EECS 230
ENGINEERING ELECTROMAGNETICS
Leland Pierce

Time-Varying Fields 2

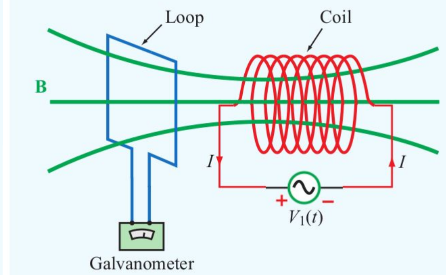
University of Michigan - Fall 2024

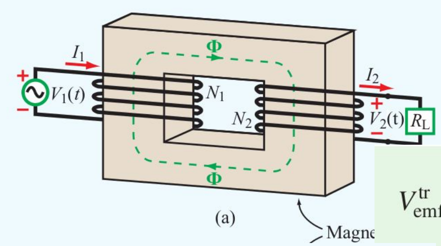
Chapter 6 Overview

- Time-Varying Fields
- Faraday's Law
- Stationary Loop in time-varying field
- Ideal Transformer
- Moving conductor in static field
- The Generator
- Moving conductor in time-varying field
- Displacement Current
- Boundary Conditions
- Charge-Current Continuity
- Free-Charge dissipation in a conductor
- Potentials

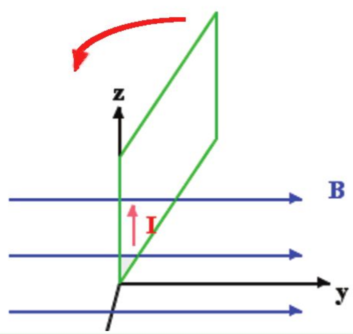
$$\nabla \cdot \mathbf{D} = \rho_v,$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$
$$\nabla \cdot \mathbf{B} = 0,$$
$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}.$$

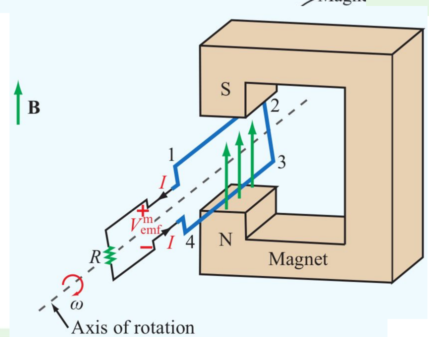
$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$$

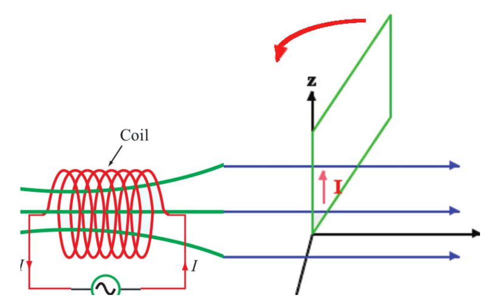




$$V_{emf}^{tr} = -N \int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}, \quad \textbf{(transformer emf)}$$



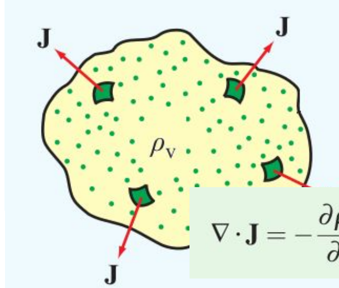




$$V_{emf}^{m} = \oint_C (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}. \quad \textbf{(motional emf)}$$

$$V_{emf} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s} + \oint_C (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}.$$

$$I_d = \int_S \mathbf{J}_d \cdot d\mathbf{s} = \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{s},$$



$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t}$$

$$\widetilde{V}(\mathbf{R}) = \frac{1}{4\pi\varepsilon} \int_{\upsilon'} \frac{\tilde{\rho}_v(\mathbf{R}_i)\, e^{-jkR'}}{R'}\, d\upsilon'$$

$$\rho_v(t) = \rho_{vo} e^{-(\sigma/\varepsilon)t}$$

Lecture Coverage

Today's lecture:

Review sections 6-1 through 6-3 of the book:

6-1: Faraday's Law
6-2: Stationary Loop in time-varying **B** Field
6-3: Ideal Transformer

Sections 6-4 through 6-6 of the book:

6-4: Moving Loop in static **B** field
6-5: The Electromagnetic Generator
6-6: Moving Loop in time-varying **B** Field

Review of Chapter 6

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q$$

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$$

$$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$$

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{s}$$

Maxwell's Equations
Empirically derived from many measurements

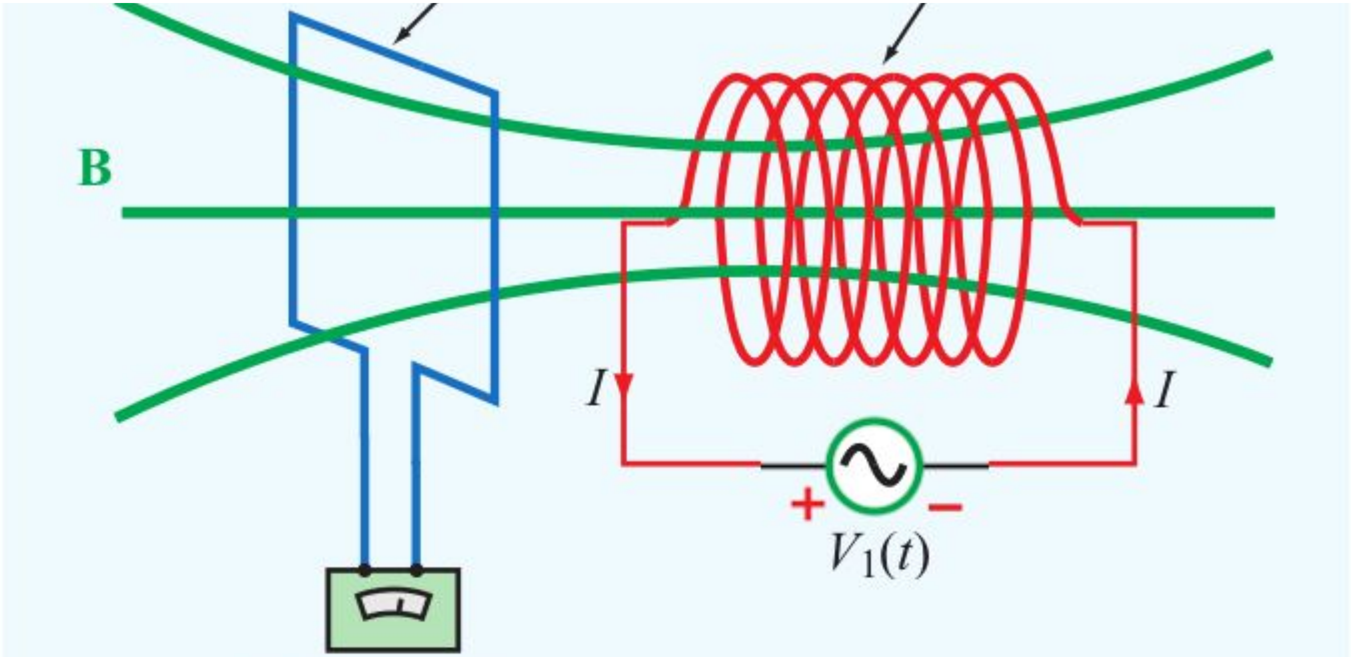
$$\mathbf{D} = \varepsilon \mathbf{E}$$

$$\mathbf{B} = \mu \mathbf{H}.$$

E: Electric Field
H: Magnetic Field
J: Current Density
ρ_v: Charge Density

Review of Chapter 6

Faraday's Law



$$V_{\text{emf}} = -N\,\frac{d\Phi}{dt} = -N\,\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{s}$$

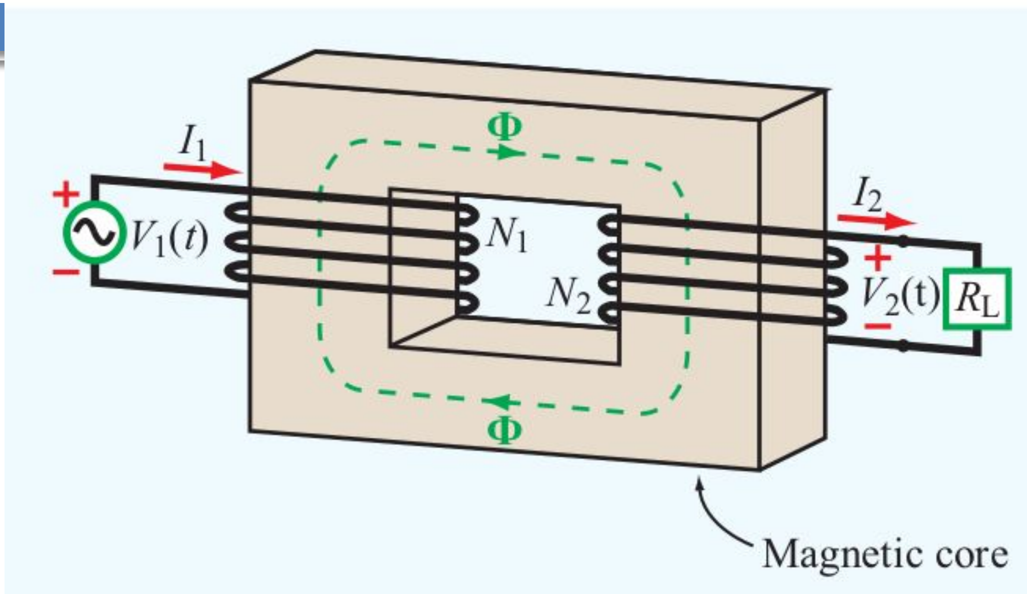
Review of Chapter 6

Time-varying **B** field, Stationary loop

$$V_{\text{emf}}^{\text{tr}} = -N \int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}, \quad \textbf{(transformer emf)}$$

Review of Chapter 6

The Ideal Transformer



For example, if $N_2=10N_1$,
Phasors:

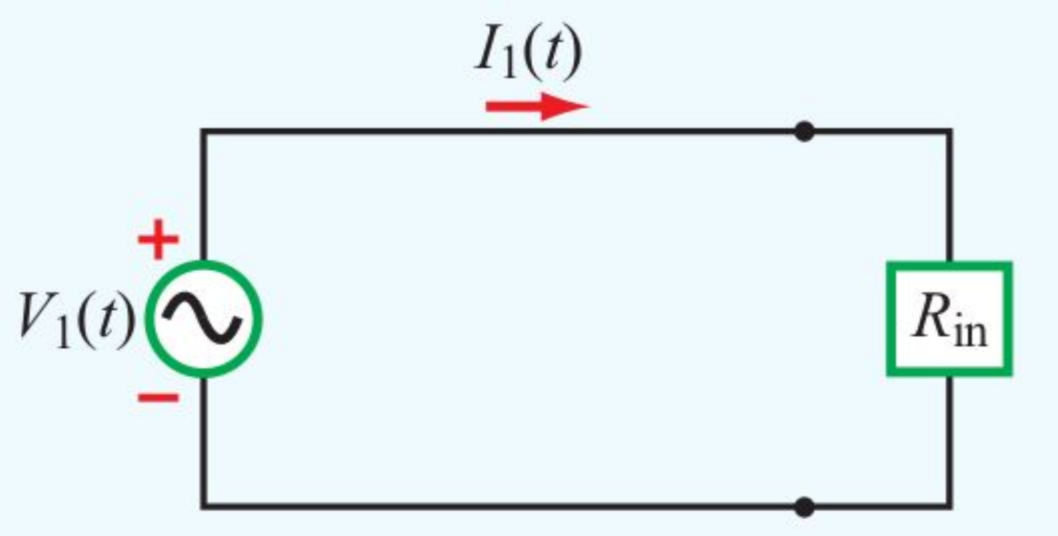
$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$V_2 = 10\ V_1$
$I_2 = 0.1\ I_1$

Review of Chapter 6

The Ideal Transformer



Equivalent Circuit:

$$R_{in} = \left(\frac{N_1}{N_2}\right)^2 R_L$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

Phasor domain:

$$Z_{in} = \left(\frac{N_1}{N_2}\right)^2 Z_L$$

Preview

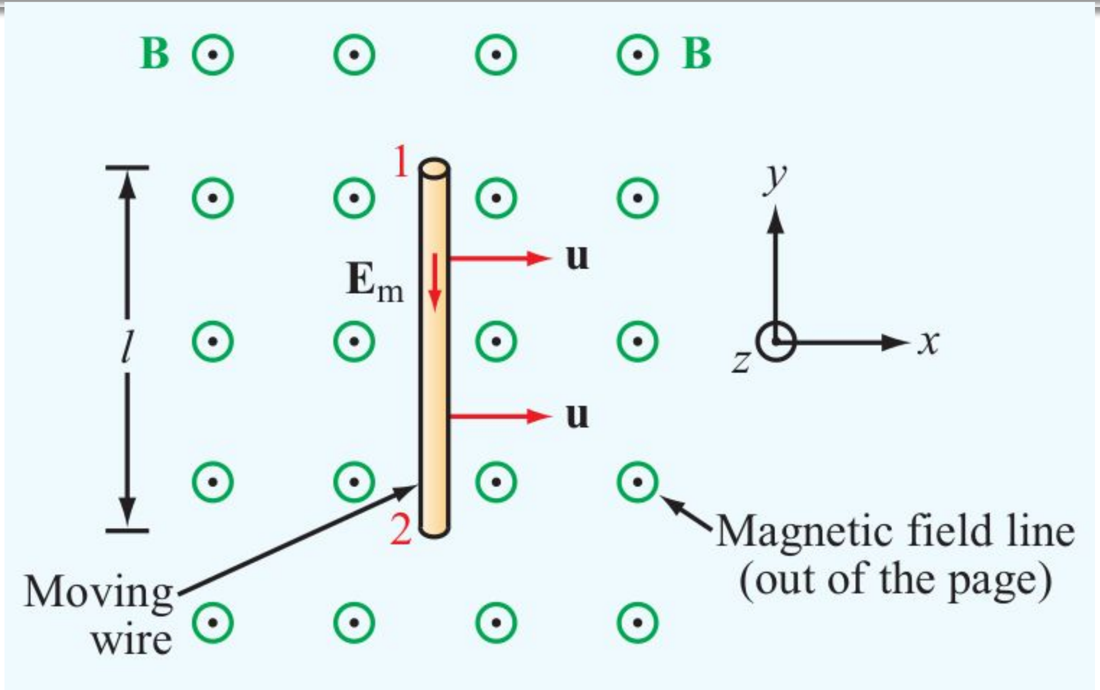
Last time: we analyzed the case:

time-varying **B** field, stationary wire loop.

Now we analyze the opposite case:

static **B** field, moving wire

6-4 Moving Conductor: Static Field



The magnetic force acting on a moving charge *q* in the wire:

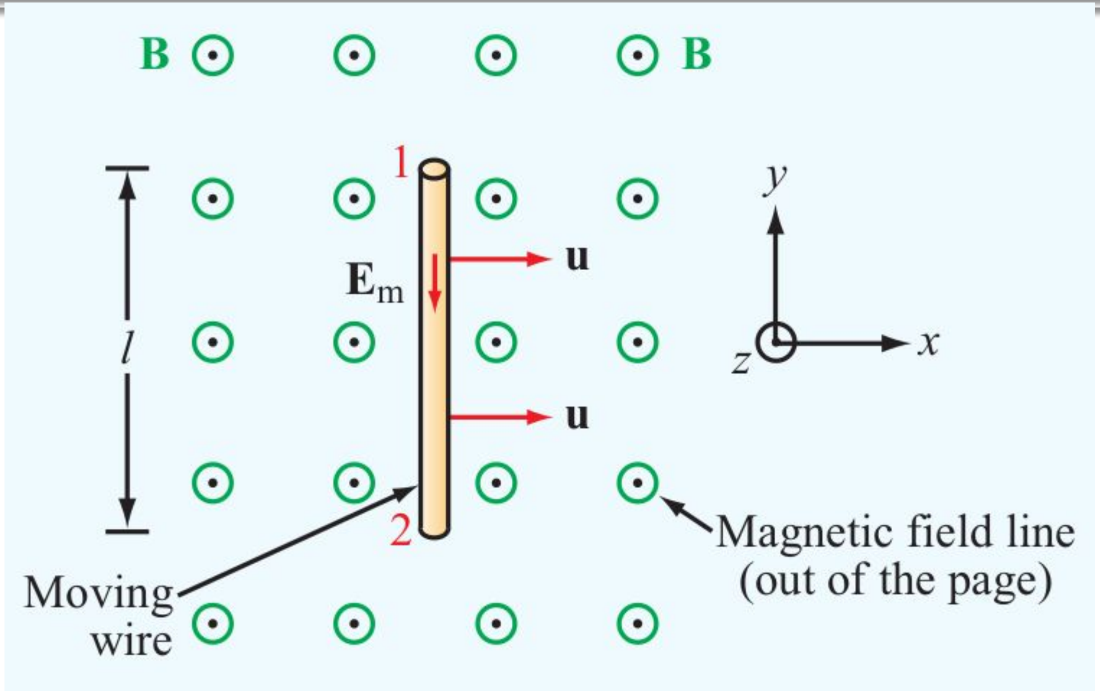
$\mathbf{F}_m = q\,(\mathbf{u} \times \mathbf{B})$

This causes the free charges to move, which generates a voltage difference between the ends of the wire:

$$V_{12} = \int_2^1 \mathbf{E}_m \cdot d\mathbf{l}$$

But what is E_m?

