EECS 230
ENGINEERING ELECTROMAGNETICS
Leland Pierce

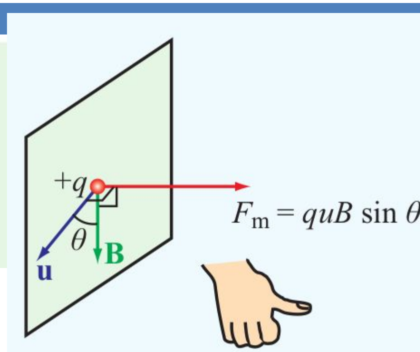
Magnetostatics 2

University of Michigan - Fall 2024

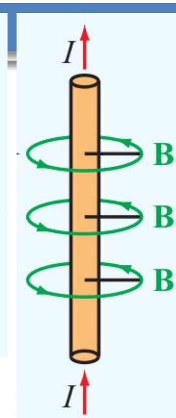
Chapter 5 Overview

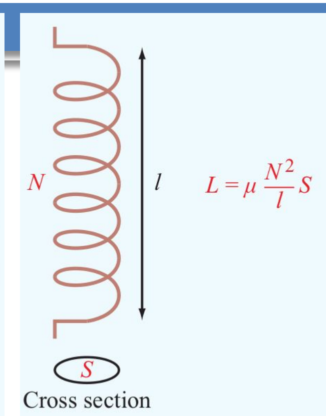
- Maxwell's Equations
  - Magnetostatics
- Magnetic Force
- Magnetic Torque
- Magnetic field from currents
- Gauss's Law for Magnetism
- Ampere's Law
- Magnetic Vector Potential Fi
  - Poisson's eqn
  - Magnetic Flux
- Magnetic Permeability
  - Hysteresis
- Magnetic Boundary Conditions
- Inductance
- Magnetic Energy

$$\nabla \cdot \mathbf{B} = 0,$$
$$\nabla \times \mathbf{H} = \mathbf{J},$$



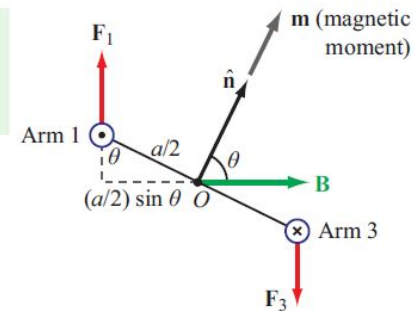
$$\mathbf{F}_m = q\mathbf{u} \times \mathbf{B}$$





$$\nabla \cdot \mathbf{B} = 0 \quad \longleftrightarrow \quad \oint_S \mathbf{B} \cdot d\mathbf{s} = 0.$$

$$\nabla \times \mathbf{H} = \mathbf{J} \quad \longleftrightarrow \quad \oint \mathbf{H} \cdot d\boldsymbol{\ell} = I,$$

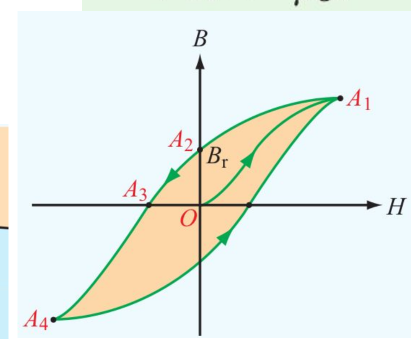


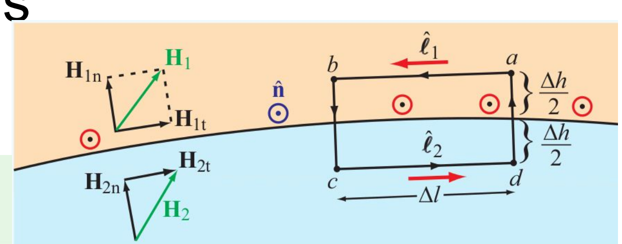
$$\mathbf{B} = \nabla \times \mathbf{A} \qquad (\text{Wb/m}^2),$$

$$\nabla^2 \mathbf{A} = -\mu \mathbf{J}.$$

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s} \qquad (\text{Wb}).$$

$$\mathbf{B} = \mu \mathbf{H},$$





$$w_m = \frac{W_m}{\upsilon} = \frac{1}{2}\mu H^2 \qquad (\text{J/m}^3).$$

Lecture Coverage

Today's lecture:

Review of Sections 5-1 and 5-2 of the book:

5-1: Magnetostatics:
Magnetic Forces and Torques
5-2: **H** due to a steady current (Biot-Savart Law)

Sections 5-3 through 5-4 of the book:

5-3: Magnetic Field from Currents
Ampere's Law
5-4: Magnetic Vector Potential Field
Poisson's eqn
Magnetic Flux

Chapter 5 Review

Static Conditions:

$$\frac{\partial \mathbf{J}}{\partial t} = 0$$

Magnetostatics:

$$\nabla \cdot \mathbf{B} = 0,$$

$$\nabla \times \mathbf{H} = \mathbf{J}.$$

magnetic flux density **B**

magnetic field intensity **H**

$$\mathbf{B} = \mu \mathbf{H}.$$

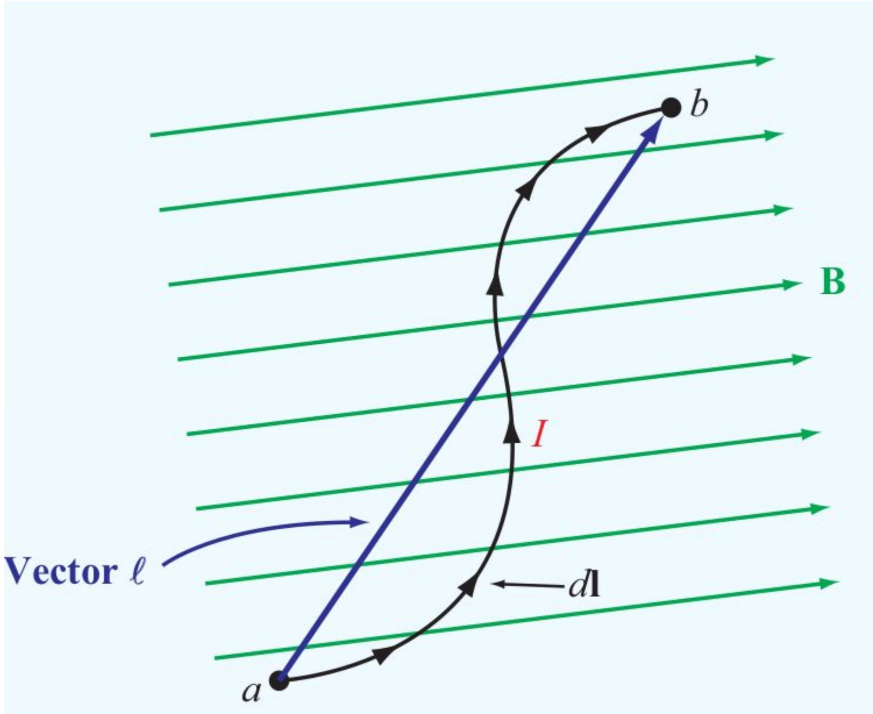
J is the current density

Chapter 5 Review

Magnetic force $\mathbf{F}_m = q\mathbf{u} \times \mathbf{B}$ (N)

Force on a current in a wire:
If **part** of wire is in uniform **B**:

$$\mathbf{F}_m = I\left(\int_\ell d\mathbf{l}\right) \times \mathbf{B} = I\boldsymbol{\ell} \times \mathbf{B},$$



Chapter 5 Review

Torque for a loop with N turns, and surface normal $\hat{\mathbf{n}}$ at angle θ relative to B direction:

$$T = N I A B_0 \sin\theta.$$

magnetic moment of the loop:

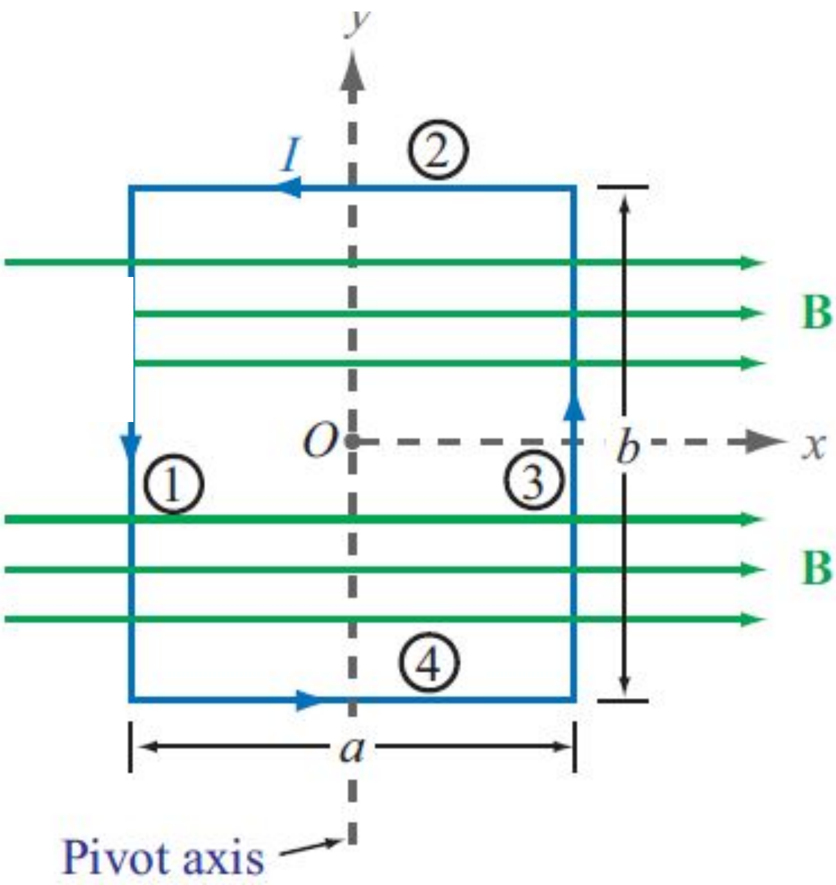
$$m = N I A$$

define:

$$\mathbf{m} = \hat{\mathbf{n}} N I A = \hat{\mathbf{n}} m \qquad (\text{A·m}^2),$$

so:

$$\mathbf{T} = \mathbf{m} \times \mathbf{B} \qquad (\text{N·m}).$$

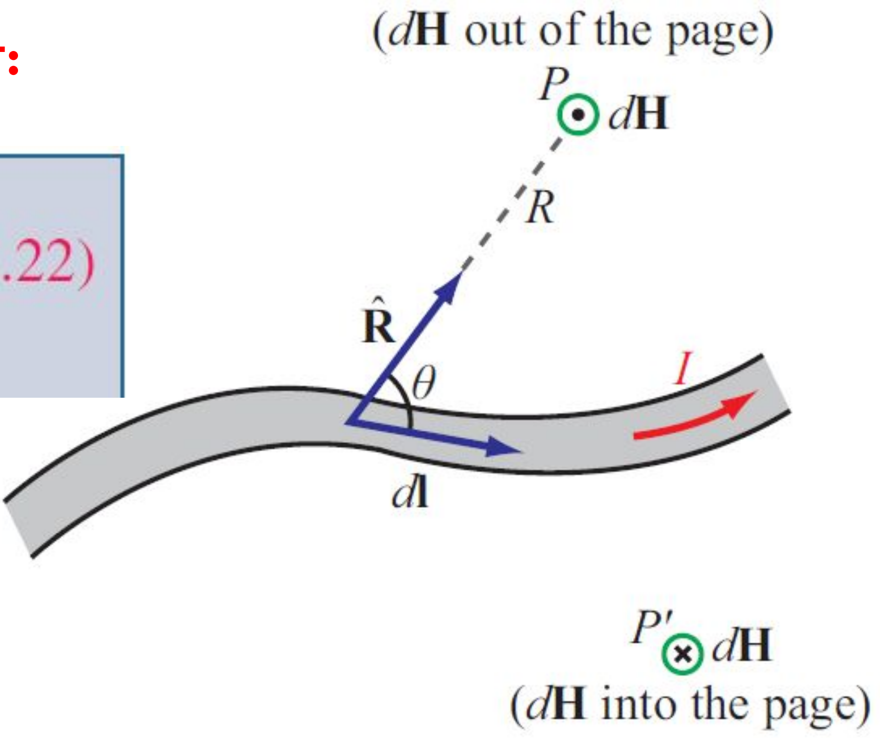


Chapter 5 Review

Biot-Savart Law:

Magnetic field induced by a current:

$$\mathbf{H} = \frac{I}{4\pi} \int_l \frac{d\mathbf{l} \times \hat{\mathbf{R}}}{R^2} \qquad \text{(A/m)}, \qquad (5.22)$$



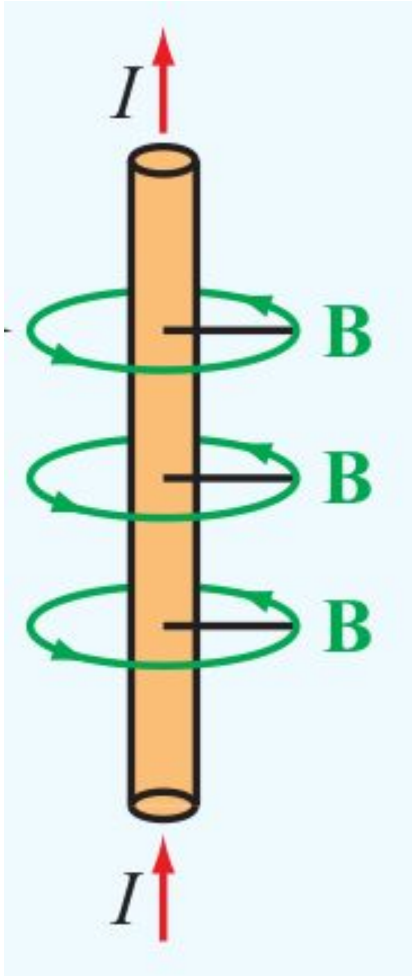
Chapter 5 Review

B due to current in straight wire:

$$\mathbf{B} = \mu_0 \mathbf{H} = \hat{\boldsymbol{\phi}} \frac{\mu_0 I l}{2\pi r\sqrt{4r^2 + l^2}}$$

For a very long wire:
In the limit as $l \to \infty$

$$\mathbf{B} = \hat{\boldsymbol{\phi}} \frac{\mu_0 I}{2\pi r}$$



Chapter 5 Review

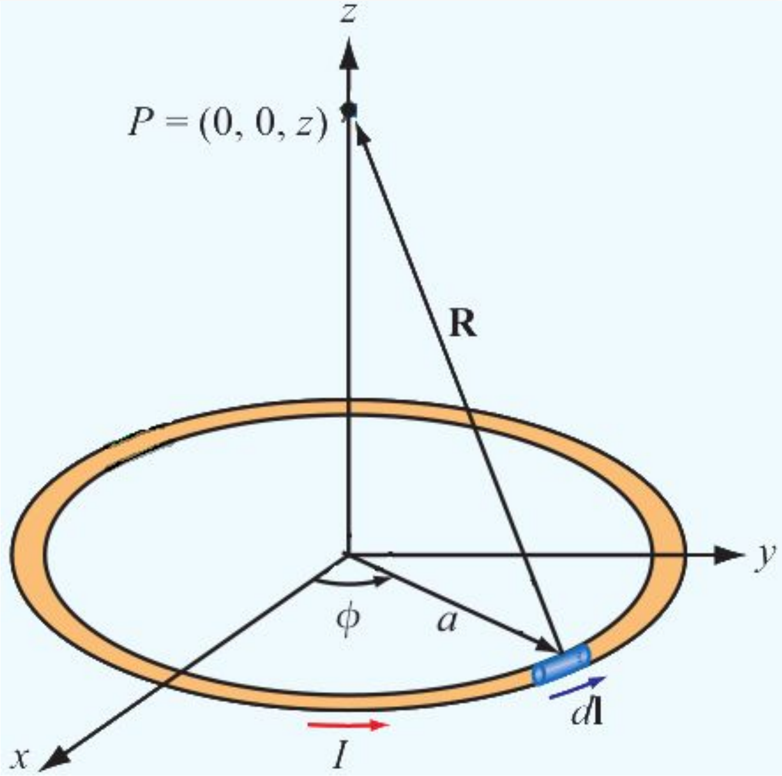
B along *z*-axis, due to circular current loop:

Since the magnetic moment of a loop in the *x*-*y* plane is:

$$\mathbf{m} = \hat{\mathbf{z}} I A$$

the magnetic moment of this loop is:

$$\mathbf{m} = \hat{\mathbf{z}} I \pi a^2$$



so:

$$\mathbf{H} = \frac{I a^2}{2R^3}\hat{\mathbf{z}} = \frac{\mathbf{m}}{2\pi R^3}\hat{\mathbf{z}}$$

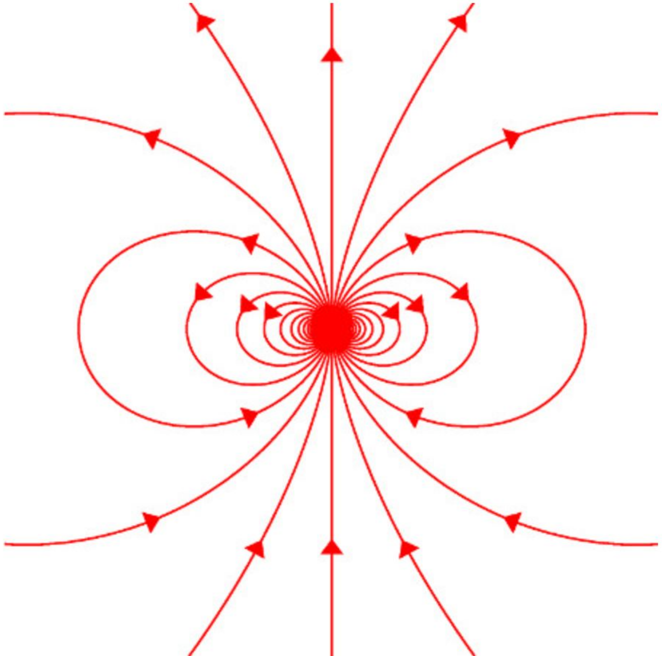
$$R = \sqrt{a^2 + z^2}$$

Chapter 5 Review

Magnetic Dipole:

