EECS 230
ENGINEERING ELECTROMAGNETICS
Leland Pierce

Electrostatics 6

Chapter 4 Overview

Maxwell's Equations
- Electrostatics
- Magnetostatics

Charge density
Current density
Electric field from charges
Gauss's Law
Electric Scalar Potential Field
- Dipole Field
- Poisson's eqn

Conductors
- current
- resistance
- joule's law

Dielectrics
- polarization

Boundary Conditions
Capacitance
Potential Energy
Image method

$$\nabla \cdot \mathbf{D} = \rho_v,\quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},\quad \nabla \cdot \mathbf{B} = 0,\quad \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}.$$

$$\mathbf{E} = \int_{v'} d\mathbf{E} = \frac{1}{4\pi\varepsilon} \int_{v'} \hat{\mathbf{R}}' \frac{\rho_v\, dv'}{R'^2}.$$

(volume distribution)

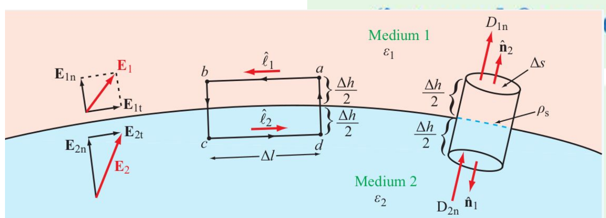
$$\nabla \cdot \mathbf{D} = \rho_v,$$

(differential form of Gauss's law)

$$\nabla \cdot \mathbf{D} = \rho_v,\quad \nabla \times \mathbf{E} = 0.$$

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q.$$

of Gauss's law)



$$V = -\int_{\infty}^{P} \mathbf{E} \cdot d\mathbf{l}$$

$$\nabla \cdot \mathbf{B} = 0,\quad \nabla \times \mathbf{H} = \mathbf{J}.$$

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon} \qquad \mathbf{E} = -\nabla V.$$

$$Q = \int_v \rho_v\, dv \qquad \text{(C)}.$$

$$\mathbf{J} = \sigma \mathbf{E} \quad (\text{A/m}^2) \quad \text{(Ohm's law)},$$

$$\mathbf{J} = \rho_v \mathbf{u} \qquad (\text{A/m}^2)$$

$$R = \frac{V}{I} = \frac{-\int_l \mathbf{E} \cdot d\mathbf{l}}{\int_S \mathbf{J} \cdot d\mathbf{s}} = \frac{-\int_l \mathbf{E} \cdot d\mathbf{l}}{\int_S \sigma \mathbf{E} \cdot d\mathbf{s}}.$$

$$P = \int_v \mathbf{E} \cdot \mathbf{J}\, dv \qquad \text{(W)}$$

$$I = \int_S \mathbf{J} \cdot d\mathbf{s} \qquad \text{(A)}.$$

Lecture Coverage

Today's lecture:

Review of Sections 4-1 through 4-10 of the book:

4-1: Maxwell's Equations
4-2: Charge and Current Distributions
4-3: Coulomb's Law
4-4: Gauss's Law
4-5: Voltage (Electric Scalar Potential)
4-6: Conductors
4-7: Dielectrics
4-8: Boundary Conditions
4-9: Capacitance
4-10: Electrostatic Potential Energy

Section 4-11 of the book:

4-11: Image Method

Chapter 4 Review

Maxwell's Equations:

$$\nabla \cdot \mathbf{D} = \rho_V,$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \cdot \mathbf{B} = 0,$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}.$$

Empirically derived from many measurements

$$\mathbf{D} = \varepsilon \mathbf{E}$$

$$\mathbf{B} = \mu \mathbf{H}.$$

E: Electric Field
H: Magnetic Field
J: Current Density
ρ_v: Charge Density

Chapter 4 Review

Static Conditions:

$$\frac{\partial \rho_v}{\partial t} = 0$$

$$\frac{\partial \mathbf{J}}{\partial t} = 0$$

Electrostatics

$$\nabla \cdot \mathbf{D} = \rho_\mathrm{v},$$

$$\nabla \times \mathbf{E} = 0.$$

Magnetostatics

$$\nabla \cdot \mathbf{B} = 0,$$

$$\nabla \times \mathbf{H} = \mathbf{J}.$$

Electric and Magnetic Fields are decoupled.

Chapter 4 Review

$$\mathbf{E} = \frac{1}{4\pi\varepsilon} \int_{\upsilon'} \hat{\mathbf{R}}' \frac{\rho_{\mathrm{v}}\, d\upsilon'}{R'^2}$$

(volume distribution)

$$\mathbf{E} = \frac{1}{4\pi\varepsilon} \int_{S'} \hat{\mathbf{R}}' \frac{\rho_{\mathrm{s}}\, ds'}{R'^2}$$

(surface distribution)

$$\mathbf{E} = \frac{1}{4\pi\varepsilon} \int_{l'} \hat{\mathbf{R}}' \frac{\rho_{\ell}\, dl'}{R'^2}$$

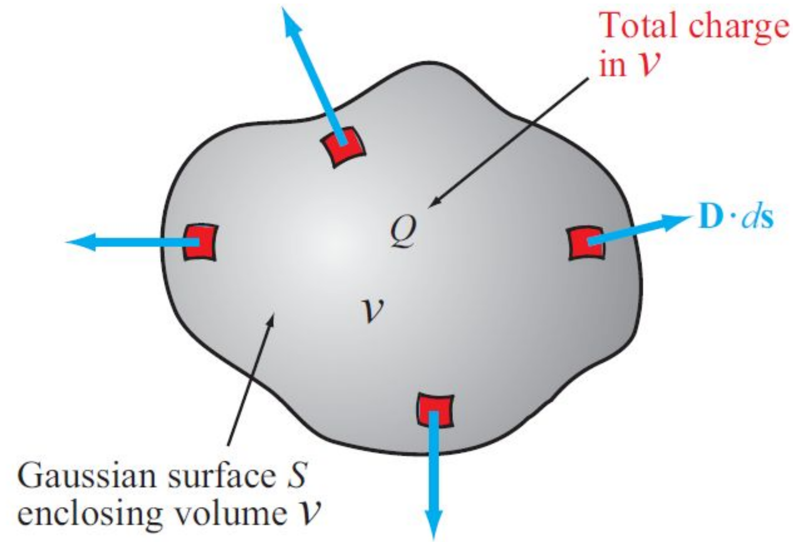
(line distribution)

Chapter 4 Review

Gauss's Law

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q \qquad (4.2$$

(Integral form of Gauss's law).



or:

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = \int_{\mathcal{V}} \rho_V \, d\mathcal{V}$$

where the closed-surface *S* is the boundary of $\mathcal{V}$

Chapter 4 Review

Voltage:

$$V = -\int_{\infty}^{P} \mathbf{E} \cdot d\mathbf{l} \qquad \text{(V)}. \qquad (4.43)$$

N Point Charges:

$$V = \frac{1}{4\pi\varepsilon} \sum_{i=1}^{N} \frac{q_i}{|\mathbf{R} - \mathbf{R}_i|} \qquad \text{(V)}.$$

Chapter 4 Review

$$V = \frac{1}{4\pi\varepsilon} \int_{\upsilon'} \frac{\rho_{\mathrm{v}}}{R'}\, d\upsilon' \quad \textbf{(volume distribution)},$$

$$V = \frac{1}{4\pi\varepsilon} \int_{S'} \frac{\rho_{\mathrm{s}}}{R'}\, ds' \quad \textbf{(surface distribution)},$$

$$V = \frac{1}{4\pi\varepsilon} \int_{l'} \frac{\rho_{\ell}}{R'}\, dl' \quad \textbf{(line distribution)}.$$

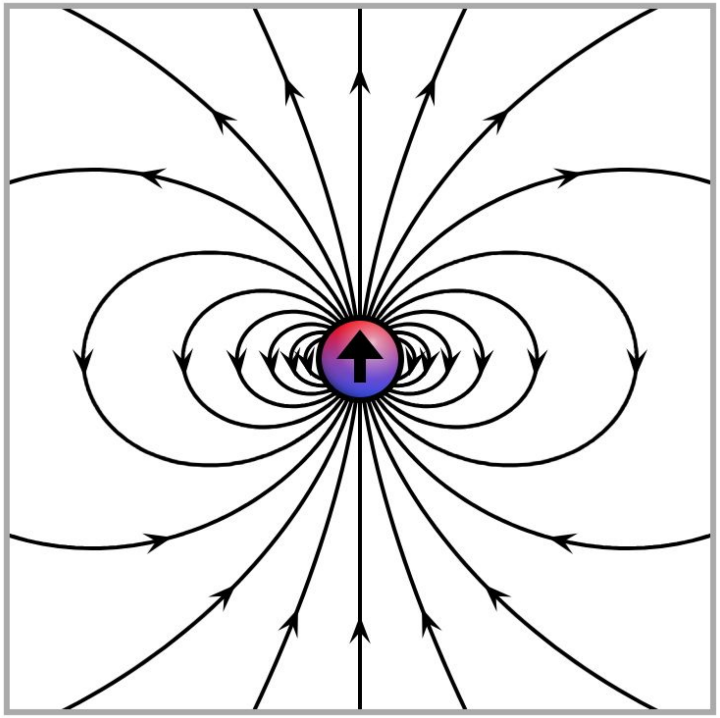
$$R' = |\mathbf{R} - \mathbf{R}_i|$$

Chapter 4 Review

$$\mathbf{E} = -\nabla V.$$

Electric Dipole:

$$\mathbf{E} = \frac{qd}{4\pi\varepsilon_0 R^3}\left(\hat{\mathbf{R}}\,2\cos\theta + \hat{\boldsymbol{\theta}}\sin\theta\right) \quad \text{(V/m)}$$



(wikipedia.org)

Chapter 4 Review

Since:

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon} \qquad \textbf{(Poisson's equation)}$$

if ρ_v=0:

$$\nabla^2 V = 0 \qquad \textbf{(Laplace's equation)}$$

Useful for problems where *V* is known on boundaries.

Chapter 4 Review

$$\mathbf{J} = \sigma \mathbf{E} \qquad (\text{A/m}^2) \quad (\text{Ohm's law})$$

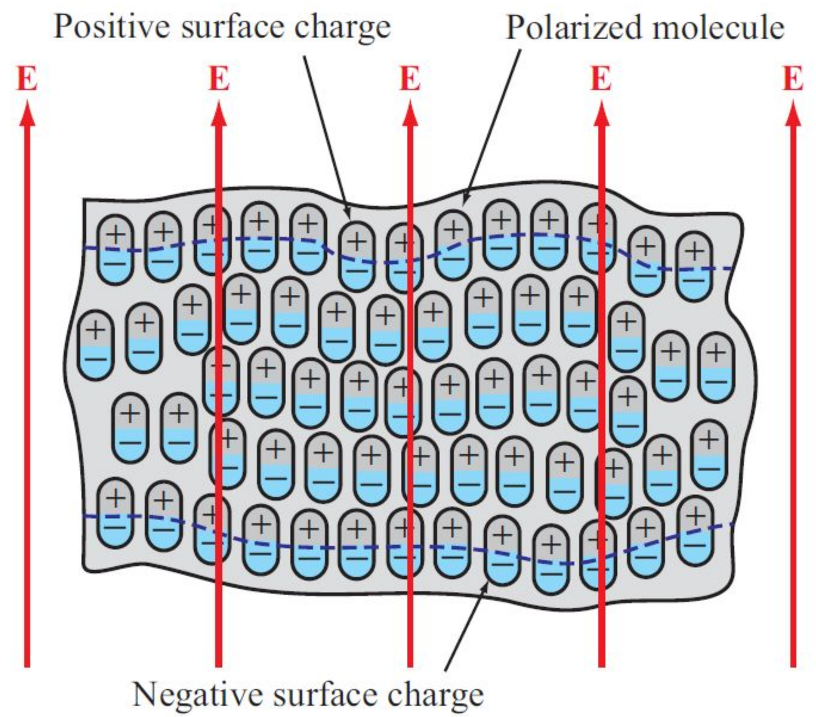
$$P = \int_V \mathbf{E} \cdot \mathbf{J} \, dV \qquad (\text{W}) \quad (\text{Joule's law})$$

