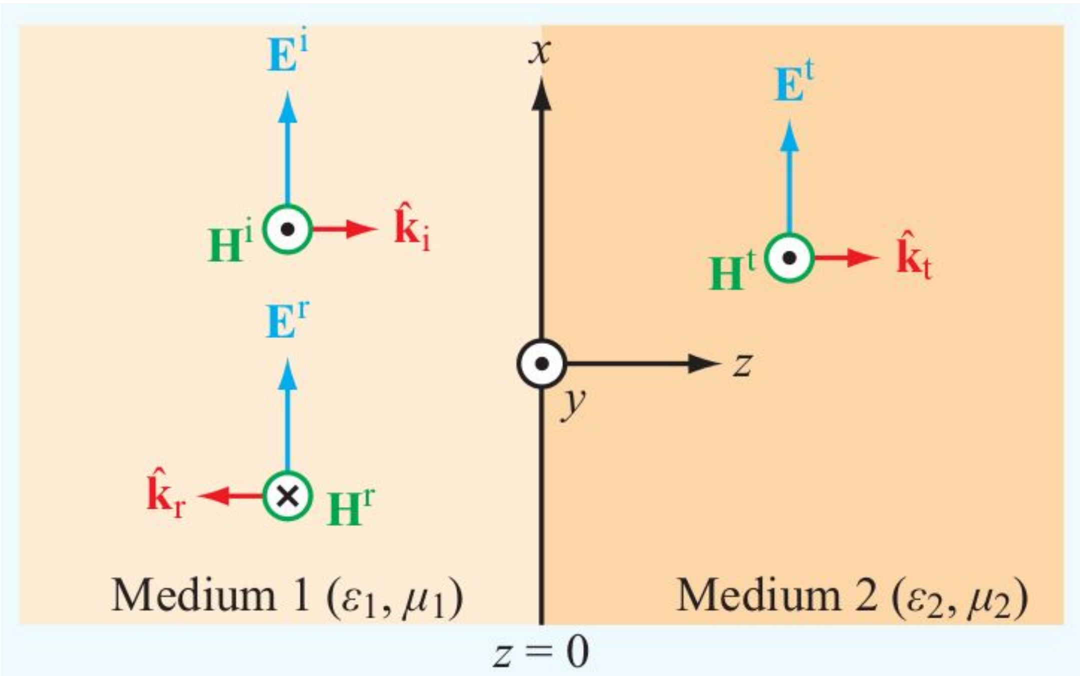


Scenario:
Know incident amplitude.

Find the reflected and transmitted amplitudes.

Solution methodology:
Apply Boundary Conditions to the **total** fields

8.1 Normal Incidence: Lossless Media



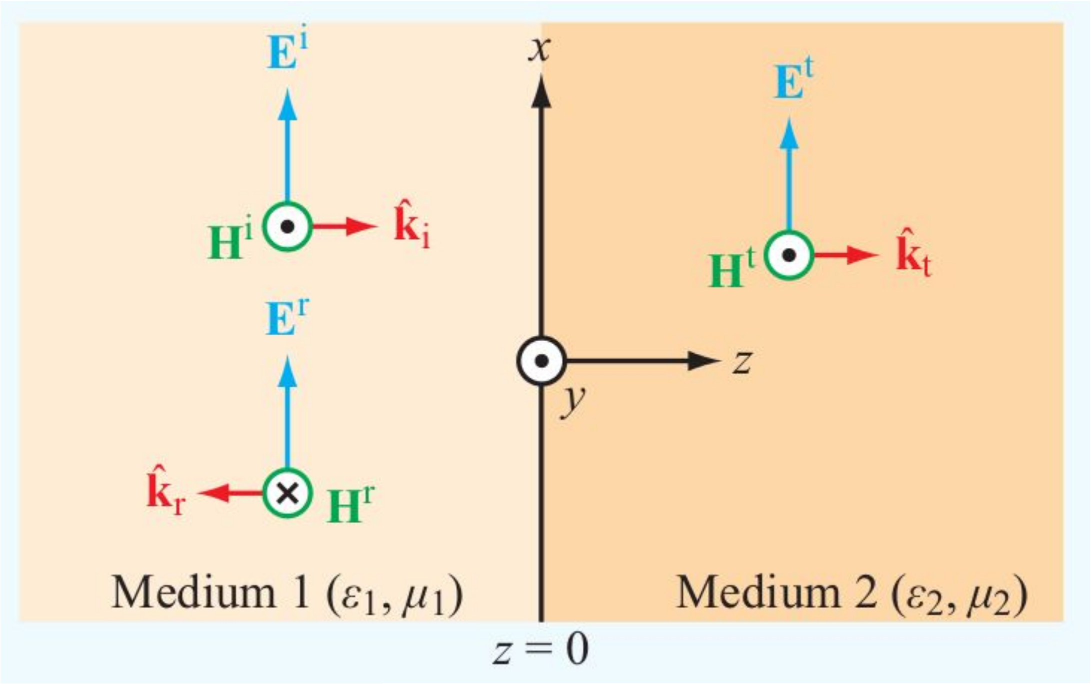
Medium 1:

Sum the incident and reflected fields:

$$\widetilde{\mathbf{E}}_1(z) = \widetilde{\mathbf{E}}^{\mathrm{i}}(z) + \widetilde{\mathbf{E}}^{\mathrm{r}}(z) = \hat{\mathbf{x}}(E_0^{\mathrm{i}} e^{-jk_1 z} + E_0^{\mathrm{r}} e^{jk_1 z}),$$

$$\widetilde{\mathbf{H}}_1(z) = \widetilde{\mathbf{H}}^{\mathrm{i}}(z) + \widetilde{\mathbf{H}}^{\mathrm{r}}(z) = \hat{\mathbf{y}} \, \frac{1}{\eta_1} \, (E_0^{\mathrm{i}} e^{-jk_1 z} - E_0^{\mathrm{r}} e^{jk_1 z}).$$

8.1 Normal Incidence: Lossless Media



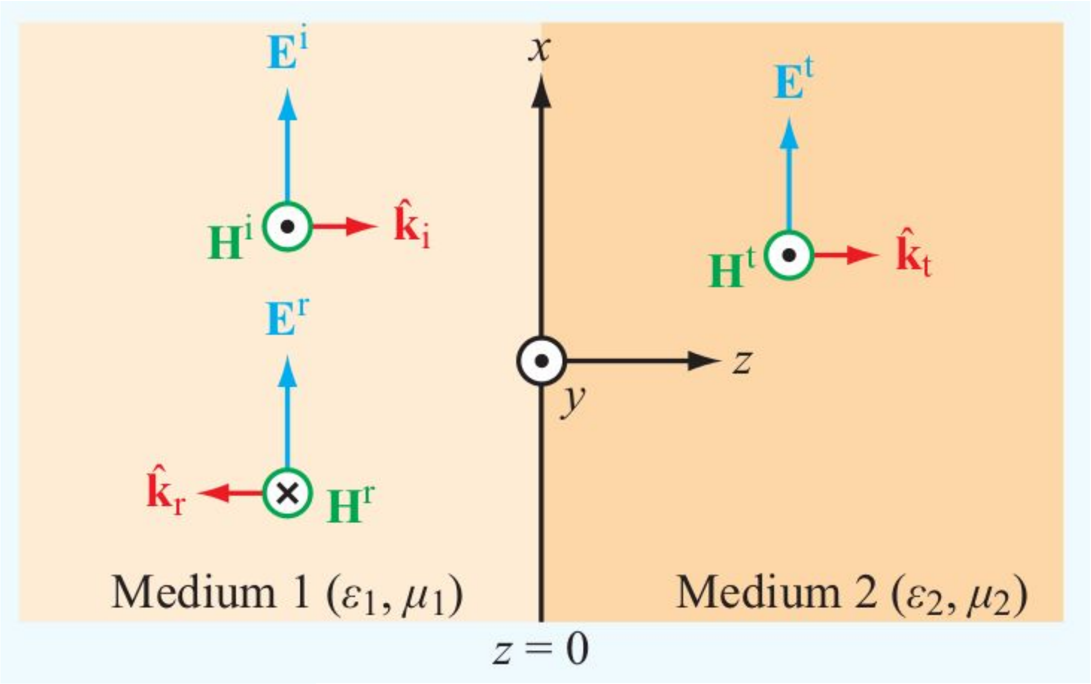
Medium 2:

Just the transmitted field:

$$\widetilde{\mathbf{E}}_2(z) = \widetilde{\mathbf{E}}^t(z) = \hat{\mathbf{x}} E_0^t e^{-jk_2 z},$$

$$\widetilde{\mathbf{H}}_2(z) = \widetilde{\mathbf{H}}^t(z) = \hat{\mathbf{y}} \frac{E_0^t}{\eta_2} e^{-jk_2 z}.$$

8.1 Normal Incidence: Lossless Media



Tangential Electric and Magnetic Fields are continuous at z=0:

$$\widetilde{\mathbf{E}}_1(0) = \widetilde{\mathbf{E}}_2(0) \quad \text{or} \quad E_0^i + E_0^r = E_0^t,$$

$$\widetilde{\mathbf{H}}_1(0) = \widetilde{\mathbf{H}}_2(0) \quad \text{or} \quad \frac{E_0^i}{\eta_1} - \frac{E_0^r}{\eta_1} = \frac{E_0^t}{\eta_2}.$$

8.1 Normal Incidence: Lossless Media

$$E_0^i + E_0^r = E_0^t$$

$$\frac{E_0^i}{\eta_1} - \frac{E_0^r}{\eta_1} = \frac{E_0^t}{\eta_2}$$

$$\frac{E_0^i}{\eta_1} - \frac{E_0^r}{\eta_1} = \frac{E_0^i + E_0^r}{\eta_2}$$

$$\frac{E_0^i - E_0^r}{\eta_1} = \frac{E_0^i + E_0^r}{\eta_2}$$

$$E_0^i - E_0^r = \frac{\eta_1}{\eta_2}(E_0^i + E_0^r)$$

8.1 Normal Incidence: Lossless Media

$$E_0^i - E_0^r = \frac{\eta_1}{\eta_2}(E_0^i + E_0^r)$$

$$E_0^i\left(1 - \frac{\eta_1}{\eta_2}\right) = E_0^r\left(1 + \frac{\eta_1}{\eta_2}\right)$$

$$E_0^i(\eta_2 - \eta_1) = E_0^r(\eta_2 + \eta_1)$$

$$E_0^r = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} E_0^i$$

8.1 Normal Incidence: Lossless Media

$$E_0^t = E_0^i + E_0^r$$

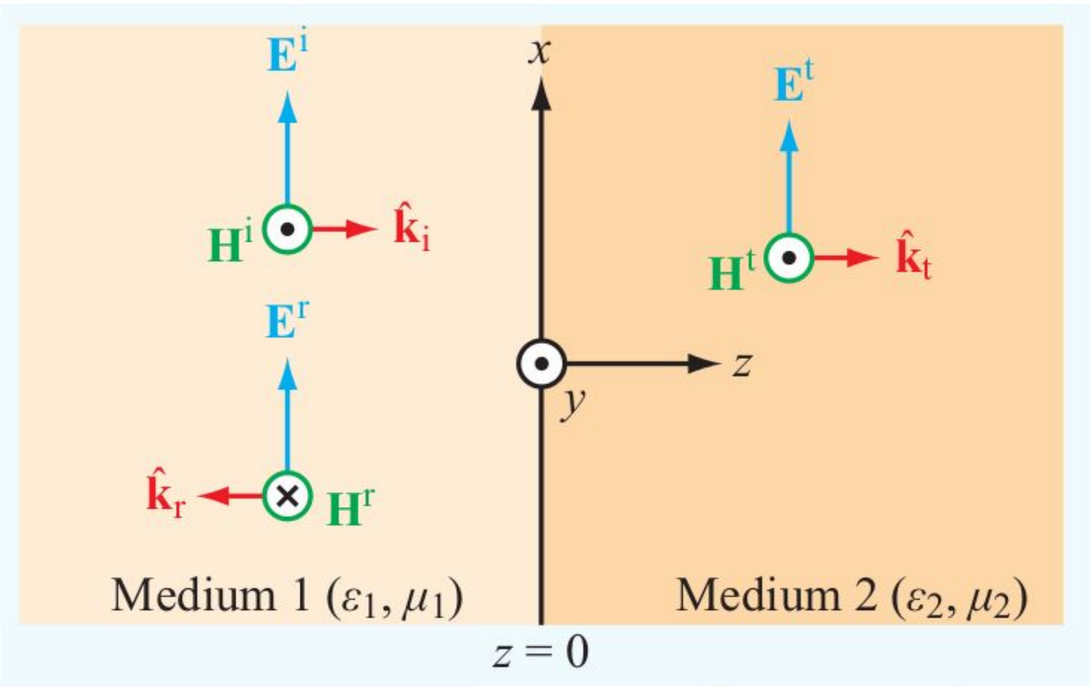
$$E_0^t = E_0^i + \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} E_0^i$$

$$E_0^t = \frac{\eta_2 + \eta_1}{\eta_2 + \eta_1} E_0^i + \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} E_0^i$$

$$E_0^t = \frac{\eta_2 + \eta_1 + \eta_2 - \eta_1}{\eta_2 + \eta_1} E_0^i$$

$$E_0^t = \frac{2\eta_2}{\eta_2 + \eta_1} E_0^i$$

8.1 Normal Incidence: Lossless Media

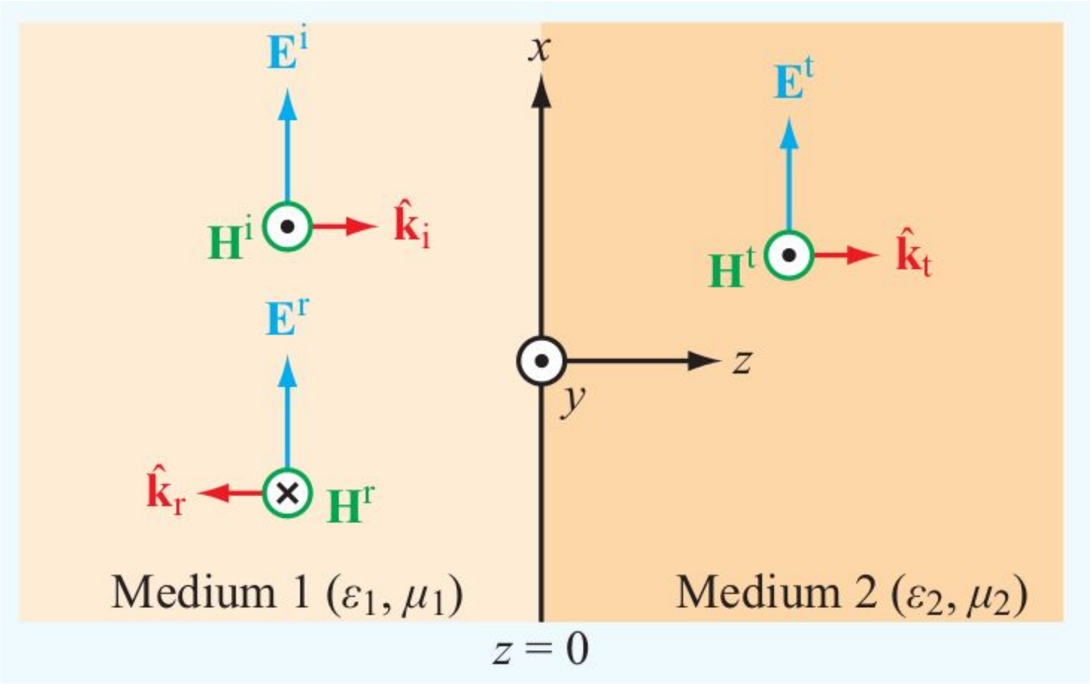


Tangential Electric and Magnetic Fields are continuous at z=0:

$$E_0^r = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right) E_0^i = \Gamma E_0^i,$$