Solving for the fields everywhere far from a current loop results in:

$$\mathbf{H} = \frac{m}{4\pi R^3}\left(\hat{\mathbf{R}}\,2\cos\theta + \hat{\boldsymbol{\theta}}\sin\theta\right) \qquad (\text{for } R \gg a).$$



(physics.stackexchange.com)

Chapter 5 Review

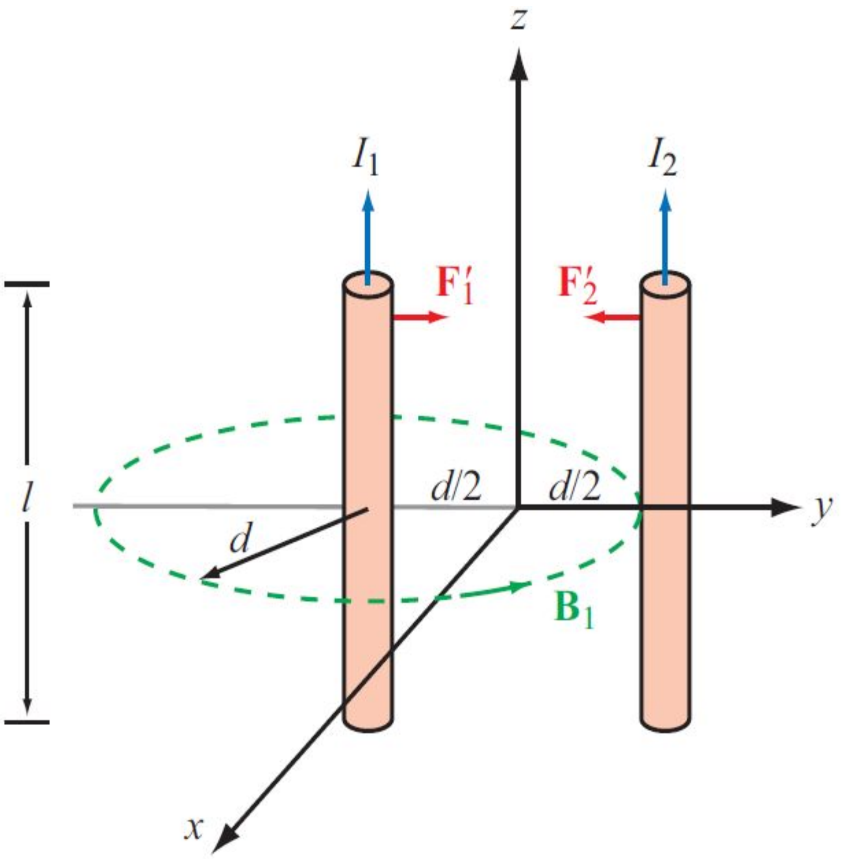
Force due to Parallel Currents:

So the force per unit length fro wire1 on wire2 is:

$$\mathbf{F}'_2 = -\hat{\mathbf{y}}\,\frac{\mu_0 I_1 I_2}{2\pi d}$$

similarly, from wire2 on wire1:

$$\mathbf{F}'_1 = \hat{\mathbf{y}}\,\frac{\mu_0 I_1 I_2}{2\pi d}$$



This force pulls the two wires together.

5-3 Maxwell's Magnetostatic Equations

Gauss's Law for Magnetism:

In Electrostatics: $\nabla \cdot \mathbf{D} = \rho_v \quad \leftrightarrow \quad \oint_S \mathbf{D} \cdot d\mathbf{s} = Q.$

In Magnetostatics: $\nabla \cdot \mathbf{B} = 0 \quad \leftrightarrow \quad \oint_S \mathbf{B} \cdot d\mathbf{s} = 0.$

This says there are no "magnetic charges" (also called: "magnetic monopoles")

Also means that magnetic field lines always form continuous closed loops.

5-3 Maxwell's Magnetostatic Equations

Ampère's Law:

In Electrostatics: $\nabla \times \mathbf{E} = 0 \quad \Leftrightarrow \quad \oint_C \mathbf{E} \cdot d\boldsymbol{\ell} = 0.$

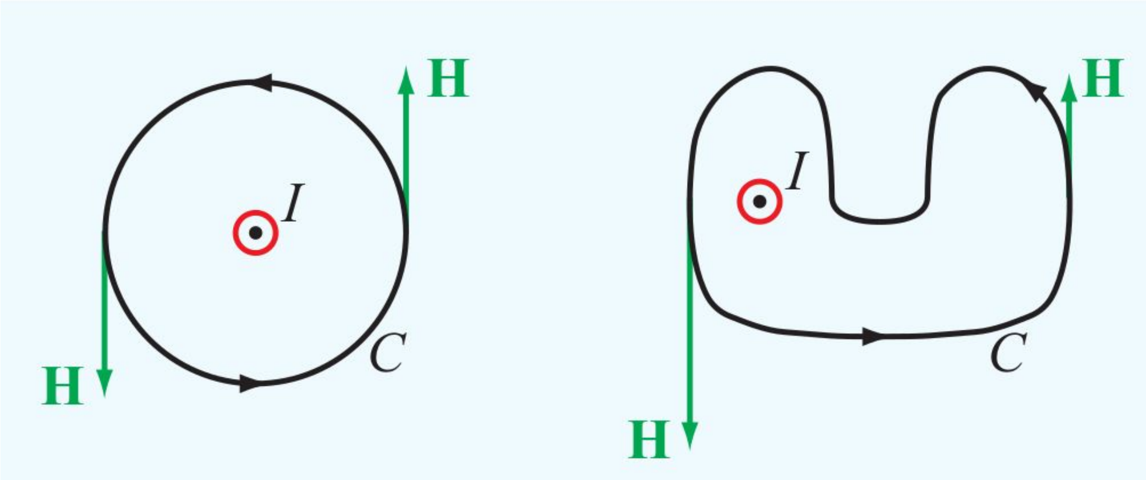
In Magnetostatics: $\nabla \times \mathbf{H} = \mathbf{J} \quad \Leftrightarrow \quad \oint_C \mathbf{H} \cdot d\boldsymbol{\ell} = I,$

I is the current **crossing** the surface of the contour *C*

Using the right-hand rule, with the thumb pointing along the direction of *C*:
Current is positive when it points along the direction of the fingers.

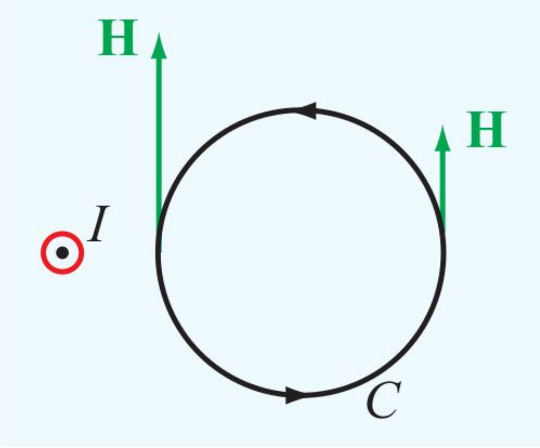
5-3 Maxwell's Magnetostatic Equations

Ampère's Law: Examples:



$$\oint_C \mathbf{H} \cdot d\boldsymbol{\ell} = I,$$

Choose contour shape to make integral easier.

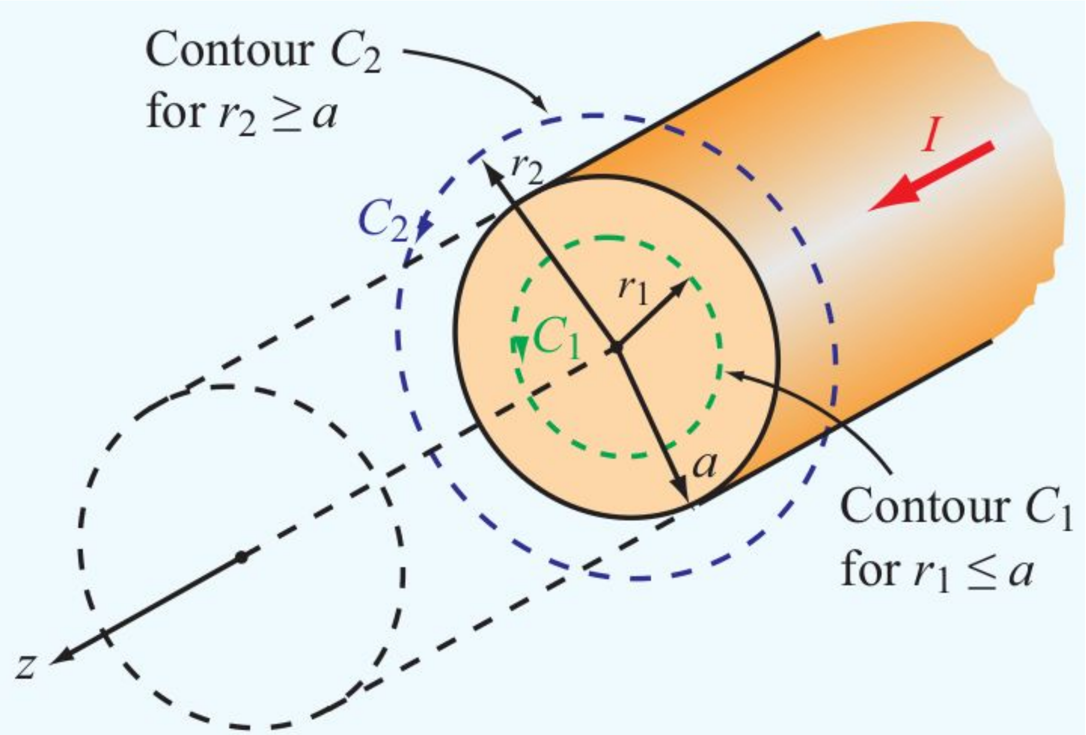


$$\oint_C \mathbf{H} \cdot d\boldsymbol{\ell} = 0.$$

Example 5-4

Magnetic Field of current in long straight wire

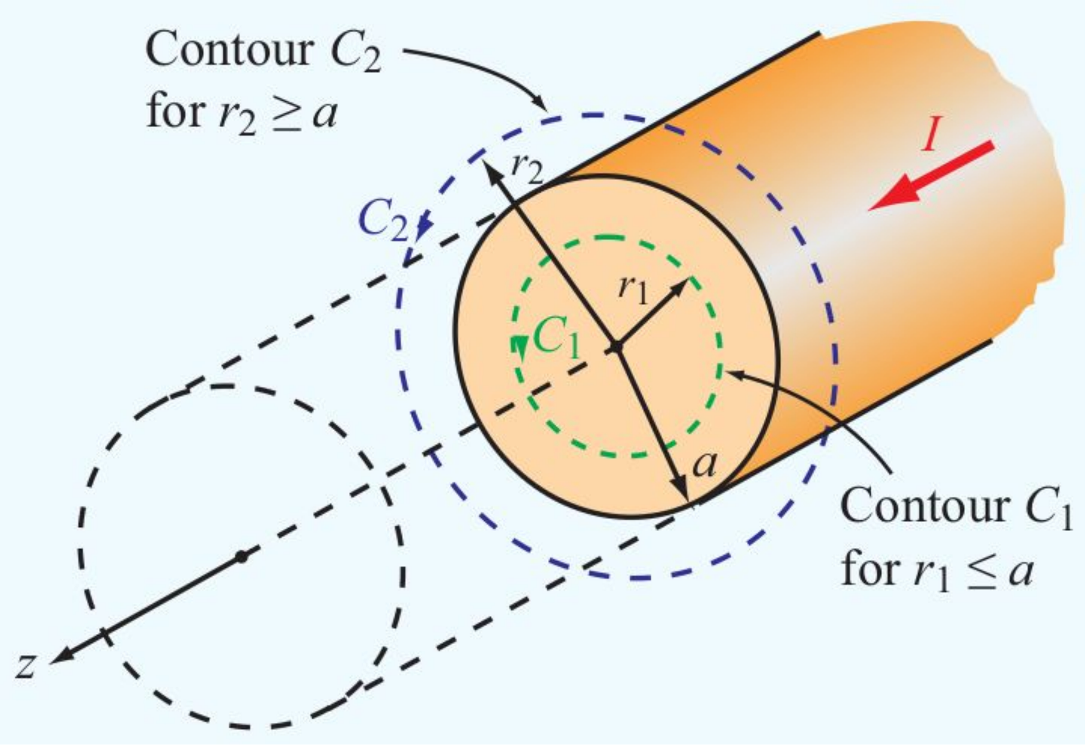
Given: wire radius: *a*
constant current, *I*
current uniformly distributed over the wire x-section



Find: **H**(r), r>0
includes both inside and outside the wire.

Example 5-4

Magnetic Field of current in long straight wire



Solution:

Use cylindrical coords:

$$\mathbf{I} = \hat{\mathbf{z}}\, I$$

within the wire:

choose circular path, C_1, with radius $r_1 < a$:

$$\oint_{C_1} \mathbf{H}_1 \cdot d\mathbf{l}_1 = I_1$$

Where I_1 is the fraction of the total current crossing C_1 within radius r_1.

Example 5-4

Magnetic Field of current in long straight wire

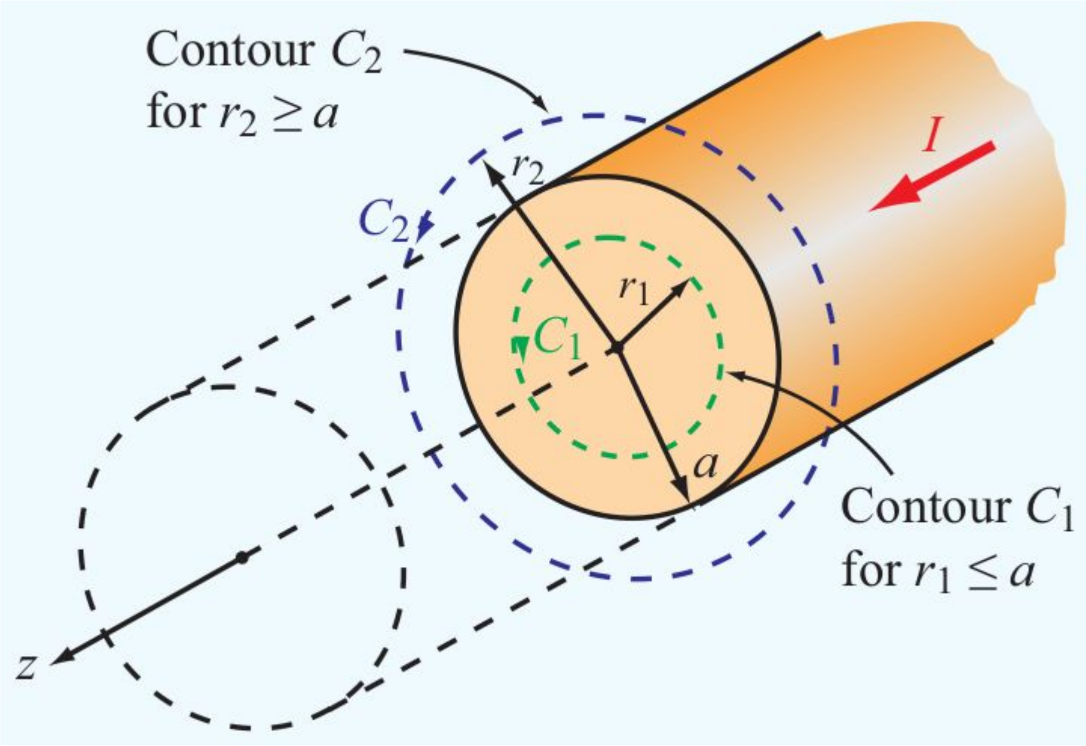
We already know that **H** must be of the form:

$$\mathbf{H}_1 = \hat{\boldsymbol{\phi}} H_1$$

with d**l** along the circle:

$$d\mathbf{l}_1 = \hat{\boldsymbol{\phi}} r_1 \, d\phi$$

so we get:

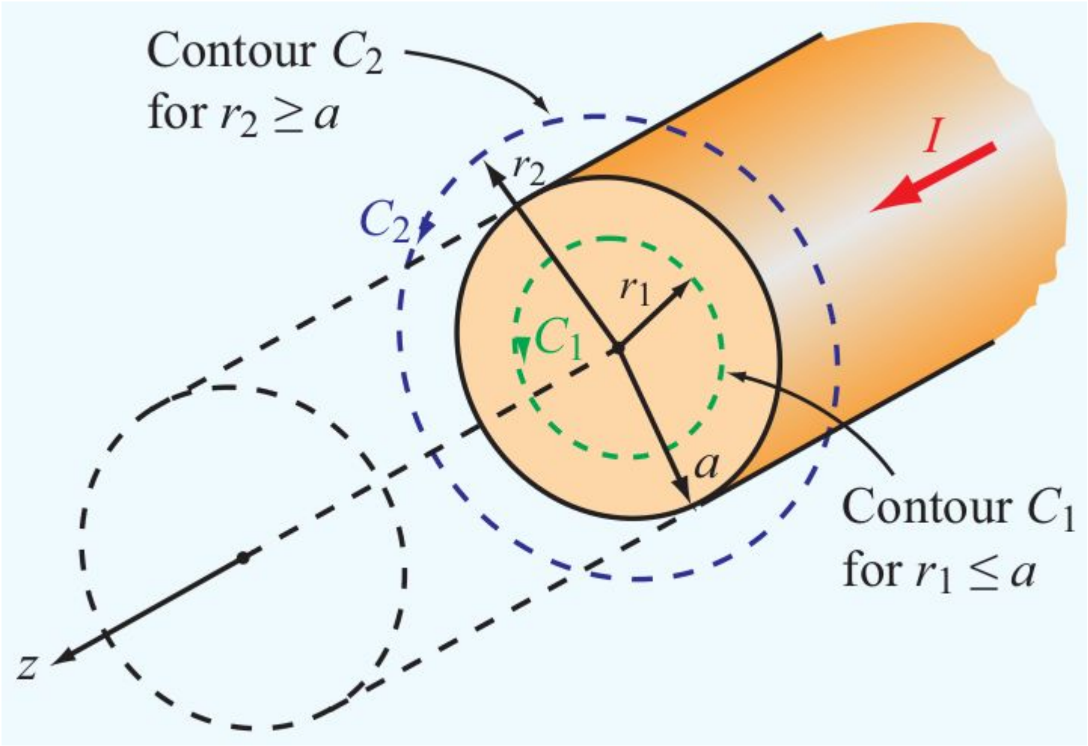


$$\oint_{C_1} \mathbf{H}_1 \cdot d\mathbf{l}_1 = \int_0^{2\pi} H_1 (\hat{\boldsymbol{\phi}} \cdot \hat{\boldsymbol{\phi}}) r_1 \, d\phi = 2\pi r_1 H_1$$

