EECS 230
ENGINEERING ELECTROMAGNETICS
Leland Pierce

Plane Waves 2

University of Michigan - Fall 2024

Announcements

HW 26 is due next monday at midnight

prelab6 is due next sunday at midnight.

Wave Polarization: since I did not have time to cover this very well last time, it is not on the final exam.

Email me if you need to take the alternate final exam.

Chapter 7 Overview

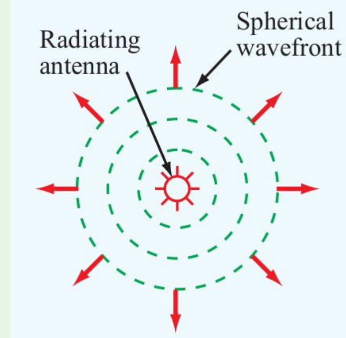
- Unbounded EM Waves
- Time-Harmonic Fields
  - Complex Permittivity
  - Wave Equations
- Plane Waves: Lossless Media
  - Uniform Plane Waves
  - Relation between E and H
- Wave Polarization
  - Linear
  - Circular
  - Elliptical
- Plane Waves: Lossy Media
  - Low Loss
  - Good Conductor
- Current in Good Conductor
- Power Density

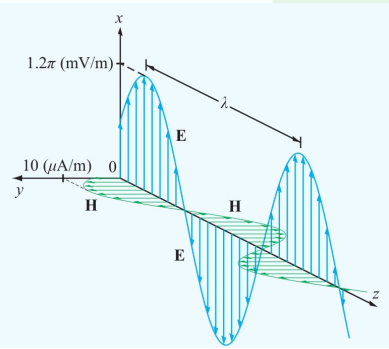
$$\varepsilon_c = \varepsilon - j\,\frac{\sigma}{\omega}$$

$$k = \omega\sqrt{\mu\varepsilon}.$$

$$\nabla^2\widetilde{\mathbf{E}} - \gamma^2\widetilde{\mathbf{E}} = 0.$$

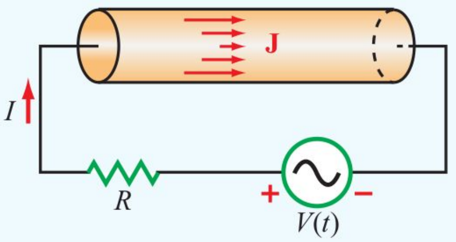
$$\nabla\cdot\widetilde{\mathbf{E}} = \tilde{\rho}_v/\varepsilon,$$
$$\nabla\times\widetilde{\mathbf{E}} = -j\omega\mu\widetilde{\mathbf{H}},$$
$$\nabla\cdot\widetilde{\mathbf{H}} = 0,$$
$$\nabla\times\widetilde{\mathbf{H}} = \widetilde{\mathbf{J}} + j\omega\varepsilon\widetilde{\mathbf{E}}.$$

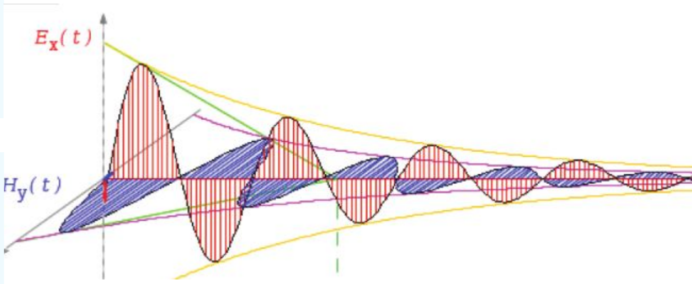




$$\widetilde{\mathbf{E}}(z) = \hat{\mathbf{x}}\widetilde{E}_x^+(z) = \hat{\mathbf{x}}E_{x0}^+e^{-jkz},$$

$$\widetilde{\mathbf{H}}(z) = \hat{\mathbf{y}}\,\frac{\widetilde{E}_x^+(z)}{\eta} = \hat{\mathbf{y}}\,\frac{E_{x0}^+}{\eta}e^{-jkz}.$$





$$\mathbf{S}_{av} = \tfrac{1}{2}\,\Re\mathfrak{e}\left[\widetilde{\mathbf{E}}\times\widetilde{\mathbf{H}}^*\right] \qquad (\mathrm{W/m^2}).$$

Lecture Coverage

Today's lecture:

Review of Sections 7-1 through 7-3 of the book:

7-1: Time-Harmonic Fields

7-2: Plane-Wave Propagation in Lossless Media

7-3: Wave Polarization

Sections 7-4 through 7-6 of the book:

7-4: Wave Propagation in Lossy Media

7-5: Current Flow in a Good Conductor

7-6: Electromagnetic Power Density

Chapter 7 Review

Complex Permittivity: $\varepsilon_c = \varepsilon - j\dfrac{\sigma}{\omega} = \varepsilon' - j\varepsilon''$,

Source-Free Medium, time-harmonic Fields:

$$\nabla \cdot \widetilde{\mathbf{E}} = 0,$$

$$\nabla \times \widetilde{\mathbf{E}} = -j\omega\mu\widetilde{\mathbf{H}},$$

$$\nabla \cdot \widetilde{\mathbf{H}} = 0,$$

$$\nabla \times \widetilde{\mathbf{H}} = j\omega\varepsilon_c\widetilde{\mathbf{E}}.$$

Chapter 7 Review

Wave Equations for Fields Propagating in unbounded media:

$$\nabla^2 \widetilde{\mathbf{E}} + \omega^2 \mu \varepsilon_c \widetilde{\mathbf{E}} = 0$$

define: $\gamma^2 = -\omega^2 \mu \varepsilon_c$ (which is complex)

get:

$$\nabla^2 \widetilde{\mathbf{E}} - \gamma^2 \widetilde{\mathbf{E}} = 0. \quad \textbf{(wave equation for } \widetilde{\mathbf{E}}\textbf{)}$$

Through a similar process we can get:

$$\nabla^2 \widetilde{\mathbf{H}} - \gamma^2 \widetilde{\mathbf{H}} = 0. \quad \textbf{(wave equation for } \widetilde{\mathbf{H}}\textbf{)}$$

Chapter 7 Review

Lossless Media: $\nabla^2 \widetilde{\mathbf{E}} + k^2 \widetilde{\mathbf{E}} = 0$ $\qquad k = \omega\sqrt{\mu\varepsilon}$

z-propagating plane wave:

$$\widetilde{\mathbf{E}}(z) = \hat{\mathbf{x}}\widetilde{E}_x^+(z) = \hat{\mathbf{x}}E_{x0}^+ e^{-jkz},$$

$$\widetilde{\mathbf{H}}(z) = \hat{\mathbf{y}}\frac{\widetilde{E}_x^+(z)}{\eta} = \hat{\mathbf{y}}\frac{E_{x0}^+}{\eta}e^{-jkz}.$$

intrinsic impedance:

$$\eta = \frac{\omega\mu}{k} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} \qquad (\Omega).$$

Chapter 7 Review

Uniform Plane Waves:

Phase velocity: (as with transmission lines)

$$u_p = \frac{\omega}{k} = \frac{\omega}{\omega\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} \qquad \text{(m/s)}.$$

Wavelength:

$$\lambda = \frac{2\pi}{k} = \frac{u_p}{f} \qquad \text{(m)}.$$

Chapter 7 Review

In a vacuum:

$$u_{\mathrm{p}} = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \qquad \text{(m/s)},$$

$$\eta = \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\ (\Omega) \approx 120\pi \quad (\Omega),$$

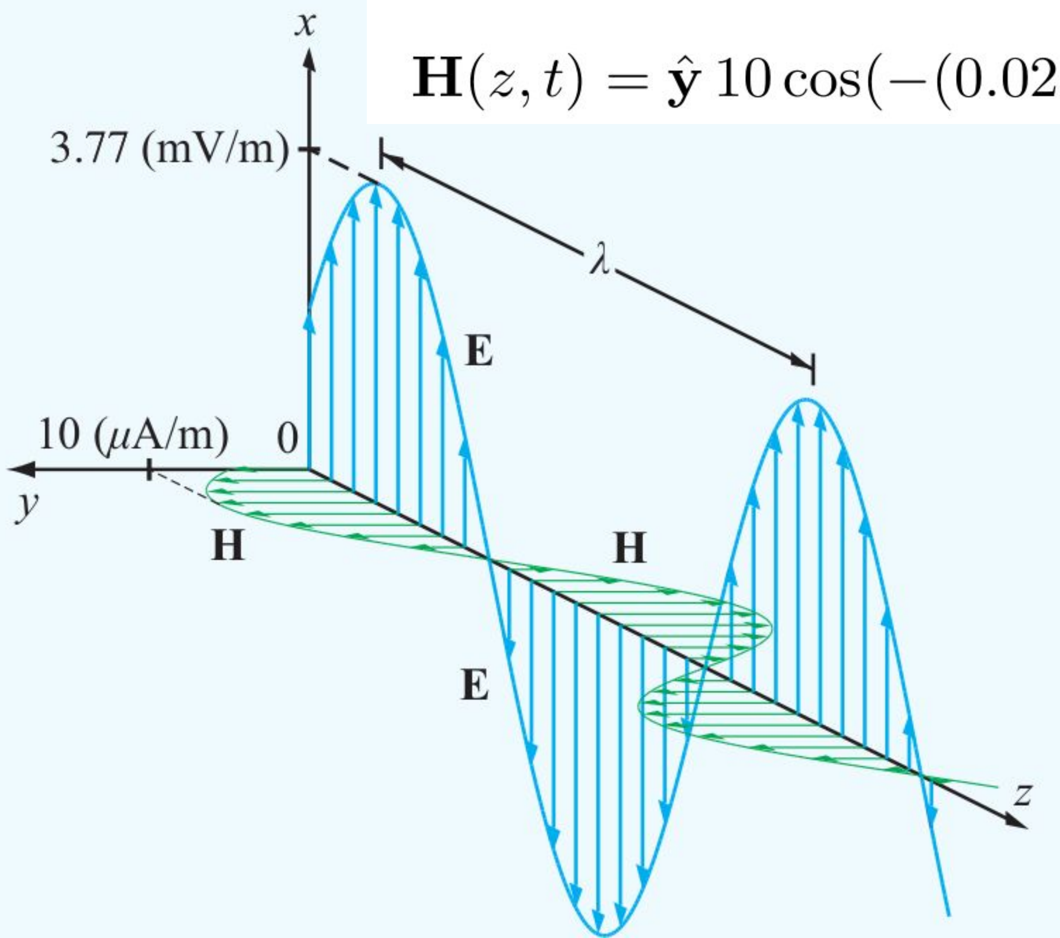
c is the velocity of light in vacuum

η_0 is the intrinsic impedance of free space

Chapter 7 Review

at t=0: $$\mathbf{E}(z,t) = \hat{\mathbf{x}}\, 3.77 \cos(-(0.021\text{ rad/m})z + 1.05\text{ rad}) \quad (\text{mV/m})$$

$$\mathbf{H}(z,t) = \hat{\mathbf{y}}\, 10 \cos(-(0.021\text{ rad/m})z + 1.05\text{ rad}) \quad (\mu\text{A/m})$$



Chapter 7 Review

General Uniform Plane Waves:

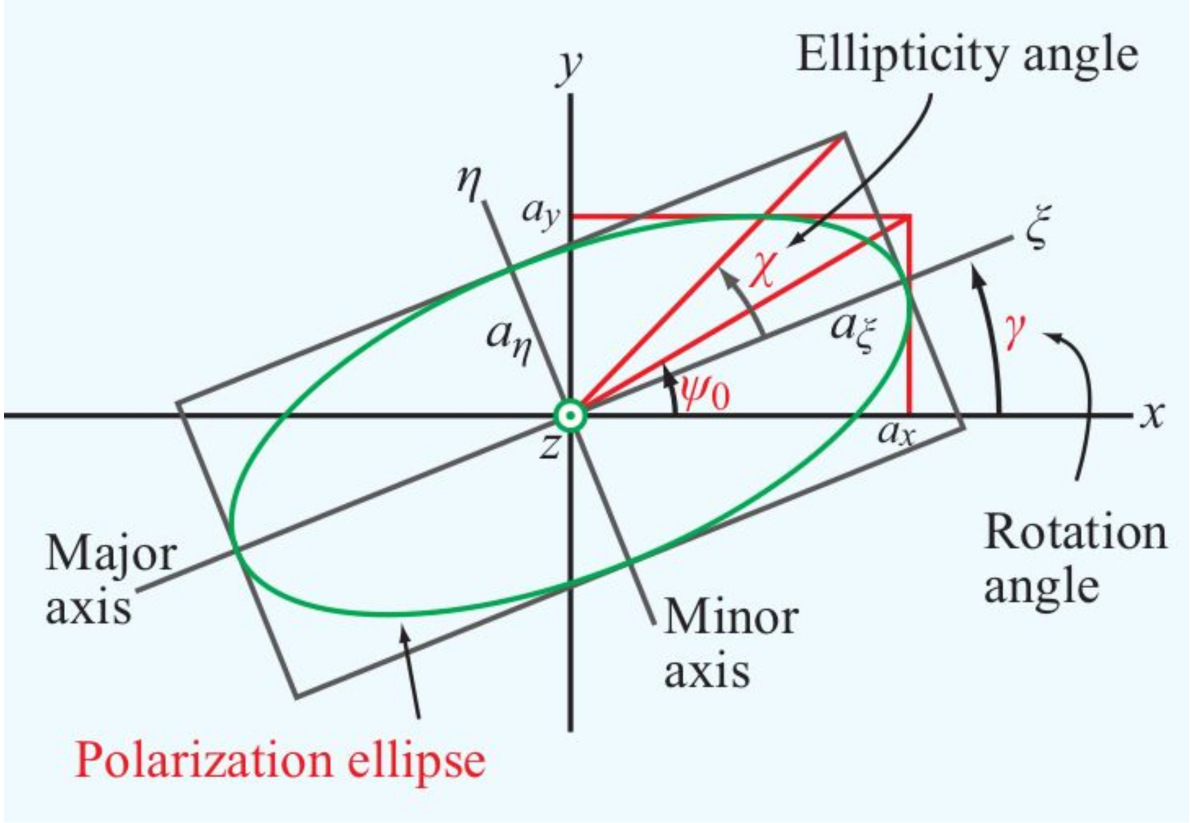
Direction of propagation: $\hat{\mathbf{k}}$,

$$\widetilde{\mathbf{H}} = \frac{1}{\eta}\,\hat{\mathbf{k}} \times \widetilde{\mathbf{E}},$$