Chapter 4 Review

In free space: $\mathbf{D} = \varepsilon_0 \mathbf{E}$

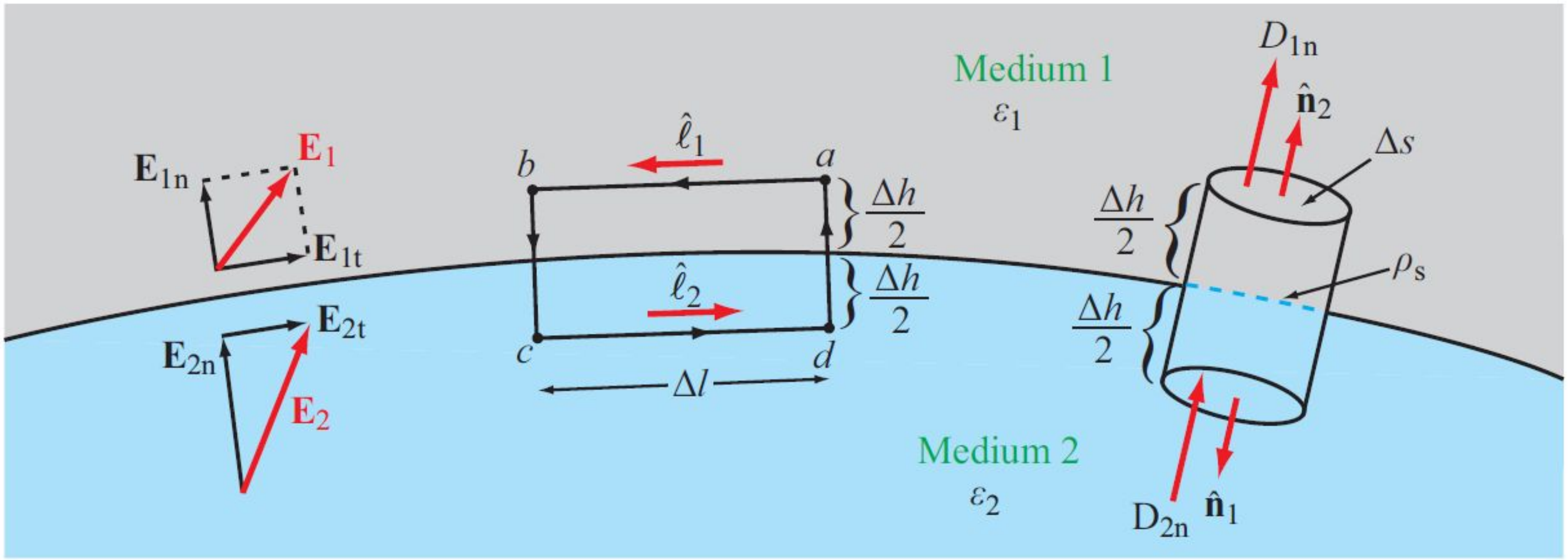
In a dielectric: $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$

$$\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E},$$



Chapter 4 Review

Two dielectric materials:

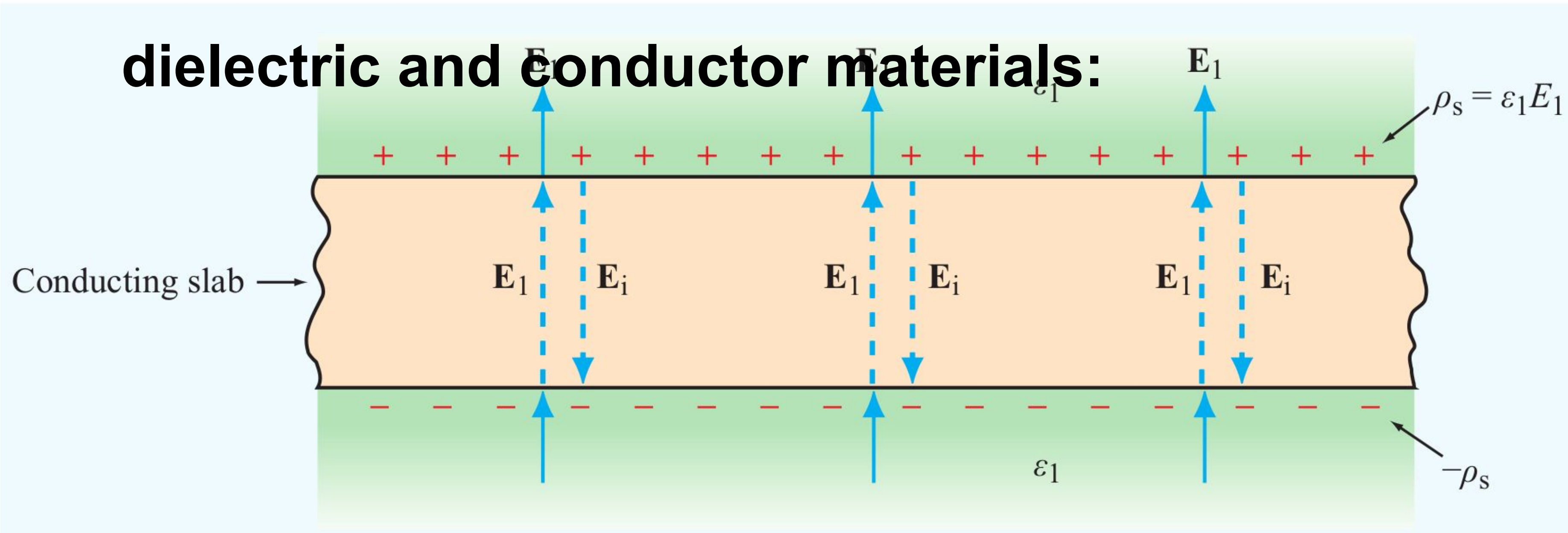


$$\mathbf{E}_{1t} = \mathbf{E}_{2t}$$

$$D_{1n} - D_{2n} = \rho_s \qquad (\text{C/m}^2).$$

Chapter 4 Review

dielectric and conductor materials:



$$E_{1t} = D_{1t} = 0, \qquad D_{1n} = \varepsilon_1 E_{1n} = \rho_s$$

$$\mathbf{D}_1 = \varepsilon_1 \mathbf{E}_1 = \hat{\mathbf{n}}\rho_s,$$

(at conductor surface)

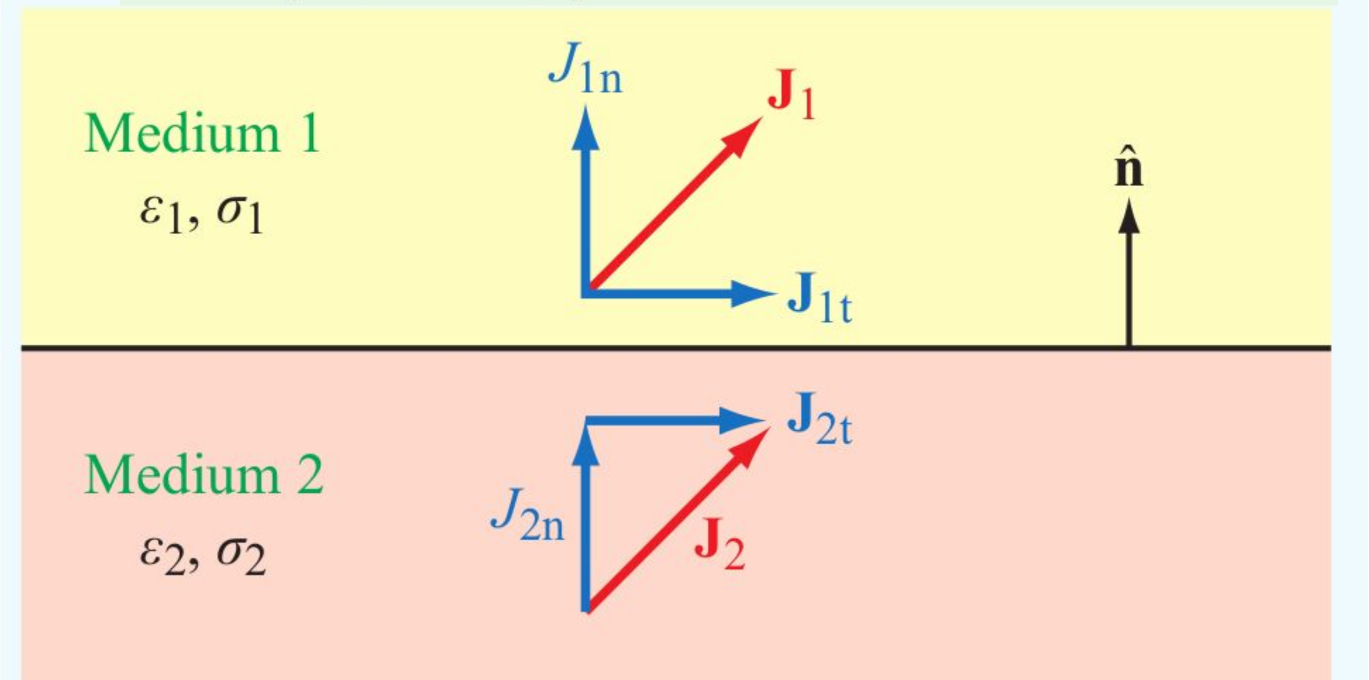
E is always normal to a conductor's surface

Chapter 4 Review

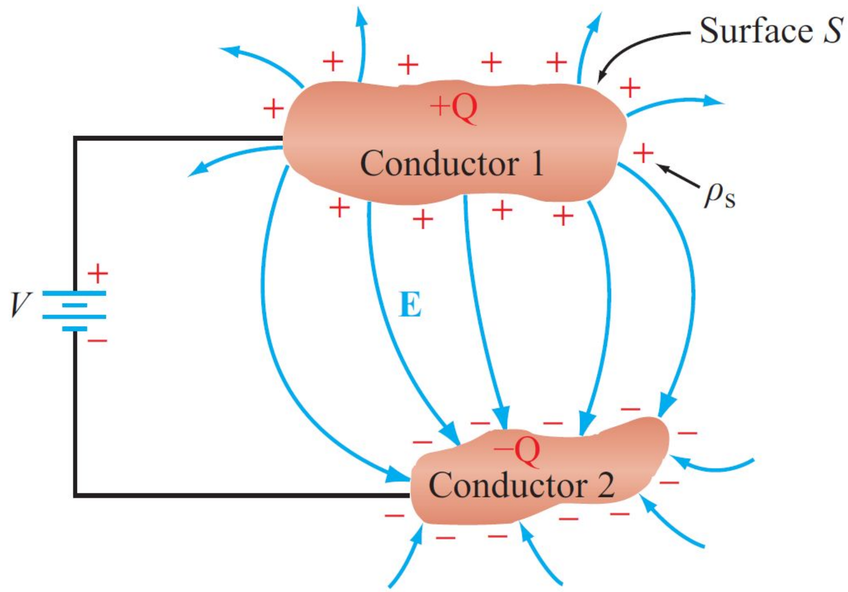
Two conducting materials:

$$\mathbf{E}_{1t} = \mathbf{E}_{2t}, \qquad \varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = \rho_s$$

$$J_{1n}\left(\frac{\varepsilon_1}{\sigma_1} - \frac{\varepsilon_2}{\sigma_2}\right) = \rho_s \quad \textbf{(electrostatics)}.$$