Example 5-4

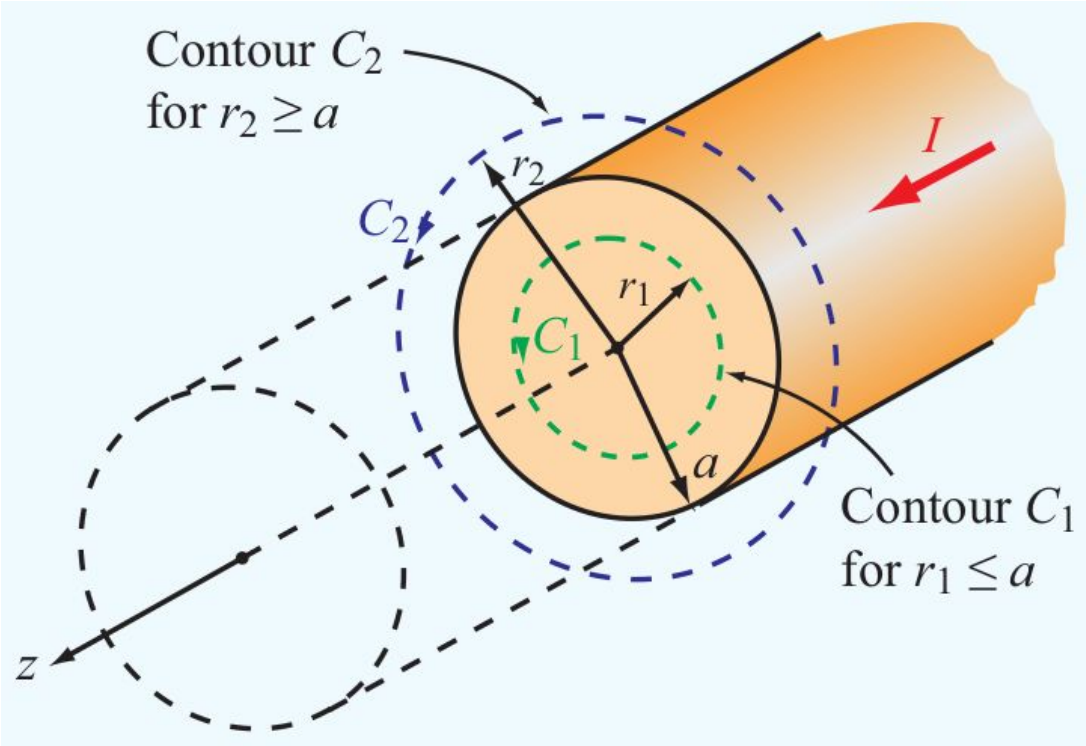
Magnetic Field of current in long straight wire

By the Ampère right-hand rule the current crossing C_1 is positive.



Example 5-4

Magnetic Field of current in long straight wire



Since I is uniform, I_1 is related to I by the ratio of the areas:

$$I_1 = \left(\frac{\pi r_1^2}{\pi a^2}\right) I = \left(\frac{r_1}{a}\right)^2 I$$

so:

$$2\pi r_1 H_1 = \frac{r_1^2}{a^2} I$$

then:

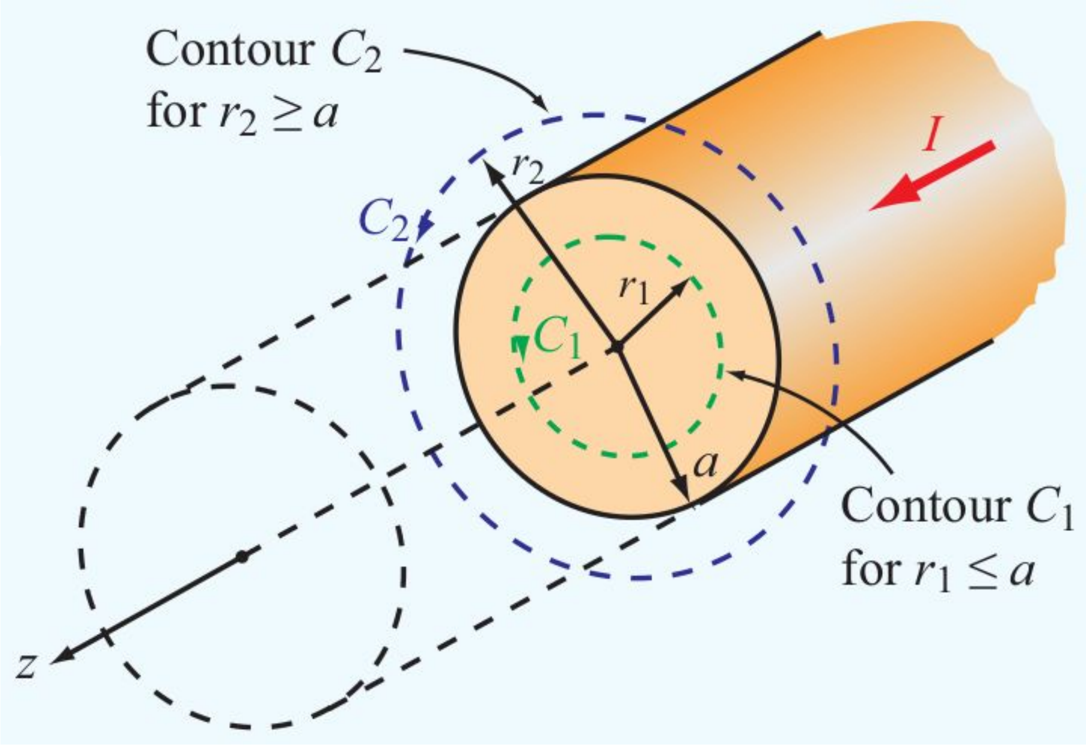
$$\mathbf{H}_1 = \hat{\boldsymbol{\phi}} H_1 = \hat{\boldsymbol{\phi}} \frac{r_1}{2\pi a^2} I \qquad (\text{for } r_1 \leq a)$$

Example 5-4

Magnetic Field of current in long straight wire

outside the wire:

$$\oint_{C_2} \mathbf{H}_2 \cdot d\mathbf{l}_2 = 2\pi r_2 H_2 = I$$



so:

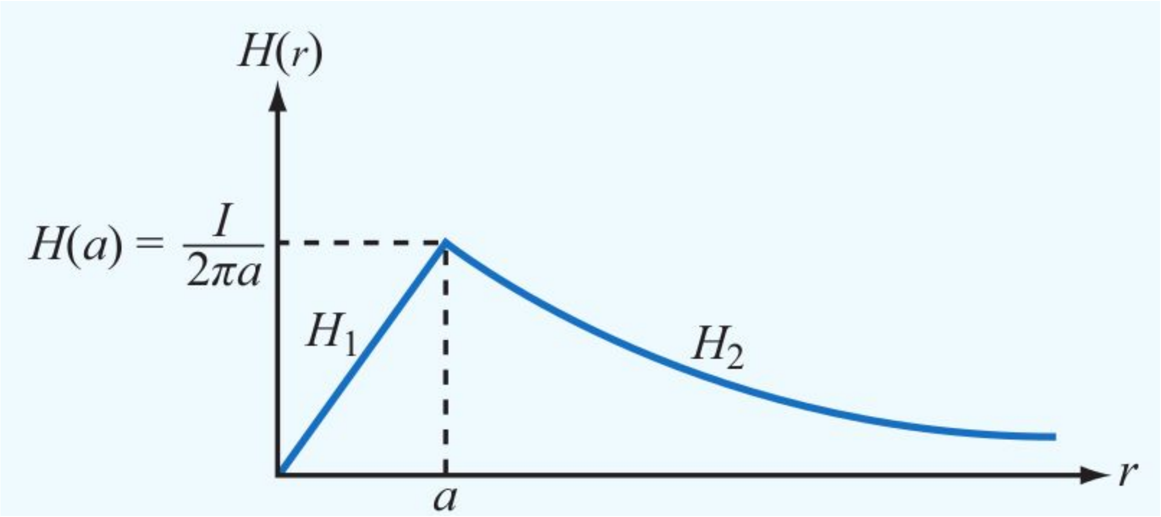
$$\mathbf{H}_2 = \hat{\boldsymbol{\phi}} H_2 = \hat{\boldsymbol{\phi}} \frac{I}{2\pi r_2} \qquad \text{(for } r_2 \geq a\text{)}$$

Example 5-4

Magnetic Field of current in long straight wire

Complete solution:

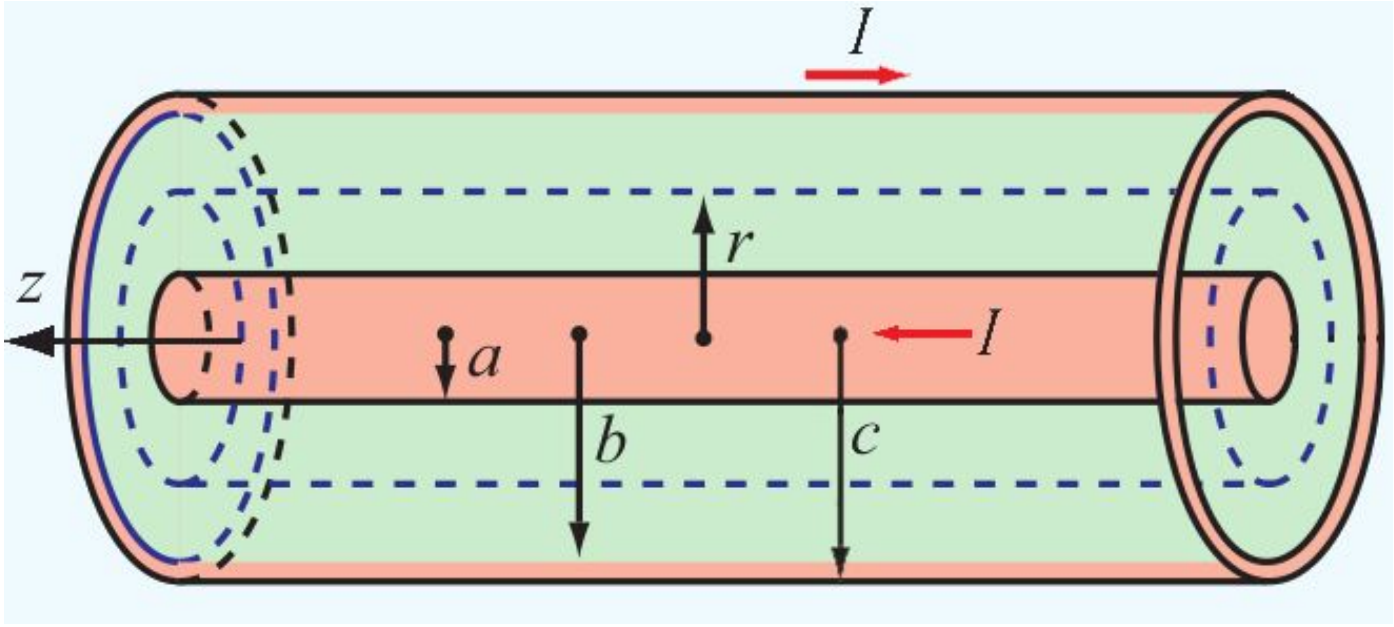
$$\mathbf{H}(r) = \begin{cases} \hat{\boldsymbol{\phi}}\,\dfrac{r}{2\pi a^2} I & 0 < r \le a \\ \hat{\boldsymbol{\phi}}\,\dfrac{1}{2\pi r} I & a \le r \end{cases}$$



Example 2

Magnetic Field of coaxial cable

Current in center conductor:
same magnitude, but
opposite direction of current
in outer conductor



Example 2

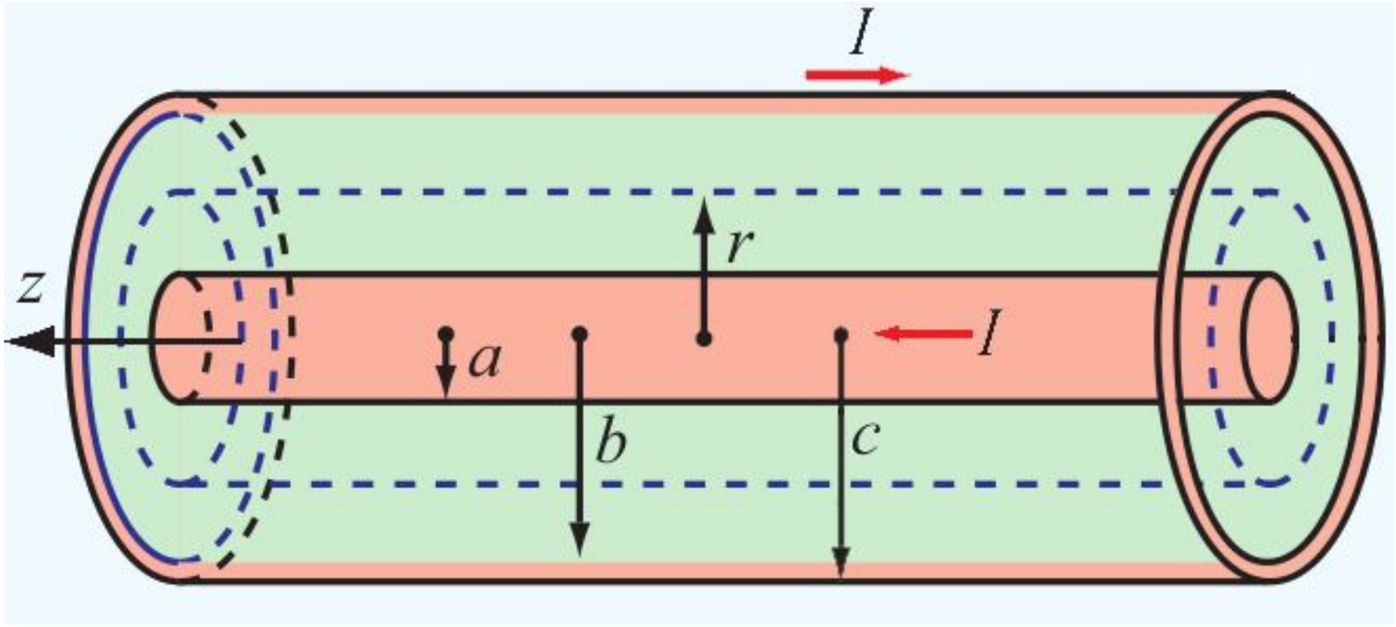
<u>Magnetic Field of coaxial cable</u>

Already solved part of this problem in Example 1:

1. Field of inner conductor
2. Field of insulation region

Still to find:

1. Field in outer conductor
2. Field outside cable



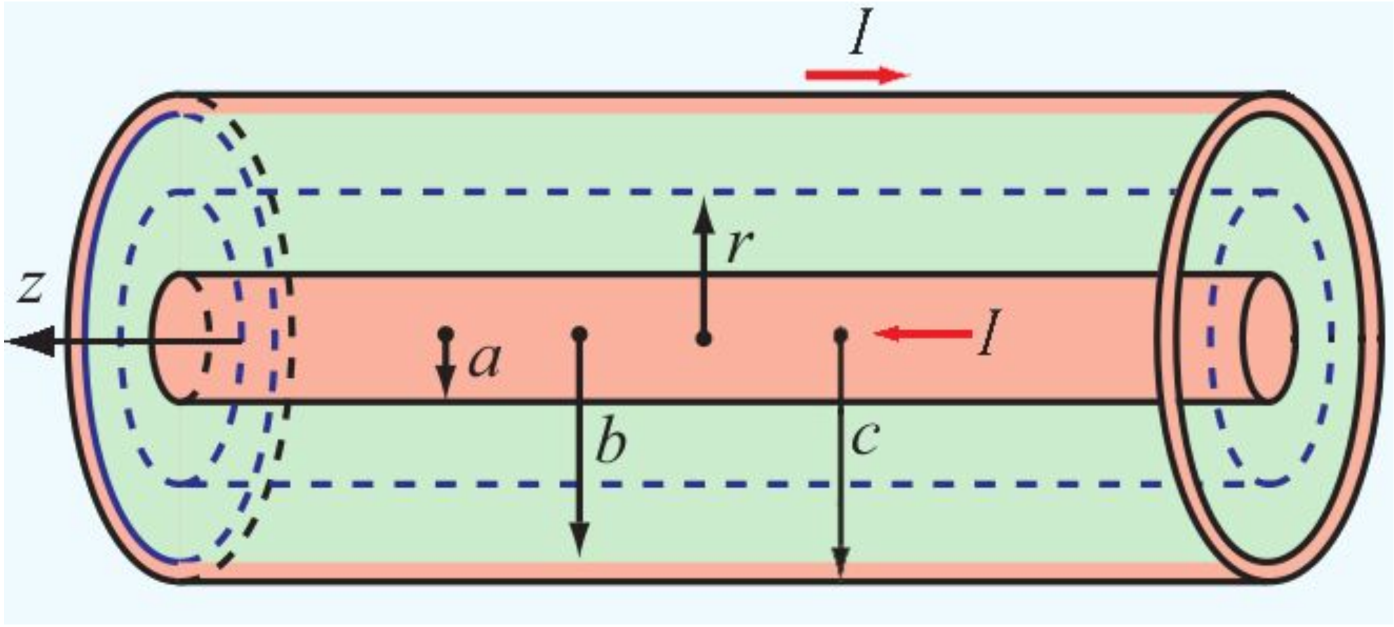
Example 2

<u>Magnetic Field of coaxial cable</u>

within the outer sheath:

choose circular path, *C*, with radius *b*<*r*<*c*:

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = I$$



Example 2

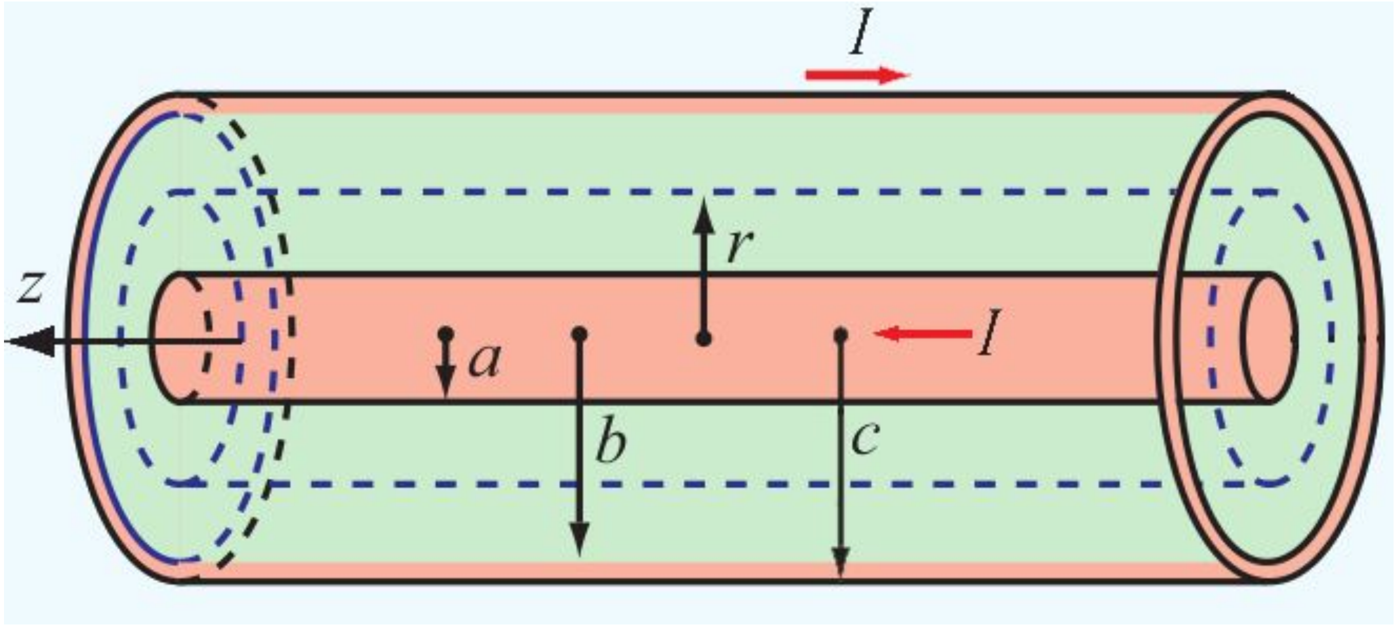
Magnetic Field of coaxial cable

determine the current enclosed:

+I from inner conductor

-I_2, where I_2 is related to I by the ratio of the areas:

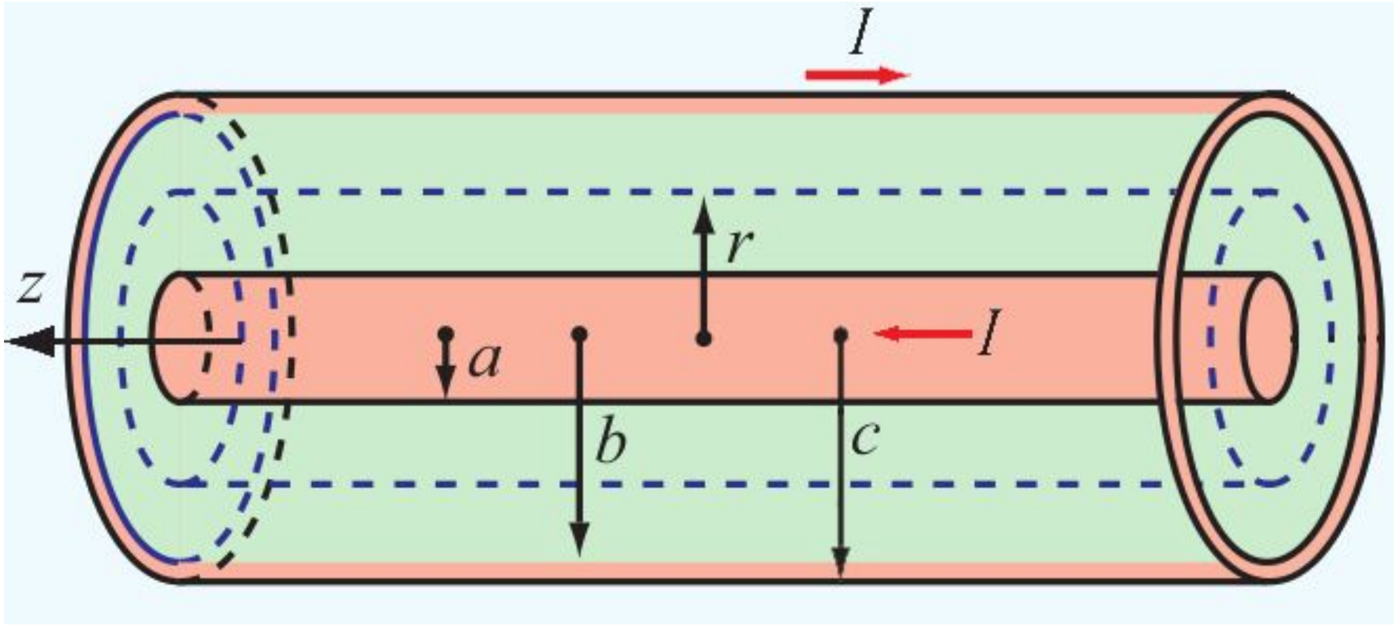
$$I_2 = \left[\frac{\pi r^2 - \pi b^2}{\pi c^2 - \pi b^2}\right] I$$



Example 2

Magnetic Field of coaxial cable

$$I_2 = \left[\frac{\pi r^2 - \pi b^2}{\pi c^2 - \pi b^2}\right] I \qquad I_{\text{enclosed}} = \left[1 - \frac{r^2 - b^2}{c^2 - b^2}\right] I$$

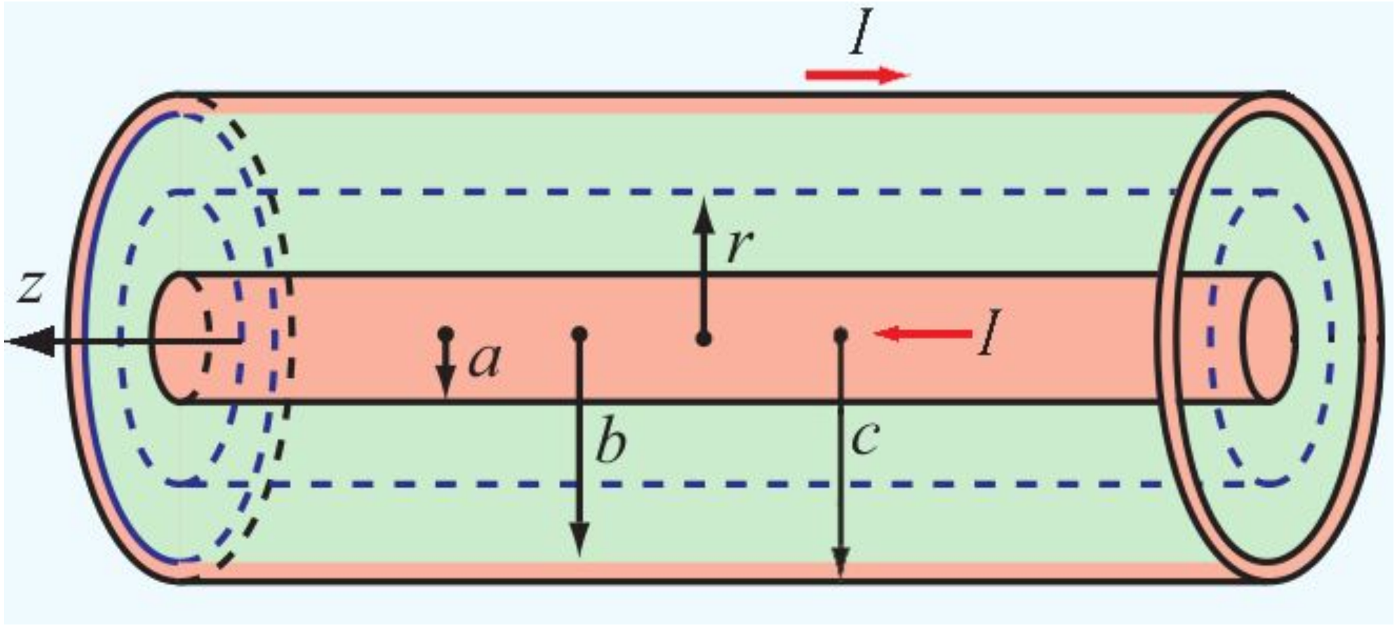


Example 2

Magnetic Field of coaxial cable

$$I_{\text{enclosed}} = \left[1 - \frac{r^2 - b^2}{c^2 - b^2}\right] I$$

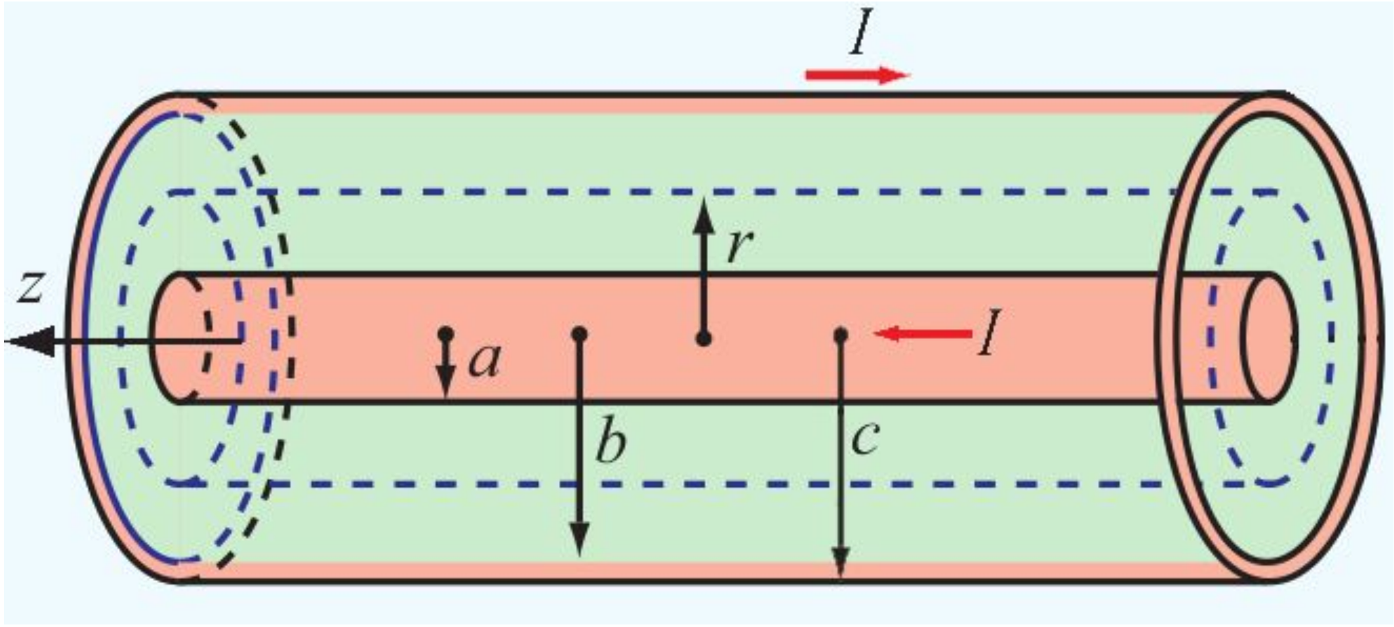
$$I_{\text{enclosed}} = \left[\frac{c^2 - r^2}{c^2 - b^2}\right] I$$



Example 2

Magnetic Field of coaxial cable

As in first example: $$\oint_C \mathbf{H} \cdot d\mathbf{l} = 2\pi r\, H_\phi$$

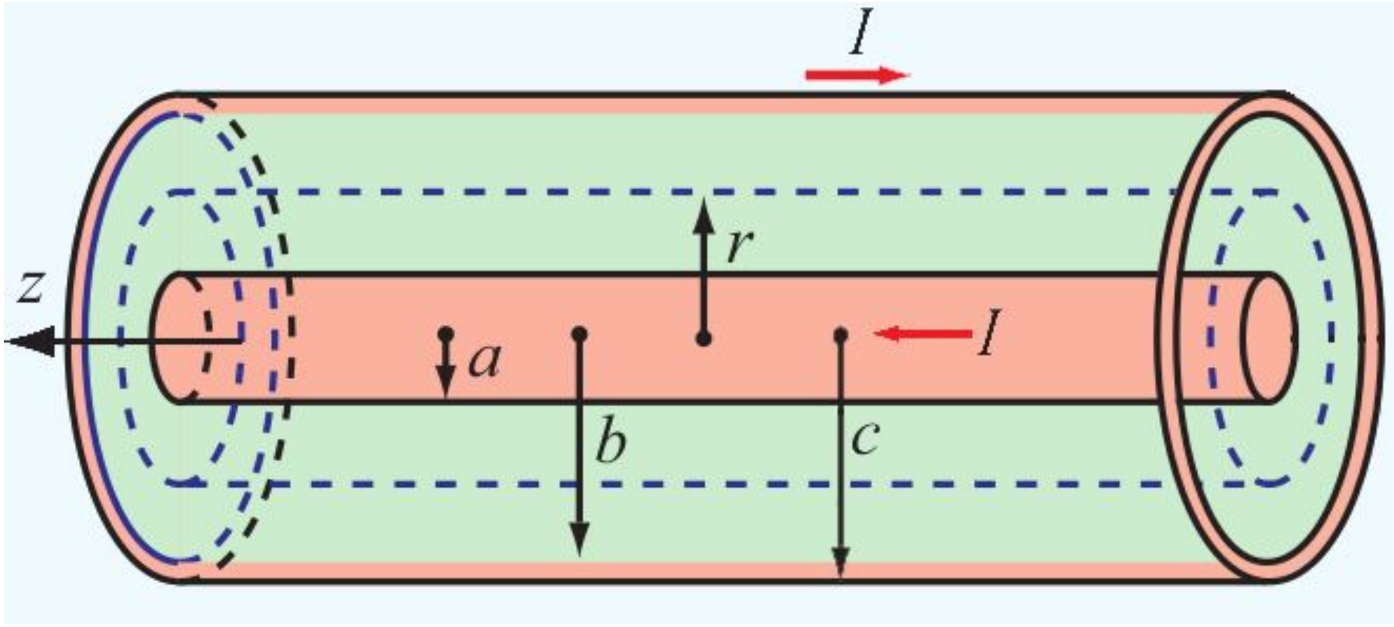


Example 2

Magnetic Field of coaxial cable in the outer conductor:

$$2\pi r\, H_\phi = \left[\frac{c^2 - r^2}{c^2 - b^2}\right] I$$

$$\mathbf{H} = \hat{\boldsymbol{\phi}}\, \frac{I}{2\pi r} \left[\frac{c^2 - r^2}{c^2 - b^2}\right]$$

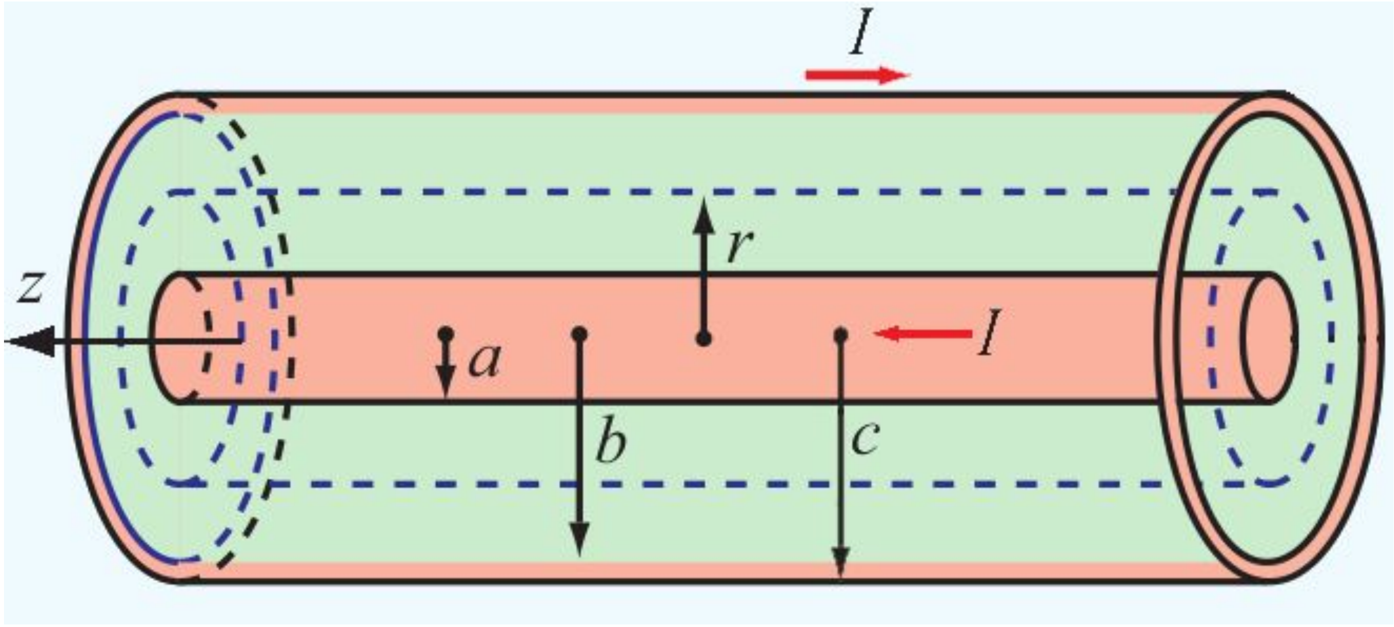


Example 2

Magnetic Field of coaxial cable

in the region outside the cable:

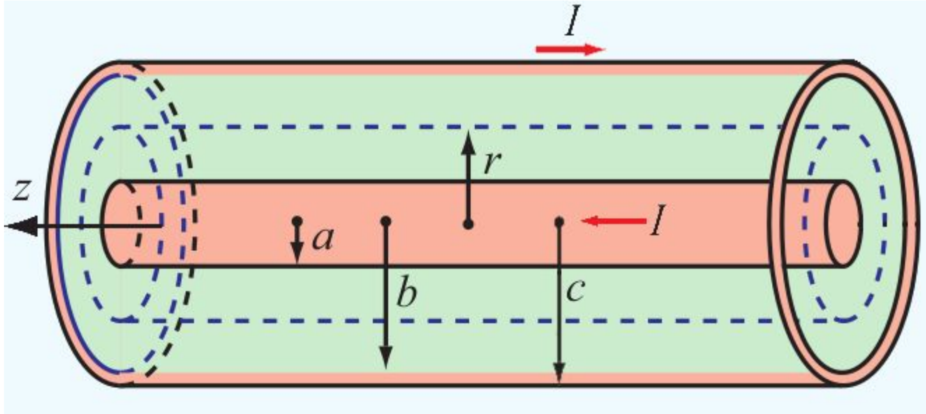
Current-enclosed is zero, so field is zero.



