

EECS 230  
*ENGINEERING ELECTROMAGNETICS*  
*Leland Pierce*

Plane Waves 1

# Chapter 7 Overview

Unbounded EM Waves  
 Time-Harmonic Fields  
 Complex Permittivity  
 Wave Equations

$$\epsilon_c = \epsilon - j \frac{\sigma}{\omega}$$

$$k = \omega \sqrt{\mu \epsilon_c}$$

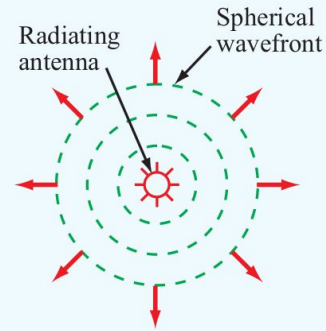
$$\nabla^2 \tilde{\mathbf{E}} - \gamma^2 \tilde{\mathbf{E}} = 0.$$

$$\nabla \cdot \tilde{\mathbf{E}} = \tilde{\rho}_v / \epsilon,$$

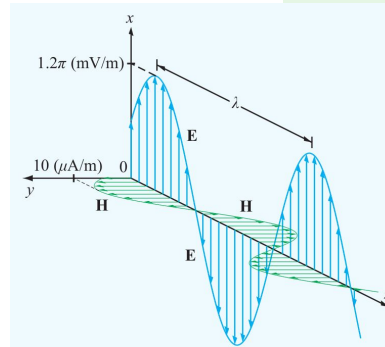
$$\nabla \times \tilde{\mathbf{E}} = -j\omega\mu\tilde{\mathbf{H}},$$

$$\nabla \cdot \tilde{\mathbf{H}} = 0,$$

$$\nabla \times \tilde{\mathbf{H}} = \tilde{\mathbf{J}} + j\omega\epsilon\tilde{\mathbf{E}}.$$



Plane Waves: Lossless Media  
 Uniform Plane Waves  
 Relation between E and H  
 Wave Polarization

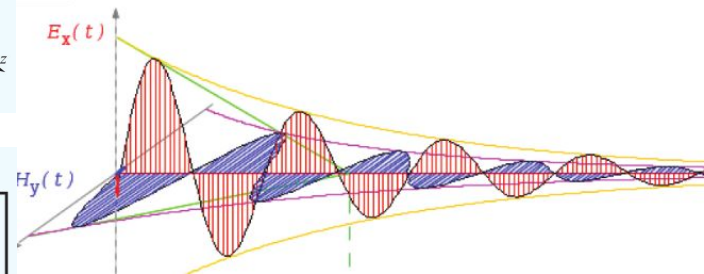


$$\tilde{\mathbf{E}}(z) = \hat{\mathbf{x}} \tilde{E}_x^+(z) = \hat{\mathbf{x}} E_{x0}^+ e^{-jkz},$$

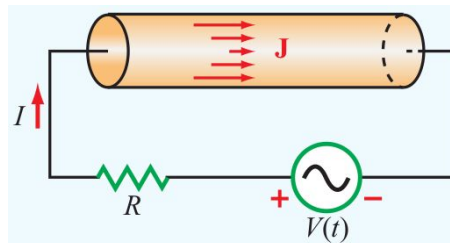
$$\tilde{\mathbf{H}}(z) = \hat{\mathbf{y}} \frac{\tilde{E}_x^+(z)}{\eta} = \hat{\mathbf{y}} \frac{E_{x0}^+}{\eta} e^{-jkz}.$$

Linear  
 Circular  
 Elliptical

Plane Waves: Lossy Media  
 Low Loss



Good Conductor  
 Current in Good Conductor  
 Power Density



$$\mathbf{S}_{av} = \frac{1}{2} \Re \left[ \tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^* \right] \quad (\text{W/m}^2).$$

# Lecture Coverage

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**Today's lecture:**

**Sections 7-1 through 7-3 of the book:**

**7-1:** Time-Harmonic Fields

**7-2:** Plane-Wave Propagation in Lossless Media

**7-3:** Wave Polarization

# 7 Unbounded EM Waves

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## Examples:

Light waves emitted by the sun, or other stars

Radio transmission by antennas

Headlights on cars

# 7 Unbounded EM Waves

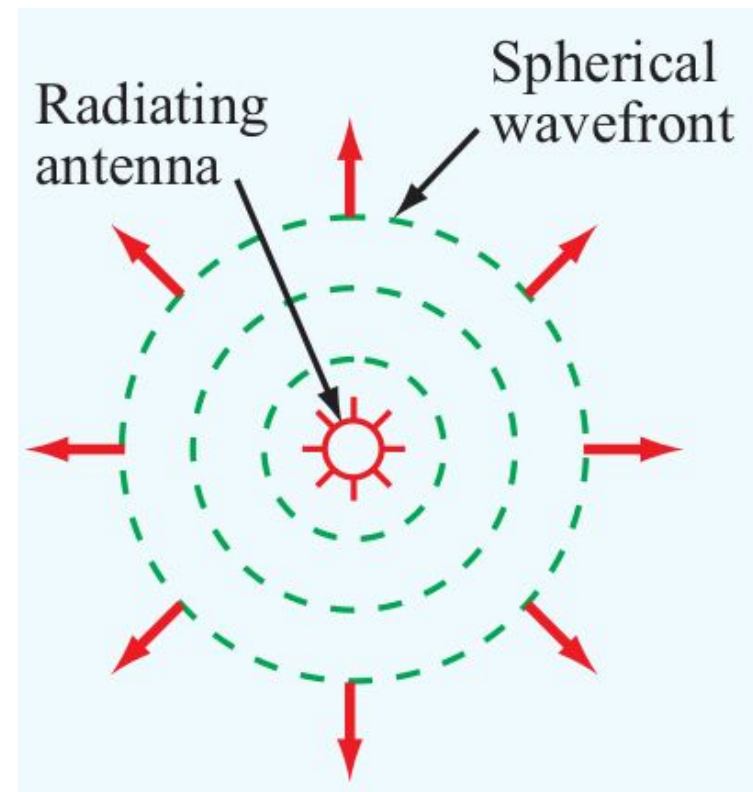
## Spherical waves:

produced by a localized source

wavefront is surface where every point is at the same distance from the source

For a point source: spheres

For single-frequency sources, a wavefront has the same phase at every point.



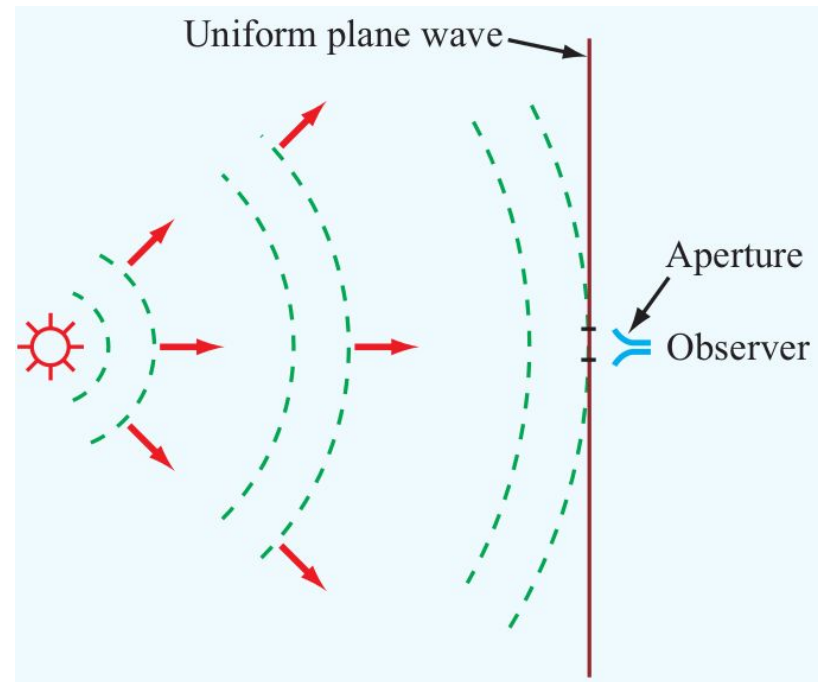
# 7 Unbounded EM Waves

## Plane waves:

produced by a distant source

a receiving antenna "sees"  
a wavefront that is nearly flat

wavefronts are still spheres,  
but in a small localized region,  
can approximate as planar.



# 7 Guided EM Waves

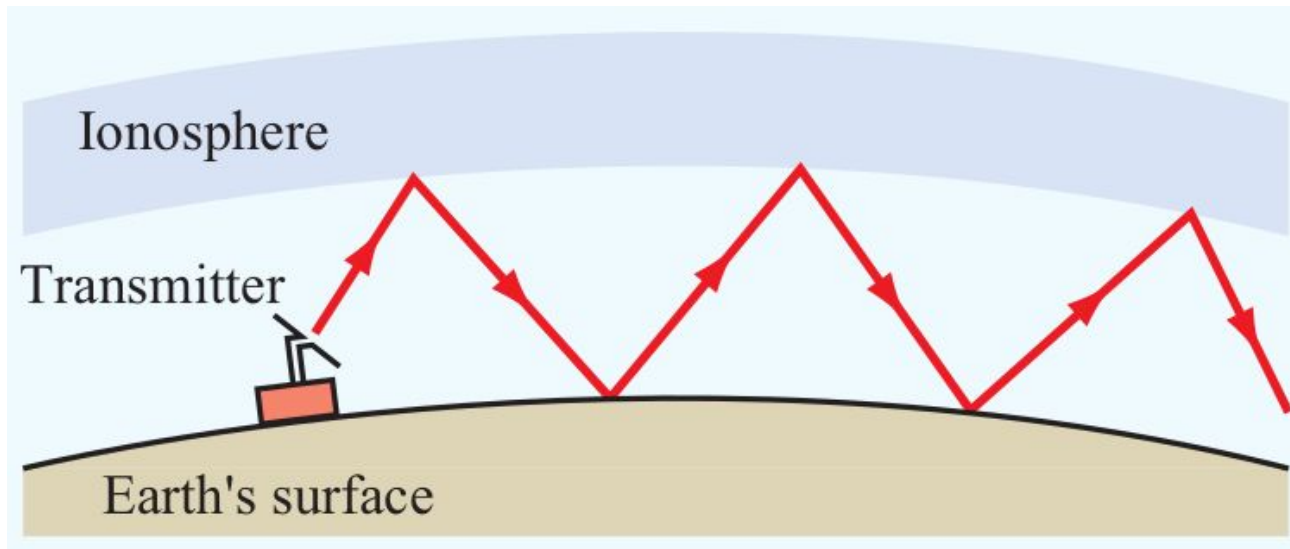
## Guided waves:

propagate along a material structure

Example:

Radio waves between  
3 and 30 MHz

reflect off the ionosphere:



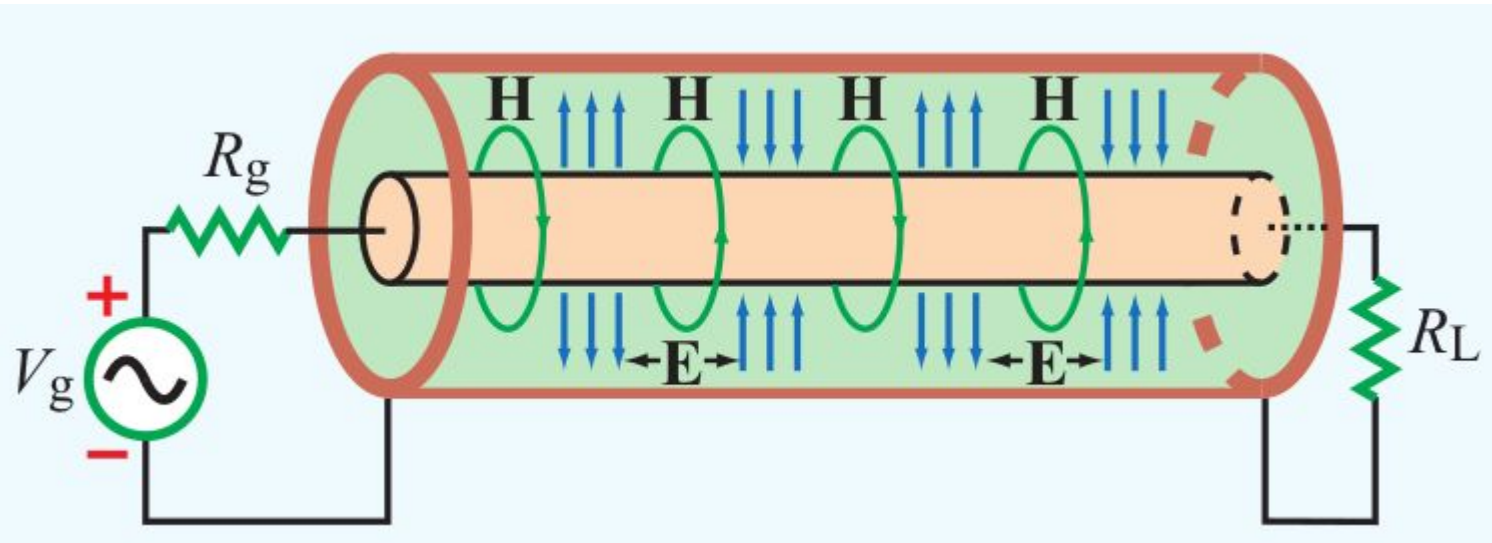
# 7 Guided EM Waves

Guided waves:

propagate along a material structure

Example:

Time-varying fields in a coaxial cable are guided



# 7-1 Time-Harmonic Fields

These are the 4 Maxwell's Equations, where we are assuming  $\mathbf{B}=\mu\mathbf{H}$  and  $\mathbf{D}=\varepsilon\mathbf{E}$ :

$$\nabla \cdot \mathbf{E} = \rho_v / \varepsilon$$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t}$$

$$\nabla \cdot \mathbf{H} = 0$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \varepsilon \frac{\partial \mathbf{E}}{\partial t}$$

# 7-1 Time-Harmonic Fields

Assume that all the fields are oscillating at the same frequency:

So, we can use the concept of phasors:

$$\mathbf{E} = \Re \left\{ \tilde{\mathbf{E}} e^{j\omega t} \right\}$$

$$\mathbf{H} = \Re \left\{ \tilde{\mathbf{H}} e^{j\omega t} \right\}$$

$$\mathbf{J} = \Re \left\{ \tilde{\mathbf{J}} e^{j\omega t} \right\}$$

# 7-1 Time-Harmonic Fields

Time-Harmonic Maxwell's eqns for media that are:  
linear, isotropic, and homogeneous  
with material properties:  $\epsilon$ ,  $\mu$ ,  $\sigma$

$$\nabla \cdot \tilde{\mathbf{E}} = \tilde{\rho}_v / \epsilon,$$

$$\nabla \times \tilde{\mathbf{E}} = -j\omega\mu\tilde{\mathbf{H}},$$

$$\nabla \cdot \tilde{\mathbf{H}} = 0,$$

$$\nabla \times \tilde{\mathbf{H}} = \tilde{\mathbf{J}} + j\omega\epsilon\tilde{\mathbf{E}}.$$

# 7-1 Complex Permittivity

$$\nabla \times \tilde{\mathbf{H}} = \tilde{\mathbf{J}} + j\omega\epsilon\tilde{\mathbf{E}} = (\sigma + j\omega\epsilon)\tilde{\mathbf{E}} = j\omega\left(\epsilon - j\frac{\sigma}{\omega}\right)\tilde{\mathbf{E}}.$$

