EECS 230
ENGINEERING ELECTROMAGNETICS
Leland Pierce

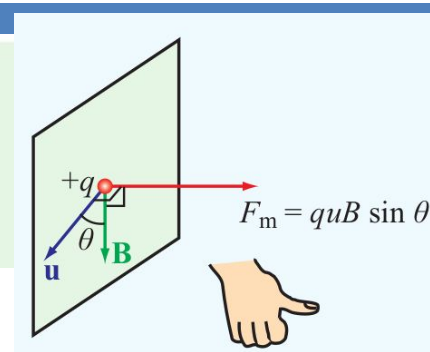
Magnetostatics 3

University of Michigan - Fall 2024

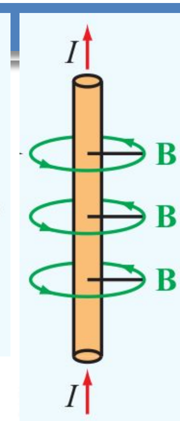
Chapter 5 Overview

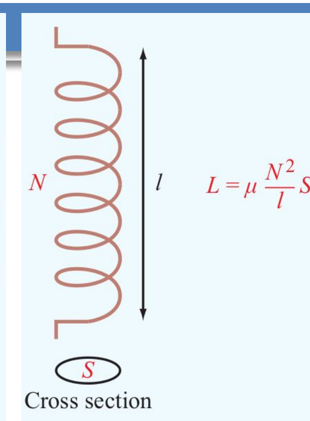
- Maxwell's Equations
  - Magnetostatics
- Magnetic Force
- Magnetic Torque
- Magnetic field from currents
- Gauss's Law for Magnetism
- Ampere's Law
- Magnetic Vector Potential Fi
  - Poisson's eqn
  - Magnetic Flux
- Magnetic Permeability
  - Hysteresis
- Magnetic Boundary Conditions
- Inductance
- Magnetic Energy

$$\nabla \cdot \mathbf{B} = 0,$$
$$\nabla \times \mathbf{H} = \mathbf{J},$$



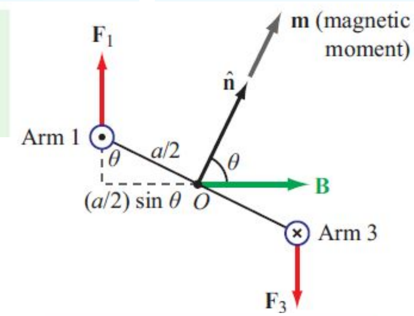
$$\mathbf{F}_m = q\mathbf{u} \times \mathbf{B}$$





$$\nabla \cdot \mathbf{B} = 0 \quad \leftrightarrow \quad \oint_S \mathbf{B} \cdot d\mathbf{s} = 0.$$

$$\nabla \times \mathbf{H} = \mathbf{J} \quad \leftrightarrow \quad \oint \mathbf{H} \cdot d\boldsymbol{\ell} = I,$$

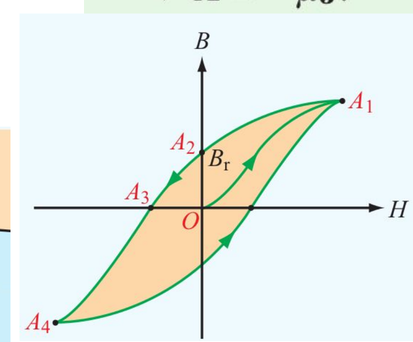


$$\mathbf{B} = \nabla \times \mathbf{A} \qquad (\mathrm{Wb/m^2}),$$

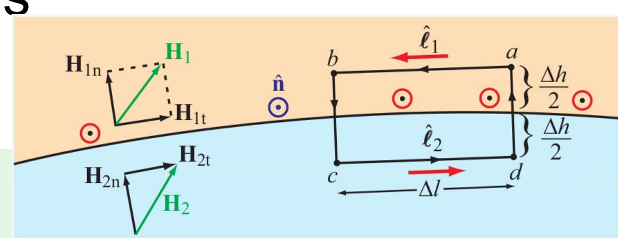
$$\nabla^2 \mathbf{A} = -\mu \mathbf{J}.$$

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s} \qquad (\mathrm{Wb}).$$

$$\mathbf{B} = \mu \mathbf{H},$$



$$w_m = \frac{W_m}{v} = \frac{1}{2}\mu H^2 \qquad (\mathrm{J/m^3}).$$



Lecture Coverage

Today's lecture:

Review of Sections 5-1 through 5-4 of the book:

5-1: Magnetostatics: Magnetic Forces and Torques
5-2: **H** due to a steady current (Biot-Savart Law)
5-3: Magnetic Field from Currents: Ampere's Law
5-4: Magnetic Vector Potential Field
Poisson's eqn, Magnetic Flux

Sections 5-5 through 5-6 of the book:

5-5: Magnetic Permeability
Hysteresis
5-6: Magnetic Boundary Conditions

Chapter 5 Review

Static Conditions:

$$\frac{\partial \mathbf{J}}{\partial t} = 0$$

Magnetostatics:

$$\nabla \cdot \mathbf{B} = 0,$$

$$\nabla \times \mathbf{H} = \mathbf{J}.$$

magnetic flux density **B**

magnetic field intensity **H**

$$\mathbf{B} = \mu \mathbf{H}.$$

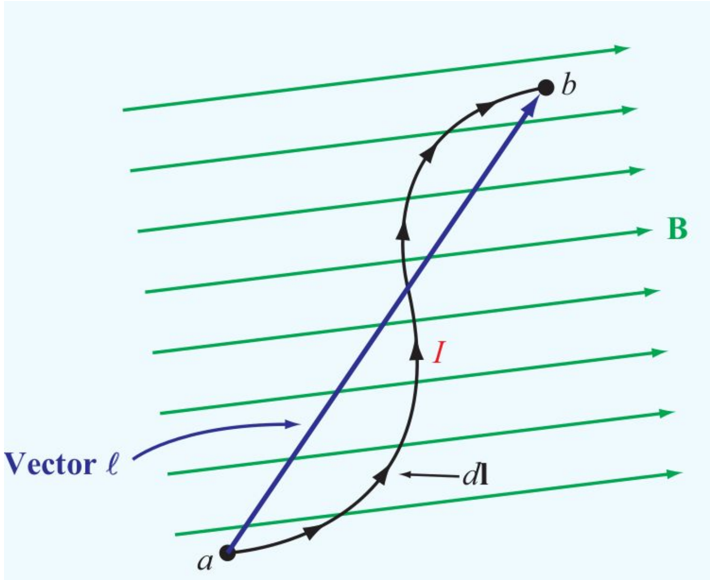
J is the current density

Chapter 5 Review

Magnetic force $\mathbf{F}_m = q\mathbf{u} \times \mathbf{B}$ (N)

Force on a current in a wire:
If **part** of wire is in uniform **B**:

$$\mathbf{F}_m = I\left(\int_\ell d\mathbf{l}\right) \times \mathbf{B} = I\boldsymbol{\ell} \times \mathbf{B},$$



Chapter 5 Review

Torque for a loop with N turns, and surface normal $\hat{\mathbf{n}}$ at angle θ relative to B direction:

$$T = NIAB_0 \sin\theta.$$

magnetic moment of the loop:

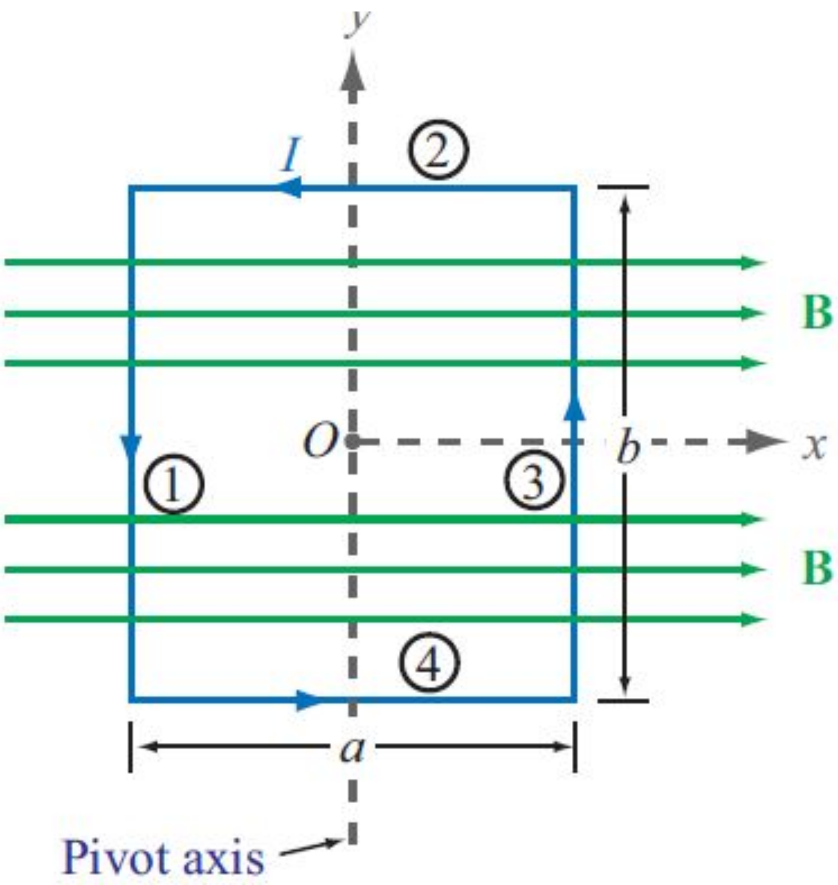
$$m = N\,I\,A$$

define:

$$\mathbf{m} = \hat{\mathbf{n}} NIA = \hat{\mathbf{n}} m \qquad (\text{A}\cdot\text{m}^2),$$

so:

$$\mathbf{T} = \mathbf{m} \times \mathbf{B} \qquad (\text{N}\cdot\text{m}).$$

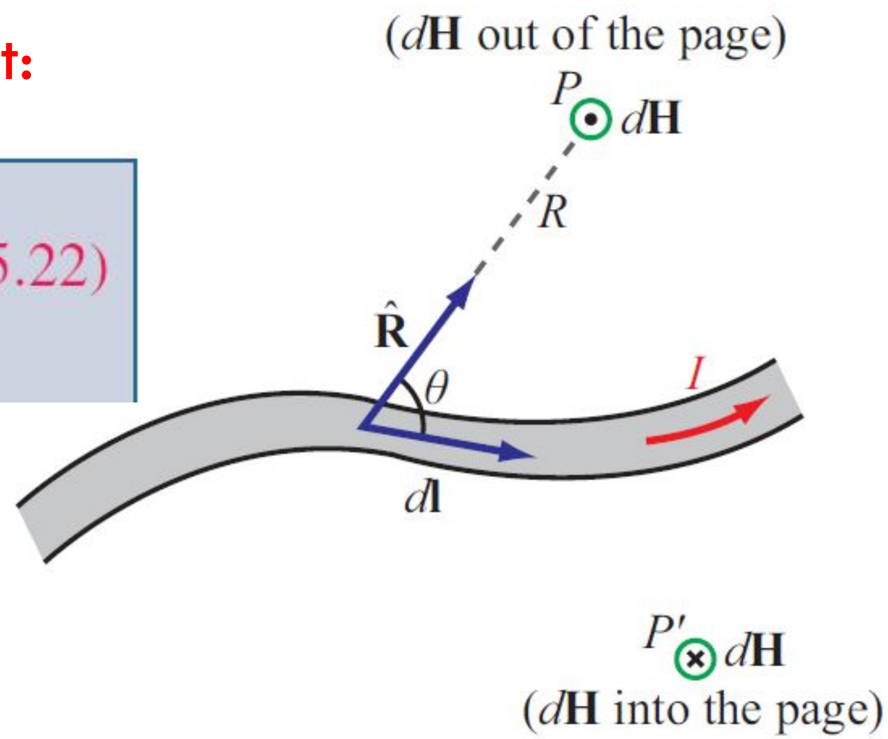


Chapter 5 Review

Biot-Savart Law:

Magnetic field induced by a current:

$$\mathbf{H} = \frac{I}{4\pi} \int_l \frac{d\mathbf{l} \times \hat{\mathbf{R}}}{R^2} \qquad \text{(A/m)}, \qquad (5.22)$$



Chapter 5 Review

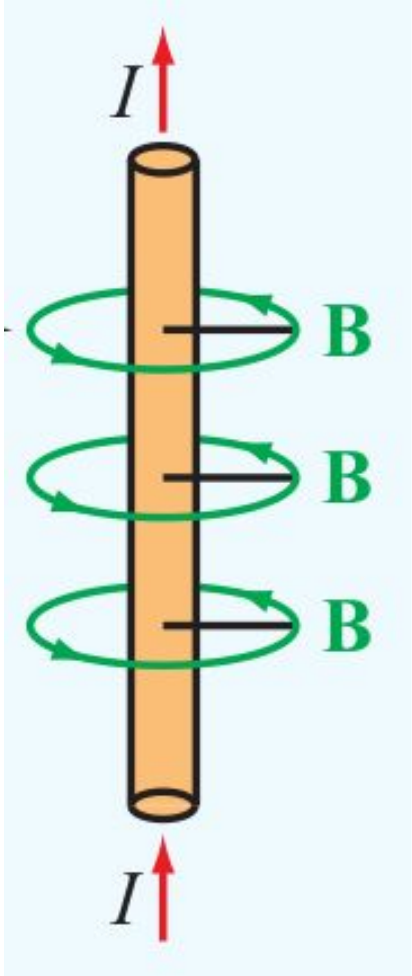
B due to current in straight wire:

$$\mathbf{B} = \mu_0 \mathbf{H} = \hat{\boldsymbol{\phi}} \frac{\mu_0 I l}{2\pi r \sqrt{4r^2 + l^2}}$$

For a very long wire:
In the limit as $l \to \infty$

$4r^2 + l^2 \to l^2$

$$\mathbf{B} = \hat{\boldsymbol{\phi}} \frac{\mu_0 I}{2\pi r}$$



Chapter 5 Review

B along *z*-axis, due to circular current loop:

Since the magnetic moment of a loop in the *x*-*y* plane is:

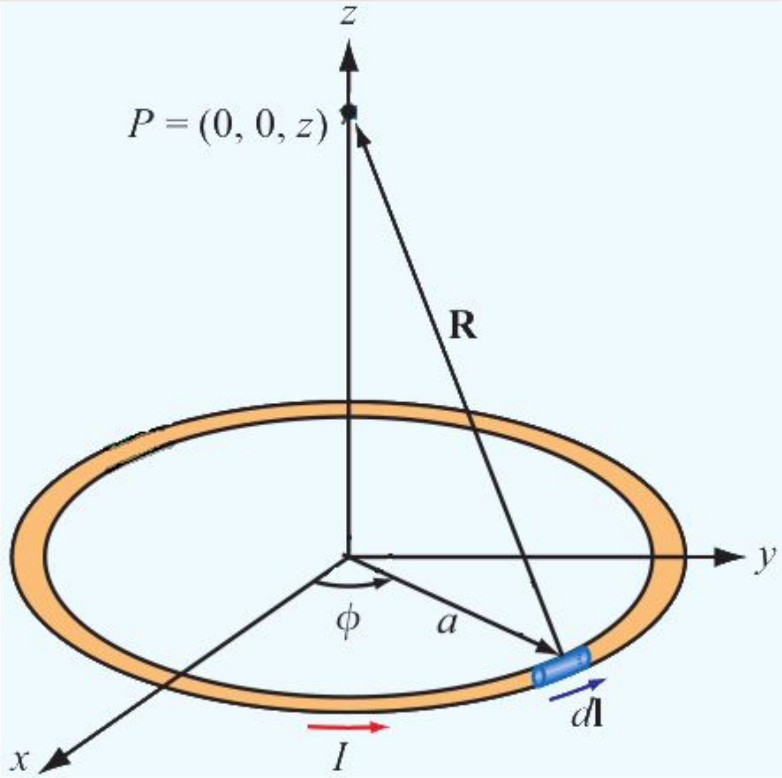
$$\mathbf{m} = \hat{\mathbf{z}} I A$$

the magnetic moment of this loop is:

$$\mathbf{m} = \hat{\mathbf{z}} I \pi a^2$$

so:

$$\mathbf{H} = \frac{I a^2}{2R^3} \hat{\mathbf{z}} = \frac{\mathbf{m}}{2\pi R^3} \hat{\mathbf{z}}$$

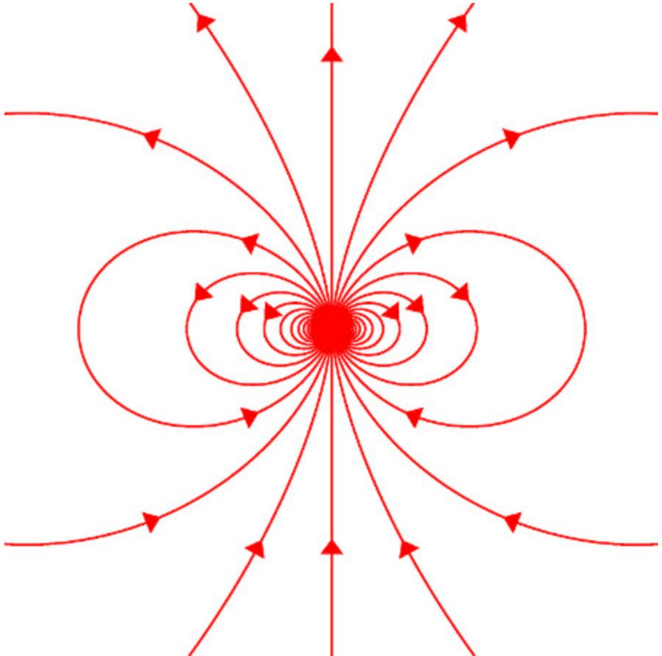


Chapter 5 Review

Magnetic Dipole:

Solving for the fields everywhere far from a current loop results in:

$$\mathbf{H} = \frac{m}{4\pi R^3}\left(\hat{\mathbf{R}}\,2\cos\theta + \hat{\boldsymbol{\theta}}\sin\theta\right) \qquad (\text{for } R \gg a).$$



(physics.stackexchange.com)

Chapter 5 Review

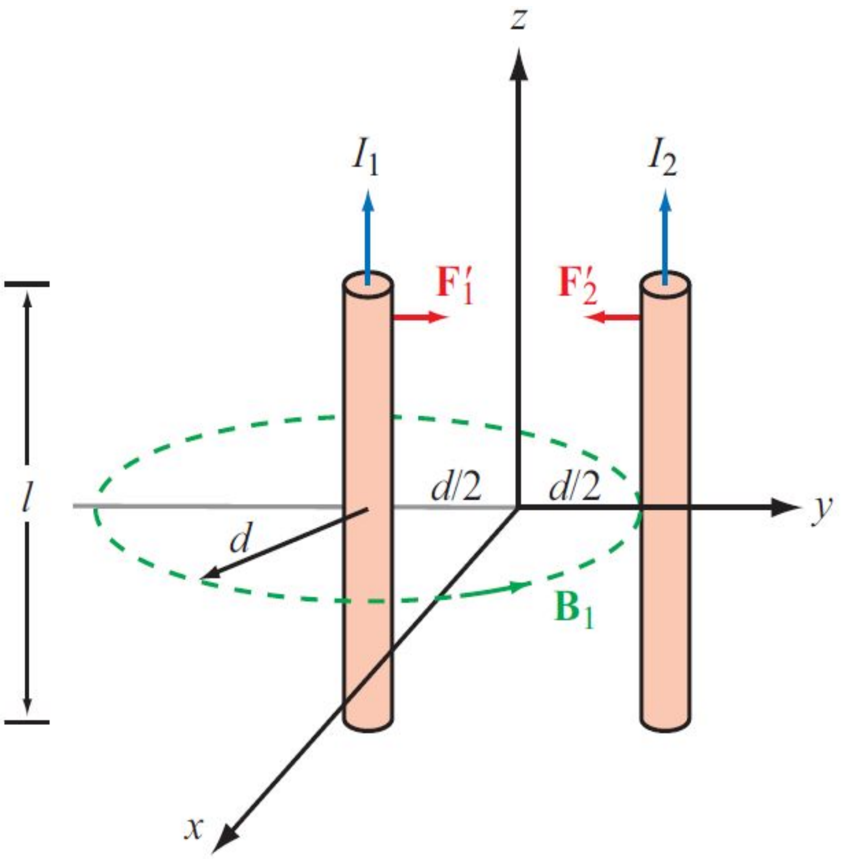
Force due to Parallel Currents:

So the force per unit length fro wire1 on wire2 is:

$$\mathbf{F}'_2 = -\hat{\mathbf{y}}\,\frac{\mu_0 I_1 I_2}{2\pi d}$$

similarly, from wire2 on wire1:

$$\mathbf{F}'_1 = \hat{\mathbf{y}}\,\frac{\mu_0 I_1 I_2}{2\pi d}$$



This force pulls the two wires together.

Chapter 5 Review

Ampère's Law:

In Electrostatics: $\nabla \times \mathbf{E} = 0 \quad \leftrightarrow \quad \oint_C \mathbf{E} \cdot d\boldsymbol{\ell} = 0.$

In Magnetostatics: $\nabla \times \mathbf{H} = \mathbf{J} \quad \leftrightarrow \quad \oint_C \mathbf{H} \cdot d\boldsymbol{\ell} = I,$

I is the current **crossing** the surface of the contour *C*

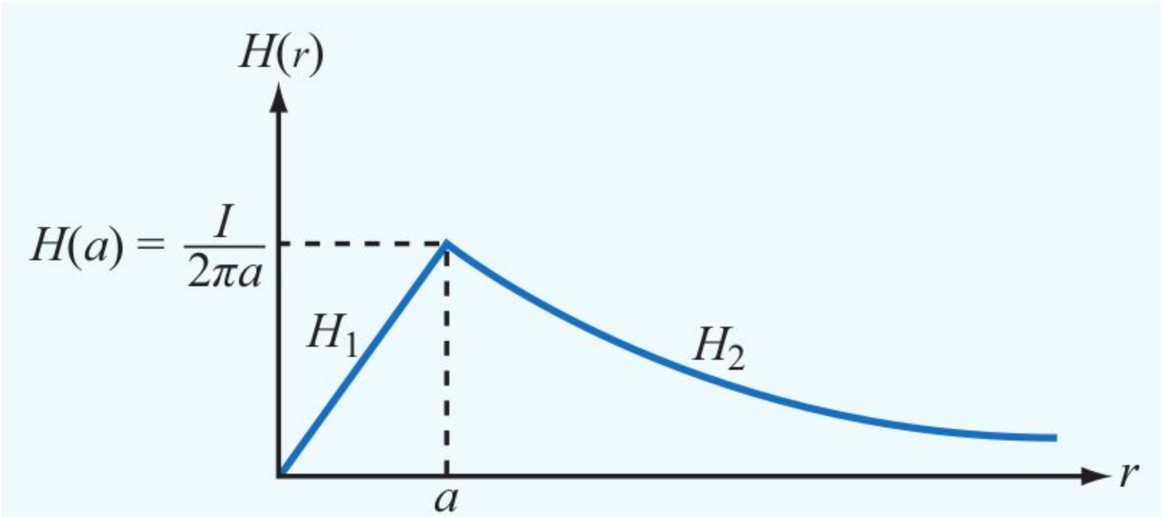
Using the right-hand rule, with the thumb pointing along the direction of C:
Current is positive if points along the direction of the fingers.

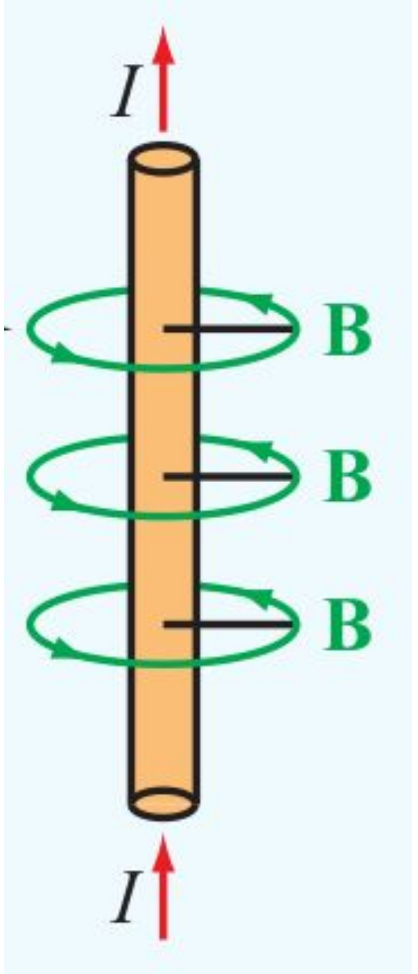
Chapter 5 Review

Magnetic Field of current in long straight wire

Complete solution:

$$\mathbf{H}(r) = \begin{cases} \hat{\boldsymbol{\phi}}\, \dfrac{r}{2\pi a^2} I & 0 < r \le a \\ \hat{\boldsymbol{\phi}}\, \dfrac{1}{2\pi r} I & a \le r \end{cases}$$

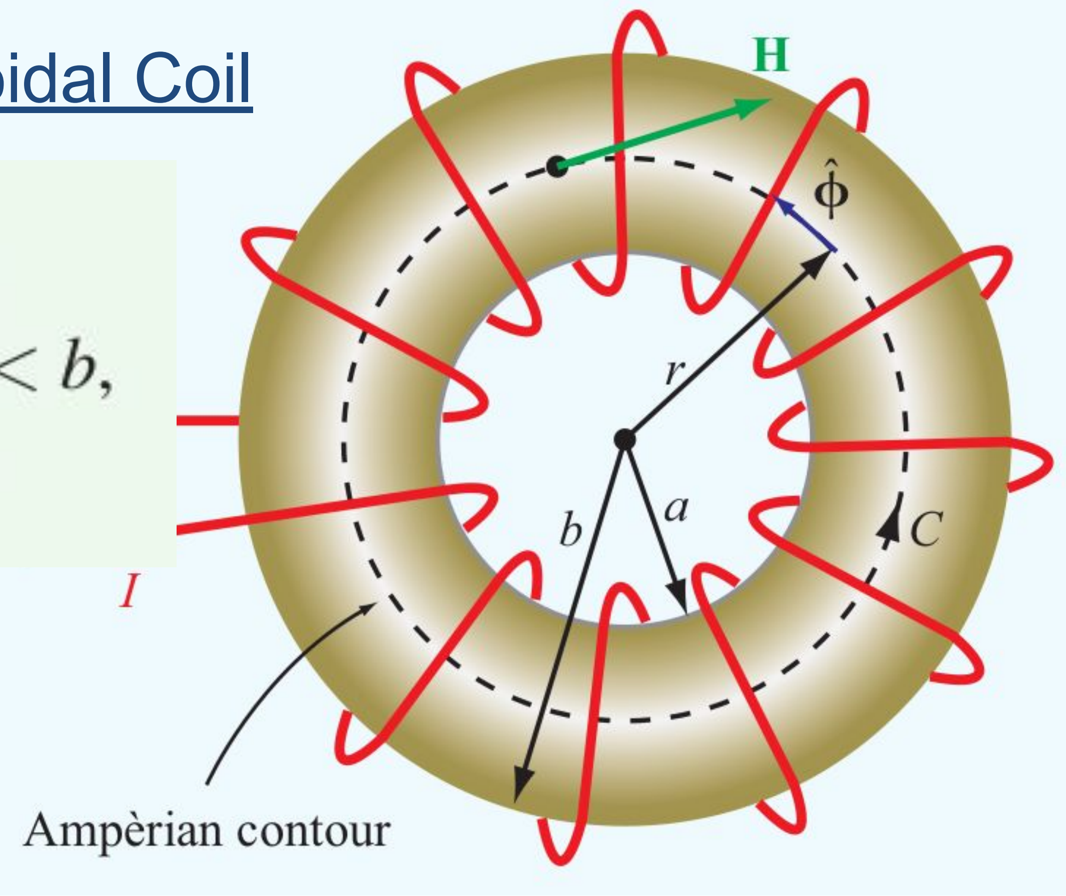




Chapter 5 Review

Magnetic Field inside Toroidal Coil

$$\mathbf{H} = \begin{cases} 0 & \text{for } r < a, \\ -\hat{\boldsymbol{\phi}}\, \dfrac{NI}{2\pi r} & \text{for } a < r < b, \\ 0 & \text{for } r > b. \end{cases}$$



Chapter 5 Review

Magnetic Vector Potential

$$\mathbf{B} = \nabla \times \mathbf{A}$$

Vector Poisson's Equation:

$$\nabla^2 \mathbf{A} = -\mu \mathbf{J}$$

$$\mathbf{A} = \frac{\mu}{4\pi} \int_{\upsilon'} \frac{\mathbf{J}}{R'} \, d\upsilon'$$

Chapter 5 Review

Total magnetic flux density passing through surface S.

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s} \qquad \text{(Wb)}$$

Using the vector magnetic potential, and Stokes' thrm:

$$\Phi = \int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{s} = \oint_C \mathbf{A} \cdot d\mathbf{l}$$

This is related to the labs on Wireless Power Transfer

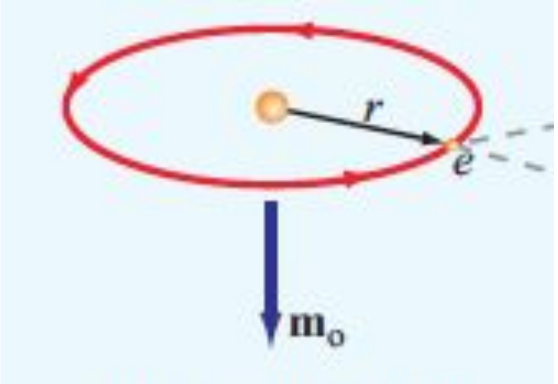
5-5 Magnetic Properties of Materials

"Classical" view of Magnetic moments in atoms:

Electron Orbital magnetic moment

one period: $T = 2\pi r/u$

current: $I = -e/T = -eu/(2\pi r)$



(a) Orbiting electron

magnetic moment:

$$m_o = IA = \left(-\frac{eu}{2\pi r}\right)(\pi r^2) = -\frac{eur}{2} = -\left(\frac{e}{2m_e}\right)L_e$$

where: $L_e = m_e u r$ is the electron's angular momentum

5-5 Magnetic Properties of Materials

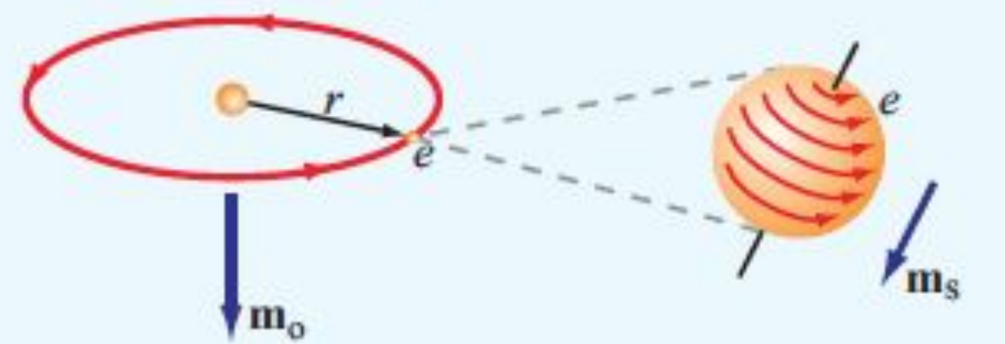
"Quantum" view of Magnetic moments in atoms:

Electron Orbital magnetic moment

L_e is quantized

Minimum L_e is therefore:

$$m_o = -\frac{e\hbar}{2m_e}$$



(a) Orbiting electron (b) Spinning electron

Electron Spin magnetic moment

$$m_s = -\frac{e\hbar}{2m_e}$$

5-5 Magnetic Permeability

Magnetization:

Because of this, for every material, the atoms and molecules can be magnetized, hence:

$$\mathbf{B} = \mu_0 \mathbf{H} + \mu_0 \mathbf{M} = \mu_0 (\mathbf{H} + \mathbf{M})$$

where **M** is the Magnetization Vector, due to the magnetic moments of the material.