Example 2

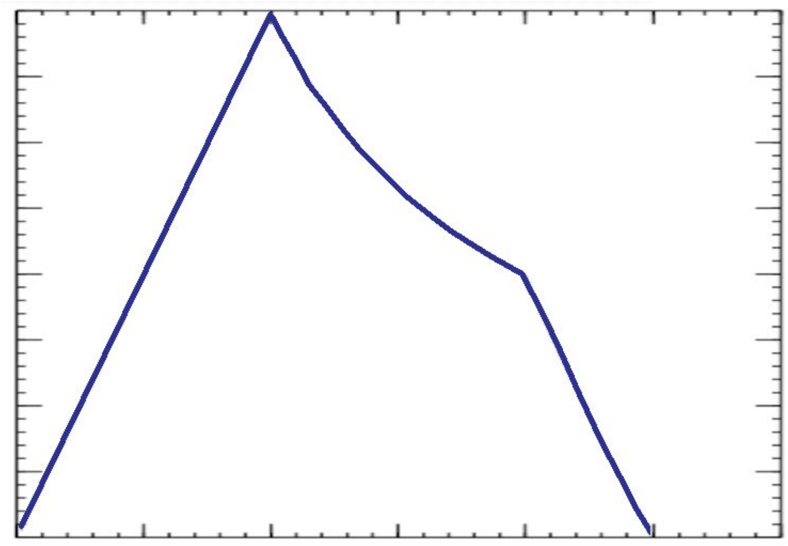
Magnetic Field of coaxial cable

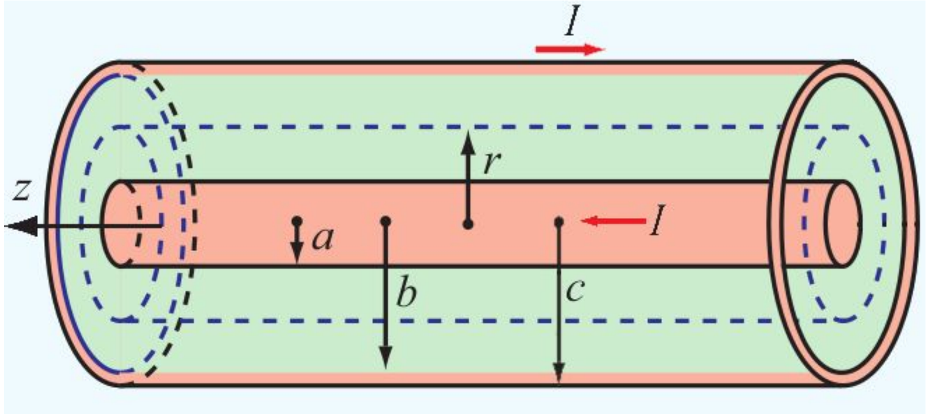
$$\mathbf{H} = \begin{cases} \hat{\boldsymbol{\phi}}\,\frac{r}{2\pi a^2} I & r < a \\ \hat{\boldsymbol{\phi}}\,\frac{1}{2\pi r} I & a < r < b \\ \hat{\boldsymbol{\phi}}\,\frac{1}{2\pi r}\left[\frac{c^2 - r^2}{c^2 - b^2}\right] I & b < r < c \\ 0 & r > c \end{cases}$$



Example 2

Magnetic Field of coaxial cable





Exercise 5-11

Maximum Current in a Superconductor

Given:
A Niobium wire of diameter 0.1mm.
It's a superconductor when cooled: $T < 9K$.

But if the magnetic field at its surface > 0.12 T
it's no longer a superconductor.

Find: Max current while still a superconductor

Exercise 5-11

Maximum Current in a Superconductor

Solution:

From Example 1: $$B = \mu_0 H = \frac{\mu_0}{2\pi a} I$$

$$I_{\max} = \frac{2\pi a B_{\max}}{\mu_0}$$

$$I_{\max} = \frac{(2\pi)(0.5 \times 10^{-3} m)(0.12 T)}{4\pi \times 10^{7} H/m}$$

$$\boxed{I_{\max} = 30 A}$$

Exercise 5-11

Maximum Current in a Superconductor

Why does this matter?

A 30A current in a 0.1mm diam copper wire would melt it

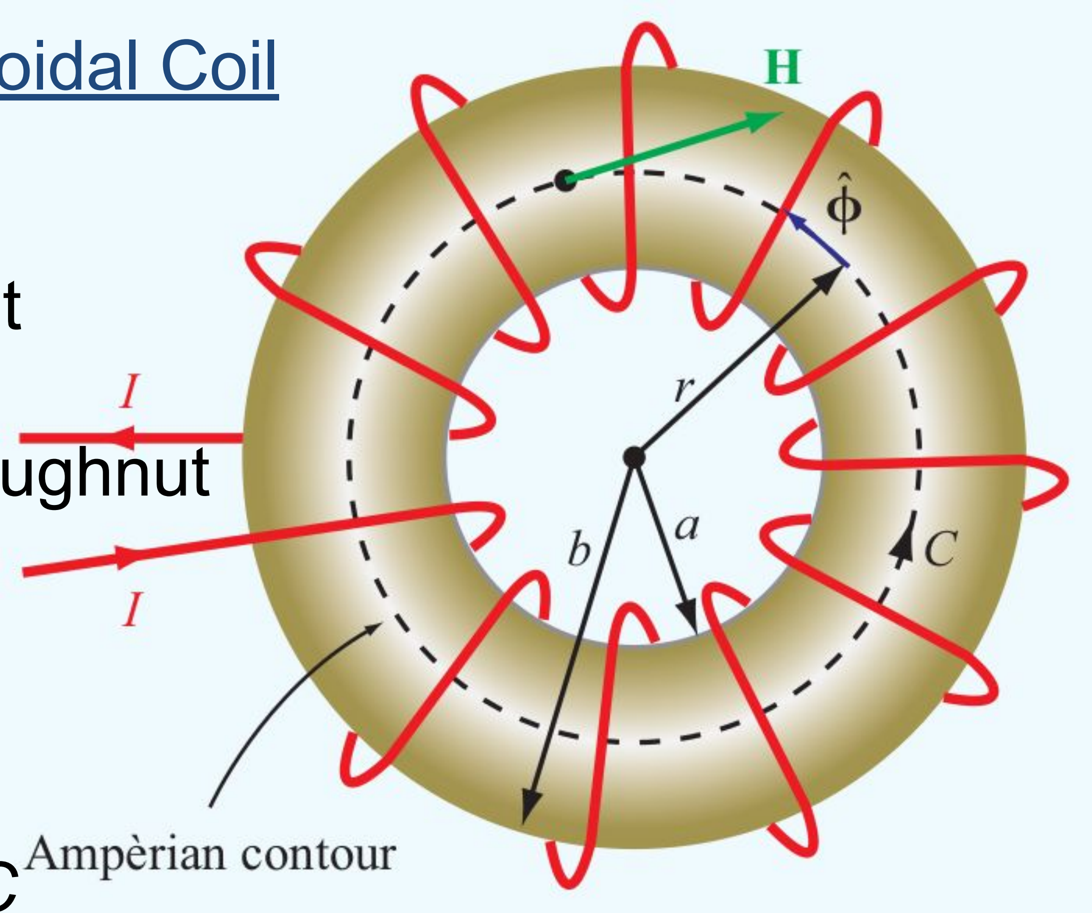
The niobium wire does not melt: no resistive heating

Can use superconductors to make electromagnets, etc
that are much smaller and more light-weight.
Just need to cool it.....

It's how they make MRI machines for medical imaging.

Example 5-5

Magnetic Field inside Toroidal Coil



Given: styrofoam doughnut
wire with current I
wrapped around doughnut
inner radius a
outer radius b

Find: **H** in the plane:
cuts thru centerline C
of the torus
in all 3 regions.

Example 5-5

Magnetic Field inside Toroidal Coil

Solution:

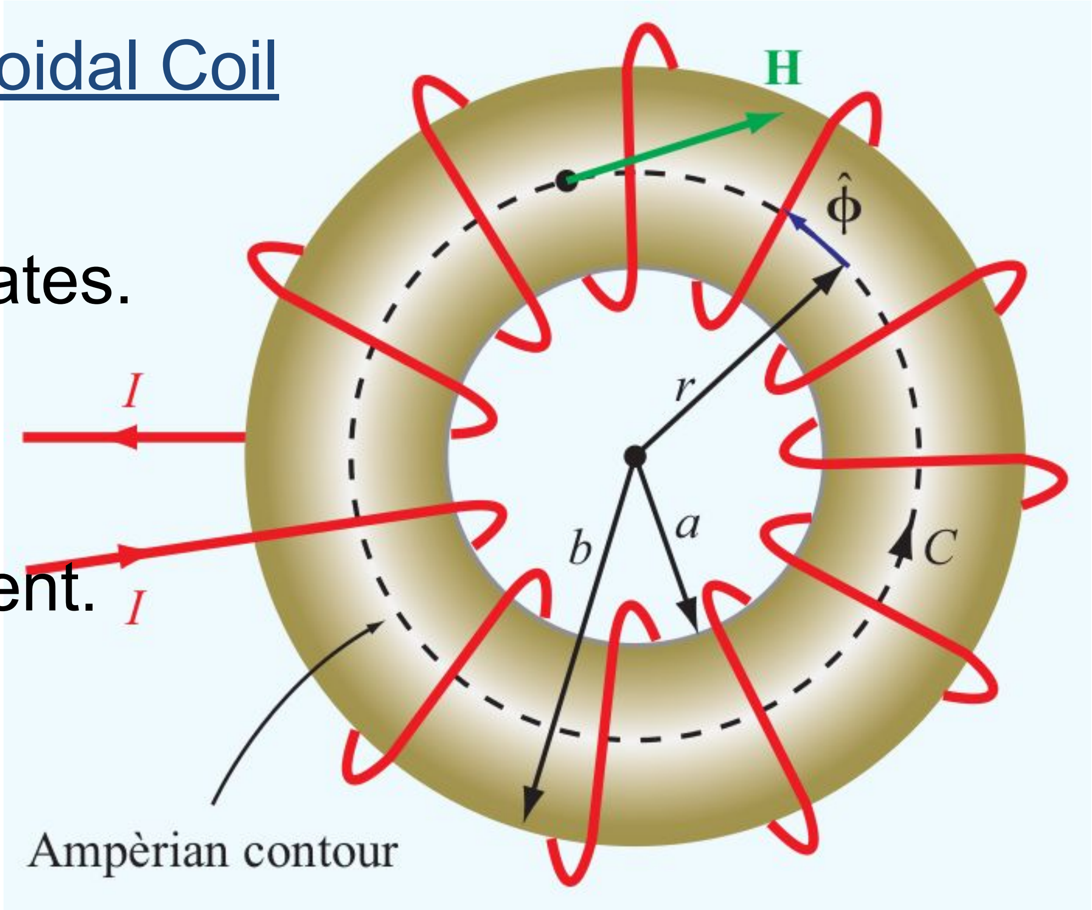
choose cylindrical coordinates.

For $r<a$:

circular contour

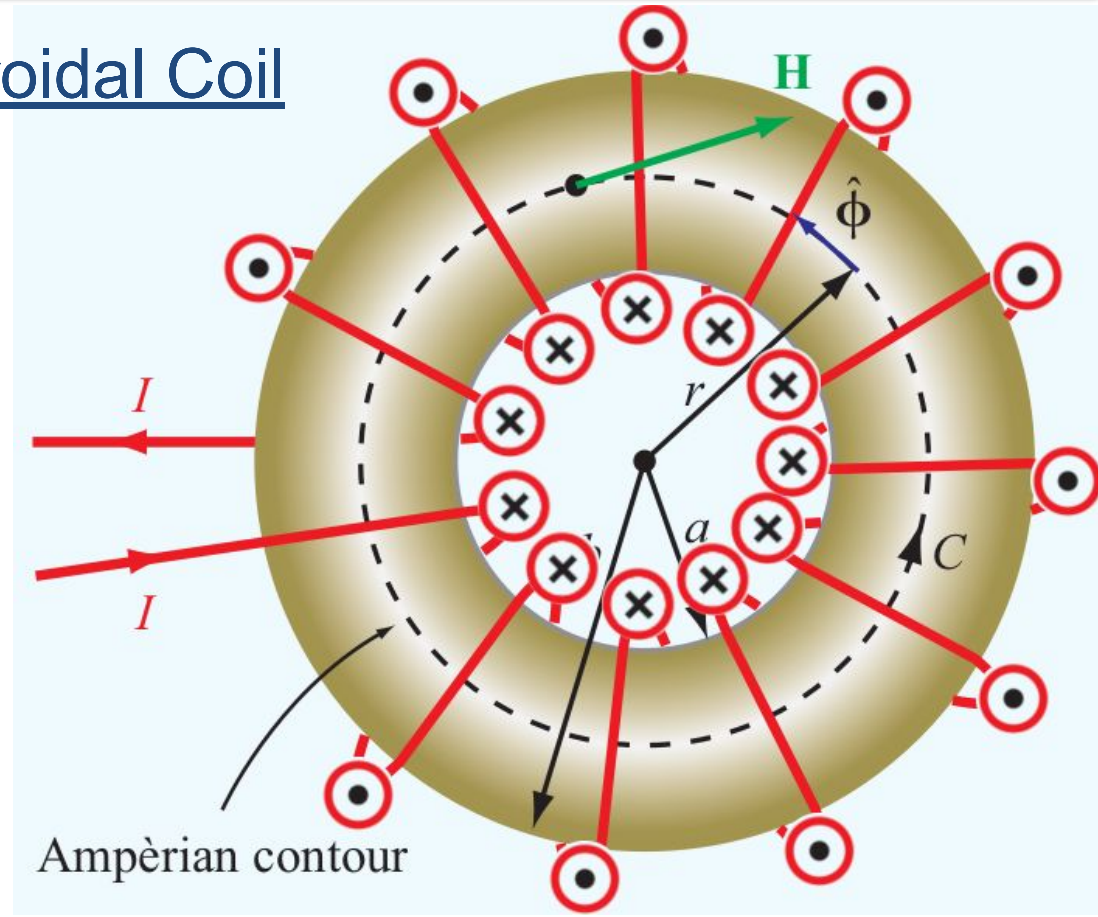
does not enclose any current.