So define the **Complex Permittivity**:

$$\epsilon_c = \epsilon - j\frac{\sigma}{\omega} = \epsilon' - j\epsilon'',$$

rewrite:

$$\nabla \times \tilde{\mathbf{H}} = j\omega\epsilon_c\tilde{\mathbf{E}}.$$

For a lossless medium:

$$\epsilon' = \epsilon \quad \epsilon'' = 0$$

# 7-1 Source-Free Medium

A source-free medium is defined with:

$$\rho_v = 0$$

Maxwell's Equations become:

$$\begin{aligned}\nabla \cdot \tilde{\mathbf{E}} &= 0, \\ \nabla \times \tilde{\mathbf{E}} &= -j\omega\mu\tilde{\mathbf{H}}, \\ \nabla \cdot \tilde{\mathbf{H}} &= 0, \\ \nabla \times \tilde{\mathbf{H}} &= j\omega\epsilon_c\tilde{\mathbf{E}}.\end{aligned}$$

# 7-1 Wave Equations

Let's derive a differential equation for  $\tilde{\mathbf{E}}$ :  
Start with one of Maxwell's Equations:

$$\nabla \times \tilde{\mathbf{E}} = -j\omega\mu\tilde{\mathbf{H}}$$

Take the curl of both sides:

$$\nabla \times (\nabla \times \tilde{\mathbf{E}}) = -j\omega\mu(\nabla \times \tilde{\mathbf{H}})$$

Use another Maxwell Eqn:

get: 
$$\nabla \times \tilde{\mathbf{H}} = j\omega\epsilon_c\tilde{\mathbf{E}}$$

$$\nabla \times (\nabla \times \tilde{\mathbf{E}}) = -j\omega\mu(j\omega\epsilon_c\tilde{\mathbf{E}}) = \omega^2\mu\epsilon_c\tilde{\mathbf{E}}.$$

# 7-1 Wave Equations

So given this:

$$\nabla \times (\nabla \times \tilde{\mathbf{E}}) = \omega^2 \mu \epsilon_c \tilde{\mathbf{E}}$$

from calculus we can say:

$$\nabla \times (\nabla \times \tilde{\mathbf{E}}) = \nabla(\nabla \cdot \tilde{\mathbf{E}}) - \nabla^2 \tilde{\mathbf{E}}$$

Because of another Maxwell Eqn:

$$\nabla \cdot \tilde{\mathbf{E}} = 0$$

we get:

$$\nabla^2 \tilde{\mathbf{E}} + \omega^2 \mu \epsilon_c \tilde{\mathbf{E}} = 0$$

# 7-1 Wave Equations

$$\nabla^2 \tilde{\mathbf{E}} + \omega^2 \mu \epsilon_c \tilde{\mathbf{E}} = 0$$

define:  $\gamma^2 = -\omega^2 \mu \epsilon_c$  (which is complex)

get:

$$\nabla^2 \tilde{\mathbf{E}} - \gamma^2 \tilde{\mathbf{E}} = 0. \quad (\text{wave equation for } \tilde{\mathbf{E}})$$

Through a similar process we can get:

$$\nabla^2 \tilde{\mathbf{H}} - \gamma^2 \tilde{\mathbf{H}} = 0. \quad (\text{wave equation for } \tilde{\mathbf{H}})$$

# 7-2 Lossless Media

Lossless media have  $\sigma=0$ ,  $\epsilon_c=\epsilon$ , so:

$$\gamma^2 = -\omega^2 \mu \epsilon \quad (\text{real})$$

and the wavenumber is defined as:

$$k = \omega \sqrt{\mu \epsilon}$$

so:

$$\gamma^2 = -k^2$$

therefore:

$$\nabla^2 \tilde{\mathbf{E}} + k^2 \tilde{\mathbf{E}} = 0$$

# 7-2 Uniform Plane Waves

The wave eqn:  $\nabla^2 \tilde{\mathbf{E}} + k^2 \tilde{\mathbf{E}} = 0$

If we have:  $\tilde{\mathbf{E}} = \hat{\mathbf{x}}\tilde{E}_x + \hat{\mathbf{y}}\tilde{E}_y + \hat{\mathbf{z}}\tilde{E}_z,$

Plugging into the wave eqn:

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) (\hat{\mathbf{x}}\tilde{E}_x + \hat{\mathbf{y}}\tilde{E}_y + \hat{\mathbf{z}}\tilde{E}_z) + k^2 (\hat{\mathbf{x}}\tilde{E}_x + \hat{\mathbf{y}}\tilde{E}_y + \hat{\mathbf{z}}\tilde{E}_z) = 0.$$

which must be true for each vector component separately.

## 7-2 Uniform Plane Waves

So we get:

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2 \right) \tilde{E}_x = 0,$$

and similar expressions for  $\tilde{E}_y$  and  $\tilde{E}_z$ .

If we have a **uniform plane wave**, with uniform properties across the  $x$ - $y$  plane, then **E** and **H** are not functions of  $x$  or  $y$ :

$$\frac{\partial \tilde{E}_x}{\partial x} = 0 \quad \frac{\partial \tilde{E}_x}{\partial y} = 0$$

so wave eqn becomes:

$$\frac{d^2 \tilde{E}_x}{dz^2} + k^2 \tilde{E}_x = 0.$$

## 7-2 Uniform Plane Waves

$$\frac{d^2 \tilde{E}_x}{dz^2} + k^2 \tilde{E}_x = 0.$$

We get similar expressions for:  $\tilde{E}_y$ ,  $\tilde{H}_x$ , and  $\tilde{H}_y$ .  
What about  $\tilde{E}_z$  ?

One of Maxwell's eqns says:

$$\nabla \times \tilde{\mathbf{H}} = j\omega\epsilon_c \tilde{\mathbf{E}}$$

The z-component is:

$$\hat{\mathbf{z}} \left( \frac{\partial \tilde{H}_y}{\partial x} - \frac{\partial \tilde{H}_x}{\partial y} \right) = \hat{\mathbf{z}} j\omega\epsilon \tilde{E}_z.$$

But since:  $\partial \tilde{H}_y / \partial x = \partial \tilde{H}_x / \partial y = 0$ , then:  $\tilde{E}_z = 0$ .  
Similarly:  $\tilde{H}_z = 0$ .

# 7-2 Uniform Plane Waves

So, a plane wave has no components in the direction of propagation.

For instance: propagating in z-direction:

$$\tilde{\mathbf{E}} = \hat{\mathbf{x}}\tilde{E}_x + \hat{\mathbf{y}}\tilde{E}_y.$$

## 7-2 Uniform Plane Waves

Let's solve the differential eqn for each component of the Electric Field. For instance:

$$\frac{d^2 \tilde{E}_x}{dz^2} + k^2 \tilde{E}_x = 0.$$

has the solution:

$$\tilde{E}_x(z) = E_{x0}^+ e^{-jkz} + E_{x0}^- e^{jkz}$$

where  $E_{x0}^+$  and  $E_{x0}^-$  are constants to be determined from boundary conditions.

This is 2 waves: one traveling in +z-direction, the other in the -z-direction.

# 7-2 Uniform Plane Waves

Let's solve for the Magnetic Field:

Simplified case:

$$\tilde{\mathbf{E}}(z) = \hat{\mathbf{x}}\tilde{E}_x^+(z) = \hat{\mathbf{x}}E_{x0}^+e^{-jkz}.$$

Use one of Maxwell's eqns:

$$\nabla \times \tilde{\mathbf{E}} = -j\omega\mu\tilde{\mathbf{H}}$$

with  $\tilde{E}_y = \tilde{E}_z = 0$ :

$$\nabla \times \tilde{\mathbf{E}} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \tilde{E}_x^+(z) & 0 & 0 \end{vmatrix} = -j\omega\mu(\hat{\mathbf{x}}\tilde{H}_x + \hat{\mathbf{y}}\tilde{H}_y + \hat{\mathbf{z}}\tilde{H}_z).$$

## 7-2 Uniform Plane Waves

So we get:  $\tilde{H}_x = 0, \tilde{H}_z = 0$

and:

$$\tilde{H}_y = \frac{1}{-j\omega\mu} \frac{\partial \tilde{E}_x^+(z)}{\partial z},$$

recall:

$$\tilde{E}_x^+(z) = E_{x0}^+ e^{-jkz}$$

so:

$$\frac{\partial \tilde{E}_x^+(z)}{\partial z} = \frac{\partial [E_{x0}^+ e^{-jkz}]}{\partial z}$$
$$\frac{\partial \tilde{E}_x^+(z)}{\partial z} = -jk E_{x0}^+ e^{-jkz}$$

## 7-2 Uniform Plane Waves

With:

$$\tilde{H}_y = \frac{1}{-j\omega\mu} \frac{\partial \tilde{E}_x^+(z)}{\partial z},$$

$$\frac{\partial \tilde{E}_x^+(z)}{\partial z} = -jk E_{x0}^+ e^{-jkz}$$

get:

$$\tilde{H}_y = \frac{1}{-j\omega\mu} -jk E_{x0}^+ e^{-jkz}$$

$$\tilde{H}_y = \frac{k}{\omega\mu} E_{x0}^+ e^{-jkz}$$

$$\tilde{H}_y = H_{y0}^+ e^{-jkz}$$

# 7-2 Uniform Plane Waves

since:

$$H_{y0}^+ = \frac{k}{\omega\mu} E_{x0}^+$$

define the **Intrinsic Impedance**:

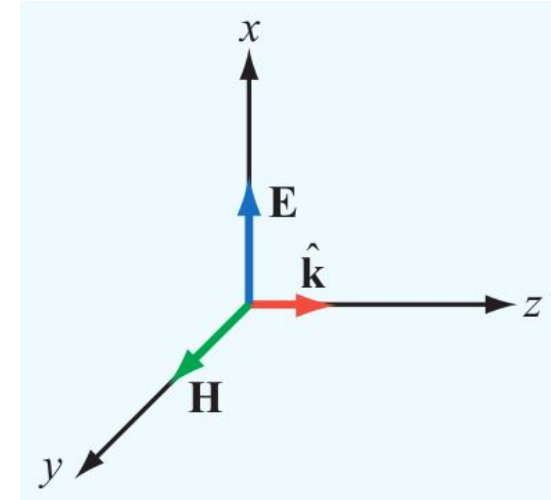
$$\eta = \frac{\omega\mu}{k} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} \quad (\Omega).$$

# 7-2 Uniform Plane Waves

so for this z-propagating wave we have:

$$\tilde{\mathbf{E}}(z) = \hat{\mathbf{x}}\tilde{E}_x^+(z) = \hat{\mathbf{x}}E_{x0}^+e^{-jkz},$$

$$\tilde{\mathbf{H}}(z) = \hat{\mathbf{y}}\frac{\tilde{E}_x^+(z)}{\eta} = \hat{\mathbf{y}}\frac{E_{x0}^+}{\eta}e^{-jkz}.$$



so  $\tilde{\mathbf{E}}$  and  $\tilde{\mathbf{H}}$  are perpendicular to each other, and to the direction of travel

Hence it is called a:

**Transverse Electromagnetic Wave (TEM)**

## 7-2 Uniform Plane Waves

Convert to time-domain:

since it's a complex number: :

$$E_{x0}^+ = |E_{x0}^+| e^{j\phi^+}.$$

we get:

$$\mathbf{E}(z,t) = \Re \left[ \tilde{\mathbf{E}}(z) e^{j\omega t} \right] = \hat{\mathbf{x}} |E_{x0}^+| \cos(\omega t - kz + \phi^+) \quad (\text{V/m}),$$

$$\mathbf{H}(z,t) = \Re \left[ \tilde{\mathbf{H}}(z) e^{j\omega t} \right] = \hat{\mathbf{y}} \frac{|E_{x0}^+|}{\eta} \cos(\omega t - kz + \phi^+) \quad (\text{A/m}).$$

Note: the  $\cos()$  terms are the same,  
so  $\mathbf{E}$  and  $\mathbf{H}$  are **in-phase** with each other.

## 7-2 Uniform Plane Waves

Phase velocity: (as with transmission lines)

$$u_p = \frac{\omega}{k} = \frac{\omega}{\omega\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}} \quad (\text{m/s})$$

Wavelength:

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\omega/u_p} = \frac{2\pi u_p}{2\pi f} = \frac{u_p}{f} \quad (\text{m})$$

## 7-2 Uniform Plane Waves

In a vacuum:

$$u_p = c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \quad (\text{m/s}),$$

$$\eta = \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377 \quad (\Omega) \approx 120\pi \quad (\Omega),$$

$c$  is the velocity of light in a vacuum

$\eta_0$  is the intrinsic impedance of free space

# Example 7-1

**Given:** Plane wave traveling in +z-direction  
frequency: 1MHz  
in air ( $\epsilon = \epsilon_0$ )

$$\mathbf{E} = \hat{\mathbf{x}}E_x$$

reaches a peak value of 3.77 mV/m at  
 $t=0, z=50\text{m}$

**Find:**  $\mathbf{E}(z,t), \mathbf{H}(z,t)$

**Solution:** Wavelength:  $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{1 \times 10^6} = 300 \text{ m},$

Wavenumber:  $k = \frac{2\pi}{300\text{m}} = 0.021 \text{ rad/m}$

# Example 7-1

General expression for  $\mathbf{E}$ :

$$\mathbf{E}(z, t) = \hat{\mathbf{x}} |E_{x0}^+| \cos(\omega t - kz + \phi^+)$$

plug in values:

$$\mathbf{E}(z, t) = \hat{\mathbf{x}} 3.77 \cos((6.28 \times 10^6 \text{ rad/sec})t - (0.021 \text{ rad/m})z + \phi^+) \quad (\text{mV/m})$$

Maximum value for  $\mathbf{E}$  occurs when  $\cos()=1$ :

Plug in  $t=0$  and  $z=50\text{m}$ :

$$(-0.021 \text{ rad/m})(50 \text{ m}) + \phi^+ = 0$$

$$\phi^+ = 1.05 \text{ rad}$$

# Example 7-1

Plug in:

$$\mathbf{E}(z, t) = \hat{\mathbf{x}} 3.77 \cos((6.28 \times 10^6 \text{ rad/sec})t - (0.021 \text{ rad/m})z + 1.05 \text{ rad}) \quad (\text{mV/m})$$

and since:

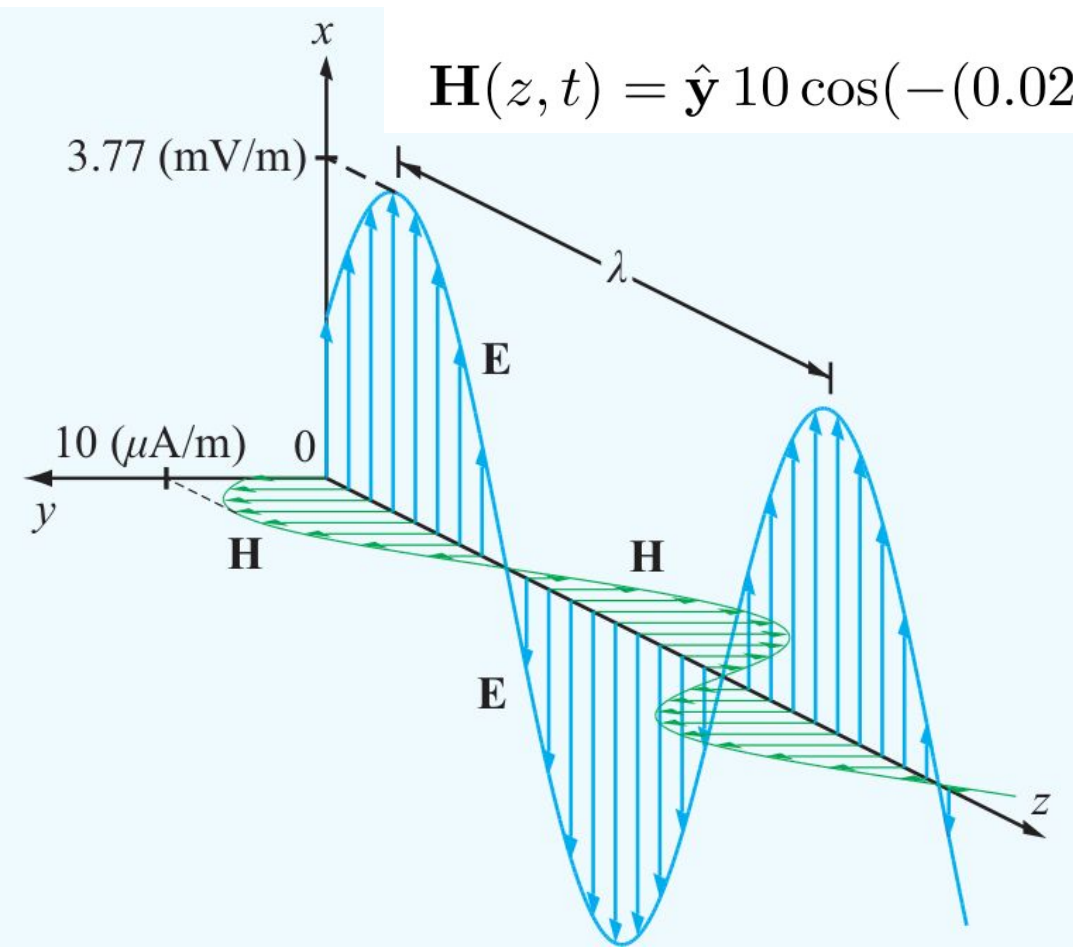
$$\mathbf{H}(z, t) = \hat{\mathbf{y}} \frac{E(z, t)}{\eta_0}$$

$$\mathbf{H}(z, t) = \hat{\mathbf{y}} 10 \cos((6.28 \times 10^6 \text{ rad/sec})t - (0.021 \text{ rad/m})z + 1.05 \text{ rad}) \quad (\mu\text{A/m})$$

# Example 7-1

at  $t=0$ :  $\mathbf{E}(z, t) = \hat{\mathbf{x}} 3.77 \cos(-(0.021 \text{ rad/m})z + 1.05 \text{ rad})$  (mV/m)

$\mathbf{H}(z, t) = \hat{\mathbf{y}} 10 \cos(-(0.021 \text{ rad/m})z + 1.05 \text{ rad})$  ( $\mu\text{A/m}$ )



## 7-2 General Uniform Plane Waves

recall for a z-propagating wave we have:

$$\tilde{\mathbf{E}}(z) = \hat{\mathbf{x}}\tilde{E}_x^+(z) = \hat{\mathbf{x}}E_{x0}^+e^{-jkz},$$

$$\tilde{\mathbf{H}}(z) = \hat{\mathbf{y}}\frac{\tilde{E}_x^+(z)}{\eta} = \hat{\mathbf{y}}\frac{E_{x0}^+}{\eta}e^{-jkz}.$$

Direction of propagation:  $\hat{\mathbf{k}} = \hat{\mathbf{z}}$

Notice that:  $\hat{\mathbf{z}} \times \hat{\mathbf{x}} = \hat{\mathbf{y}}$

So:

$$\frac{1}{\eta}\hat{\mathbf{k}} \times \tilde{\mathbf{E}} = \tilde{\mathbf{H}}$$

## 7-2 General Uniform Plane Waves

Similarly:  $\hat{\mathbf{z}} \times \hat{\mathbf{y}} = -\hat{\mathbf{x}}$

so:  $-\eta \hat{\mathbf{k}} \times \tilde{\mathbf{H}} = \tilde{\mathbf{E}}$

Turns out, this is true for **all** plane waves:

$$\tilde{\mathbf{H}} = \frac{1}{\eta} \hat{\mathbf{k}} \times \tilde{\mathbf{E}},$$
$$\tilde{\mathbf{E}} = -\eta \hat{\mathbf{k}} \times \tilde{\mathbf{H}}.$$

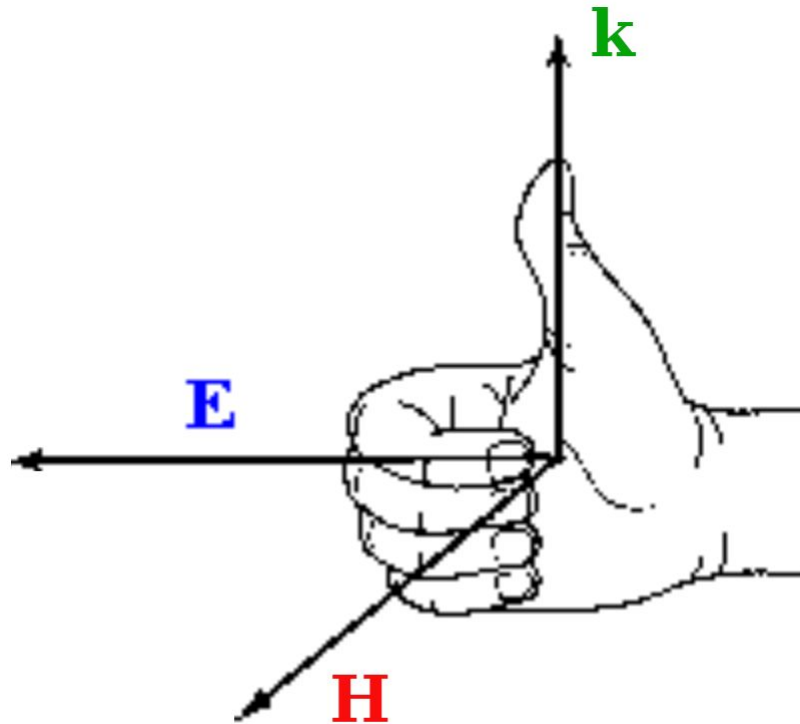
Direction of propagation:  $\hat{\mathbf{k}}$

# 7-2 General Uniform Plane Waves

Direction of propagation:  $\hat{\mathbf{k}}$

Can get this using the right-hand rule:

Curl fingers from  $\mathbf{E}$  to  $\mathbf{H}$ , thumb points:  $\hat{\mathbf{k}}$



## 7-2 General Uniform Plane Waves

**Case 1:**  $\hat{\mathbf{k}} = \hat{\mathbf{z}}$  and  $\tilde{\mathbf{E}} = \hat{\mathbf{x}} \tilde{E}_x^+(z)$

$$\tilde{\mathbf{H}} = \frac{1}{\eta} \hat{\mathbf{k}} \times \tilde{\mathbf{E}} = \frac{1}{\eta} (\hat{\mathbf{z}} \times \hat{\mathbf{x}}) \tilde{E}_x^+(z) = \hat{\mathbf{y}} \frac{\tilde{E}_x^+(z)}{\eta}$$

**Case 2:**  $\hat{\mathbf{k}} = -\hat{\mathbf{z}}$  and  $\tilde{\mathbf{E}} = \hat{\mathbf{x}} \tilde{E}_x^-(z)$

$$\tilde{\mathbf{H}} = \frac{1}{\eta} (-\hat{\mathbf{z}} \times \hat{\mathbf{x}}) \tilde{E}_x^-(z) = -\hat{\mathbf{y}} \frac{\tilde{E}_x^-(z)}{\eta} :$$

## 7-2 General Uniform Plane Waves

**Case 3:**  $\hat{\mathbf{k}} = \hat{\mathbf{z}}$  and  $\tilde{\mathbf{E}} = \hat{\mathbf{x}}\tilde{E}_x^+(z) + \hat{\mathbf{y}}\tilde{E}_y^+(z)$

$$\tilde{\mathbf{H}} = \hat{\mathbf{x}}\tilde{H}_x^+(z) + \hat{\mathbf{y}}\tilde{H}_y^+(z)$$

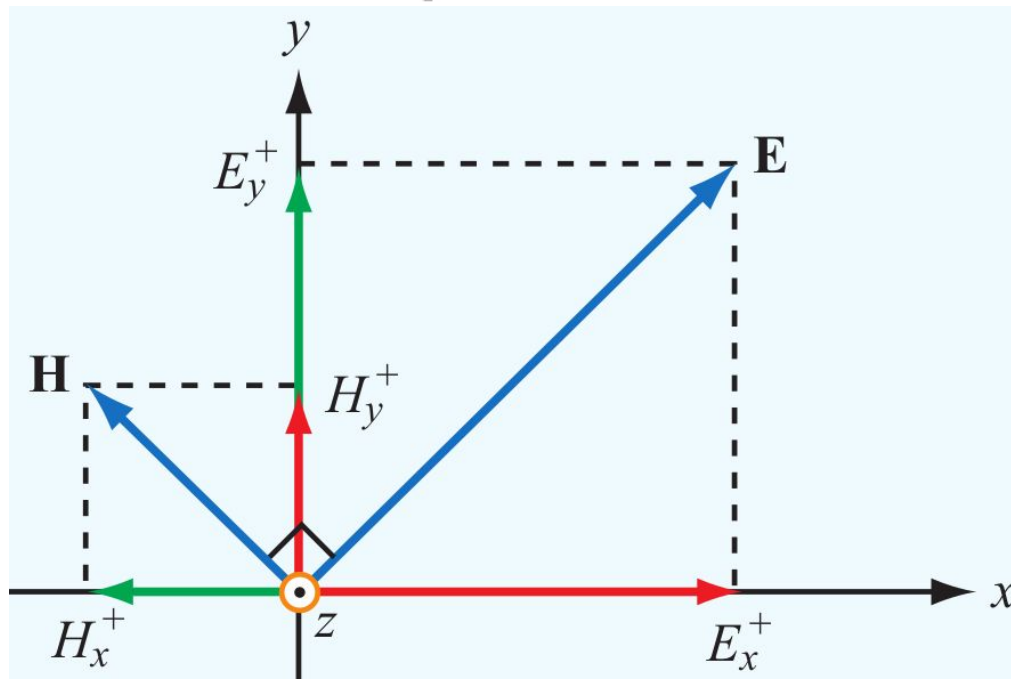
$$\tilde{\mathbf{H}} = \frac{1}{\eta} \hat{\mathbf{z}} \times \tilde{\mathbf{E}} = -\hat{\mathbf{x}} \frac{\tilde{E}_y^+(z)}{\eta} + \hat{\mathbf{y}} \frac{\tilde{E}_x^+(z)}{\eta}$$

$$\tilde{H}_x^+(z) = -\frac{\tilde{E}_y^+(z)}{\eta}, \quad \tilde{H}_y^+(z) = \frac{\tilde{E}_x^+(z)}{\eta}$$

# 7-2 General Uniform Plane Waves

**Case 3:**  $\hat{\mathbf{k}} = \hat{\mathbf{z}}$  and  $\tilde{\mathbf{E}} = \hat{\mathbf{x}}\tilde{E}_x^+(z) + \hat{\mathbf{y}}\tilde{E}_y^+(z)$

$$\tilde{H}_x^+(z) = -\frac{\tilde{E}_y^+(z)}{\eta}, \quad \tilde{H}_y^+(z) = \frac{\tilde{E}_x^+(z)}{\eta}$$



# 7-2 General Uniform Plane Waves

**Module 7.1 Linking E to H** Select the directions and magnitudes of **E** and **H** and observe the resultant wave vector.

Module 7.1 **Linking E to H**

**Frequency = 10.0 [GHz]**

**$\epsilon_r = 1.0$**

**$\theta = 45.0^\circ$**        **$\phi = 0.0^\circ$**

**$|E| = 10.0$  [V/m]**       **$\psi = 0.0^\circ$**

Select E       Select H

---

**$\eta = 376.991$  [ $\Omega$ ]**

**$|k| = 209.43951$  [ $\text{rad m}^{-1}$ ]**

**$k_x = 148.096098$  [ $\text{rad m}^{-1}$ ]**

**$k_y = 0.0$  [ $\text{rad m}^{-1}$ ]**

**$k_z = 148.096098$  [ $\text{rad m}^{-1}$ ]**

**$|E| = 10.0$  [V/m]**

**$E_x = 0.0$  [V/m]**

**$E_y = 10.0$  [V/m]**

**$E_z = 0.0$  [V/m]**


**$|H| = 0.026526$  [A/m]**


**$H_x = -0.018757$  [A/m]**


**$H_y = 0.0$  [A/m]**

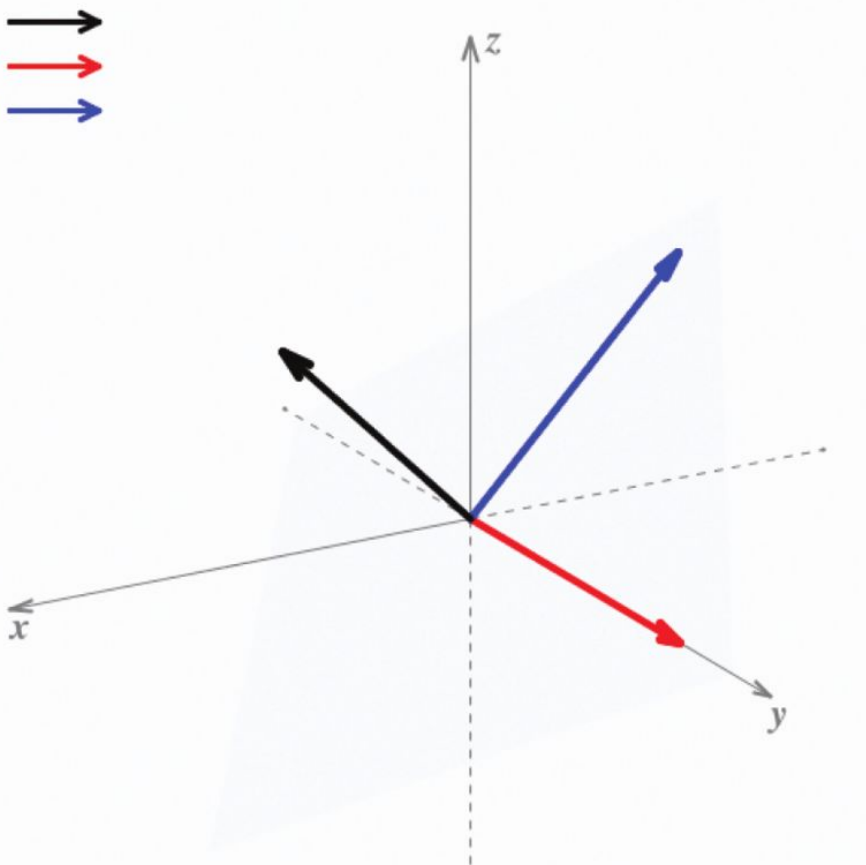
**$H_z = 0.018757$  [A/m]**

[Instructions](#)

**$k$**  

**$E$**  

**$H$**  



# Exercise 7-1

**Given:** Uniform plane wave

$$f = 10\text{MHz}$$

$$\sigma=0, \mu=\mu_0, \epsilon_r=9$$

**Find:** phase velocity,  $u_p$

wavenumber,  $k$

wavelength,  $\lambda$

intrinsic impedance,  $\eta$

**Solution:**

$$u_p = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\epsilon_0}\sqrt{\epsilon_r}} = \frac{c}{\sqrt{\epsilon_r}}$$

$$u_p = \frac{3 \times 10^8}{\sqrt{9}} = 1 \times 10^8 \text{ (m/s)}$$

# Exercise 7-1

$$k = \frac{\omega}{u_p} = \frac{2\pi \times 10^7}{10^8} = 0.628 \text{ (rad/m)}$$

$$k = \frac{2\pi}{\lambda}, \quad \lambda = \frac{2\pi}{k} = \frac{2\pi}{0.2\pi} = 10 \text{ (m)}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \cdot \frac{1}{\sqrt{\epsilon_r}} = \frac{377}{3} = 125.67 \text{ } \Omega.$$