6-4 Moving Conductor: Static Field



Recall that the force on a charge due to an electric field is:

$$\mathbf{F}_e = q\,\mathbf{E}$$

Equating $\mathbf{F}_e$ and $\mathbf{F}_m$:

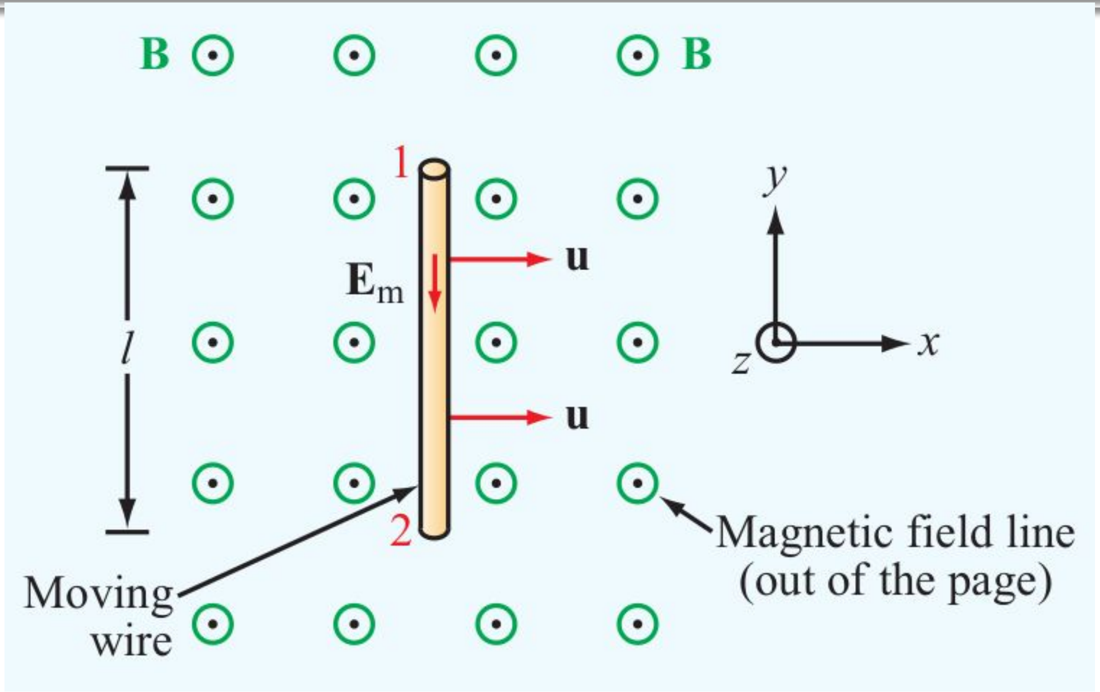
$$V_{12} = \int_2^1 \mathbf{E}_m \cdot d\mathbf{l} = \int_2^1 (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}.$$

Since this gives us the right units... it's useful.

6-4 Moving Conductor: Static Field

This is called the Motional EMF:

$$V_{\text{emf}}^{\text{m}} = \oint_C (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}.$$



For this example:

$\mathbf{u} = \hat{\mathbf{x}}u$

$\mathbf{B} = \hat{\mathbf{z}}\, B_0$

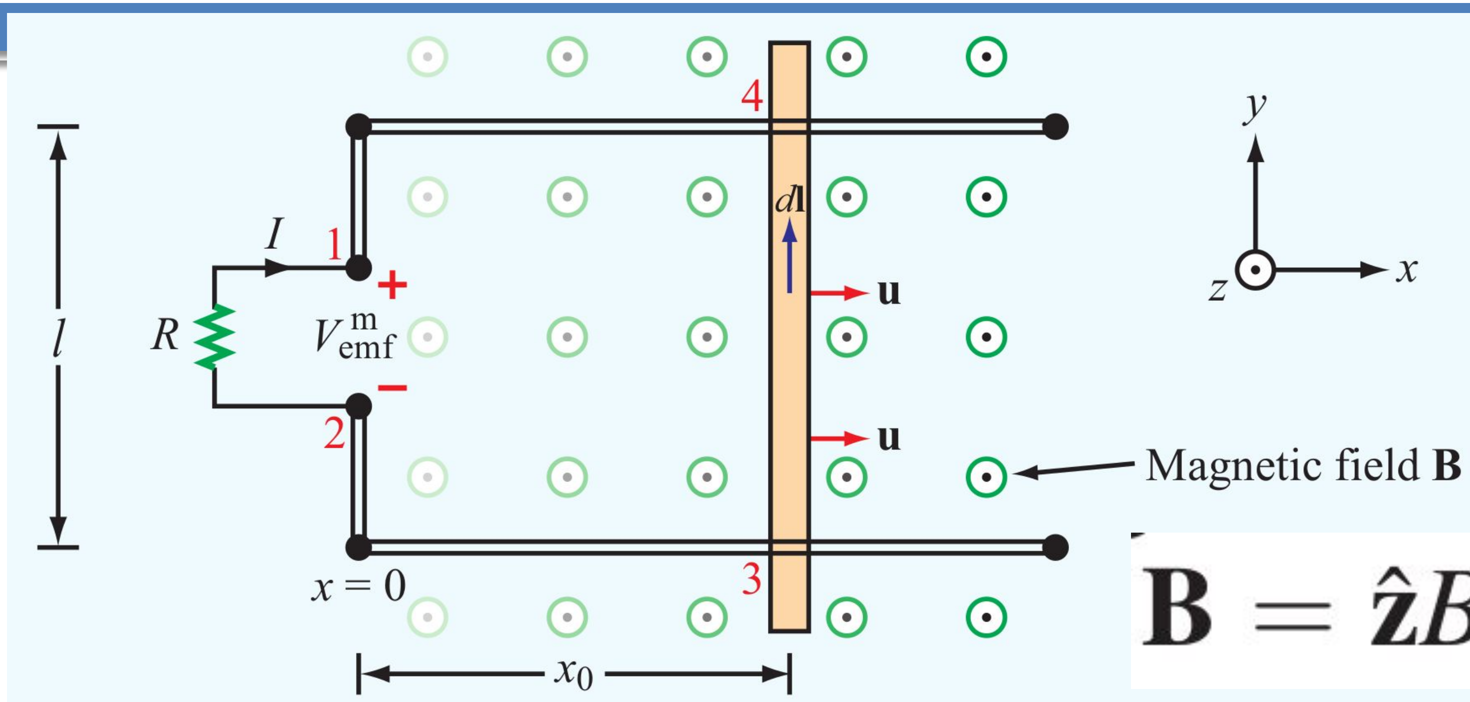
so $\mathbf{u} \times \mathbf{B} = -\hat{\mathbf{y}}\ uB_0$

and: $d\mathbf{l}$ $\hat{\mathbf{y}}$ dy

so:

$$V_{\text{emf}}^{\text{m}} = -uB_0 l.$$

Example 6-3: Sliding Bar in Static Field



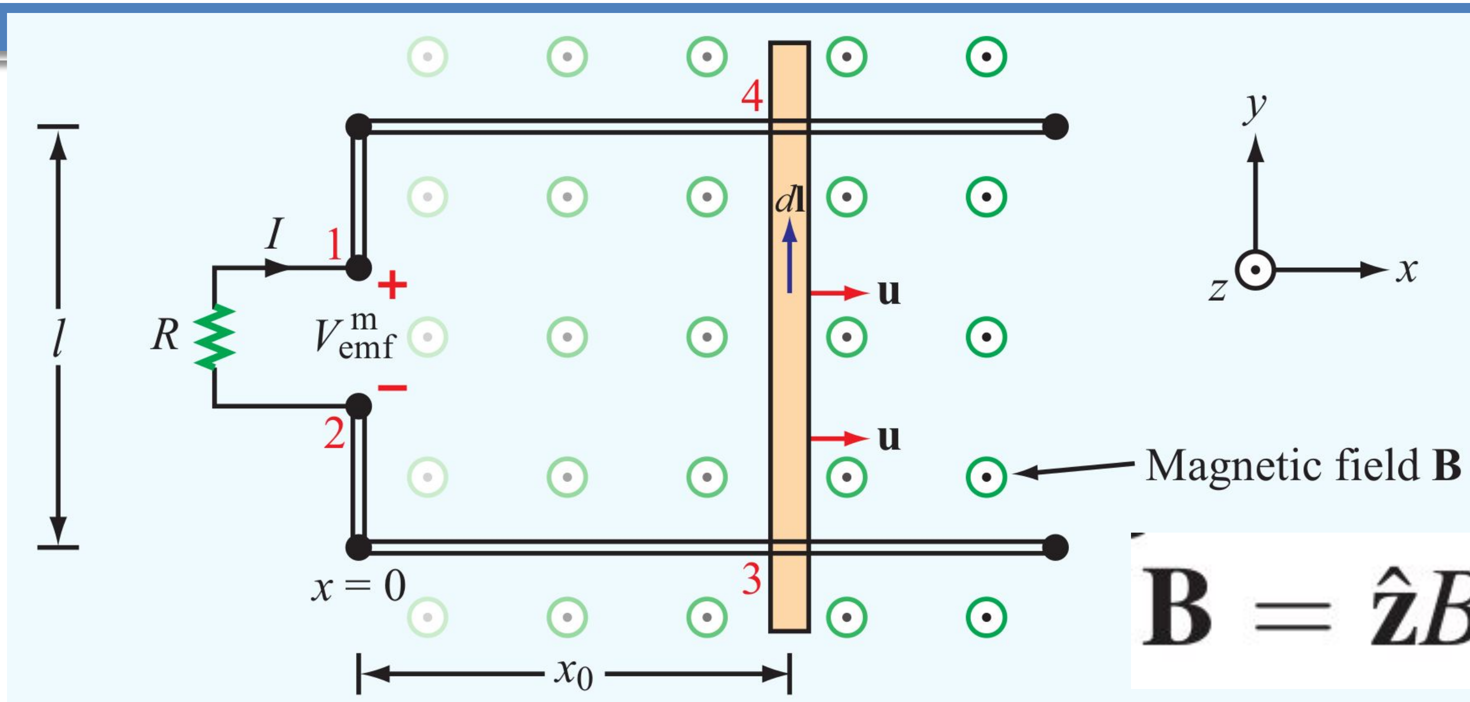
$$\mathbf{B} = \hat{\mathbf{z}} B_0 x.$$

(can ignore loop resistance)

Motional EMF:

$$V_{\text{emf}}^{\text{m}} = V_{12} = V_{43} = \int_3^4 (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$$

Example 6-3: Sliding Bar in Static Field



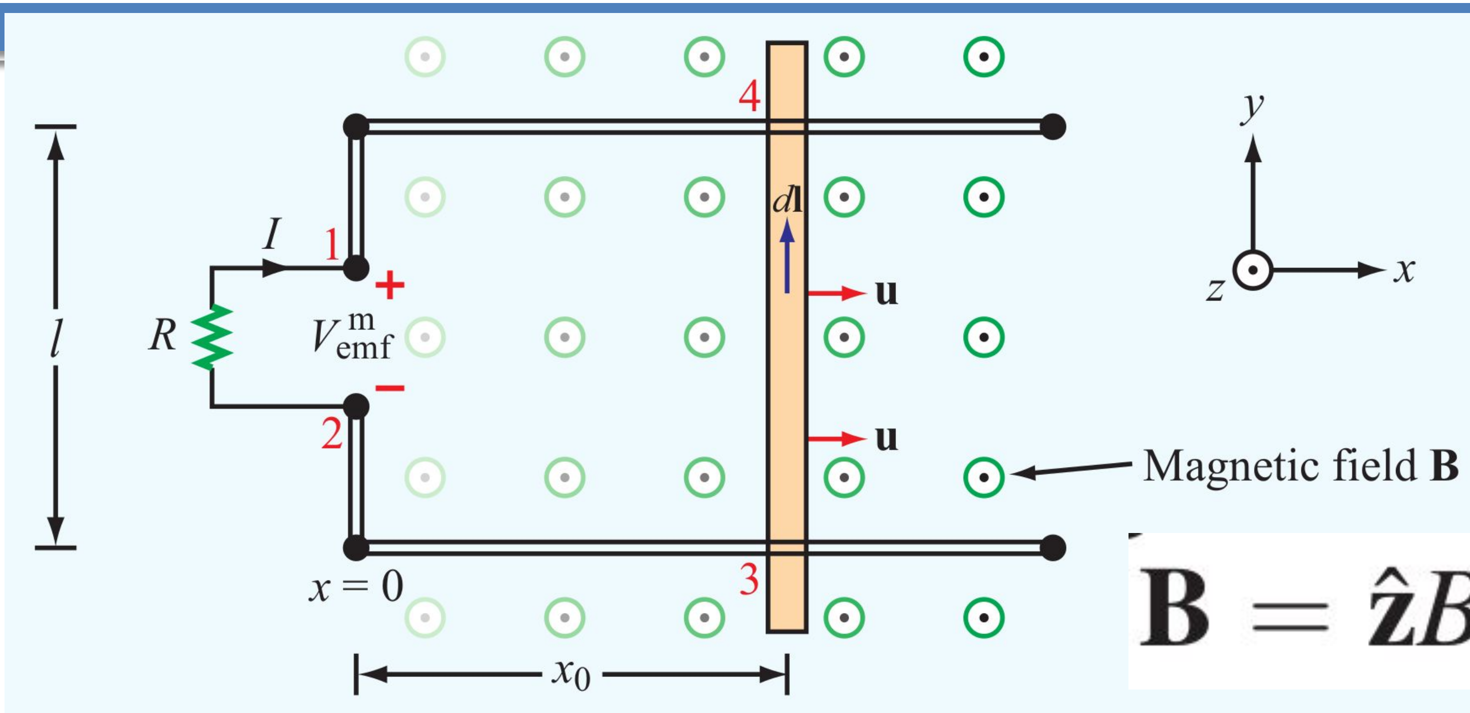
$$\mathbf{B} = \hat{\mathbf{z}} B_0 x.$$

at one moment during the sliding process:

$$V_{\mathrm{emf}}^{\mathrm{m}} = V_{12} = V_{43} = \int_3^4 (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$$

$$= \int_3^4 (\hat{\mathbf{x}} u \times \hat{\mathbf{z}} B_0 x_0) \cdot \hat{\mathbf{y}}\, dl = -u B_0 x_0 l.$$