$\mathbf{H}=0$



Example 5-5

Magnetic Field inside Toroidal Coil



For *r>b*:
equal current crosses the plane of the enclosing contour in both directions:

current into page: 

current out of page: 

net enclosed current is zero:
H=0

Example 5-5

Magnetic Field inside Toroidal Coil

Inside the toroid:

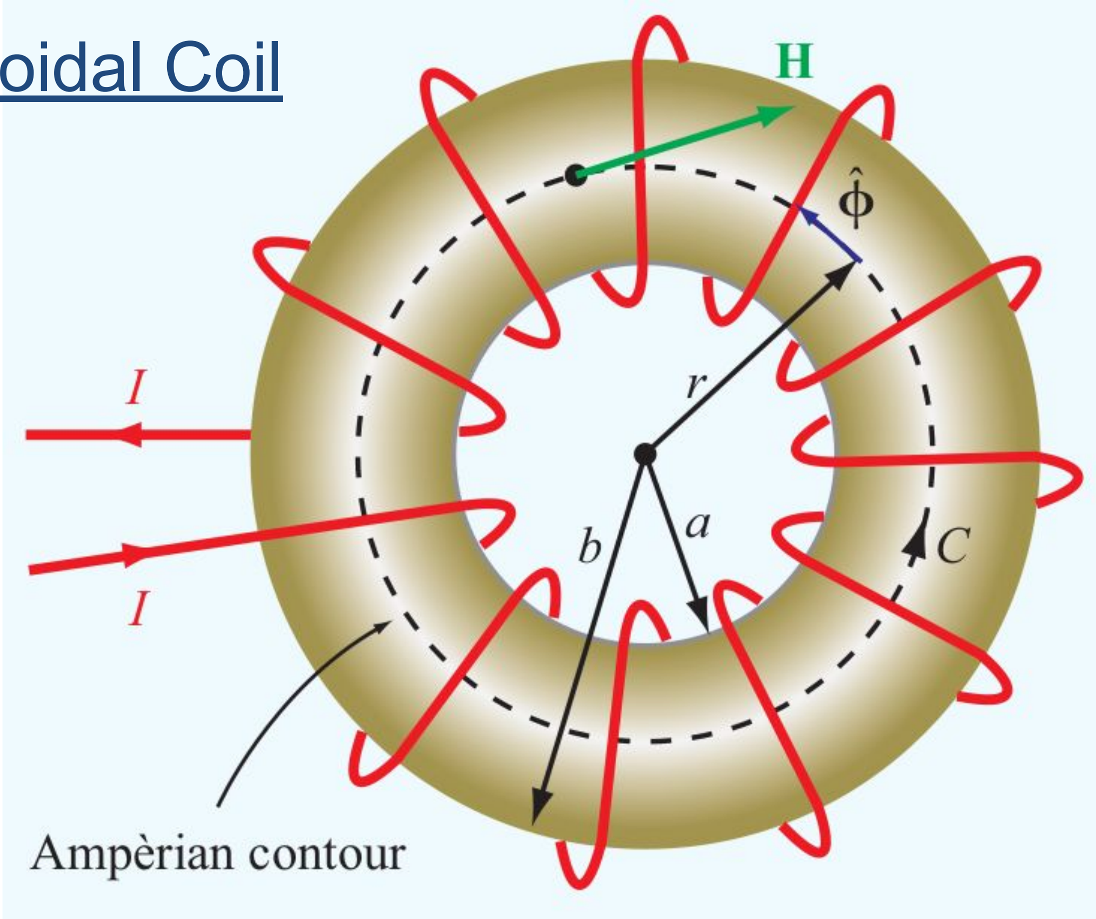
a<r<b

Given the direction of the current, using right-hand rule:

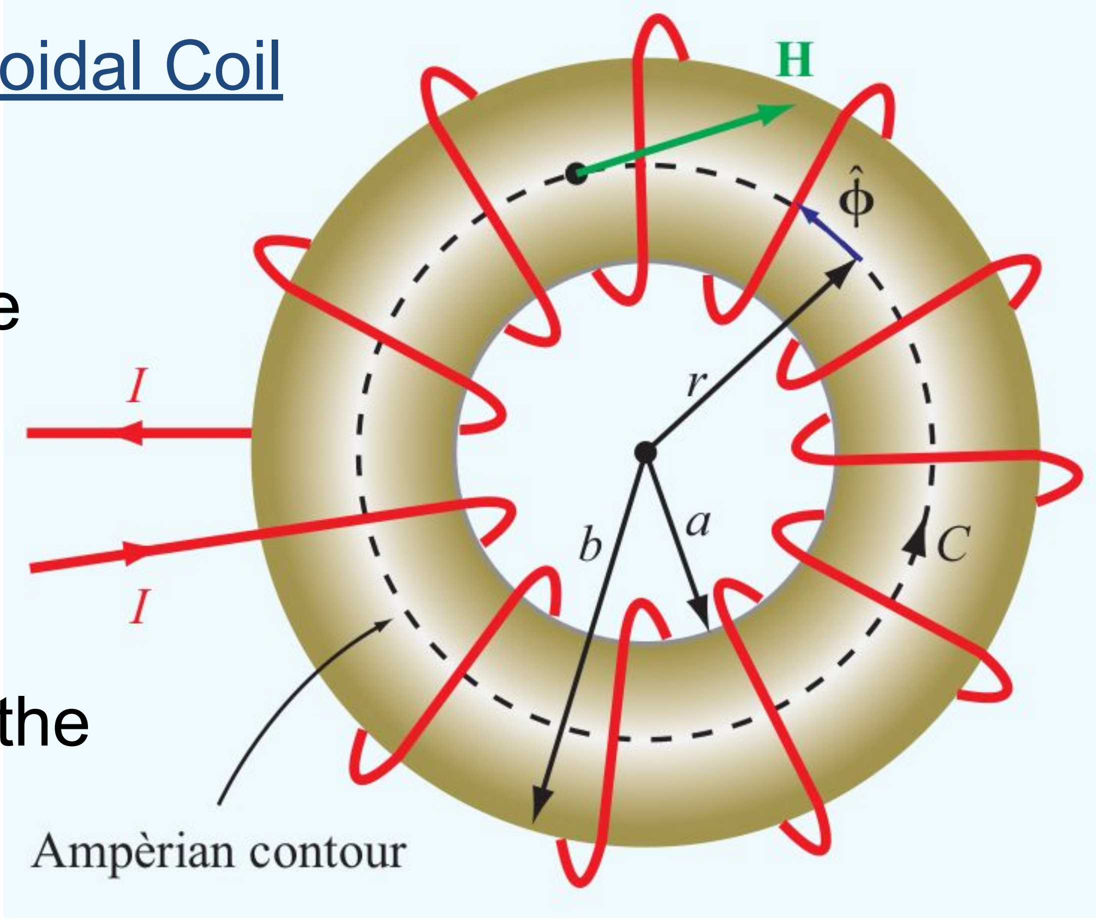
$$\mathbf{H} = -\hat{\boldsymbol{\phi}} H$$

and as in example 1:

$$d\mathbf{l} = \hat{\boldsymbol{\phi}} r \, d\phi$$



Example 5-5



[Magnetic Field inside Toroidal Coil]()

Given the direction of the contour, the right-hand rule for Ampère's law would have a positive current if it's **out of the page.**

Since the current crosses the contour *N* times, and is directed **into the page**:

Current enclosed is $-NI$

Example 5-5

<u>Magnetic Field inside Toroidal Coil</u>

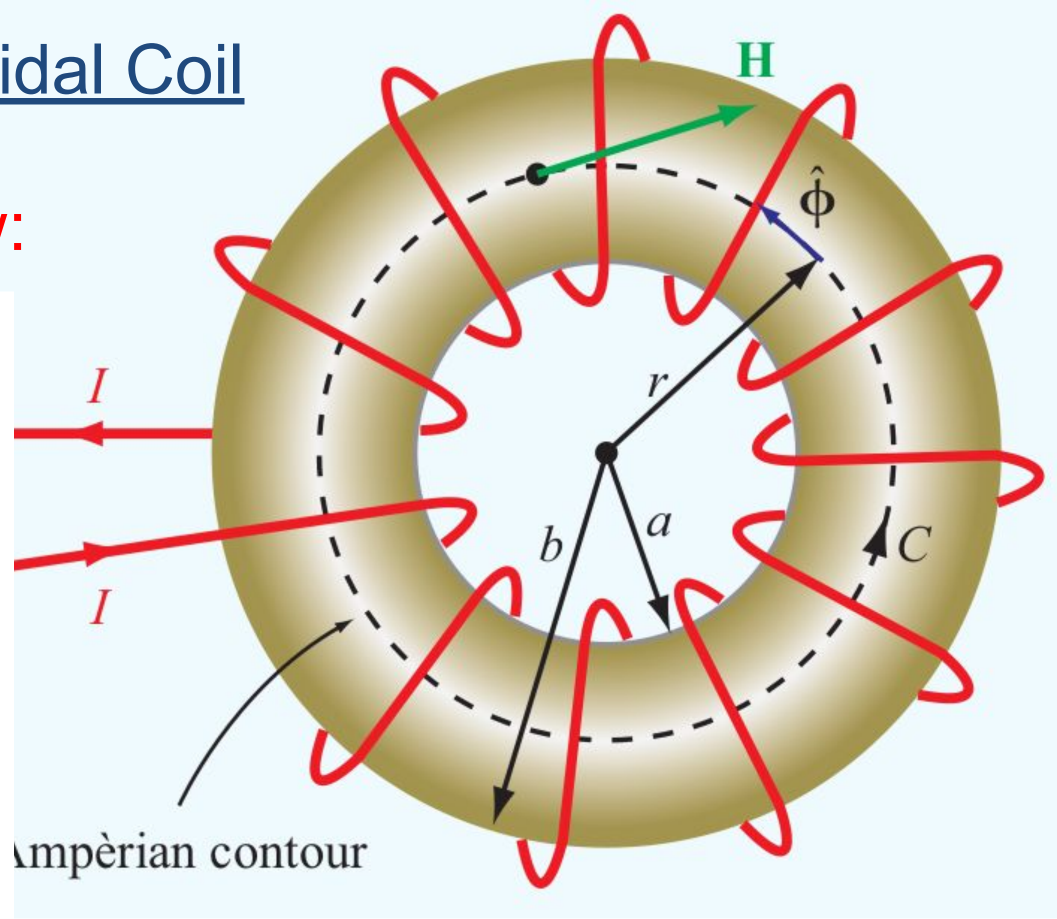
Plugging into Ampere's Law:

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = I$$

$$\int_0^{2\pi} (-\hat{\boldsymbol{\phi}}\, H) \cdot \hat{\boldsymbol{\phi}}\, r\, d\phi = -NI$$

$$-2\pi r H = -NI$$

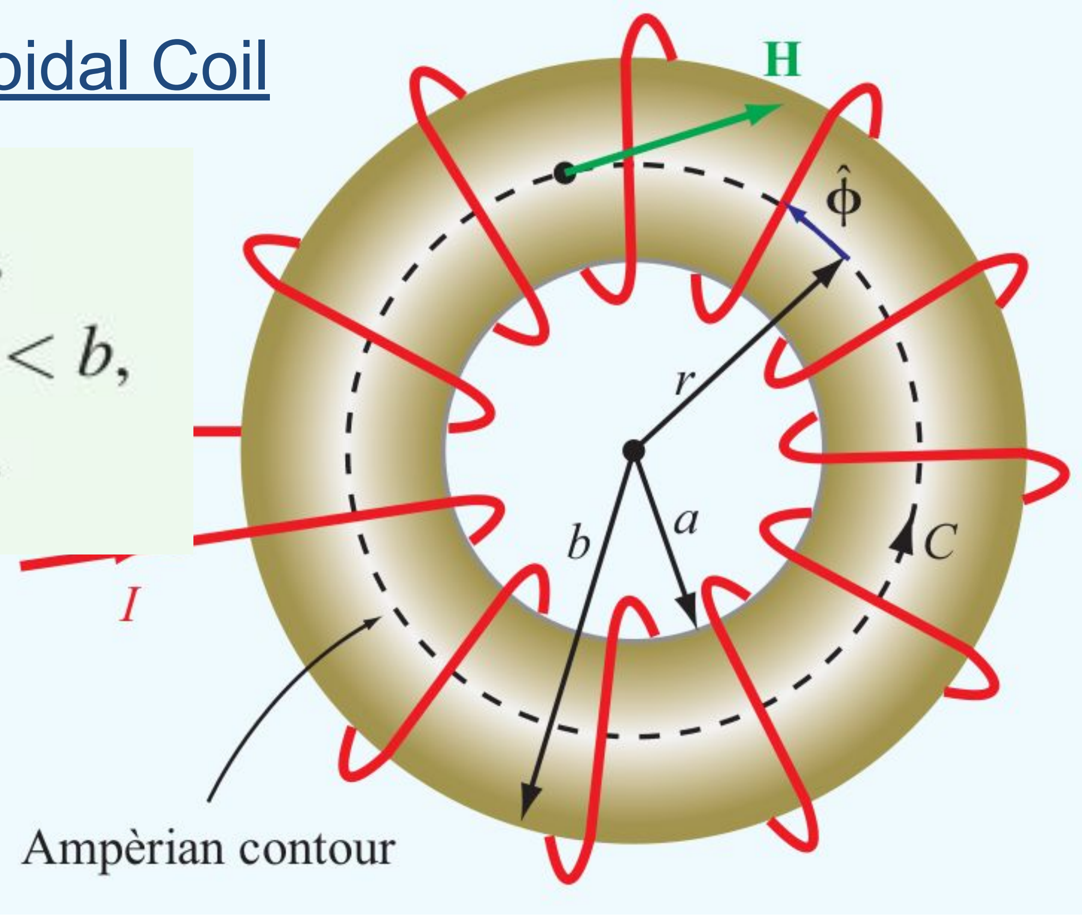
$$H = \frac{NI}{2\pi r}$$



Example 5-5

Magnetic Field inside Toroidal Coil

$$\mathbf{H} = \begin{cases} 0 & \text{for } r < a, \\ -\hat{\boldsymbol{\phi}}\,\dfrac{NI}{2\pi r} & \text{for } a < r < b, \\ 0 & \text{for } r > b. \end{cases}$$

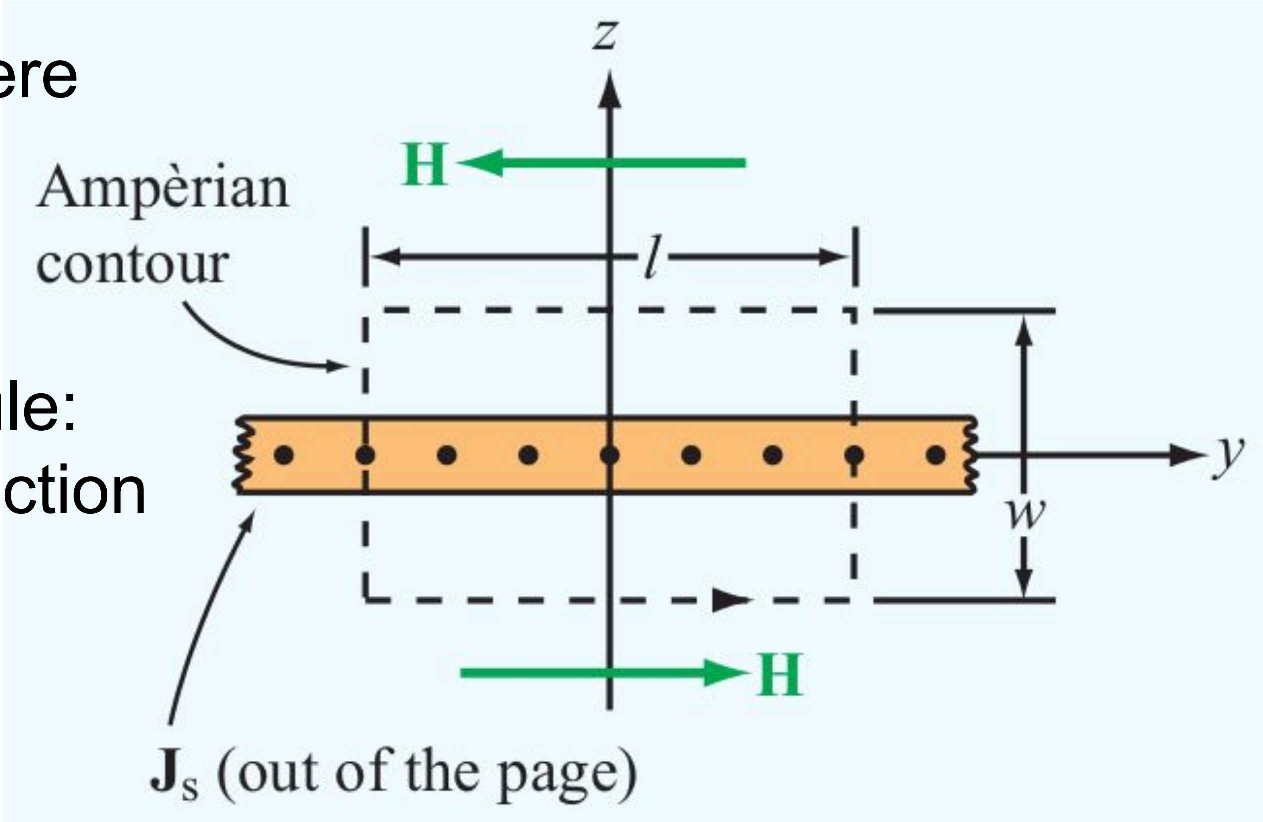


Example 5-6

Given: x-y plane contains an infinite current sheet with surface current density: $\mathbf{J}_s = \hat{\mathbf{x}} J_s$

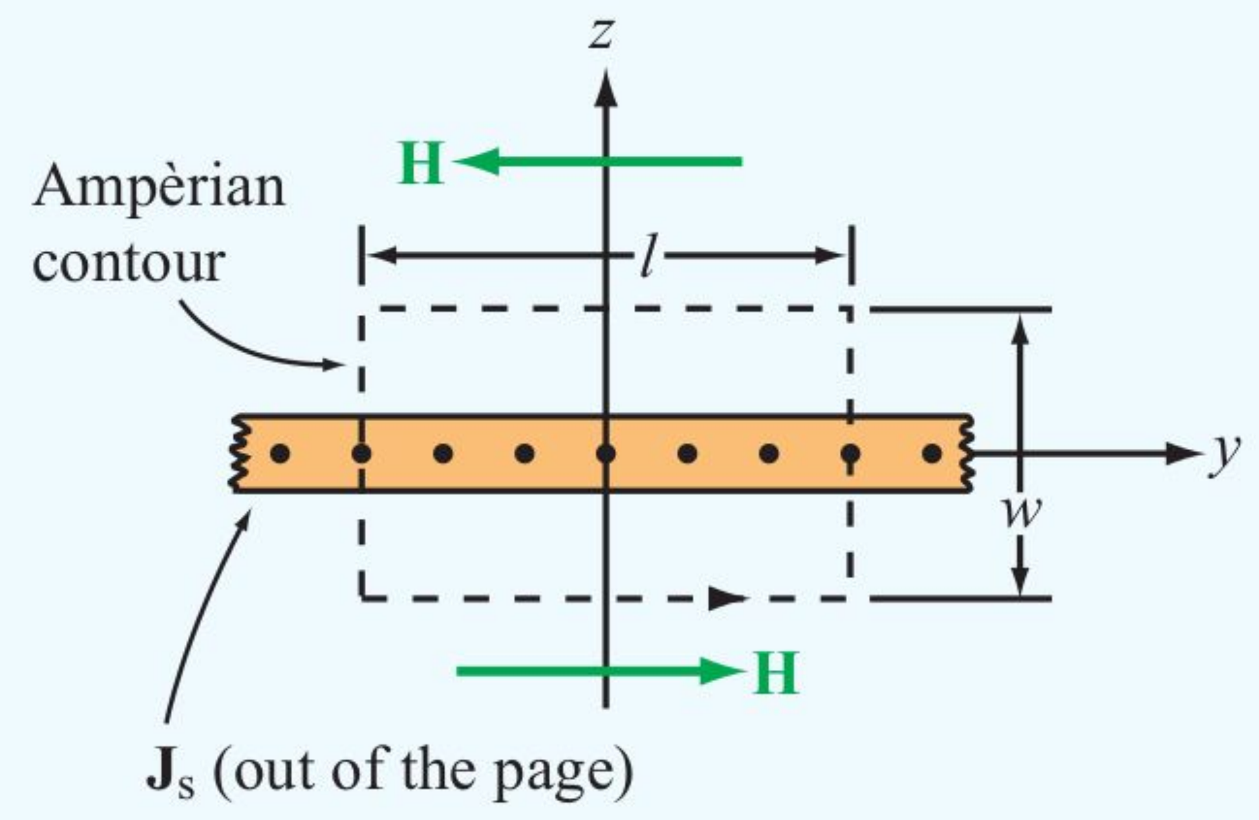
Find: **H** everywhere

Solution:
due to symmetry and right-hand rule:
H must be in direction shown in **green** on the figure



Example 5-6

Solution: Choose a rectangular path around the sheet
Total current crossing this surface: $I = J_s l$.