# Exercise 7-2

**Given:** Uniform plane wave

Lossless medium

$$\eta = 188.5 \, \Omega, \quad \mu = \mu_0$$

$$\tilde{\mathbf{E}} = \hat{\mathbf{z}} 10 e^{-j4\pi y} \text{ (mV/m)}.$$

**Find:**  $\tilde{\mathbf{H}}$ ,  $\mathbf{E}(y, t)$

**Solution:** because  $E$  is a function of  $-y$ ,  $\mathbf{k} = +\hat{\mathbf{y}}$ .

$$\tilde{\mathbf{H}} = \frac{1}{\eta} \hat{\mathbf{k}} \times \tilde{\mathbf{E}} = \frac{1}{\eta} (\hat{\mathbf{y}} \times \hat{\mathbf{z}}) \tilde{E}_z^+(y) = \hat{\mathbf{x}} \frac{\tilde{E}_z^+(y)}{\eta}$$

## Exercise 7-2

$$\tilde{\mathbf{H}} := \hat{\mathbf{x}} \frac{\tilde{E}_z^+(y)}{\eta}$$

$$\tilde{\mathbf{H}} = \hat{\mathbf{x}} \frac{10 \times 10^{-3}}{188.5} e^{-j4\pi y} = \hat{\mathbf{x}} 53 e^{-j4\pi y} \quad (\mu\text{A/m}).$$

Since we expect the functional form to be:

$$e^{-jk y}$$

$$k=4\pi = 12.57 \text{ rad/m}$$

## Exercise 7-2

Since:  $\eta = \eta_0 / \sqrt{\epsilon_r}$ ,

$$\epsilon_r = \left( \frac{\eta_0}{\eta} \right)^2 = \left( \frac{377}{188.5} \right)^2 = 4.$$

$$u_p = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{2} = 1.5 \times 10^8 \quad (\text{m/s}).$$

$$\omega = ku_p = 4\pi \times 1.5 \times 10^8 = 18.85 \times 10^8 \quad (\text{rad/sec})$$

## Exercise 7-2

$$\begin{aligned}\mathbf{E}(z, t) &= \Re[\tilde{\mathbf{E}}(z) e^{j\omega t}] \\ &= \hat{\mathbf{z}} 10 \cos(\omega t - ky)\end{aligned}$$

$$= \hat{\mathbf{z}} 10 \cos((18.85 \times 10^8 \text{ rad/sec})t - (12.57 \text{ rad/m})y) \quad (\text{mV/m})$$

# Plane Wave Module

Module 7.2

Plane Wave

$t = 0.653T$

$\omega t = 235^\circ$



Input/Output

Phase Planes

Instructions

|Phasors|

Input

Frequency  $f = 1.0E9$  Hz

Conductivity  $\sigma = 0.0$  S/m

Relative Permittivity  $\epsilon_r = 1.0$

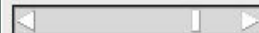
Relative Permeability  $\mu_r = 1.0$

E-field Amplitude (z=0)  $E_0 = 1.0$  V/m

E-field Phase (z=0)  $\varphi = 0.0$  rad

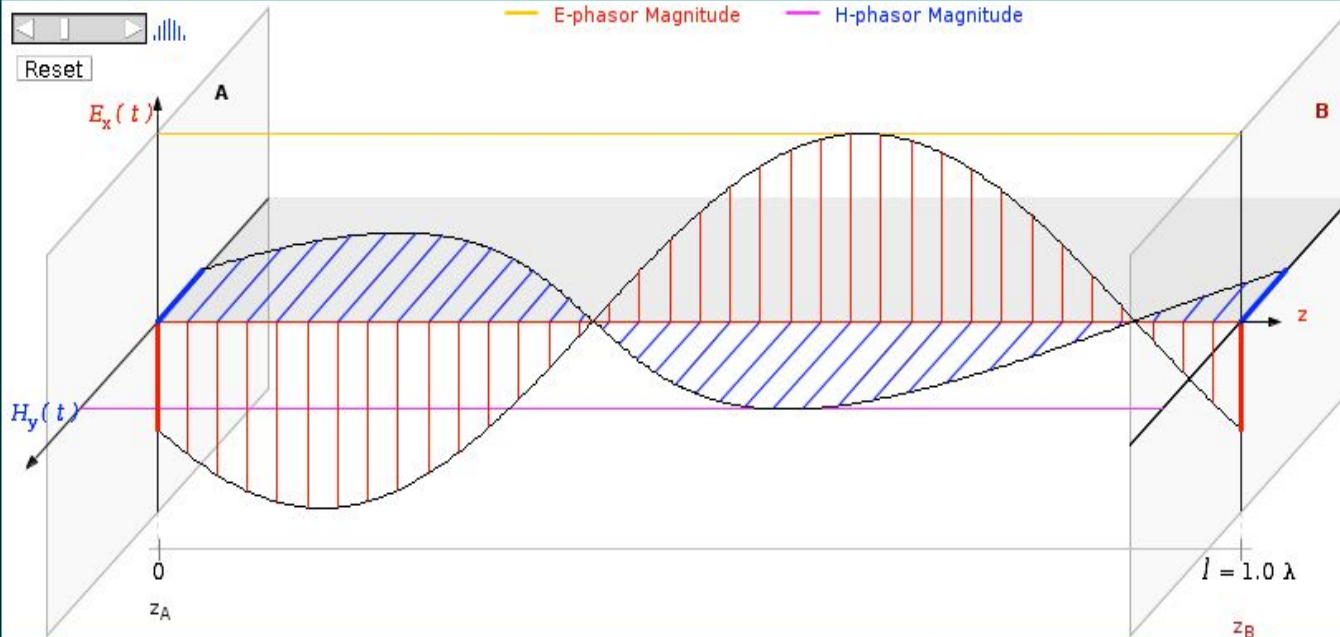
Length Displayed  $l = 1.0$   $\lambda$

[A] & [B] Windows Area = 1.0 m<sup>2</sup>



Animation speed

Update



Output

Wave Properties

WaveLength  $\lambda = 30.0$  [ cm ]

Phase Velocity  $v_p = 3.0 \times 10^8$  [ m / s ]

Period  $T = 1.0 \times 10^{-9}$  [ s ]

Impedance of the Medium [  $\Omega$  ]

$\eta = 376.991118 + j 0.0$

$= 376.991118 \angle 0.0$  rad

$= 376.991118 \angle 0.0^\circ$

Penetration (Skin) Depth

$\delta_s = \infty$

Phase and Attenuation Constants

$\beta = 20.94395$  [ m<sup>-1</sup> ]

$\alpha = 0.0$  [ Ne / m ]

$\sigma / \omega \epsilon = 0.0$

The material is vacuum (perfect dielectric)



A)  $z_A = 0.0 \lambda = 0.0$  [ m ]  
 $|E_A| = 1.0$  [ V / m ]  
 $\angle E_A = 0.0$  [ rad ]  
 $|H_A| = 2.65258 \times 10^{-3}$  [ A / m ]  
 $\angle H_A = 0.0$  [ rad ]

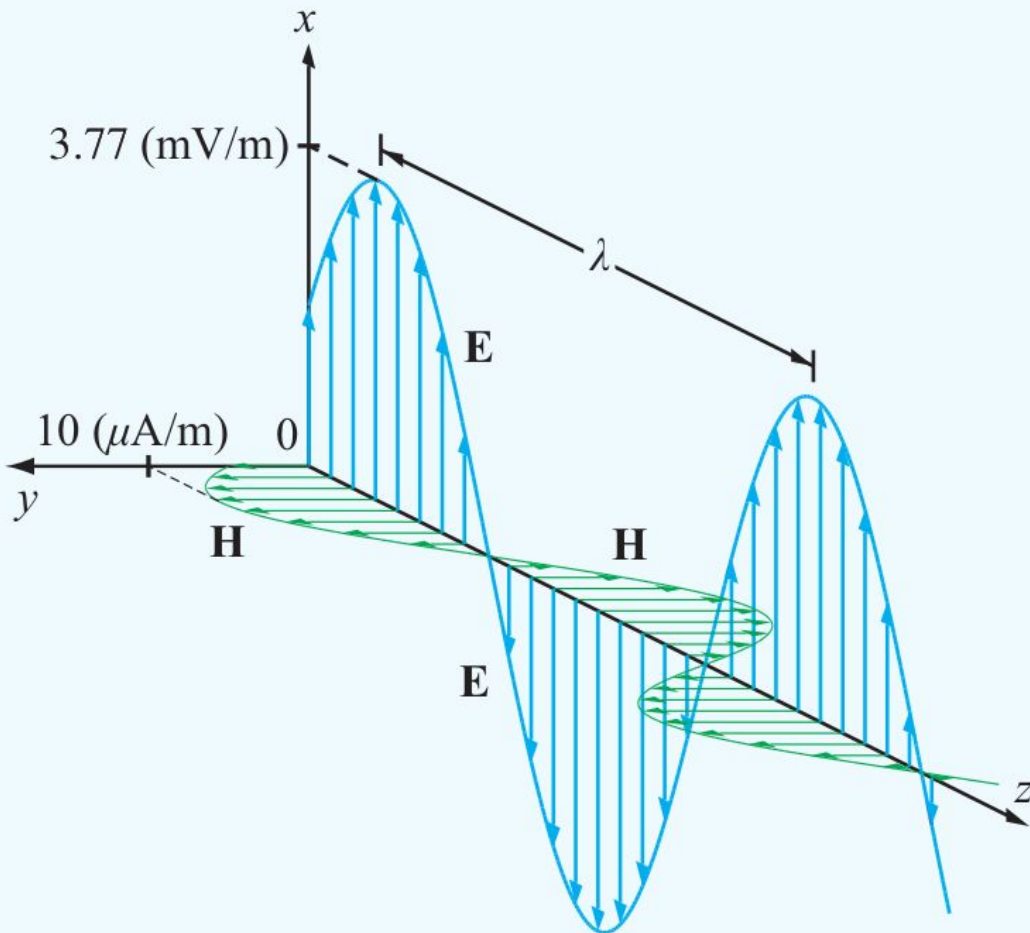
$f = 1.0$  GHz  
 $l = 1.0 \lambda = 30.0$  [ cm ]

Phasor fields on selected phase planes

$E_x(t)$    $H_y(t)$

B)  $z_B = 1.0 \lambda = 30.0$  [ cm ]  
 $|E_B| = 1.0$  [ V / m ]  
 $\angle E_B = -6.28319$  [ rad ]  
 $|H_B| = 2.65258 \times 10^{-3}$  [ A / m ]  
 $\angle H_B = -6.28319$  [ rad ]

# 7-3 Polarization

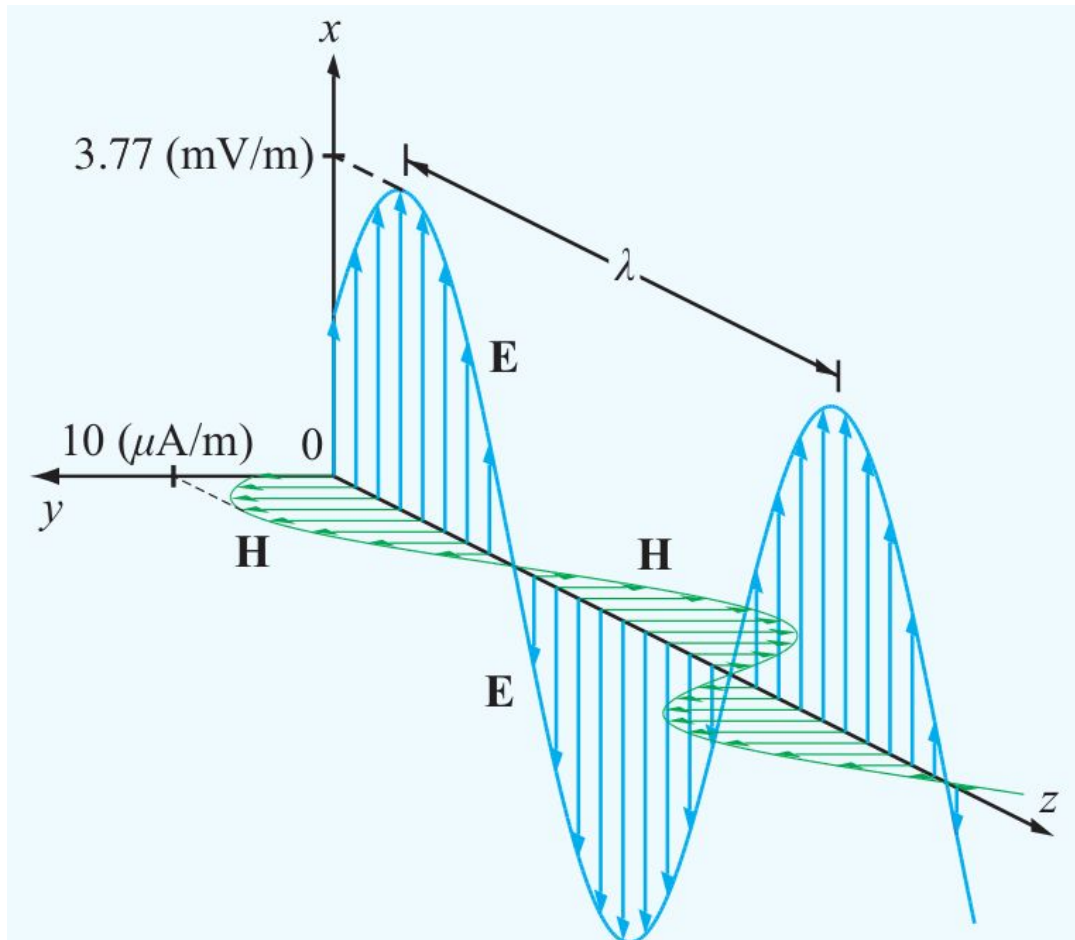


Look at the  $\mathbf{E}$  field of a plane wave at  $z=0$ , as a function of time.

This wave has  $E$  varying between  $+E_0$  and  $-E_0$ ,

Looking at it along  $z$ : it just appears as a point tracing out a vertical line: up-down-up-down... forever.

# 7-3 Polarization



This wave has  $E$  varying between  $+E_0$  and  $-E_0$

Looking at it along  $z$ : it just appears as a point tracing out a vertical line: up-down-up-down... forever.

Other shapes possible

## 7-3 Polarization

Recall for a z-directed plane wave:

$$\tilde{\mathbf{E}}(z) = \hat{\mathbf{x}}\tilde{E}_x(z) + \hat{\mathbf{y}}\tilde{E}_y(z)$$

where:

$$\tilde{E}_x(z) = E_{x0}e^{-jkz},$$

$$\tilde{E}_y(z) = E_{y0}e^{-jkz},$$

$E_{x0}$  and  $E_{y0}$  are complex,  
each with an amplitude and phase.

Polarization depends on both  $E_{x0}$  and  $E_{y0}$ .

# 7-3 Polarization

Polarization depends on the amplitude and phase of both the  $x$ - and  $y$ -components.

But it depends only on the **relative** phase, so define:

$$E_{x0} = a_x,$$

$$E_{y0} = a_y e^{j\delta},$$

where  $a_x$  and  $a_y$  are positive real numbers  
 $\delta$  is the phase difference between the  
 $x$ - and  $y$ -components.