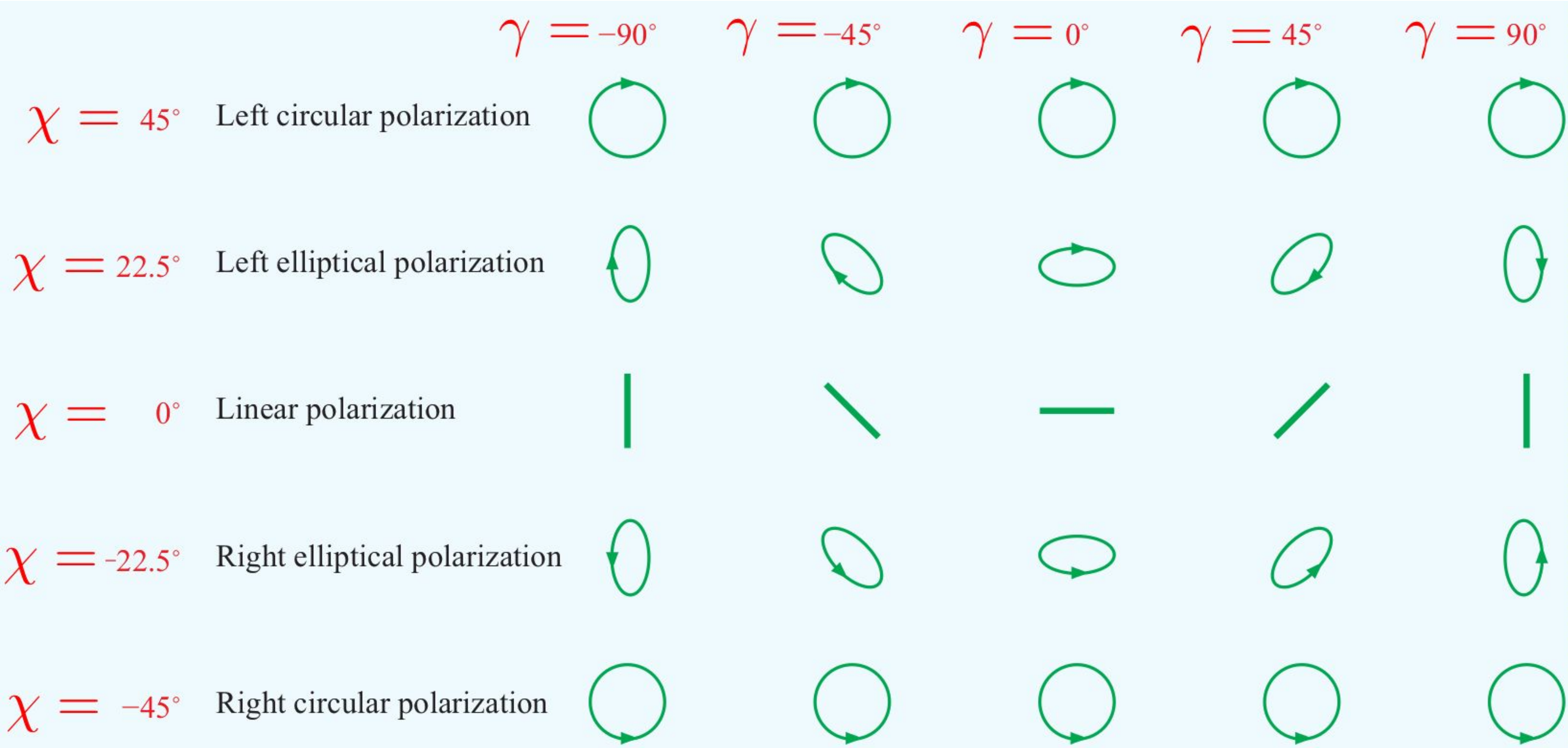
$$\widetilde{\mathbf{E}} = -\eta\,\hat{\mathbf{k}} \times \widetilde{\mathbf{H}}.$$

Works for lossless and lossy media.
Just use appropriate expression for η

Chapter 7 Review



Chapter 7 Review



7-4 Plane-Wave Propagation in Lossy Media

Recall: $$\nabla^2 \widetilde{\mathbf{E}} + \omega^2 \mu \varepsilon_c \widetilde{\mathbf{E}} = 0$$

where: $$\gamma^2 = -\omega^2 \mu \varepsilon_c = -\omega^2 \mu (\varepsilon' - j\varepsilon'')$$

$$\varepsilon' = \varepsilon \text{ and } \varepsilon'' = \sigma / \omega$$

set: $$\gamma = \alpha + j\beta$$

get:

$$(\alpha + j\beta)^2 = (\alpha^2 - \beta^2) + j2\alpha\beta = -\omega^2 \mu \varepsilon' + j\omega^2 \mu \varepsilon''$$

Equate Real parts: $$\alpha^2 - \beta^2 = -\omega^2 \mu \varepsilon'$$

Equate Imaginary parts: $$2\alpha\beta = \omega^2 \mu \varepsilon''$$

7-4 Plane-Wave Propagation in Lossy Media

So:

$$\alpha^2 - \beta^2 = -\omega^2 \mu \varepsilon'$$

$$2\alpha\beta = \omega^2 \mu \varepsilon''$$

Giving:

$$\alpha = \omega \left\{ \frac{\mu \varepsilon'}{2} \left[\sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'} \right)^2} - 1 \right] \right\}^{1/2}$$

(Np/m),

$$\beta = \omega \left\{ \frac{\mu \varepsilon'}{2} \left[\sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'} \right)^2} + 1 \right] \right\}^{1/2}$$

(rad/m).

7-4 Plane-Wave Propagation in Lossy Media

Here's the algebra for showing the previous result:

$$\alpha^2 - \beta^2 = -\omega^2 \mu \varepsilon'$$

$$2\alpha\beta = \omega^2 \mu \varepsilon''$$

Use second equation to solve for α:

$$\alpha = \frac{1}{2\beta} \omega^2 \mu \varepsilon''$$

Plug this expression for α into first equation:

$$\left[\omega^2 \mu \varepsilon''\right]^2 - (2\beta)^2 \beta^2 = -(2\beta)^2 \omega^2 \mu \varepsilon'$$

7-4 Plane-Wave Propagation in Lossy Media

$$\left[\frac{1}{2\beta}\right]^2 \left[\omega^2\mu\varepsilon''\right]^2 - \beta^2 = -\omega^2\mu\varepsilon'$$

$$\left[\omega^2\mu\varepsilon''\right]^2 - (2\beta)^2\beta^2 = -(2\beta)^2\omega^2\mu\varepsilon'$$

Let $q = \beta^2$:

$$\left[\omega^2\mu\varepsilon''\right]^2 - 4q^2 = -4q\omega^2\mu\varepsilon'$$

$$(-4)q^2 + (4\omega^2\mu\varepsilon')q + \left[\omega^2\mu\varepsilon''\right]^2 = 0$$

Use the quadratic formula to solve for q:

$$aq^2 + bq + c = 0$$

$$q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

7-4 Plane-Wave Propagation in Lossy Media

$$(-4)q^2 + (4\omega^2\mu\varepsilon')q + \left[\omega^2\mu\varepsilon''\right]^2 = 0$$

$$q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$q = \frac{-4\omega^2\mu\varepsilon' \pm \sqrt{(4\omega^2\mu\epsilon')^2 - 4(-4)(\omega^2\mu\epsilon'')^2}}{2(-4)}$$

$$q = \frac{1}{2}\omega^2\mu\varepsilon' \pm \frac{4\sqrt{(\omega^2\mu\epsilon')^2 + (\omega^2\mu\epsilon'')^2}}{-8}$$

$$q = \frac{1}{2}\omega^2\mu\varepsilon' \pm \frac{1}{2}\sqrt{(\omega^2\mu\epsilon')^2\left[1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2\right]}$$

$$q = \frac{1}{2}\omega^2\mu\varepsilon'\left[1 \pm \sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2}\right]$$

7-4 Plane-Wave Propagation in Lossy Media

$$q = \frac{1}{2}\omega^2 \mu\varepsilon' \left[1 \pm \sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2}\right]$$

$$\beta = \omega \left\{ \frac{\mu\varepsilon'}{2} \left[1 \pm \sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2}\right] \right\}^{1/2}$$

For β real, need square-root of a positive number:

$$\beta = \omega \left\{ \frac{\mu\varepsilon'}{2} \left[1 + \sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'}\right)^2}\right] \right\}^{1/2}$$

7-4 Plane-Wave Propagation in Lossy Media

Get α using first equation:

$$\alpha^2 - \beta^2 = -\omega^2\mu\varepsilon'$$

$$\alpha^2 = \beta^2 - \omega^2\mu\varepsilon'$$

$$\alpha^2 = \frac{1}{2}\omega^2\mu\varepsilon\left\{1+\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}\right\} - \omega^2\mu\varepsilon'$$

$$\alpha^2 = \frac{1}{2}\omega^2\mu\varepsilon\left\{-1+\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}\right\}$$

$$\alpha = \omega\left\{\frac{\mu\varepsilon'}{2}\left[-1+\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}\right]\right\}^{1/2}$$

7-4 Plane-Wave Propagation in Lossy Media

For both α and β we want the positive square-root.

7-4 Plane-Wave Propagation in Lossy Media

For a uniform plane wave propagating in the +*z*-direction:

$$\widetilde{\mathbf{E}}(z) = \hat{\mathbf{x}} E_{x0} e^{-\alpha z} e^{-j\beta z}$$

$$\widetilde{\mathbf{H}}(z) = \hat{\mathbf{y}} \frac{E_{x0}}{\eta_c} e^{-\alpha z} e^{-j\beta z}$$

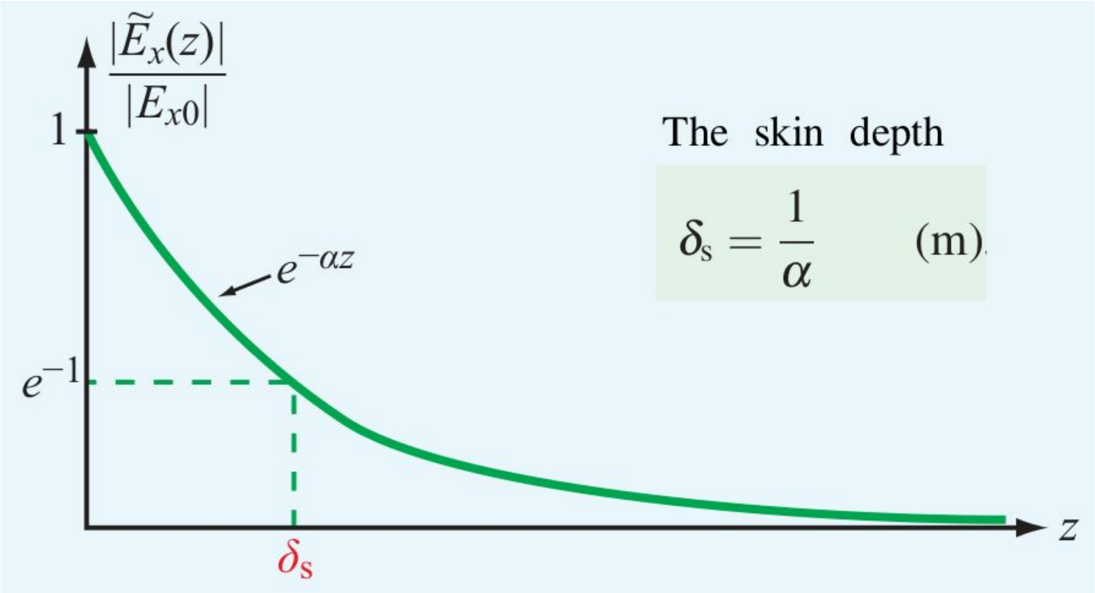
$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon'}} \left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2} \quad (\Omega)$$

7-4 Plane-Wave Propagation in Lossy Media

Look at the loss:

$$|\widetilde{E}_x(z)| = |E_{x0}e^{-\alpha z}e^{-j\beta z}| = |E_{x0}|e^{-\alpha z}$$

The energy in the wave is converted into heat due to the conduction in the medium.