Chapter 4 Review

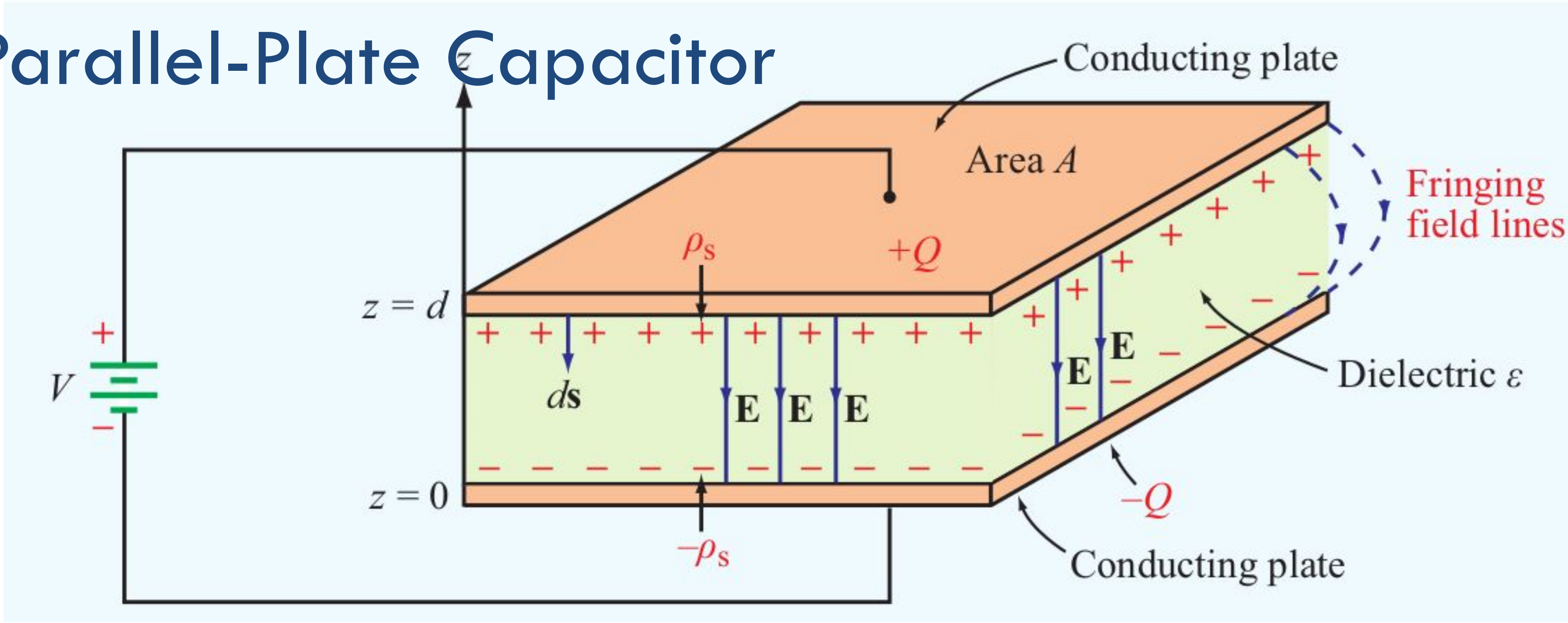


The ***capacitance*** of a two-conductor configuration is defined as

$$C = \frac{Q}{V} \qquad \text{(C/V or F)}, \qquad (4.105)$$

Chapter 4 Review

Parallel-Plate Capacitor



Idealized: lossless dielectric

E exists only in dielectric and is uniform

uniform surface charge density

Chapter 4 Review

Parallel-Plate Capacitor

$$\mathrm{E} = \rho_s/\varepsilon = \frac{Q}{\varepsilon A}$$

$$V = -\int_0^d \mathbf{E} \cdot d\mathbf{l} = -\int_0^d (-\hat{\mathbf{z}}E) \cdot \hat{\mathbf{z}}\, dz = Ed.$$

$$C = \frac{Q}{V} = \frac{Q}{Ed} = \frac{\varepsilon A}{d}$$

Chapter 4 Review

Capacitance of a Coaxial Transmission Line

$$V = -\int_a^b \mathbf{E} \cdot d\mathbf{l} = -\int_a^b \left(-\hat{\mathbf{r}} \, \frac{Q}{2\pi\varepsilon r l} \right) \cdot (\hat{\mathbf{r}} \, dr)$$

$$= \frac{Q}{2\pi\varepsilon l} \ln\left(\frac{b}{a}\right).$$

then:

$$C = \frac{Q}{V} = \frac{2\pi\varepsilon l}{\ln(b/a)}$$

so:

$$C' = \frac{C}{l} = \frac{2\pi\varepsilon}{\ln(b/a)} \qquad \text{(F/m)}$$

Chapter 4 Review

Electrostatic Potential Energy:

$$W_e = \frac{1}{2} C V^2$$

which should be familiar from EECS 215.

For a parallel-plate capacitor with:

$$C = \varepsilon A / d \qquad V = Ed$$

this becomes:

$$W_e = \frac{1}{2} \frac{\varepsilon A}{d} (Ed)^2 = \frac{1}{2} \varepsilon E^2 (Ad)$$

Chapter 4 Review

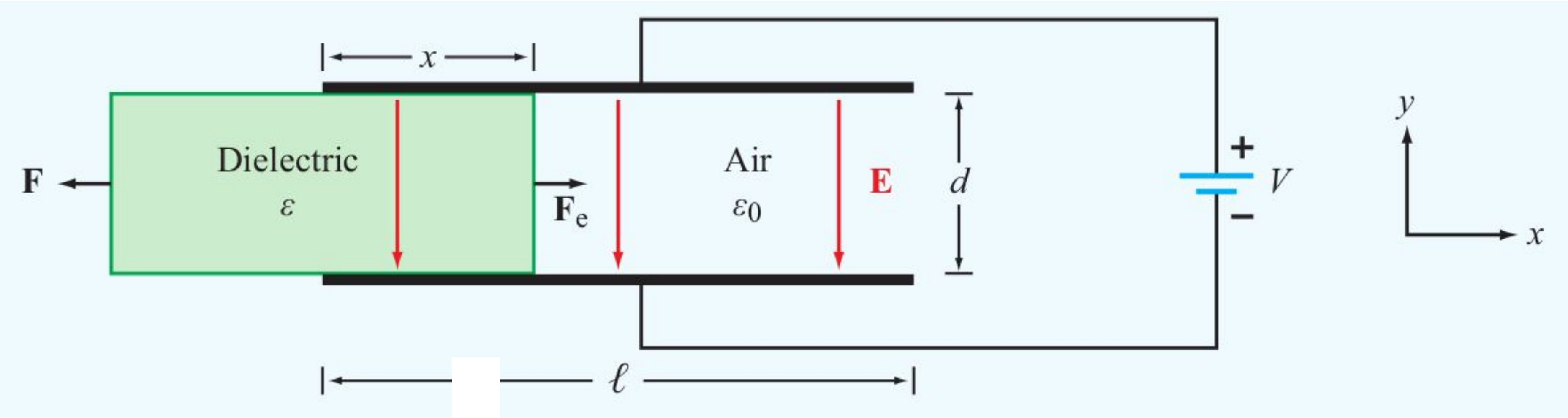
So, we get the electrostatic force acting on the upper plate:

$$\mathbf{F}_e = -\hat{\mathbf{z}}\,\frac{1}{2}\,\varepsilon A\,\frac{V^2}{d^2} \qquad \text{(N)}.$$

(parallel-plate capacitor)

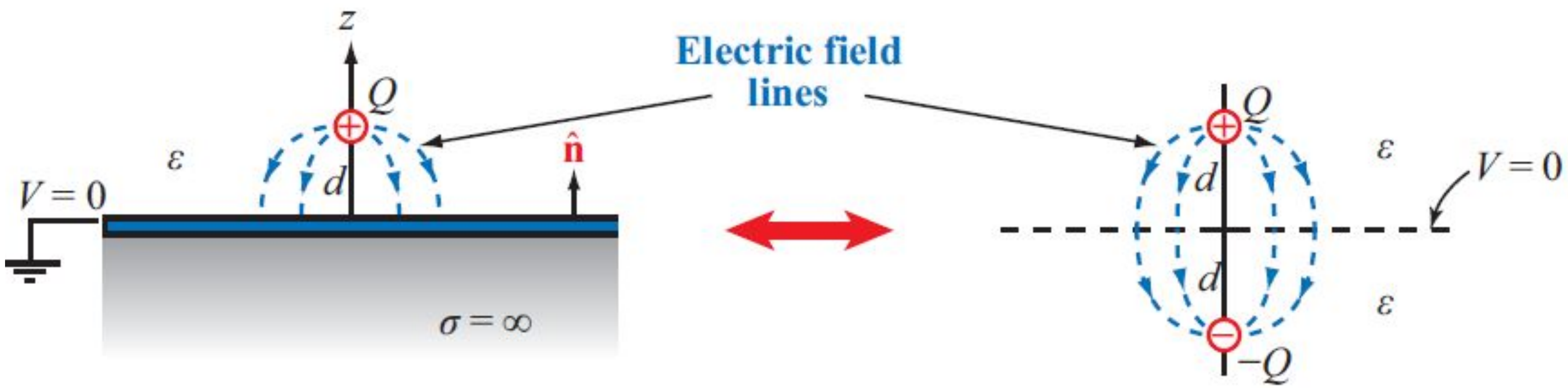
Chapter 4 Review

Capacitor as an Actuator:



$$\mathbf{F}_e = \hat{\mathbf{x}} \frac{1}{2} \frac{V^2}{d} w(\varepsilon - \varepsilon_0).$$

Image Method

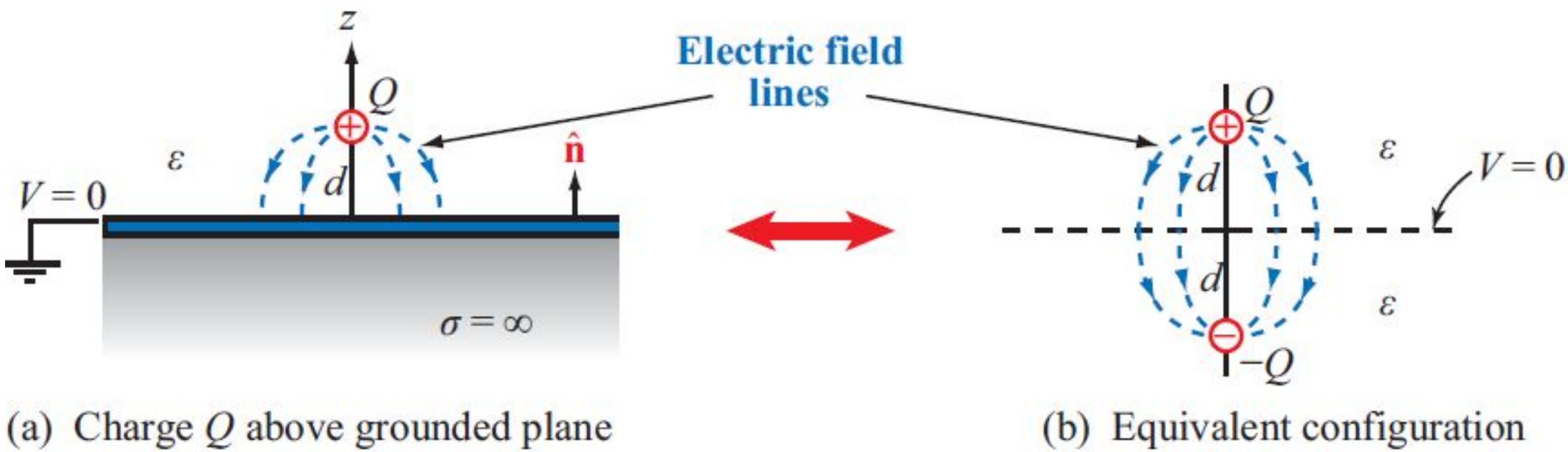


(a) Charge Q above grounded plane

(b) Equivalent configuration

Image method simplifies calculation for **E** and *V* due to charges near conducting planes.