Example 6-3: Sliding Bar in Static Field



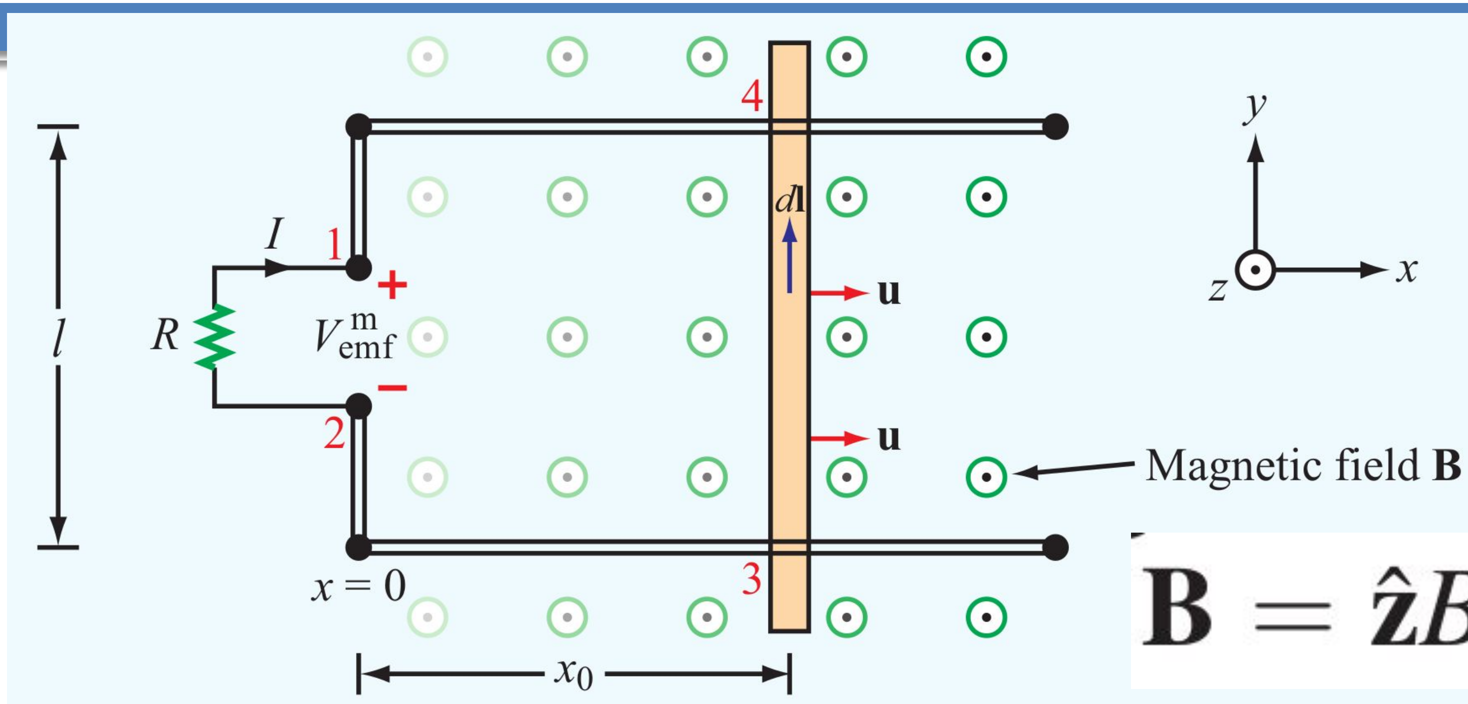
$$\mathbf{B} = \hat{\mathbf{z}} B_0 x.$$

for any time during the sliding process:

Since $x_0 = u\,t$

$$V_{\text{emf}}^{\text{m}} = -B_0 u^2 l t$$

Example 6-3: Sliding Bar in Static Field



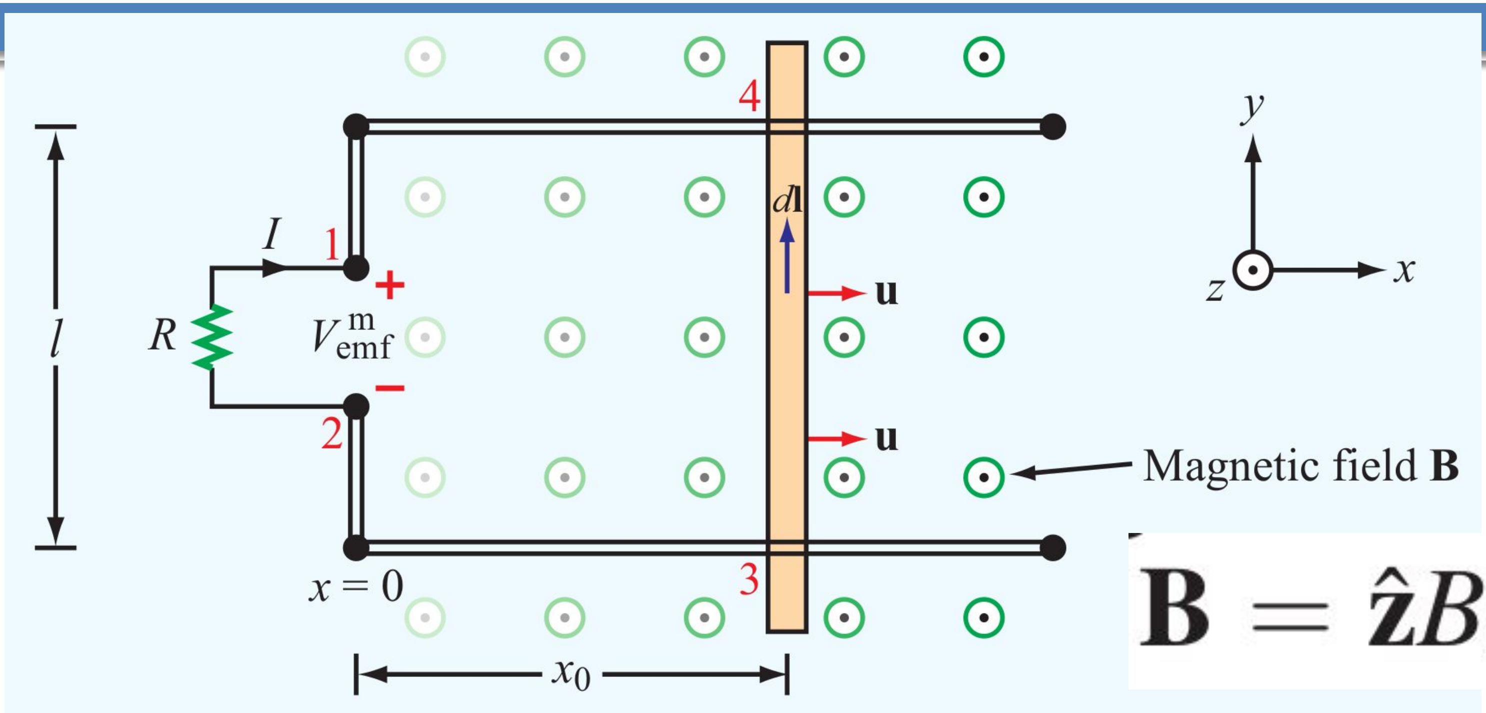
$$\mathbf{B} = \hat{\mathbf{z}} B_0 x.$$

Since we have a wire loop:

Can use Faraday's law instead: S: surface of loop

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s} = \int_S (\hat{\mathbf{z}} B_0 x) \cdot \hat{\mathbf{z}}\, dx\, dy = B_0 l \int_0^{x_0} x\, dx = \frac{B_0 l x_0^2}{2}.$$

Example 6-3: Sliding Bar in Static Field



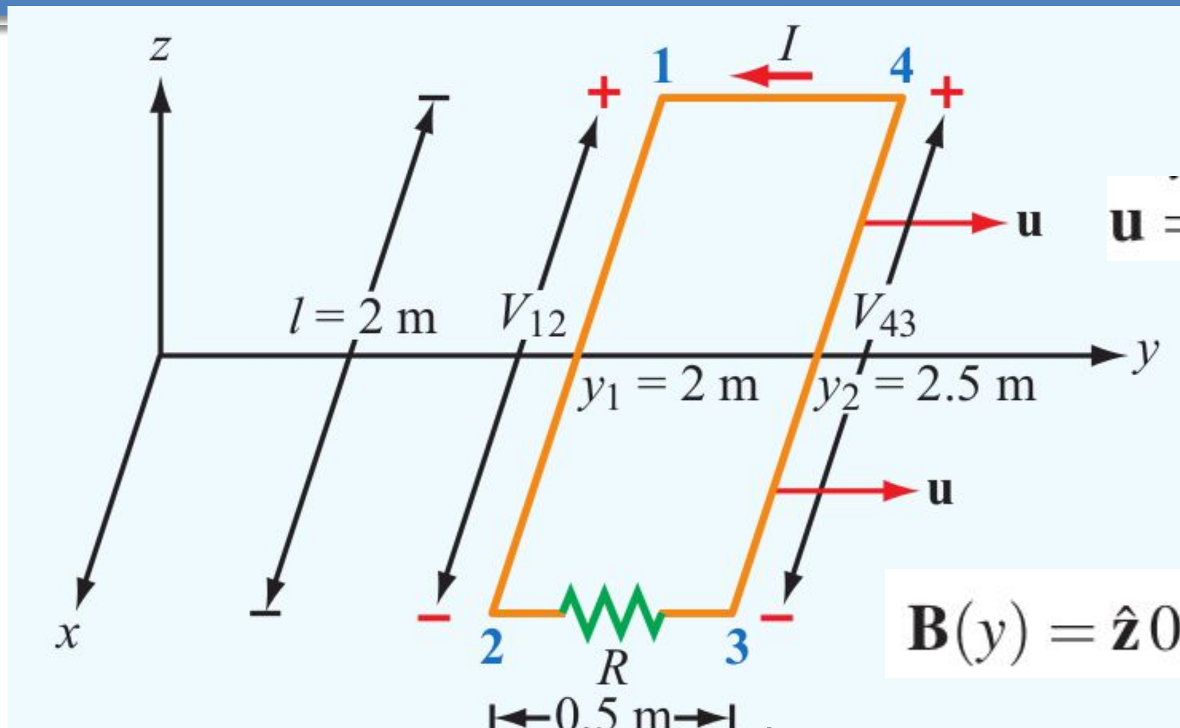
$$\mathbf{B} = \hat{\mathbf{z}} B_0 x.$$

And so V_{EMF} is:

$$V_{emf} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left(\frac{B_0 l u^2 t^2}{2}\right) = \boxed{-B_0 u^2 l t \quad \text{(V)}}$$

(same as before)

Example 6-4: Moving Loop in Static Field



$\mathbf{u} = \hat{\mathbf{y}}5$ (m/s)

$R = 5\ \Omega$,

$\mathbf{B}(y) = \hat{\mathbf{z}}0.2e^{-0.1y}$ (T).

Find the current at the moment shown:

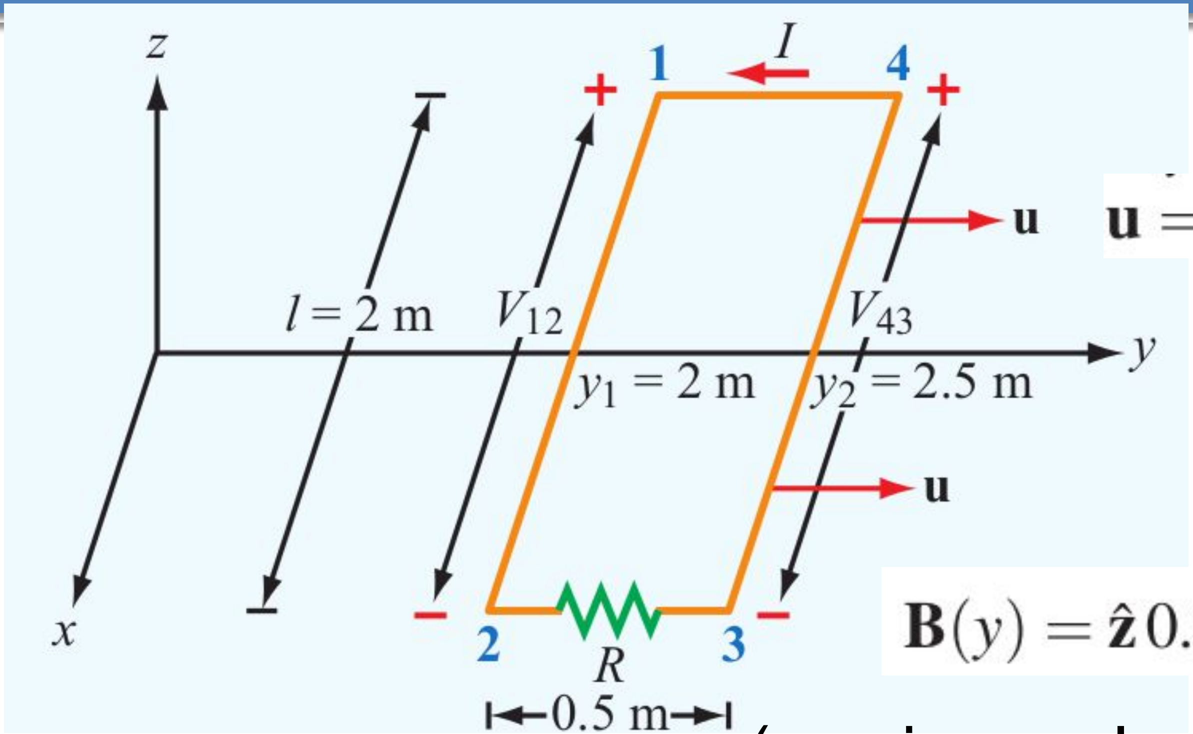
(can ignore loop resistance)

uXB $\propto\ \hat{\mathbf{y}} \times \hat{\mathbf{z}}$: along *x*-direction:

Voltages induced along *x*-directed segments only:

segments 1-2 and 3-4

Example 6-4: Moving Loop in Static Field



$\mathbf{u} = \hat{\mathbf{y}}5$ (m/s)

$R = 5\ \Omega$,

$$\mathbf{B}(y) = \hat{\mathbf{z}}0.2e^{-0.1y} \quad \text{(T)}.$$

Note: **B** is different for the 2 segments.

(can ignore loop resistance)

Get *V* for side 1-2 first:

$$\mathbf{B}(y_1) = \hat{\mathbf{z}}0.2e^{-0.1y_1} = \hat{\mathbf{z}}0.2e^{-0.2} \quad \text{(T)}.$$

Example 6-4: Moving Loop in Static Field

V for side 1-2 is:

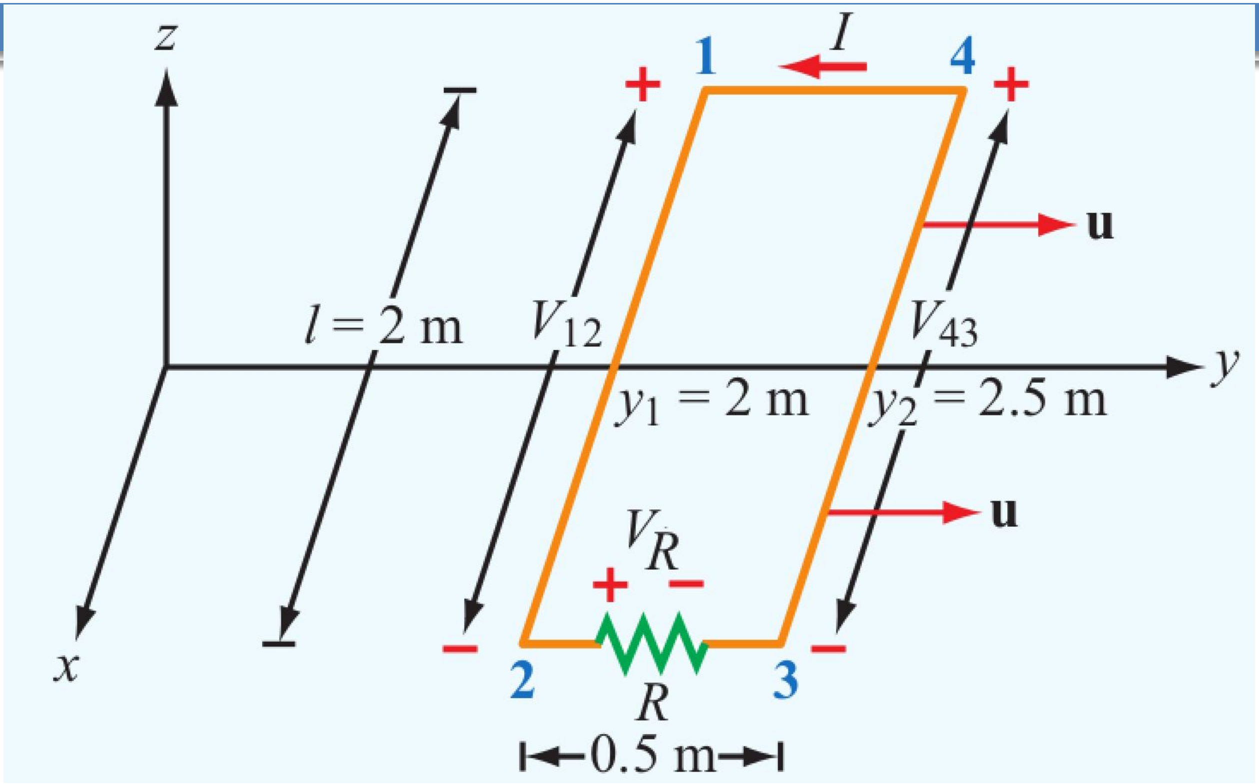
$$\begin{aligned} V_{12} &= \int_2^1 [\mathbf{u} \times \mathbf{B}(y_1)] \cdot d\mathbf{l} \\ &= \int_{l/2}^{-l/2} (\hat{\mathbf{y}}5 \times \hat{\mathbf{z}}0.2e^{-0.2}) \cdot \hat{\mathbf{x}}\, dx \\ &= -e^{-0.2} l \\ &= -2e^{-0.2} \\ &= -1.637 \qquad \text{(V)}. \end{aligned}$$