For many materials: $\mathbf{M} = \chi_m \mathbf{H}$

and so: $\mathbf{B} = \mu_0 (\mathbf{H} + \chi_m \mathbf{H}) = \mu_0 (1 + \chi_m) \mathbf{H},$

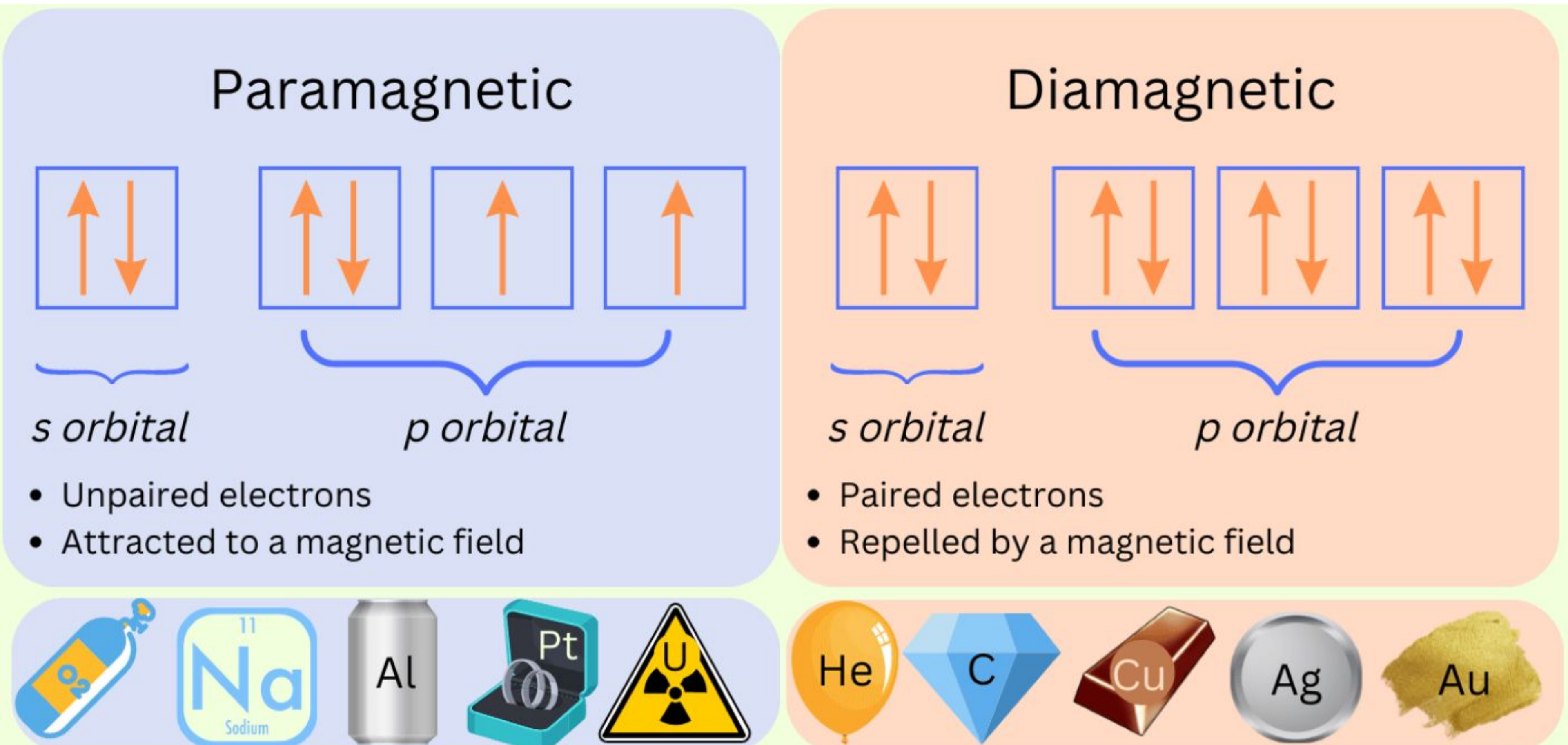
$$\mathbf{B} = \mu \mathbf{H},$$

5-5 Magnetic Permeability

Classes of Magnetization:

χ_m >0, linear

χ_m <0, linear



5-5 Magnetic Permeability

Classes of Magnetization:

Ferromagnetic: $\chi_m > 0$, Nonlinear

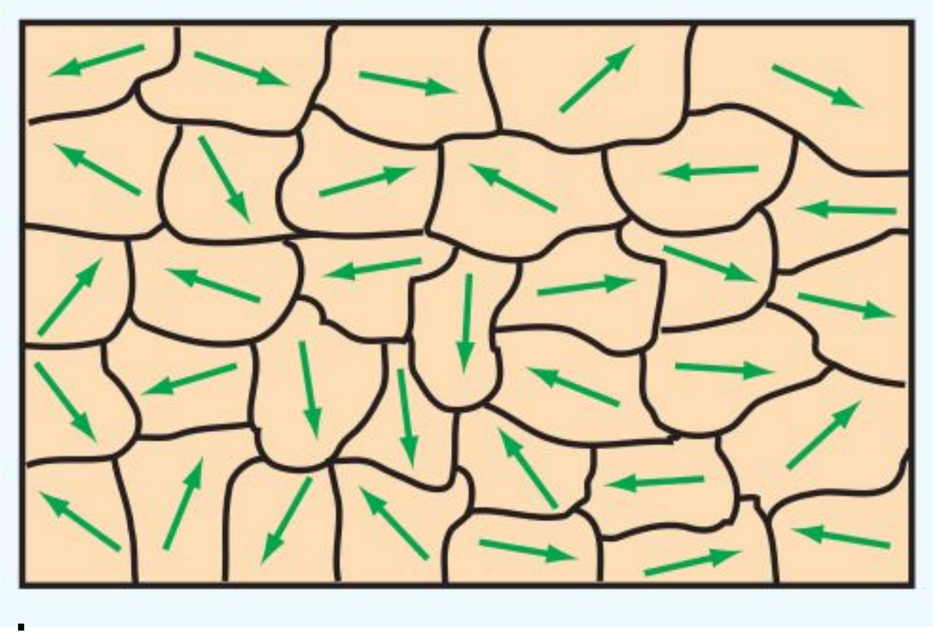
Depends on the magnetization history of the material.

Engineers have exploited this property to make new devices.

Full understanding requires quantum mechanics

5-5 Magnetic Permeability

Classes of Magnetization:

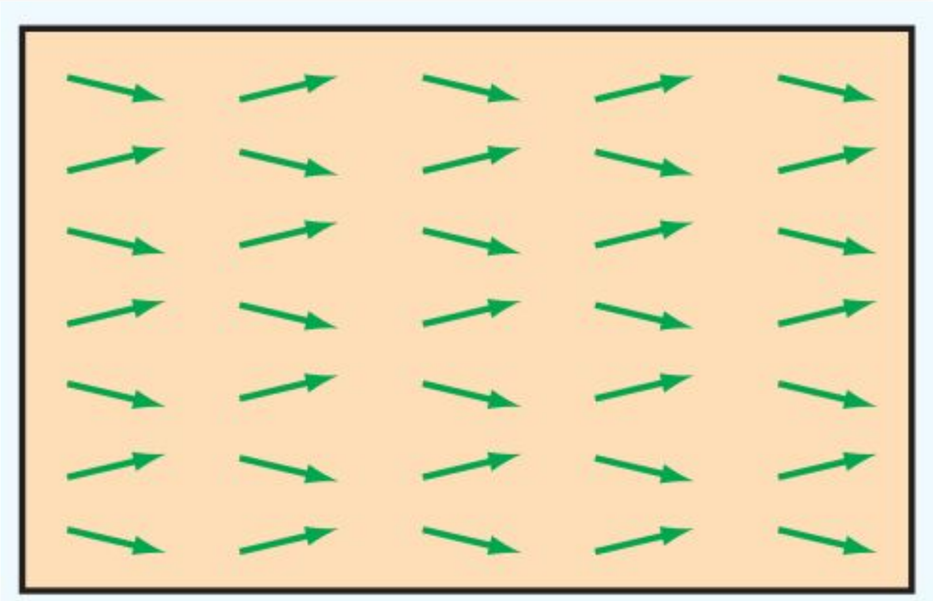


"Classical" view of Ferromagnetic Materials:

Unmagnetized material:
made up of many domains:
each domain is microscopic
all atoms in one domain have aligned magnetic moments
domain magnetic moments vary randomly:
$\chi_m = 0$

5-5 Magnetic Permeability

Classes of Magnetization:



"Classical" view of Ferromagnetic Materials:

Magnetized material:

domain magnetic moments partially align with applied magnetic field:

$\chi_m > 0$

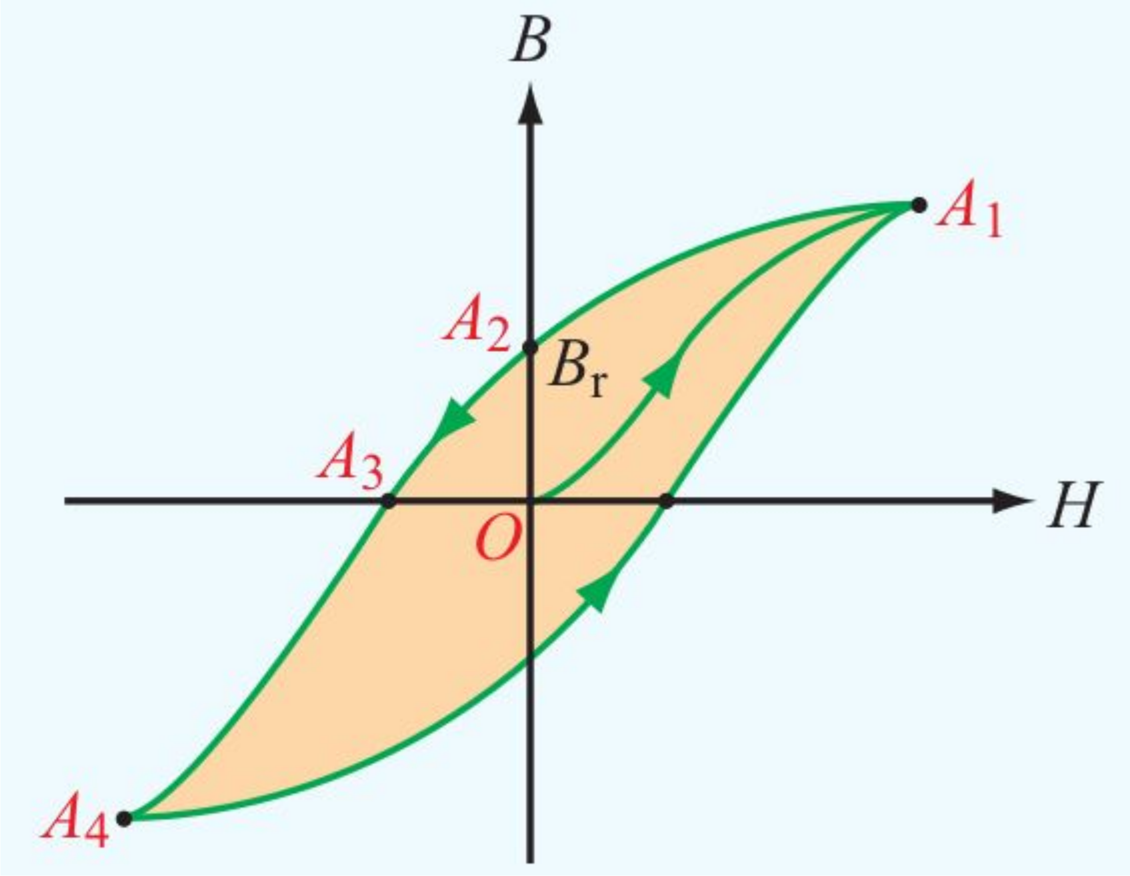
depends on time history of applied magnetic fields

5-5 Magnetic Permeability

Hysteresis

Follow the green line to trace out a typical time-history of magnetization in a ferromagnetic material.

H is the applied field
B is the field within the material



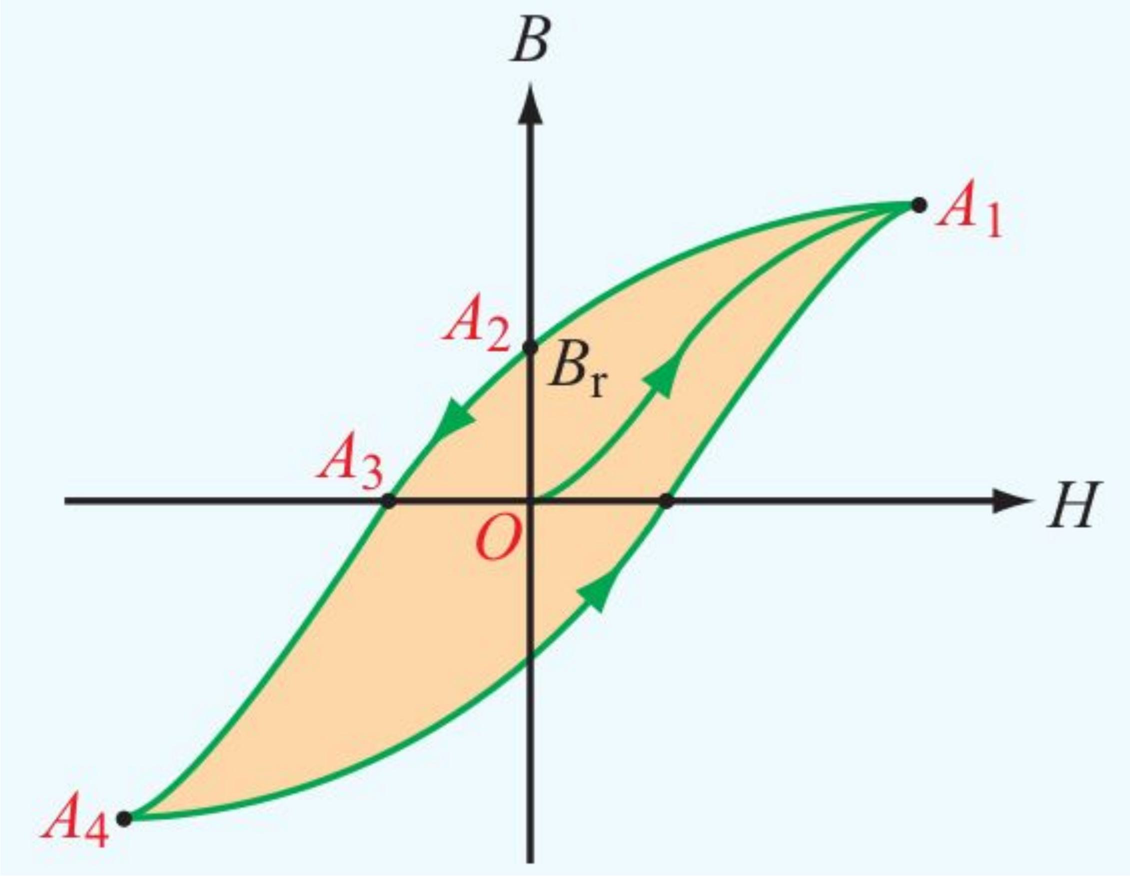
5-5 Magnetic Permeability

Hysteresis

Start with an unmagnetized material:
H=0
B=0

at the origin: O

Domains: unaligned



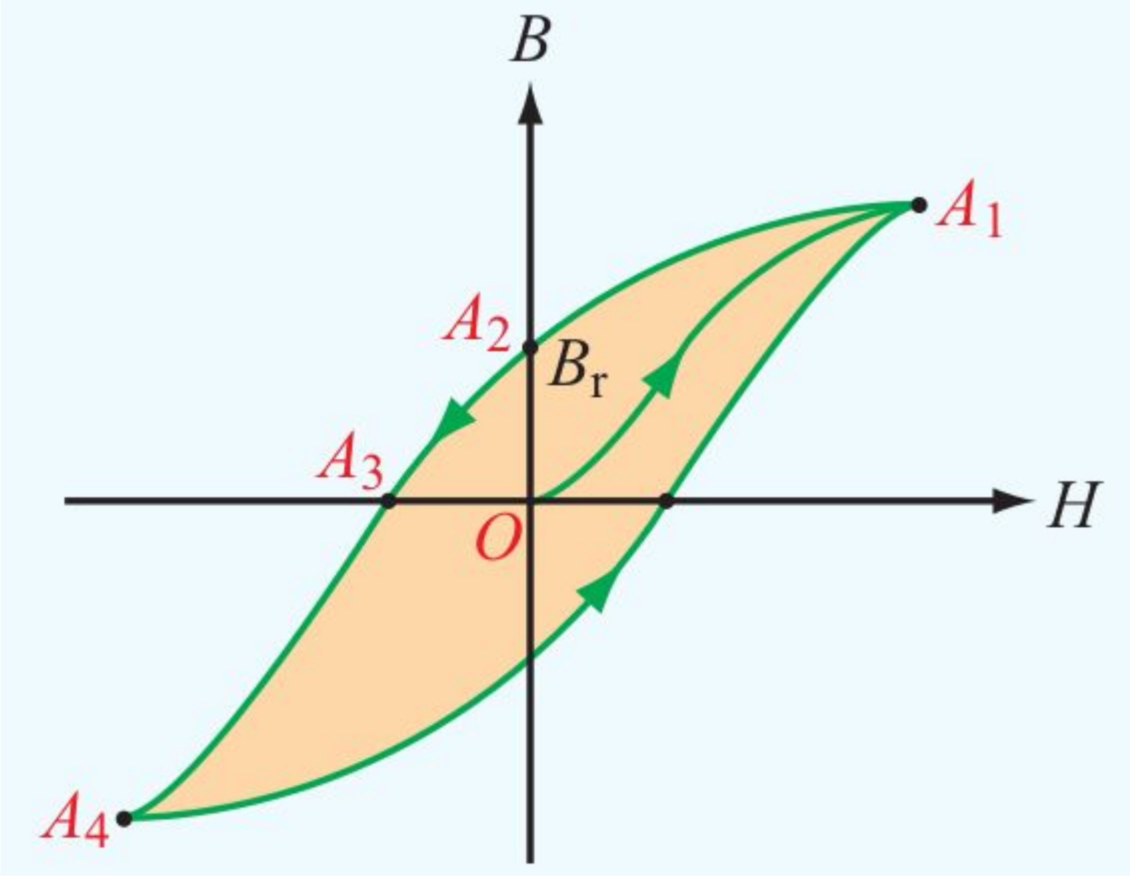
5-5 Magnetic Permeability

Hysteresis

Increase applied **H** field:
B increases to a maximum.

to the point: A_1

Domains: mostly aligned.