$$E_0^t = \left(\frac{2\eta_2}{\eta_2 + \eta_1}\right) E_0^i = \tau E_0^i,$$

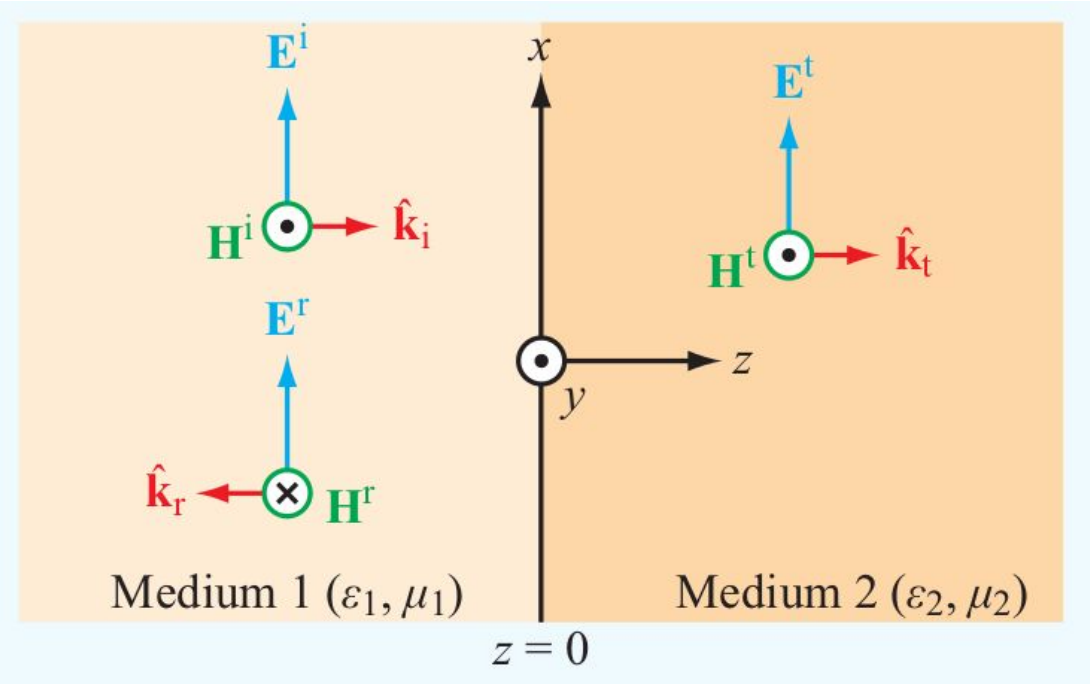
8.1 Normal Incidence: Lossless Media



Reflection Coefficient:

$$\Gamma = \frac{E_0^r}{E_0^i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad \text{(normal incidence)},$$

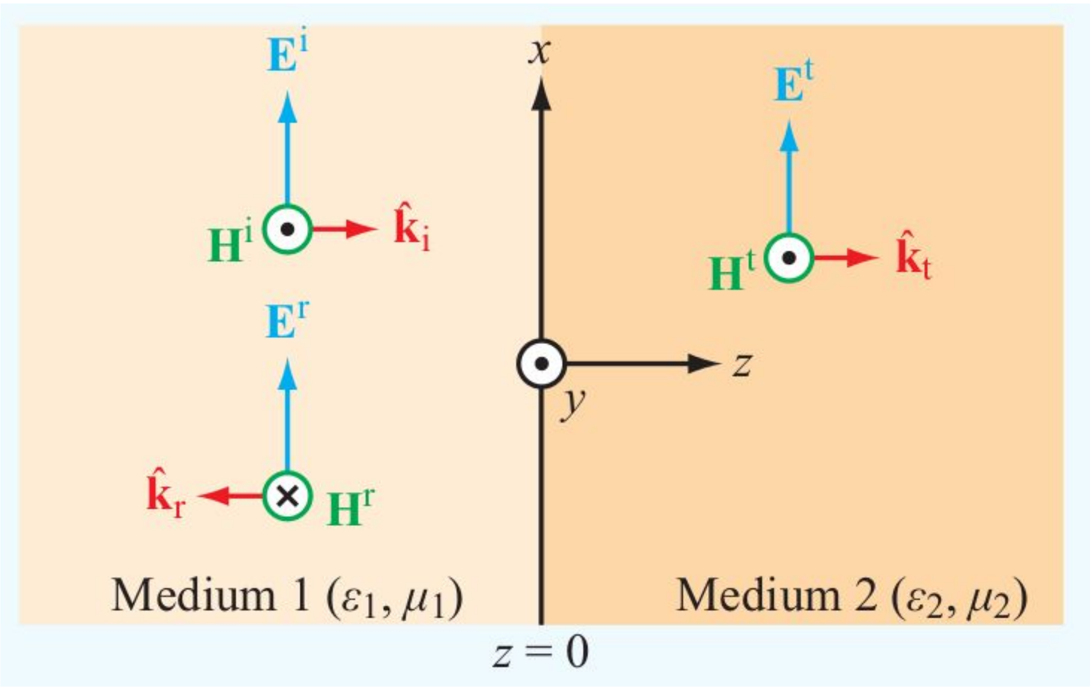
8.1 Normal Incidence: Lossless Media



Transmission Coefficient:

$$\tau = \frac{E_0^t}{E_0^i} = \frac{2\eta_2}{\eta_2 + \eta_1} \quad \text{(normal incidence).}$$

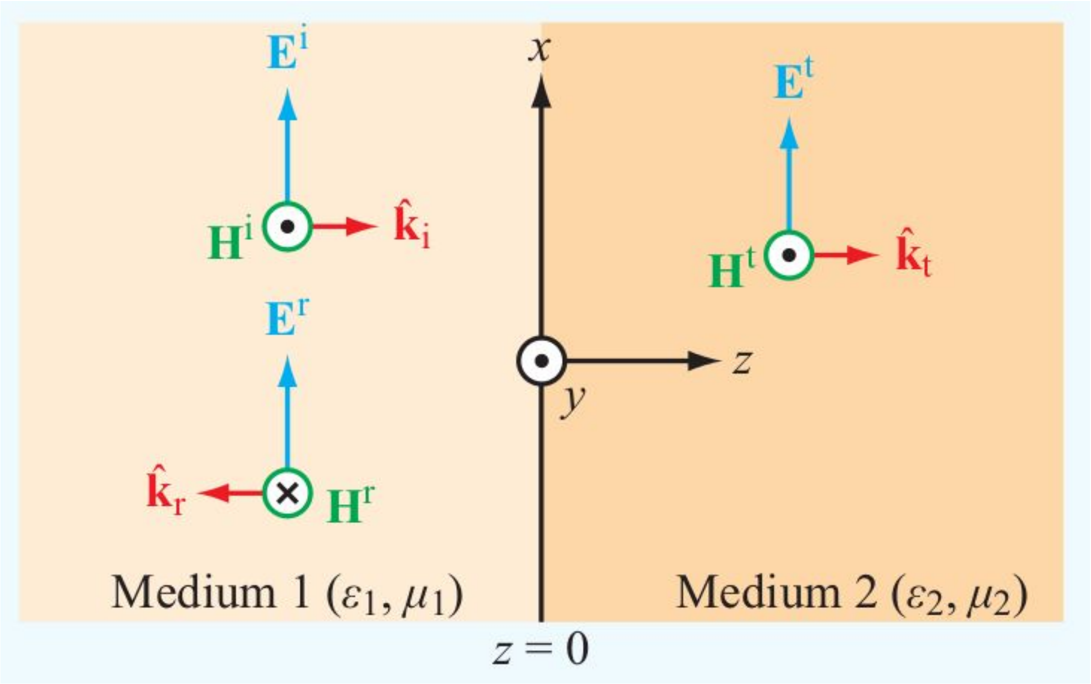
8.1 Normal Incidence: Lossless Media



Reflection and Transmission Coefficients:

$$\tau = 1 + \Gamma \qquad \text{(normal incidence).}$$

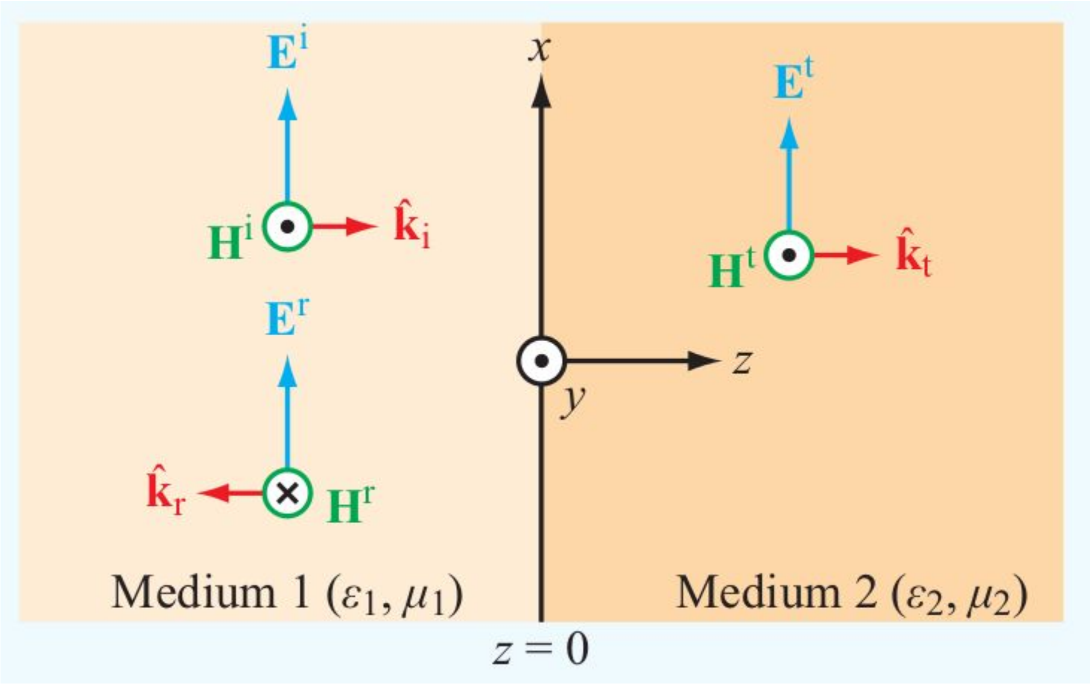
8.1 Normal Incidence: Lossless Media



Special case:
$\boldsymbol{\mu} = \boldsymbol{\mu}_0$:

$$\eta_1 = \frac{\eta_0}{\sqrt{\varepsilon_{r_1}}}, \quad \text{and} \quad \eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{r_2}}},$$

8.1 Normal Incidence: Lossless Media

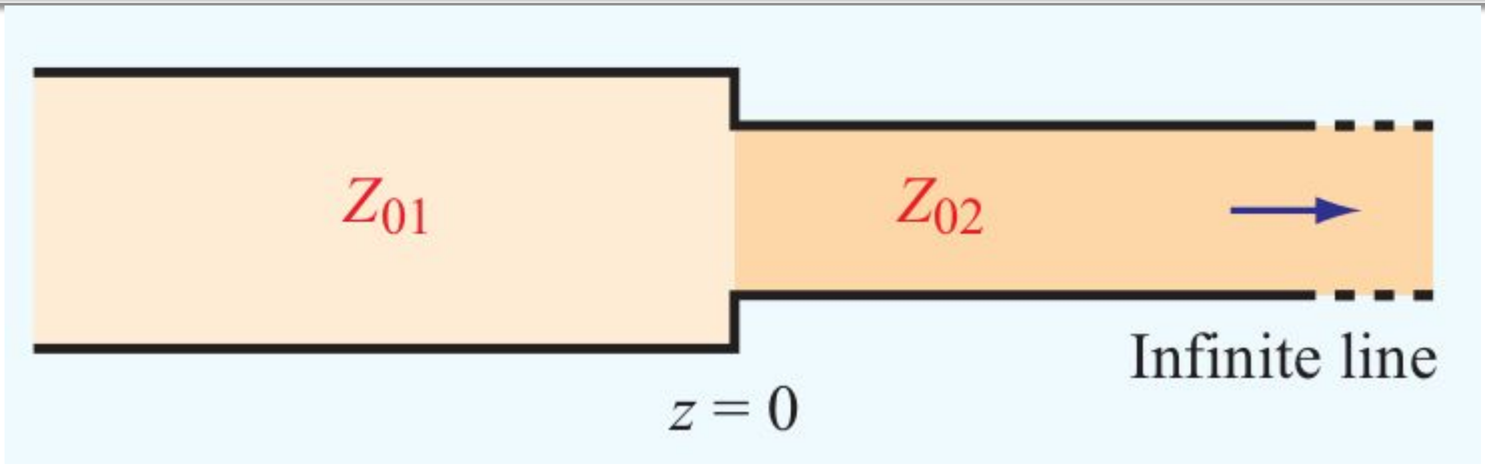


Special case:

$\boldsymbol{\mu} = \boldsymbol{\mu}_0$:

$$\Gamma = \frac{\sqrt{\varepsilon_{r_1}} - \sqrt{\varepsilon_{r_2}}}{\sqrt{\varepsilon_{r_1}} + \sqrt{\varepsilon_{r_2}}} \quad \textbf{(nonmagnetic media)}.$$

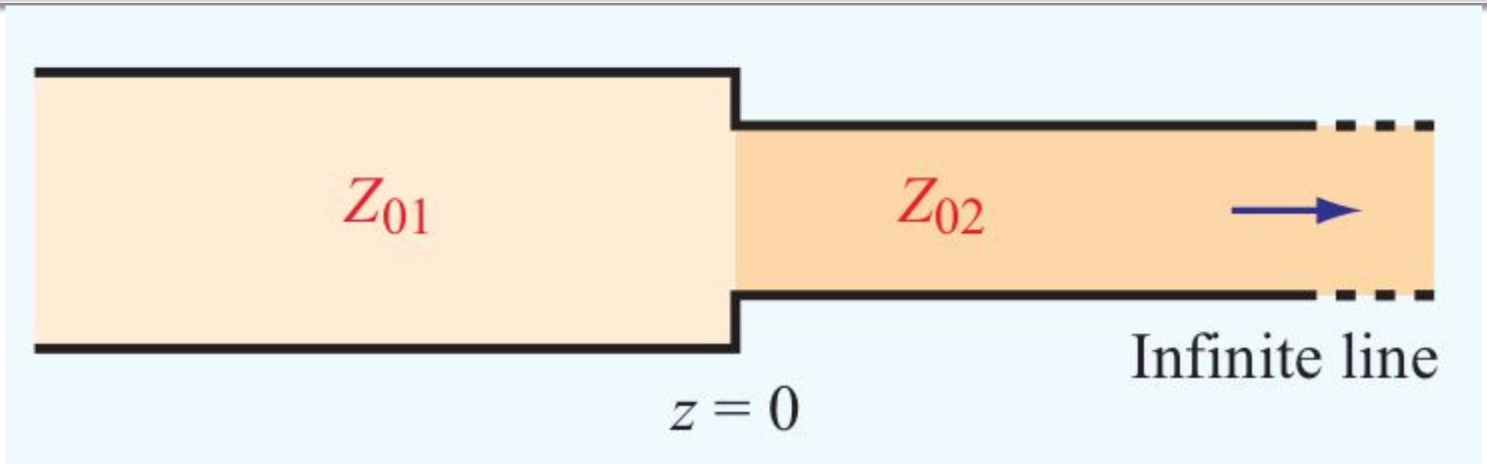
8-1 Normal Incidence: Transmission-Line Analogy



Consider this special case of 2 transmission lines: Since Z_{in} of an infinite line is Z_0, we get the voltage reflection coefficient at z=0 to be:

$$\Gamma = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}},$$

8-1 Normal Incidence: Transmission-Line Analogy



Transmission-Line equations are identical to the Plane-Wave equations

when these substitutions are made:

$$\begin{array}{ccc} \tilde{E} & \longleftrightarrow & \tilde{V} \\ \tilde{H} & \longleftrightarrow & \tilde{I} \\ k & \longleftrightarrow & \beta \\ \eta & \longleftrightarrow & Z_0 \end{array}$$