Example 6-4: Moving Loop in Static Field

Similarly, *V* for side 3-4 is:

$$
\begin{aligned}
V_{43} &= -uB(y_2)\,l \\
&= -5 \times 0.2e^{-0.25} \times 2 \\
&= -1.558 \qquad \text{(V)}.
\end{aligned}
$$

Example 6-4: Moving Loop in Static Field



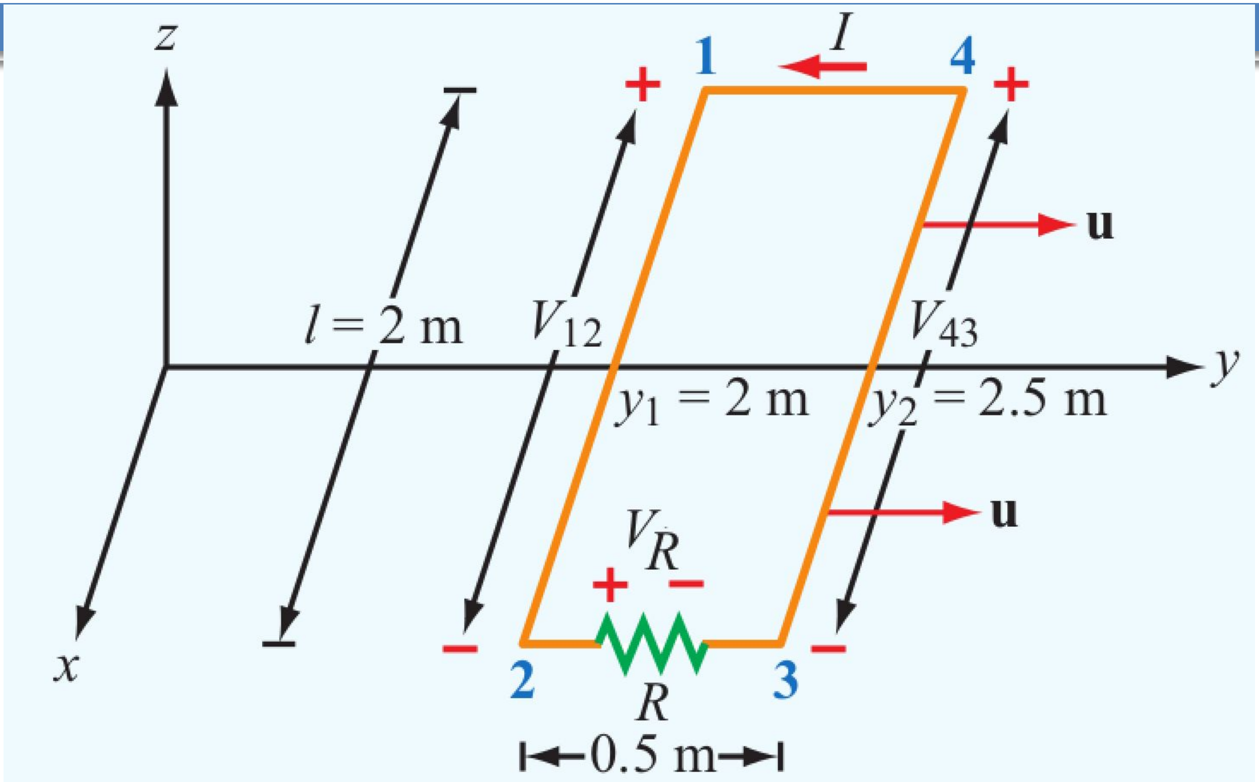
KVL counter-clockwise around the loop:

$V_R - V_{43} + V_{12} = 0$

$V_R = V_{43} - V_{12}$

$I = V/R = (V_{43} - V_{12}) / R$

Example 6-4: Moving Loop in Static Field



$I = V/R = (V_{43} - V_{12}) / R$

$= (-1.558\text{V} - -1.637\text{V}) / 5\Omega$

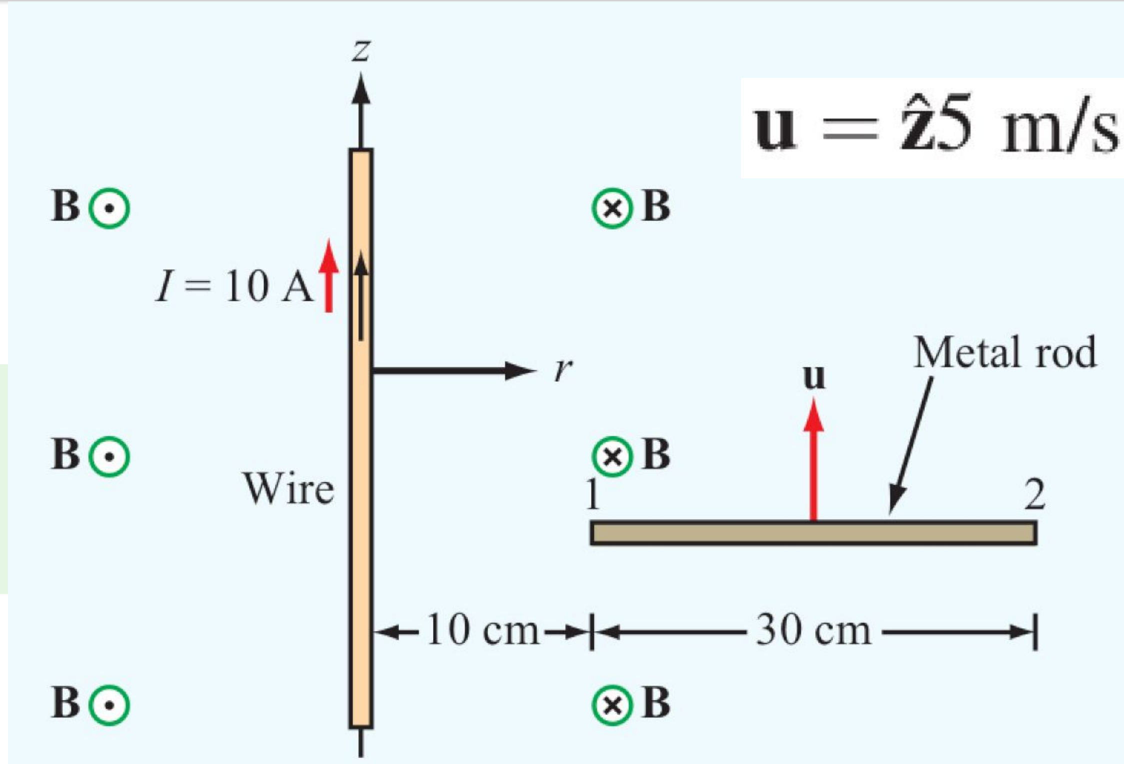
$= 0.079\text{V} / 5\Omega$

$I = 15.8 \text{ mA}$

Example 6-5: Moving Rod near Straight Current

Find the voltage induced across the ends of the rod.

$$V_{\text{emf}}^{\text{m}} = \oint_C (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}.$$



B due to the current in the wire:

$$\mathbf{B} = \hat{\boldsymbol{\phi}} \frac{\mu_0 I}{2\pi r}$$

and:

$$d\mathbf{l} = \hat{\mathbf{r}} \, dr$$

Example 6-5: Moving Rod near Straight Current

$$V_{12} = \int_{40\text{ cm}}^{10\text{ cm}} (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$$

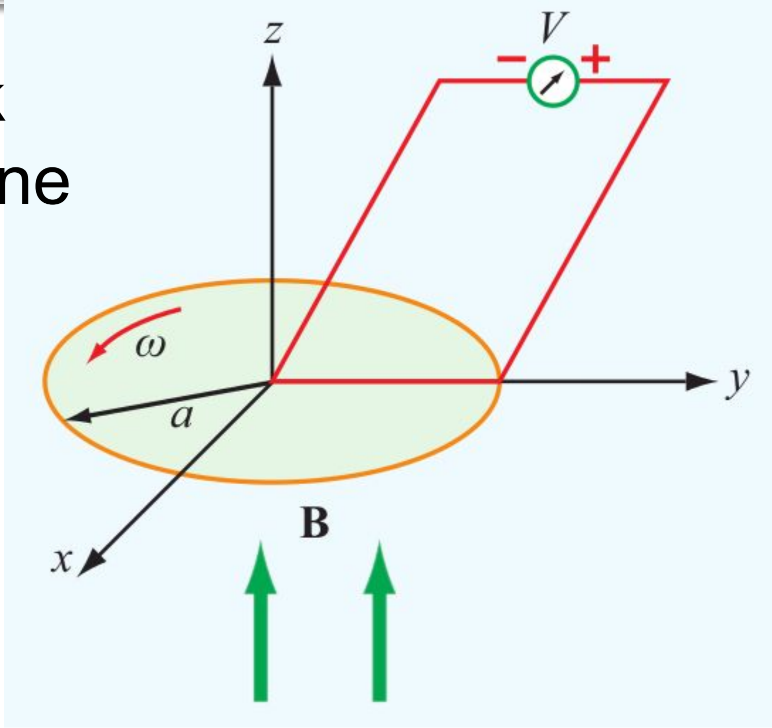
$$= \int_{40\text{ cm}}^{10\text{ cm}} \left(\hat{\mathbf{z}}5 \times \hat{\boldsymbol{\phi}} \frac{\mu_0 I}{2\pi r} \right) \cdot \hat{\mathbf{r}}\, dr$$

$$= -\frac{5\mu_0 I}{2\pi} \int_{40\text{ cm}}^{10\text{ cm}} \frac{dr}{r}$$

$$= -\frac{5 \times 4\pi \times 10^{-7} \times 10}{2\pi} \times \ln\left(\frac{10}{40}\right)$$

$$= 13.9 \qquad (\mu\text{V}).$$

6-4 Rotating Disk in Uniform Field



Given: A circular conducting disk centered at origin, *x*-*y* plane spinning about *z*-axis: ω radius *a* uniform $\mathbf{B} = \hat{\mathbf{z}} B_0$

Find: V_{EMF} between rim & center

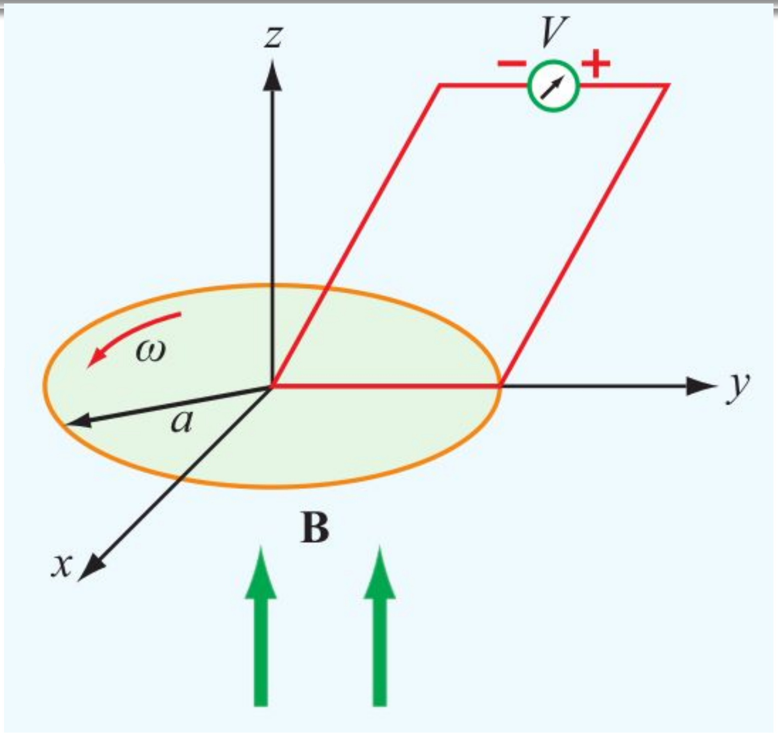
Solution: know:

$$V_{\text{emf}}^{\text{m}} = \oint_C (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$$

6-4 Rotating Disk in Uniform Field

Solution: know:

$$V_{\text{emf}}^{\text{m}} = \oint_C (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$$



The contour *C* is a line from the center to the edge.
so:

$$d\mathbf{l} = \hat{\mathbf{r}}\, dr$$

The velocity varies with r:

$$\mathbf{u} = \hat{\boldsymbol{\phi}}\, \omega r$$

6-4 Rotating Disk in Uniform Field

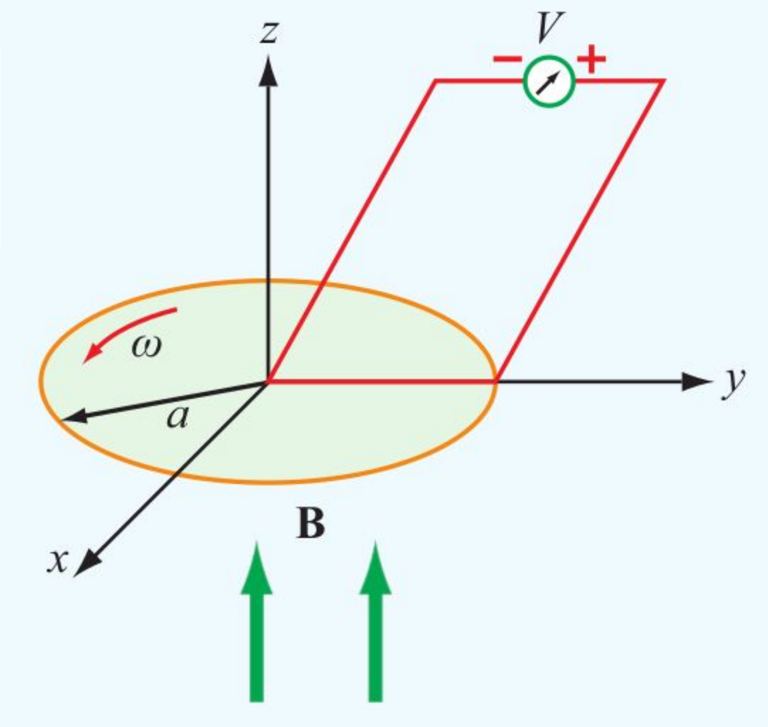
know:

$$V_{\text{emf}}^{\text{m}} = \oint_C (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$$

$$d\mathbf{l} = \hat{\mathbf{r}}\, dr$$

$$\mathbf{u} = \hat{\boldsymbol{\phi}}\, \omega r$$

$$\mathbf{B} = \hat{\mathbf{z}} B_0$$



Plug in:

$$V = \int_0^a [(\hat{\boldsymbol{\phi}}\, \omega r) \times \hat{\mathbf{z}} B_0] \cdot \hat{\mathbf{r}}\, dr$$

6-4 Rotating Disk in Uniform Field

$$V = \int_0^a [(\hat{\boldsymbol{\phi}}\, \omega r) \times \hat{\mathbf{z}} B_0] \cdot \hat{\mathbf{r}}\, dr$$