7-4 Plane-Wave Propagation in Lossy Media

Examples:

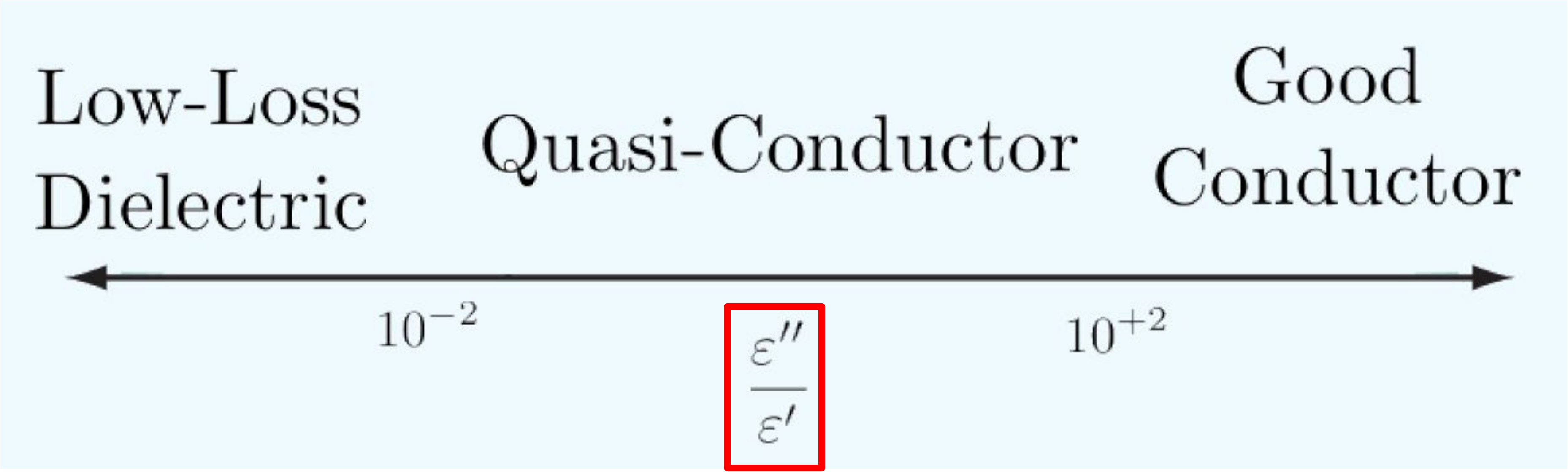
1. Perfect Dielectric: σ=0, ε''=0
   so: α=0   and skin depth: δ_s=∞
   propagates without loss

2. Perfect Conductor: σ=∞
   so: α=∞   and skin depth: δ_s=0
   cannot propagate

3. Coaxial Cable: outer conductor, high σ
   designed to be several skin depths thick
   prevents energy from leaking: both directions.

7-4 Plane-Wave Propagation in Lossy Media

Categories:



7-4 Plane-Wave Propagation in Lossy Media

Low-Loss Dielectrics

in general: $$\gamma = j\omega\sqrt{\mu\varepsilon'}\left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{1/2}.$$

when $\varepsilon''/\varepsilon' \ll 1$, use the approximation:

$$\gamma \approx j\omega\sqrt{\mu\varepsilon'}\left(1 - j\frac{\varepsilon''}{2\varepsilon'}\right)$$

which leads to:

$$\alpha \approx \frac{\omega\varepsilon''}{2}\sqrt{\frac{\mu}{\varepsilon'}} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} \qquad \text{(Np/m)},$$

$$\beta \approx \omega\sqrt{\mu\varepsilon'} = \omega\sqrt{\mu\varepsilon} \qquad \text{(rad/m)}.$$

(low-loss medium)

$$\eta_c \approx \sqrt{\frac{\mu}{\varepsilon}}.$$

7-4 Plane-Wave Propagation in Lossy Media

Good Conductor

in general: $$\gamma = j\omega\sqrt{\mu\varepsilon'}\left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{1/2}.$$

when $\varepsilon''/\varepsilon' >> 100$, use the approximations:

$$\alpha \approx \omega\sqrt{\frac{\mu\varepsilon''}{2}} = \omega\sqrt{\frac{\mu\sigma}{2\omega}} = \sqrt{\pi f \mu\sigma} \quad \text{(Np/m)},$$

$$\beta \approx \alpha \approx \sqrt{\pi f \mu\sigma} \quad \text{(rad/m)},$$

$$\eta_c \approx \sqrt{j\frac{\mu}{\varepsilon''}} = (1+j)\sqrt{\frac{\pi f\mu}{\sigma}} = (1+j)\frac{\alpha}{\sigma} \quad (\Omega).$$

(good conductors)

Example 7-4

Plane Wave in Seawater

Given: Plane wave travels in +*z*-direction

$\varepsilon_r = 80$, $\mu_r = 1$, $\sigma = 4$ S/m

at $z=0$:

$$\mathbf{H}(0,t) = \hat{\mathbf{y}}100\cos(2\pi \times 10^3 \text{rad/sec}\, t + 15^\circ) \quad \text{mA/m}$$

Find: **E**(*z,t*), **H**(*z,t*), skin depth, δ_s

Solution: propagation direction is along *z*,

H is along *y*, so:

E must be along *x*

Example 7-4

Plane Wave in Seawater

Can write the form of the answer:

$$\widetilde{\mathbf{E}}(z) = \hat{\mathbf{x}} E_{x0} e^{-\alpha z} e^{-j\beta z},$$

$$\widetilde{\mathbf{H}}(z) = \hat{\mathbf{y}} \frac{E_{x0}}{\eta_c} e^{-\alpha z} e^{-j\beta z}.$$

Need to determine the 4 parameters:

α, β, η_c, and E_{x0}

Start by determining:

$\varepsilon'' / \varepsilon'$

Example 7-4

Plane Wave in Seawater

Since: $\omega = 2\pi \times 10^3 \text{ (rad/s)}$

$$\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon} = \frac{\sigma}{\omega\varepsilon_r\varepsilon_0} = \frac{4}{2\pi \times 10^3 \times 80 \times (10^{-9}/36\pi)} = 9 \times 10^5.$$

So, we can use the "Good Conductor" approximation.

Example 7-4

Plane Wave in Seawater

$$\alpha = \sqrt{\pi f \mu \sigma}$$
$$= \sqrt{\pi \times 10^3 \times 4\pi \times 10^{-7} \times 4}$$
$$= 0.126 \qquad \text{(Np/m)},$$
$$\beta = \alpha = 0.126 \qquad \text{(rad/m)},$$
$$\eta_c = (1 + j)\frac{\alpha}{\sigma}$$
$$= (\sqrt{2}\, e^{j\pi/4})\frac{0.126}{4} = 0.044 e^{j\pi/4} \qquad (\Omega).$$

Example 7-4

Plane Wave in Seawater

Plug in, and convert to time-domain:

$$\mathbf{E}(z,t) = \mathfrak{Re}\left[\hat{\mathbf{x}}|E_{x0}|e^{j\phi_0}e^{-\alpha z}e^{-j\beta z}e^{j\omega t}\right]$$

$$= \hat{\mathbf{x}}|E_{x0}|e^{-0.126z}\cos(2\pi \times 10^3 t - 0.126z + \phi_0) \quad \text{(V/m)},$$

$$\mathbf{H}(z,t) = \mathfrak{Re}\left[\hat{\mathbf{y}}\,\frac{|E_{x0}|e^{j\phi_0}}{0.044e^{j\pi/4}}e^{-\alpha z}e^{-j\beta z}e^{j\omega t}\right]$$

$$= \hat{\mathbf{y}}22.5|E_{x0}|e^{-0.126z}\cos(2\pi \times 10^3 t - 0.126z + \phi_0 - 45^\circ) \quad \text{(A/m)}.$$

Example 7-4

Plane Wave in Seawater

Comparing our expression for **H** with the one given, we can conclude:

$$22.5|E_{x0}| = 100 \times 10^{-3}$$ so: $$|E_{x0}| = 4.44 \quad \text{(mV/m)}$$

and:

$$\phi_0 - 45^\circ = 15^\circ$$ so: $$\phi_0 = 60^\circ.$$

Example 7-4

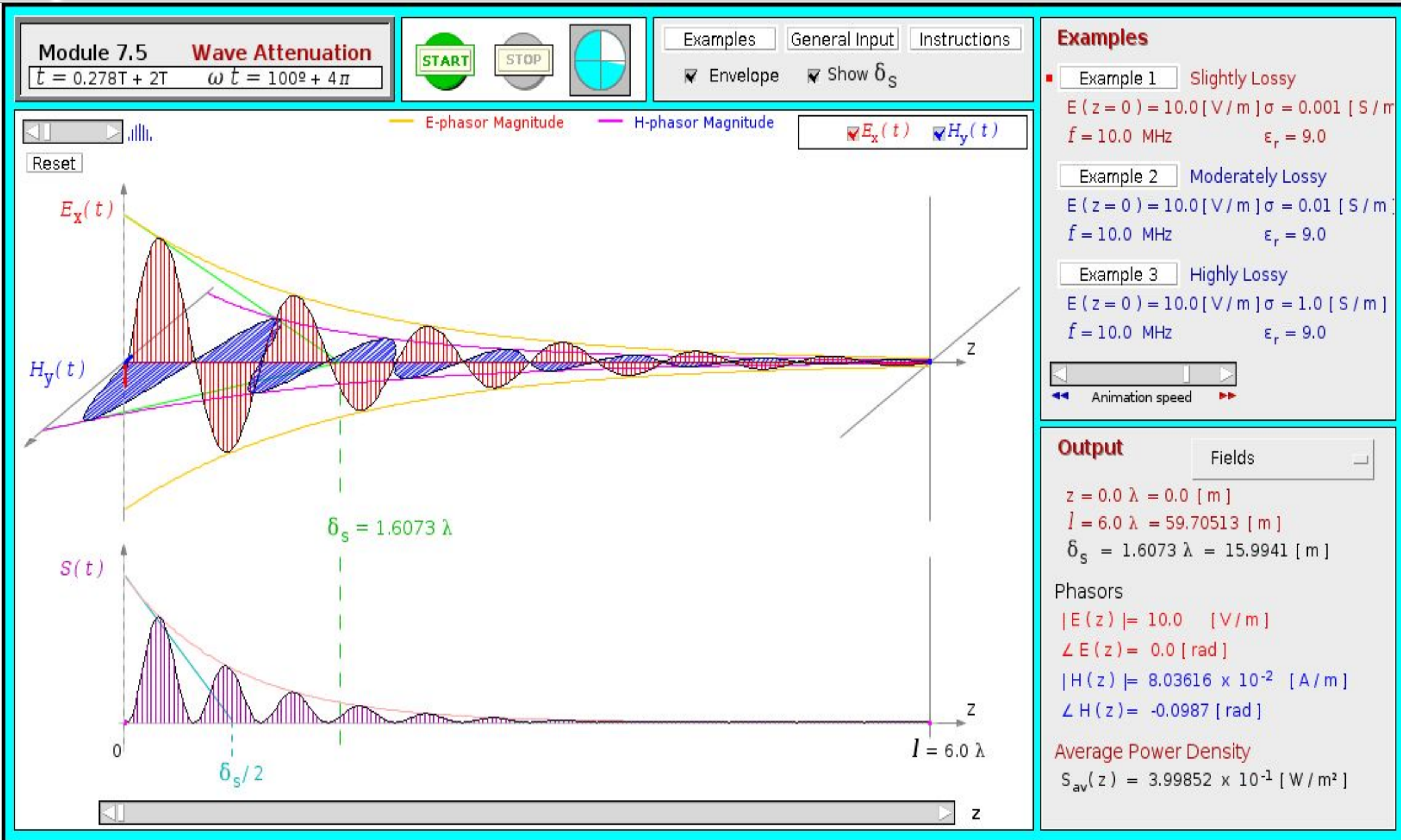
Plane Wave in Seawater

$$\mathbf{E}(z,t) = \hat{\mathbf{x}}(4.44\,\text{mV/m})\, e^{-0.126\text{m}^{-1}z} \cos((6.26\times 10^3\text{rad/sec})t - (0.126\text{rad/m})z + 60^\circ)$$

$$\mathbf{H}(z,t) = \hat{\mathbf{y}}(100\,\text{mA/m})\, e^{-0.126\text{m}^{-1}z} \cos((6.26\times 10^3\text{rad/sec})t - (0.126\text{rad/m})z + 15^\circ)$$