Image Method



1. For each charge Q *at d*, add an image charge –Q *at -d*
2. Remove conducting plane
3. Calculate field due to all charges

Solution for E and V is applicable only in the upper region

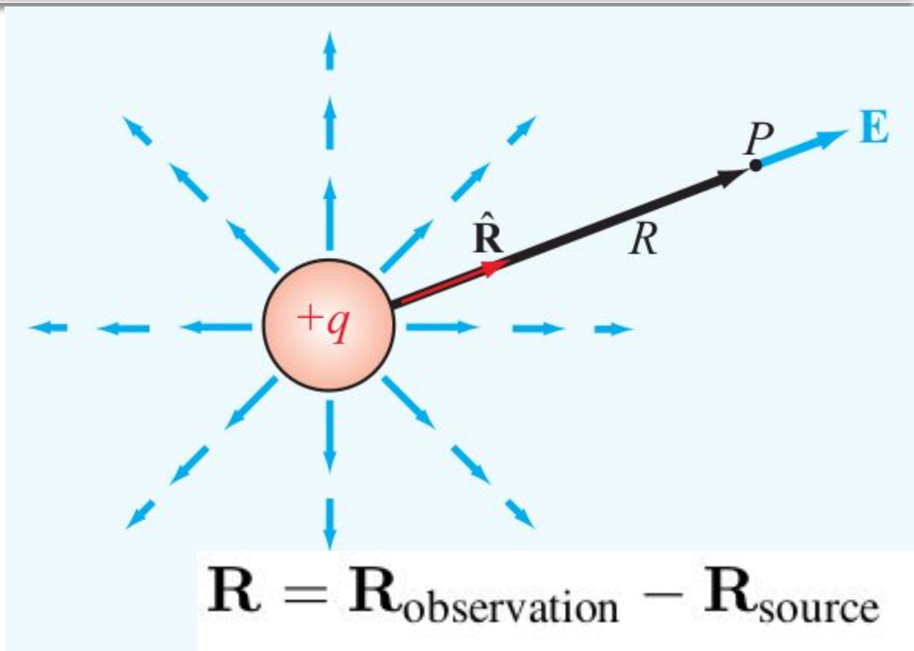
Review: E Field for a charge

Coulomb's Law:

$$F = \frac{1}{4\pi\epsilon}\frac{q_1 q_2}{R^2}$$

$$\mathbf{F} = \hat{\mathbf{R}}\frac{1}{4\pi\epsilon}\frac{q_1 q_2}{R^2}$$

$$\mathbf{F} = \mathbf{R}\frac{1}{4\pi\epsilon}\frac{q_1 q_2}{R^3}$$



Define the Electric Field:

$$\mathbf{F} = q\mathbf{E}$$

$$\mathbf{E} = \mathbf{R}\frac{1}{4\pi\epsilon}\frac{q_1}{R^3}$$

Review: E Field for charges

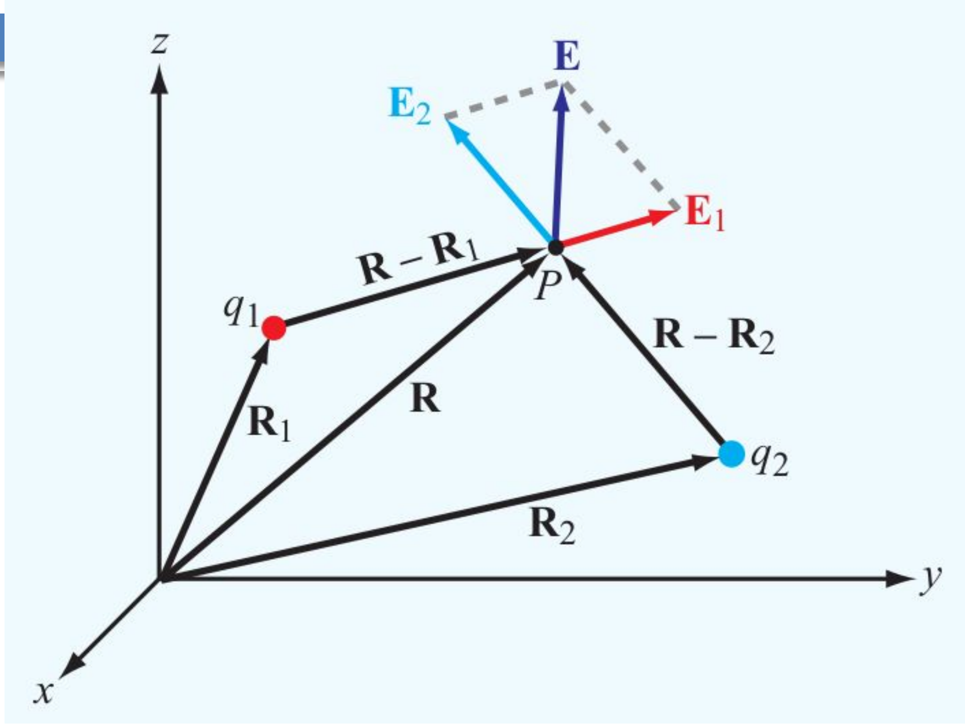
Multiple charges:

$$\mathbf{E} = \frac{1}{4\pi\epsilon}\sum_i \frac{q_i \mathbf{R}_i}{R_i^3}$$

$$\mathbf{R}_i = \mathbf{P} - \mathbf{Q}_i$$

$$R_i = \left| \mathbf{P} - \mathbf{Q}_i \right|$$

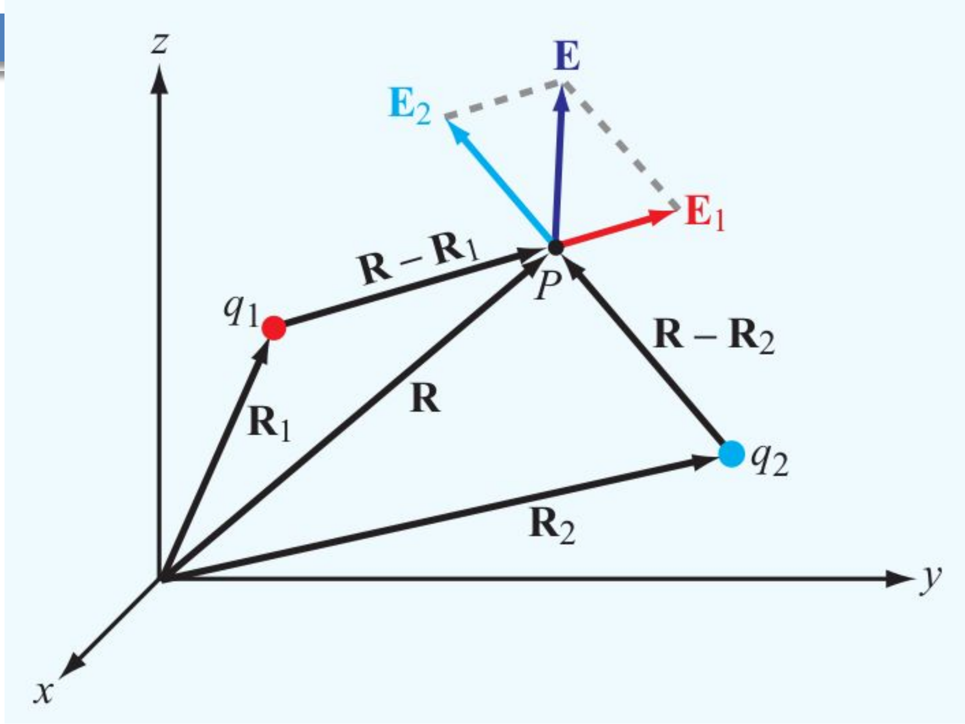
$$R_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}$$



Review: E Field for charges

Multiple charges:

$$\mathbf{E} = \frac{1}{4\pi\epsilon}\sum_i \frac{q_i \mathbf{R}_i}{R_i^3}$$



$$Q_1 = +Q$$

$$\mathbf{Q}_1 = (0, 0, d)$$

$$\mathbf{Q}_1 = \hat{\mathbf{x}}0 + \hat{\mathbf{y}}0 + \hat{\mathbf{z}}d$$

$$\mathbf{P} = \hat{\mathbf{x}}x + \hat{\mathbf{y}}y + \hat{\mathbf{z}}z$$

$$\mathbf{R}_1 = \mathbf{P} - \mathbf{Q}_1$$

$$\mathbf{R}_1 = \hat{\mathbf{x}}(x-0) + \hat{\mathbf{y}}(y-0) + \hat{\mathbf{z}}(z-d)$$

$$\mathbf{R}_1 = \hat{\mathbf{x}}x + \hat{\mathbf{y}}y + \hat{\mathbf{z}}(z-d)$$

$$R_1 = \sqrt{x^2 + y^2 + (z-d)^2}$$

Review: E Field for charges

Multiple charges:

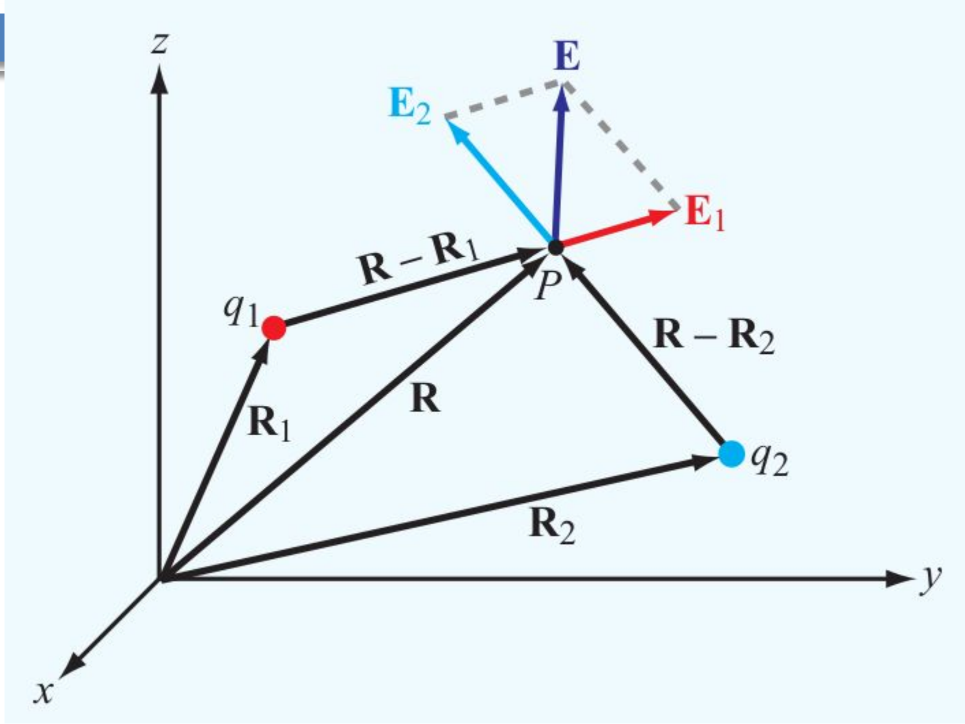
$$\mathbf{E} = \frac{1}{4\pi\epsilon} \sum_i \frac{q_i \mathbf{R}_i}{R_i^3}$$

$$Q_2 = -Q$$

$$\mathbf{Q}_2 = (0, 0, -d)$$

$$\mathbf{Q}_2 = \hat{\mathbf{x}}0 + \hat{\mathbf{y}}0 + \hat{\mathbf{z}}(-d)$$

$$\mathbf{P} = \hat{\mathbf{x}}x + \hat{\mathbf{y}}y + \hat{\mathbf{z}}z$$



$$\mathbf{R}_2 = \mathbf{P} - \mathbf{Q}_2$$

$$\mathbf{R}_2 = \hat{\mathbf{x}}(x-0) + \hat{\mathbf{y}}(y-0) + \hat{\mathbf{z}}(z-(-d))$$