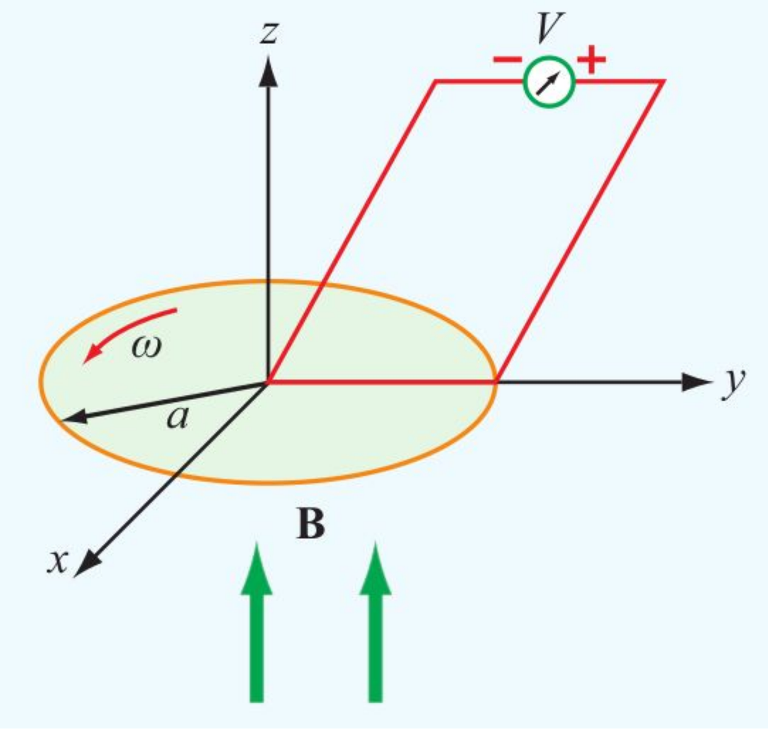
since:

$$\hat{\boldsymbol{\phi}} \times \hat{\mathbf{z}} = \hat{\mathbf{r}}$$

get:

$$V = \omega B_0 \int_0^a r\, dr$$

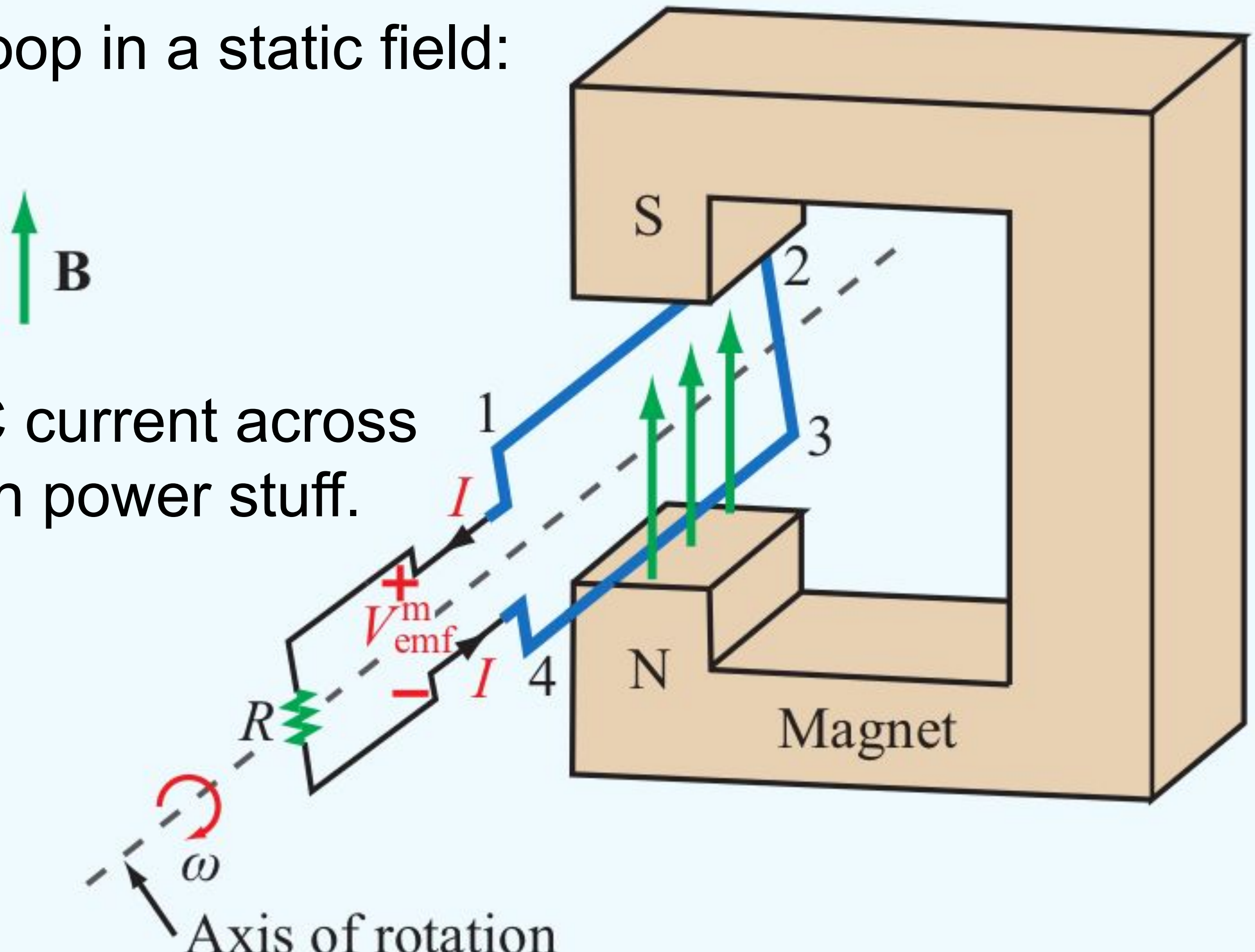
$$V = \frac{\omega B_0 a^2}{2}$$



6-5 Electromagnetic Generator

A rotating loop in a static field:

Induced AC current across the load can power stuff.

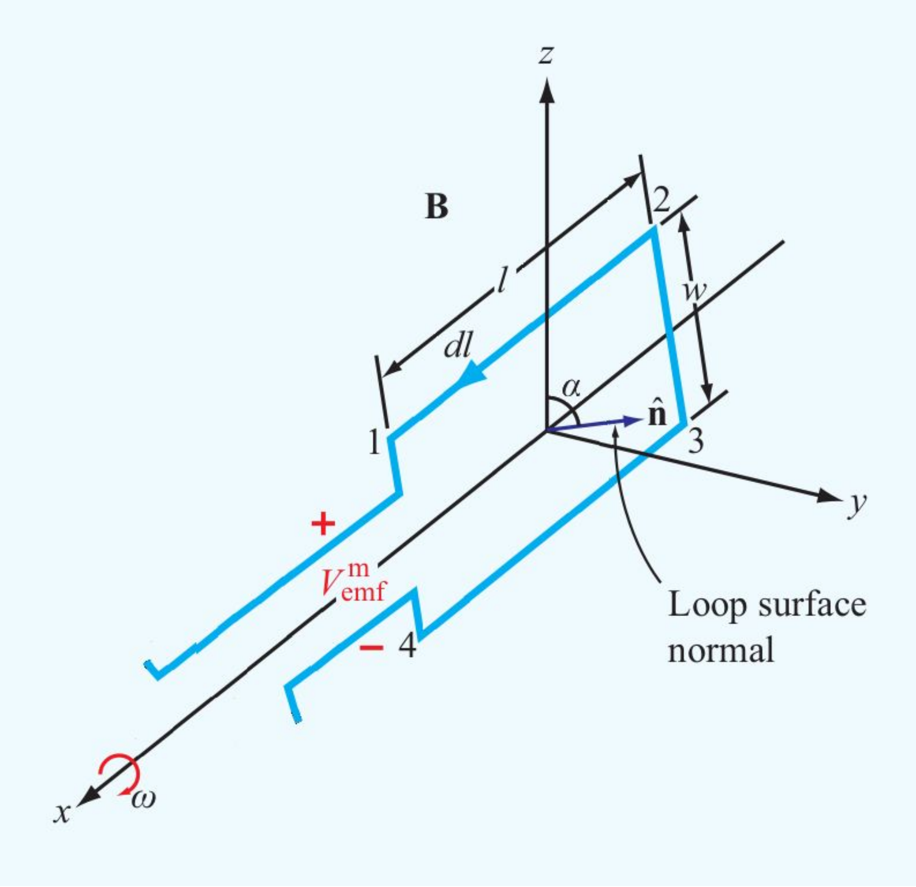


6-5 Electromagnetic Generator

Cartesian coords:
loop axis is the *x*-axis

loop segments 1-2 & 3-4
along *x*-axis
each with length ℓ
spaced *w* apart

Loop is rotated
(somehow) at an
angular rate of ω, in
radians/sec



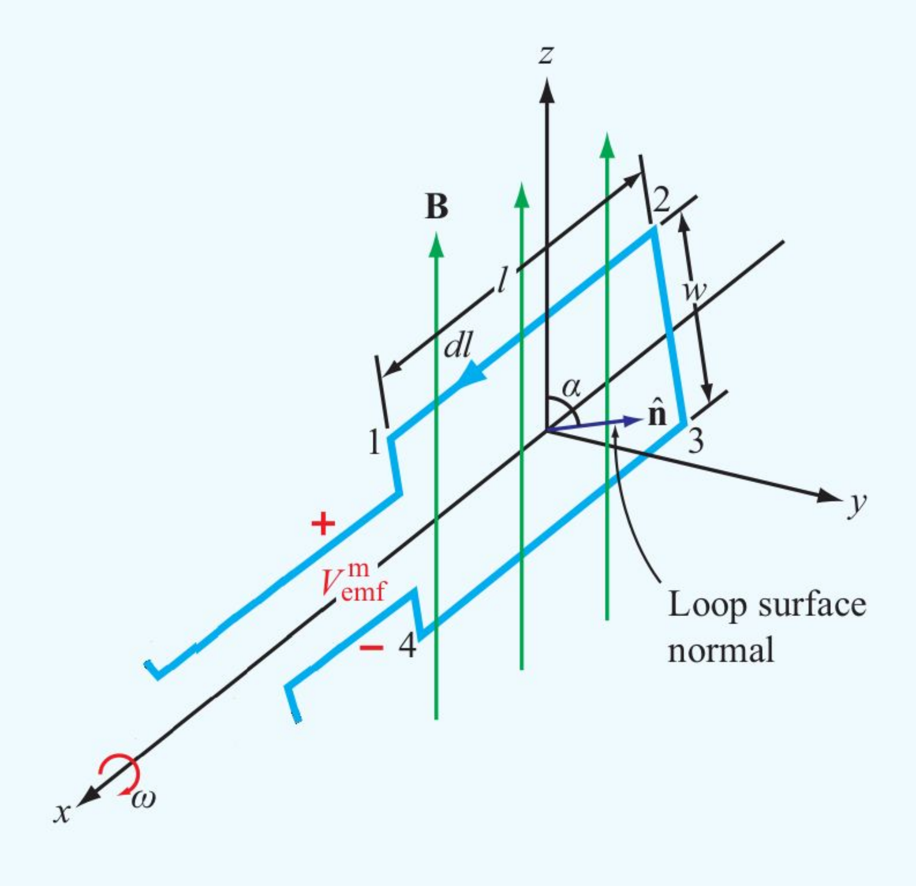
Loop normal makes angle α
with *z*-axis.

6-5 Electromagnetic Generator

Add vertical **B field:**

Only segments 1-2 & 3-4 cross the **B** field:
so only those segments contribute to the V_{EMF}.

$d\alpha = \omega\, dt$
$\omega = d\alpha/dt$



As loop rotates, segment 1-2 moves with velocity **u**:

$$\mathbf{u} = \hat{\mathbf{n}} \frac{d\alpha}{dt} r = \hat{\mathbf{n}} \omega \frac{\mathrm{w}}{2}$$

6-5 Electromagnetic Generator

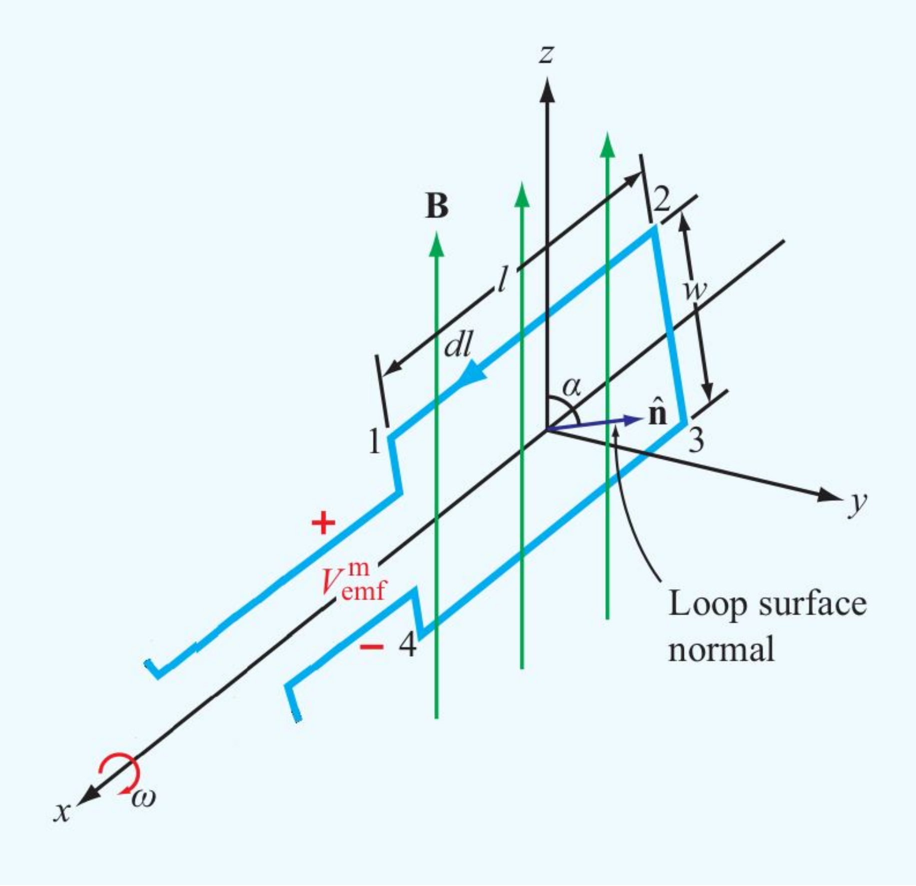
Figure out the loop normal:
Special cases:

$$\alpha = 0 \qquad \hat{\mathbf{n}} = \hat{\mathbf{z}}$$

$$\alpha = 90^\circ \qquad \hat{\mathbf{n}} = \hat{\mathbf{y}}$$

So:

$$\hat{\mathbf{n}} = \hat{\mathbf{z}} \cos\alpha + \hat{\mathbf{y}} \sin\alpha$$



6-5 Electromagnetic Generator

since:

$$\hat{\mathbf{n}} = \hat{\mathbf{z}} \cos\alpha + \hat{\mathbf{y}} \sin\alpha$$

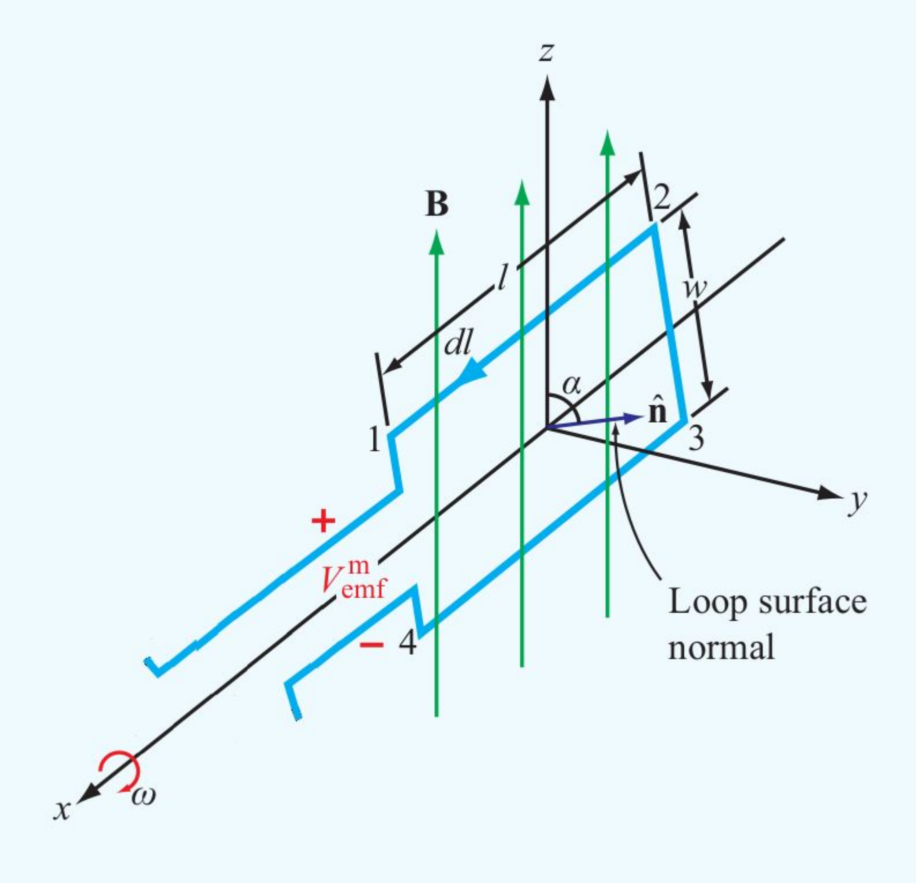
will need:

$$\hat{\mathbf{n}} \times \hat{\mathbf{z}} = \hat{\mathbf{x}} \sin\alpha.$$

Segment 3-4 moves with velocity -**u**

d**l** is along each segment:

$$d\mathbf{l} = \hat{\mathbf{x}} dx$$



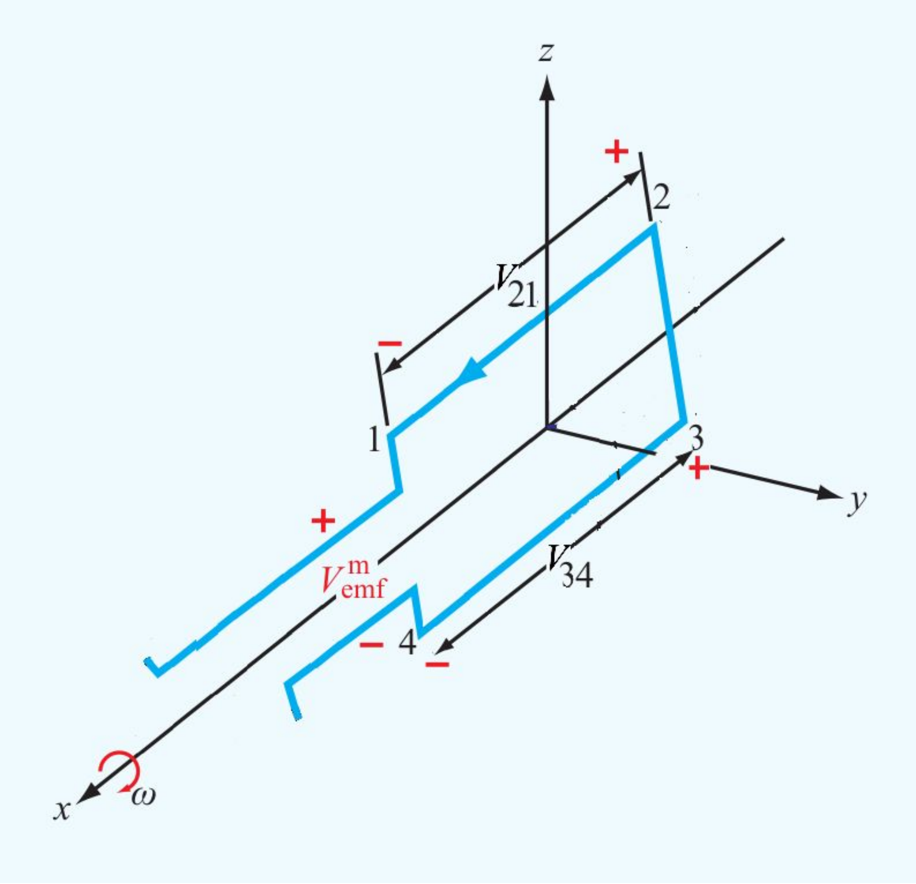
6-5 Electromagnetic Generator

KVL counter-clockwise:

$$V_{EMF} - V_{34} + V_{21} = 0$$

so:

$$V_{EMF} = V_{34} - V_{21}$$

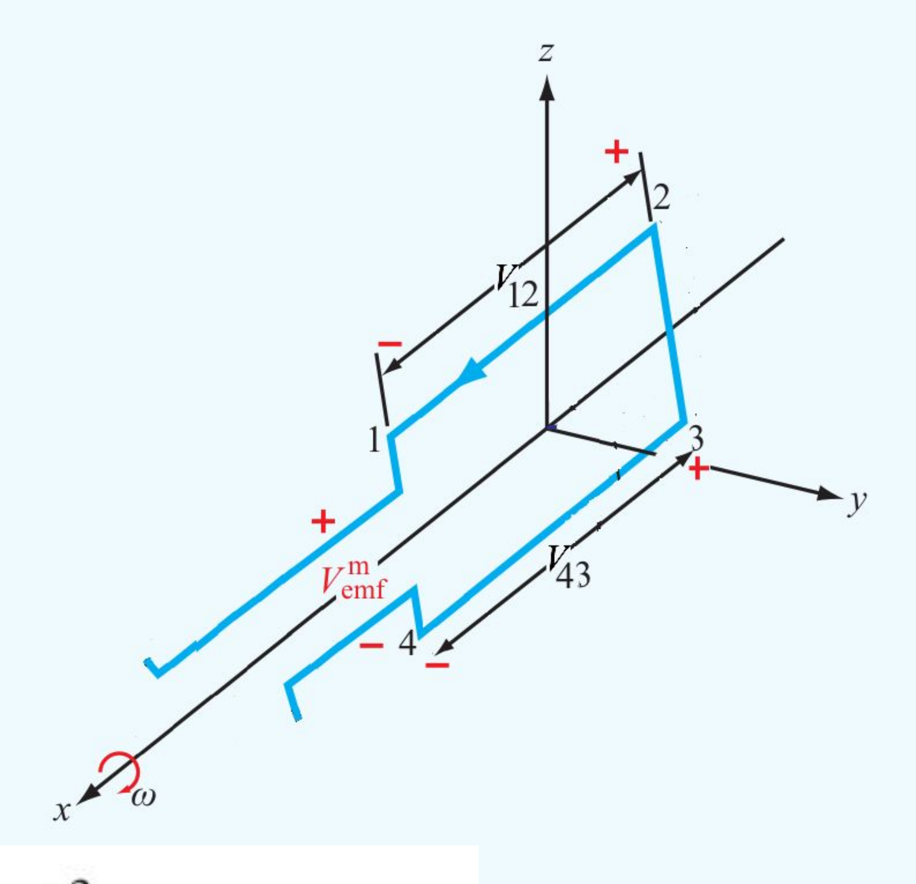


6-5 Electromagnetic Generator

$$V_{21} = \int_1^2 [\mathbf{u} \times \mathbf{B}] \cdot d\mathbf{l}$$

$$-V_{21} = \int_2^1 [\mathbf{u} \times \mathbf{B}] \cdot d\mathbf{l}$$

$$V_{34} = \int_4^3 [\mathbf{u} \times \mathbf{B}] \cdot d\mathbf{l}$$



$$V_{\rm emf}^{\rm m} = \int_2^1 (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l} + \int_4^3 (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$$

6-5 Electromagnetic Generator

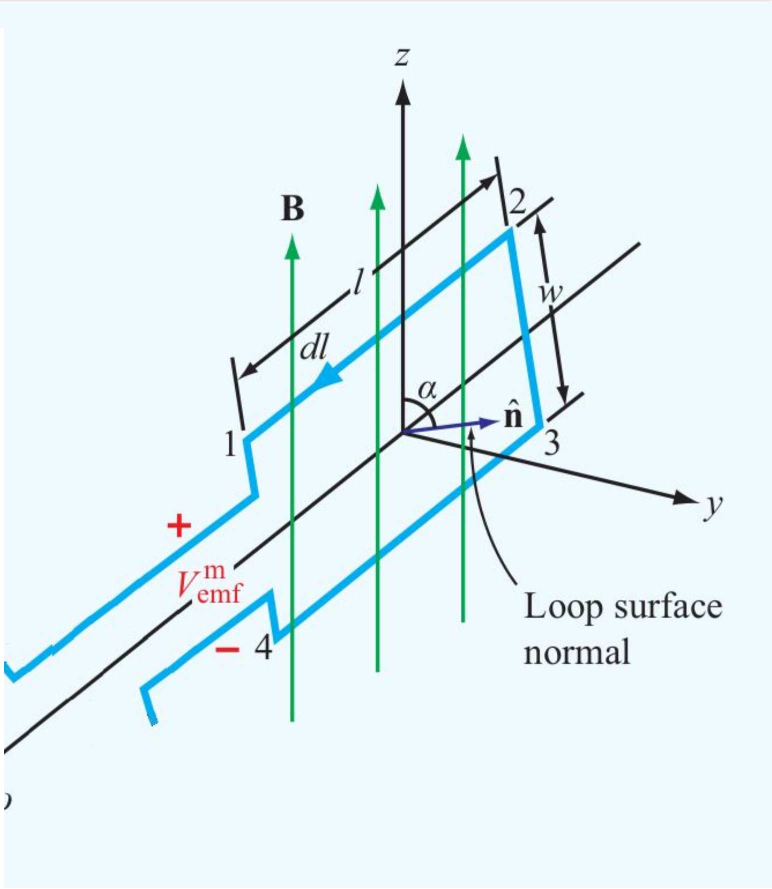
$$V_{EMF} = \int_2^1 [\mathbf{u} \times \mathbf{B}] \cdot d\mathbf{l} + \int_4^3 [\mathbf{u} \times \mathbf{B}] \cdot d\mathbf{l}$$

$$= \int_{-l/2}^{+l/2} \left[\left(+\hat{\mathbf{n}}\omega \frac{\text{w}}{2} \right) \times \hat{\mathbf{z}} B_0 \right] \cdot \hat{\mathbf{x}} dx$$

$$+ \int_{+l/2}^{-l/2} \left[\left(-\hat{\mathbf{n}}\omega \frac{\text{w}}{2} \right) \times \hat{\mathbf{z}} B_0 \right] \cdot \hat{\mathbf{x}} dx$$

$$= \omega \frac{\text{w}}{2} B_0 \left\{ \int_{-l/2}^{+l/2} [\hat{\mathbf{x}} \sin\alpha] \cdot \hat{\mathbf{x}} dx - \int_{+l/2}^{-l/2} [\hat{\mathbf{x}} \sin\alpha] \cdot \hat{\mathbf{x}} dx \right\}$$

$$= \omega \frac{\text{w}}{2} B_0 \{ l \sin\alpha - (-l) \sin\alpha \}$$

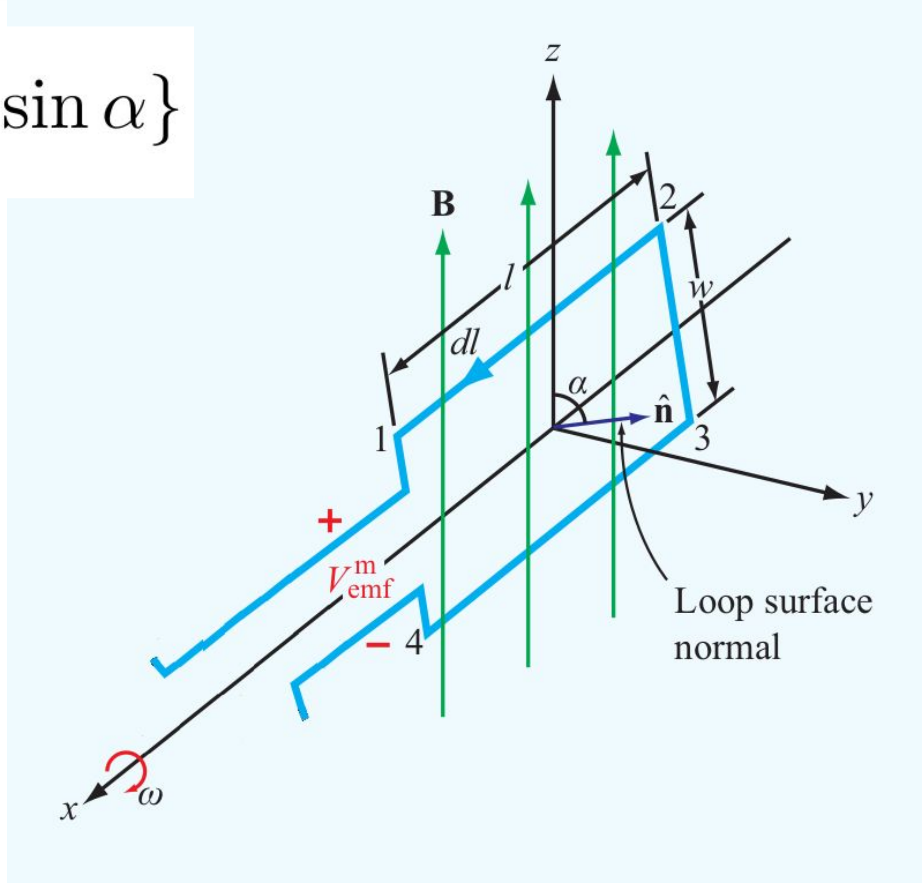


6-5 Electromagnetic Generator

$$V_{EMF} = \omega \frac{\mathrm{w}}{2} B_0 \{l \sin\alpha - (-l)\sin\alpha\}$$

$$V_{EMF} = \mathrm{w} l \omega B_0 \sin\alpha$$

$$V_{EMF} = A\omega B_0 \sin(\omega t)$$

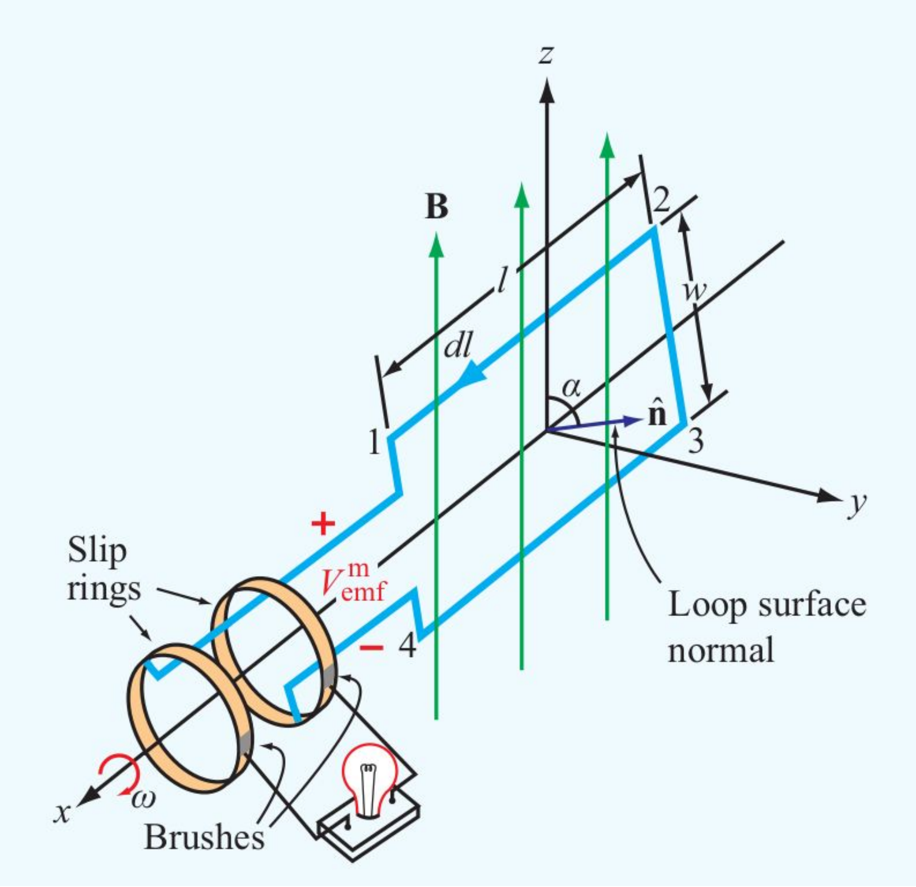


6-5 Electromagnetic Generator

To get current to something stationary requires 2 things:
1. Slip rings
2. Brushes

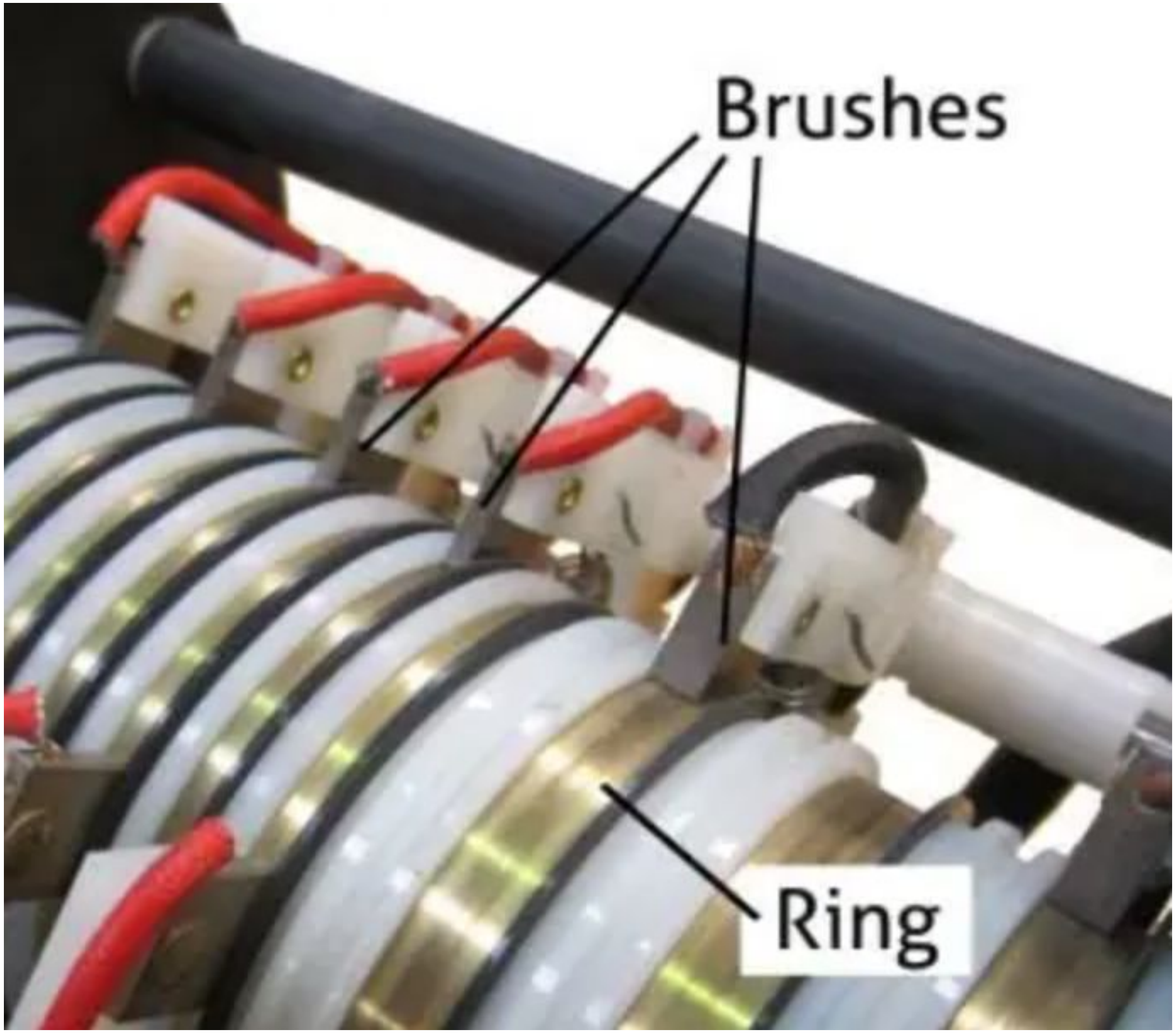
Slip rings are attached to the wires and rotate with them