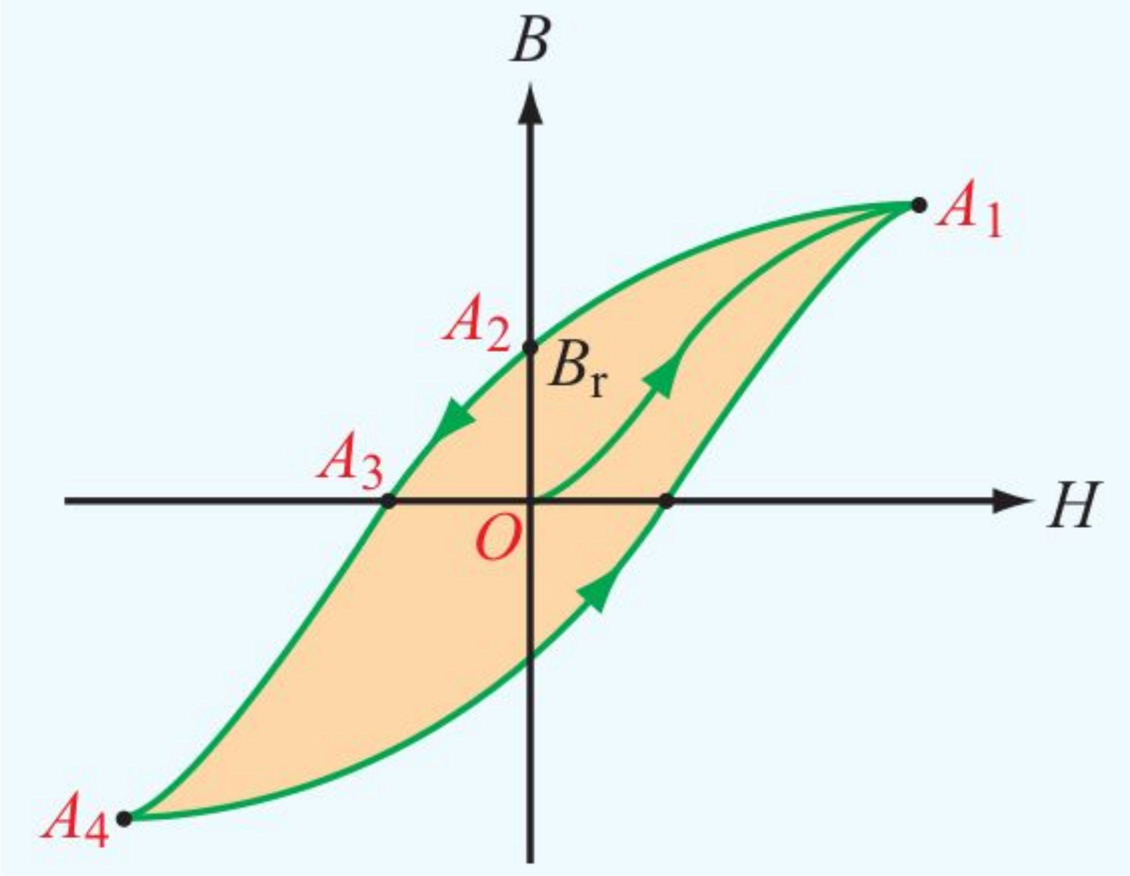
5-5 Magnetic Permeability

Hysteresis

Decrease applied **H** field back to zero.
B decreases but **not** to zero.

to the point: A_2

Domains: many **remain aligned** with applied field

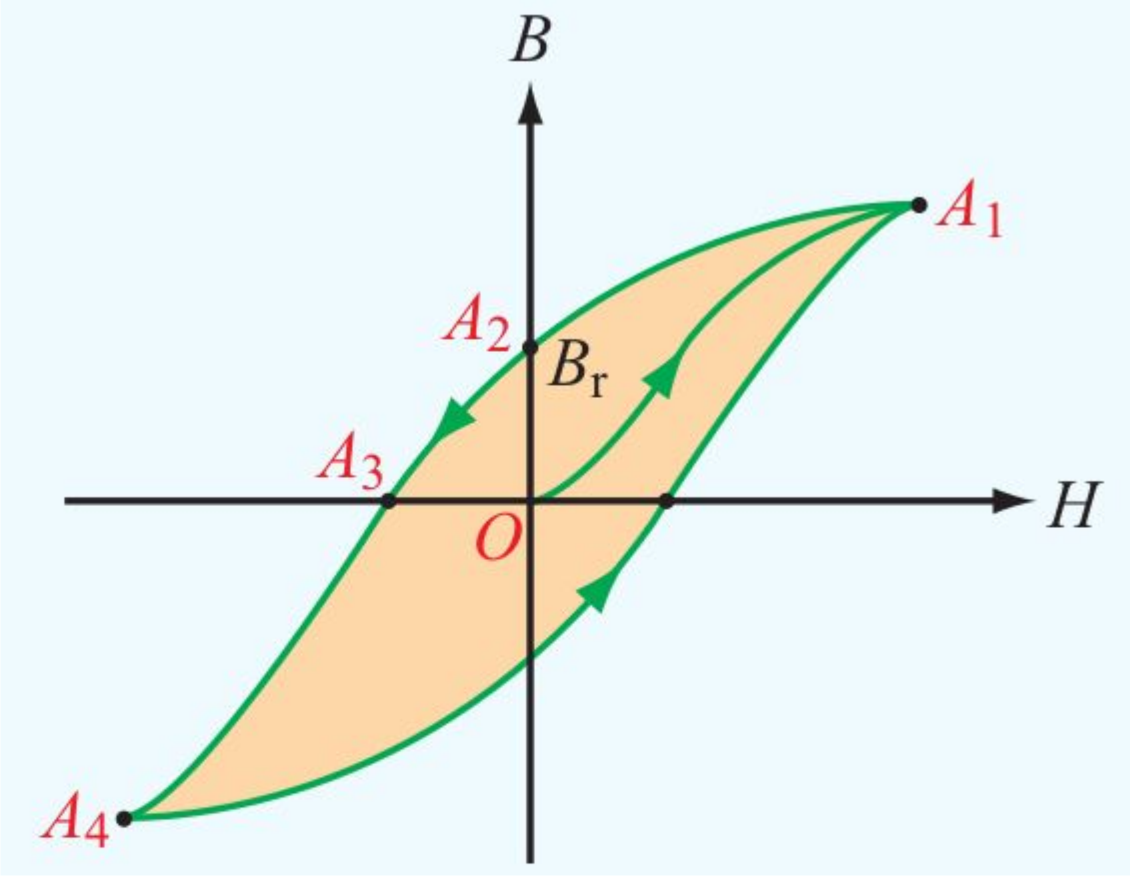


5-5 Magnetic Permeability

Hysteresis

This is where we stop in creation of a **permanent magnet.**

It has an internal magnetic field strength of $\boldsymbol{B}_r$, the "Residual Magnetization"



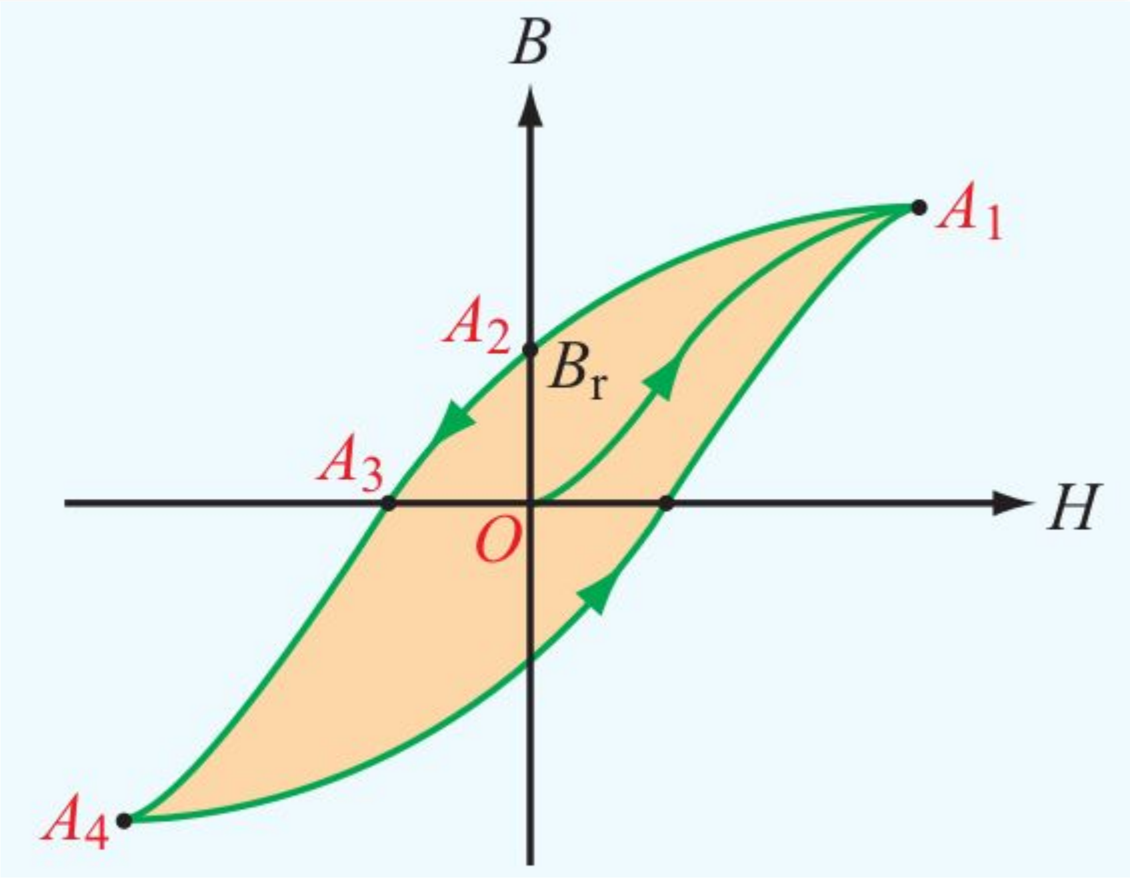
5-5 Magnetic Permeability

Hysteresis

Reverse the direction of applied **H** field.
B decreases to maximum in the opposite direction.

to the point: A_4

Domains: mostly aligned



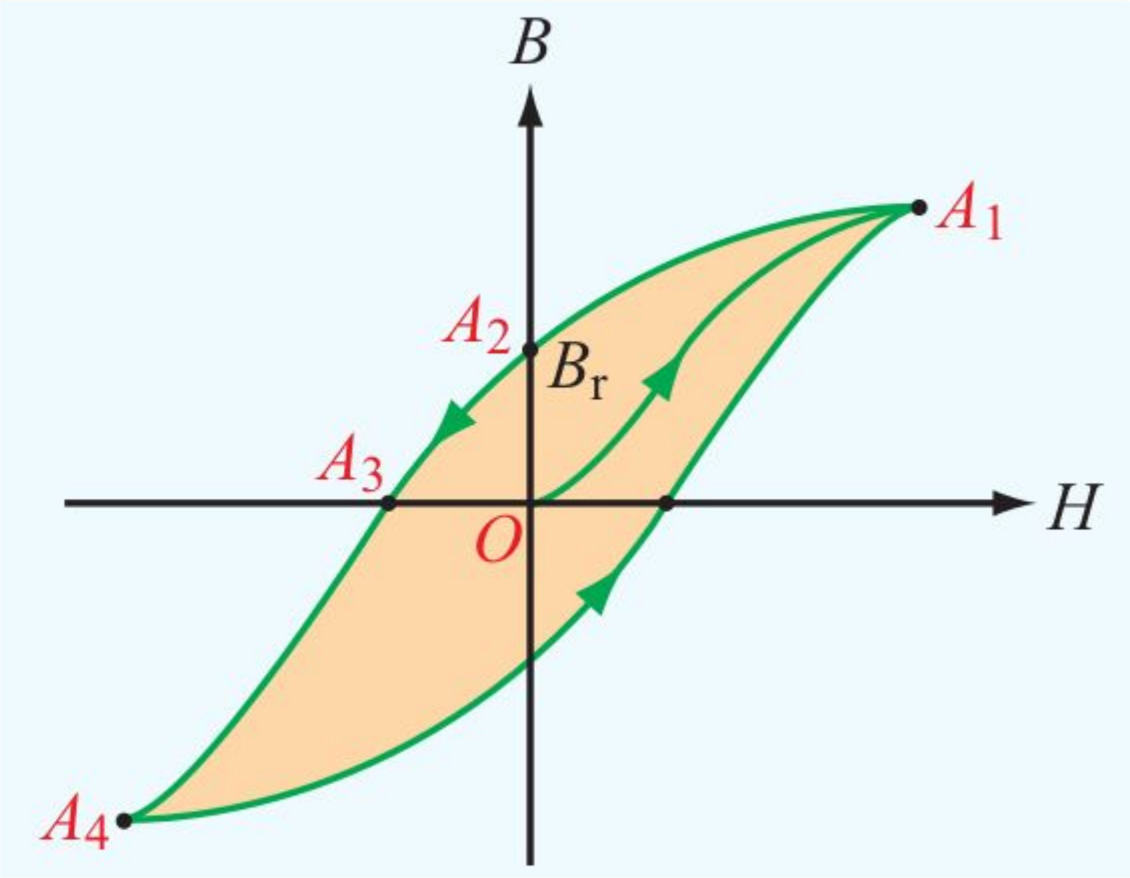
5-5 Magnetic Permeability

Hysteresis

Change **H** from max negative back to max positive.
B increases to zero, then to the maximum positive value.

to the point: A_1

Domains: mostly aligned

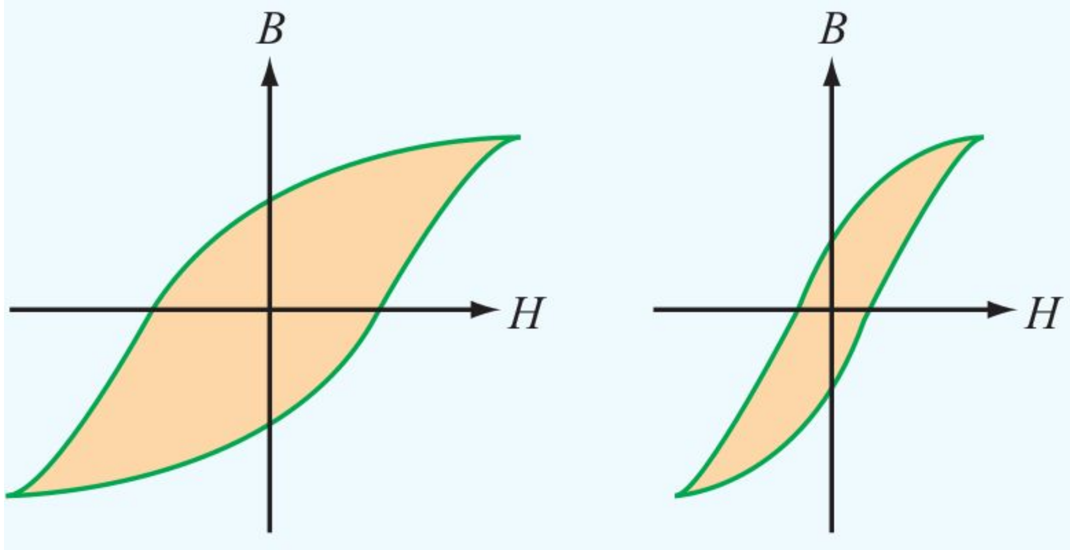


5-5 Magnetic Permeability

Hysteresis Types

"Hard" ferromagnetic materials have large $\boldsymbol{B}_r$, and are difficult to demagnetize.

"Soft" ferromagnetic materials: smaller $\boldsymbol{B}_r$, easier to demagnetize.



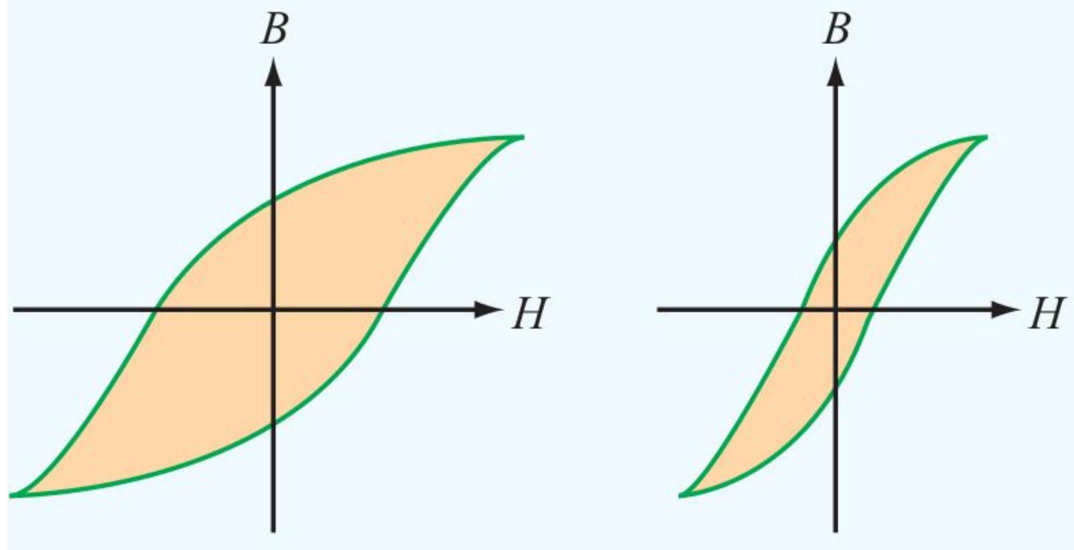
(a) Hard material (b) Soft material

5-5 Magnetic Permeability

Hysteresis Types

Magnetization:
Apply high $\mathbf{H}_{max}$, then turn it off: left with B_r in material.

Demagnetization:
Apply several hysteresis cycles, gradually decreasing $\mathbf{H}_{max}$.