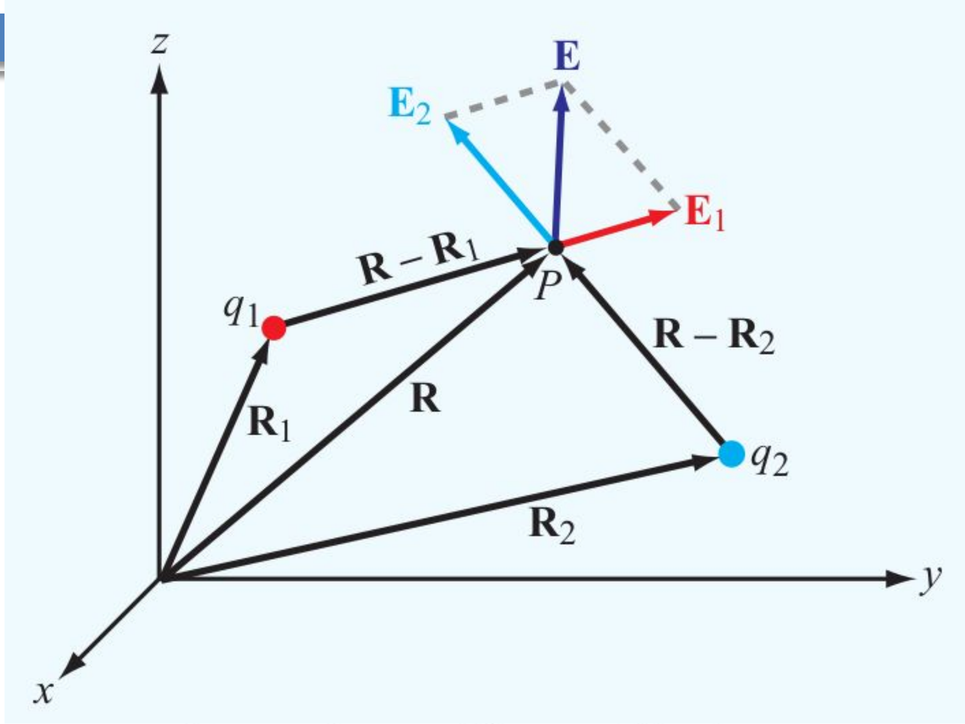
$$\mathbf{R}_2 = \hat{\mathbf{x}}x + \hat{\mathbf{y}}y + \hat{\mathbf{z}}(z+d)$$

$$R_2 = \sqrt{x^2 + y^2 + (z+d)^2}$$

Review: E Field for charges

Multiple charges:

$$\mathbf{E} = \frac{1}{4\pi\epsilon}\sum_i \frac{q_i \mathbf{R}_i}{R_i^3}$$



$$\mathbf{E} = \frac{Q}{4\pi\epsilon}\left[\frac{\mathbf{R}_1}{R_1^3} - \frac{\mathbf{R}_2}{R_2^3}\right]$$

$$\mathbf{E} = \frac{Q}{4\pi\epsilon}\left[\frac{\hat{\mathbf{x}}x + \hat{\mathbf{y}}y + \hat{\mathbf{z}}(z-d)}{(x^2+y^2+(z-d)^2)^{3/2}} - \frac{\hat{\mathbf{x}}x + \hat{\mathbf{y}}y + \hat{\mathbf{z}}(z+d)}{(x^2+y^2+(z+d)^2)^{3/2}}\right]$$

Image Method

Positive Charge above ground plane: replace ground plane with -Q, same distance below the plane.

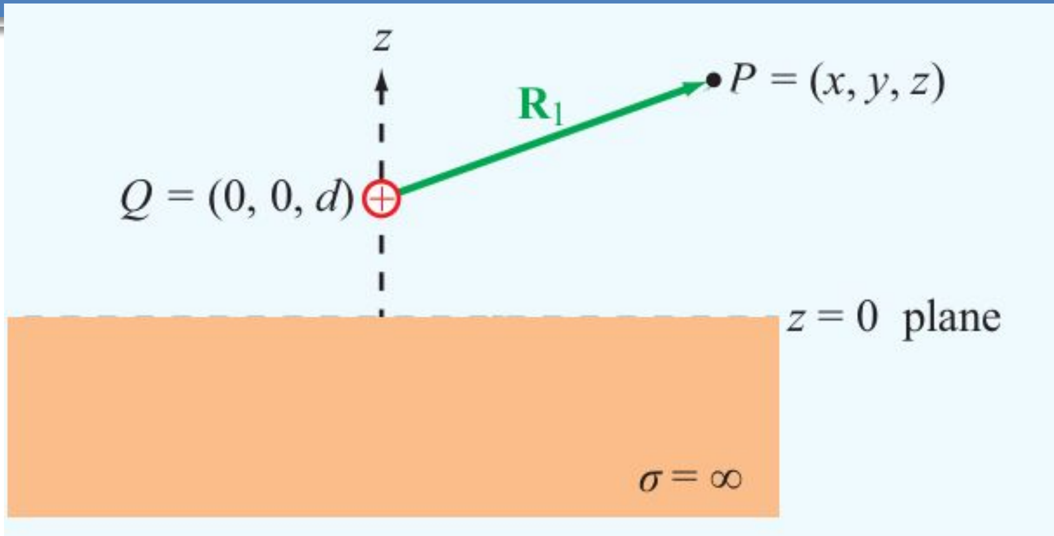
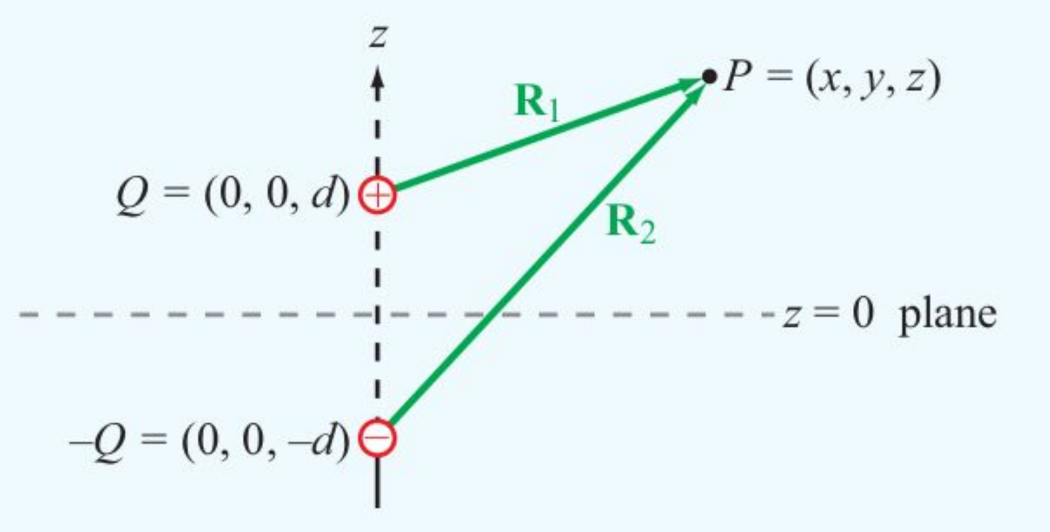


Image Method

Positive Charge above ground plane: replace ground plane with -Q, same distance below the plane.



$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q\mathbf{R}_1}{R_1^3} + \frac{-Q\mathbf{R}_2}{R_2^3}\right)$$

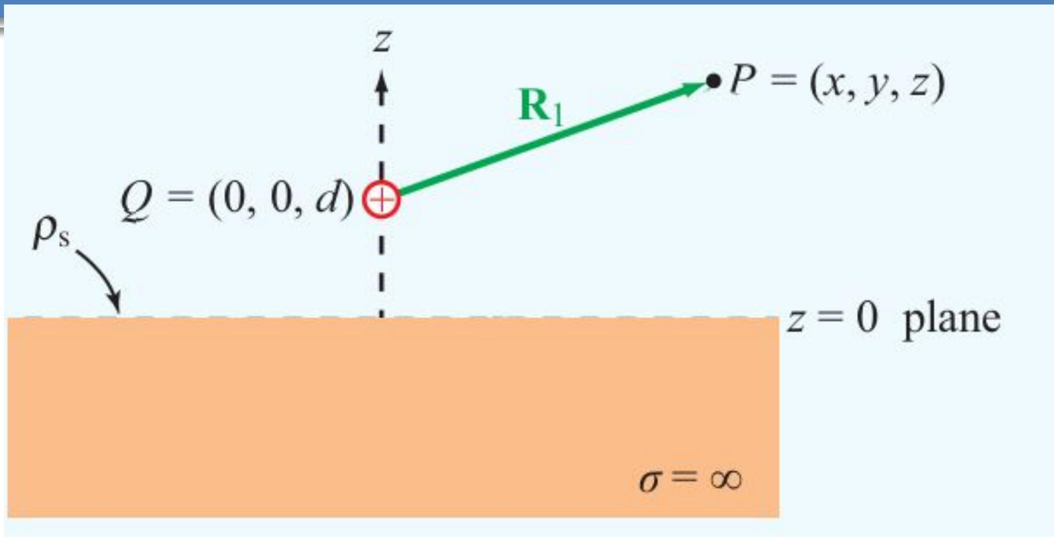
$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{\hat{\mathbf{x}}x + \hat{\mathbf{y}}y + \hat{\mathbf{z}}(z-d)}{[x^2+y^2+(z-d)^2]^{3/2}} - \frac{\hat{\mathbf{x}}x + \hat{\mathbf{y}}y + \hat{\mathbf{z}}(z+d)}{[x^2+y^2+(z+d)^2]^{3/2}}\right]$$

for $z \geq 0$.

Image Method

Charge Density on the ground plane:

$$\rho_s = (\hat{\mathbf{n}} \cdot \mathbf{E})\varepsilon_0,$$



First find **E** at *z*=0:

$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{\hat{\mathbf{x}}x + \hat{\mathbf{y}}y + \hat{\mathbf{z}}(z-d)}{[x^2+y^2+(z-d)^2]^{3/2}} - \frac{\hat{\mathbf{x}}x + \hat{\mathbf{y}}y + \hat{\mathbf{z}}(z+d)}{[x^2+y^2+(z+d)^2]^{3/2}}\right]$$

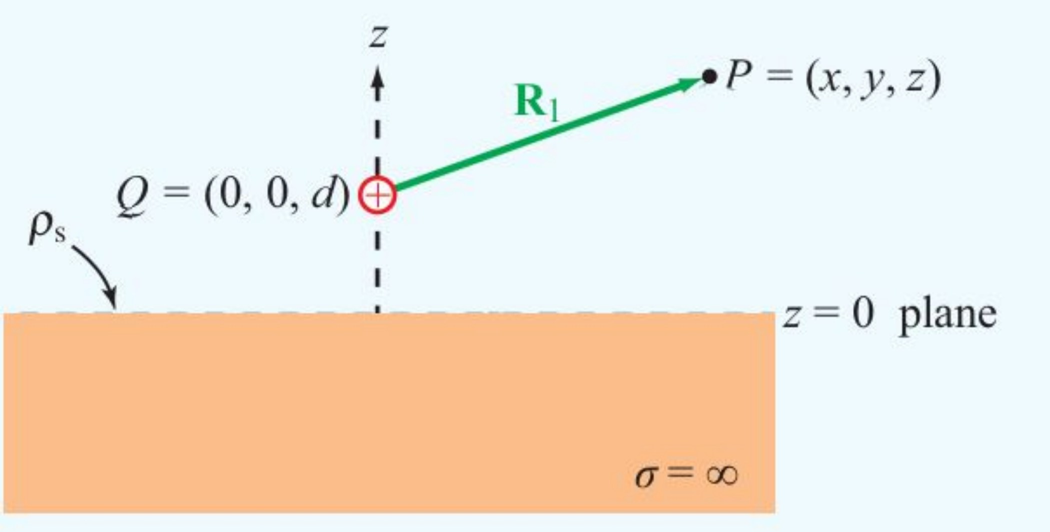
$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{\hat{\mathbf{x}}x + \hat{\mathbf{y}}y + \hat{\mathbf{z}}(-d)}{[x^2+y^2+d^2]^{3/2}} - \frac{\hat{\mathbf{x}}x + \hat{\mathbf{y}}y + \hat{\mathbf{z}}(d)}{[x^2+y^2+d^2]^{3/2}}\right]$$

$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_0}\frac{\hat{\mathbf{z}}(-2d)}{[x^2+y^2+d^2]^{3/2}}$$

Image Method

Charge Density on the ground plane:

$$\rho_s = (\hat{\mathbf{n}} \cdot \mathbf{E})\varepsilon_0,$$



First find **E** at *z*=0:

$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_0} \frac{\hat{\mathbf{z}}(-2d)}{[x^2 + y^2 + d^2]^{3/2}}$$

$$\mathbf{E} = -\hat{\mathbf{z}} \frac{Qd}{2\pi\varepsilon_0} \frac{1}{[x^2 + y^2 + d^2]^{3/2}}$$