## 7-3 Polarization

The total  $\mathbf{E}$  field phasor is:

$$\tilde{\mathbf{E}}(z) = (\hat{\mathbf{x}}a_x + \hat{\mathbf{y}}a_y e^{j\delta})e^{-jkz},$$

with the corresponding time-domain field:

$$\begin{aligned}\mathbf{E}(z,t) &= \Re \left[ \tilde{\mathbf{E}}(z) e^{j\omega t} \right] \\ &= \hat{\mathbf{x}}a_x \cos(\omega t - kz) + \hat{\mathbf{y}}a_y \cos(\omega t - kz + \delta)\end{aligned}$$

For given  $z,t$  this is a vector in the  $x$ - $y$  plane.

Let's look at the magnitude and direction of this vector.

# 7-3 Polarization

Magnitude of the **E** field:

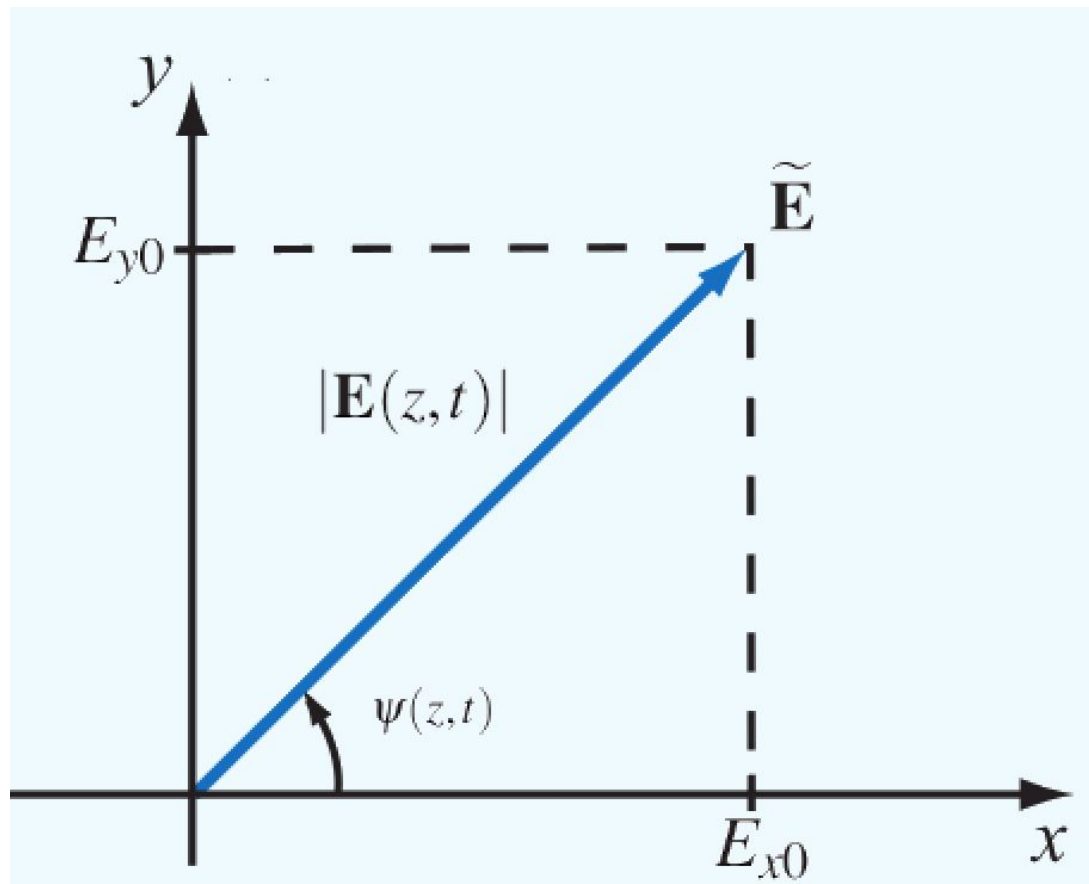
$$\begin{aligned} |\mathbf{E}(z,t)| &= [E_x^2(z,t) + E_y^2(z,t)]^{1/2} \\ &= [a_x^2 \cos^2(\omega t - kz) + a_y^2 \cos^2(\omega t - kz + \delta)]^{1/2} \end{aligned}$$

The direction of the **E** field is defined by the **inclination angle,  $\psi$** :

$$\psi(z,t) = \tan^{-1} \left[ \frac{E_y(z,t)}{E_x(z,t)} \right]$$

# 7-3 Polarization

The Electric Field vector in  $x$ - $y$  space at one moment in time:



shown are  
the  
component  
magnitudes  
and  
inclination  
angle,  $\psi$

# 7-3 Polarization

## Linear Polarization

As time varies, the tip of the arrow traces a straight line.

Happens if:  $\delta=0$  (in-phase) or  $\delta=\pi$  (out-of-phase)

$$\mathbf{E}(z, t) = (\hat{\mathbf{x}}a_x + \hat{\mathbf{y}}a_y) \cos(\omega t - kz), \quad \text{(in phase)}$$

$$\mathbf{E}(z, t) = (\hat{\mathbf{x}}a_x - \hat{\mathbf{y}}a_y) \cos(\omega t - kz). \quad \text{(out of phase)}$$

## 7-3 Polarization

**Linear Polarization:** out-of-phase case:

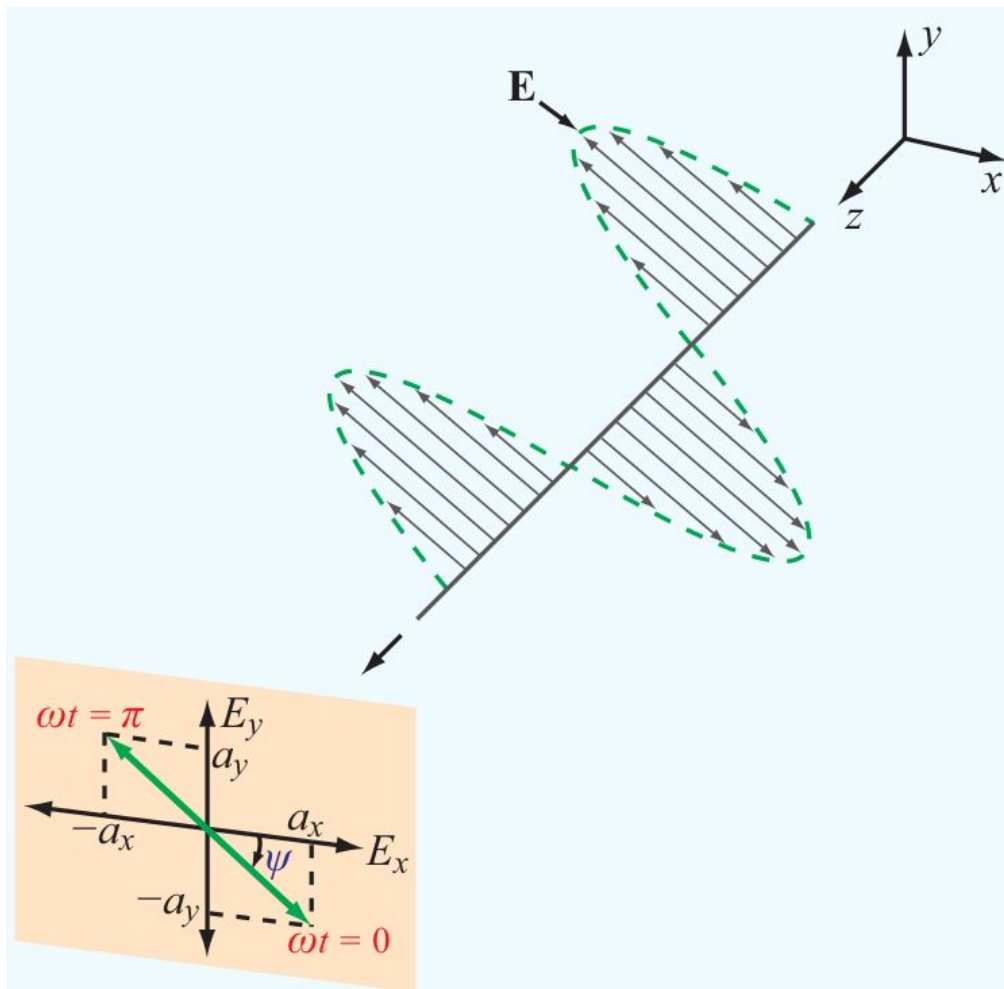
$$\mathbf{E}(z, t) = (\hat{\mathbf{x}}a_x - \hat{\mathbf{y}}a_y) \cos(\omega t - kz).$$

$$|\mathbf{E}(z, t)| = [a_x^2 + a_y^2]^{1/2} |\cos(\omega t - kz)|.$$

$$\psi = \tan^{-1} \left( \frac{-a_y}{a_x} \right)$$

# 7-3 Polarization

**Linear Polarization:** out-of-phase case:



The green line shows the trace of the tip of the  $\mathbf{E}$  field vector as a function of  $z$ .

Depending on the relative magnitudes of the  $x$ - and  $y$ -components this line could be at any angle,  $\psi$

# 7-3 Polarization

## Circular Polarization

As time varies, the tip of the arrow traces a circle.

Happens if:

x- and y-components have equal magnitudes,  
and:  $\delta = \pm \pi/2$

**case 1:**

with:  $a_x = a_y = a$  and  $\delta = \pi/2$ ,

get:

$$\tilde{\mathbf{E}}(z) = (\hat{\mathbf{x}}a + \hat{\mathbf{y}}ae^{j\pi/2})e^{-jkz} = a(\hat{\mathbf{x}} + j\hat{\mathbf{y}})e^{-jkz},$$

# 7-3 Polarization

## Circular Polarization

with:  $a_x = a_y = a$  and  $\delta = \pi/2$ ,

get:

$$\tilde{\mathbf{E}}(z) = (\hat{\mathbf{x}}a + \hat{\mathbf{y}}ae^{j\pi/2})e^{-jkz} = a(\hat{\mathbf{x}} + j\hat{\mathbf{y}})e^{-jkz},$$

$$\begin{aligned}\mathbf{E}(z,t) &= \Re \left[ \tilde{\mathbf{E}}(z) e^{j\omega t} \right] \\ &= \hat{\mathbf{x}}a \cos(\omega t - kz) + \hat{\mathbf{y}}a \cos(\omega t - kz + \pi/2) \\ &= \hat{\mathbf{x}}a \cos(\omega t - kz) - \hat{\mathbf{y}}a \sin(\omega t - kz).\end{aligned}$$

$$\begin{aligned}|\mathbf{E}(z,t)| &= [E_x^2(z,t) + E_y^2(z,t)]^{1/2} \\ &= [a^2 \cos^2(\omega t - kz) + a^2 \sin^2(\omega t - kz)]^{1/2} = a\end{aligned}$$

# 7-3 Polarization

## Circular Polarization

with:  $a_x = a_y = a$  and  $\delta = \pi/2$ ,

get:  $|\mathbf{E}(z,t)| = a$

$$\begin{aligned}\psi(z,t) &= \tan^{-1} \left[ \frac{E_y(z,t)}{E_x(z,t)} \right] \\ &= \tan^{-1} \left[ \frac{-a \sin(\omega t - kz)}{a \cos(\omega t - kz)} \right] = -(\omega t - kz).\end{aligned}$$

at  $z=0$ :  $\psi = -\omega t$

the inclination angle decreases with time.

# 7-3 Polarization

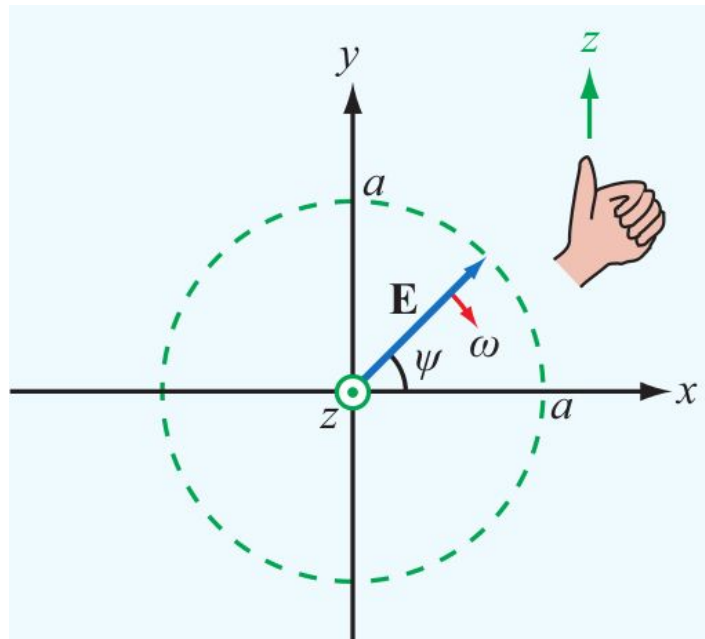
## Circular Polarization: Left-hand Circular (LHC)

with:  $a_x = a_y = a$  and  $\delta = \pi/2$ ,

get:  $|\mathbf{E}(z,t)| = a$

at  $z=0$ :  $\psi = -\omega t$

the inclination angle **decreases** with time:



With the **left hand**,  
point the thumb in  
the z-direction:  
the fingers give the  
direction of rotation

# 7-3 Polarization

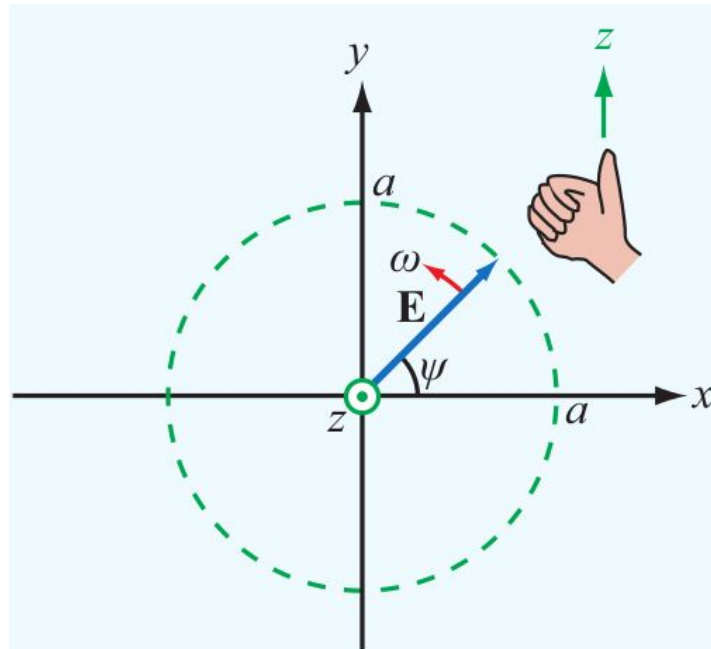
## Circular Polarization: Right-Hand Circular (RHC)

with:  $a_x = a_y = a$  and  $\delta = -\pi/2$

get:  $|\mathbf{E}(z,t)| = a$

at  $z=0$ :  $\psi = +\omega t$

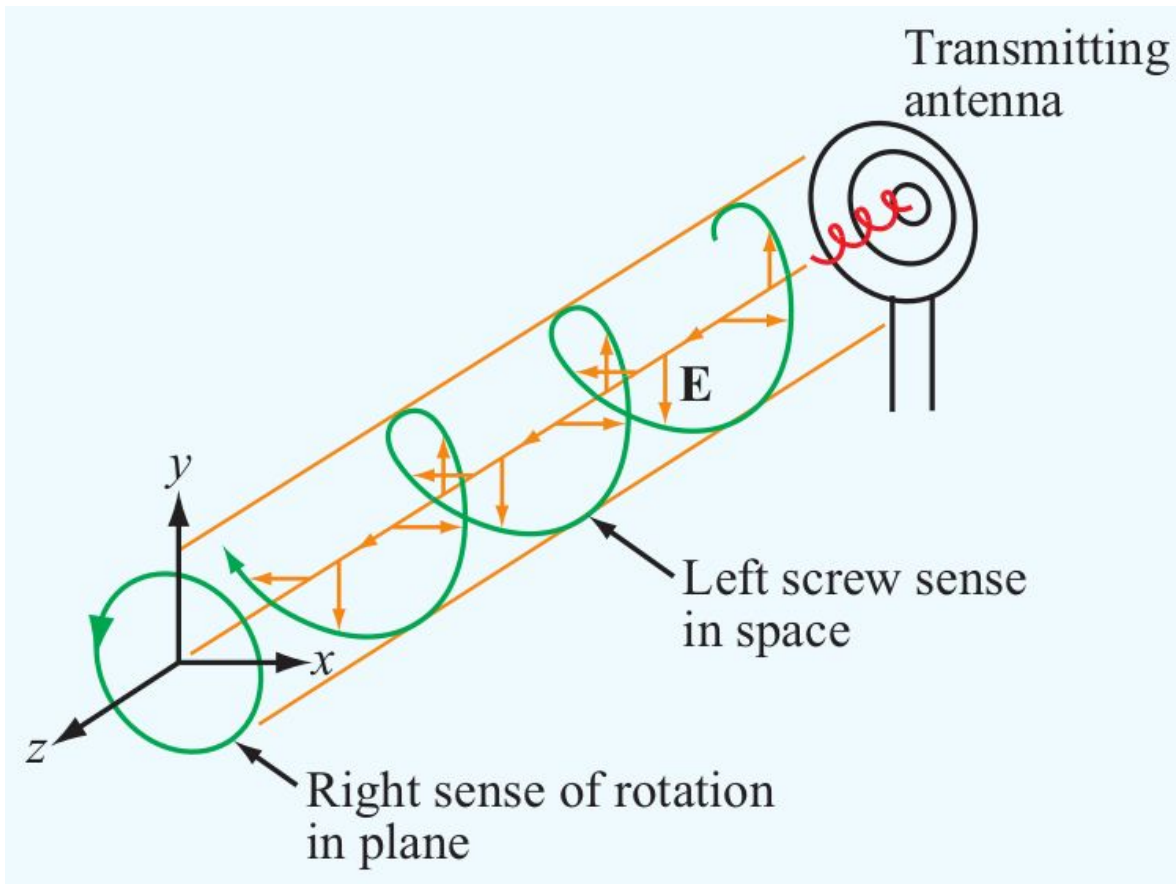
the inclination angle **increases** with time:



With the **right hand**,  
point the thumb in  
the z-direction:  
the fingers give the  
direction of rotation

# 7-3 Polarization

## Circular Polarization: Right-Hand Circular (RHC)



Note that the polarization is defined at a **constant position**.

When looking at the wave at a **constant time**, it can appear to have a different direction of rotation.

# Example 7-2

**Given:** RHC-polarized plane wave  
traveling in  $+y$  direction

$$|\mathbf{E}| = 3 \text{ mV/m}$$

$$\varepsilon = 4\varepsilon_0, \mu = \mu_0, \sigma = 0$$

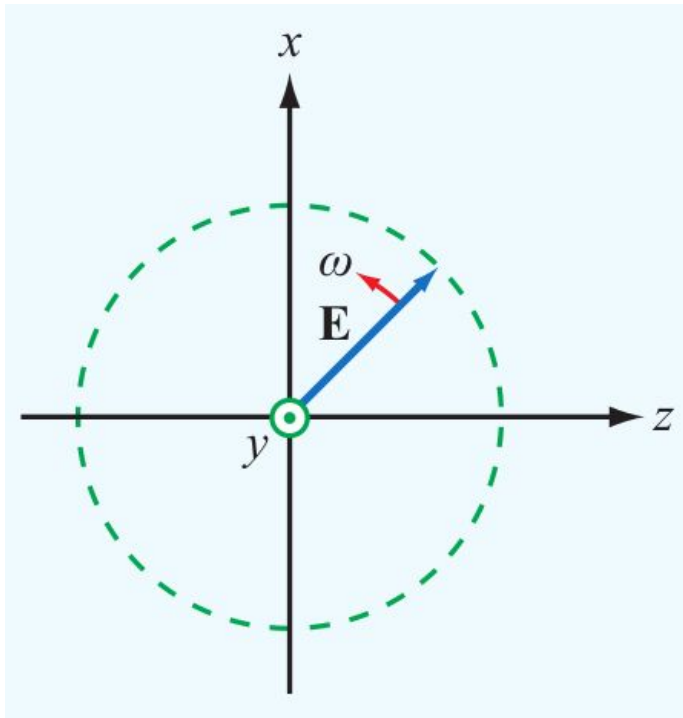
$$f = 100 \text{ MHz}$$

**Find:**  $\mathbf{E}(y,t)$ ,  $\mathbf{H}(y,t)$

**Solution:** since traveling in  $+y$  direction:  
components must be in  $x, z$

# Example 7-2

**Solution:** since traveling in +y direction:  
components must be in x, z  
since RHC:



So assign:

$$E_z = a$$
$$E_x = ae^{-j\pi/2}$$

where  $a = 3 \text{ mV/m}$

# Example 7-2

**Solution:**

$$\begin{aligned}\tilde{\mathbf{E}}(y) &= \hat{\mathbf{x}}\tilde{E}_x + \hat{\mathbf{z}}\tilde{E}_z = \hat{\mathbf{x}}ae^{-j\pi/2}e^{-jky} + \hat{\mathbf{z}}ae^{-jky} \\ &= (-\hat{\mathbf{x}}j + \hat{\mathbf{z}})3e^{-jky} \quad (\text{mV/m}),\end{aligned}$$

$$\begin{aligned}\tilde{\mathbf{H}}(y) &= \frac{1}{\eta} \hat{\mathbf{y}} \times \tilde{\mathbf{E}}(y) = \frac{1}{\eta} \hat{\mathbf{y}} \times (-\hat{\mathbf{x}}j + \hat{\mathbf{z}})3e^{-jky} \\ &= \frac{3}{\eta} (\hat{\mathbf{z}}j + \hat{\mathbf{x}})e^{-jky} \quad (\text{mA/m}).\end{aligned}$$

# Example 7-2

**Solution:** need  $\omega$ ,  $k$  and  $\eta$

$$\omega = 2\pi f = 6.28 \times 10^8 \text{ (rad/s)}$$

$$k = \frac{\omega\sqrt{\epsilon_r}}{c} = \frac{2\pi \times 10^8 \sqrt{4}}{3 \times 10^8} = 4.19 \quad \text{(rad/m)}$$

$$\eta = \frac{\eta_0}{\sqrt{\epsilon_r}} \approx \frac{120\pi}{\sqrt{4}} = 188.5 \quad (\Omega)$$

# Example 7-2

**Solution:** in the time-domain:

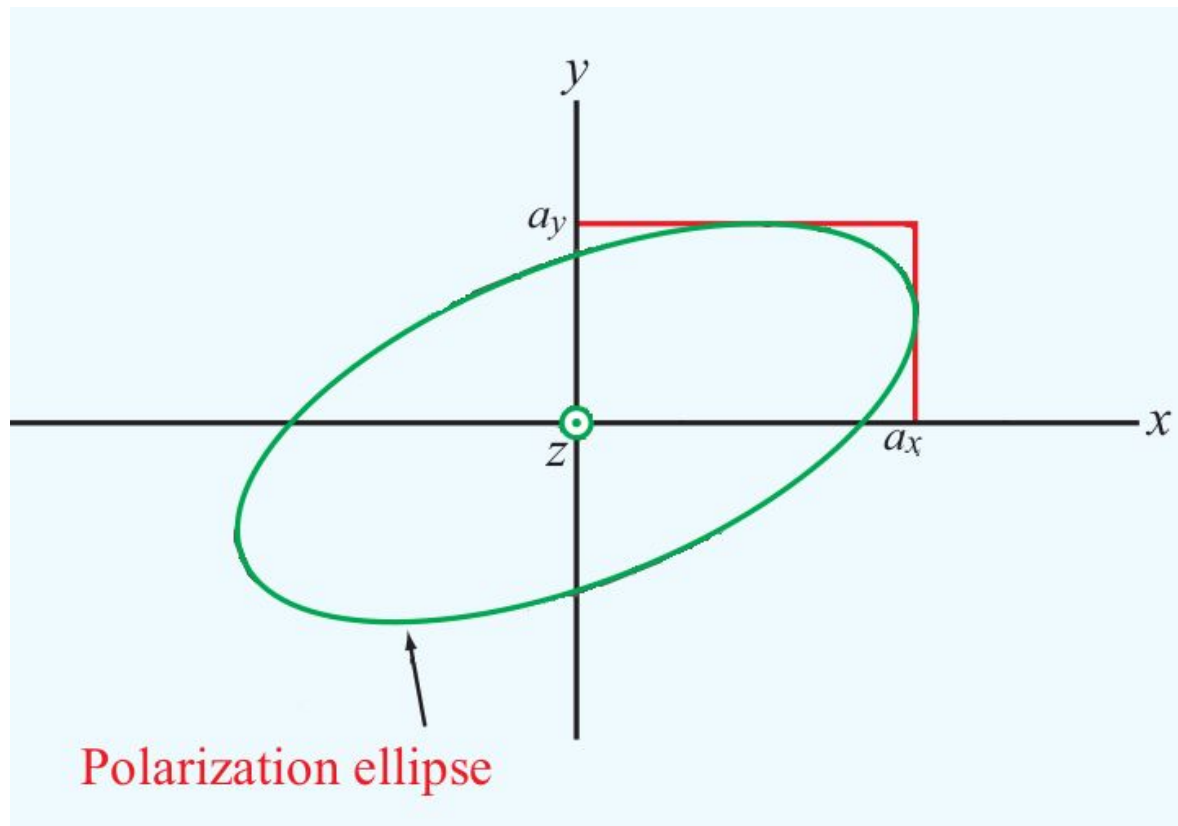
$$\begin{aligned}\mathbf{E}(y,t) &= \Re \left[ \tilde{\mathbf{E}}(y) e^{j\omega t} \right] \\ &= \Re \left[ (-\hat{\mathbf{x}}j + \hat{\mathbf{z}})3e^{-jky} e^{j\omega t} \right] \\ &= 3[\hat{\mathbf{x}} \sin(\omega t - ky) + \hat{\mathbf{z}} \cos(\omega t - ky)] \quad (\text{mV/m})\end{aligned}$$

$$\begin{aligned}\mathbf{H}(y,t) &= \Re \left[ \tilde{\mathbf{H}}(y) e^{j\omega t} \right] \\ &= \Re \left[ \frac{3}{\eta} (\hat{\mathbf{z}}j + \hat{\mathbf{x}})e^{-jky} e^{j\omega t} \right] \\ &= 0.016 [\hat{\mathbf{x}} \cos(\omega t - ky) - \hat{\mathbf{z}} \sin(\omega t - ky)] \quad (\text{mA/m})\end{aligned}$$

# 7-3 Polarization

## Elliptical Polarization

most general: **E** field traces a tilted ellipse:

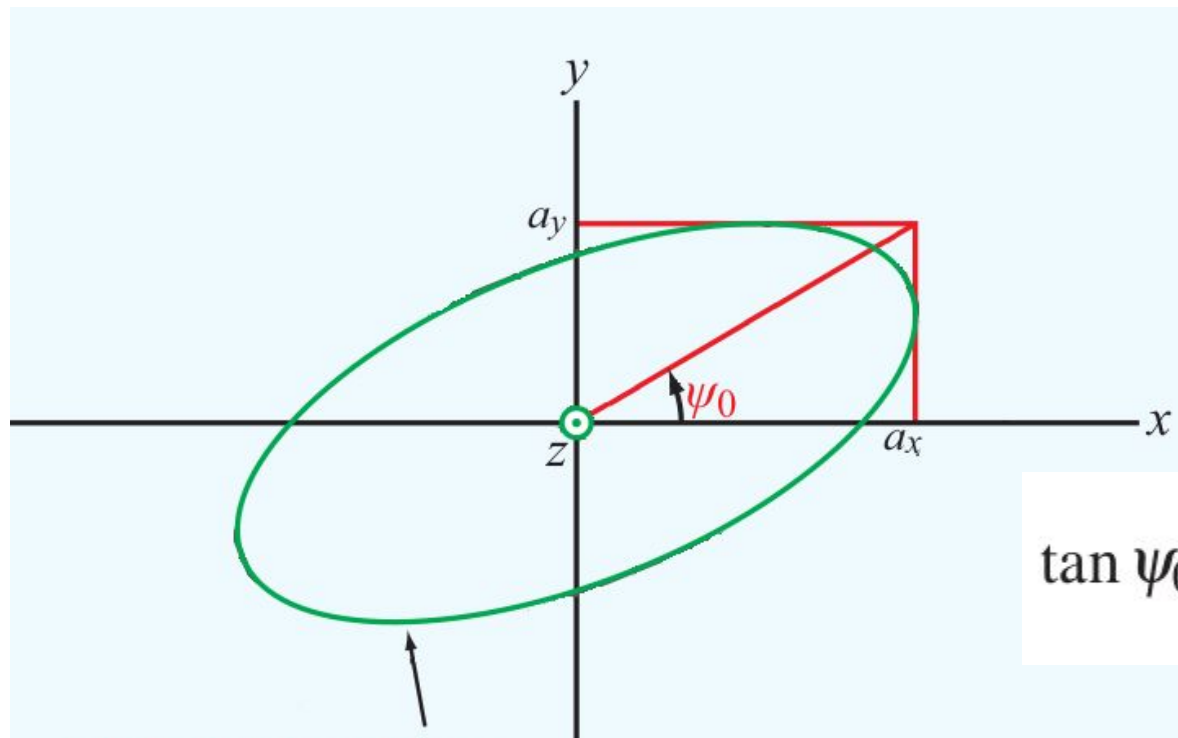


$a_x$  and  $a_y$  are the components along the x- and y-axes

# 7-3 Polarization

## Elliptical Polarization

most general: **E** field traces a tilted ellipse:



Polarization ellipse

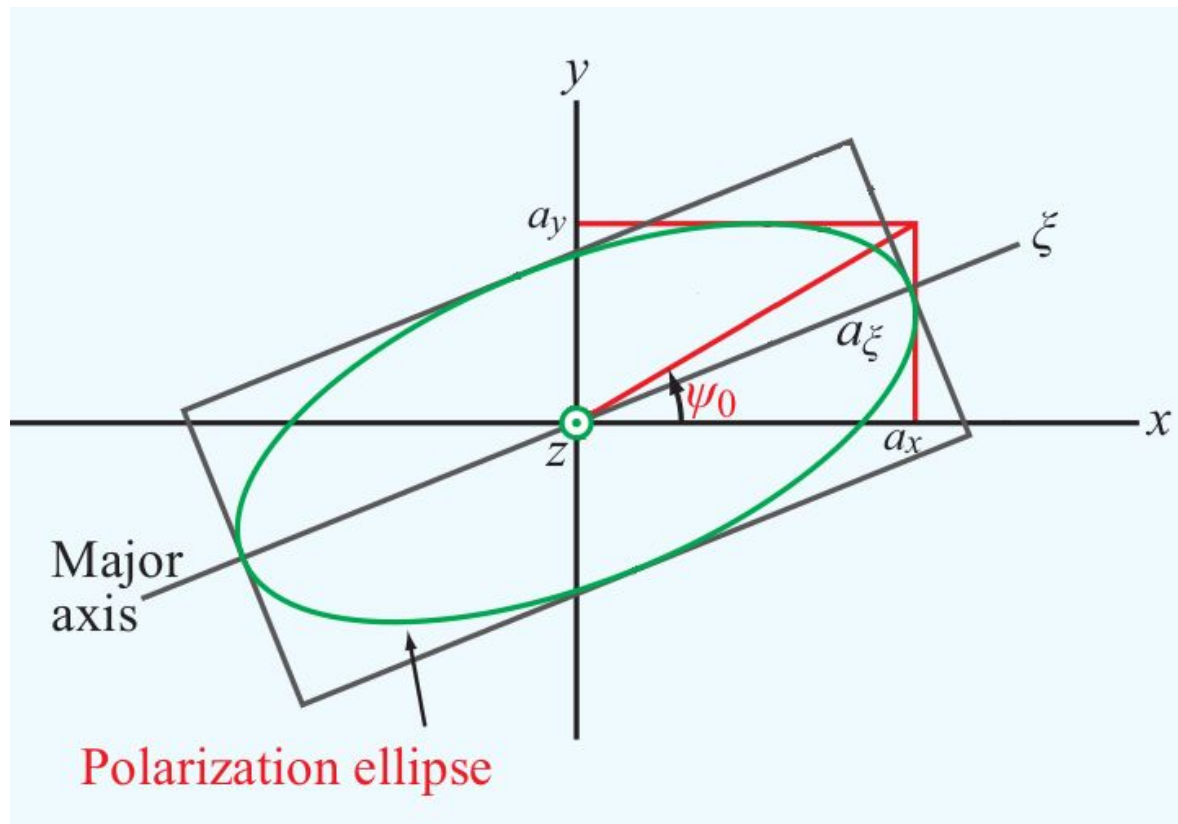
no longer the inclination angle, now called the **auxiliary angle**:

$$\tan \psi_0 = \frac{a_y}{a_x} \quad \left( 0 \leq \psi_0 \leq \frac{\pi}{2} \right)$$