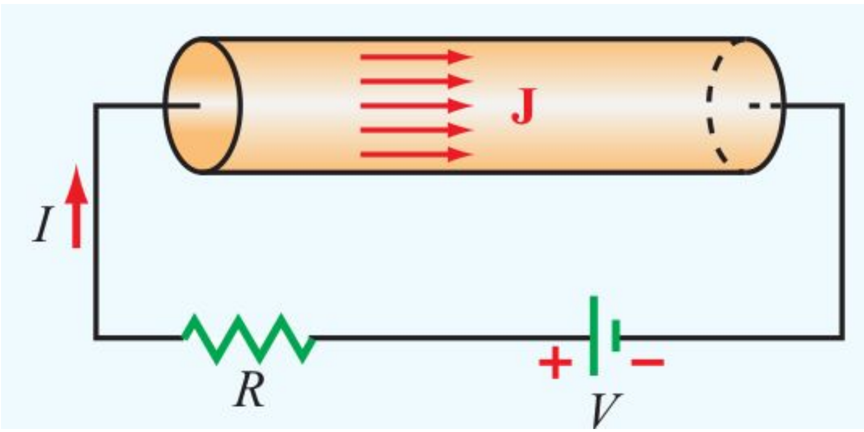
with a skin depth of: $\delta_s = 1/\alpha = 1/(0.126\text{m}^{-1})$

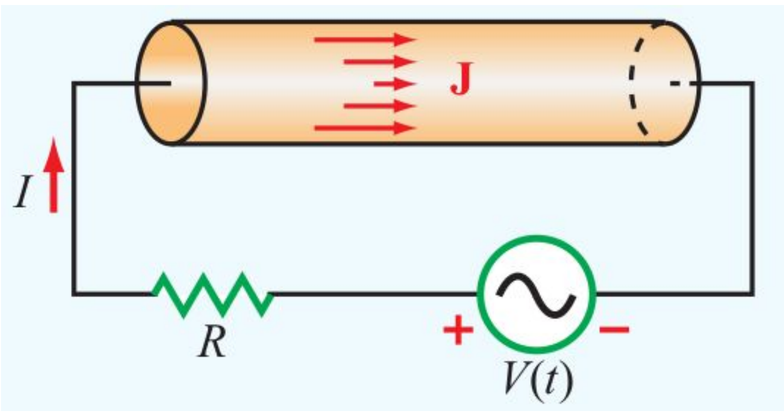
$\delta_s = 7.9\text{m}$



Module 7.5 Wave Attenuation
t = 0.278T + 2T ω t = 100º + 4π
START
STOP
Examples
General Input
Instructions
Envelope
Show δs
E-phasor Magnitude
H-phasor Magnitude
Ex(t)
Hy(t)
Reset
Ex(t)
Hy(t)
z
δs = 1.6073 λ
S(t)
z
0
l = 6.0 λ
δs / 2
z
Examples
Example 1 Slightly Lossy
E (z = 0) = 10.0 [V / m] σ = 0.001 [S / m
f = 10.0 MHz εr = 9.0
Example 2 Moderately Lossy
E (z = 0) = 10.0 [V / m] σ = 0.01 [S / m]
f = 10.0 MHz εr = 9.0
Example 3 Highly Lossy
E (z = 0) = 10.0 [V / m] σ = 1.0 [S / m]
f = 10.0 MHz εr = 9.0
Animation speed
Output
Fields
z = 0.0 λ = 0.0 [m]
l = 6.0 λ = 59.70513 [m]
δs = 1.6073 λ = 15.9941 [m]
Phasors
| E (z) |= 10.0 [V / m]
∠ E (z) = 0.0 [rad]
| H (z) |= 8.03616 x 10-2 [A / m]
∠ H (z) = -0.0987 [rad]
Average Power Density
Sav(z) = 3.99852 x 10-1 [W / m²]

7-5 Current Flow in a Good Conductor





DC Current:
J uniform in wire

AC Current:
J decays from surface

This is due to the skin depth.
Let's investigate the details...

7-5 Current Flow in a Good Conductor

Conducting half-plane

We know:

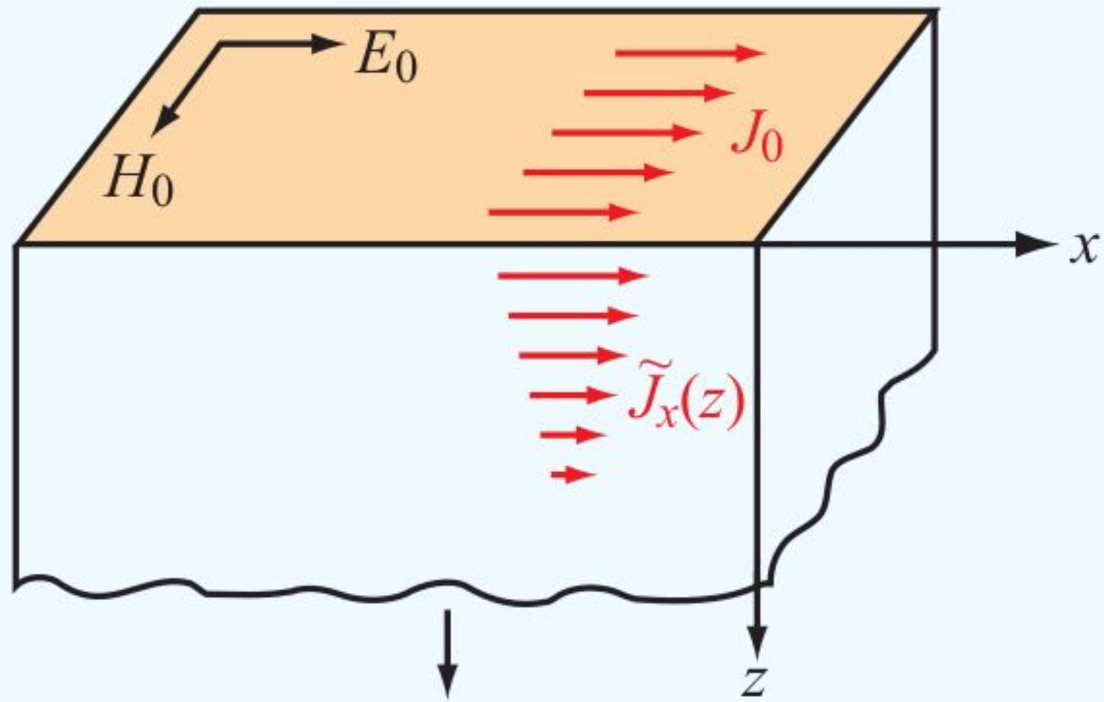
$$\widetilde{\mathbf{E}}(z) = \hat{\mathbf{x}} E_0 e^{-\alpha z} e^{-j\beta z},$$

$$\widetilde{\mathbf{H}}(z) = \hat{\mathbf{y}} \frac{E_0}{\eta_c} e^{-\alpha z} e^{-j\beta z}.$$

so:

$$\widetilde{\mathbf{J}}(z) = \hat{\mathbf{x}} \widetilde{J}_x(z),$$

$$\widetilde{J}_x(z) = \sigma E_0 e^{-\alpha z} e^{-j\beta z}$$



Exponentially decaying $\widetilde{J}_x(z)$

7-5 Current Flow in a Good Conductor

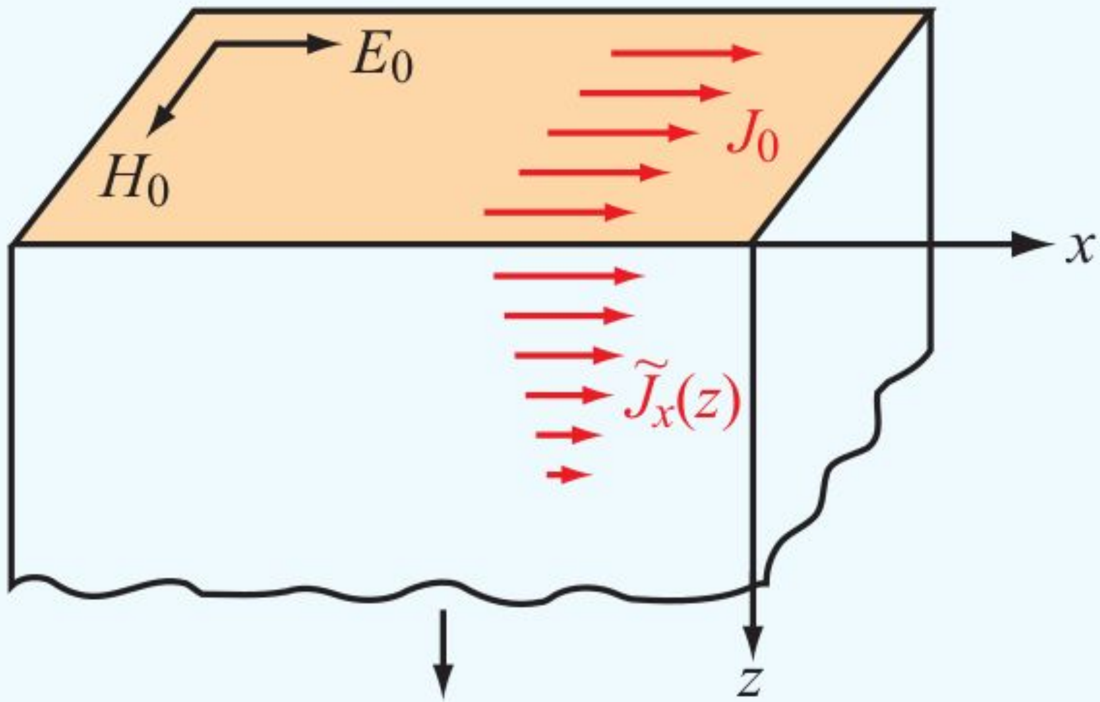
Conducting half-plane

Since:

$$\widetilde{J}_x(z) = \sigma E_0 e^{-\alpha z} e^{-j\beta z} = J_0 e^{-\alpha z} e^{-j\beta z}$$

and $\alpha=\beta=1/\delta_s$ for a good conductor:

$$\widetilde{J}_x(z) = J_0 e^{-(1+j)z/\delta_s}$$



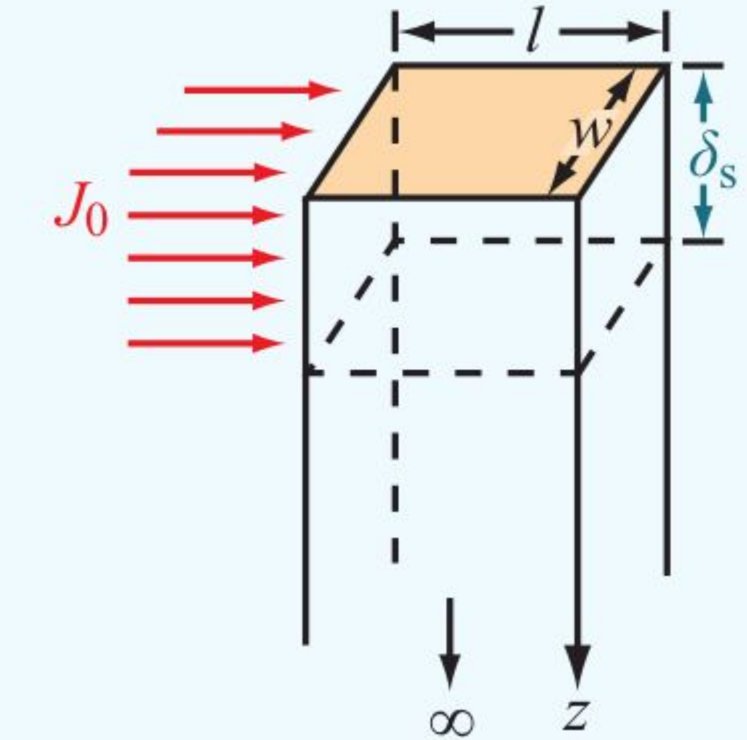
Exponentially decaying $\widetilde{J}_x(z)$