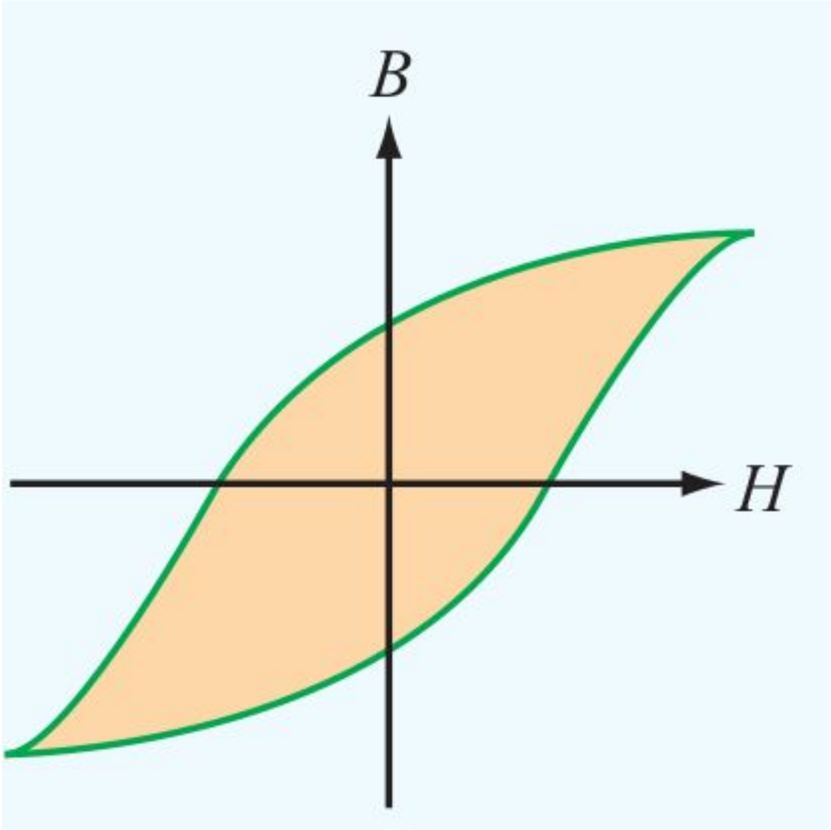


(a) Hard material (b) Soft material

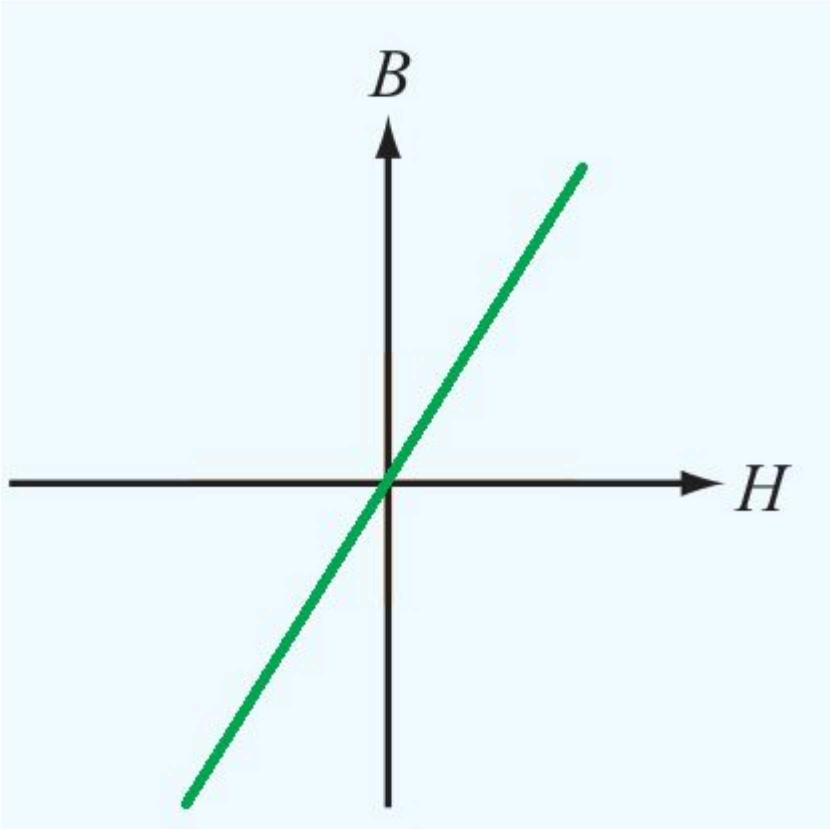
5-5 Magnetic Permeability

Linearity

μ depends on H, history

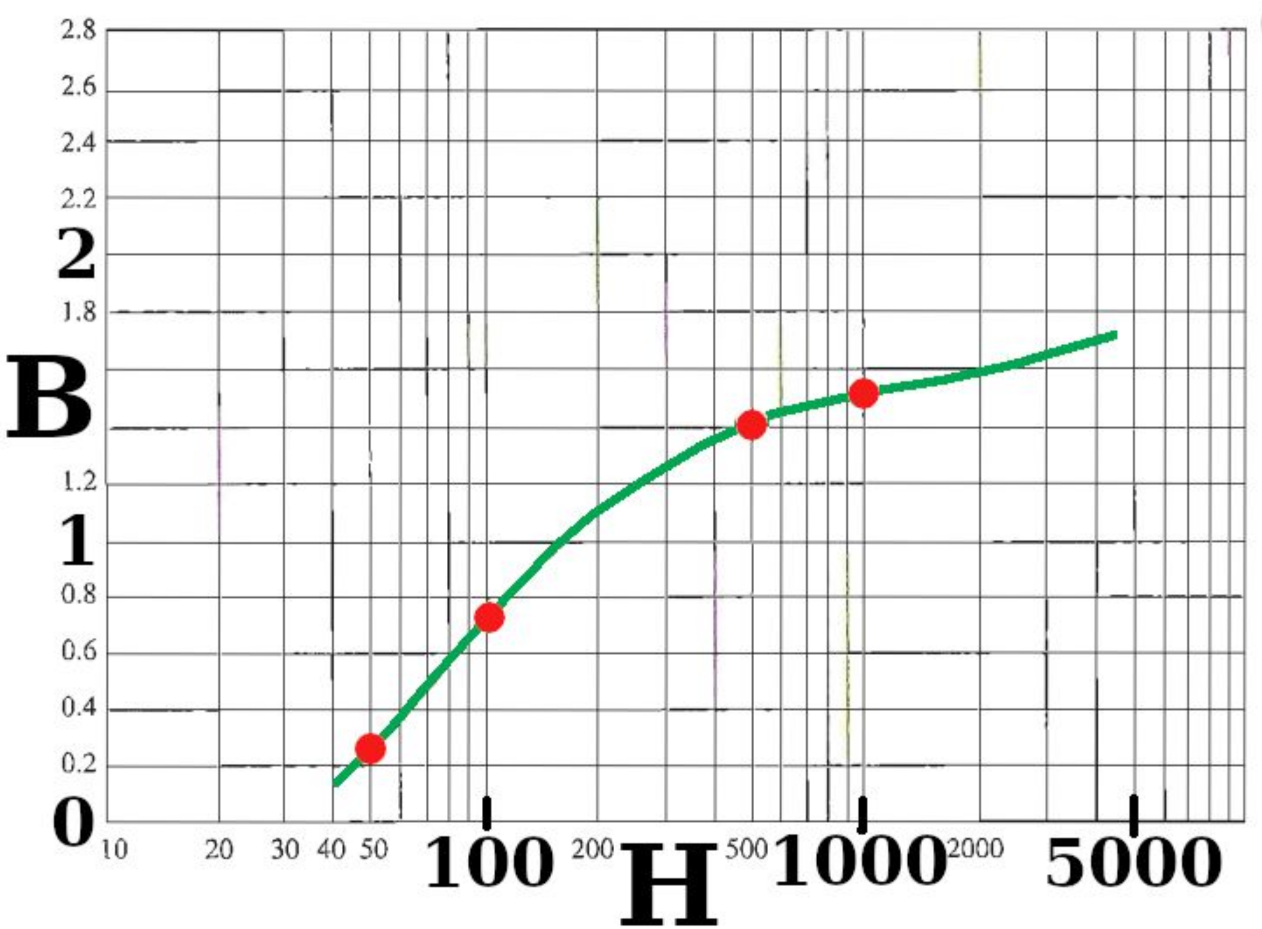


μ a constant



5-5 Magnetic Permeability

Numbers



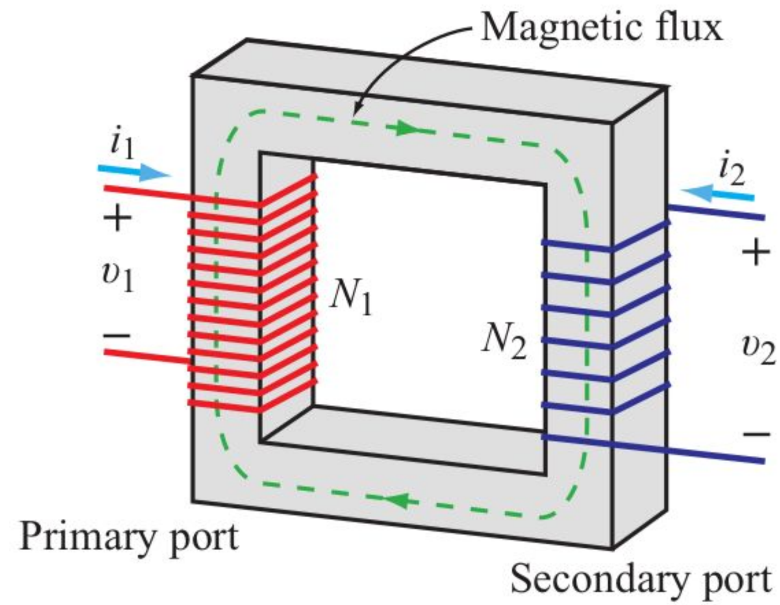
H (A/m)	μ_r
50	3980
100	5730
500	2230
1000	1200

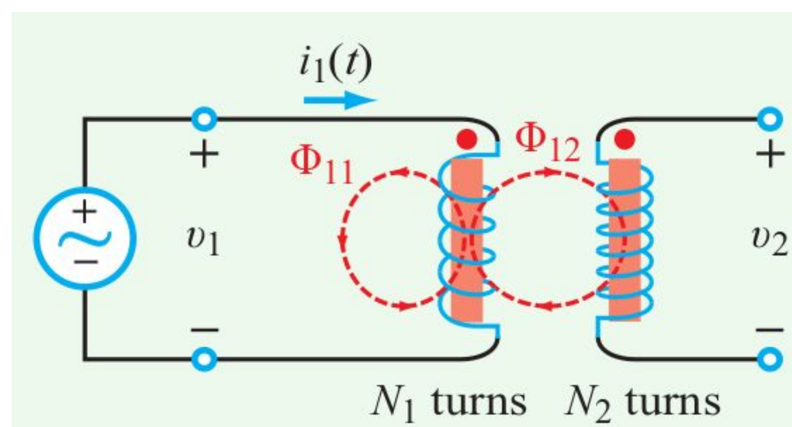
5-5 Magnetic Permeability

Engineering Application

Ferromagnetic materials tend to have very high permeabilities: $\mu_r \approx 10^5$.

This concentrates the magnetic flux and makes transformers and other circuit elements possible.

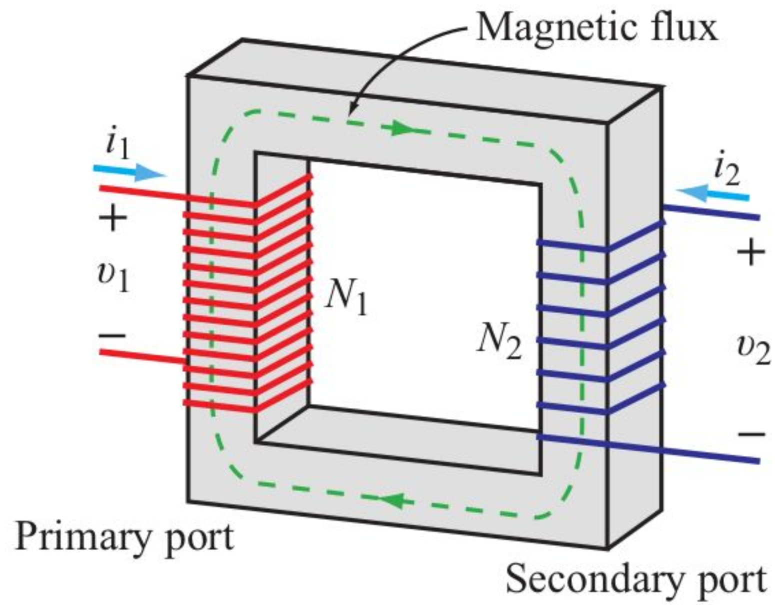




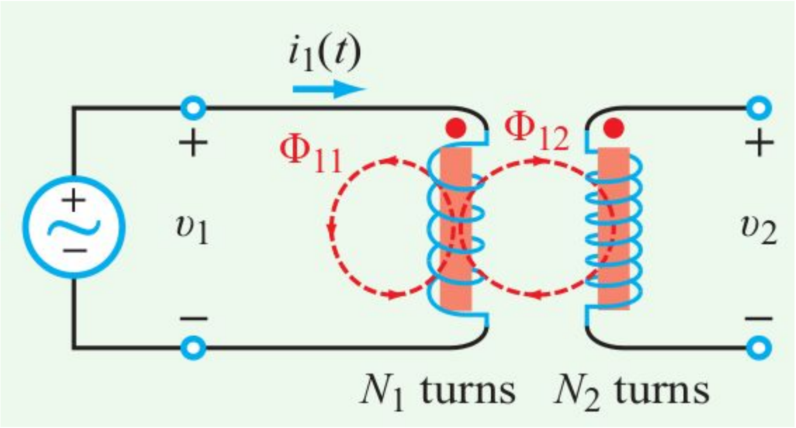
5-5 Magnetic Permeability

Engineering Application

Magnetic flux is channeled in the ferromagnetic core of a transformer.



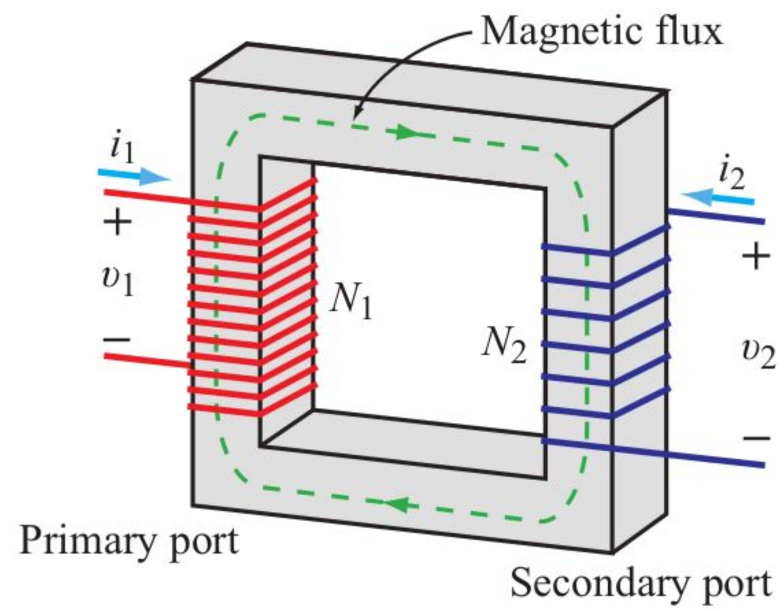
This makes the power transferred to the secondary coil much greater as compared to an air-core device

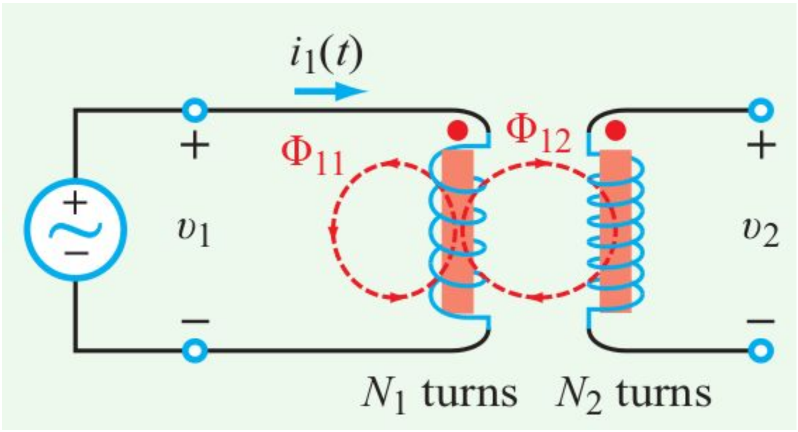


5-5 Magnetic Permeability

Engineering Application

More on transformers in Chapter 6.





5-5 Magnetic Permeability

Ferrite RF Choke on USB Cable

Filters out RF Noise

A cylinder of high-μ material surrounding the wires.

RF signals from phones and motors induce currents on wires.
These induced currents are noise.

5-5 Magnetic Permeability

Ferrite RF Choke on USB Cable

These induced currents produce a magnetic field that is concentrated in the ferrite cylinder.

Since the ferrite has some resistance, some of the energy is converted to heat.