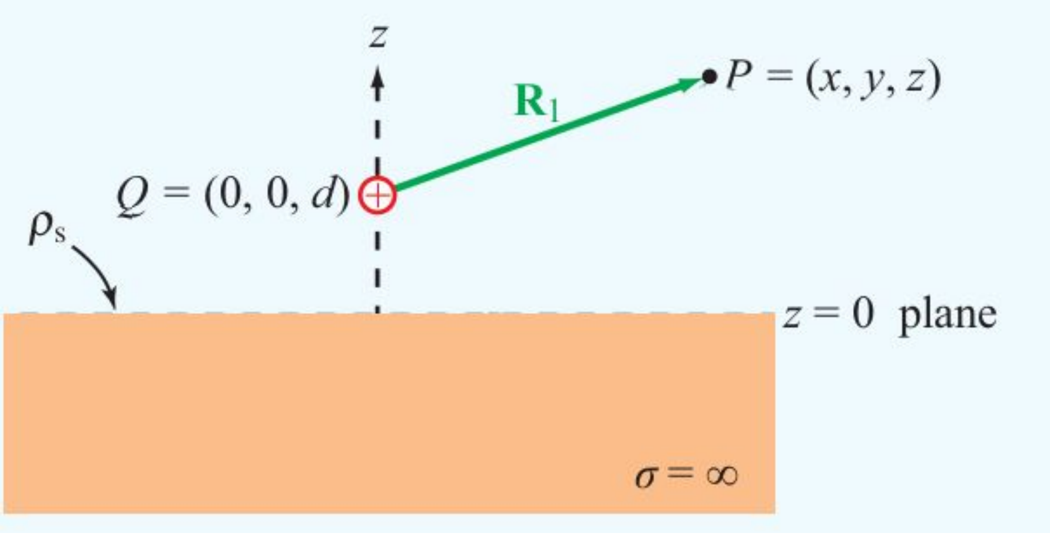
$$\rho_s = \hat{\mathbf{z}} \cdot \left[-\hat{\mathbf{z}} \frac{Qd}{2\pi\varepsilon_0} \frac{1}{[x^2 + y^2 + d^2]^{3/2}} \right] \varepsilon_0$$

$$\rho_s = -\frac{Qd}{2\pi} \frac{1}{[x^2 + y^2 + d^2]^{3/2}}$$

Image Method

Charge Density on the ground plane:

$$\rho_s = -\frac{Qd}{2\pi}\frac{1}{[x^2+y^2+d^2]^{3/2}}$$



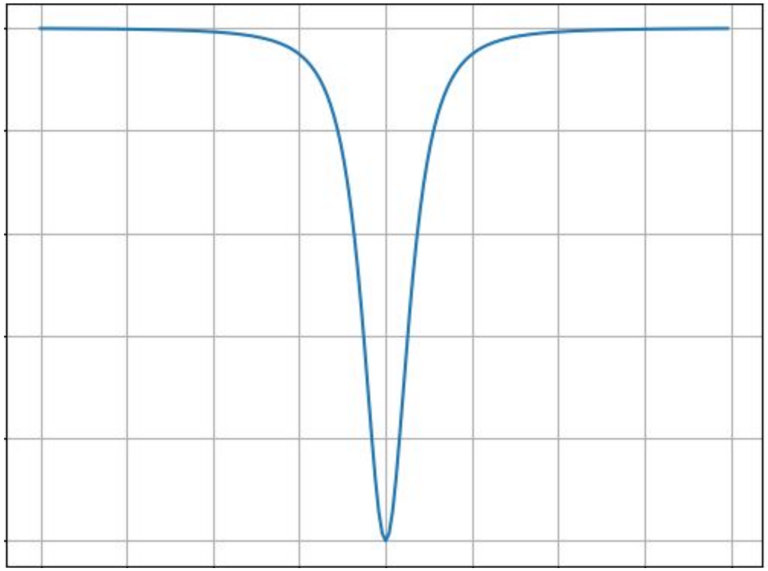
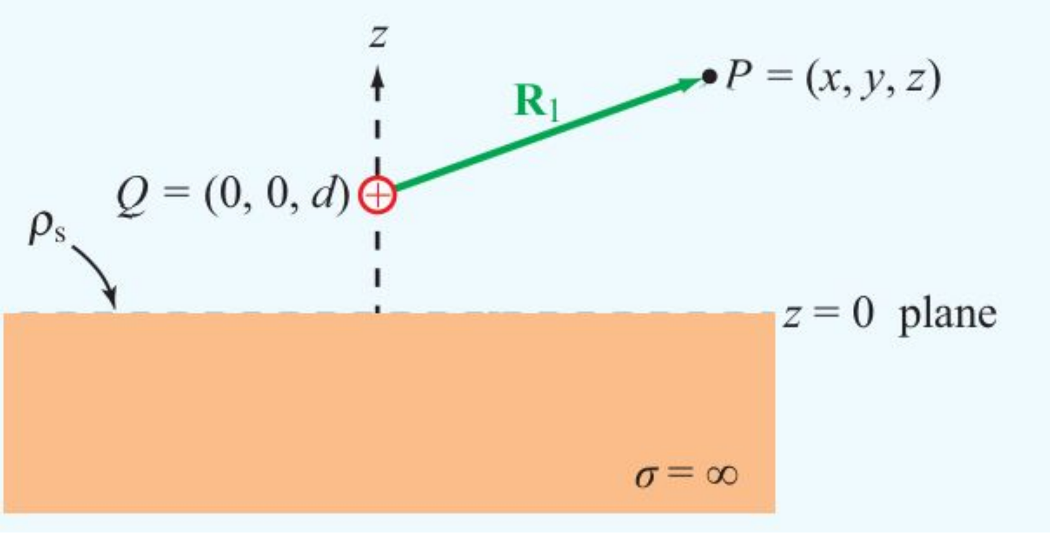


Image Method

Charge Density on the ground plane:



$$\mathbf{F} = q\mathbf{E}$$

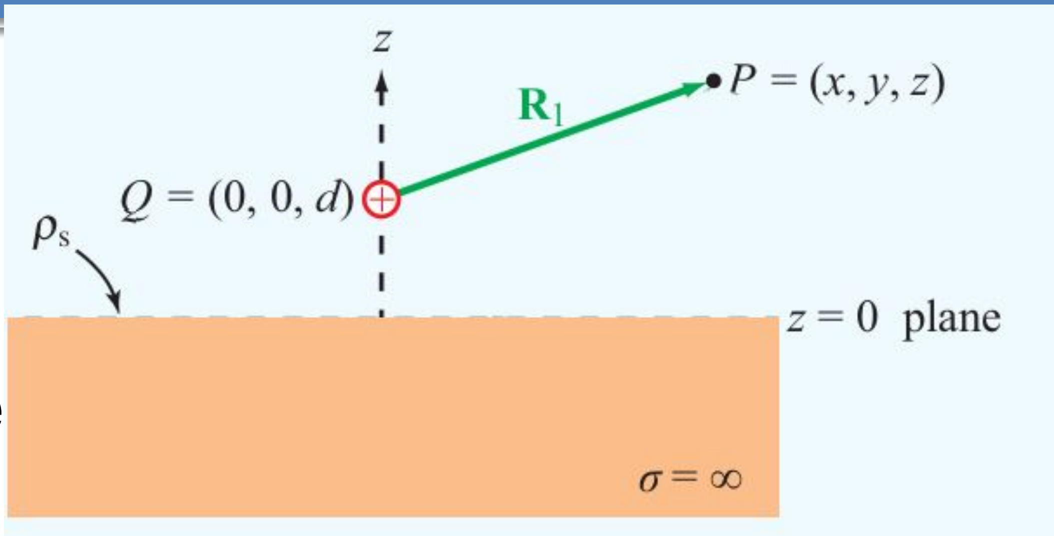
$$\mathbf{F} = -\hat{\mathbf{z}}\frac{Q^2 d}{2\pi\varepsilon_0}\frac{1}{[x^2+y^2+d^2]^{3/2}}$$

So the positive charge is pulled closer to the ground plane.

Image Method

Charge Density on the ground plane:

What is the total charge on the ground plane?



$$\text{Total Charge} = \int_S \rho_s ds$$

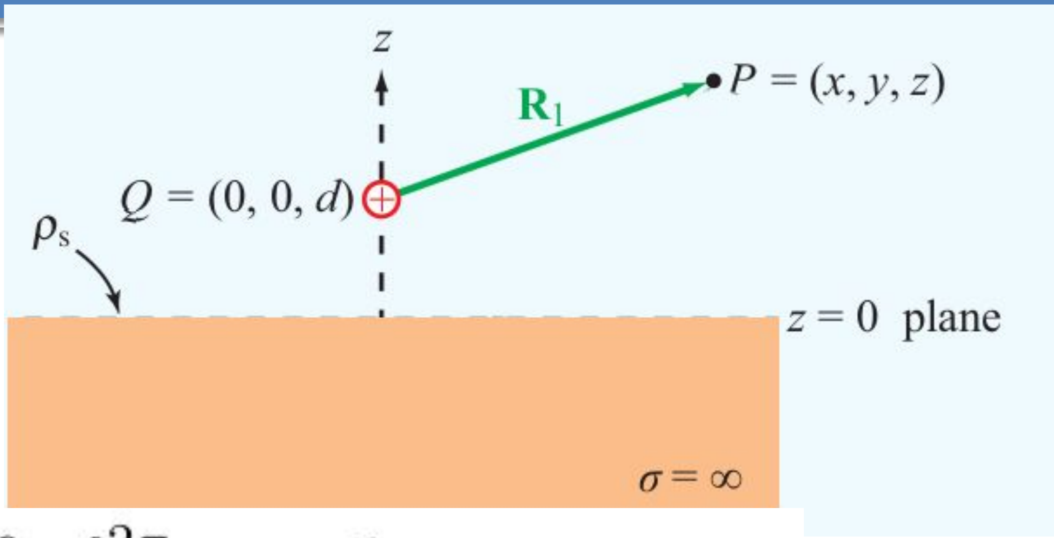
$$\text{Total Charge} = -\frac{Qd}{2\pi}\int_{x=-\infty}^{+\infty}\int_{y=-\infty}^{+\infty}\frac{1}{[x^2+y^2+d^2]^{3/2}}\,dx\,dy$$

too difficult: convert to cylindrical:

Image Method

Charge Density on the ground plane:

in cylindrical:



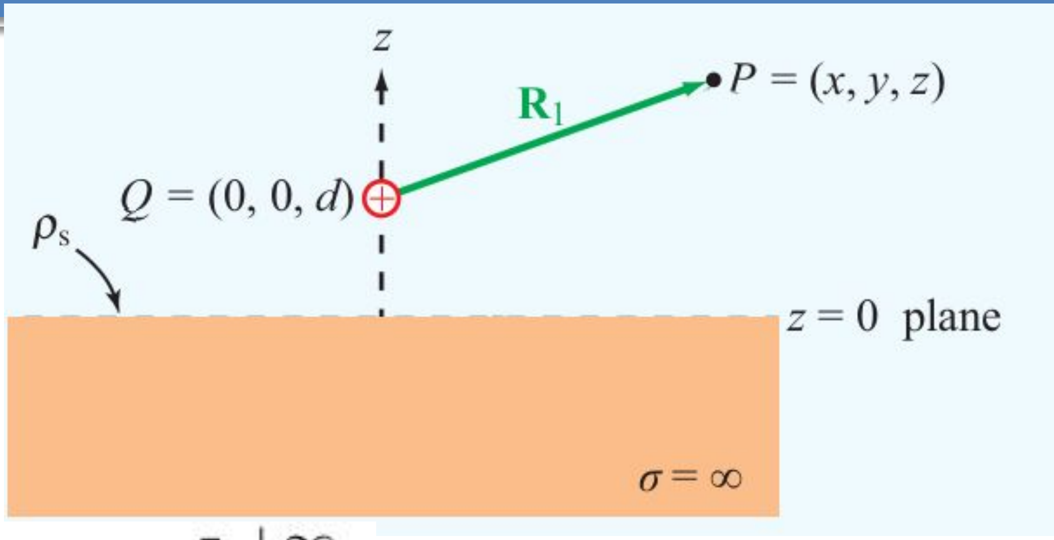
$$\text{Total Charge} = -\frac{Qd}{2\pi}\int_{r=0}^{+\infty}\int_{\phi=0}^{2\pi}\frac{1}{[r^2+d^2]^{3/2}}\,r\,dr\,d\phi$$

$$\text{Total Charge} = -Qd\int_{r=0}^{+\infty}\frac{1}{[r^2+d^2]^{3/2}}\,r\,dr$$

$$\text{Total Charge} = -Qd\left[\frac{-1}{\sqrt{r^2+d^2}}\right]_{r=0}^{+\infty}$$