Brushes are stationary and make sliding contact with the slip rings



Both are conductive

6-5 Electromagnetic Generator

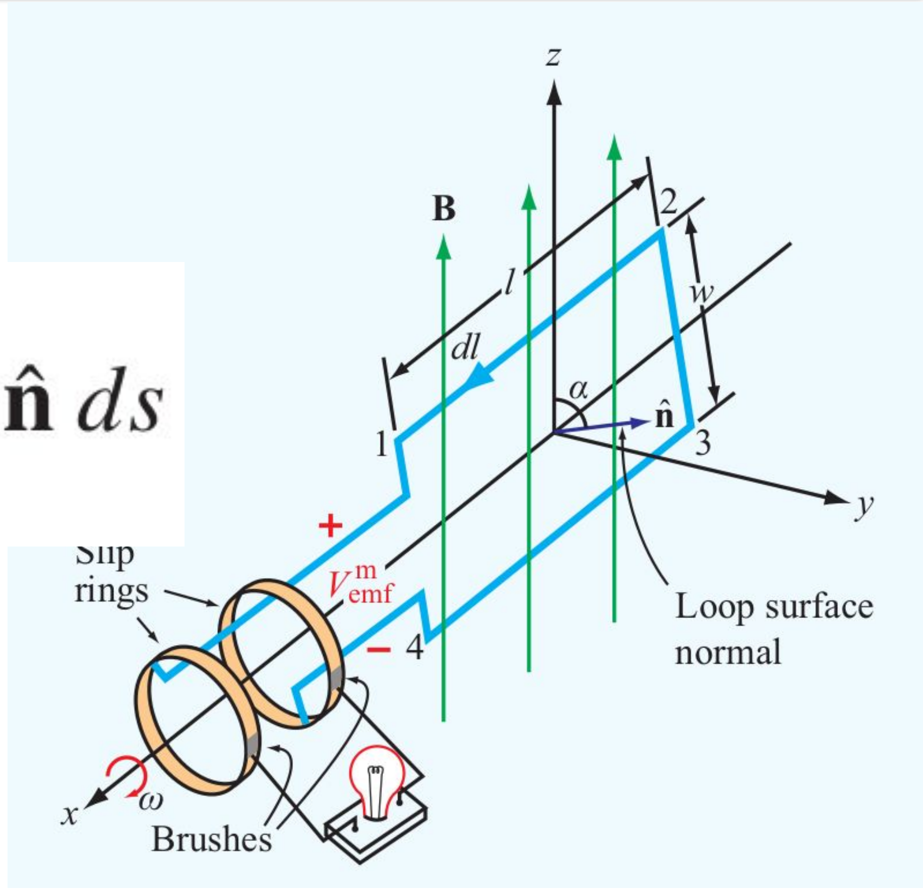


(grandslipring.com)

6-5 Electromagnetic Generator

Alternate Derivation:
use Faraday's Law:

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s} = \int_S \hat{\mathbf{z}} B_0 \cdot \hat{\mathbf{n}}\, ds$$



recall:

$$\hat{\mathbf{n}} = \hat{\mathbf{z}} \cos\alpha + \hat{\mathbf{y}} \sin\alpha$$

$$\Phi = B_0 A \cos\alpha$$

$$= B_0 A \cos(\omega t + C_0)$$

C_0 accounts for an arbitrary phase angle.

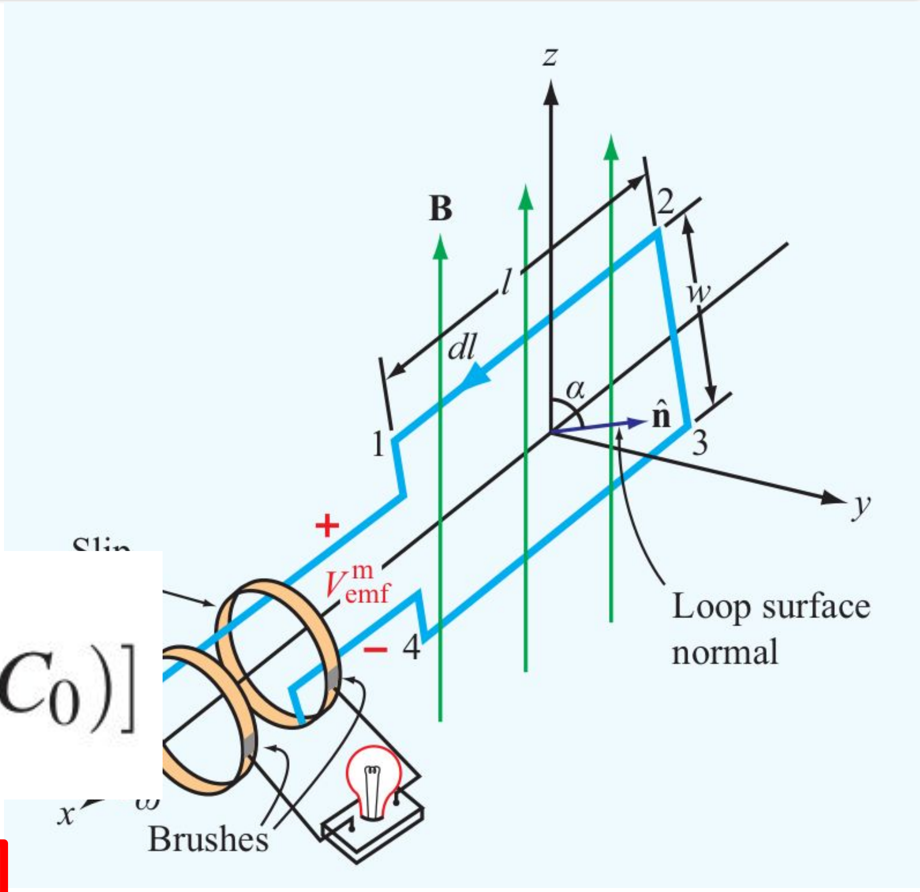
6-5 Electromagnetic Generator

Alternate Derivation:

use Faraday's Law:

$$V_{\text{emf}} = -\frac{d\Phi}{dt}$$

$$= -\frac{d}{dt}[B_0 A \cos(\omega t + C_0)]$$

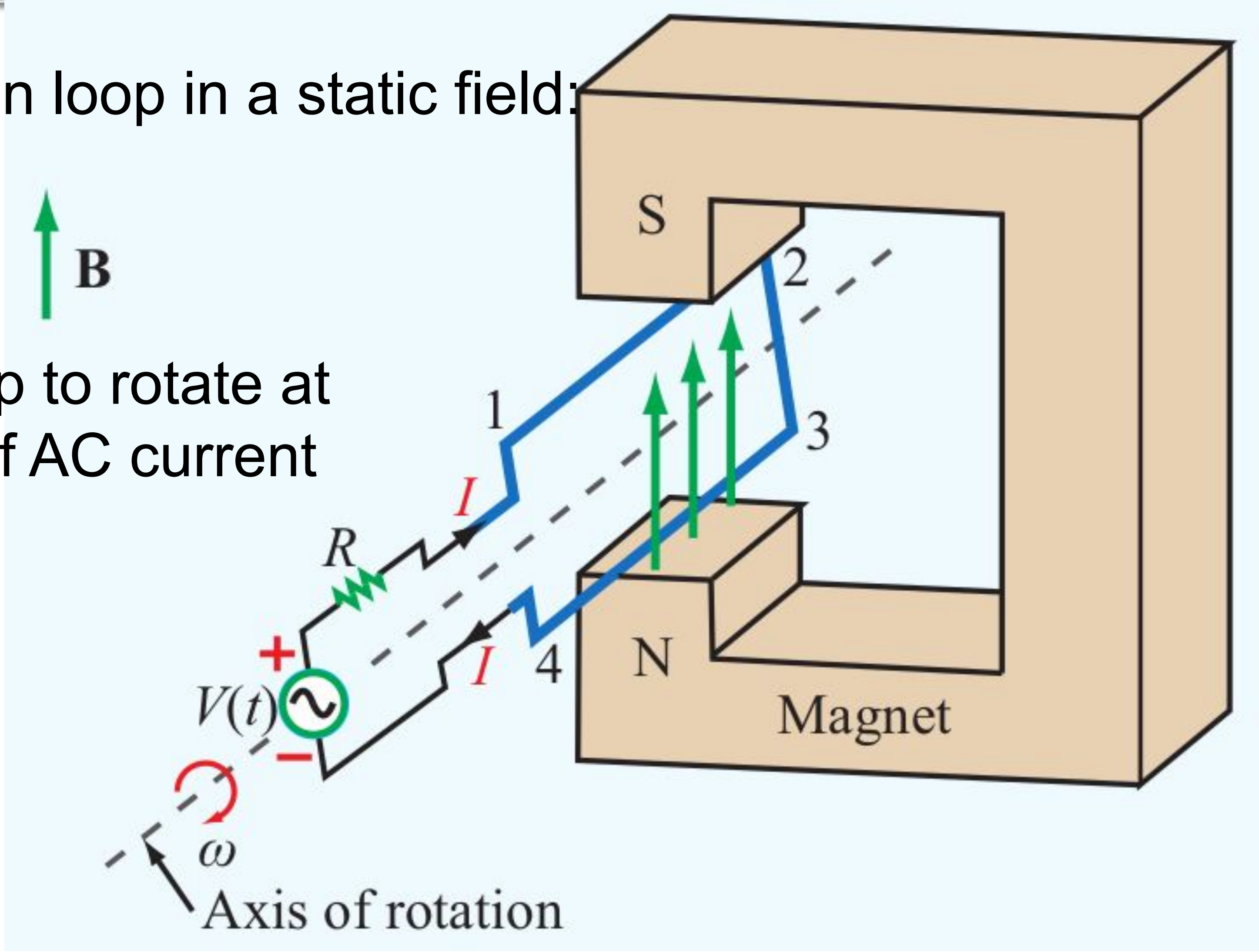


$$V_{\text{emf}} = A\omega B_0 \sin(\omega t + C_0)$$

(same as before)

6-5 AC Electromagnetic Motor

AC current in loop in a static field:



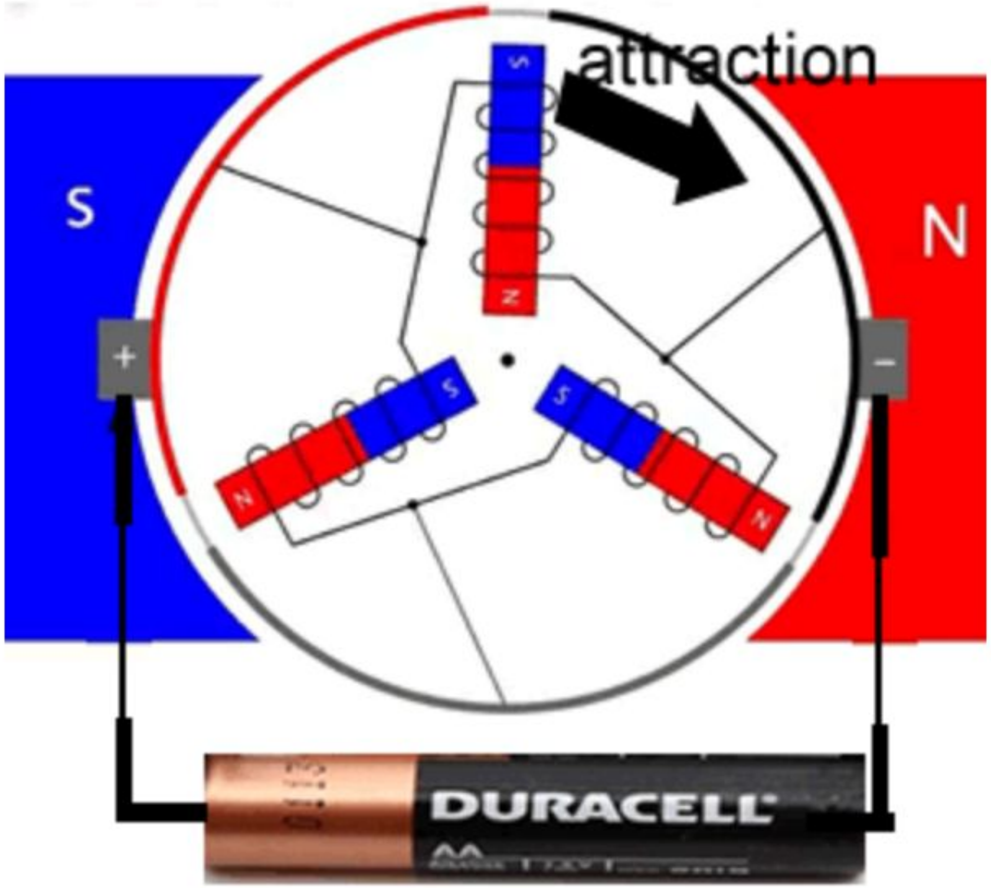
Induces loop to rotate at frequency of AC current

6-5 DC Electromagnetic Motor

Dozens of ways to make this work.
This is one way.

Electromagnets on a spinning shaft.

Surrounded by constant magnetic field.

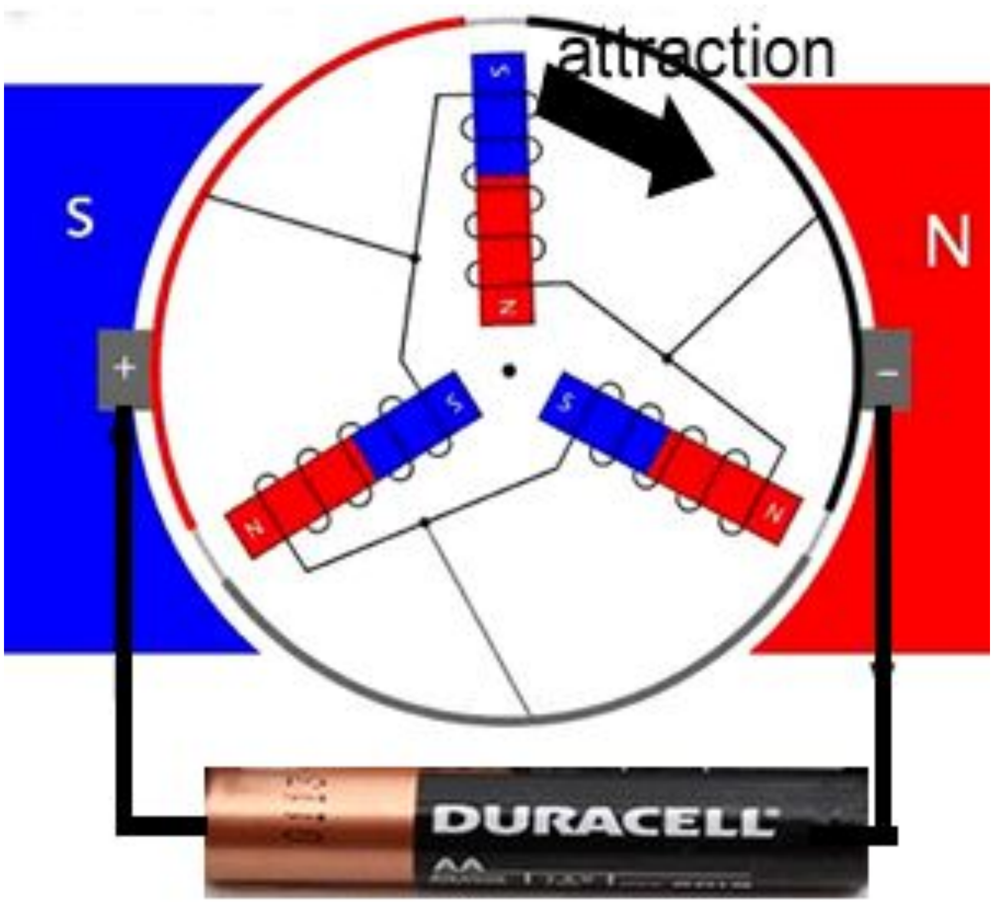


(dynamicscience.com.au)

6-5 DC Electromagnetic Motor

Power to electromagnets is made to depend on the rotation angle.