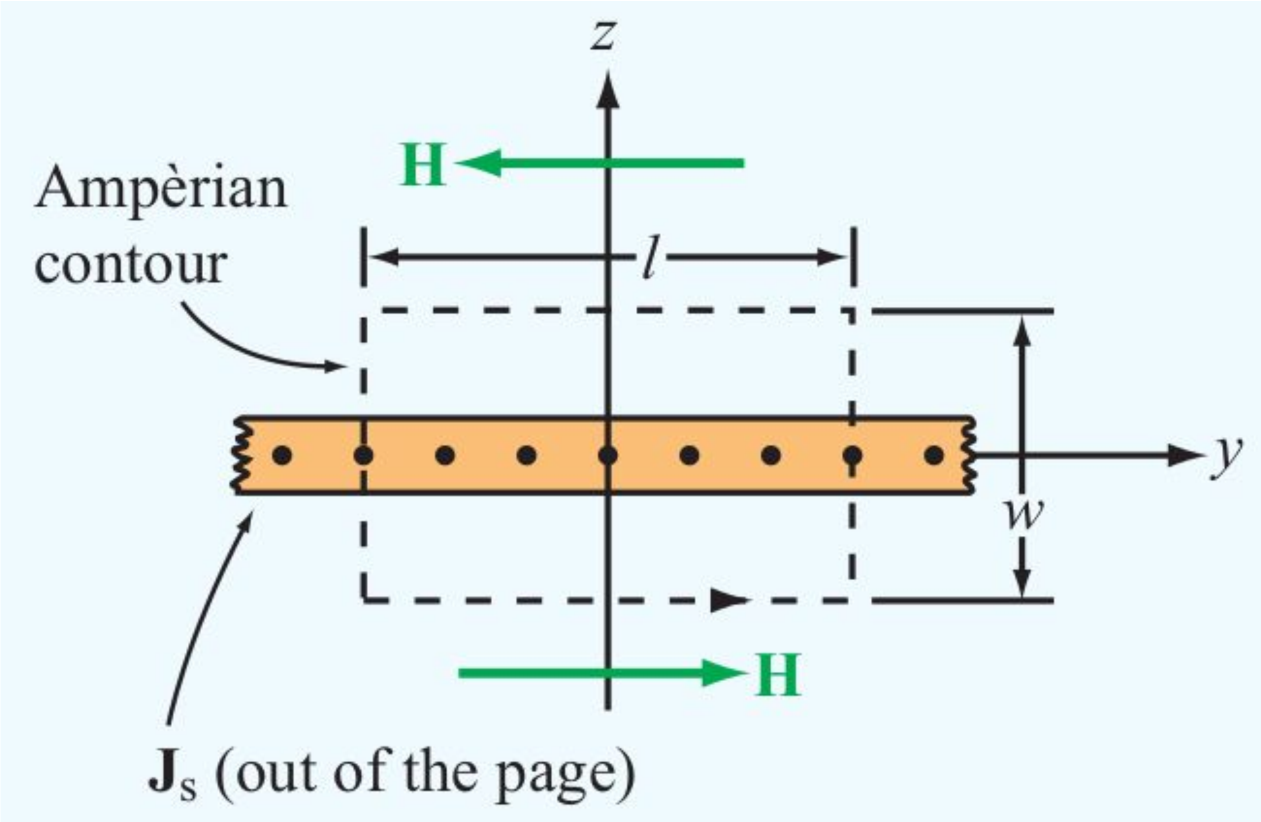
Example 5-6

Solution: Apply Ampère's Law around loop in counter-clockwise direction:

$$\oint_C \mathbf{H} \cdot d\boldsymbol{\ell} = I,$$

because **H** perp to path along w, $\mathbf{H} \cdot d\boldsymbol{\ell}$ = 0, so:

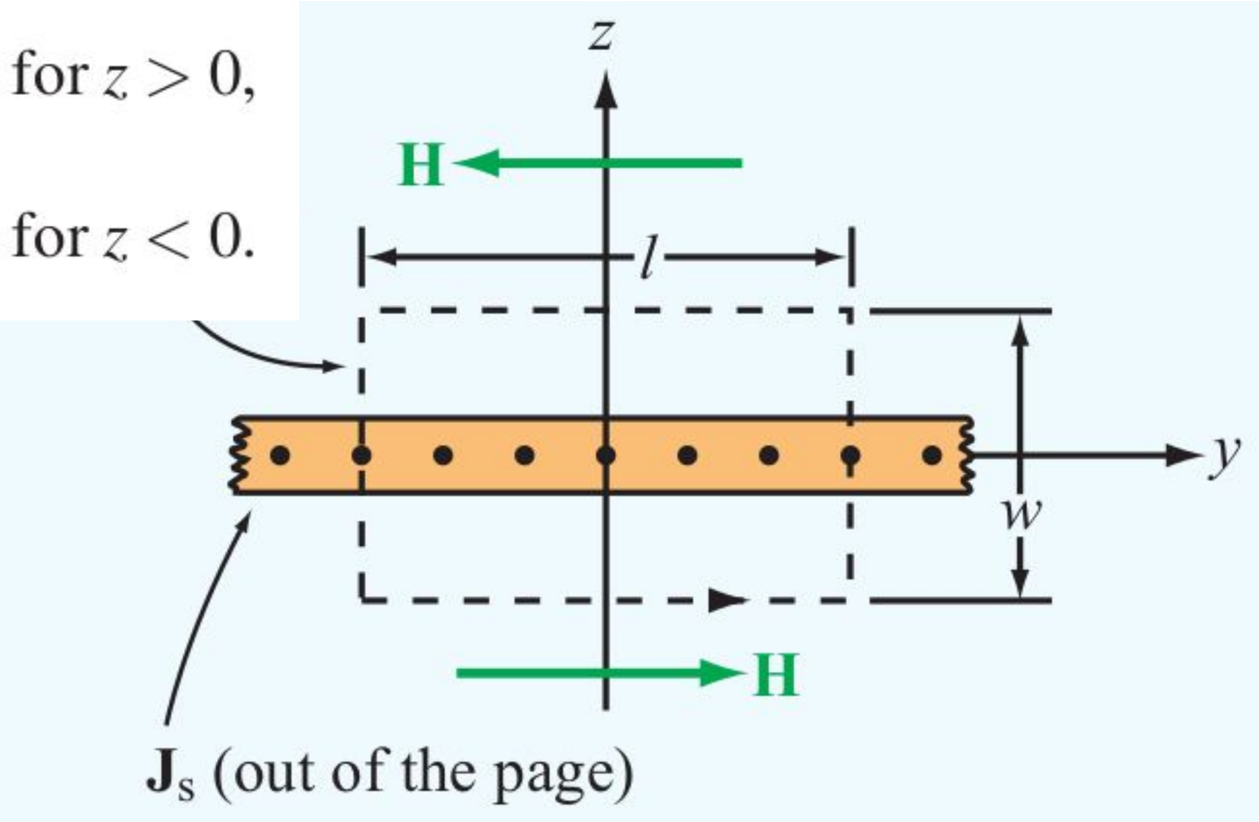
(H)(2ℓ) = J_s ℓ



Example 5-6

Solution: Since (H)(2ℓ) = J_s ℓ
we get:

$$\mathbf{H} = \begin{cases} -\hat{\mathbf{y}}\,\dfrac{J_s}{2} & \text{for } z > 0, \\ \hat{\mathbf{y}}\,\dfrac{J_s}{2} & \text{for } z < 0. \end{cases}$$



5-4 Magnetic Vector Potential **A**

In Electrostatics the potential is scalar and is defined in terms of the **E** field.

$$\mathbf{E} = -\nabla V.$$

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$$

What about the potential in Magnetostatics?

Since for any vector A: $\nabla \cdot (\nabla \times \mathbf{A}) = 0$

if we form: $\mathbf{B} = \nabla \times \mathbf{A}$

we are guaranteed that: $\nabla \cdot \mathbf{B} = 0$

(which is one of Maxwell's eqns)

5-4 Magnetic Vector Potential **A**

Looking at another of Maxwell's Eqns:

$$\nabla \times \mathbf{B} = \mu \mathbf{J}$$

Plug in our new definition of **B**:

$$\nabla \times (\nabla \times \mathbf{A}) = \mu \mathbf{J}$$

rewrite using vector identities:

$$\nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} = \mu \mathbf{J}$$

Since we are specifying **A**, force/choose: $\nabla \cdot \mathbf{A} = 0$

get:

$$\nabla^2 \mathbf{A} = -\mu \mathbf{J}$$

(vector Poisson's eqn)

5-4 Vector Poisson's Equation

$$\nabla^2 \mathbf{A} = -\mu \mathbf{J}$$ (vector Poisson's eqn)

Can compare to Electrostatic Poisson's eqn:

$$\nabla^2 V = -\frac{\rho_{\mathrm{v}}}{\varepsilon}$$

which gave, for a volume charge distribution:

$$V = \frac{1}{4\pi\varepsilon} \int_{\upsilon'} \frac{\rho_{\mathrm{v}}}{R'} \, d\upsilon'$$

Similarly, we get:

$$\mathbf{A} = \frac{\mu}{4\pi} \int_{\upsilon'} \frac{\mathbf{J}}{R'} \, d\upsilon'$$

5-4 Magnetic Vector Potential **A**

Electrostatics	Magnetostatics
$\mathbf{E} = -\nabla V$	$\mathbf{B} = \nabla \times \mathbf{A} \qquad (\text{Wb/m}^2),$
$\nabla^2 V = -\dfrac{\rho_\text{v}}{\varepsilon}$	$\nabla^2 \mathbf{A} = -\mu \mathbf{J}.$
$V = \dfrac{1}{4\pi\varepsilon} \displaystyle\int_{\mathcal{V}'} \dfrac{\rho_\text{v}}{R'}\, d\mathcal{V}'$	$\mathbf{A} = \dfrac{\mu}{4\pi} \displaystyle\int_{\mathcal{V}'} \dfrac{\mathbf{J}}{R'}\, d\mathcal{V}' \qquad (\text{Wb/m}).$

Example 6

Given: $\mathbf{A} = \hat{\mathbf{z}}\, A_0 \dfrac{x^2 + y^2}{4}$

Find: **J,** using the vector Poisson's Equation

Solution:

The vector Poisson's Equation is:

$$\nabla^2 \mathbf{A} = -\mu \mathbf{J}$$

so:

$$\nabla^2 \mathbf{A} = \hat{\mathbf{z}}\, \frac{A_0}{4} \left[\frac{\partial^2 (x^2 + y^2)}{\partial x^2} + \frac{\partial^2 (x^2 + y^2)}{\partial y^2} \right]$$

Example 6

have:

$$\nabla^2 \mathbf{A} = \hat{\mathbf{z}}\,\frac{A_0}{4}\left[\frac{\partial^2(x^2+y^2)}{\partial x^2} + \frac{\partial^2(x^2+y^2)}{\partial y^2}\right]$$

get:

$$\nabla^2 \mathbf{A} = \hat{\mathbf{z}}\,\frac{A_0}{4}\,[2+2]$$

so:

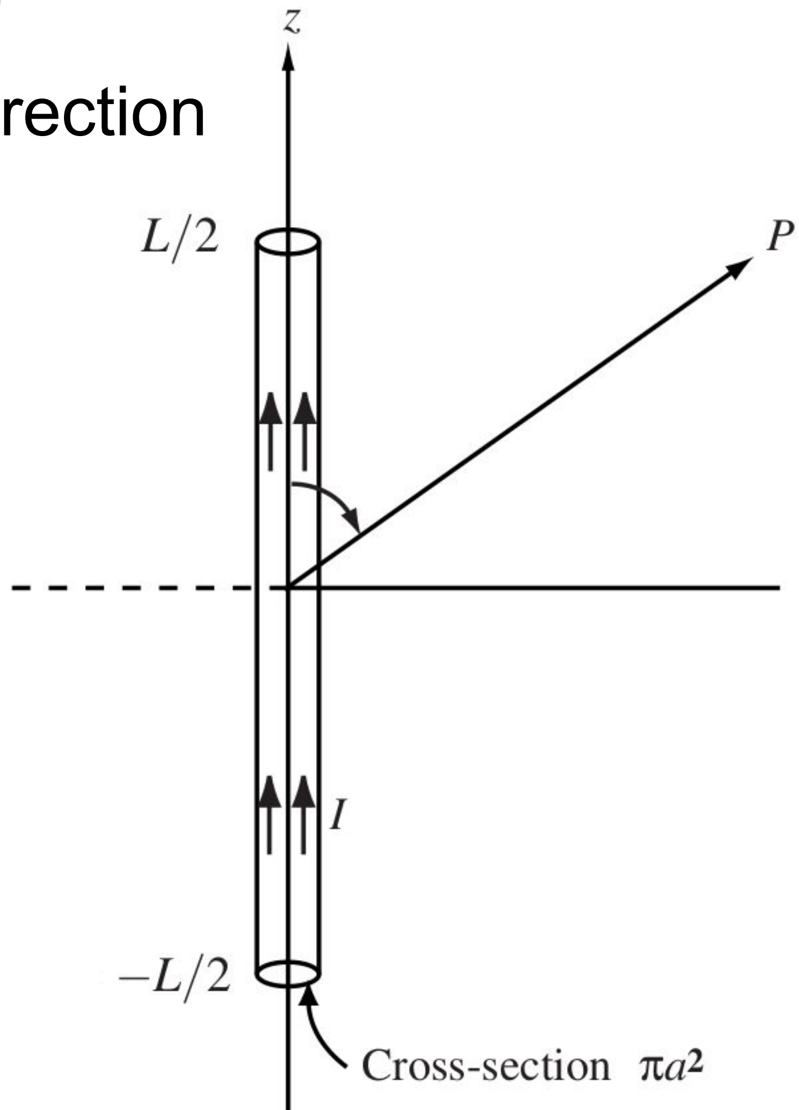
$$\nabla^2 \mathbf{A} = \hat{\mathbf{z}}\,A_0 = -\mu \mathbf{J}$$

$$\boxed{\mathbf{J} = -\hat{\mathbf{z}}\,\frac{A_0}{\mu}}$$

Example 7

Given: Current directed in +z direction
Wire of length *L*,
circular cross-section,
radius *a*

Find: **A** at point *P*,
distance to *P* is large
such that distance from
every point on wire to *P*
is "the same".



Example 7

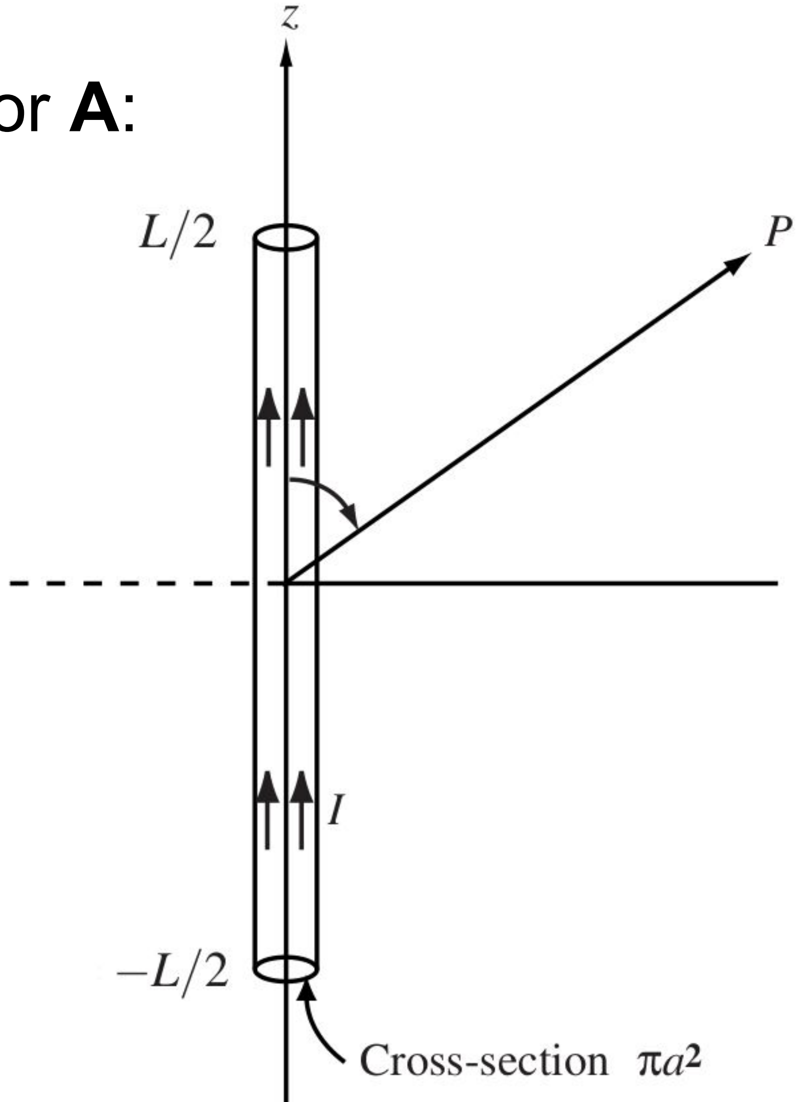
Solution: Use the integral eqn for **A**:

$$\mathbf{A} = \frac{\mu_0}{4\pi} \int_{\upsilon'} \frac{\mathbf{J}}{R'}\, d\upsilon'$$