# 7-3 Polarization

## Elliptical Polarization

most general: **E** field traces a tilted ellipse:



major axis:

length  $a_\xi$

along the

$\xi$  direction

Aα

ALPHA

Bβ

BETA

Γγ

GAMMA

Δδ

DELTA

Εε

EPSILON

Ζζ

ZETA

Ηη

ETA

Θθ

THETA

Ιι

IOTA

Κκ

KAPPA

Λλ

LAMBDA

Μμ

MU

Νν

NU

Ξξ

XI

Οο

OMICRON

Ππ

PI

Ρρ

RHO

Σσς

SIGMA

Ττ

TAU

Υυ

UPSILON

Φφ

PHI

Χχ

CHI

Ψψ

PSI

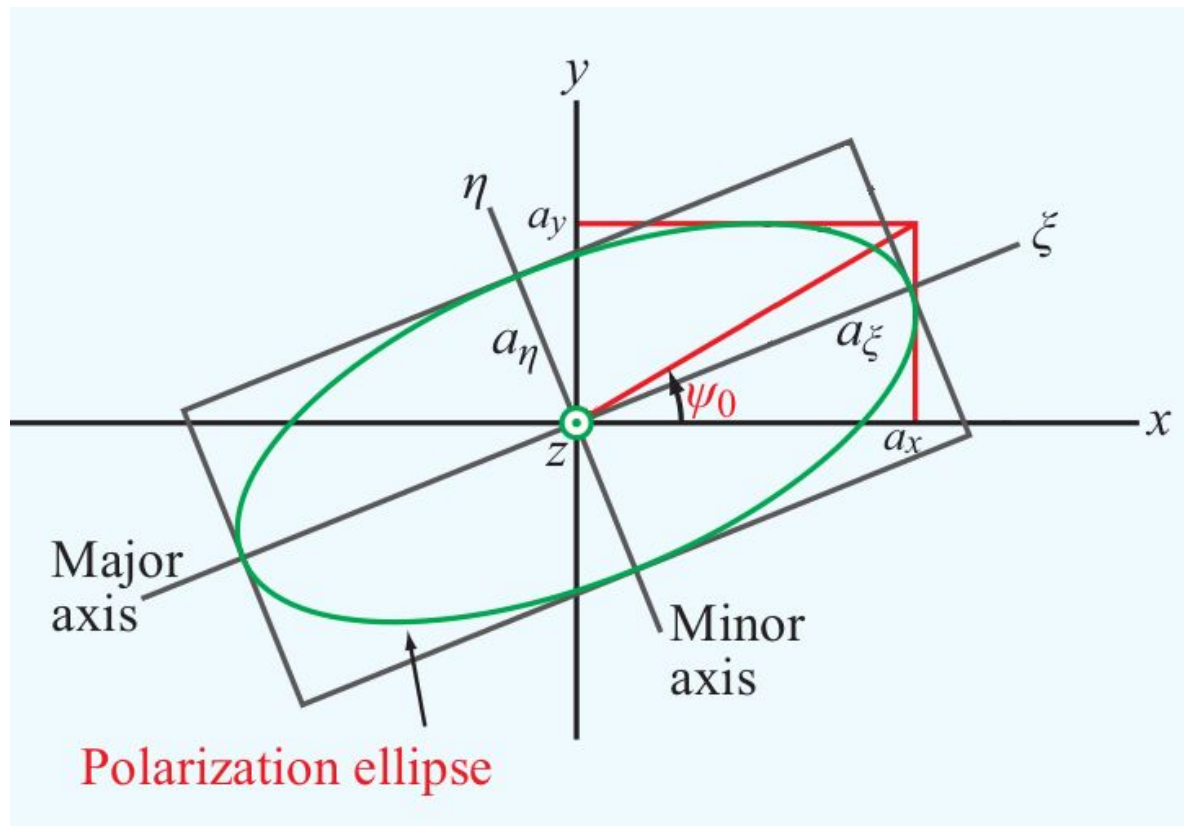
Ωω

OMEGA

# 7-3 Polarization

## Elliptical Polarization

most general:  $\mathbf{E}$  field traces a tilted ellipse:

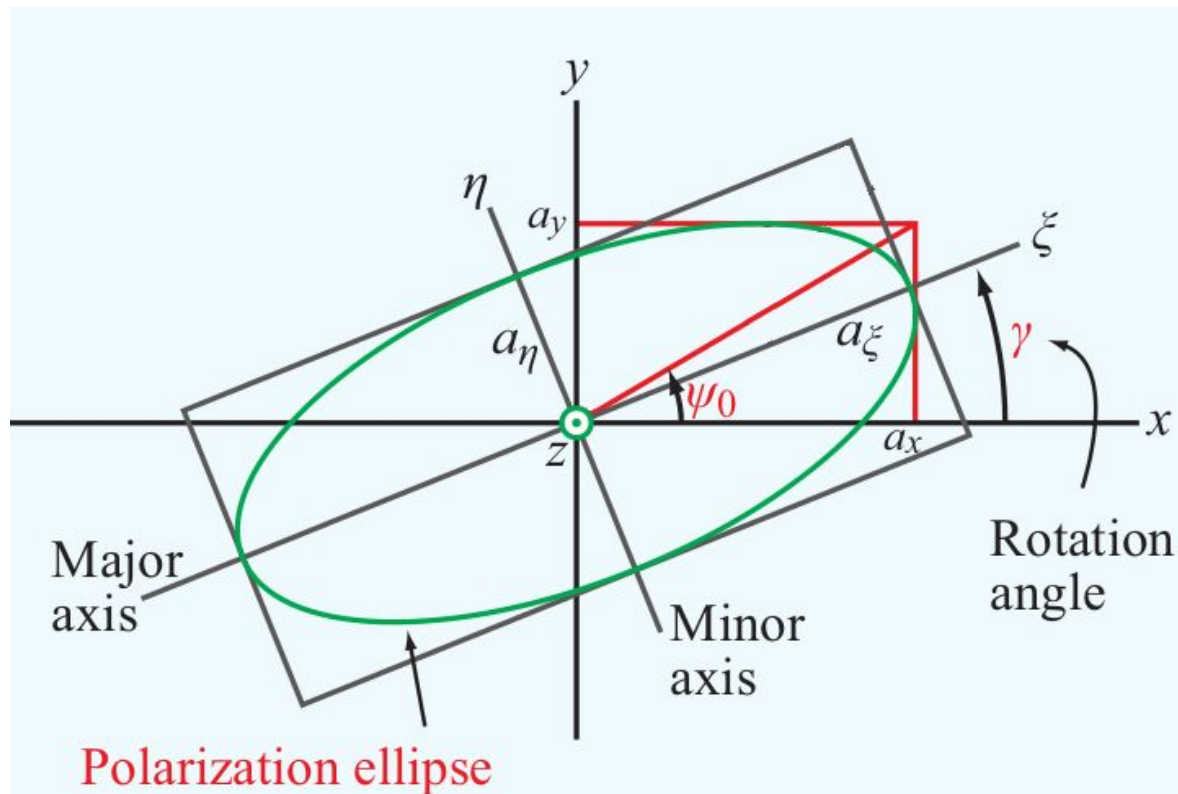


minor axis:  
length  $a_\eta$   
along the  
 $\eta$  direction.

# 7-3 Polarization

## Elliptical Polarization

most general:  $\mathbf{E}$  field traces a tilted ellipse:



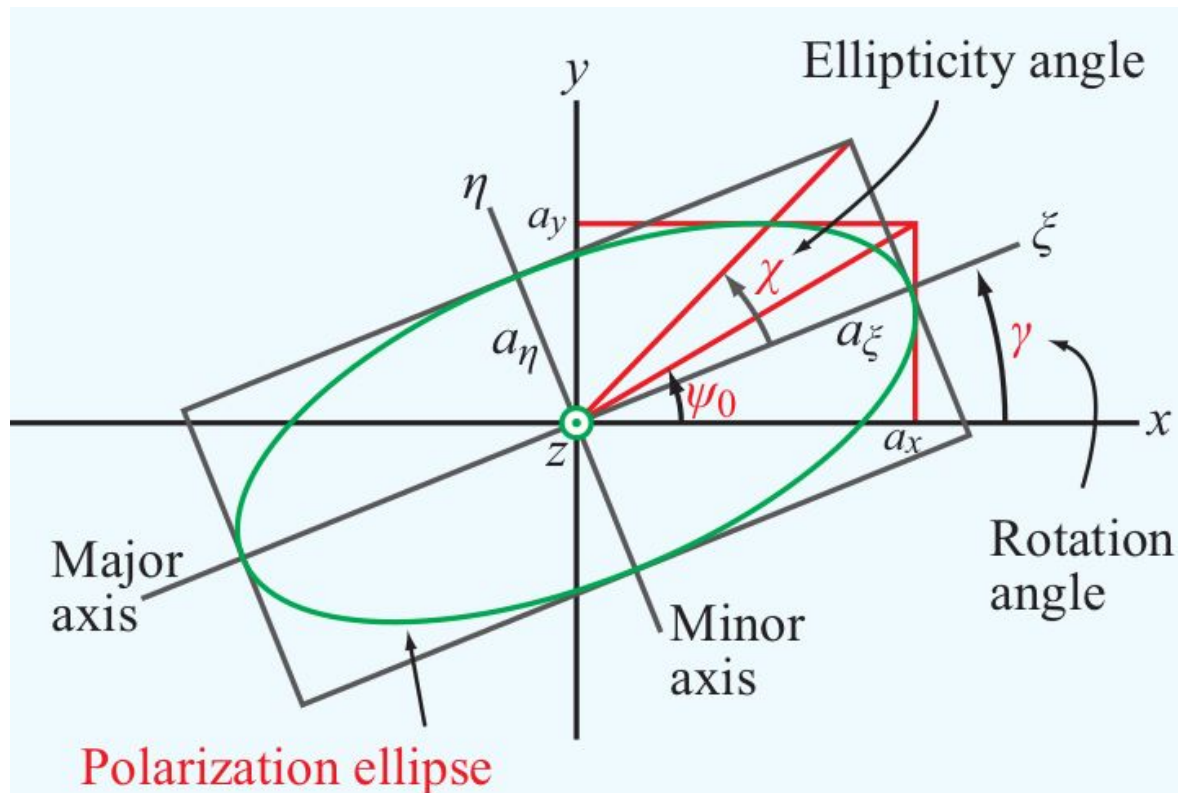
**rotation angle:**

$$-\pi/2 \leq \gamma \leq \pi/2.$$

# 7-3 Polarization

## Elliptical Polarization

most general: **E** field traces a tilted ellipse:



**ellipticity angle:**

$$\tan \chi = \pm \frac{a_\eta}{a_\xi} = \pm \frac{1}{R}$$

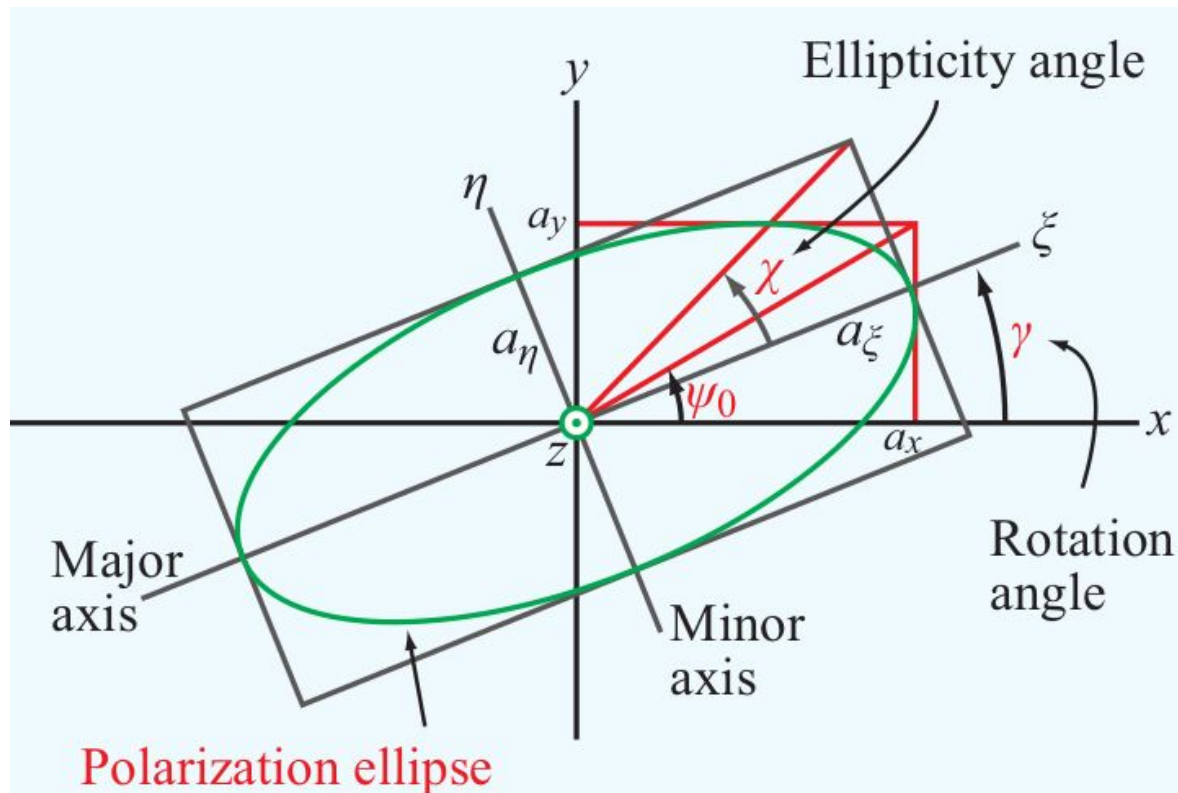
+ : left-handed rotation  
- : right-handed rotation



# 7-3 Polarization

## Elliptical Polarization

most general:  $\mathbf{E}$  field traces a tilted ellipse:



**ellipticity angle:**

$$\tan \chi = \pm \frac{a_\eta}{a_\xi} = \pm \frac{1}{R}$$

**axial ratio:**

$$R = a_\xi / a_\eta$$

=1: circular plzn

= $\infty$ : linear plzn

# 7-3 Polarization

## Elliptical Polarization

$$\tan 2\gamma = (\tan 2\psi_0) \cos \delta \quad (-\pi/2 \leq \gamma \leq \pi/2).$$

$$\sin 2\chi = (\sin 2\psi_0) \sin \delta \quad (-\pi/4 \leq \chi \leq \pi/4).$$


























$$\gamma > 0 \text{ if } \cos \delta > 0,$$

$$\gamma < 0 \text{ if } \cos \delta < 0.$$

$$\chi > 0 \text{ if } \sin \delta > 0,$$

$$\chi < 0 \text{ if } \sin \delta < 0.$$

# 7-3 Polarization

		$\gamma = -90^\circ$	$\gamma = -45^\circ$	$\gamma = 0^\circ$	$\gamma = 45^\circ$	$\gamma = 90^\circ$
$\chi = 45^\circ$	Left circular polarization					
$\chi = 22.5^\circ$	Left elliptical polarization					
$\chi = 0^\circ$	Linear polarization					
$\chi = -22.5^\circ$	Right elliptical polarization					
$\chi = -45^\circ$	Right circular polarization					

# Example 7-3

**Given:** plane wave with electric field:

$$\mathbf{E}(z, t) = \hat{\mathbf{x}} 3 \cos(\omega t - kz + 30^\circ) - \hat{\mathbf{y}} 4 \sin(\omega t - kz + 45^\circ) \quad (\text{mV/m}).$$

**Find:** The polarization state

**Solution:** First we need to find the phasor representation.  
So, convert the sin() to a cos()

$$\begin{aligned} \mathbf{E} &= \hat{\mathbf{x}} 3 \cos(\omega t - kz + 30^\circ) - \hat{\mathbf{y}} 4 \cos(\omega t - kz + 45^\circ - 90^\circ) \\ &= \hat{\mathbf{x}} 3 \cos(\omega t - kz + 30^\circ) - \hat{\mathbf{y}} 4 \cos(\omega t - kz - 45^\circ). \end{aligned}$$

# Example 7-3

So the phasor is:

$$\begin{aligned}\tilde{\mathbf{E}}(z) &= \hat{\mathbf{x}}3e^{-jkz}e^{j30^\circ} - \hat{\mathbf{y}}4e^{-jkz}e^{-j45^\circ} \\ &= \hat{\mathbf{x}}3e^{-jkz}e^{j30^\circ} + \hat{\mathbf{y}}4e^{-jkz}e^{-j45^\circ}e^{j180^\circ} \\ &= \hat{\mathbf{x}}3e^{-jkz}e^{j30^\circ} + \hat{\mathbf{y}}4e^{-jkz}e^{j135^\circ},\end{aligned}$$

so:  $\delta_x = 30^\circ$  and  $\delta_y = 135^\circ$

$$\delta = \delta_y - \delta_x = 135^\circ - 30^\circ = 105^\circ$$

$$\psi_0 = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{4}{3}\right) = 53.1^\circ$$

## Example 7-3

$$\tan 2\gamma = (\tan 2\psi_0) \cos \delta = \tan 106.2^\circ \cos 105^\circ = 0.89,$$

2 solutions:  $\gamma = 20.8^\circ$  and  $-69.2^\circ$ .

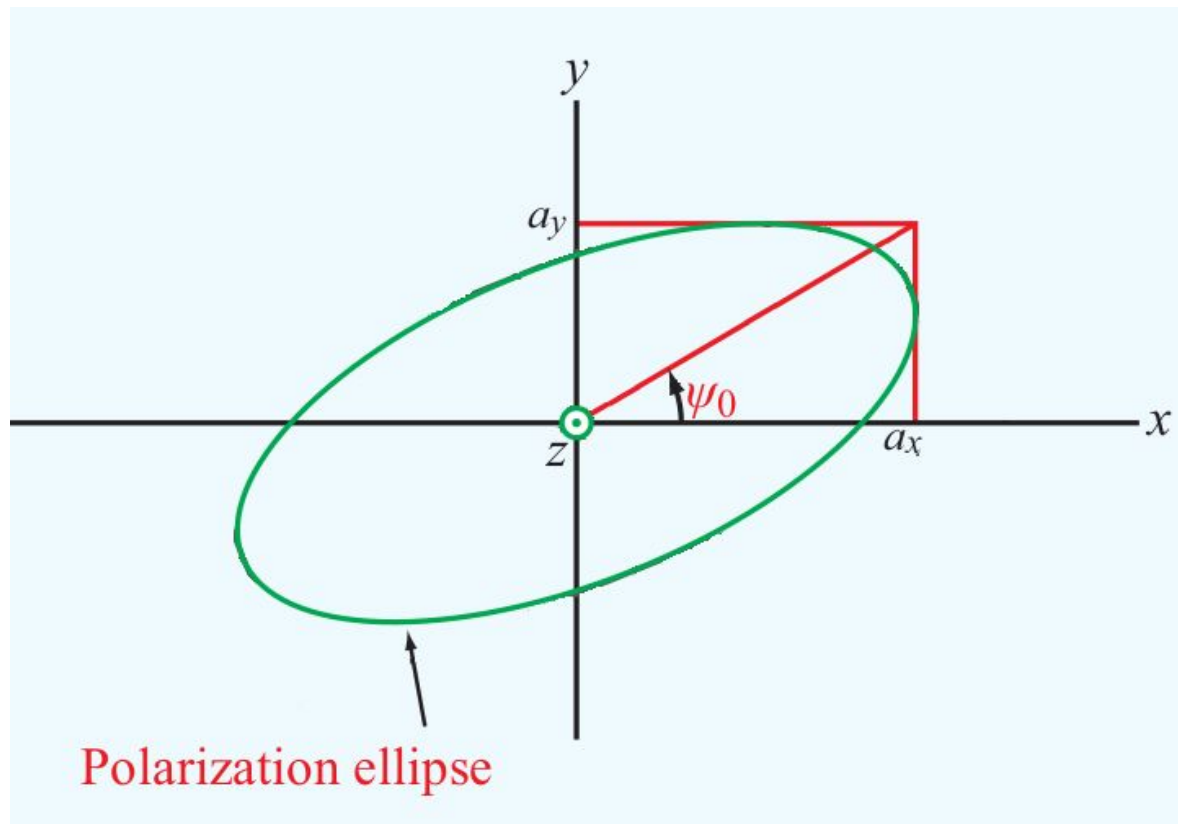
since  $\cos \delta < 0$ , choose  $-69.2^\circ$

$$\begin{aligned} \sin 2\chi &= (\sin 2\psi_0) \sin \delta \\ &= \sin 106.2^\circ \sin 105^\circ = 0.93 \quad \text{or} \quad \chi = 34.0^\circ. \end{aligned}$$

So, it's **elliptically polarized** and because  $\chi$  is positive, it means it's **left-handed**.

# 7-3 Calculating Polarization Angles

$$\tan \psi_0 = \frac{a_y}{a_x} \quad \left(0 \leq \psi_0 \leq \frac{\pi}{2}\right)$$



## 7-3 Calculating Polarization Angles

$$\tan 2\gamma = (\tan 2\psi_0) \cos \delta \quad (-\pi/2 \leq \gamma \leq \pi/2),$$

$$\sin 2\chi = (\sin 2\psi_0) \sin \delta \quad (-\pi/4 \leq \chi \leq \pi/4),$$

$$\gamma > 0 \text{ if } \cos \delta > 0,$$

$$\gamma < 0 \text{ if } \cos \delta < 0.$$

$$\chi > 0 \text{ if } \sin \delta > 0,$$

$$\chi < 0 \text{ if } \sin \delta < 0.$$

# 7-3 Calculating Polarization Angles

**Since arctan and arcsin are multi-valued:**

Need to look for the "appropriate" solution, given the solution from your calculator

Generally can go  $\pm 180^\circ$  from the calculated answer.

In our case, since we have  $\tan(2\gamma)$  and  $\sin(2\chi)$ , we should try  $\pm 90^\circ$  on  $\gamma$  and  $\chi$  in order to satisfy the sign constraint.

## Module 7.3 Polarization I

The polarization is ELLIPTICAL ( right-handed )

Instructions

Amplitude X

Phase X  deg  rad

Amplitude Y

Phase Y  deg  rad

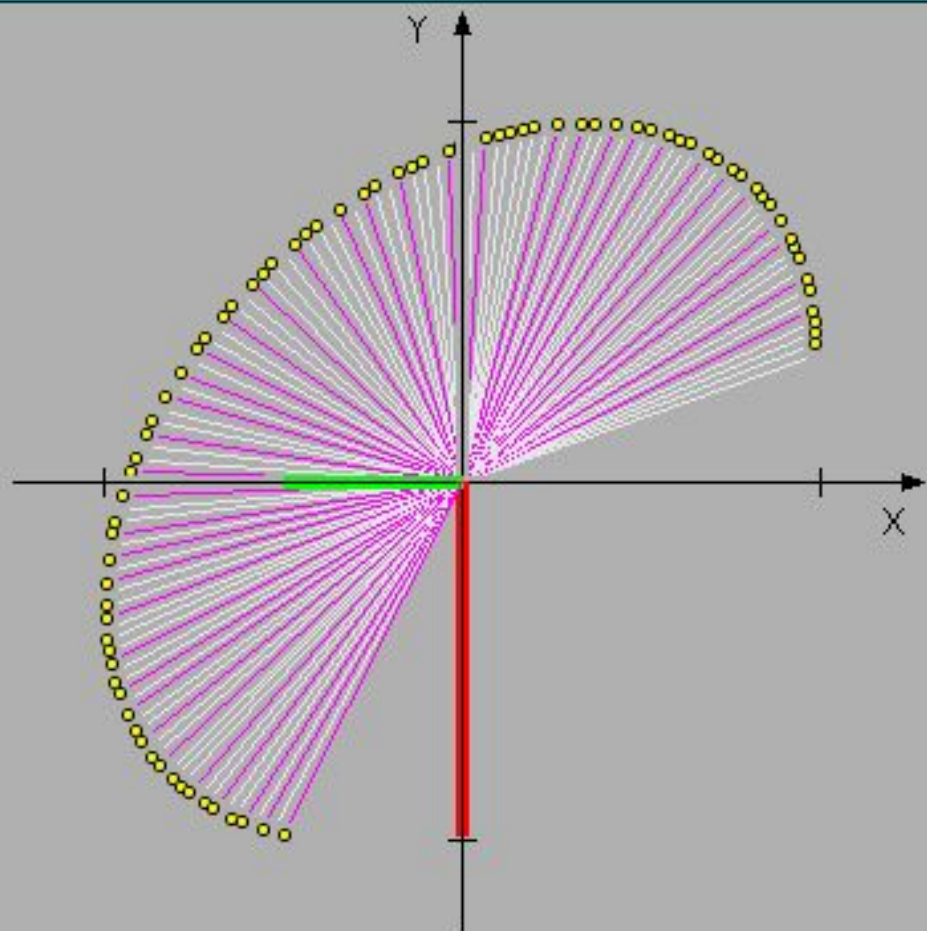
Animation speed

Update

Trace:  On  Off

START

STOP



The wave travels in the z-direction (towards the viewer)

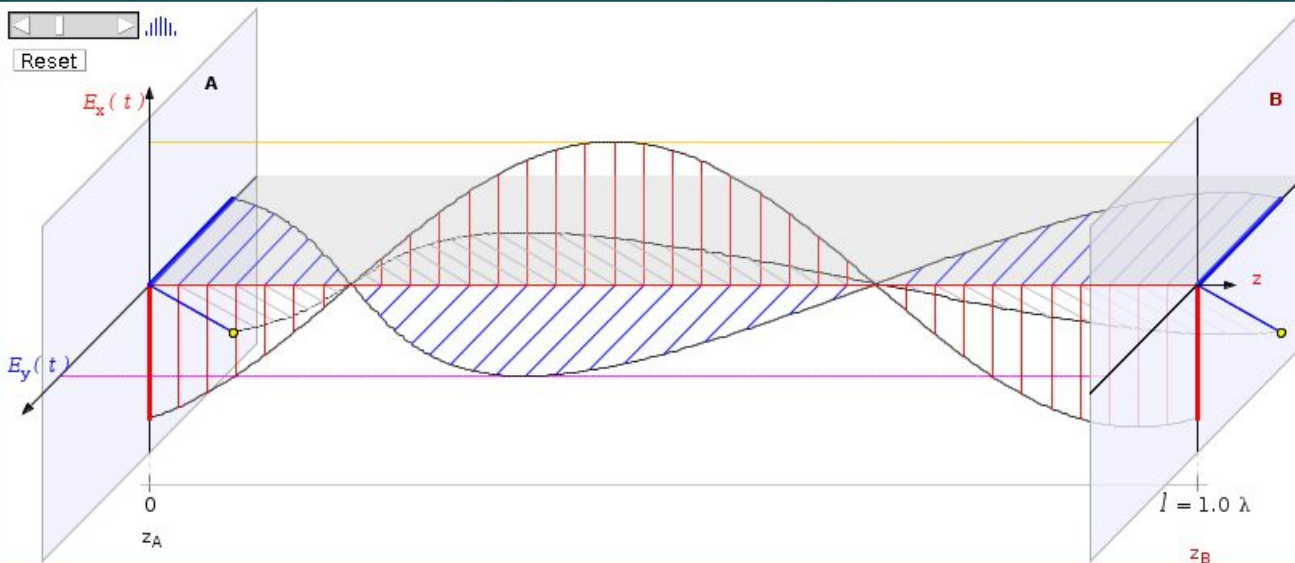
Module 7.4 Polarization II

$t = 0.444T$        $\omega t = 160^\circ$



Input/Output    Phase Planes    Instructions

Animation speed



**Input** Update

Frequency  $f = 1.0E9$  [ Hz ]

Relative Permittivity  $\epsilon_r = 1.0$

Relative Permeability  $\mu_r = 1.0$

Reference Amplitude  $E_0 = 1.0$  [ V / m ]

Reference Phase (z=0)  $\varphi = 0.0$  [ rad ]

Length Displayed  $l = 1.0$  [  $\lambda$  ]

$E_x = 1.0 E_0$        $E_y = 1.0 E_0$

$\varphi(E_y) - \varphi(E_x) = 0.0^\circ$

**Output**

WaveLength  $\lambda = 30.0$  [ cm ]

Phase Velocity  $v_p = 3.0 \times 10^8$  [ m / s ]

Period  $T = 1.0 \times 10^{-9}$  [ s ]

Impedance of the Medium [  $\Omega$  ]

$\eta = 376.991118$

Phase Constant

$\beta = 20.94395$  [  $m^{-1}$  ]

LINEAR POLARIZATION

A A

B B

**A)**  $z_A = 0.0 \lambda = 0.0$  [ m ]

$|E_x| = 1.0$  [ V / m ]

$\angle E_x = 0.0$  [ rad ]

$|E_y| = 1.0$  [ V / m ]

$\angle E_y = 0.0$  [ rad ]

$f = 1.0$  GHz

$l = 1.0 \lambda = 30.0$  [ cm ]

$E_x(t)$      $E_y(t)$      $E_{tot}(t)$

**B)**  $z_B = 1.0 \lambda = 30.0$  [ cm ]

$|E_x| = 1.0$  [ V / m ]

$\angle E_x = -6.28319$  [ rad ]

$|E_y| = 1.0$  [ V / m ]

$\angle E_y = -6.28319$  [ rad ]

# Homework

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**Homework 25 is due tomorrow at midnight.**

**submit to gradescope via the canvas site.**

# Next Time



## **Sections 7-4 through 7-6:**

Wave Propagation in Lossy Media

Current Flow in a Good Conductor

Electromagnetic Power Density