Image Method

Charge Density on the ground plane:



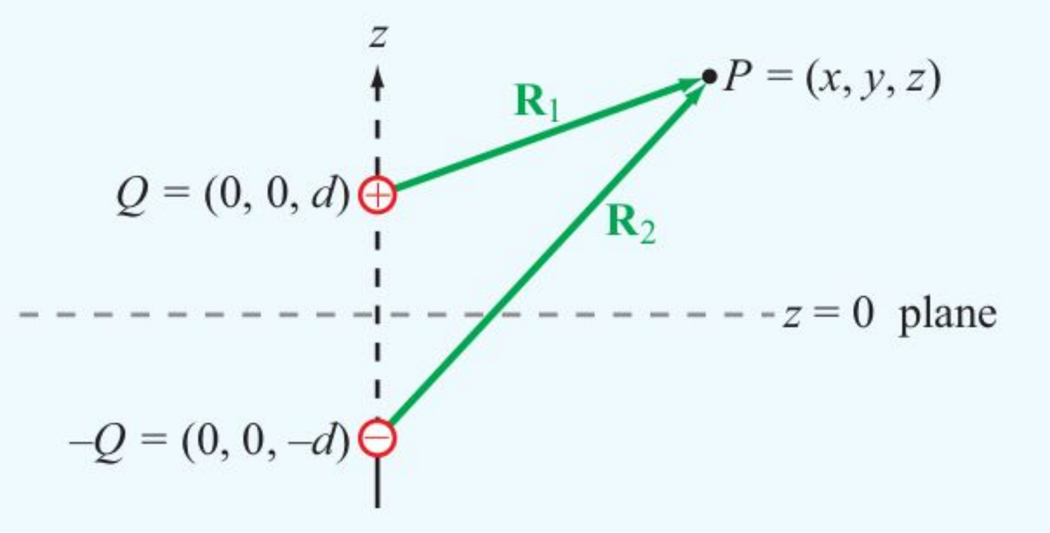
$$\text{Total Charge} = -Qd\left[\frac{-1}{\sqrt{r^2+d^2}}\right]_{r=0}^{+\infty}$$

$$\text{Total Charge} = -Qd\left[\frac{-1}{\infty} - \frac{-1}{d}\right]$$

$$\text{Total Charge} = -Q$$

Charges are balanced

Image Method



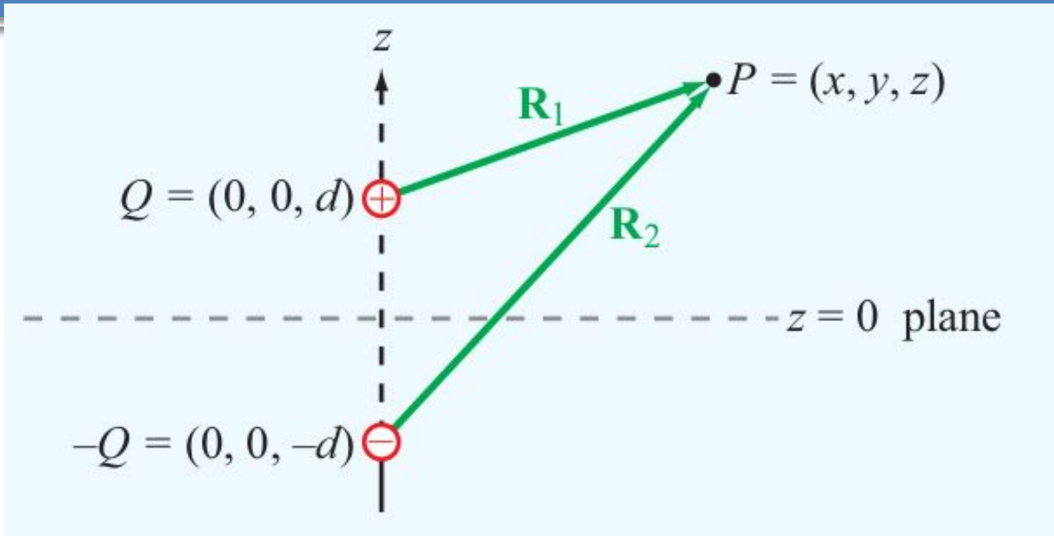
Voltage for a charge near a ground plane:

Apply the same process as before: adding a charge below the ground plane.

Use equation for Voltage of *N* charges:

$$V = \frac{1}{4\pi\varepsilon} \sum_{i=1}^{N} \frac{q_i}{R_i}$$

Image Method



Voltage for a charge near a ground plane:

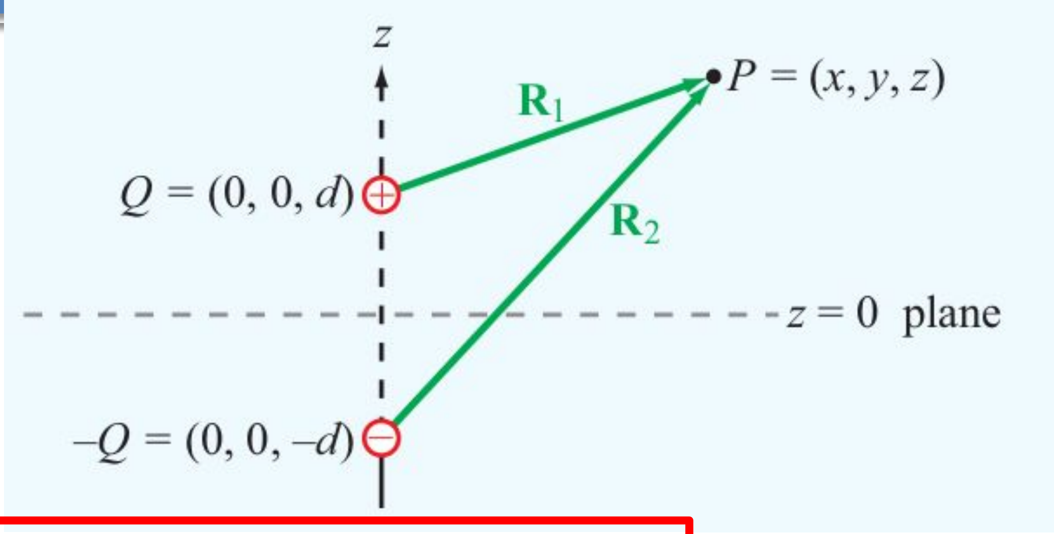
Apply the same process as before: adding a charge below the ground plane. Use equation for Voltage of *N* charges:

$$V = \frac{1}{4\pi\varepsilon} \sum_{i=1}^{N} \frac{q_i}{R_i}$$

$$V = \frac{1}{4\pi\varepsilon} \left[\frac{Q}{\sqrt{r^2 + (z-d)^2}} + \frac{-Q}{\sqrt{r^2 + (z+d)^2}} \right]$$

Image Method

Voltage for a charge near a ground plane:



$$V = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{\sqrt{r^2+(z-d)^2}} - \frac{1}{\sqrt{r^2+(z+d)^2}}\right]$$

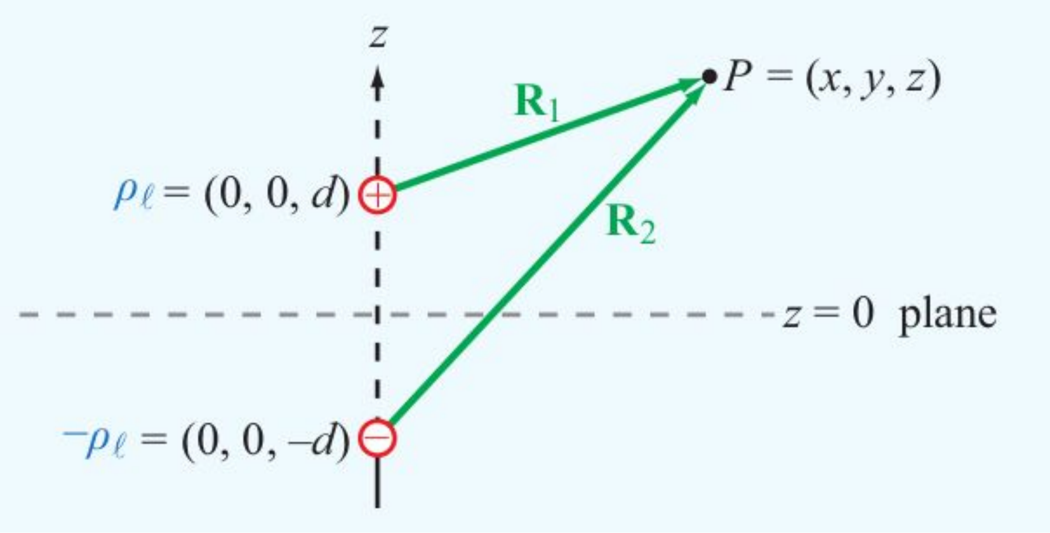
$$V(z=0) = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{\sqrt{r^2+(-d)^2}} - \frac{1}{\sqrt{r^2+(d)^2}}\right]$$

$$V(z=0) = 0$$

Voltage=0 on ground plane, as expected.

Image Method

Positive Line Charge above ground plane:
replace ground plane with $-\rho_l$, same distance below the plane.



Single line charge: $\mathbf{E} = \hat{\mathbf{r}} \dfrac{\rho_l}{2\pi\varepsilon_0 r}$

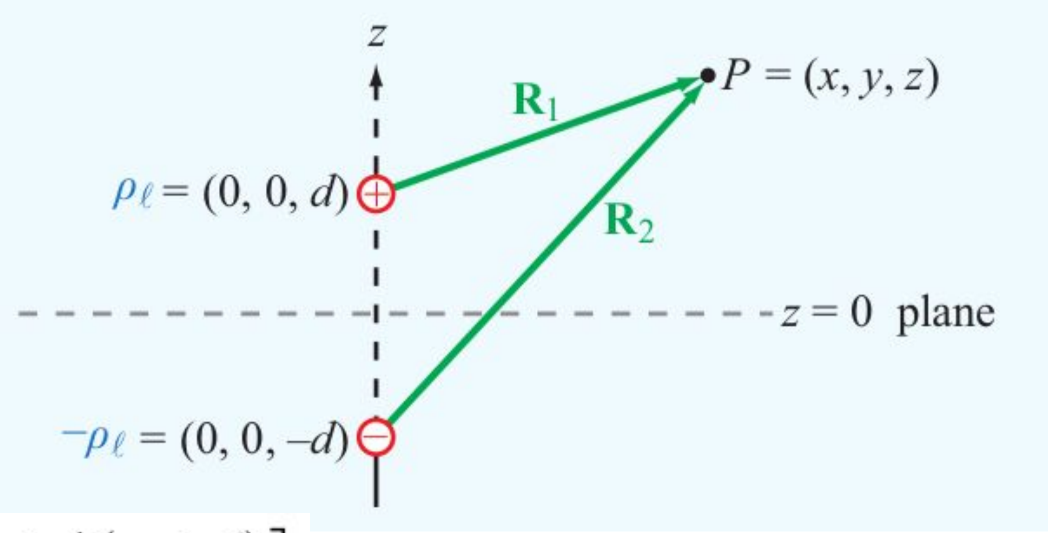
2 line charges:

$$\mathbf{E} = \hat{\mathbf{R}}_1 \frac{+\rho_l}{2\pi\varepsilon_0 R_1} + \hat{\mathbf{R}}_2 \frac{-\rho_l}{2\pi\varepsilon_0 R_2}$$

$$\mathbf{E} = \frac{\rho_l}{2\pi\varepsilon_0}\left[\frac{\hat{\mathbf{r}}r + \hat{\mathbf{z}}(z-d)}{r^2+(z-d)^2} - \frac{\hat{\mathbf{r}}r + \hat{\mathbf{z}}(z+d)}{r^2+(z+d)^2}\right]$$