5-5 Magnetic Permeability

Ferrite RF Choke on USB Cable

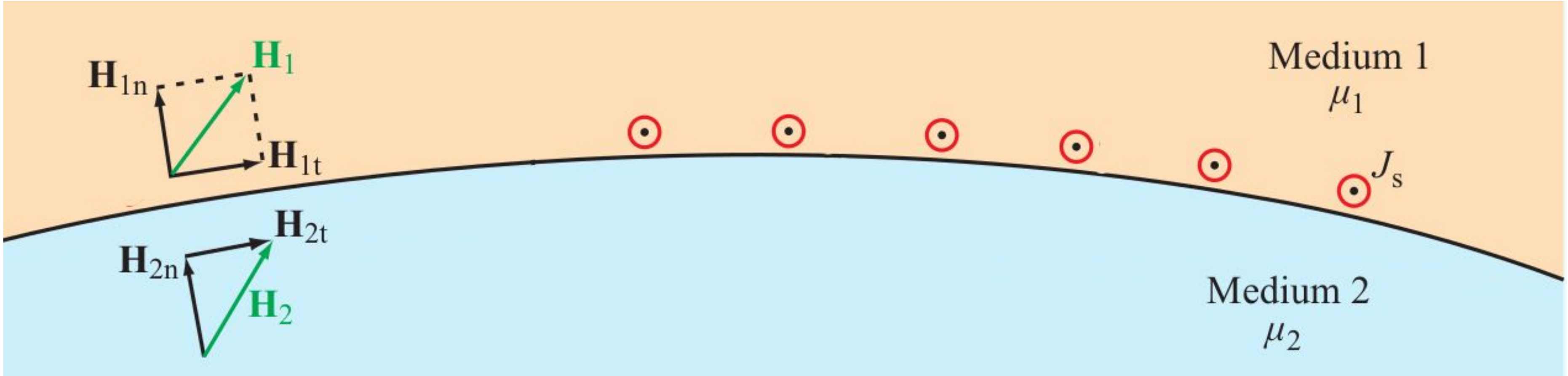
This attenuates the noise,
but not the desired signal.
Because the desired signal
does not produce an
external magnetic field.

Only the noise does.

The parameters of the choke are
such that noise at a few MHz and
above is attenuated.

5-6 Magnetic Boundary Conditions

- How do the fields (**H**, **B**, **J**) change across a boundary?
- Boundary defined by: different materials: μ
- ...and possibly a surface current



5-6 Magnetic Boundary Conditions

From math we know:

$$\iiint_{\mathcal{V}} \nabla \times \mathbf{F}\, d\mathcal{V} = -\oiint_{S} \mathbf{F} \times d\mathbf{s}$$

From Maxwell's equations (magnetostatics):

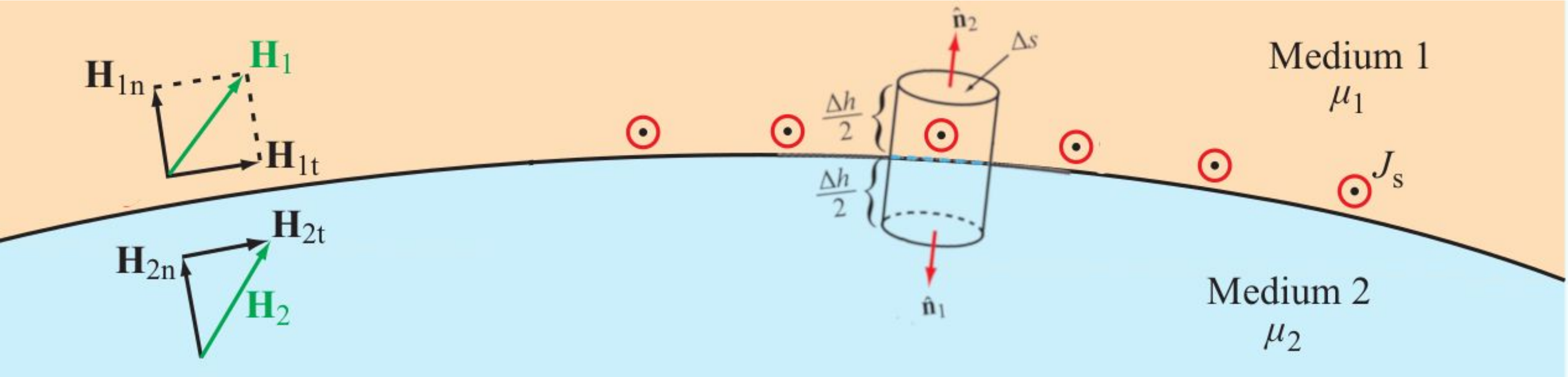
$$\nabla \times \mathbf{H} = \mathbf{J},$$

Plug in:

$$-\oiint_{S} \mathbf{H} \times d\mathbf{s} = \iiint_{\mathcal{V}} \mathbf{J}\, d\mathcal{V}$$

5-6 Magnetic Boundary Conditions

Choose the surface to be a box that spans the boundary:



In the limit as $\Delta h \rightarrow 0$, we are left with the two end-caps.
The "vector" surface d**s** uses the outward normal:

in medium 2: $\hat{\mathbf{n}}_1$

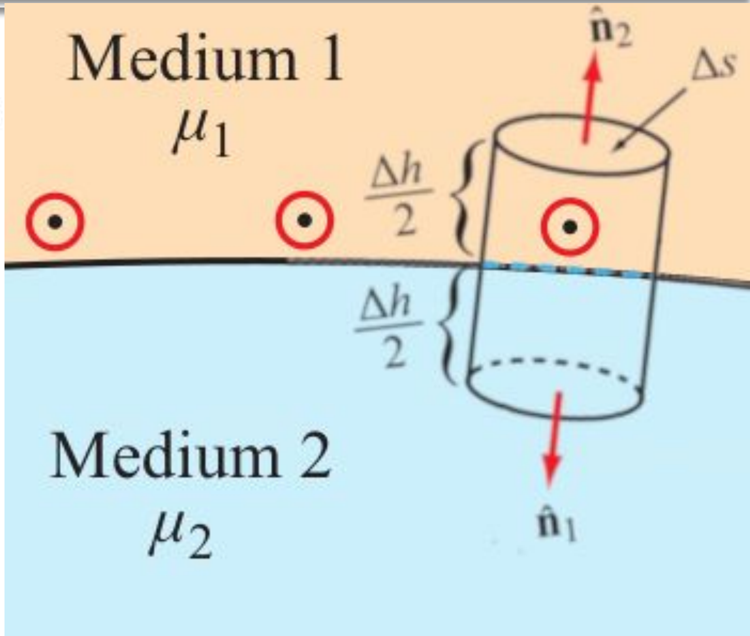
in medium 1: $\hat{\mathbf{n}}_2$

5-6 Magnetic Boundary Conditions

$$\oiint_{S_1} \mathbf{H}_1 \times \hat{\mathbf{n}}_2 \, ds + \oiint_{S_2} \mathbf{H}_2 \times \hat{\mathbf{n}}_1 \, ds = -\iiint_{\mathcal{V}} \mathbf{J} \, d\mathcal{V}$$

Since $\hat{\mathbf{n}}_1 = -\hat{\mathbf{n}}_2$:

$$\oiint_{S_1} \mathbf{H}_1 \times \hat{\mathbf{n}}_2 \, ds - \oiint_{S_2} \mathbf{H}_2 \times \hat{\mathbf{n}}_2 \, ds = -\iiint_{\mathcal{V}} \mathbf{J} \, d\mathcal{V}$$



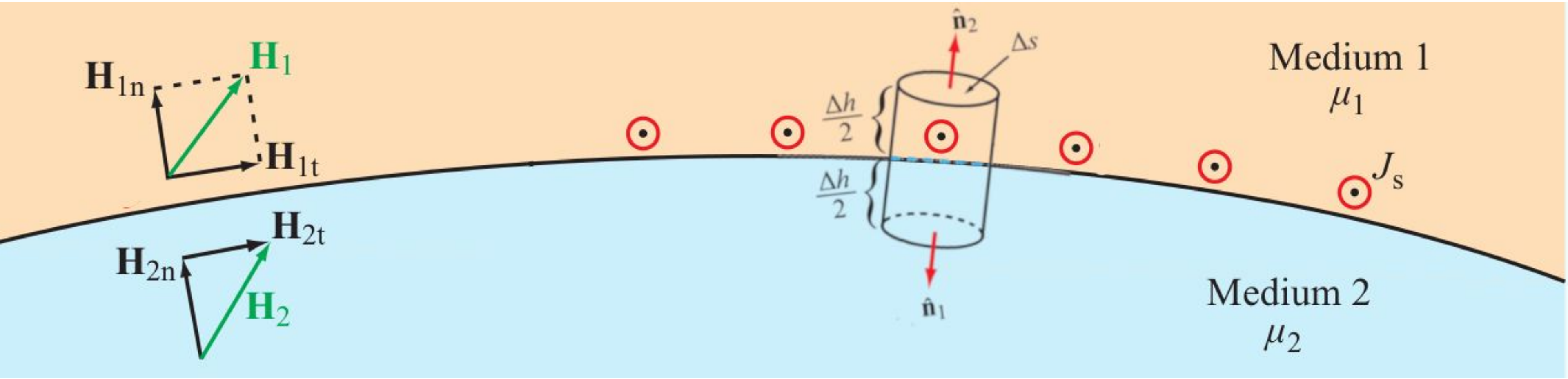
Very small: so nothing changing inside the surface:

$$\mathbf{H}_1 \times \hat{\mathbf{n}} \, \Delta s - \mathbf{H}_2 \times \hat{\mathbf{n}} \, \Delta s = -\mathbf{J}_s \, \Delta s$$

$$(\mathbf{H}_1 - \mathbf{H}_2) \times \hat{\mathbf{n}} = -\mathbf{J}_s$$

where: $\mathbf{H} \times \hat{\mathbf{n}} = \mathbf{H}_{\text{tangential}}$

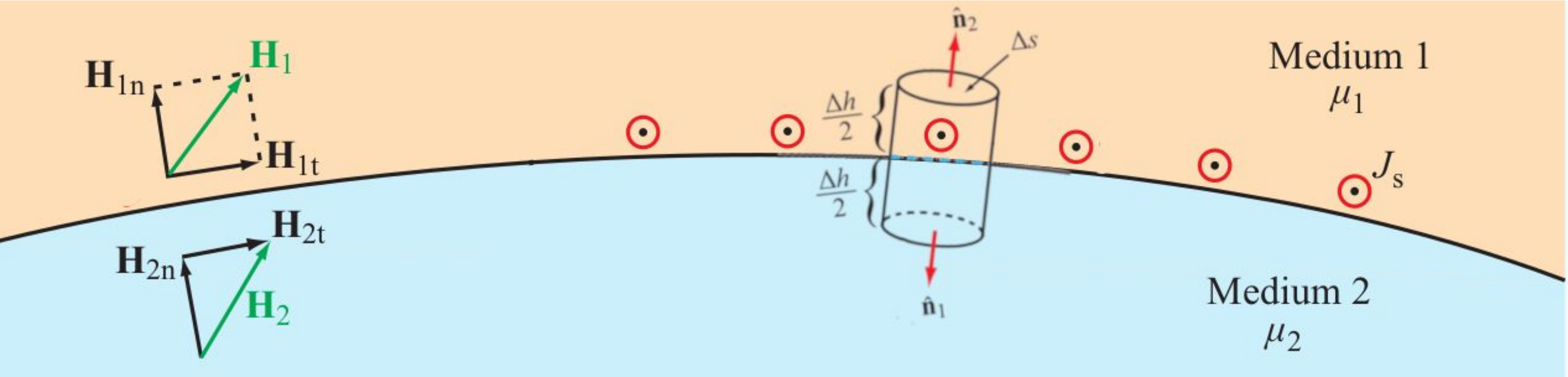
5-6 Magnetic Boundary Conditions



$$(\mathbf{H}_1 - \mathbf{H}_2) \times \hat{\mathbf{n}} = -\mathbf{J}_s$$

in words: Tangential **H** is discontinuous by the surface current density

5-6 Magnetic Boundary Conditions



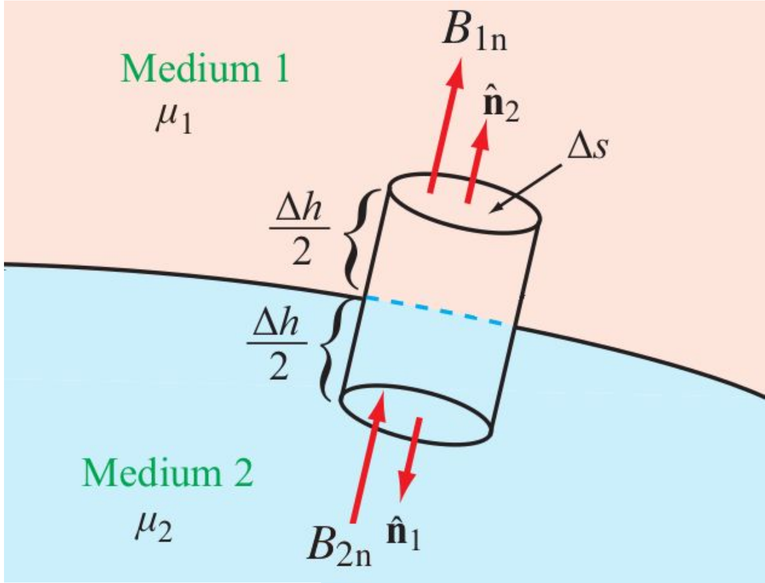
$$(\mathbf{H}_1 - \mathbf{H}_2) \times \hat{\mathbf{n}} = 0$$

in words: Tangential **H** is continuous when there is zero surface current density (true for finite conductivities)

5-6 Magnetic Boundary Conditions

Consider tangential and normal components separately

BC for normal components of **H, B**



$$\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$$

For the cylindrical surface,

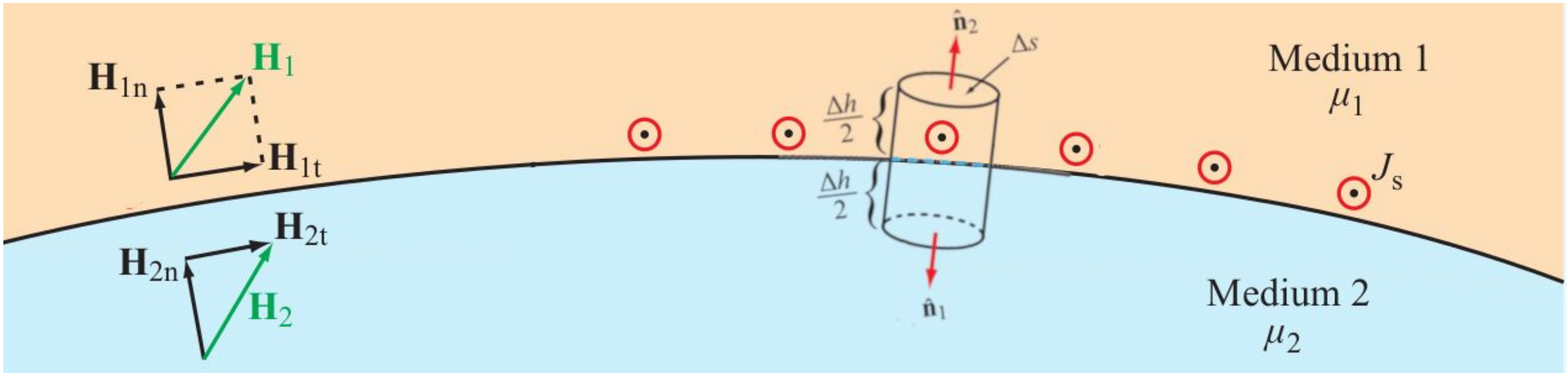
in the limit as $\Delta h \to 0$:

$$\oint_S \mathbf{B} \cdot d\mathbf{s} = \int_{\text{top}} \mathbf{B}_1 \cdot \hat{\mathbf{n}}_2 \, ds + \int_{\text{bottom}} \mathbf{B}_2 \cdot \hat{\mathbf{n}}_1 \, ds = 0$$

$$\hat{\mathbf{n}}_2 \cdot (\mathbf{B}_1 - \mathbf{B}_2) = 0$$

So: $\mathbf{B}_{1n} = \mathbf{B}_{2n}$

5-6 Magnetic Boundary Conditions



$$H_{1t} = H_{2t}.$$

$$\mathbf{B}_{1n} = \mathbf{B}_{2n}$$

Tangential **H** is continuous (finite conductivities)

Normal **B** is continuous

Example 5-7

Given: 2 regions separated by a tilted planar boundary, defined by: $y + 2x = 2$
no surface current

$$\mathbf{B}_1 = \hat{\mathbf{x}}2 + \hat{\mathbf{y}}3$$

Find: $\mathbf{B}_2$, then evaluate for the case: $\mu_2 = 2\mu_1$

Example 5-7

Solution:

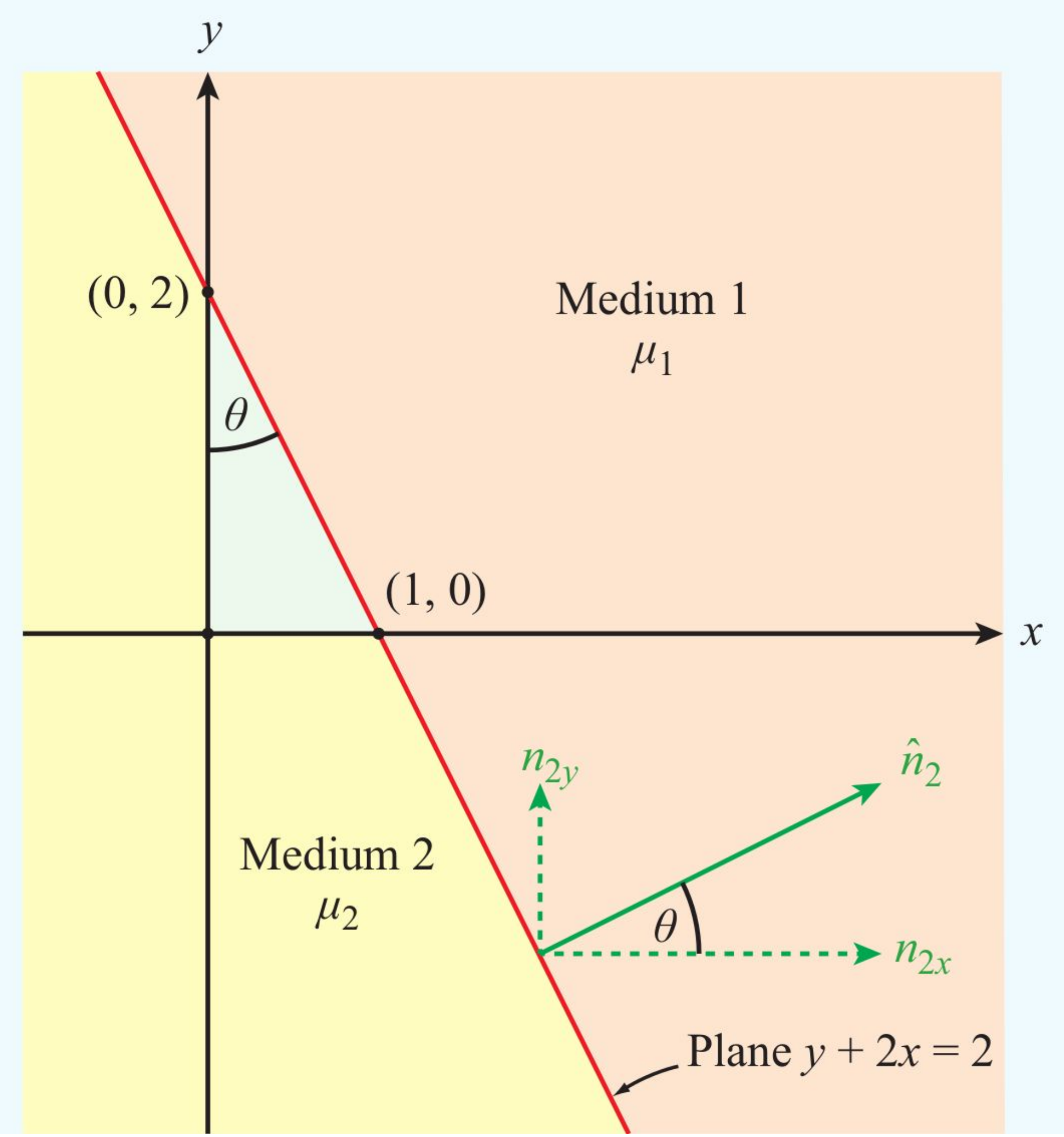
1. find an eqn for $\hat{\mathbf{n}}_2$

2. use that to find:

$$\mathbf{B}_1 = \mathbf{B}_{1n} + \mathbf{B}_{1t}$$

3. Apply BCs for normal case, then for tangential.

4. assemble $\mathbf{B}_2$



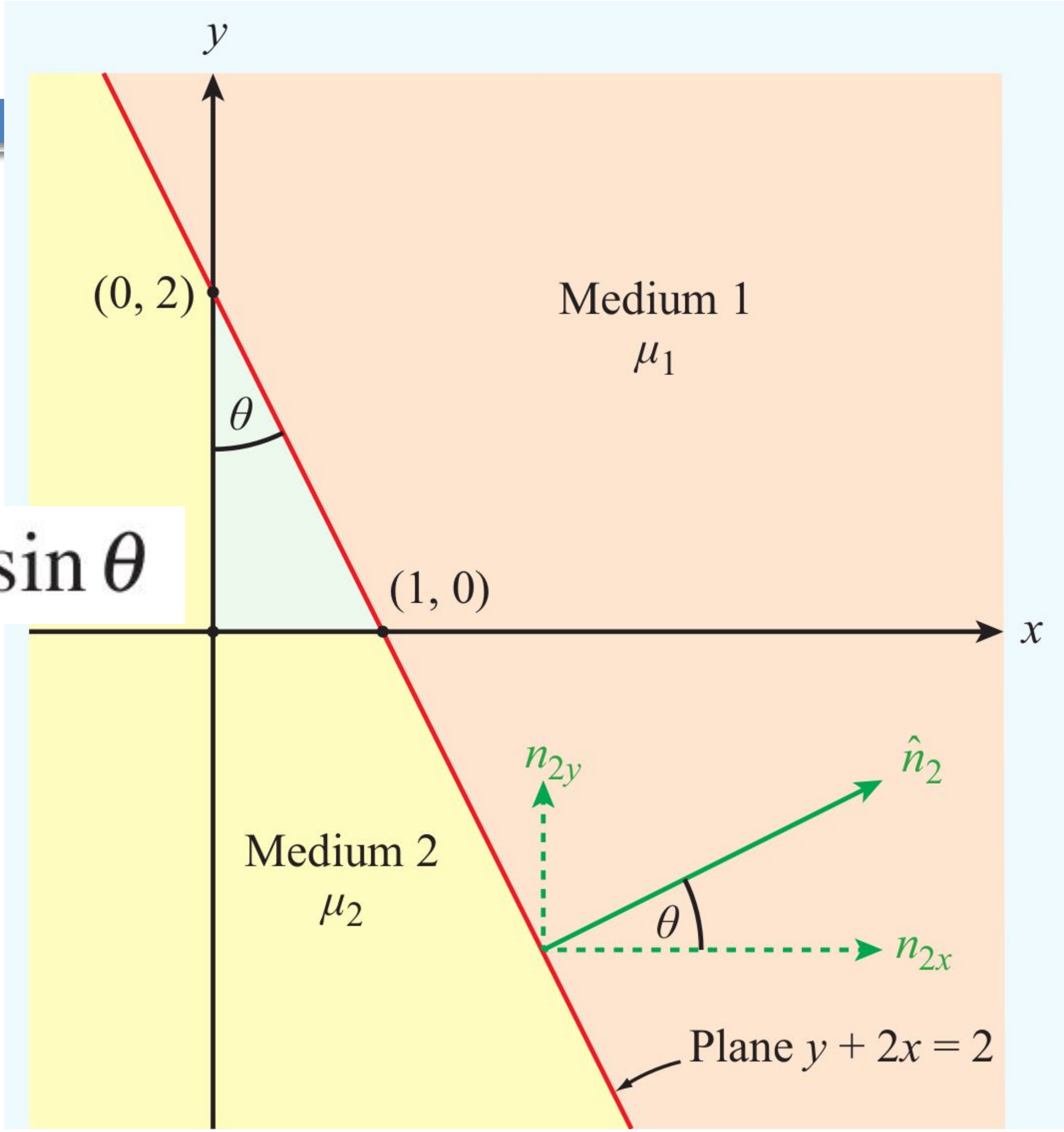
Example 5-7

Solution:

1. find an eqn for $\hat{\mathbf{n}}_2$

$$\hat{\mathbf{n}}_2 = \hat{\mathbf{x}} \cos\theta + \hat{\mathbf{y}} \sin\theta$$

$$\theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.57^\circ.$$



Example 5-7

Solution:

1. find an eqn for $\hat{\mathbf{n}}_2$

alternate method:

y = mx + b

y = -2x +2

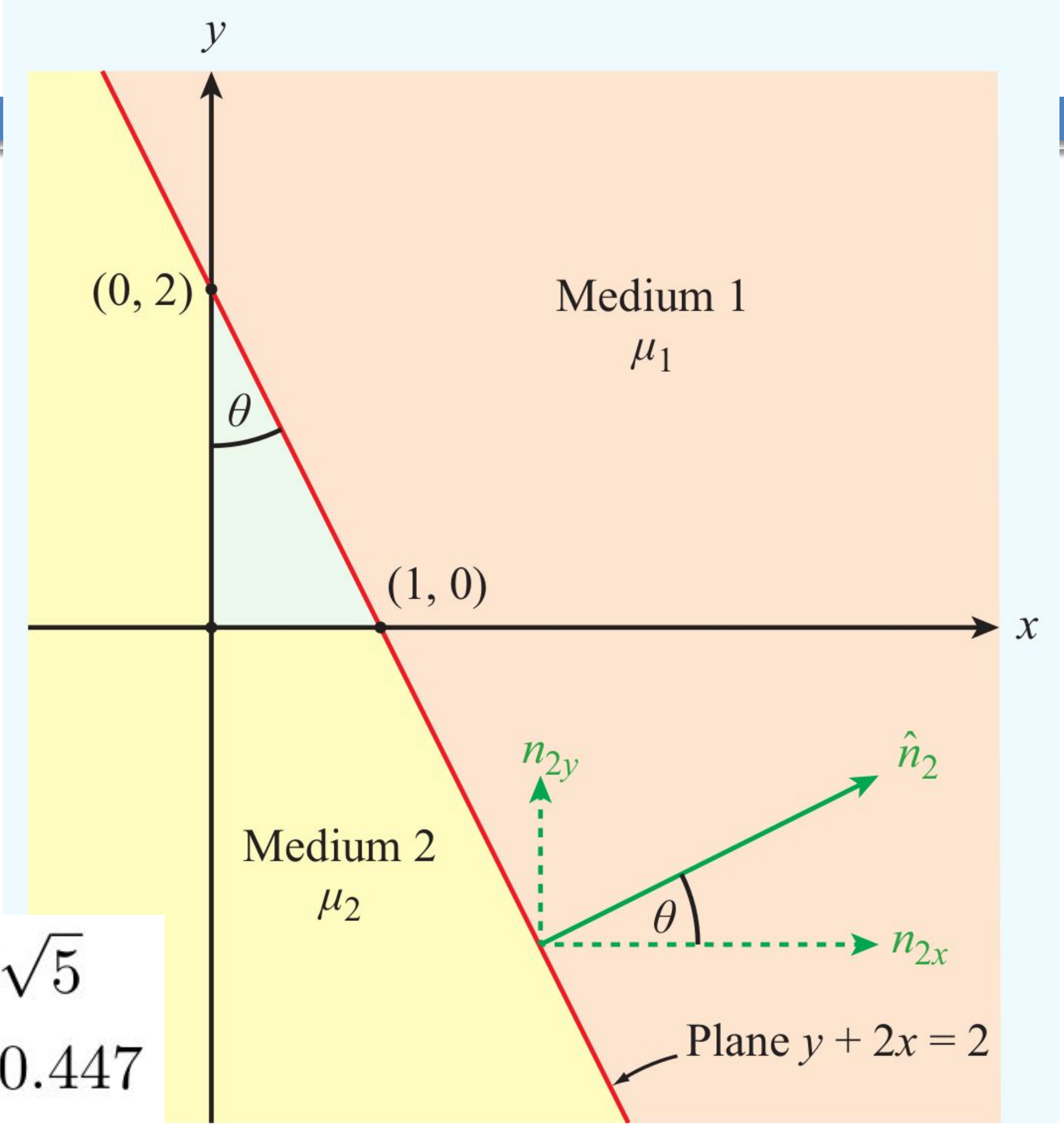
slope=-2

perp: slope=+1/2

so:

$$\hat{\mathbf{n}}_2 = (\hat{\mathbf{x}}2 + \hat{\mathbf{y}}1)/\sqrt{5}$$

$$\hat{\mathbf{n}}_2 = \hat{\mathbf{x}}0.894 + \hat{\mathbf{y}}0.447$$



Example 5-7

Solution:

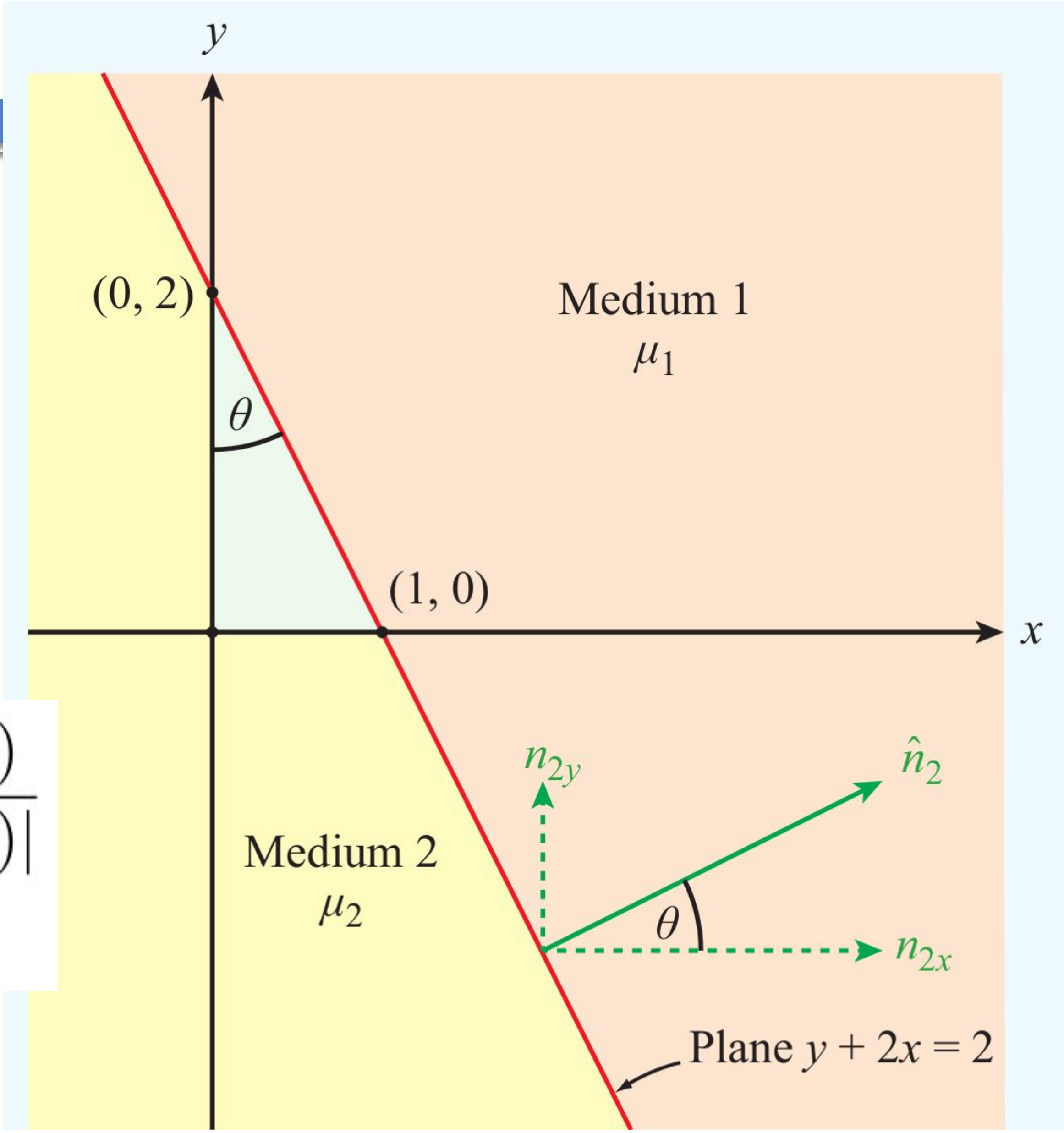
1. find an eqn for $\hat{\mathbf{n}}_2$

alternate method:

gradient of plane eqn is normal vector:

$$\hat{\mathbf{n}}_2 = \frac{\nabla(y + 2x - 2)}{|\nabla(y + 2x - 2)|}$$

$$\hat{\mathbf{n}}_2 = (\hat{\mathbf{y}}1 + \hat{\mathbf{x}}2)/\sqrt{5}$$



Example 5-7

Solution:

2. use that to find

$$\mathbf{B}_1 = \mathbf{B}_{1n} + \mathbf{B}_{1t}$$

$$\mathbf{B}_{1n} = B_{1n}\hat{\mathbf{n}}_2$$

$$B_{1n} = \hat{\mathbf{n}}_2 \cdot \mathbf{B}_1$$

$$= (\hat{\mathbf{x}}\cos\theta + \hat{\mathbf{y}}\sin\theta) \cdot (\hat{\mathbf{x}}2 + \hat{\mathbf{y}}3)$$

$$= 2\cos\theta + 3\sin\theta.$$

Example 5-7

Solution:

2. tangential component:

$$\begin{aligned}\mathbf{B}_{1t} &= \mathbf{B}_1 - \mathbf{B}_{1n} \\ &= \mathbf{B}_1 - B_{1n}\hat{\mathbf{n}}_2 \\ &= (\hat{\mathbf{x}}2 + \hat{\mathbf{y}}3) - (2\cos\theta + 3\sin\theta)(\hat{\mathbf{x}}\cos\theta + \hat{\mathbf{y}}\sin\theta) \\ &= \hat{\mathbf{x}}(2 - 2\cos^2\theta - 3\sin\theta\cos\theta) \\ &\quad + \hat{\mathbf{y}}(3 - 2\cos\theta\sin\theta - 3\sin^2\theta).\end{aligned}$$

Example 5-7

Solution:

3. Apply Boundary Conditions:

$$\mathbf{B}_{1n} = \mathbf{B}_{2n}$$

$$\frac{\mathbf{B}_{1t}}{\mu_1} = \frac{\mathbf{B}_{2t}}{\mu_2}$$

Example 5-7

Solution:

4. Assemble $\mathbf{B}_2$:

$$\mathbf{B}_2 = \mathbf{B}_{2n} + \mathbf{B}_{2t} = \mathbf{B}_{1n} + \frac{\mu_2}{\mu_1}\,\mathbf{B}_{1t}$$

$$= B_{1n}\hat{\mathbf{n}}_2 + \frac{\mu_2}{\mu_1}\,\mathbf{B}_{1t}$$

$$= (2\cos\theta + 3\sin\theta)(\hat{\mathbf{x}}\cos\theta + \hat{\mathbf{y}}\sin\theta)$$

$$+ \frac{\mu_2}{\mu_1}\,[\hat{\mathbf{x}}(2 - 2\cos^2\theta - 3\sin\theta\cos\theta)$$

$$+ \hat{\mathbf{y}}(3 - 2\cos\theta\sin\theta - 3\sin^2\theta)]$$

Example 5-7

Solution:

4. Assemble $\mathbf{B}_2$:

$$= \hat{\mathbf{x}}\left[2\cos^2\theta + 3\sin\theta\cos\theta + \frac{\mu_2}{\mu_1}\left(2 - 2\cos^2\theta - 3\sin\theta\cos\theta\right)\right] + \hat{\mathbf{y}}\left[2\cos\theta\sin\theta + 3\sin^2\theta + \frac{\mu_2}{\mu_1}\left(3 - 2\cos\theta\sin\theta - 3\sin^2\theta\right)\right].$$

Example 5-7

Solution:

4. Assembled $\mathbf{B}_2$: specific case

For $\theta = 26.57°$ and $\mu_2 = 2\mu_1$,

$$\mathbf{B}_2 = \hat{\mathbf{x}}1.2 + \hat{\mathbf{y}}4.6 \qquad \text{(T)}.$$

Exercise 5-13

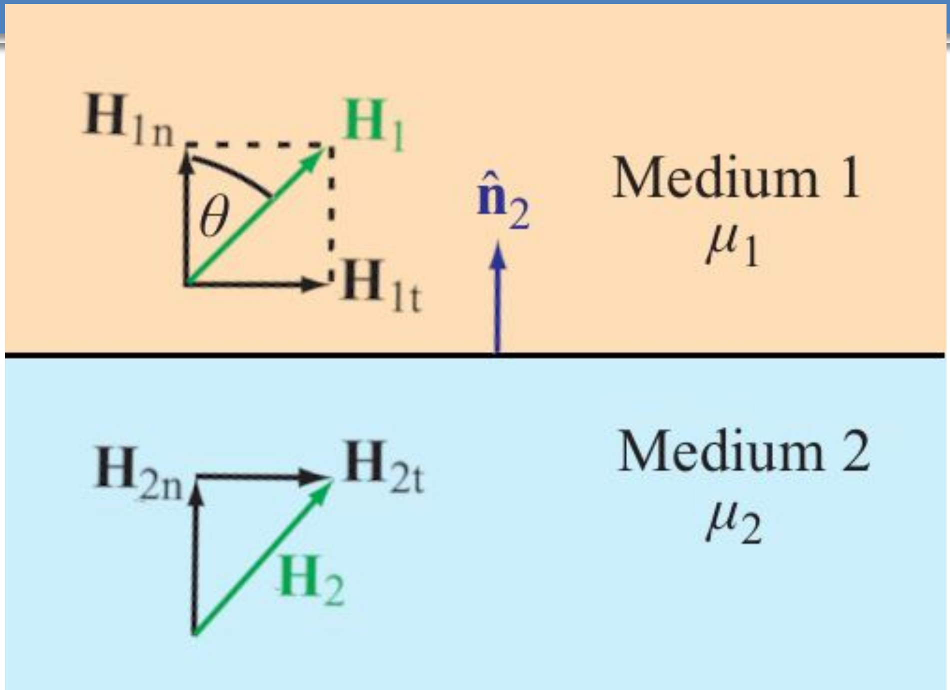
Given: $\hat{\mathbf{n}}_2 = \hat{\mathbf{z}}$

$$\mathbf{H}_2 = (\hat{\mathbf{x}}3 + \hat{\mathbf{z}}2) \text{ (A/m)},$$

$$\mu_{r_1} = 2, \quad \mu_{r_2} = 8,$$

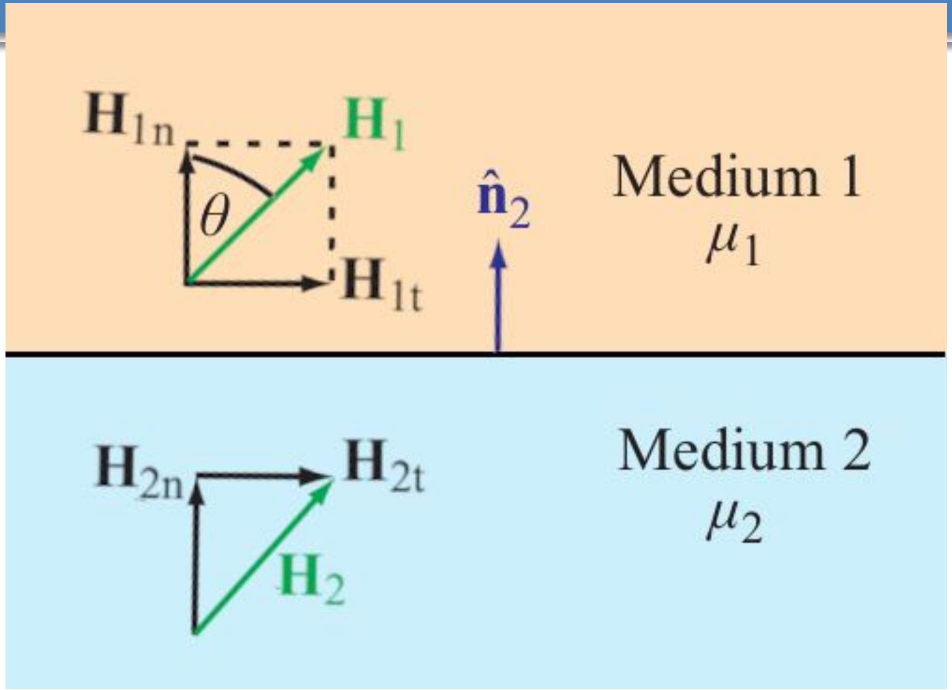
no surface current

Find: angle θ



Exercise 5-13

Solution:



Tangential **H** is continuous:

$$H_{1x} = H_{2x} = 3$$

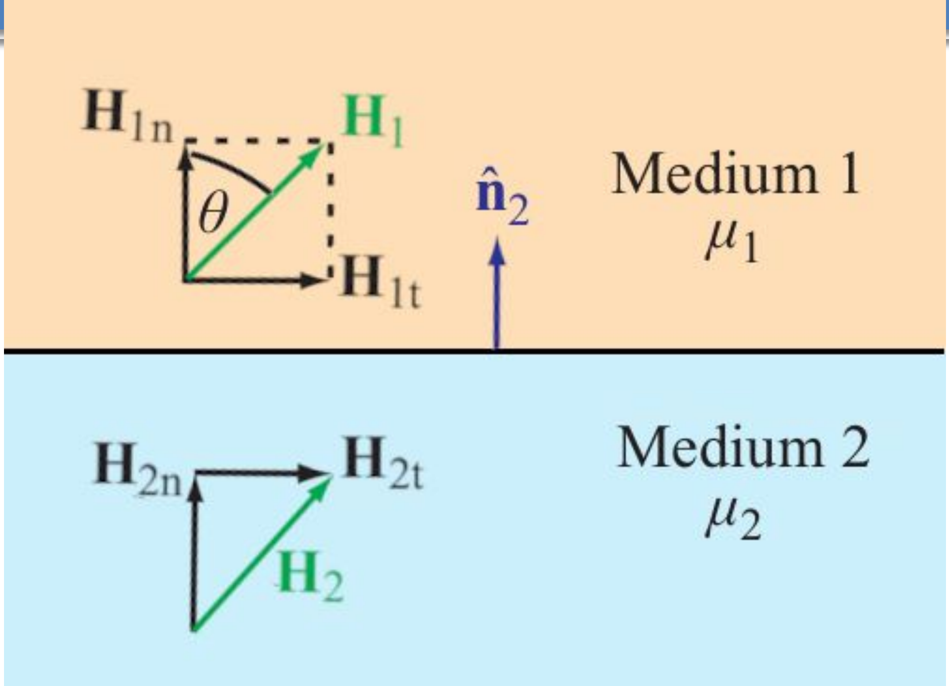
Normal **B** is continuous:

$$\mu_1 H_{1z} = \mu_2 H_{2z}$$

$$H_{1z} = \frac{\mu_2}{\mu_1} H_{2z} = \frac{8}{2} \times 2 = 8$$

Exercise 5-13

Solution:



Assemble $\mathbf{H}_1$:

$$\mathbf{H}_1 = \hat{\mathbf{x}}3 + \hat{\mathbf{z}}8$$

Get angle:

$$\mathbf{H}_1 \cdot \hat{\mathbf{z}} = H_1 \cos\theta$$

$$\cos\theta = \frac{\mathbf{H}_1 \cdot \hat{\mathbf{z}}}{H_1} = \frac{8}{\sqrt{9+64}} = \frac{8}{\sqrt{73}} = 0.936$$

$$\theta = 20.6^\circ.$$

Example 3

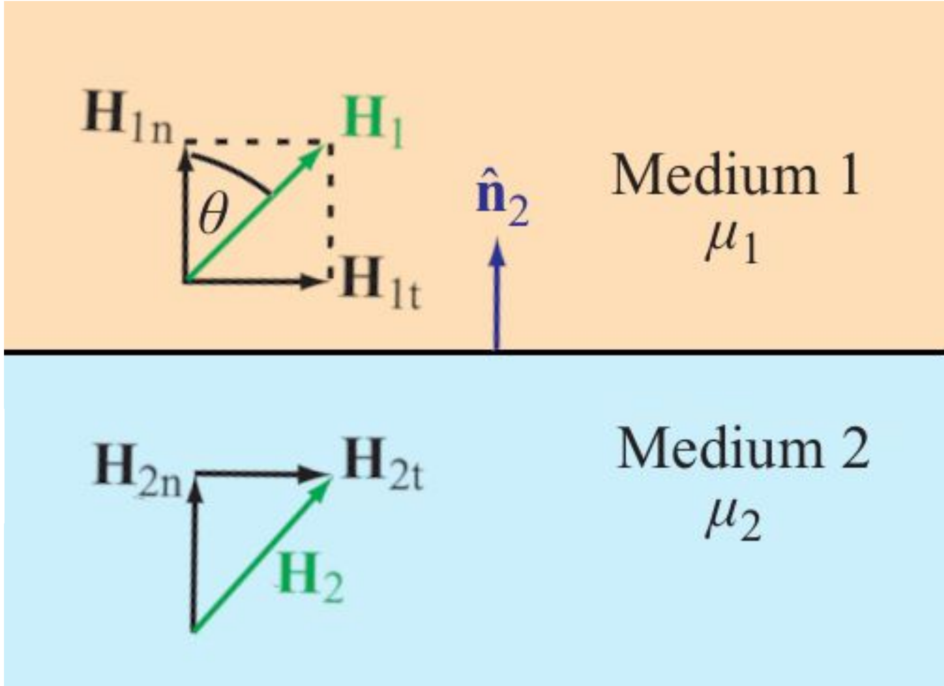
Given:

Two half-planes:

Region 1: μ_{r1}=2, 2x+3y-4z>1

Region 2: μ_{r2}=5, 2x+3y-4z<1

$$\mathbf{H}_1 = 50\hat{\mathbf{x}} - 30\hat{\mathbf{y}} + 20\hat{\mathbf{z}} \;\; \text{A/m}$$



Find: $\mathbf{H}_{1normal}$, $\mathbf{H}_{1tangential}$, $\mathbf{H}_2$

Solution:

First find the normal vector:

Gradient of the plane-eqn points to increasing values, hence is $\hat{\mathbf{n}}_2$

Example 3

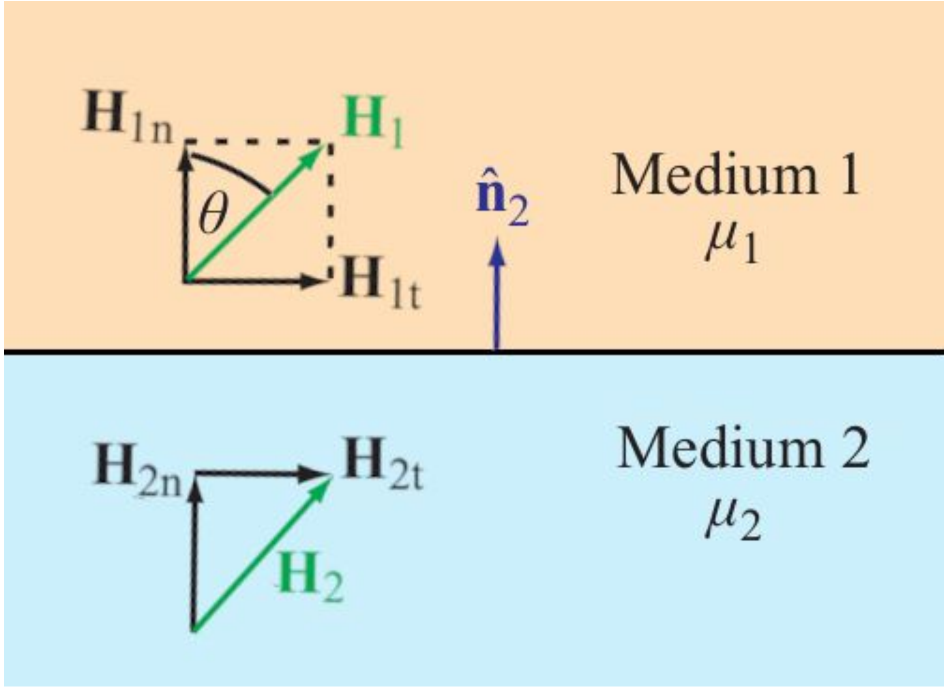
Solution:

First find the normal vector:

$$\hat{\mathbf{n}}_2 = \frac{\nabla(2x + 3y - 4z)}{|\nabla(2x + 3y - 4z)|}$$

$$\hat{\mathbf{n}}_2 = \frac{2\hat{\mathbf{x}} + 3\hat{\mathbf{y}} - 4\hat{\mathbf{z}}}{\sqrt{2^2 + 3^2 + 4^2}}$$

$$\hat{\mathbf{n}}_2 = 0.37\hat{\mathbf{x}} + 0.56\hat{\mathbf{y}} - 0.74\hat{\mathbf{z}}$$



Example 3

Solution:

$$\mathbf{H}_{1\text{normal}} = \hat{\mathbf{n}}_2[\mathbf{H}_1 \cdot \hat{\mathbf{n}}_2]$$

$$\mathbf{H}_{1\text{normal}} = (0.37\hat{\mathbf{x}} + 0.56\hat{\mathbf{y}} - 0.74\hat{\mathbf{z}})$$
$$[(50\hat{\mathbf{x}} - 30\hat{\mathbf{y}} + 20\hat{\mathbf{z}}) \cdot (0.37\hat{\mathbf{x}} + 0.56\hat{\mathbf{y}} - 0.74\hat{\mathbf{z}})]$$

$$\mathbf{H}_{1\text{normal}} = (0.37\hat{\mathbf{x}} + 0.56\hat{\mathbf{y}} - 0.74\hat{\mathbf{z}})$$
$$[(50)(0.37) + (-30)(0.56) + (20)(-0.74)]$$

$$\mathbf{H}_{1\text{normal}} = (0.37\hat{\mathbf{x}} + 0.56\hat{\mathbf{y}} - 0.74\hat{\mathbf{z}})[-13.1]$$

$$\boxed{\mathbf{H}_{1\text{normal}} = -4.85\hat{\mathbf{x}} - 7.33\hat{\mathbf{y}} + 9.7\hat{\mathbf{z}} \;\; \text{A/m}}$$

Example 3

Solution:

$$\mathbf{H}_{1\text{tangential}} = \mathbf{H}_1 - \mathbf{H}_{1\text{normal}}$$

$$\mathbf{H}_{1\text{tangential}} = (50\hat{\mathbf{x}} - 30\hat{\mathbf{y}} + 20\hat{\mathbf{z}}) - (-4.85\hat{\mathbf{x}} - 7.33\hat{\mathbf{y}} + 9.7\hat{\mathbf{z}})$$

$$\mathbf{H}_{1\text{tangential}} = 54.85\hat{\mathbf{x}} - 22.67\hat{\mathbf{y}} + 10.3\hat{\mathbf{z}} \ \ \text{A/m}$$

Example 3

Solution:

Normal **B** is continuous: $\mu_{r2}\mathbf{H}_{2normal} = \mu_{r1}\mathbf{H}_{1normal}$

$$\mathbf{H}_2 = \mathbf{H}_{2\text{tangential}} + \mathbf{H}_{2\text{normal}}$$

$$\mathbf{H}_2 = \mathbf{H}_{1\text{tangential}} + \frac{\mu_1}{\mu_2}\mathbf{H}_{1\text{normal}}$$

$$\mathbf{H}_2 = (54.85\hat{\mathbf{x}} - 22.67\hat{\mathbf{y}} + 10.3\hat{\mathbf{z}}) + \frac{2}{5}[-4.85\hat{\mathbf{x}} - 7.33\hat{\mathbf{y}} + 9.7\hat{\mathbf{z}}]$$

$$\mathbf{H}_2 = 52.91\hat{\mathbf{x}} - 25.6\hat{\mathbf{y}} + 14.18\hat{\mathbf{z}} \ \ \mathrm{A/m}$$

Homework

Homework 20 is due tomorrow at midnight.

submit to gradescope via the canvas site.

Next Time

Sections 5-7 through 5-8:

Inductance
Magnetic Energy