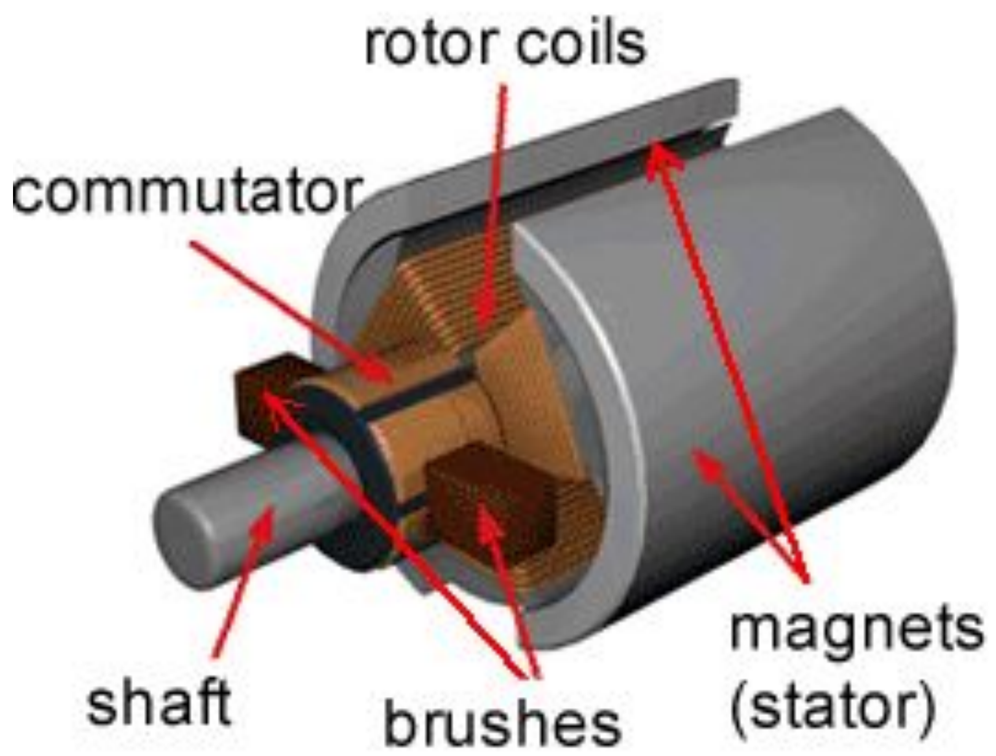
Depends on momentum
to keep it rotating when direction of electromagnets switches.



(dynamicscience.com.au)

6-5 DC Electromagnetic Motor

What it would really look like when built:



(dynamicscience.com.au)

6-5 DC Electromagnetic Motor

Back EMF

Recall the induced voltage on a moving wire in a uniform magnetic field:

$$V_{\text{emf}}^{\text{m}} = -uB_0 l.$$

Depends on: the velocity of the wire
the magnetic field strength
the length of the wire

So, when we apply voltage to the solenoid/electromagnet to make the motor turn, the turning produces a voltage on the wire that opposes this: call the **Back EMF**

6-6 Moving Loop in Time-Varying Field

When **BOTH**: loop is moving *and* field is varying:

$$V_{\mathrm{emf}} = V_{\mathrm{emf}}^{\mathrm{tr}} + V_{\mathrm{emf}}^{\mathrm{m}}$$

$$= -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s} + \oint_C (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}.$$

General expression of Faraday's Law still valid:

$$V_{\mathrm{emf}} = -\frac{d\Phi}{dt} = -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{s}$$

6-6 Generator with Time-Varying Field

With sinusoidal external **B** field:

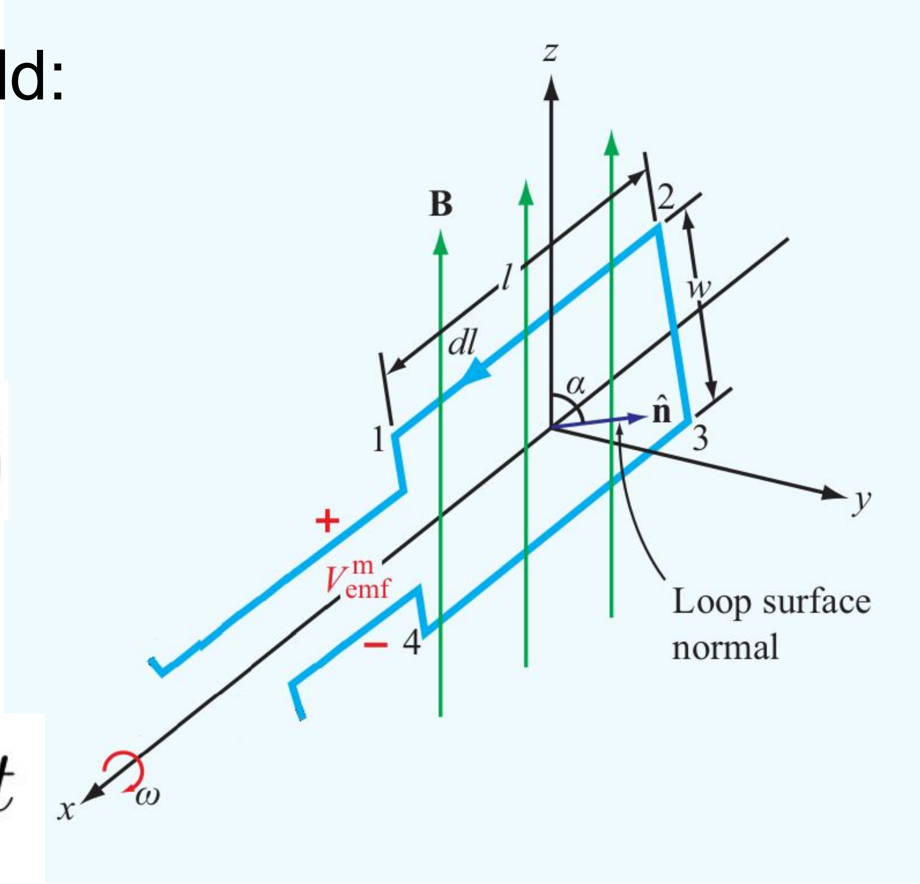
$$\mathbf{B} = \hat{\mathbf{z}} B_0 \cos \omega t$$

For constant **B**, we had:

$$\Phi = B_0 A \cos(\omega t + C_0)$$

plug in the time-varying B:

$$\Phi = (B_0 \cos \omega t) A \cos \omega t$$
$$= B_0 A \cos^2 \omega t$$



6-6 Generator with Time-Varying Field

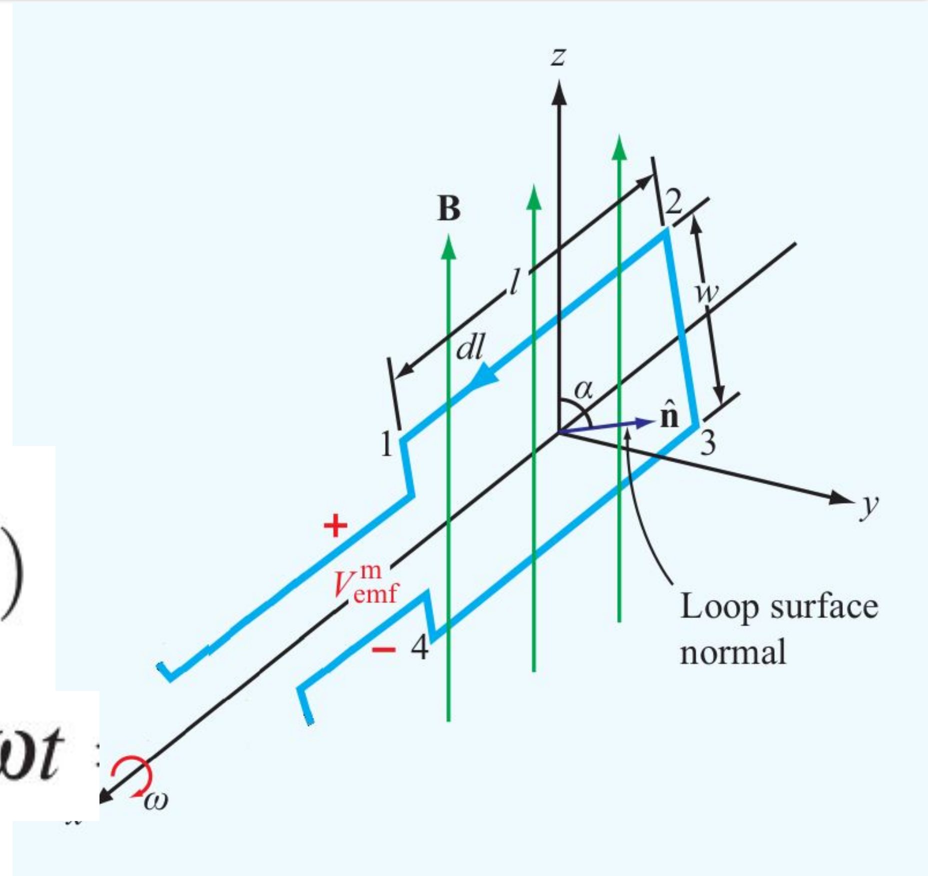
Solve for the EMF:

$$V_{emf} = -\frac{d\Phi}{dt}$$

$$= -\frac{d}{dt}\left(B_0 A \cos^2 \omega t\right)$$

$$= 2B_0 A \omega \cos \omega t \sin \omega t$$

$$V_{emf} = B_0 A \omega \sin 2\omega t$$



Example 1

Given: circular loop:

conducting rubber

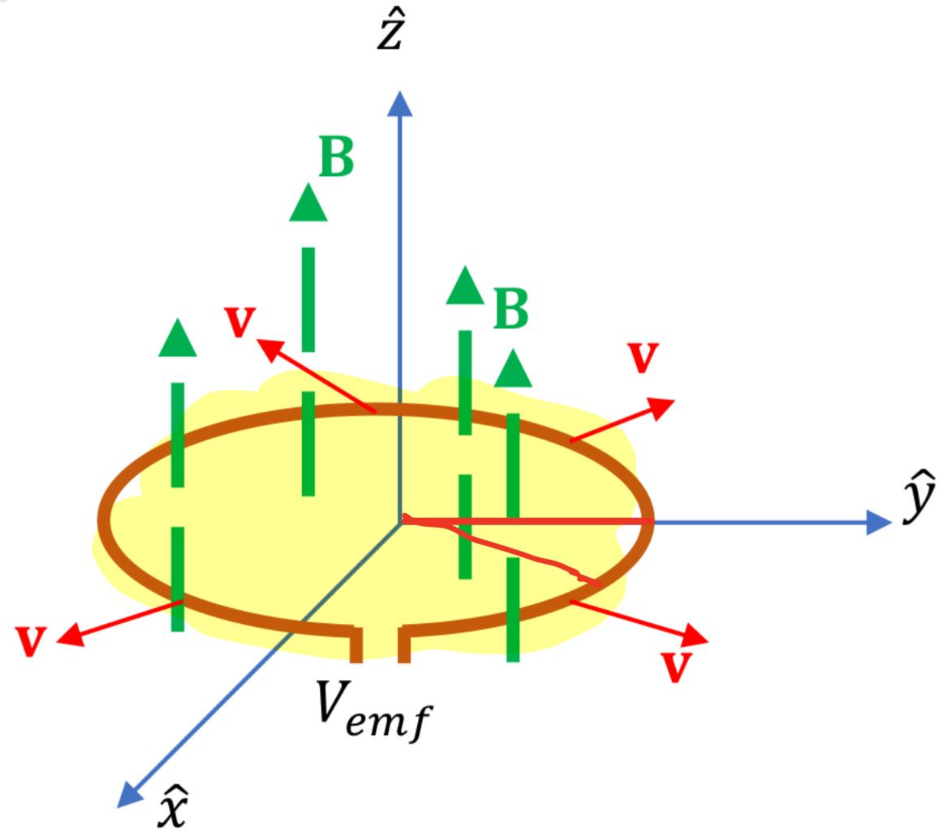
expanding:

$$\mathbf{v} = \hat{\mathbf{r}}\, v_0$$

uniform magnetic field:

$$\mathbf{B} = \hat{\mathbf{z}}\, B_0\, t^2$$

Find: V_{EMF}

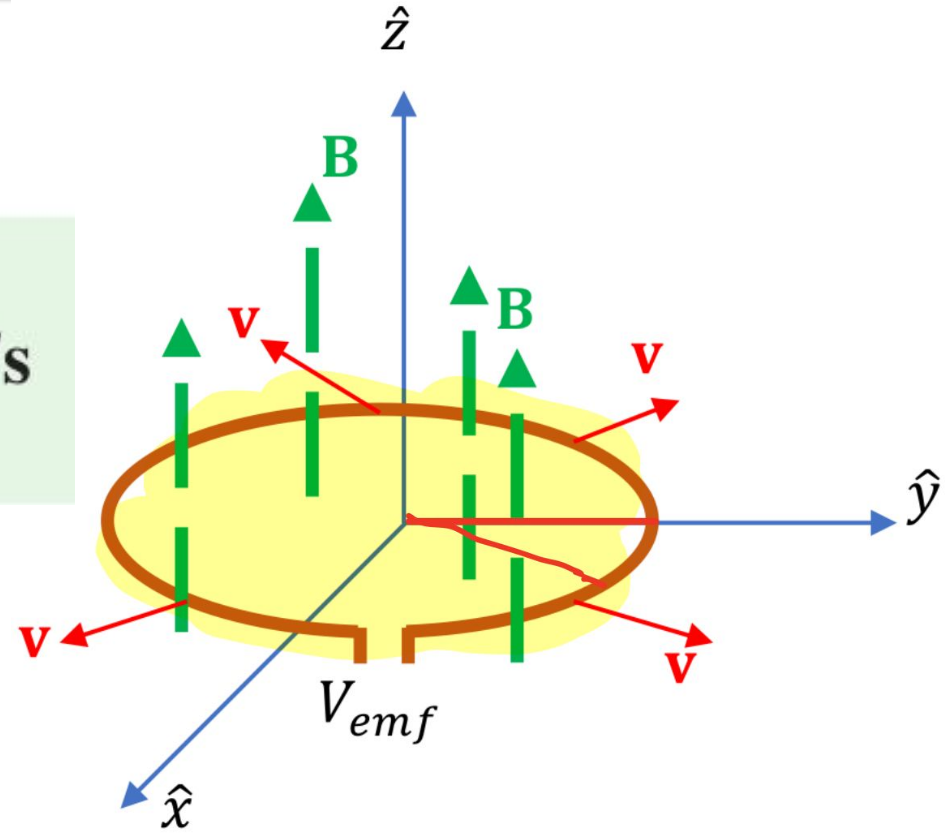


Example 1

Solution:

Use Faraday's Law:

$$V_{\text{emf}} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{s}$$



know:

S is disk bounded by r_{loop}

$r_{loop} = v_0\, t$

$$d\mathbf{s} = \hat{\mathbf{z}}\, r\, dr\, d\phi$$

Example 1

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s}$$

$$= \int_{\phi=0}^{2\pi} \int_{r=0}^{r_{\text{loop}}} [\hat{\mathbf{z}}\, B_0 t^2] \cdot \hat{\mathbf{z}}\, r dr d\phi$$

$$= 2\pi\, B_0 t^2 \int_{r=0}^{r_{\text{loop}}} r dr$$

$$= 2\pi\, B_0 t^2 r_{\text{loop}}^2 / 2$$

Example 1

since $r_{\text{loop}} = v_0 t$:

$$\Phi = \pi B_0 v_0^2 t^4$$

$$V_{EMF} = -\frac{d\Phi}{dt} = -\pi B_0 v_0^2 4 t^3$$

$$\boxed{V_{EMF} = -4\pi B_0 v_0^2 t^3}$$

Homework

Homework 23 is due tomorrow at midnight.

submit to gradescope via the canvas site.

Next Time

Sections 6-7 through 6-9:

Displacement Current

Electromagnetic Boundary Conditions

Charge-Current Continuity Relation