R' is constant since we are assuming it's the same for all points in wire

J is current/area: $\hat{\mathbf{z}} \dfrac{I}{(\pi a^2)}$

$d\mathcal{V}' = \text{Area } dz = \pi a^2\, dz$



Example 7

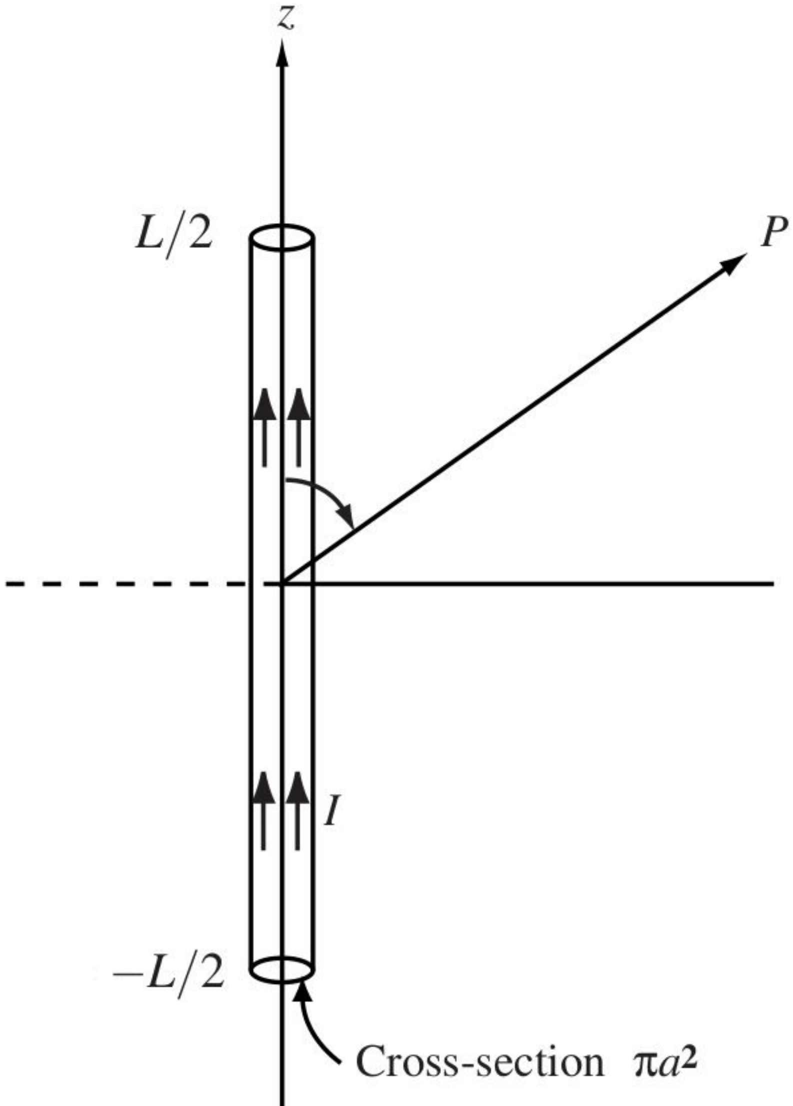
$$\mathbf{A} = \frac{\mu_0}{4\pi} \int_{\upsilon'} \frac{\mathbf{J}}{R'} \, d\upsilon'$$

$$= \frac{\mu_0}{4\pi R} \int_{\upsilon'} \hat{\mathbf{z}} \frac{I}{(\pi a^2)} \pi a^2 \, dz$$

$$= \hat{\mathbf{z}} \frac{\mu_0 I}{4\pi R} \int_{-L/2}^{L/2} dz$$

$$\boxed{\mathbf{A} = \hat{\mathbf{z}} \frac{\mu_0 I L}{4\pi R}}$$

(R>>L)



5-4 Magnetic Flux Φ

Defined as the total "magnetic flux density" passing through surface S.

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s} \qquad \text{(Wb)}$$

Using the vector magnetic potential, and Stokes' thrm:

$$\Phi = \int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{s} = \oint_C \mathbf{A} \cdot d\mathbf{l}$$

where *S* is usually a planar area, with edge *C*.

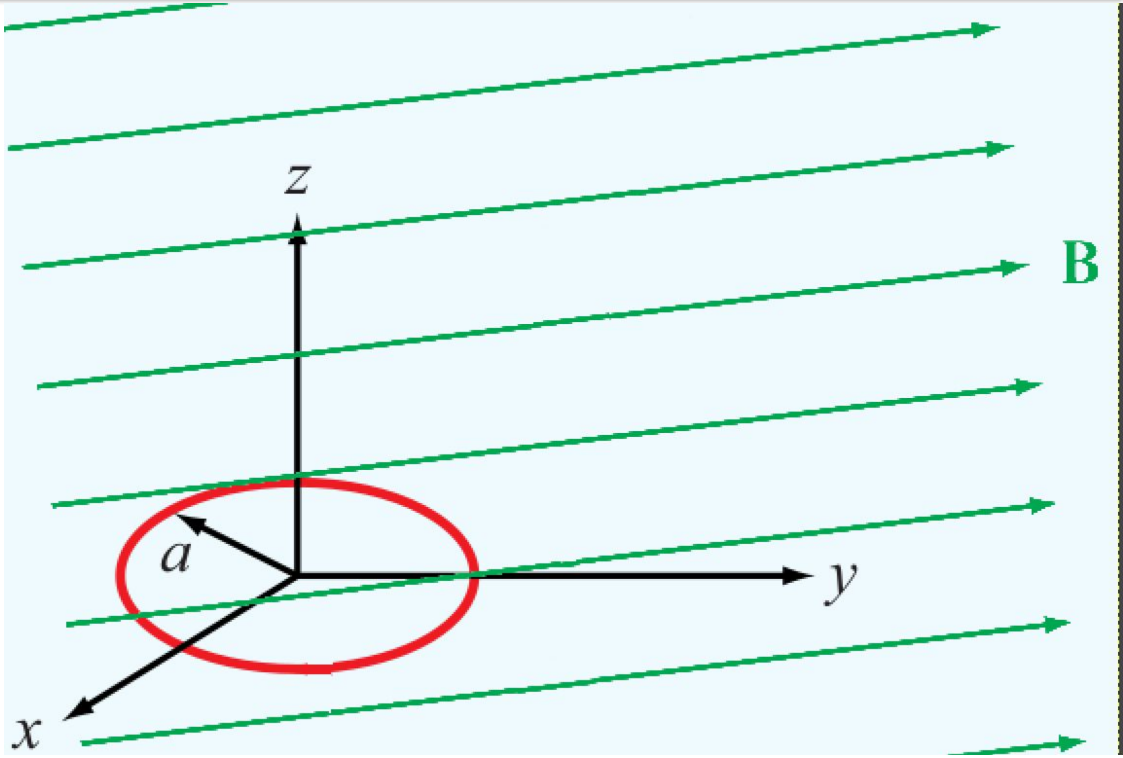
This is related to the Wireless Power Transfer labs

Example 8

Given:
Circular loop,
radius *a*,
in *x*-*y* plane

$$\mathbf{B} = \hat{\mathbf{y}} B_y + \hat{\mathbf{z}} B_z$$

Find: Φ



Example 8

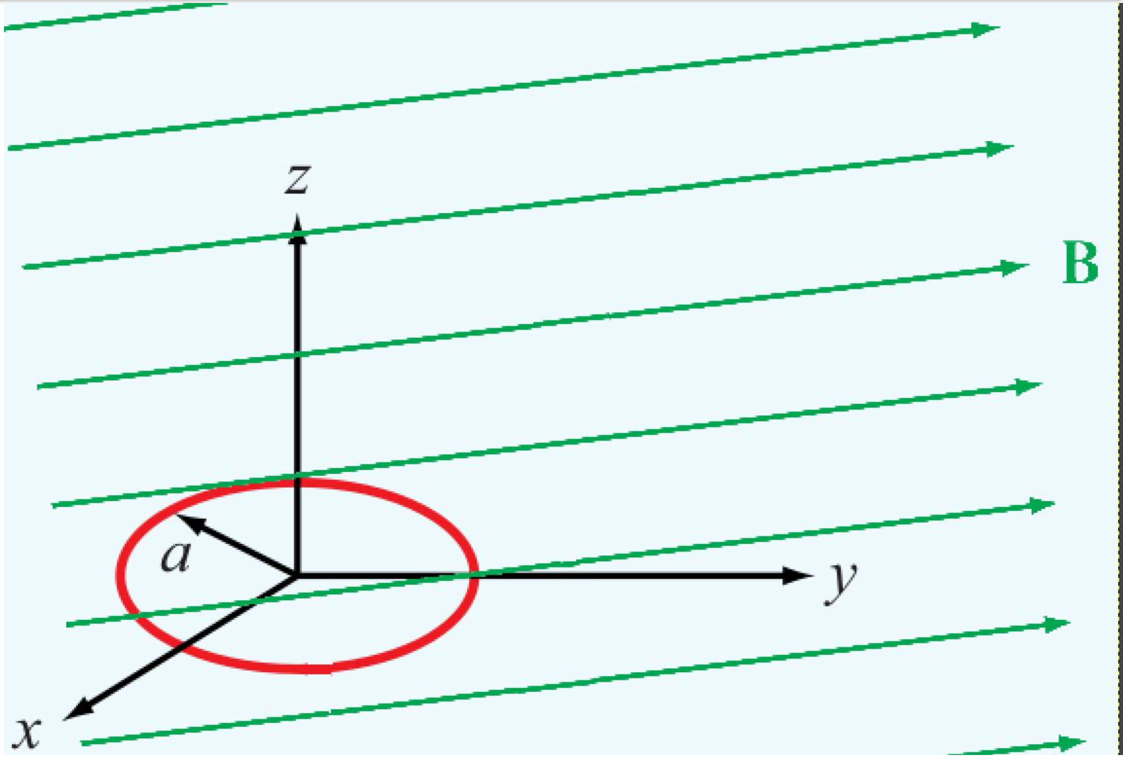
Solution:

Use the integral eqn:

$$\Phi = \oint_S \mathbf{B} \cdot d\mathbf{s}$$

where *S* is the surface of the loop:

$$d\mathbf{s} = \hat{\mathbf{z}}\, r\, dr\, d\phi$$



Example 8

$$\Phi = \oint_S \mathbf{B} \cdot d\mathbf{s}$$

$$\Phi = \int_{r=0}^{a} \int_{\phi=0}^{2\pi} [\hat{\mathbf{y}} B_y + \hat{\mathbf{z}} B_z] \cdot \hat{\mathbf{z}}\, r\, dr\, d\phi$$

$$\Phi = \int_{r=0}^{a} \int_{\phi=0}^{2\pi} B_z\, r\, dr\, d\phi$$

$$\Phi = B_z \int_{r=0}^{a} r\, dr \int_{\phi=0}^{2\pi} d\phi$$

Example 8

$$\Phi = B_z \int_{r=0}^{a} r\,dr \int_{\phi=0}^{2\pi} d\phi$$

$$\Phi = 2\pi B_z \int_{r=0}^{a} r\,dr$$

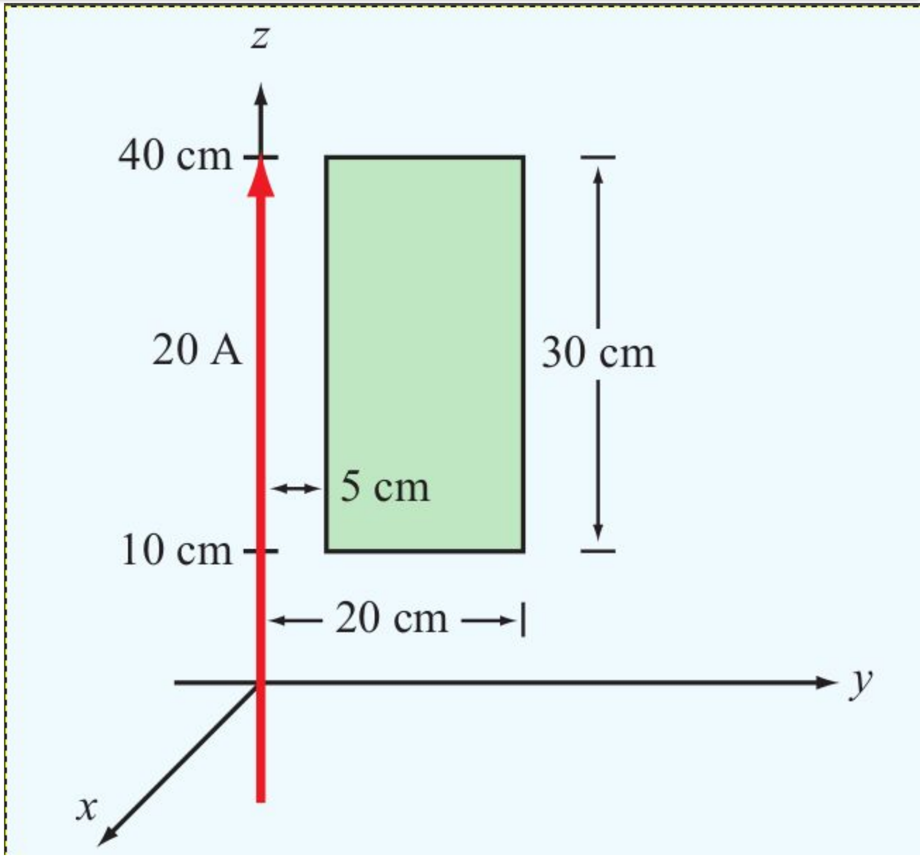
$$\Phi = 2\pi B_z \left[\frac{r^2}{2}\right]_{r=0}^{a}$$

$$\Phi = 2\pi B_z \frac{a^2}{2}$$

$$\boxed{\Phi = \pi a^2 B_z}$$

Example 9

Given:
rectangular conducting loop coplanar with an infinitely-long straight wire carrying a current I=20A



Find: Flux through the loop

Solution:

$$\Phi = \oint_S \mathbf{B} \cdot d\mathbf{s}$$

Example 9

$$\Phi = \oint_S \mathbf{B} \cdot d\mathbf{s}$$

S is the area enclosed by the loop, so d**s** is:

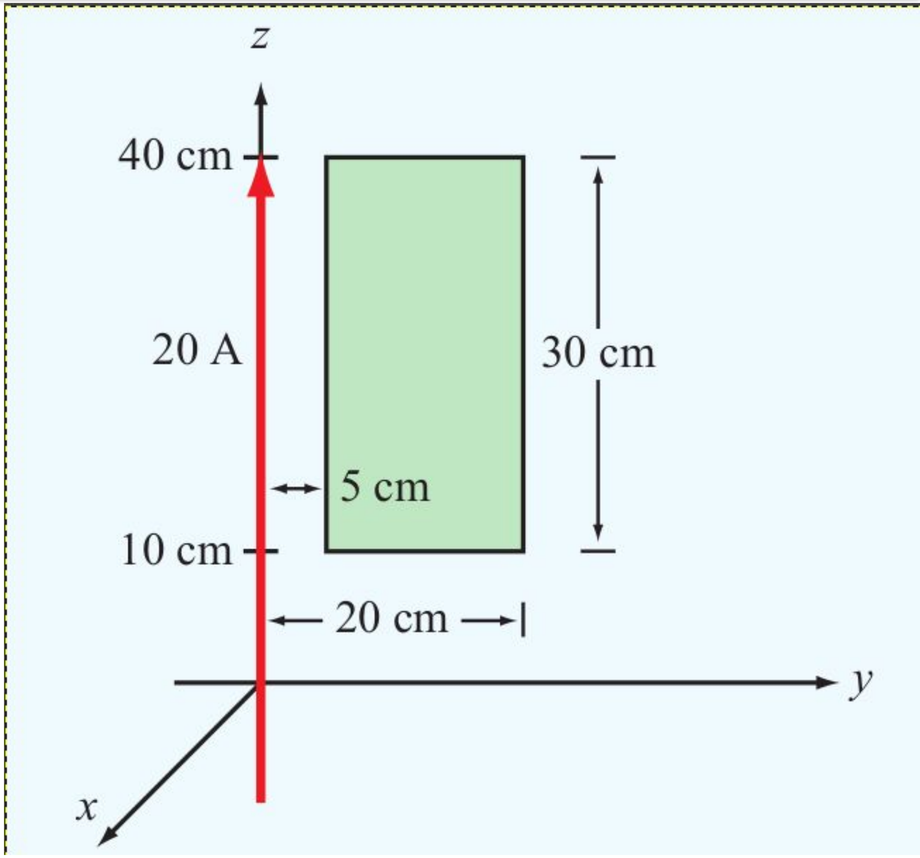
$$\hat{\mathbf{x}}\, dy\, dz.$$

B is the magnetic field due to the wire:

$$\mathbf{B} = \hat{\boldsymbol{\phi}} \frac{\mu I}{2\pi r}$$

for our problem:

$$\mathbf{B} = -\hat{\mathbf{x}} \frac{\mu_0 I}{2\pi y}$$



Example 9

$$\Phi = \oint_S \mathbf{B} \cdot d\mathbf{s}$$

$$\Phi = \int_{y=0.05\text{m}}^{0.2\text{m}} \int_{z=0.1\text{m}}^{0.4\text{m}} \left(-\hat{\mathbf{x}} \frac{\mu_0 I}{2\pi y} \right) \cdot (\hat{\mathbf{x}}\, dy\, dz)$$

$$\Phi = -\frac{\mu_0 I}{2\pi} (0.3) \int_{y=0.05\text{m}}^{0.2\text{m}} \left(\frac{1}{y} \right) dy$$

$$\Phi = -\frac{(4\pi \times 10^{-7}\text{H/m})(10\text{A})}{2\pi} (0.3) \ln \frac{0.2}{0.05}$$

$$\Phi = -2.8\mu\text{Wb}$$

Homework

Homework 19 is due tomorrow at midnight.

submit to gradescope via the canvas site.

Next Time

Sections 5-5 through 5-6:

Magnetic Permeability
    Hysteresis

Magnetic Boundary Conditions