7-5 Current Flow in a Good Conductor

Conducting half-plane

Total current flowing through a rectangular strip of width *w* along the *y*-direction, and from 0 to ∞ in *z*-direction is:

$$\widetilde{I} = w \int_0^\infty \widetilde{J}_x(z)\, dz$$



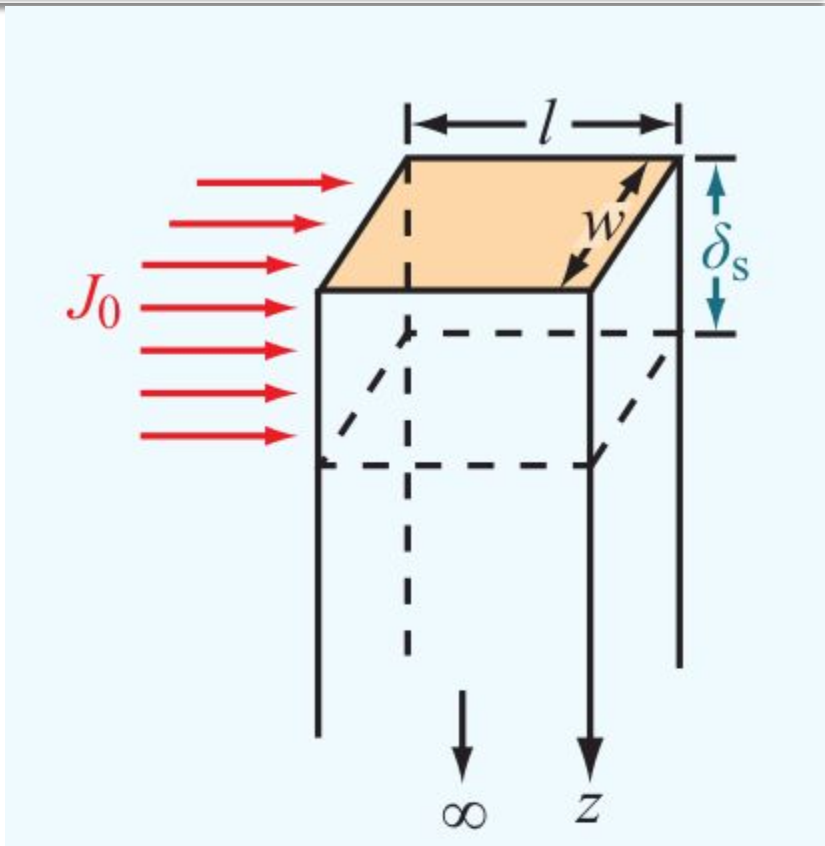
7-5 Current Flow in a Good Conductor

Conducting half-plane

Total current flowing through a rectangular strip of width *w* along the *y*-direction, and from 0 to ∞ in *z*-direction is:

$$\widetilde{I} = w \int_0^\infty \widetilde{J}_x(z)\, dz$$

$$= w \int_0^\infty J_0 e^{-(1+j)z/\delta_s}\, dz$$



7-5 Current Flow in a Good Conductor

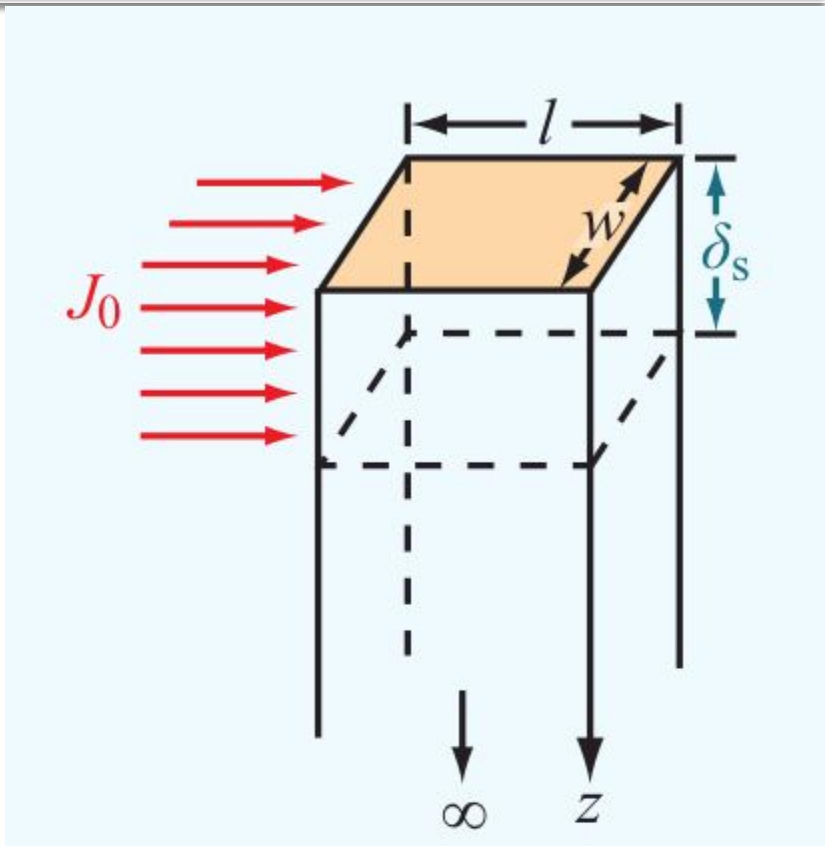
Conducting half-plane

Total current flowing through a rectangular strip of width *w* along the *y*-direction, and from 0 to ∞ in *z*-direction is:

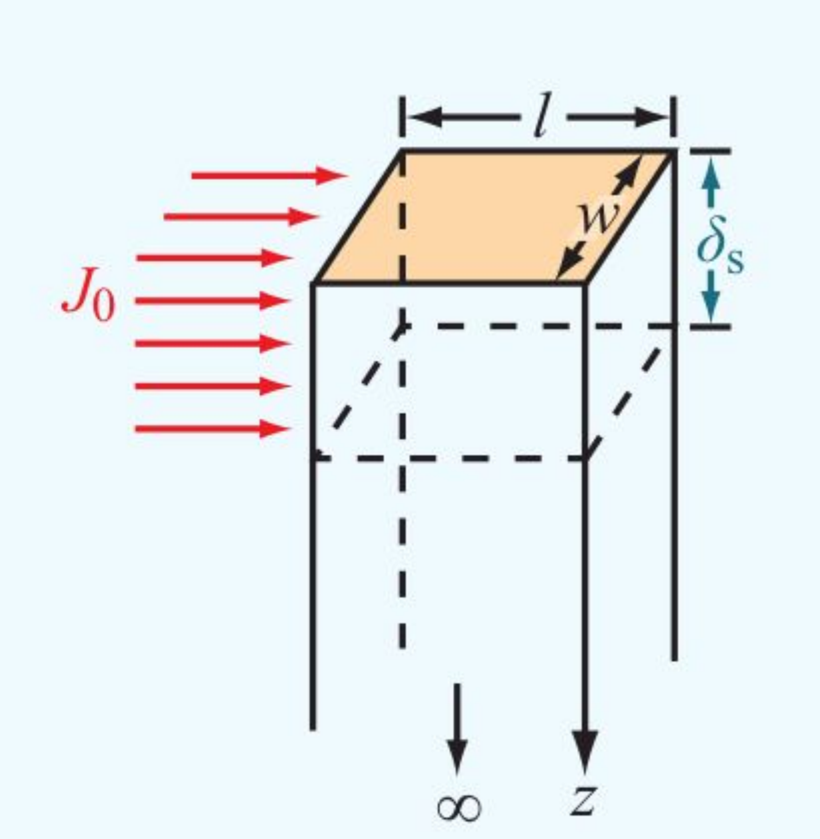
$$\tilde{I} = w \int_0^\infty \tilde{J}_x(z)\, dz$$

$$= w \int_0^\infty J_0 e^{-(1+j)z/\delta_s}\, dz$$

$$\tilde{I} = \frac{J_0 w \delta_s}{(1+j)} \qquad \text{(A)}$$



7-5 Current Flow in a Good Conductor



Conducting half-plane

Voltage in the *x*-direction, across a length, l, at the surface is:

$$\widetilde{V} = E_0 l = \frac{J_0}{\sigma} l.$$

So the impedance of a slab: width *w*, length *l*, depth > $5\delta_s$:

$$Z = \frac{\widetilde{V}}{\widetilde{I}} = \frac{1+j}{\sigma\delta_s} \frac{l}{w} \qquad (\Omega)$$

7-5 Current Flow in a Good Conductor

Conducting half-plane

So the impedance of a slab:

width w, length l, depth > $5\delta_s$:

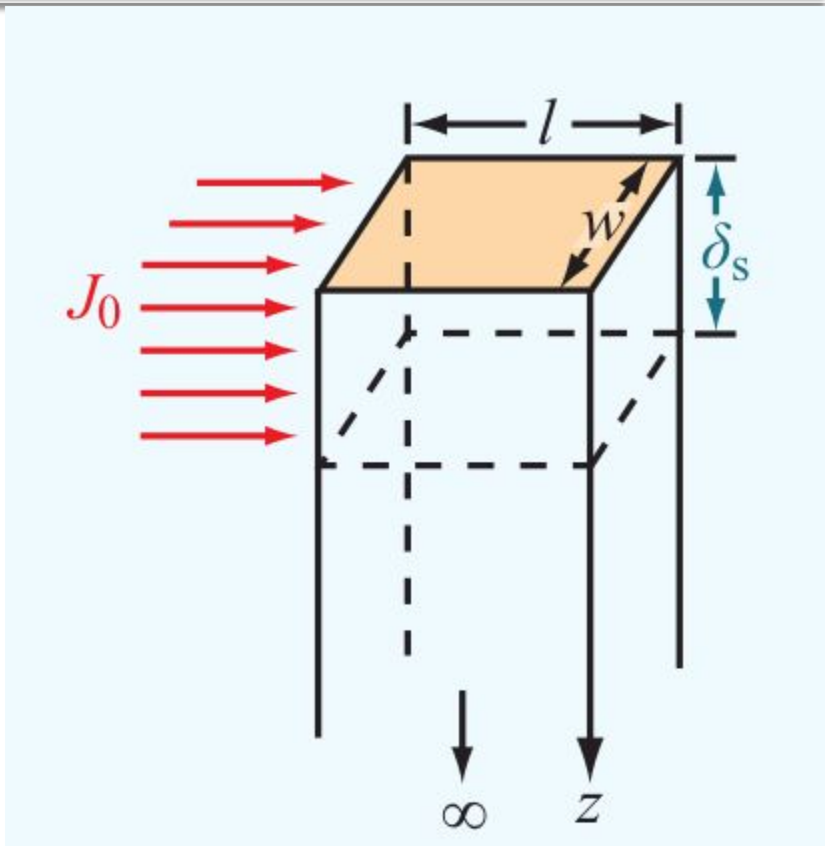
$$Z = \frac{\tilde{V}}{\tilde{I}} = \frac{1+j}{\sigma\delta_s}\,\frac{l}{w} \qquad (\Omega)$$

Define the surface impedance:

$$Z_s = \frac{1+j}{\sigma\delta_s} \qquad (\Omega).$$

so:

$$Z = Z_s\,\frac{l}{w}$$



7-5 Current Flow in a Good Conductor

Conducting half-plane

The surface impedance:

$$Z_s = \frac{1+j}{\sigma \delta_s} \qquad (\Omega).$$

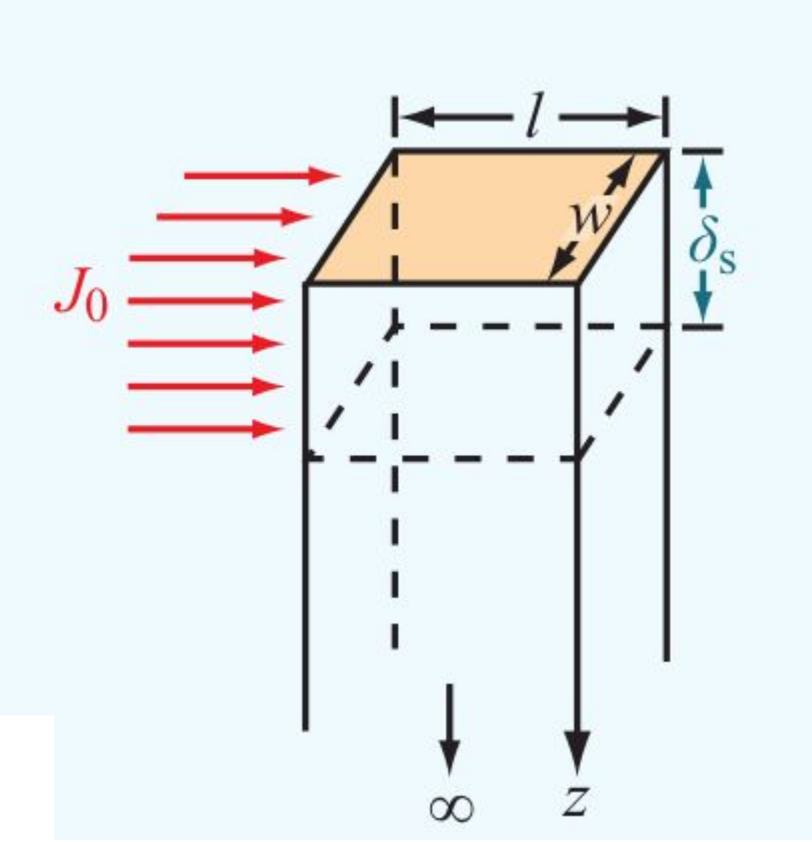
Since the imaginary part is > 0:

$$Z_s = R_s + j\omega L_s$$

and:

$$R_s = \frac{1}{\sigma \delta_s} = \sqrt{\frac{\pi f \mu}{\sigma}} \qquad (\Omega),$$

$$L_s = \frac{1}{\omega \sigma \delta_s} = \frac{1}{2}\sqrt{\frac{\mu}{\pi f \sigma}} \qquad (\text{H}),$$



we used:

$$\delta_s = 1/\alpha \approx 1/\sqrt{\pi f \mu \sigma}$$

7-5 Current Flow in a Good Conductor

Conducting half-plane

Define the **AC Resistance**:

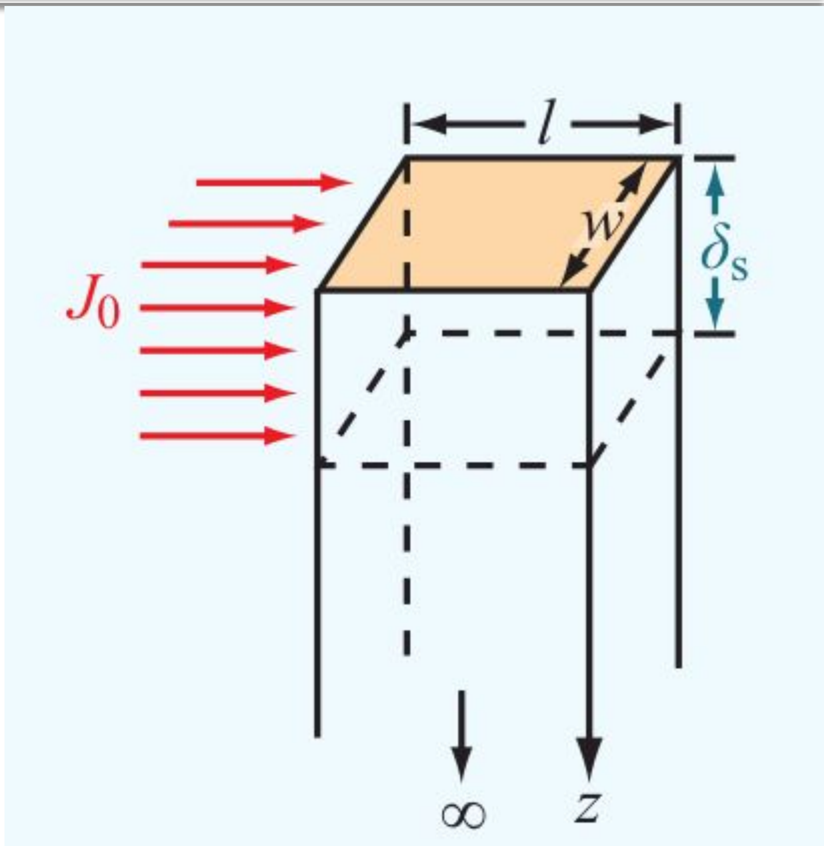
$$R = R_s \frac{l}{w} = \frac{l}{\sigma \delta_s w} \qquad (\Omega).$$

which is the same as the DC Resistance...

$$R = \frac{l}{\sigma A} \qquad (\Omega)$$

but with the cross-sectional area:

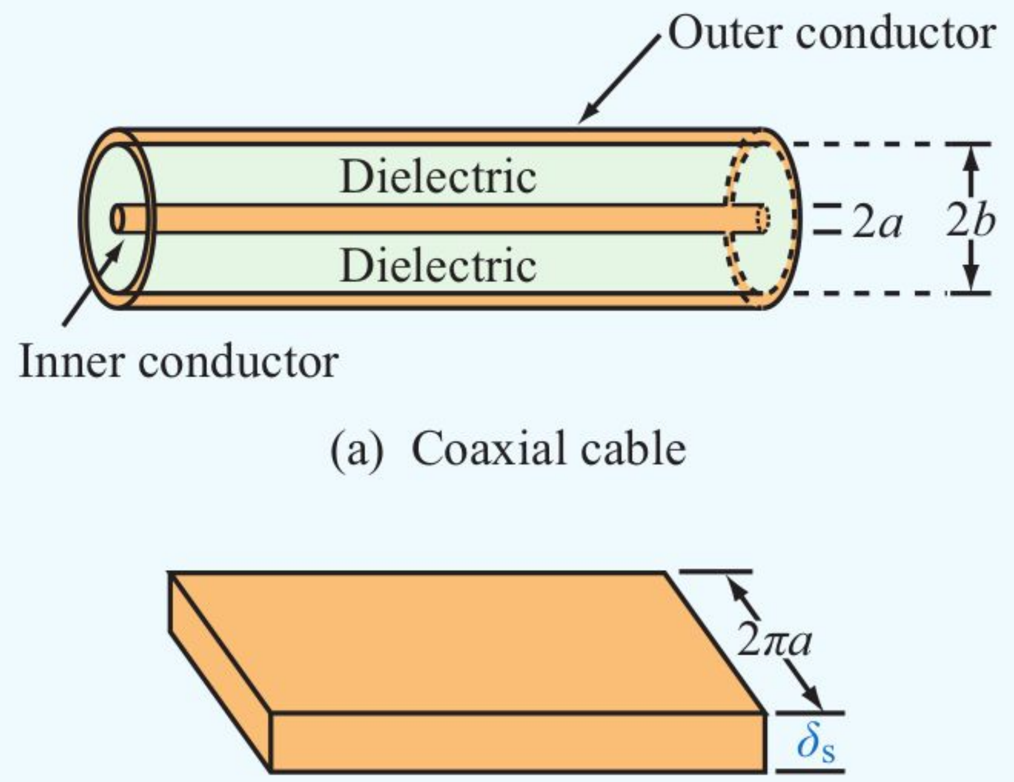
$A = \delta_s w$



7-5 Current Flow in a Good Conductor

AC Resistance of Coax

Inner conductor has a thin layer where there is actually current.



(a) Coaxial cable

(b) Equivalent inner conductor

Unwrap this thin layer into a slab: width $2\pi a$

since: $R = R_s \dfrac{l}{w}$

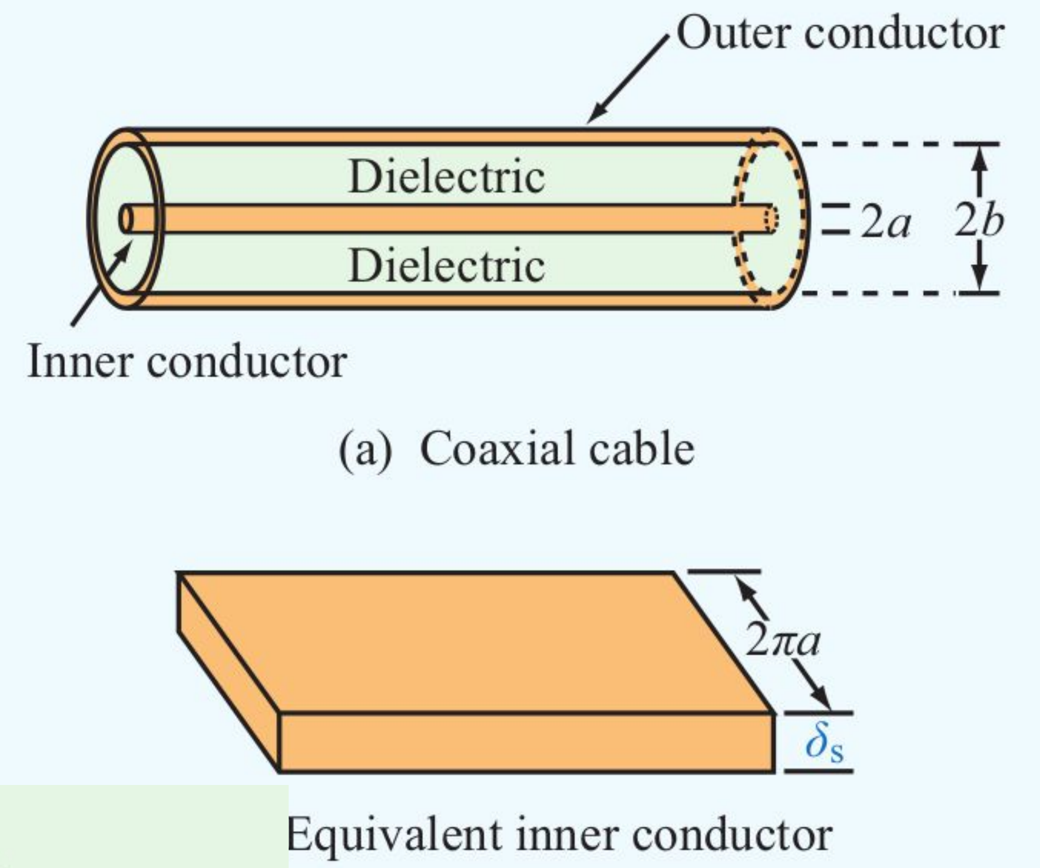
$$R'_1 = \frac{R}{l} = \frac{R_s}{2\pi a} \qquad (\Omega/\text{m}).$$

7-5 Current Flow in a Good Conductor

AC Resistance of Coax

Similarly for the outer conductor:

$$R'_2 = \frac{R_s}{2\pi b} \qquad (\Omega/\text{m})$$



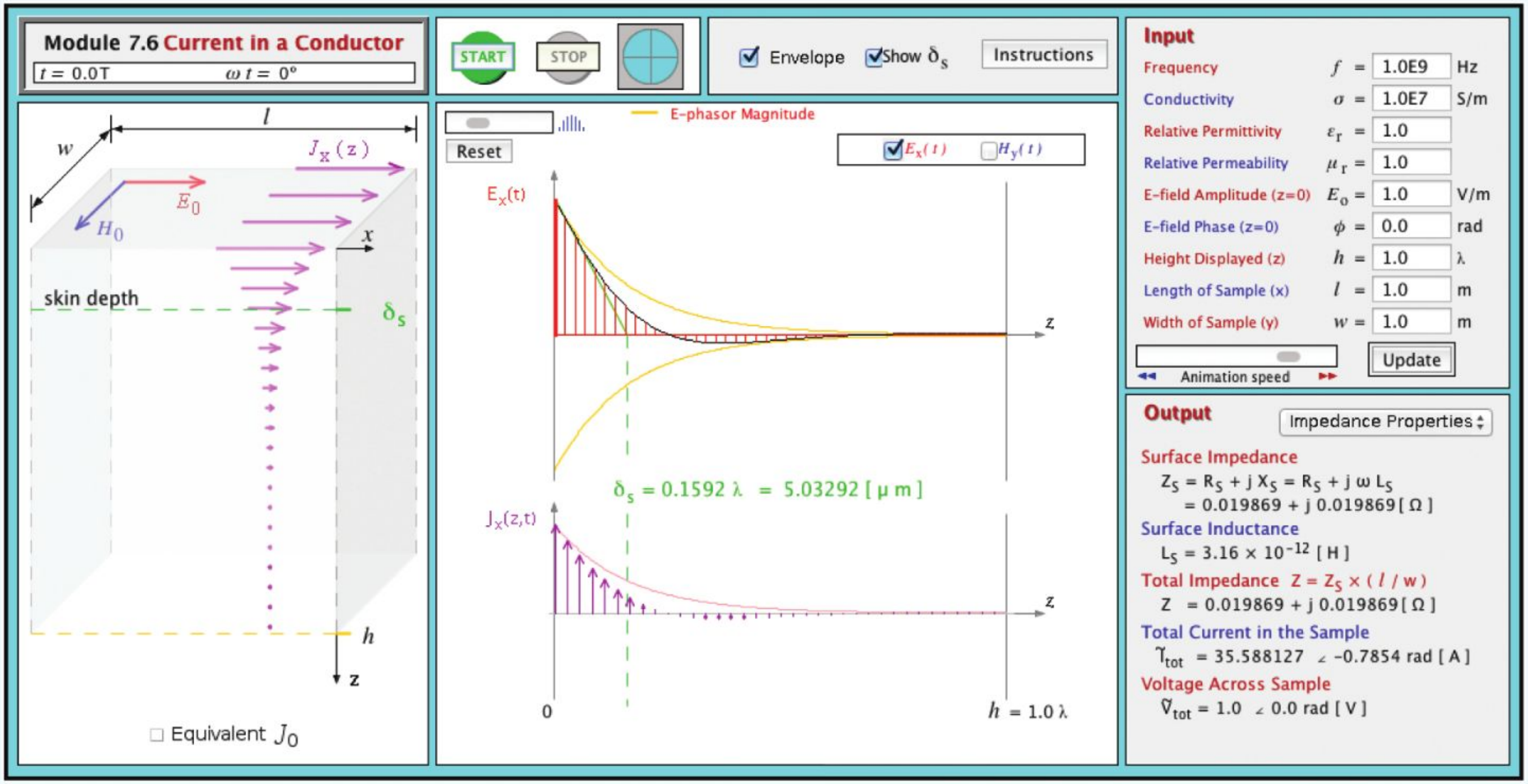
Total AC resistance is the sum:

$$R' = R'_1 + R'_2 = \frac{R_s}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right) \qquad (\Omega/\text{m}).$$

(as in chapter 2)

7-5 Current Flow in a Good Conductor

Module 7.6 Current in a Conductor Module displays exponential decay of current density in a conductor.