8-1 Normal Incidence: Transmission-Line Analogy

Plane Waves

$$\widetilde{\mathbf{E}}_1(z) = \hat{\mathbf{x}} E_0^{\mathrm{i}} (e^{-jk_1 z} + \Gamma e^{jk_1 z})$$

$$\widetilde{\mathbf{H}}_1(z) = \hat{\mathbf{y}} \frac{E_0^{\mathrm{i}}}{\eta_1} (e^{-jk_1 z} - \Gamma e^{jk_1 z})$$

$$\widetilde{\mathbf{E}}_2(z) = \hat{\mathbf{x}} \tau E_0^{\mathrm{i}} e^{-jk_2 z}$$

$$\widetilde{\mathbf{H}}_2(z) = \hat{\mathbf{y}} \tau \frac{E_0^{\mathrm{i}}}{\eta_2} e^{-jk_2 z}$$

Transmission Lines

$$\widetilde{V}_1(z) = V_0^+ (e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z})$$

$$\widetilde{I}_1(z) = \frac{V_0^+}{Z_{01}} (e^{-j\beta_1 z} - \Gamma e^{j\beta_1 z})$$

$$\widetilde{V}_2(z) = \tau V_0^+ e^{-j\beta_2 z}$$

$$\widetilde{I}_2(z) = \tau \frac{V_0^+}{Z_{02}} e^{-j\beta_2 z}$$

8-1 Normal Incidence: Transmission-Line Analogy

Plane Waves	Transmission Lines
$\Gamma = (\eta_2 - \eta_1)/(\eta_2 + \eta_1)$	$\Gamma = (Z_{02} - Z_{01})/(Z_{02} + Z_{01})$
$\tau = 1 + \Gamma$	$\tau = 1 + \Gamma$
$k_1 = \omega\sqrt{\mu_1\varepsilon_1}\,, \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$	$\beta_1 = \omega\sqrt{\mu_1\varepsilon_1}\,, \qquad \beta_2 = \omega\sqrt{\mu_2\varepsilon_2}$
$\eta_1 = \sqrt{\mu_1/\varepsilon_1}\,, \qquad \eta_2 = \sqrt{\mu_2/\varepsilon_2}$	Z_{01} and Z_{02} depend on transmission-line parameters

8-1 Normal Incidence: Transmission-Line Analogy

Plane Waves | Transmission Lines

$$S = \frac{|\widetilde{E}_1|_{\max}}{|\widetilde{E}_1|_{\min}} = \frac{1+|\Gamma|}{1-|\Gamma|}.$$

Special Cases:

If $\eta_1 = \eta_2$ (matched): $\Gamma = 0$, $S = 1$

if $\eta_2 = 0$ (perfect conductor or short): $\Gamma = -1$, $S = \infty$

8-1 Normal Incidence: Transmission-Line Analogy

Plane Waves | Transmission Lines

Electric Field Maximum | Voltage Maximum

$$l_{\max} = \frac{\theta_r + 2n\pi}{2k_1} = \frac{\theta_r \lambda_1}{4\pi} + \frac{n\lambda_1}{2},$$

$$\begin{cases} n = 1, 2, \ldots & \text{if } \theta_r < 0, \\ n = 0, 1, 2, \ldots & \text{if } \theta_r \geq 0, \end{cases}$$

$$\lambda_1 = 2\pi / k_1$$

Spacing between adjacent maxima is $\lambda_1/2$
Nearest minimum is a distance $\lambda_1/4$ from a maximum

8-1 Normal Incidence: Power Flow

Medium 1:

We know: (for lossless media)

$$\mathbf{S}_{\mathrm{av}_1}(z) = \frac{1}{2}\mathfrak{Re}[\widetilde{\mathbf{E}}_1(z) \times \widetilde{\mathbf{H}}_1^*(z)]$$

$$= \frac{1}{2}\mathfrak{Re}\left[\hat{\mathbf{x}}E_0^{\mathrm{i}}(e^{-jk_1z} + \Gamma e^{jk_1z}) \times \hat{\mathbf{y}}\,\frac{E_0^{\mathrm{i}*}}{\eta_1}(e^{jk_1z} - \Gamma^* e^{-jk_1z})\right]$$

$$\mathbf{S}_{av}(z) = \frac{1}{2}\hat{\mathbf{z}}\frac{E_o^i E_o^{i*}}{\eta_1}\Re e\left[e^{-jk_1z}e^{+jk_1z} + e^{-jk_1z}(-\Gamma^*)e^{-jk_1z}\right.$$

$$\left. + \Gamma e^{+jk_1z}e^{+jk_1z} + \Gamma e^{+jk_1z}(-\Gamma^*)e^{-jk_1z}\right]$$

8-1 Normal Incidence: Power Flow

Medium 1:

We know: (for lossless media)

$$\mathbf{S}_{av}(z) = \frac{1}{2}\hat{\mathbf{z}}\frac{E_o^i E_o^{i*}}{\eta_1}\Re e\left[e^{-jk_1 z}e^{+jk_1 z} + e^{-jk_1 z}(-\Gamma^*)e^{-jk_1 z}\right.$$
$$\left. +\Gamma e^{+jk_1 z}e^{+jk_1 z} + \Gamma e^{+jk_1 z}(-\Gamma^*)e^{-jk_1 z}\right]$$

$$\mathbf{S}_{av}(z) = \frac{1}{2}\hat{\mathbf{z}}\frac{|E_o^i|^2}{\eta_1}\left[1 - |\Gamma|^2 + \Re e\left\{(-\Gamma^*)e^{-j2k_1 z}\right\} + \Re e\left\{\Gamma e^{+j2k_1 z}\right\}\right]$$

$$\mathbf{S}_{av}(z) = \frac{1}{2}\hat{\mathbf{z}}\frac{|E_o^i|^2}{\eta_1}\left[1 - |\Gamma|^2 + -\Re e(\Gamma)\cos(-2k_1 z) + \Re e(\Gamma)\cos(+2k_1 z)\right]$$

$$\mathbf{S}_{av}(z) = \frac{1}{2}\hat{\mathbf{z}}\frac{|E_o^i|^2}{\eta_1}\left[1 - |\Gamma|^2 + -\Re e(\Gamma)\cos(2k_1 z) + \Re e(\Gamma)\cos(2k_1 z)\right]$$

8-1 Normal Incidence: Power Flow

Medium 1:

We know: (for lossless media)

$$\mathbf{S}_{av}(z) = \frac{1}{2}\hat{\mathbf{z}}\frac{|E_o^i|^2}{\eta_1}\left[1 - |\Gamma|^2 + -\Re e(\Gamma)\cos(2k_1 z) + \Re e(\Gamma)\cos(2k_1 z)\right]$$

$$= \hat{\mathbf{z}}\,\frac{|E_0^{\rm i}|^2}{2\eta_1}\,(1 - |\Gamma|^2),$$

8-1 Normal Incidence: Power Flow

Medium 1:

$$\mathbf{S}_{\mathrm{av}_1} = \mathbf{S}^{\mathrm{i}}_{\mathrm{av}} + \mathbf{S}^{\mathrm{r}}_{\mathrm{av}}$$

$$\mathbf{S}^{\mathrm{i}}_{\mathrm{av}} = \hat{\mathbf{z}} \frac{|E^{\mathrm{i}}_0|^2}{2\eta_1},$$

$$\mathbf{S}^{\mathrm{r}}_{\mathrm{av}} = -\hat{\mathbf{z}}|\Gamma|^2 \frac{|E^{\mathrm{i}}_0|^2}{2\eta_1} = -|\Gamma|^2 \mathbf{S}^{\mathrm{i}}_{\mathrm{av}}.$$

8-1 Normal Incidence: Power Flow

Medium 2:

$$\mathbf{S}_{\text{av}_2}(z) = \frac{1}{2}\Re\mathfrak{e}[\widetilde{\mathbf{E}}_2(z) \times \widetilde{\mathbf{H}}_2^*(z)]$$

$$\mathbf{S}_{\text{av}_2}(z) = \frac{1}{2}\Re\mathfrak{e}\left[\hat{\mathbf{x}}\tau E_0^{\text{i}} e^{-jk_2 z} \times \hat{\mathbf{y}}\tau^* \frac{E_0^{\text{i}*}}{\eta_2} e^{jk_2 z}\right]$$

$$\mathbf{S}_{\text{av}_2}(z) = \hat{\mathbf{z}}|\tau|^2 \frac{|E_0^{\text{i}}|^2}{2\eta_2}$$

8-1 Normal Incidence: Power Flow

Since we expect power to be conserved in lossless media:

$$\mathbf{S}_{\text{av}_1} = \mathbf{S}_{\text{av}_2}$$

$$\hat{\mathbf{z}} \frac{|E_0^i|^2}{2\eta_1}(1 - |\Gamma|^2) = \hat{\mathbf{z}} \frac{|E_0^i|^2}{2\eta_2} |\tau|^2$$

$$\frac{(1 - |\Gamma|^2)}{\eta_1} = \frac{|\tau|^2}{\eta_2}$$

$$\frac{\tau^2}{\eta_2} = \frac{1 - \Gamma^2}{\eta_1}, \qquad \textbf{(lossless media)}$$

(τ, Γ, η's are real for lossless media)

8-1 Normal Incidence: Power Flow

Look at the ratio of
average power transmitted to average power incident:

$$\frac{S^t_{\mathrm{av}}}{S^i_{\mathrm{av}}} = \frac{|\tau|^2|E^i_0|^2/2\eta_2}{|E^i_0|^2/2\eta_1}$$

$$\frac{S^t_{\mathrm{av}}}{S^i_{\mathrm{av}}} = \frac{|\tau|^2/\eta_2}{1/\eta_1}$$

$$\frac{S^t_{\mathrm{av}}}{S^i_{\mathrm{av}}} = \frac{|\tau|^2\eta_1}{\eta_2}$$

8-1 Normal Incidence: Power Flow

$$\frac{S^t_{\text{av}}}{S^i_{\text{av}}} = \frac{|\tau|^2 \eta_1}{\eta_2}$$

since:

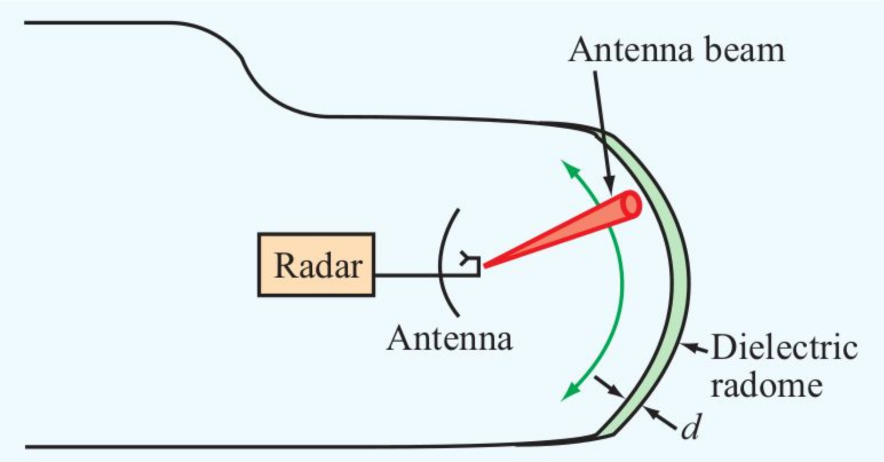
$$\tau^2 = (1 - \Gamma^2)\frac{\eta_2}{\eta_1}$$

get:

$$\frac{S^t_{\text{av}}}{S^i_{\text{av}}} = 1 - \Gamma^2$$

For lossless media

Example 8-1



Radar Radome Design

Given: 10 GHz

narrow-beam antenna

$\varepsilon_r = 9$, $\sigma = 0$, $\mu_r = 1$

Assume: radome is close to planar over the narrow beam width

Find: thickness, *d*, so that the radome appears transparent to the radar

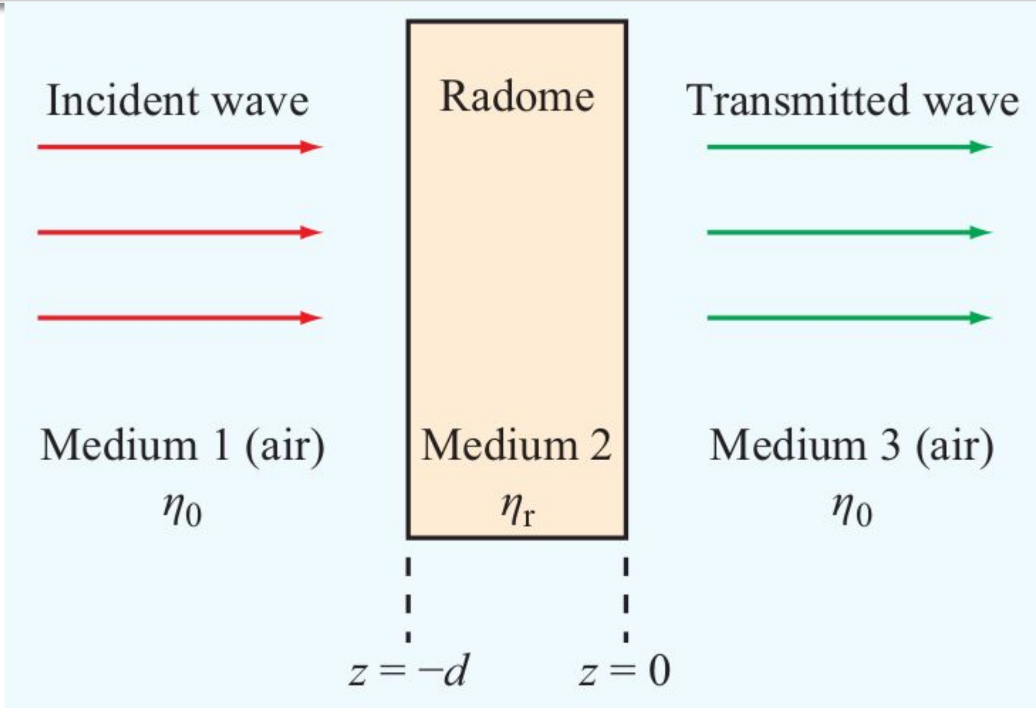
d>2.3cm for structural integrity

Example 8-1

Radar Radome Design

Solution:

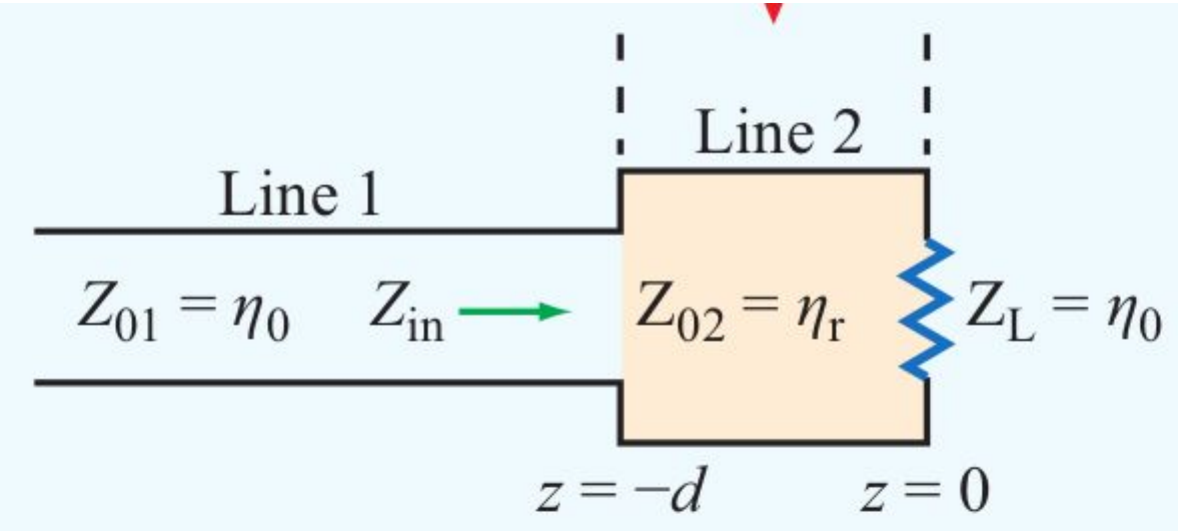
Approximate geometry is normal incidence on a planar boundary:



Example 8-1

Radar Radome Design

Equivalent Transmission-Line Model:



Transparent means no reflection:

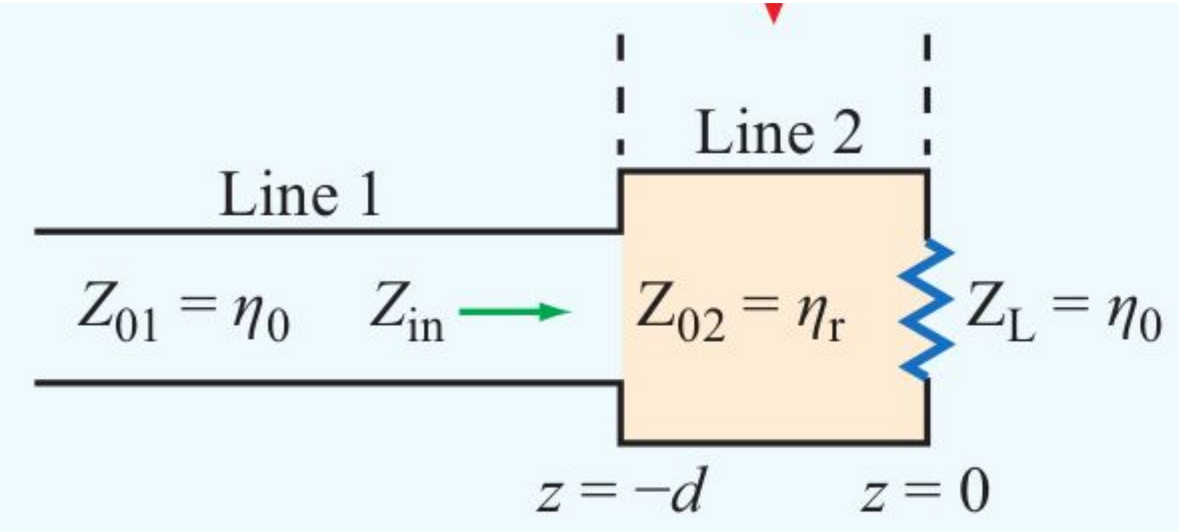
$$\Gamma(z=-d) = 0$$

hence require: $Z_{in} = \eta_0$

Example 8-1

Radar Radome Design

hence require: $Z_{in} = \eta_0$



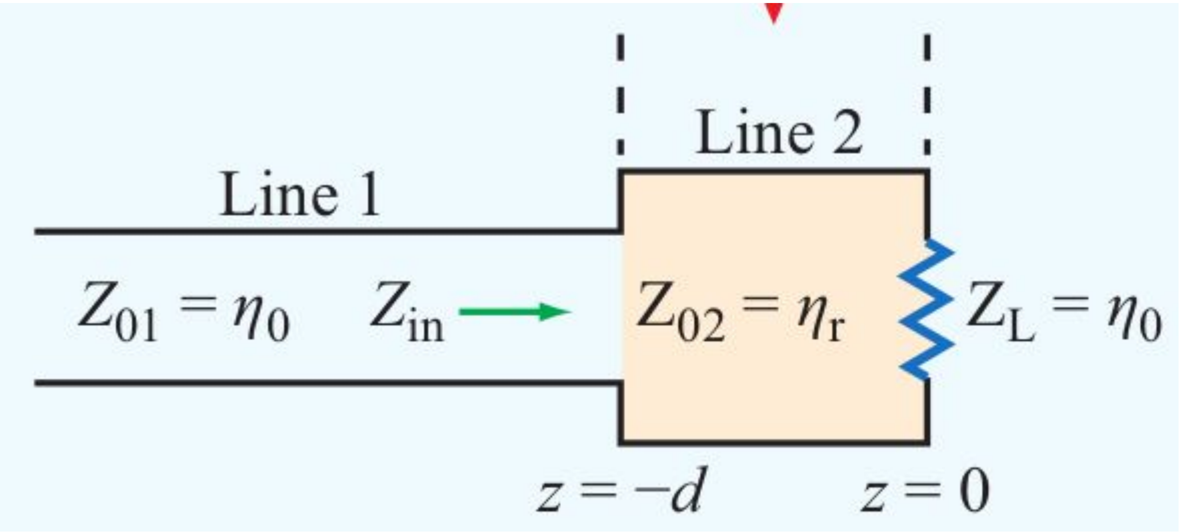
Use idea that $\lambda/2$ section has $Z_{in} = Z_L$

So choose n so that $n\lambda_2/2 > 2.3$cm

Example 8-1

Radar Radome Design

So choose *n* so that $n\lambda_2/2 > 2.3$cm



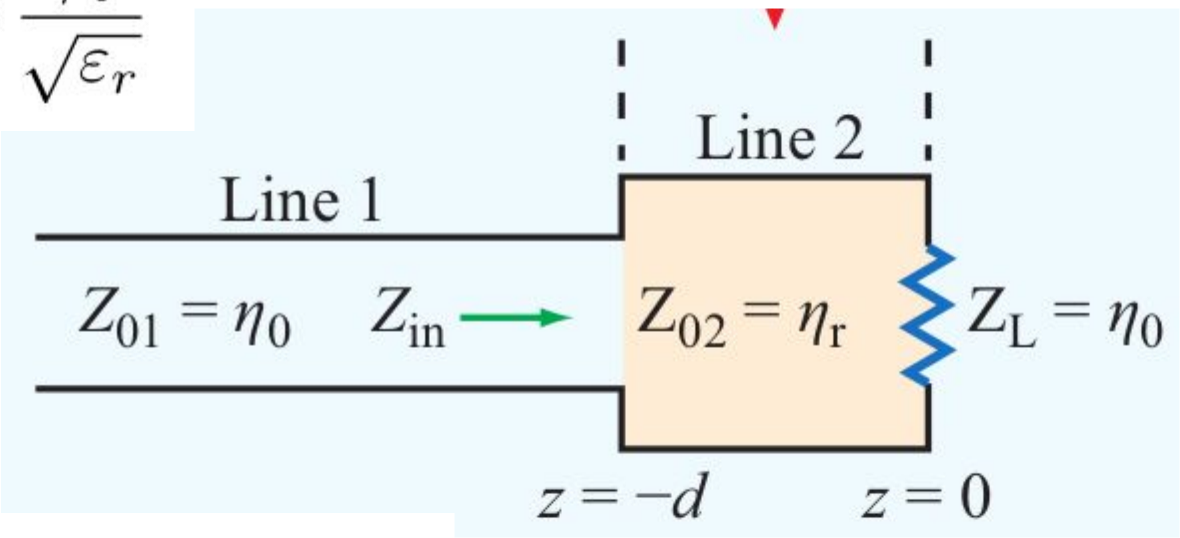
know:

$$\lambda_2 = \frac{c/f}{\sqrt{\varepsilon_r}}$$

Example 8-1

Radar Radome Design

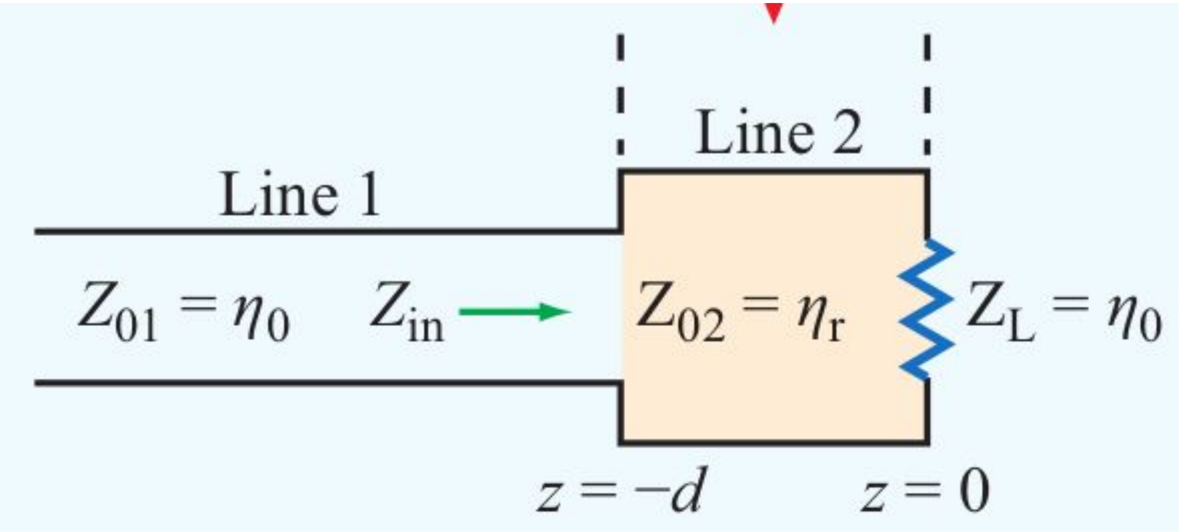
$$\lambda_2 = \frac{c/f}{\sqrt{\varepsilon_r}}$$



$$\lambda_2 = \frac{3 \times 10^8 \text{m/sec}}{(10 \times 10^9 \text{Hz})3} = 1\,\text{cm}$$

Example 8-1

Radar Radome Design



with $\lambda_2 = 1\ \text{cm}$

choose: $d = 5\lambda_2/2 = 2.5\ \text{cm}$

Example 8-2

Light Incident on Glass

Given: λ =0.6 μm

ε_r=2.25, σ=0, μ_r=1

Assume: thick enough=∞

Find: (a) locations of electric field maxima

(b) voltage standing-wave ratio: *S*

(c) fraction of incident power transmitted into the glass

Solution: know: $$l_{\max} = \frac{\theta_r \lambda_1}{4\pi} + n\frac{\lambda_1}{2}$$

so need θ_r, hence Γ, which means we need η_1 and η_2

Example 8-2

Light Incident on Glass

$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 120\pi \ (\Omega),$$

$$\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \frac{1}{\sqrt{\varepsilon_r}} \approx \frac{120\pi}{\sqrt{2.25}} = 80\pi \ (\Omega),$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{80\pi - 120\pi}{80\pi + 120\pi} = -0.2.$$

Example 8-2

Light Incident on Glass

Hence, $|\Gamma| = 0.2$ and $\theta_r = \pi$.

giving:

$$l_{max} = \frac{\theta_r \lambda_1}{4\pi} + n\frac{\lambda_1}{2} = \frac{\lambda_1}{4} + n\,\frac{\lambda_1}{2}$$

where: $\lambda_1 = 0.6\ \mu$m

also:

$$S = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.2}{1-0.2} = \boxed{1.5.}$$

Example 8-2

Light Incident on Glass

get:

$$\frac{S_{\text{av}_2}}{S^{\text{i}}_{\text{av}}} = 1 - |\Gamma|^2 = 1 - (0.2)^2 = 0.96, \text{ or } \boxed{96\%}$$

= the fraction of incident power transmitted into the glass.

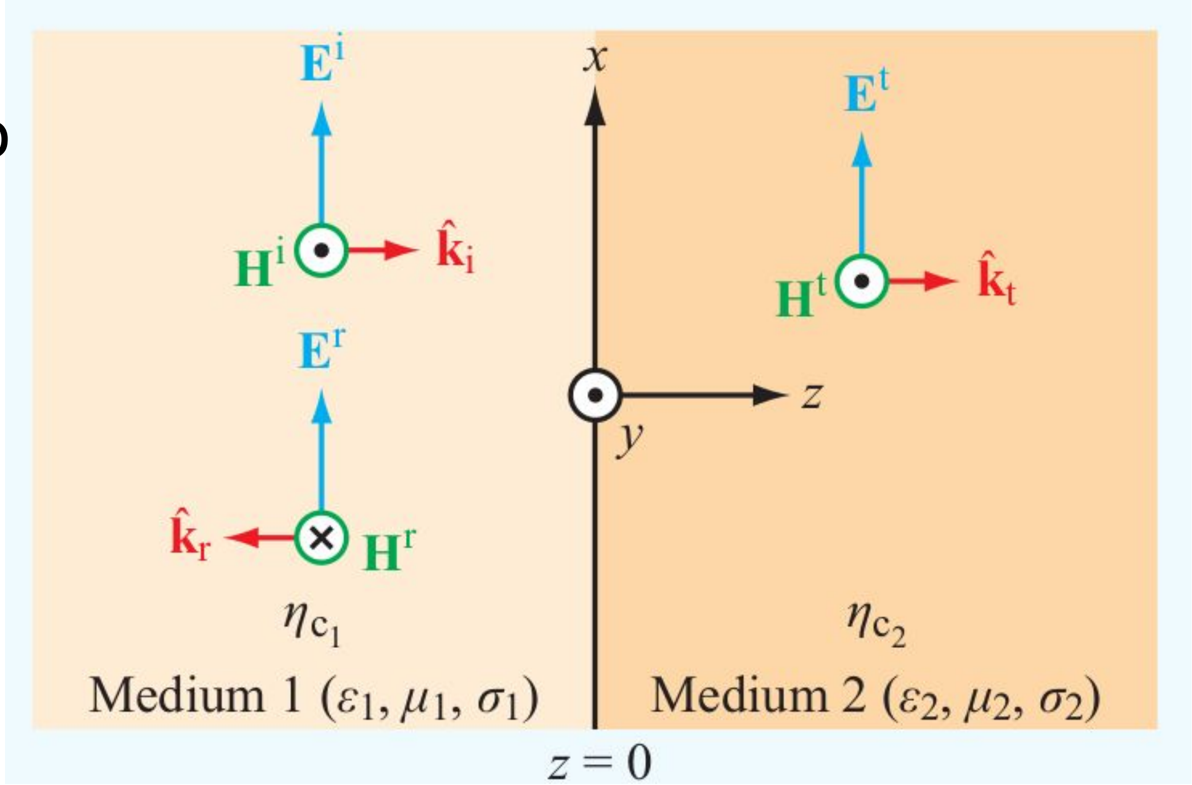
8-1 Normal Incidence: Boundary Between Lossy Media

Allow both media to have non-zero conductivity:

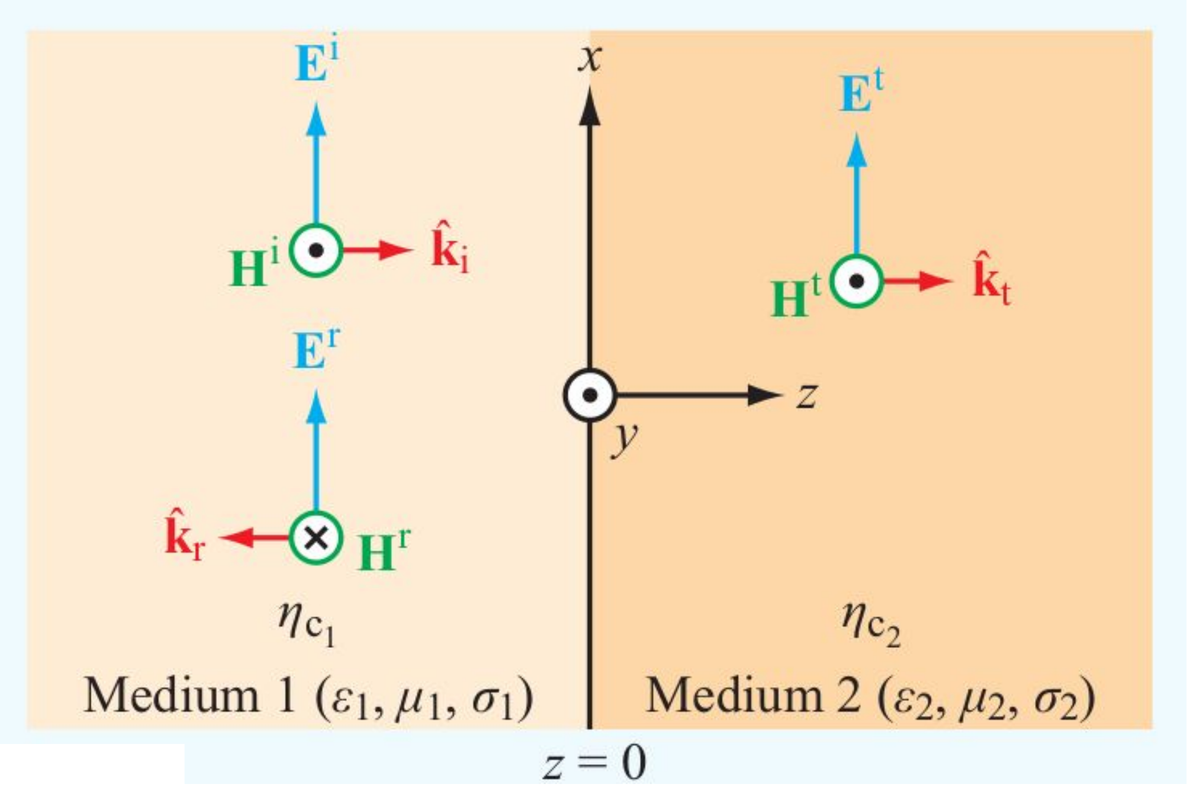
must use:

γ instead of k

η_c instead of η



8-1 Normal Incidence: Boundary Between Lossy Media

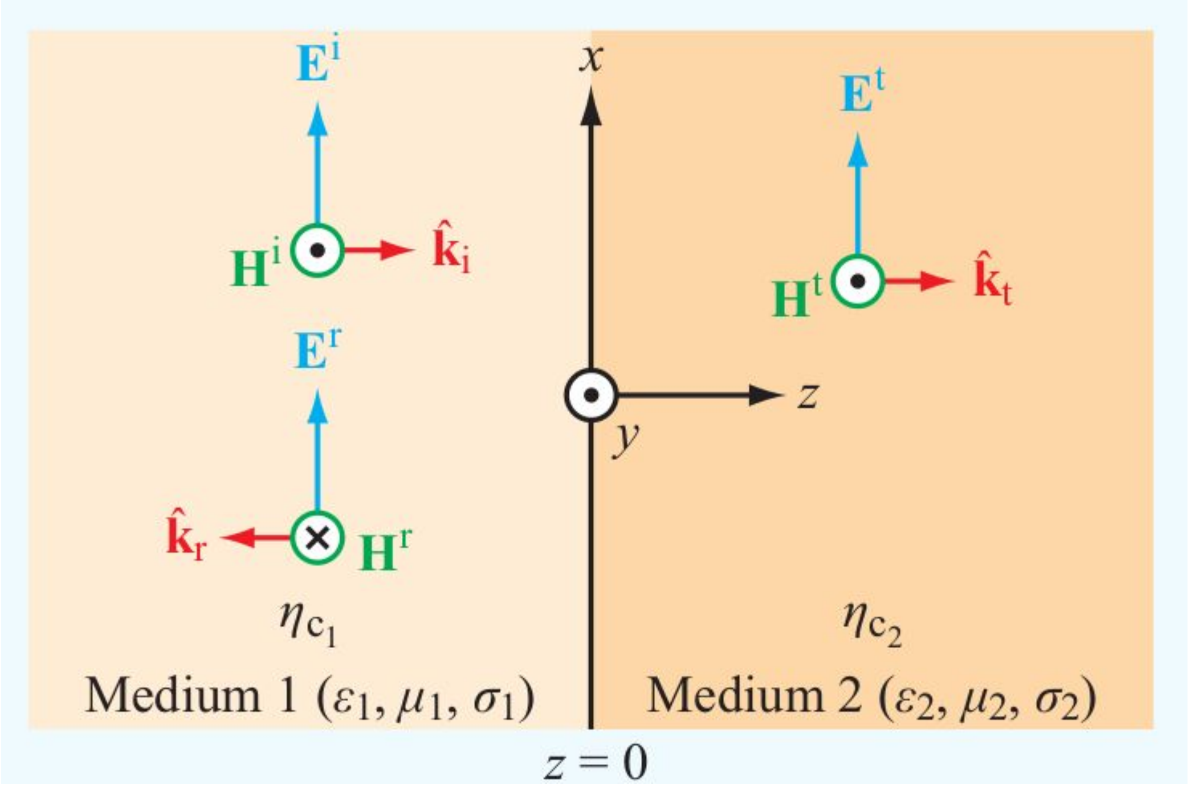


Medium 1

$$\widetilde{\mathbf{E}}_1(z) = \hat{\mathbf{x}} E_0^i (e^{-\gamma_1 z} + \Gamma e^{\gamma_1 z}),$$

$$\widetilde{\mathbf{H}}_1(z) = \hat{\mathbf{y}} \frac{E_0^i}{\eta_{c_1}} (e^{-\gamma_1 z} - \Gamma e^{\gamma_1 z}),$$

8-1 Normal Incidence: Boundary Between Lossy Media



Medium 2

$$\widetilde{\mathbf{E}}_2(z) = \hat{\mathbf{x}}\tau E_0^i e^{-\gamma_2 z},$$

$$\widetilde{\mathbf{H}}_2(z) = \hat{\mathbf{y}}\tau \frac{E_0^i}{\eta_{c_2}} e^{-\gamma_2 z}.$$

8-1 Normal Incidence: Boundary Between Lossy Media

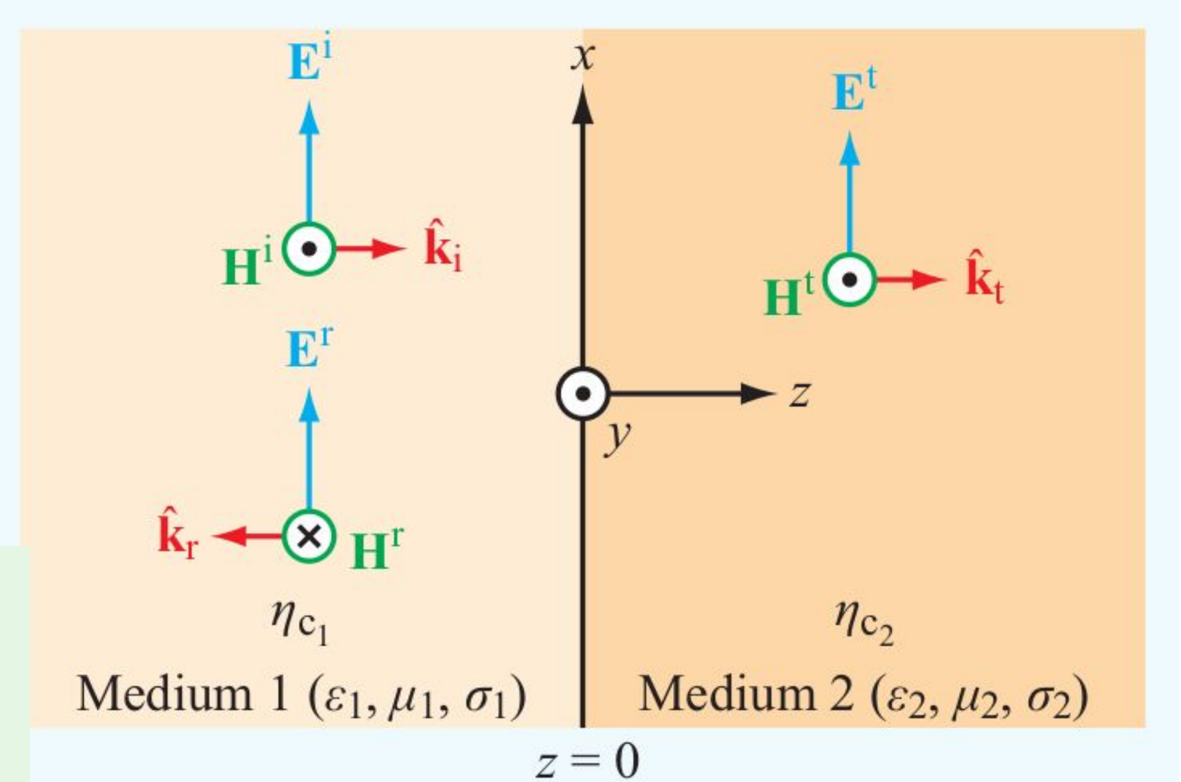
Complex:

$$\gamma_1 = \alpha_1 + j\beta_1,$$

$$\gamma_2 = \alpha_2 + j\beta_2,$$

$$\Gamma = \frac{\eta_{c_2} - \eta_{c_1}}{\eta_{c_2} + \eta_{c_1}},$$

$$\tau = 1 + \Gamma = \frac{2\eta_{c_2}}{\eta_{c_2} + \eta_{c_1}}$$



Example 8-3

Normal Incidence on Metal

Given: 1 GHz
x-polarized plane wave
traveling in +z-direction
in air
incident on copper:
$\varepsilon_r=1$, $\mu_r=1$, $\sigma = 5.8 \text{ X } 10^7$ S/m
$|E^i| = 12$ mV/m

Assume: metal is several skin depths thick

Find: $\mathbf{E}_1$ and $\mathbf{H}_1$

Example 8-3

Normal Incidence on Metal

Solution:

need γ, η, Γ:

in medium 1 (air) α=0

$$\beta = k_1 = \frac{\omega}{c} = \frac{2\pi \times 10^9}{3 \times 10^8} = \frac{20\pi}{3} = 20.94 \quad \text{rad/m}$$

$$\eta_1 = \eta_0 = 377 \ (\Omega), \qquad \lambda = \frac{2\pi}{k_1} = 0.3 \text{ m}.$$

Example 8-3

Normal Incidence on Metal

Use good conductor formula for η_{c2}:

$$\eta_{c_2} = (1+j)\sqrt{\frac{\pi f \mu}{\sigma}}$$

$$\eta_{c_2} = (1+j)\left[\frac{\pi \times 10^9 \times 4\pi \times 10^{-7}}{5.8 \times 10^7}\right]^{1/2}$$

$$\eta_{c_2} = 8.25(1+j) \qquad (\text{m}\Omega).$$

Example 8-3

Normal Incidence on Metal

For air: $\eta_{c1} = \eta_0 = 377\ \Omega$

Calculate Γ: since $\eta_{c2} << \eta_0$:

$$\Gamma = \frac{\eta_{c_2} - \eta_0}{\eta_{c_2} + \eta_0} \approx -1.$$

plug in:

$$\widetilde{\mathbf{E}}_1(z) = \hat{\mathbf{x}} E_0^{\mathrm{i}} (e^{-jk_1 z} - e^{jk_1 z}) = -\hat{\mathbf{x}} j2E_0^{\mathrm{i}} \sin k_1 z,$$

$$\widetilde{\mathbf{H}}_1(z) = \hat{\mathbf{y}}\, \frac{E_0^{\mathrm{i}}}{\eta_1}\, (e^{-jk_1 z} + e^{jk_1 z}) = \hat{\mathbf{y}} 2\, \frac{E_0^{\mathrm{i}}}{\eta_1} \cos k_1 z.$$

Example 8-3

Normal Incidence on Metal

$$\mathbf{E}_1(z,t) = \mathfrak{Re}[\widetilde{\mathbf{E}}_1(z)\, e^{j\omega t}]$$

$$= \hat{\mathbf{x}}\, 2E_0^{\mathrm{i}} \sin k_1 z \sin \omega t$$

$$\mathbf{E}_1(z,t) = \hat{\mathbf{x}}\, 24 \sin((20.9\text{rad/m})z) \sin((6.28\text{rad/sec})t) \quad \text{mV/m}$$

$$\mathbf{H}_1(z,t) = \mathfrak{Re}[\widetilde{\mathbf{H}}_1(z)\, e^{j\omega t}]$$

$$= \hat{\mathbf{y}}\, 2\, \frac{E_0^{\mathrm{i}}}{\eta_1} \cos k_1 z \cos \omega t$$

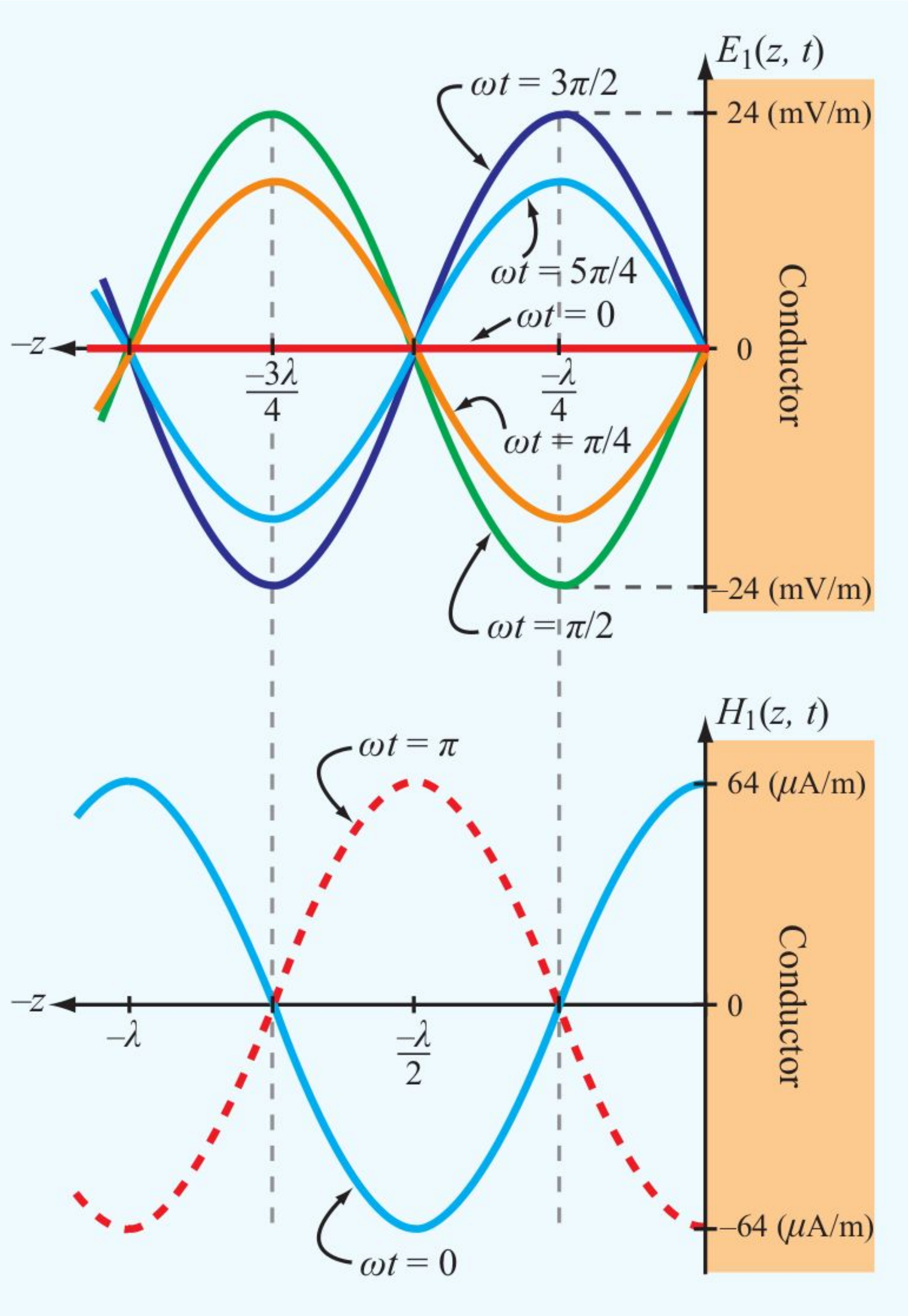
$$\mathbf{H}_1(z,t) = \hat{\mathbf{y}}\, 64 \cos((20.9\text{rad/m})z) \cos((6.28\text{rad/sec})t) \quad \mu\text{A/m}$$