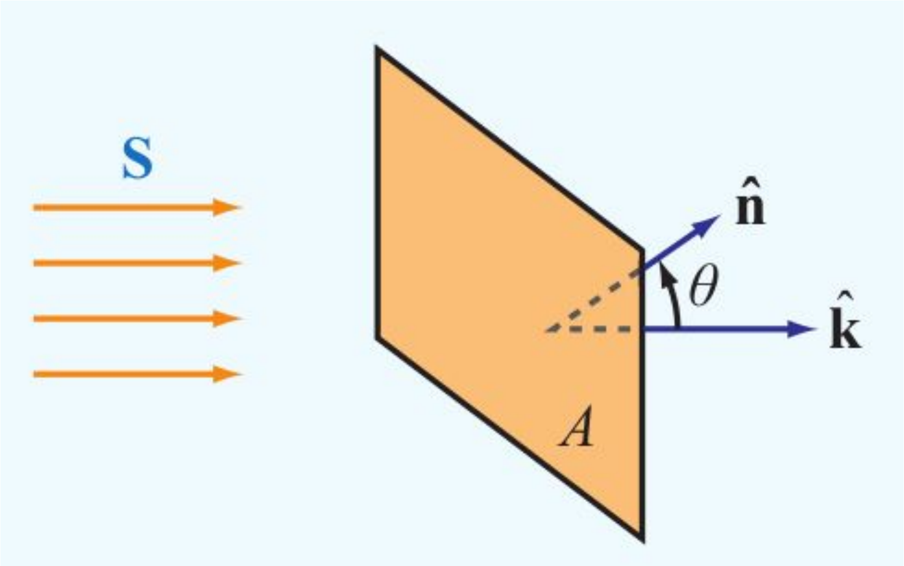
7-6 Electromagnetic Power Density

Define: $\mathbf{S} = \mathbf{E} \times \mathbf{H}$

Called the **Poynting Vector**

with units of Watts/m^2.

Define a planar area in space:



Total Power flowing thru this aperture:

$$P = \int_A \mathbf{S} \cdot \hat{\mathbf{n}} \, dA \qquad \text{(W)}.$$

7-6 Electromagnetic Power Density

Time-Average Power Density:

$$\mathbf{S}_{av} = \frac{1}{2}\,\mathfrak{Re}\left[\widetilde{\mathbf{E}} \times \widetilde{\mathbf{H}}^*\right] \qquad (\text{W/m}^2)$$

Note the complex-conjugate on **H**.

Recall the definition of complex conjugate:

if $\quad Z = a + jb$

then $Z^* = a - jb$

so: just replace every j with a $-j$

7-6 Electromagnetic Power Density

For a plane wave in a **lossless** medium:

$$\tilde{\mathbf{E}} = (\hat{\mathbf{x}} E_{x0} + \hat{\mathbf{y}} E_{y0}) e^{-jkz}$$

$$\tilde{\mathbf{H}}^* = \frac{1}{\eta} (-\hat{\mathbf{x}} E_{y0}^* + \hat{\mathbf{y}} E_{x0}^*) e^{+jkz}$$

$$\mathbf{S}_{\text{av}} = \frac{1}{2} \mathfrak{Re} \left[\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^* \right]$$

$$\mathbf{S}_{\text{av}} = \frac{1}{2\eta} \mathfrak{Re} \left[(\hat{\mathbf{x}} E_{x0} + \hat{\mathbf{y}} E_{y0}) \times (-\hat{\mathbf{x}} E_{y0}^* + \hat{\mathbf{y}} E_{x0}^*) \right]$$

7-6 Electromagnetic Power Density

For a plane wave in a **lossless** medium:

$$\mathbf{S}_{\text{av}} = \frac{1}{2\eta}\mathfrak{Re}\left[(\hat{\mathbf{x}}E_{x0} + \hat{\mathbf{y}}E_{y0}) \times (-\hat{\mathbf{x}}E^*_{y0} + \hat{\mathbf{y}}E^*_{x0})\right]$$

$$\mathbf{S}_{\text{av}} = \frac{1}{2\eta}\hat{\mathbf{z}}(E_{x0}E^*_{x0} + E_{y0}E^*_{y0})$$

$$\mathbf{S}_{\text{av}} = \frac{1}{2\eta}\hat{\mathbf{z}}(|E_{x0}|^2 + |E_{y0}|^2)$$

$$\mathbf{S}_{\text{av}} = \frac{1}{2\eta}\hat{\mathbf{z}}(|\tilde{\mathbf{E}}|^2)$$

Example 7-5

Solar Power

Given: Solar power on the Earth's surface is S_{av}=1 kW/m^2

The radius of the Earth's orbit: R_s=1.5X10^8 km

The radius of the Earth: R_e=6380 km

Find: (a) Total power radiated by the sun, P_{sun}

(b) Total power intercepted by the Earth, P_{int}

Example 7-5

Solution:

Assume the sun radiates equally in all directions.

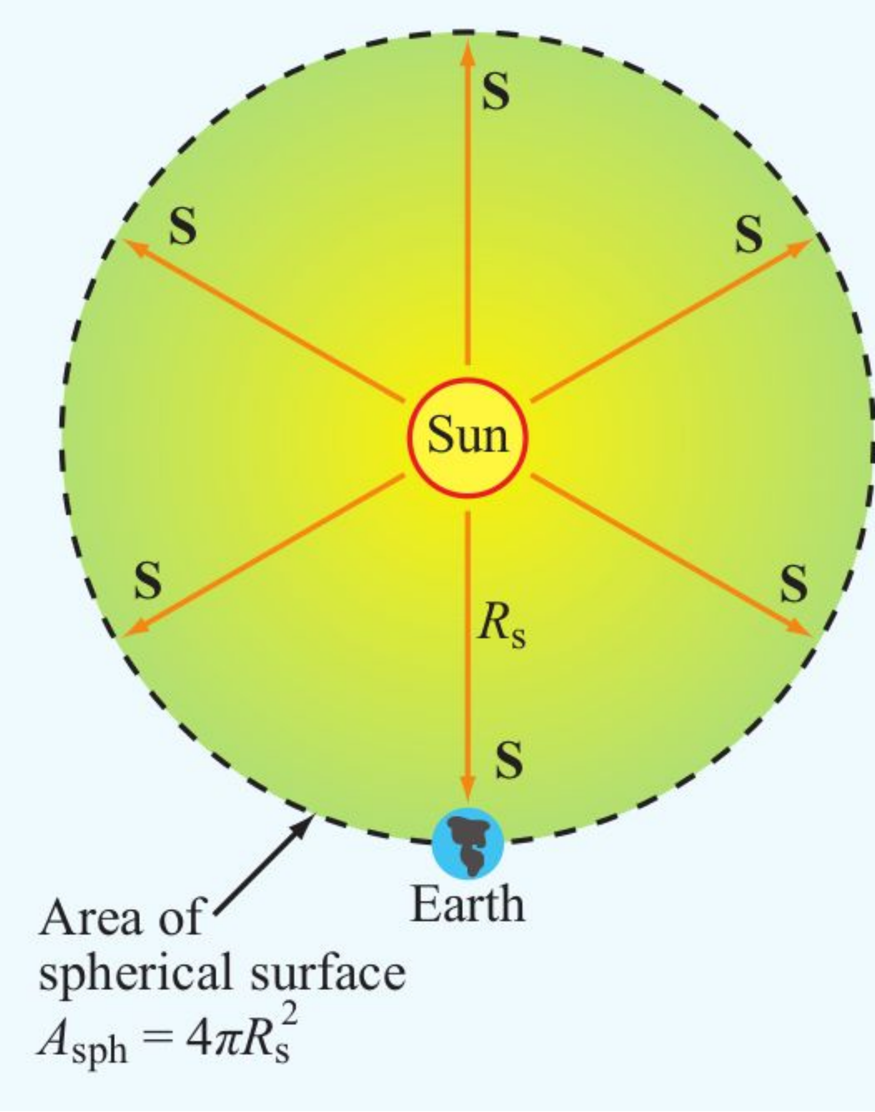
$P_{sun} = S_{av} A_{sphere}$

$A_{sphere} = 4\pi R^2$

so:

$P_{sun} = (1\ kW/m^2) 4\pi (1.5 X 10^{11} m)^2$

$P_{sun} = 2.8\ X\ 10^{26}$ Watts



Example 7-5

Assume the power intercepted by the Earth is approximately the same as intercepted by a circular disk with the same radius as the Earth:

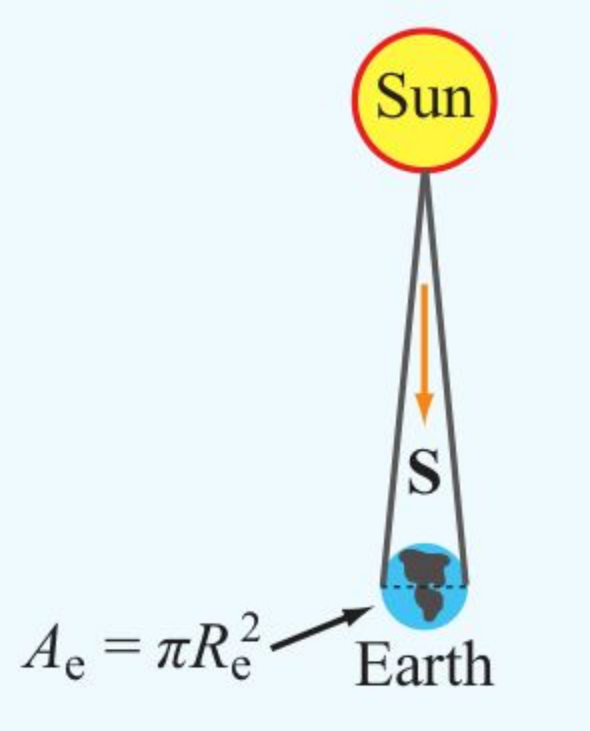
$$A_e = \pi R_e^2$$

so:

$$P_{int} = S_{av}(\pi R_e^2)$$

P_{int}=(1 kW/m^2)π(6380X10^{3}m)^2

P_{int}= 1.28 X 10^{17} Watts



7-6 Electromagnetic Power Density

For a plane wave in a **lossy** medium:

$$\tilde{\mathbf{E}} = (\hat{\mathbf{x}} E_{x0} + \hat{\mathbf{y}} E_{y0}) e^{-\alpha z} e^{-j\beta z}$$

$$\tilde{\mathbf{H}}^* = \frac{1}{\eta_c^*} (-\hat{\mathbf{x}} E_{y0}^* + \hat{\mathbf{y}} E_{x0}^*) e^{-\alpha z} e^{+j\beta z}$$

Calculate the time-average power density:

$$\mathbf{S}_{\text{av}} = \frac{1}{2} \mathfrak{Re} \left[\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^* \right]$$

7-6 Electromagnetic Power Density

$$\tilde{\mathbf{E}} = (\hat{\mathbf{x}} E_{x0} + \hat{\mathbf{y}} E_{y0}) e^{-\alpha z} e^{-j\beta z}$$

$$\tilde{\mathbf{H}}^* = \frac{1}{\eta_c^*} (-\hat{\mathbf{x}} E_{y0}^* + \hat{\mathbf{y}} E_{x0}^*) e^{-\alpha z} e^{+j\beta z}$$

$$\mathbf{S}_{\text{av}} = \frac{1}{2} \mathfrak{Re} \left[\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^* \right]$$

$$\mathbf{S}_{\text{av}} = \frac{1}{2} \Re e \left[(\hat{\mathbf{x}} E_{x0} + \hat{\mathbf{y}} E_{y0}) \times (-\hat{\mathbf{x}} E_{y0}^* + \hat{\mathbf{y}} E_{x0}^*) \right.$$

$$\left. e^{-\alpha z} e^{-j\beta z} \frac{1}{\eta_c^*} e^{-\alpha z} e^{+j\beta z} \right]$$

$$\mathbf{S}_{\text{av}} = \frac{1}{2} \Re e \left[\hat{\mathbf{z}} (E_{x0} E_{x0}^* + E_{y0} E_{y0}^*) e^{-2\alpha z} \frac{1}{\eta_c^*} \right]$$

$$\mathbf{S}_{\text{av}} = \frac{\hat{\mathbf{z}} (|E_{x0}|^2 + |E_{y0}|^2)}{2} e^{-2\alpha z} \Re e \left[\frac{1}{\eta_c^*} \right]$$

7-6 Electromagnetic Power Density

$$\mathbf{S}_{\text{av}} = \frac{\hat{\mathbf{z}}(|E_{x0}|^2 + |E_{y0}|^2)}{2} e^{-2\alpha z} \Re e \left[\frac{1}{\eta_c^*} \right]$$

$$\Re e \left[\frac{1}{\eta_c^*} \right] = \Re e \left[\frac{1}{|\eta_c| \, e^{-j\theta_\eta}} \right]$$

$$\Re e \left[\frac{1}{\eta_c^*} \right] = \Re e \left[\frac{1}{|\eta_c|} \, e^{+j\theta_\eta} \right]$$

$$\Re e \left[\frac{1}{\eta_c^*} \right] = \frac{1}{|\eta_c|} \cos(\theta_\eta)$$

$$\mathbf{S}_{\text{av}} = \frac{\hat{\mathbf{z}}(|E_{x0}|^2 + |E_{y0}|^2)}{2|\eta_c|} e^{-2\alpha z} \cos(\theta_\eta)$$