Example 8-3

Normal Incidence on Metal

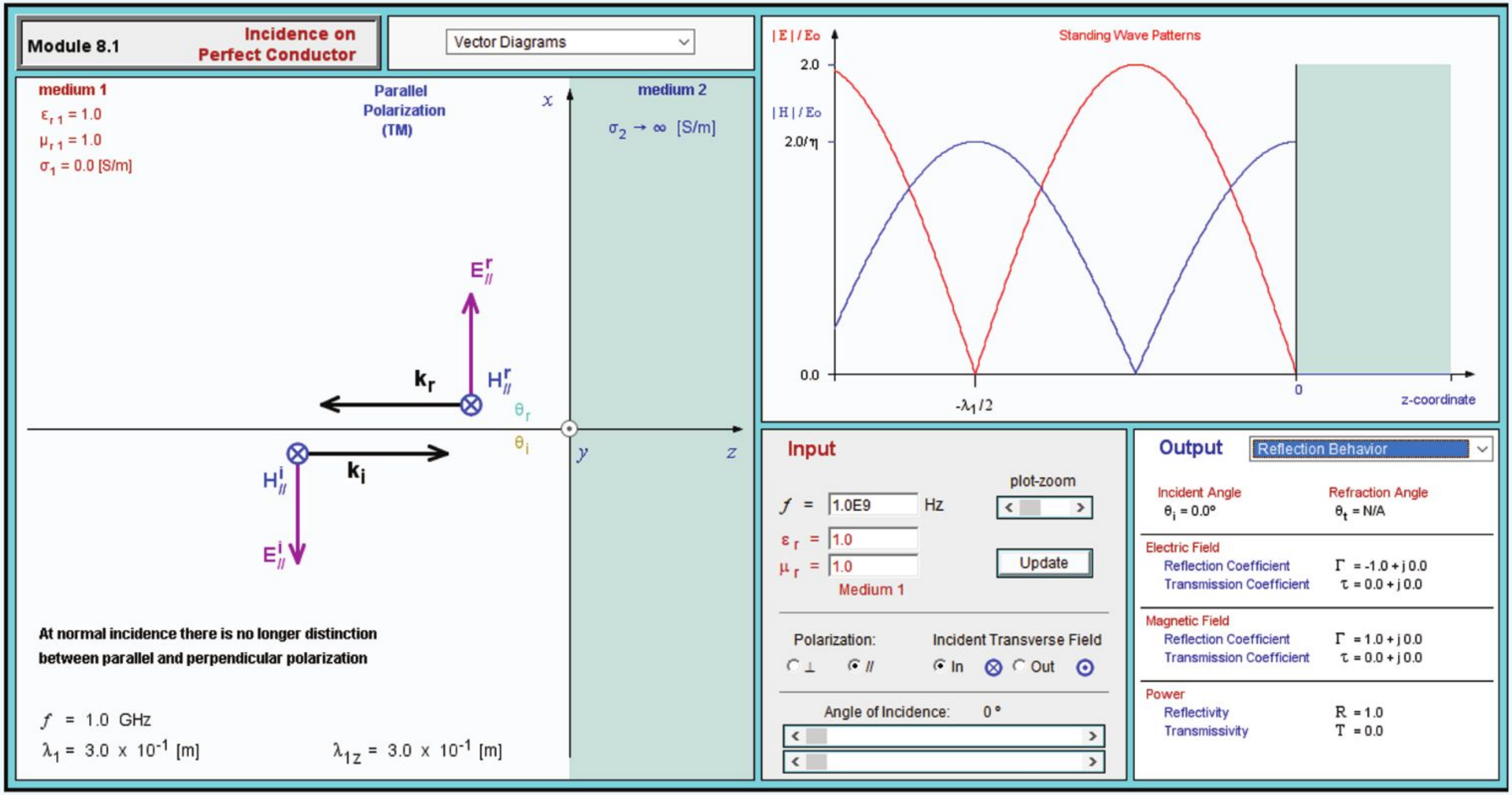
Standing waves:
envelope repeats every $\lambda/2$
E and **H:** 90° out of phase

Just like for V and I on a shorted transmission line



Normal Incidence: Perfect Conductor

Module 8.1 Incidence on Perfect Conductor Observe the standing-wave pattern created by the combination of a wav
incident normally onto the plane surface of a conductor and its reflection.

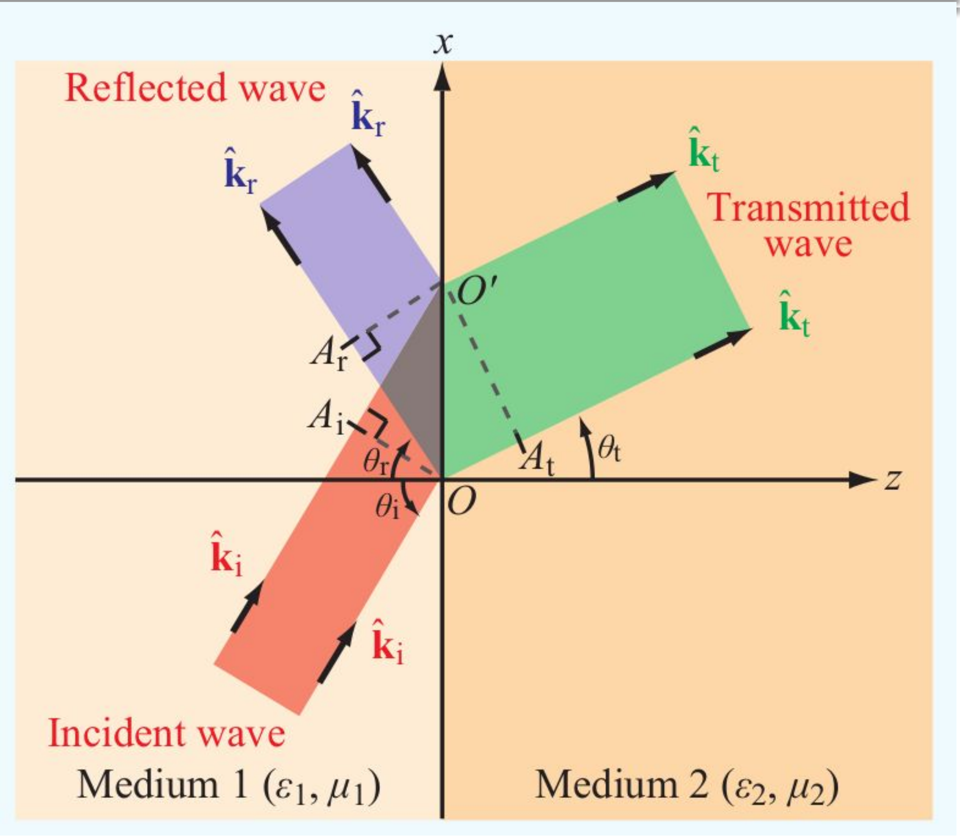


8-2 Snell's Laws

Now consider the Oblique Incidence Case:

Lossless Media

Rays are directed along the propagation vectors:



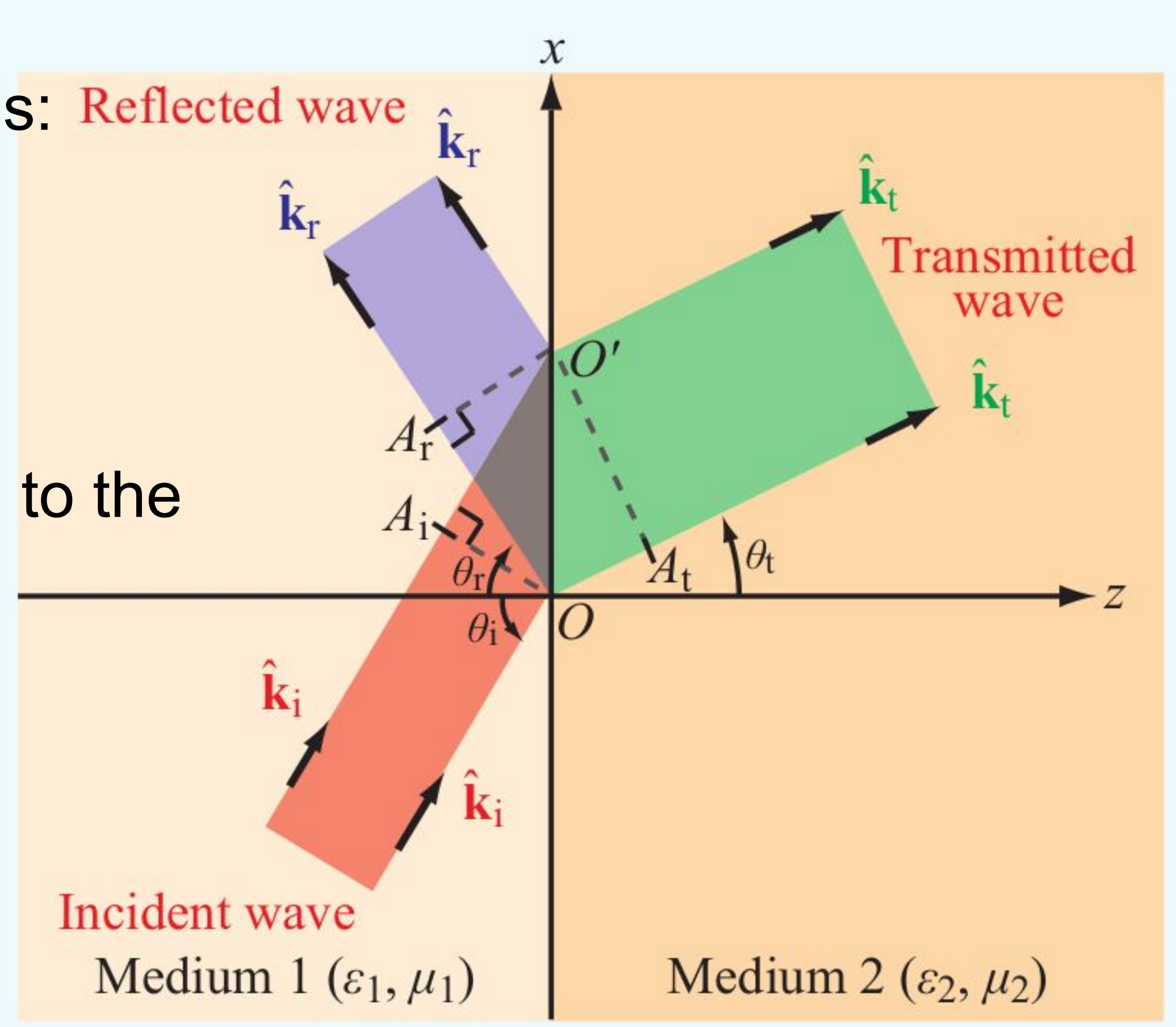
Still have the 3 directions:
Incident
Reflected
Transmitted (sometimes called Refracted)

8-2 Snell's Laws

These 3 directions:
Incident
Reflected
Transmitted

No longer normal to the boundary.

Define angles with respect to the normal to the boundary



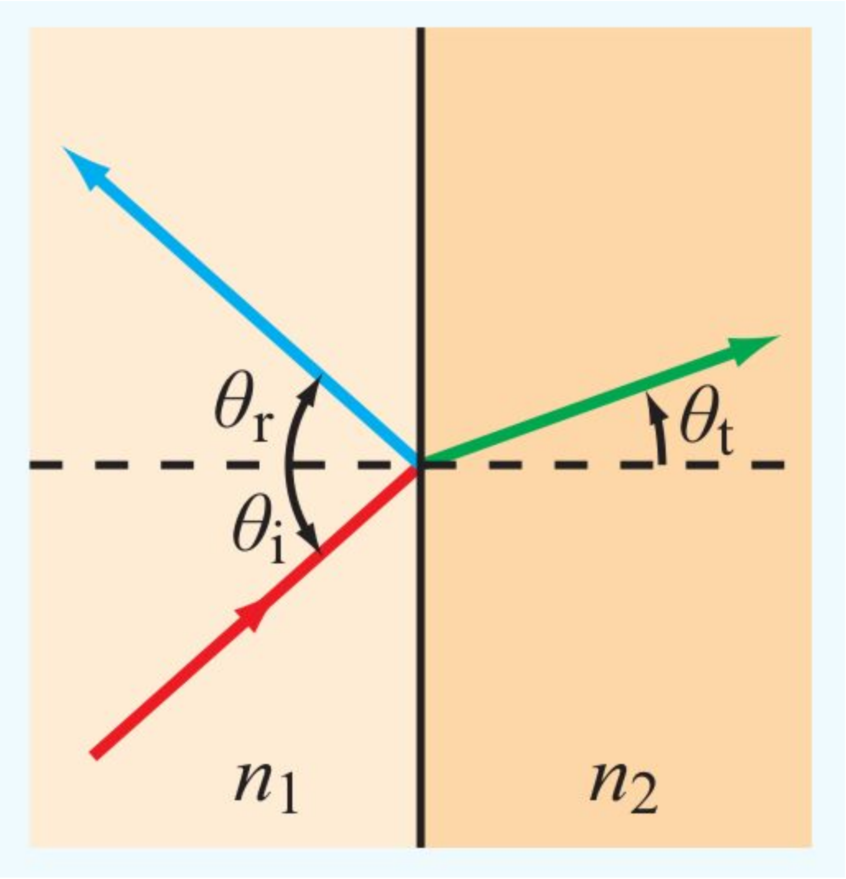
8-2 Snell's Laws

Normal to surface is the dashed line

Angle of Incidence: θ_i

Angle of Reflection: θ_r

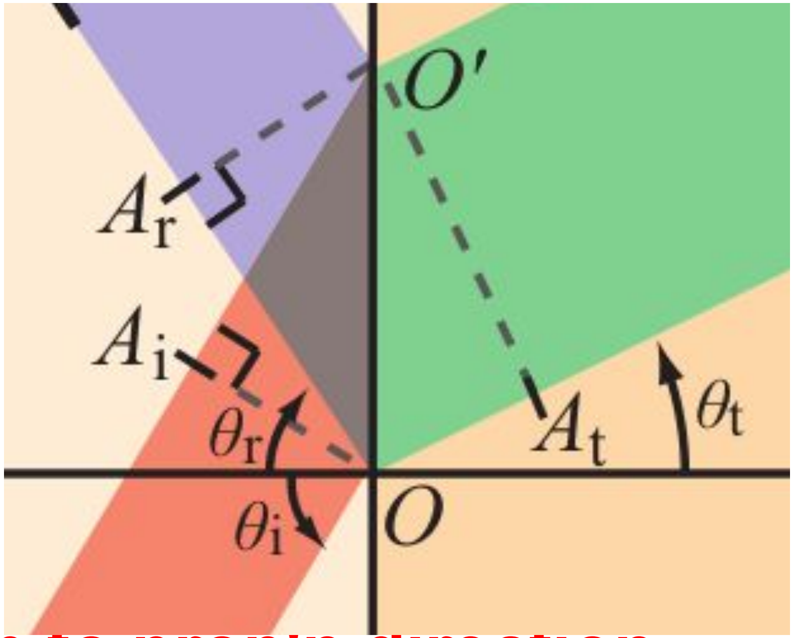
Angle of Transmission: θ_t



8-2 Snell's Laws

Let's derive the relationship between these angles

Use geometry and velocities for each ray at the edges of the beams



1. Wavefronts:

constant phase surfaces: perp to prop'n direction

$O\,A_i$

$O'\,A_r$

$O'\,A_t$

8-2 Snell's Laws

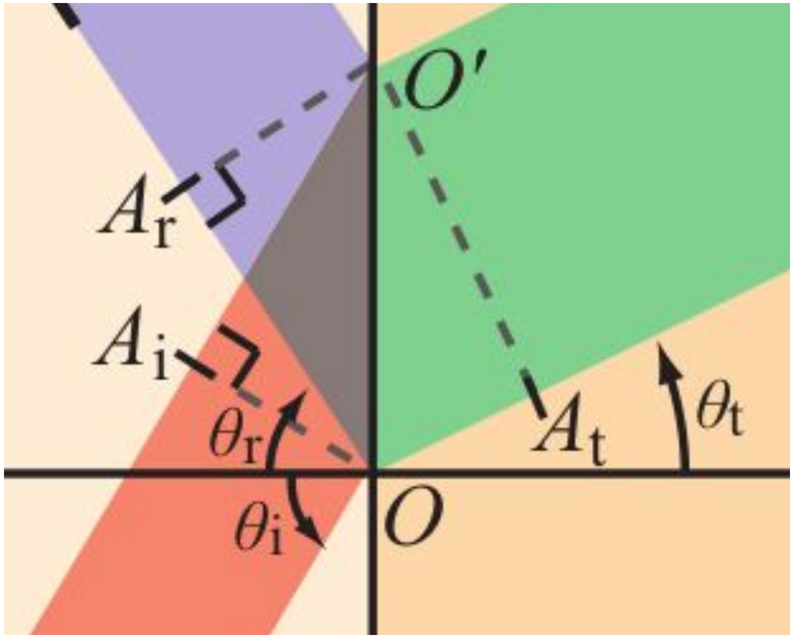
2. Phase velocities:

medium 1 (on left):

$$u_{p_1} = 1/\sqrt{\mu_1 \varepsilon_1},$$

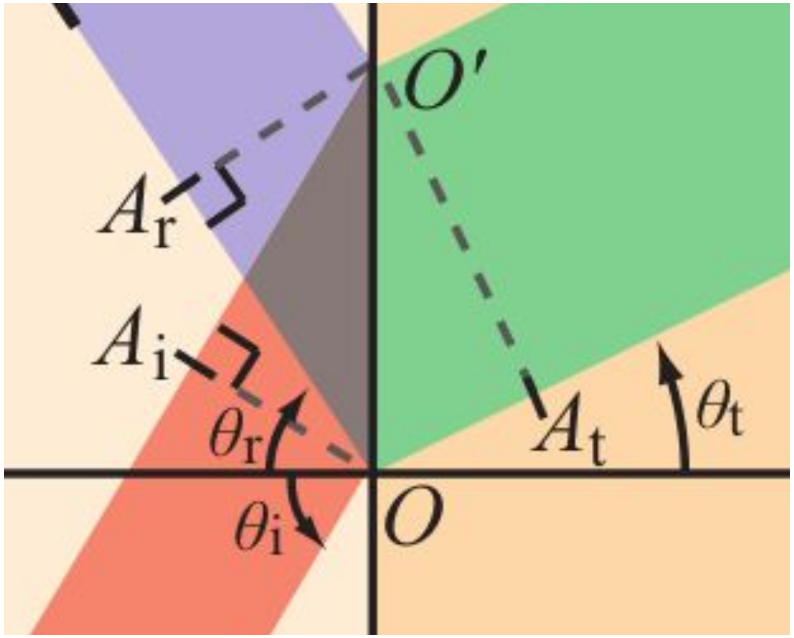
medium 2 (on right):

$$u_{p_2} = 1/\sqrt{\mu_2 \varepsilon_2}.$$



8-2 Snell's Laws

3. Propagation times:



Since the line segment

$O'A_r$

is constant phase:

propagation times must be equal for lengths:

length(OA_r) and length(A_iO')

and:

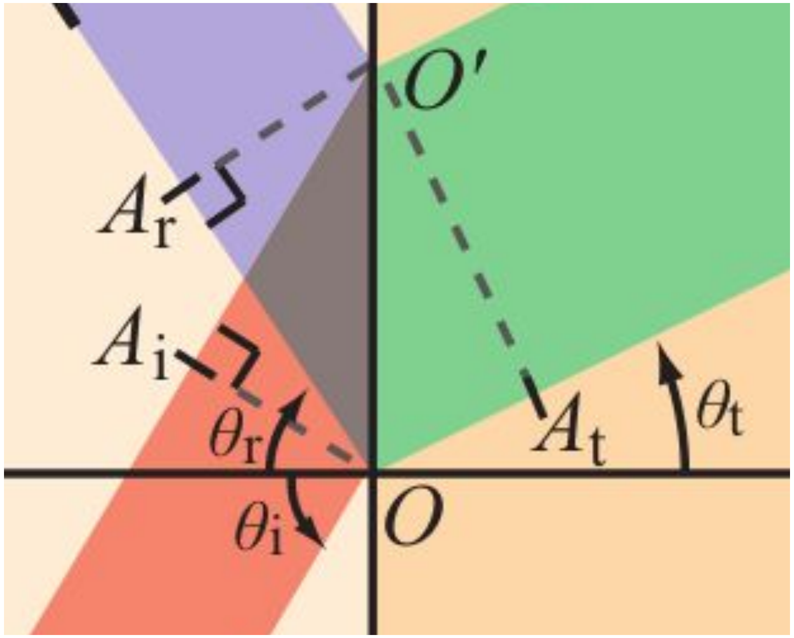
length(OA_t) and length(A_iO')

8-2 Snell's Laws

4. Propagation distances:

since: time = distance / velocity

$$\frac{\text{len}(O\,A_r)}{u_{p1}} = \frac{\text{len}(O\,A_t)}{u_{p2}} = \frac{\text{len}(O'\,A_i)}{u_{p1}}$$

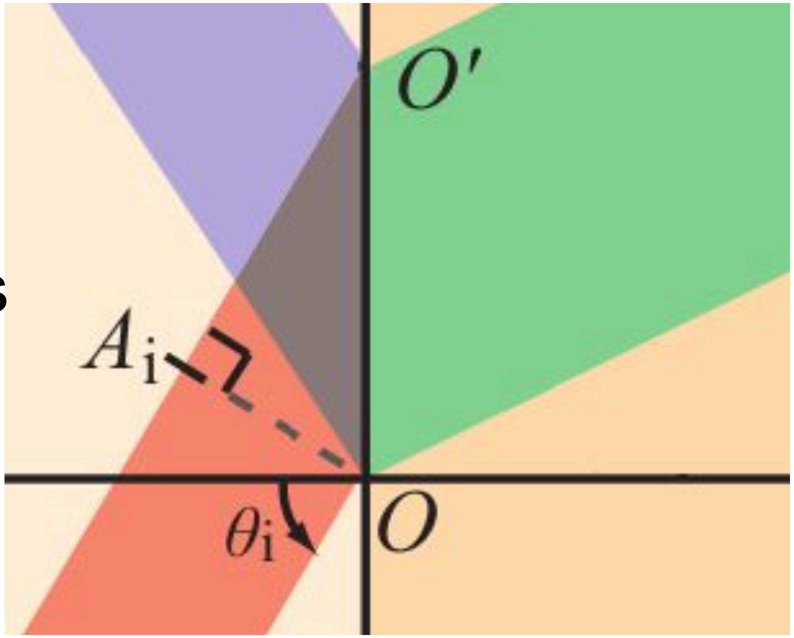


8-2 Snell's Laws

5. Angles:

Want to get expressions for angles as function of the lengths we've already defined.

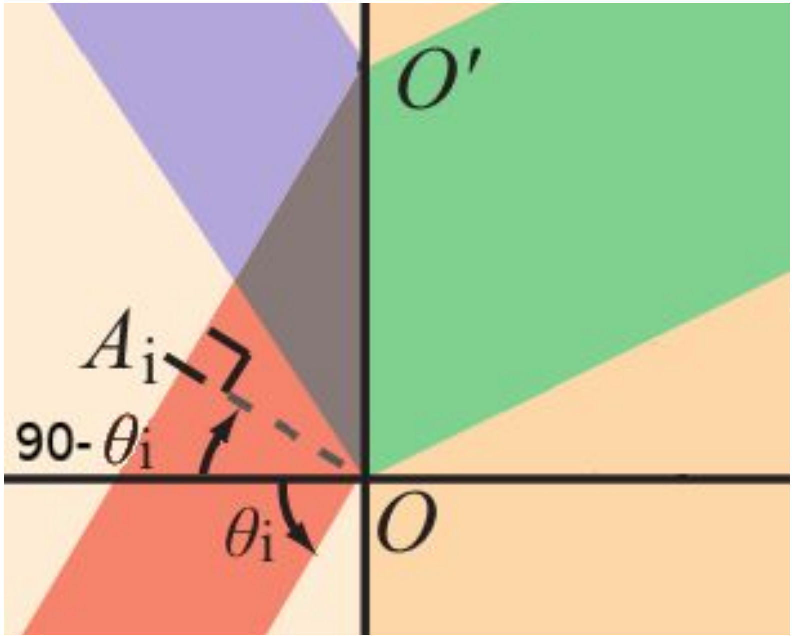
Start with the incidence angle:



8-2 Snell's Laws

5. Angles:

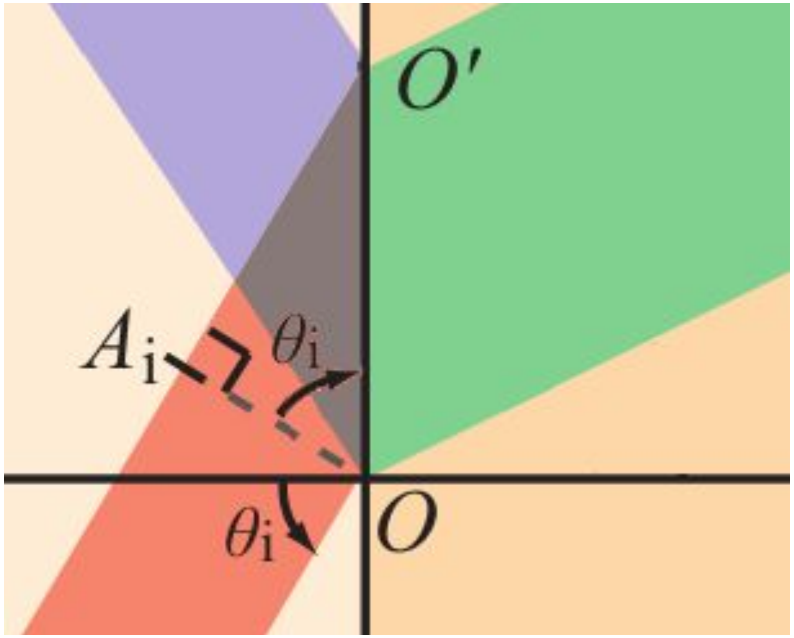
Because the wavefront is perpendicular to the ray



8-2 Snell's Laws

5. Angles:

Because the axes are perpendicular

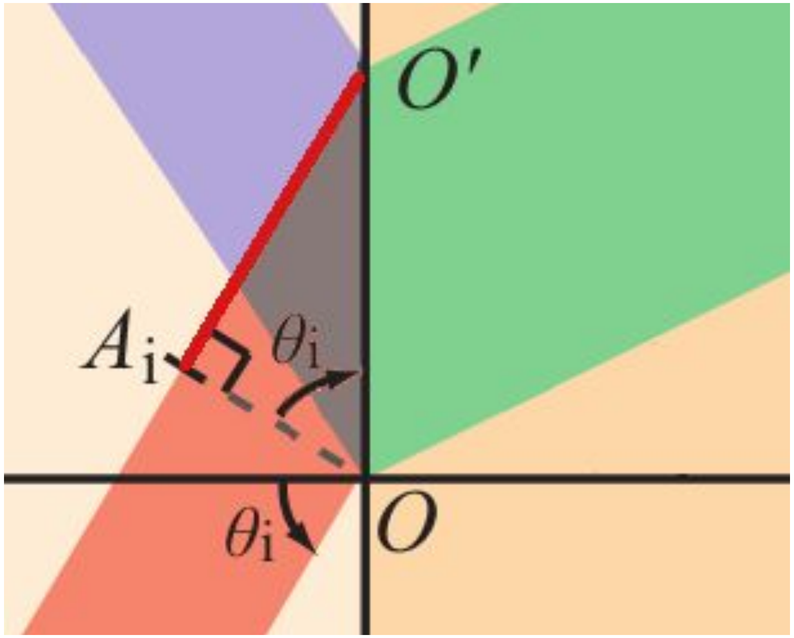


8-2 Snell's Laws

5. Angles:

sin θ_i = opposite / hypotenuse

sin θ_i = length(A_i O') / hyp.

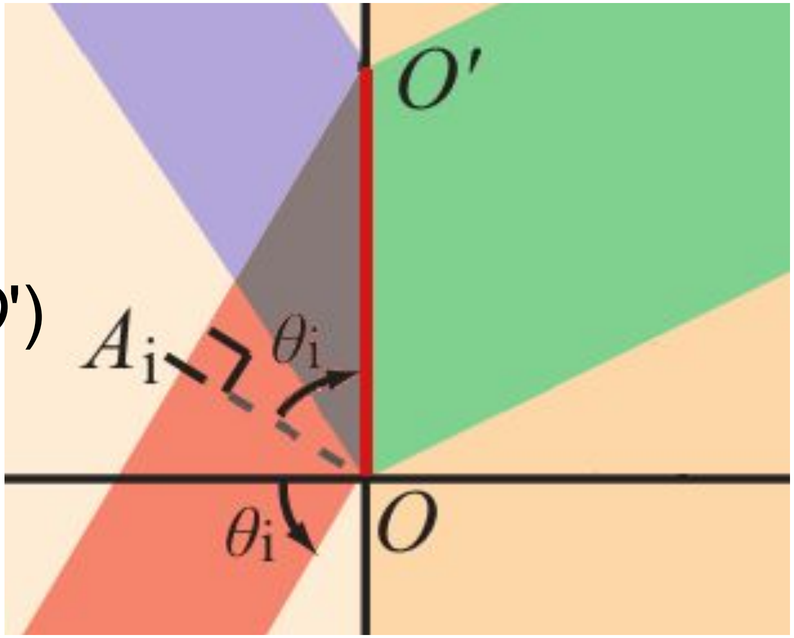


8-2 Snell's Laws

5. Angles:

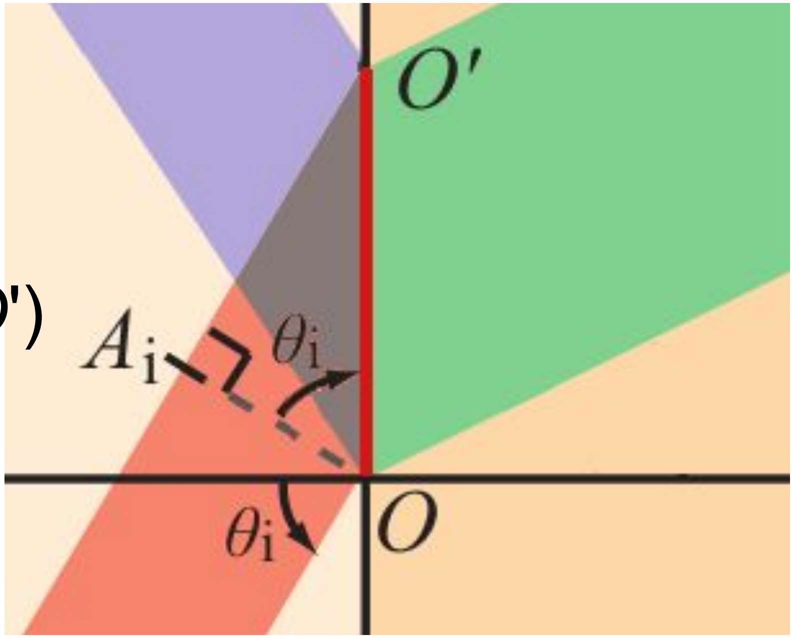
$\sin \theta_i$ = opposite / hypotenuse

$\sin \theta_i$ = length(A_i O') / length(OO')



8-2 Snell's Laws

5. Angles:



$\sin \theta_i$ = opposite / hypotenuse

$\sin \theta_i$ = length(A_i O') / length(OO')

so:

length(A_i O') = length(OO') $\sin\theta_i$

similarly for the other angles:

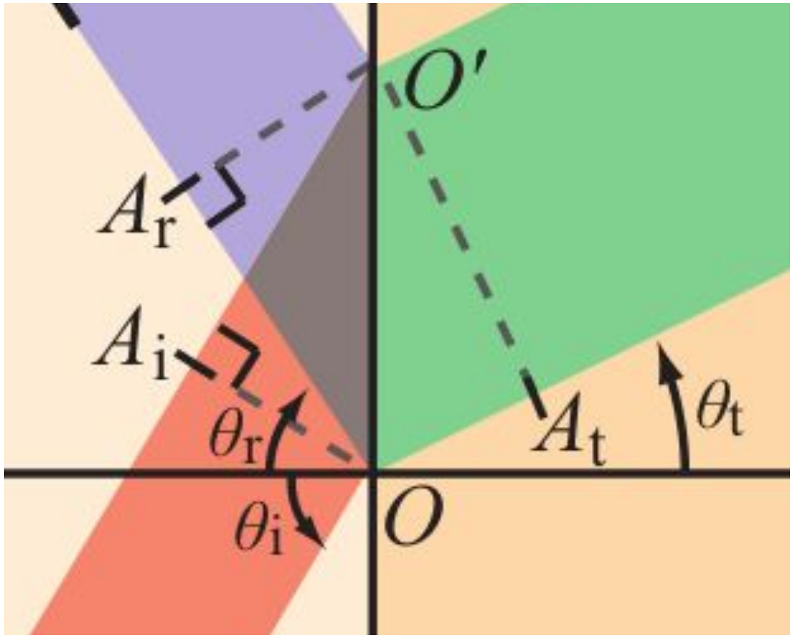
length(O A_r) = length(OO') $\sin\theta_r$

length(O A_t) = length(OO') $\sin\theta_t$

8-2 Snell's Laws

5. Angles:

plug into distance formulas:



$$\frac{\text{length}(OO')\sin\theta_i}{u_{p1}} =$$

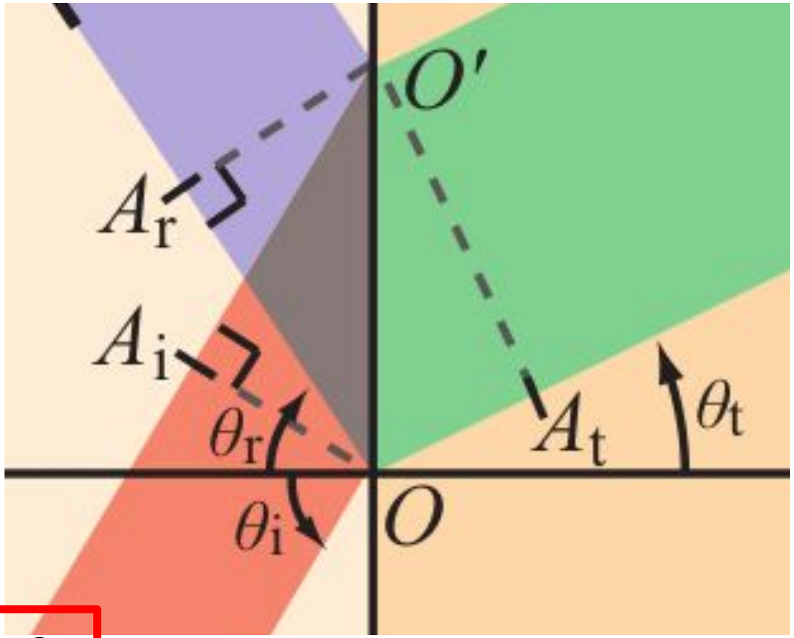
$$\frac{\text{length}(OO')\sin\theta_r}{u_{p1}} = \frac{\text{length}(OO')\sin\theta_t}{u_{p2}}$$

8-2 Snell's Laws

5. Angles:

simplify:

$$\frac{\sin\theta_i}{u_{p1}} = \frac{\sin\theta_r}{u_{p1}} = \frac{\sin\theta_t}{u_{p2}}$$



The first equation says: $\theta_i = \theta_r$

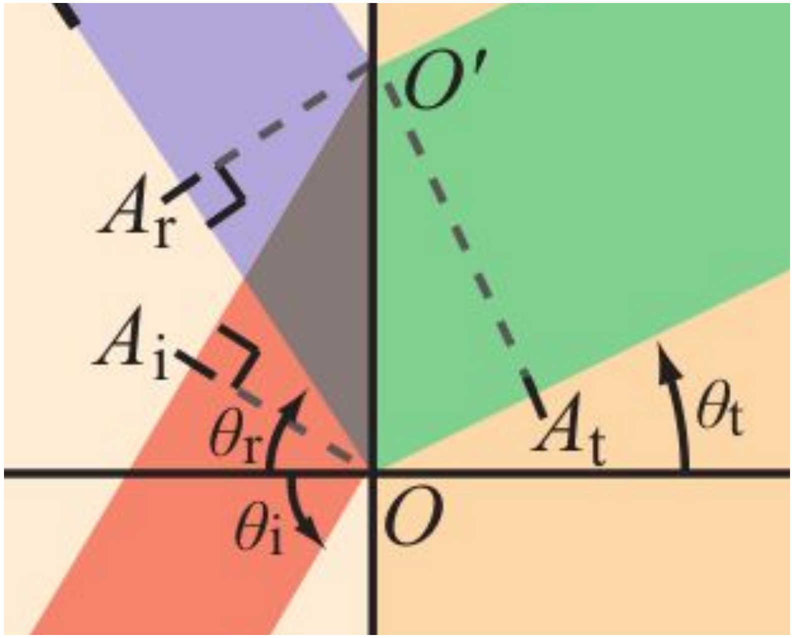
The first and last terms say: $\frac{\sin\theta_t}{\sin\theta_i} = \frac{u_{p2}}{u_{p1}} = \sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}$

8-2 Snell's Laws

Summary:

$$\theta_i = \theta_r$$

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\mu_1 \varepsilon_1}{\mu_2 \varepsilon_2}}$$



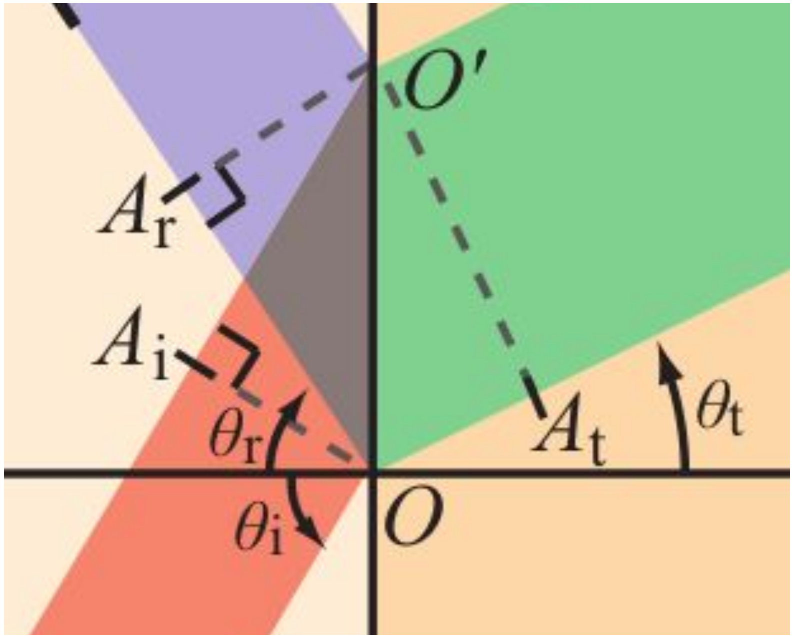
8-2 Snell's Laws

Parameterize Differently:

define:

$$n = \frac{c}{u_p} = \sqrt{\frac{\mu\varepsilon}{\mu_0\varepsilon_0}} = \sqrt{\mu_r\varepsilon_r}\,.$$

(index of refraction)



leads to:

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{n_1}{n_2} = \sqrt{\frac{\mu_{r_1}\varepsilon_{r_1}}{\mu_{r_2}\varepsilon_{r_2}}}$$

for non-magnetic materials:

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{n_1}{n_2} = \sqrt{\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}}} = \frac{\eta_2}{\eta_1} \quad (\text{for } \mu_1 = \mu_2).$$

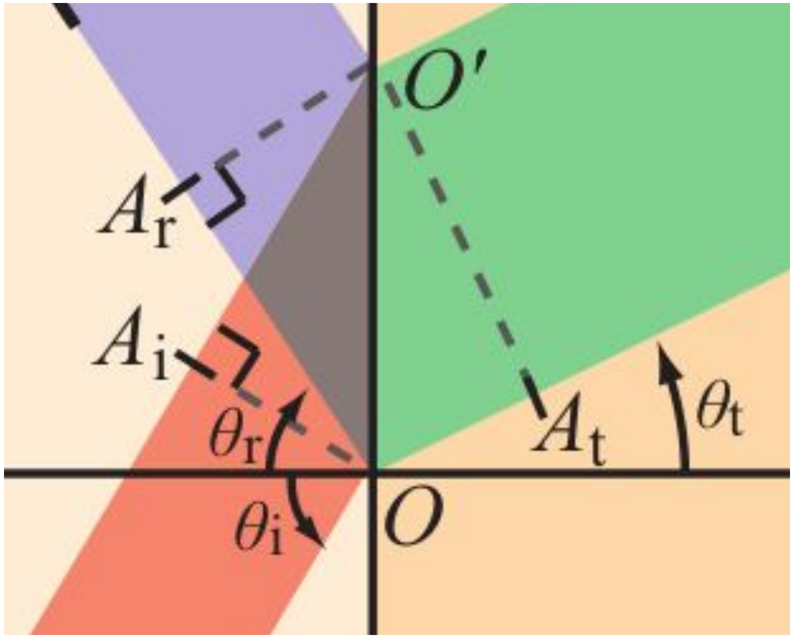
8-2 Snell's Laws

Critical Angle:

when $\theta_t = 90^o$ (or π/2)

Refracted wave propagates along the boundary

No energy transmitted into medium 2.



The critical angle is the value of θ_i that makes this happen:

$$\sin\theta_c = \frac{n_2}{n_1} = \sqrt{\frac{\varepsilon_{r_2}}{\varepsilon_{r_1}}} \qquad (\text{for } \mu_1 = \mu_2).$$

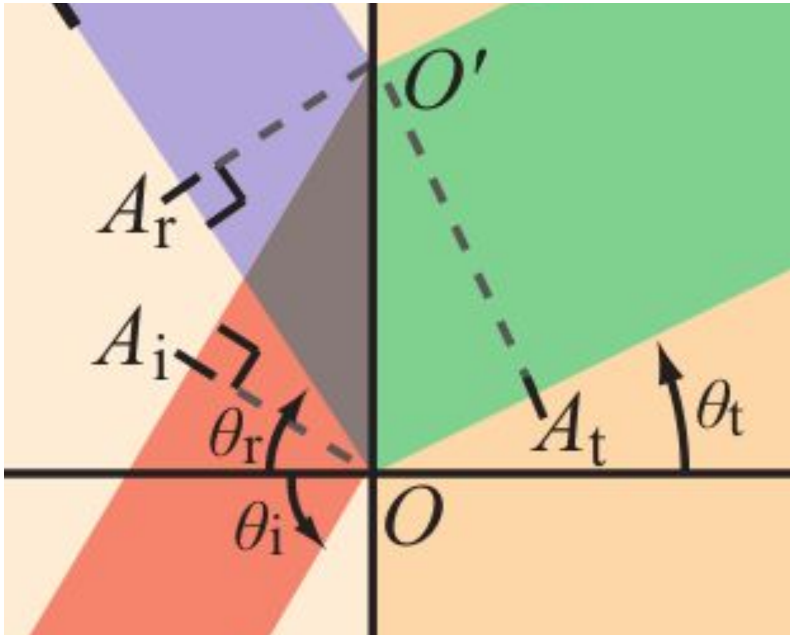
8-2 Snell's Laws

Critical Angle:

For $\theta_i > \theta_c$

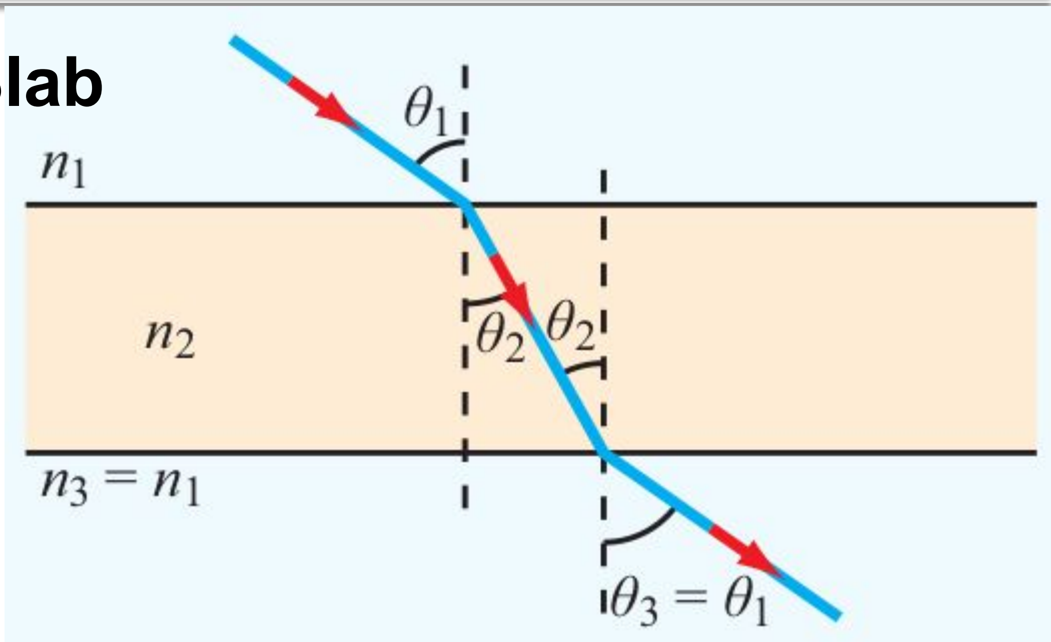
Incident wave totally reflected.

Refracted wave is a decaying "surface wave"



This is called Total Internal Reflection (TIR)

Example 8-4



Light Passing through Slab

Given: dielectric slab: n_2

surrounding medium: n_1

Assume: $\theta_i < \theta_c$

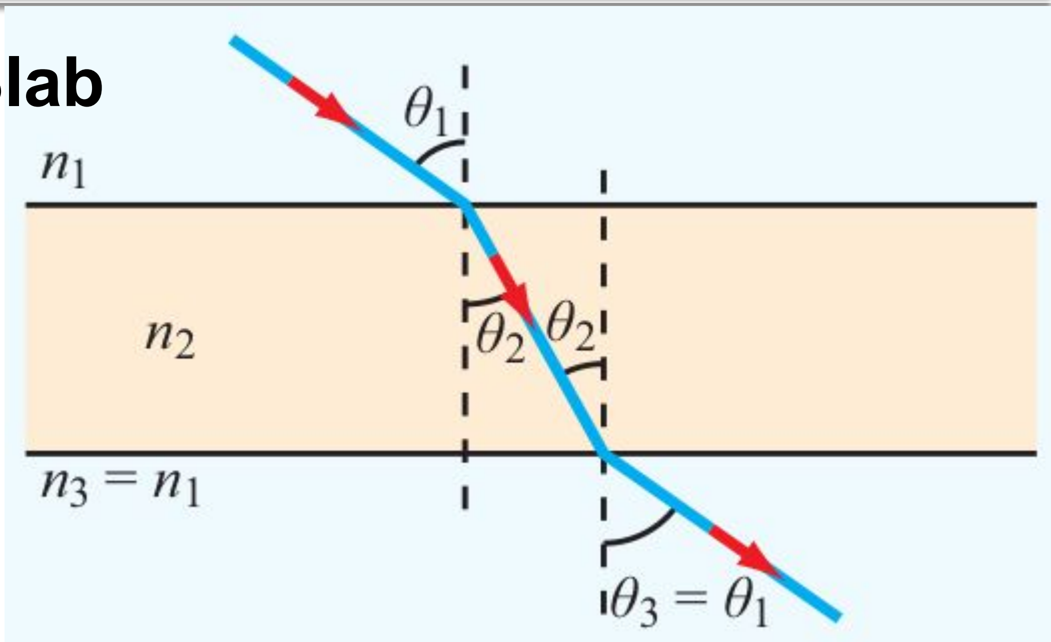
Show: $\theta_3 = \theta_1$

Solution: Apply Snell's Law at Upper Surface:

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$$

Example 8-4

Light Passing through Slab



At Upper Surface:

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$$

At Lower Surface:

$$\sin\theta_3 = \frac{n_2}{n_3}\sin\theta_2 = \frac{n_2}{n_1}\sin\theta_2$$

Together:

$$\sin\theta_3 = \left(\frac{n_2}{n_1}\right)\left(\frac{n_1}{n_2}\right)\sin\theta_1 = \sin\theta_1$$

So: $\boxed{\theta_3 = \theta_1}$

Homework

There is no Homework 27

Next Time

Review of the entire semester.