7-6 Electromagnetic Power Density

For a plane wave in a **lossy** medium:

$$\mathbf{S}_{\text{av}} = \frac{\hat{\mathbf{z}}(|E_{x0}|^2 + |E_{y0}|^2)}{2|\eta_c|} e^{-2\alpha z} \cos(\theta_\eta)$$

since: $|\tilde{E}(z=0)|^2 = |E_{x0}|^2 + |E_{y0}|^2$

$$\mathbf{S}_{\text{av}}(z) = \hat{\mathbf{z}} \frac{|\widetilde{E}(0)|^2}{2|\eta_c|} e^{-2\alpha z} \cos\theta_\eta \quad (\text{W/m}^2),$$

Example 7-6

Submarine Communications

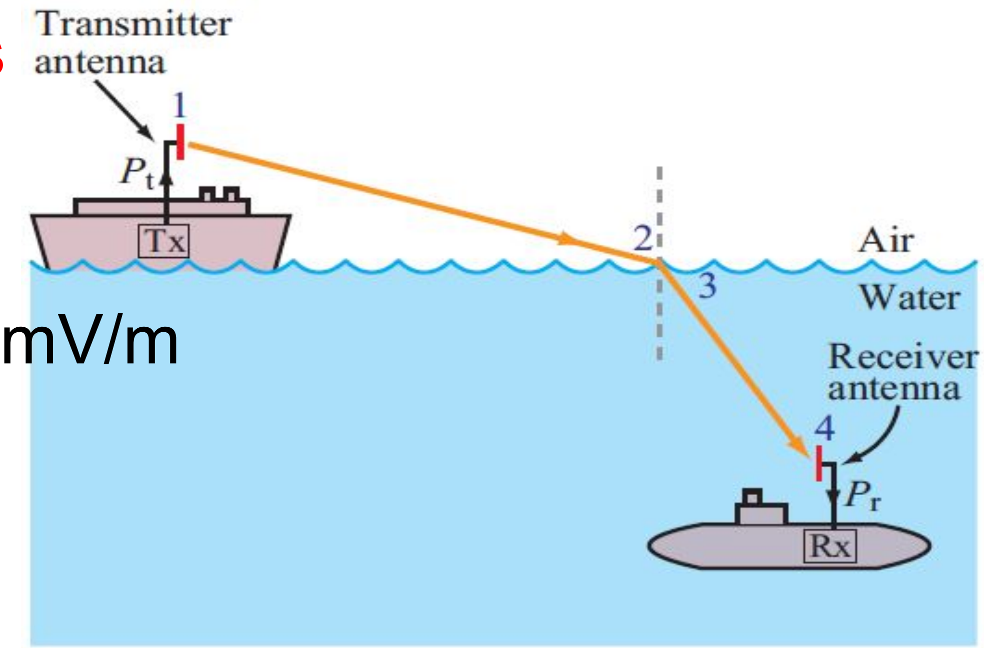
Given: depth: 200m

f = 1KHz

$|\tilde{E}(z=0)|^2 =$ 4.44mV/m

α = 0.126 Np/m

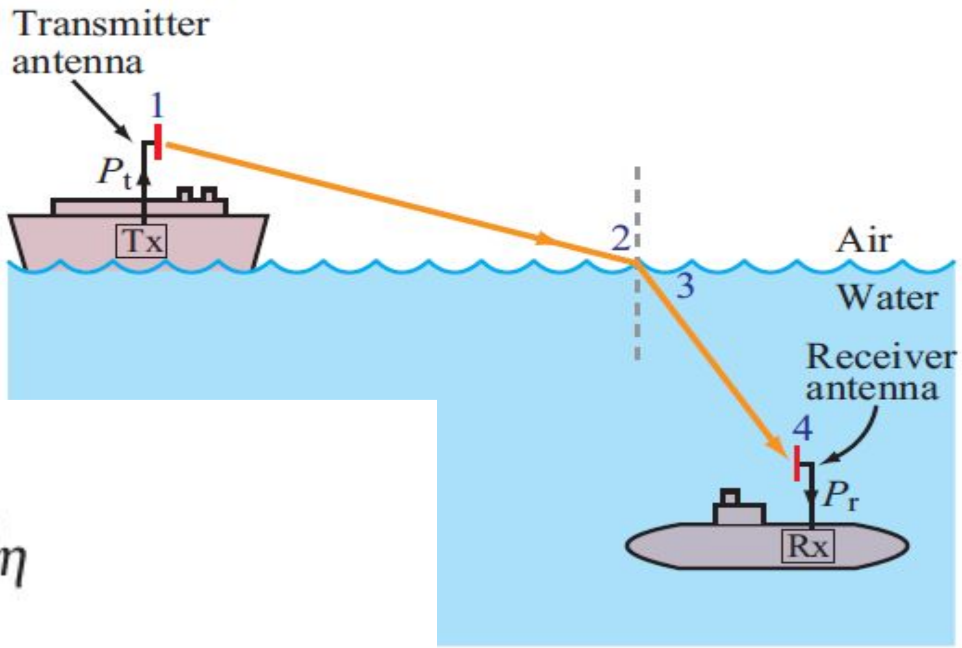
η_c = 0.044 ∠45° Ω



Find: Power density incident on the submarine's antenna

Example 7-6

Submarine Communications

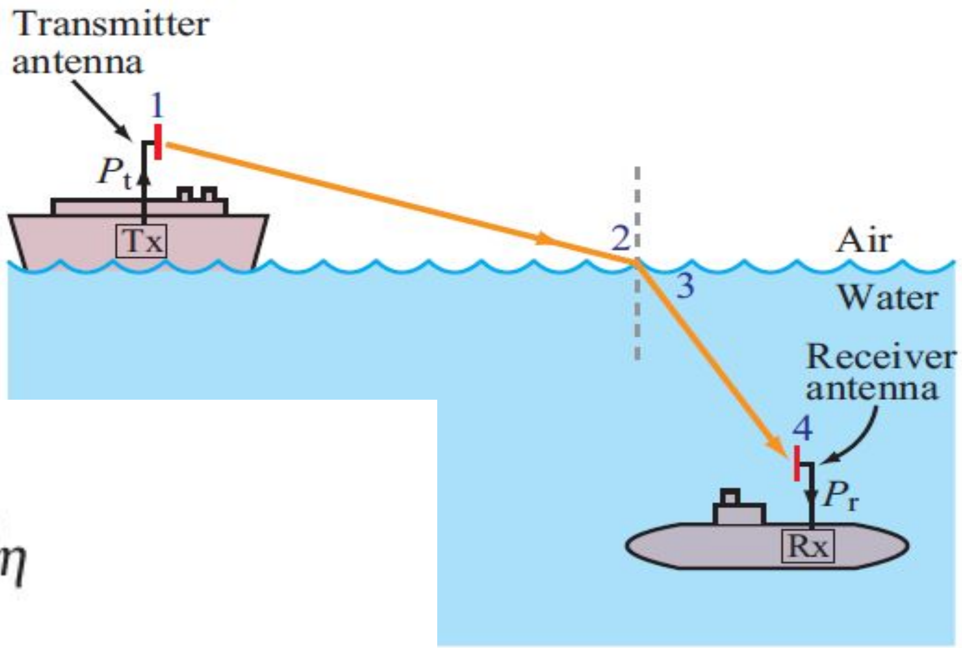


Solution:

$$\mathbf{S}_{av}(z) = \hat{\mathbf{z}} \frac{|E_0|^2}{2|\eta_c|} e^{-2\alpha z} \cos\theta_\eta$$

Example 7-6

Submarine Communications



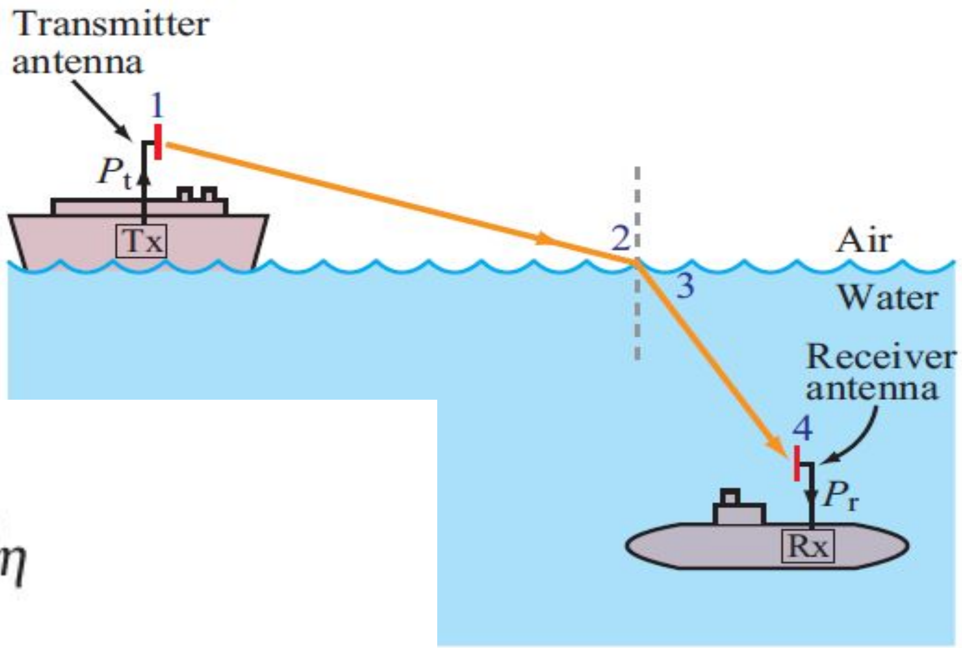
Solution:

$$\mathbf{S}_{av}(z) = \hat{\mathbf{z}} \frac{|E_0|^2}{2|\eta_c|} e^{-2\alpha z} \cos\theta_\eta$$

$$= \hat{\mathbf{z}} \frac{(4.44 \times 10^{-3})^2}{2 \times 0.044} e^{-0.252z} \cos 45^\circ$$

Example 7-6

Submarine Communications



Solution:

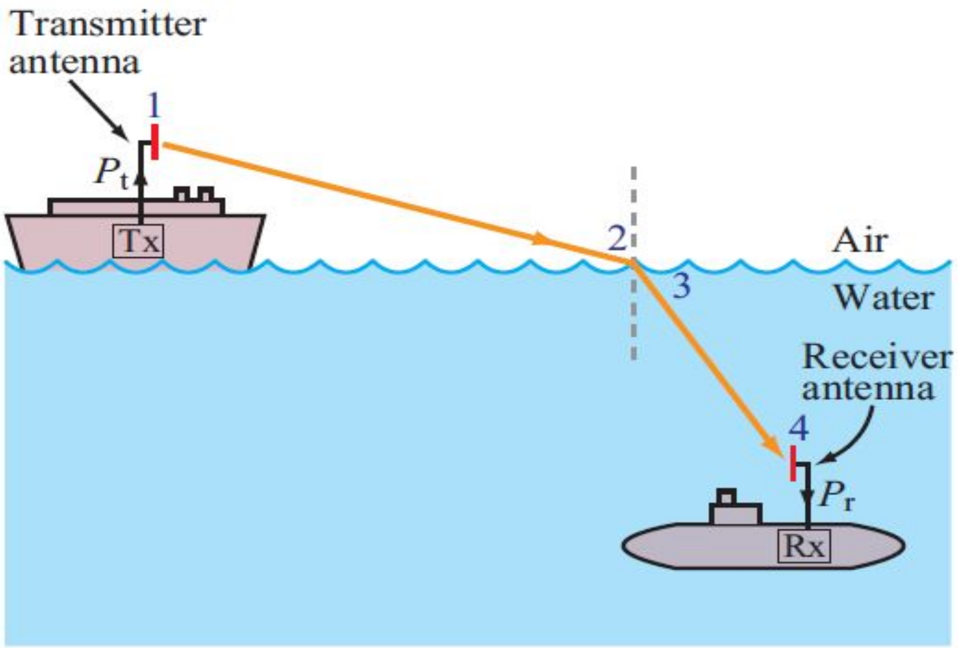
$$\mathbf{S}_{av}(z) = \hat{\mathbf{z}} \frac{|E_0|^2}{2|\eta_c|} e^{-2\alpha z} \cos\theta_\eta$$

$$= \hat{\mathbf{z}} \frac{(4.44 \times 10^{-3})^2}{2 \times 0.044} e^{-0.252z} \cos 45^\circ$$

$$= \hat{\mathbf{z}} 0.16 e^{-0.252z} \qquad (\text{mW/m}^2).$$

Example 7-6

Submarine Communications



Solution:

plug in z=200m

$$\mathbf{S}_{av} = \hat{\mathbf{z}}(0.16 \times 10^{-3} e^{-0.252 \times 200})$$
$$= 2.1 \times 10^{-26} \quad (\text{W/m}^2).$$

Note that this is so small that it is below the noise power in the system, and so it would not work.

This is why submarines surface in order to communicate.

Homework

Homework 26 is due tomorrow at midnight.

submit to gradescope via the canvas site.

Next Time

Sections 8-1 through 8-3:

Wave Reflection and Transmission: Normal Incidence

Snell's Laws