Image Method

Positive Line Charge above ground plane:



get surface charge:
E at z=0:

$$\mathbf{E} = \frac{\rho_l}{2\pi\varepsilon_0}\left[\frac{\hat{\mathbf{r}}r + \hat{\mathbf{z}}(z-d)}{r^2+(z-d)^2} - \frac{\hat{\mathbf{r}}r + \hat{\mathbf{z}}(z+d)}{r^2+(z+d)^2}\right]$$

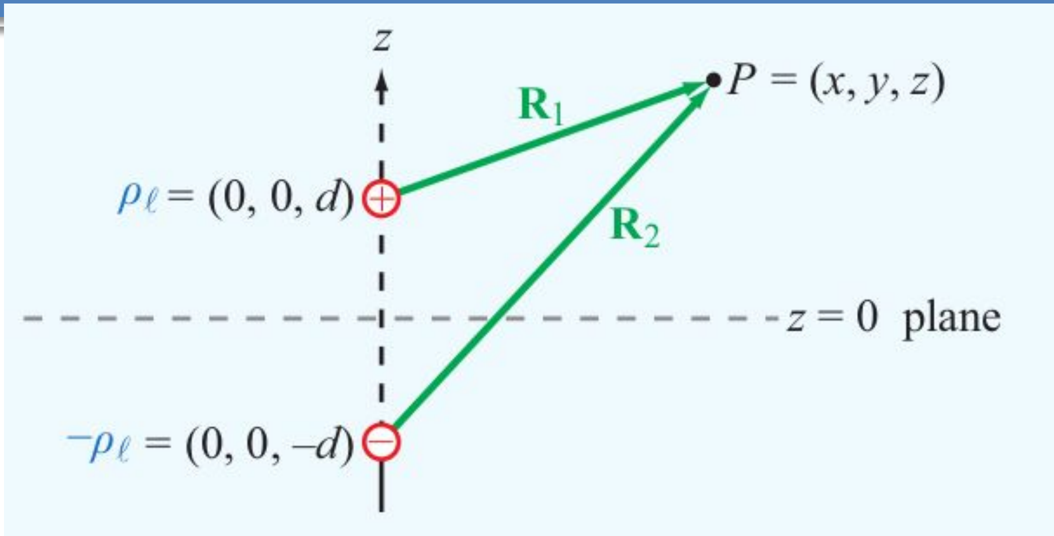
$$\mathbf{E} = \frac{\rho_l}{2\pi\varepsilon_0}\left[\frac{\hat{\mathbf{r}}r + \hat{\mathbf{z}}(-d)}{r^2+(-d)^2} - \frac{\hat{\mathbf{r}}r + \hat{\mathbf{z}}(d)}{r^2+(d)^2}\right]$$

$$\mathbf{E} = \frac{\rho_l}{2\pi\varepsilon_0}\left[\frac{\hat{\mathbf{z}}(-2d)}{r^2+d^2}\right]$$

$$\mathbf{E} = -\hat{\mathbf{z}}\frac{\rho_l d}{\pi\varepsilon_0}\frac{1}{r^2+d^2}$$

Image Method

Positive Line Charge above ground plane:



E at z=0:

$$\mathbf{E} = \frac{\rho_l}{2\pi\varepsilon_0}\left[\frac{\hat{\mathbf{z}}(-2d)}{r^2 + d^2}\right]$$

$$\mathbf{E} = -\hat{\mathbf{z}}\frac{\rho_l d}{\pi\varepsilon_0}\frac{1}{r^2 + d^2}$$

Charge density on ground plane:

$$\rho_s = \hat{\mathbf{z}} \cdot \left[-\hat{\mathbf{z}}\frac{\rho_l d}{\pi\varepsilon_0}\frac{1}{r^2 + d^2}\right]\varepsilon_0$$

$$\rho_s = -\frac{\rho_l d}{\pi}\frac{1}{r^2 + d^2}$$

Image Method

Positive Charge near grounded inside corner:

determine the Voltage

Assume the ground planes are infinite.

first: form image of the charge in the horizontal plane.

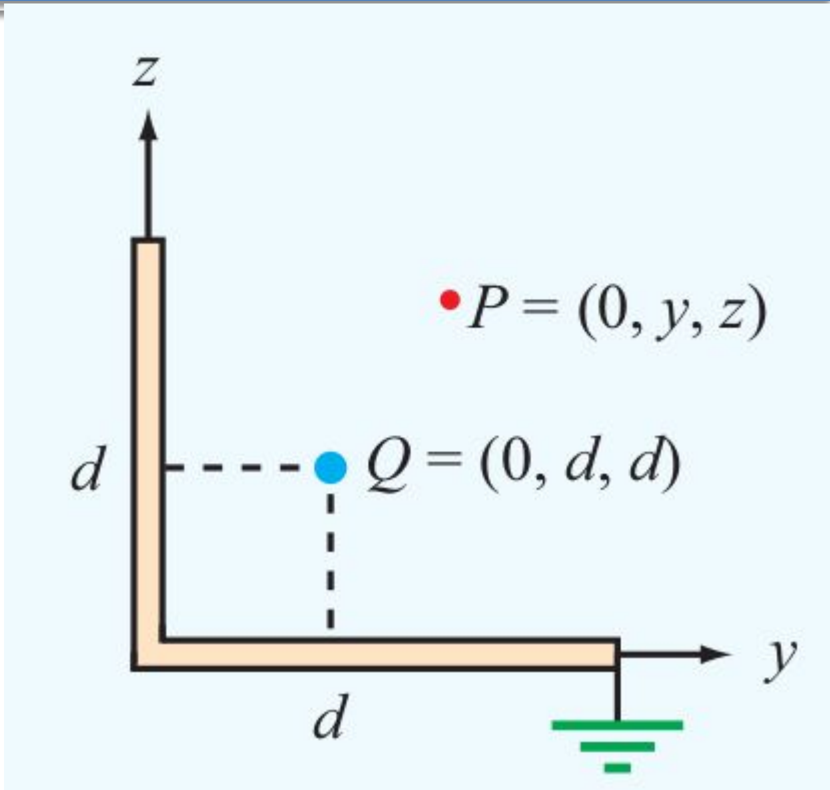


Image Method

Positive Charge near grounded inside corner:

determine the Voltage

Assume the ground planes are infinite.

first: form image of the charge in the horizontal plane.

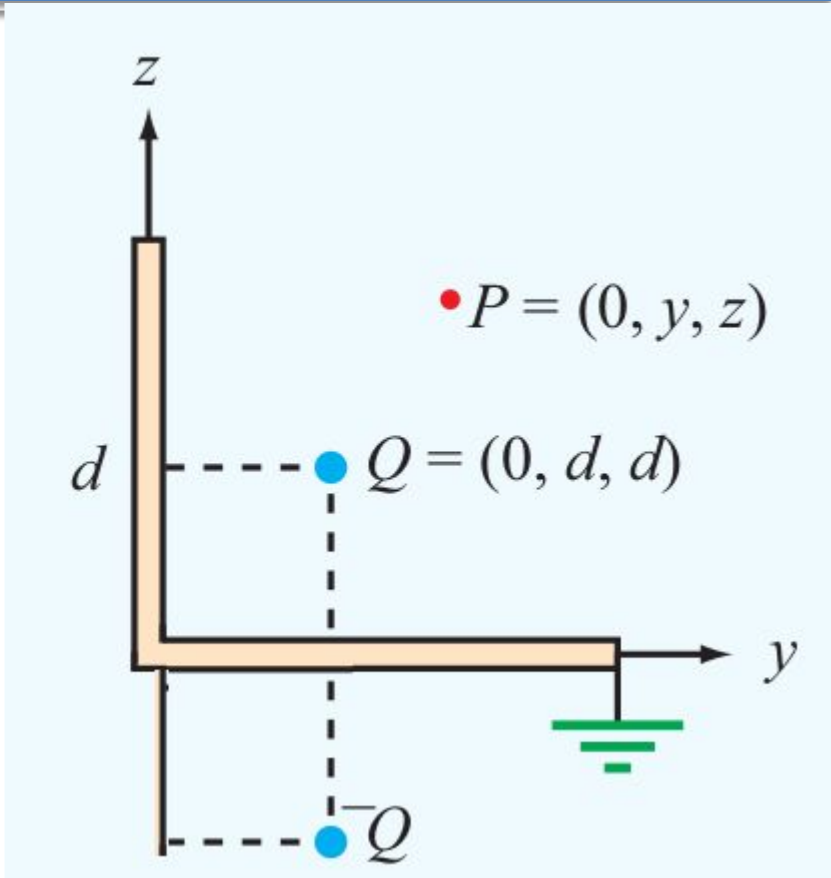


Image Method

Positive Charge near grounded inside corner:

determine the Voltage

Next: form the image of BOTH charges in the vertical ground plane.

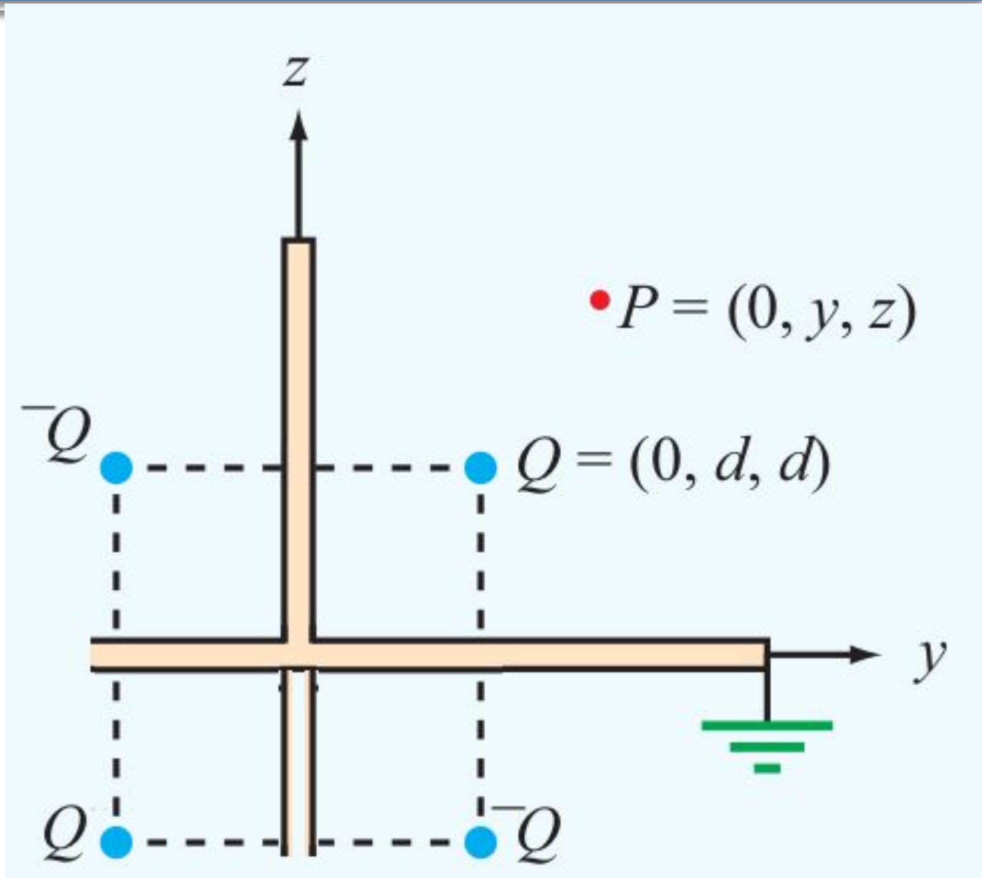
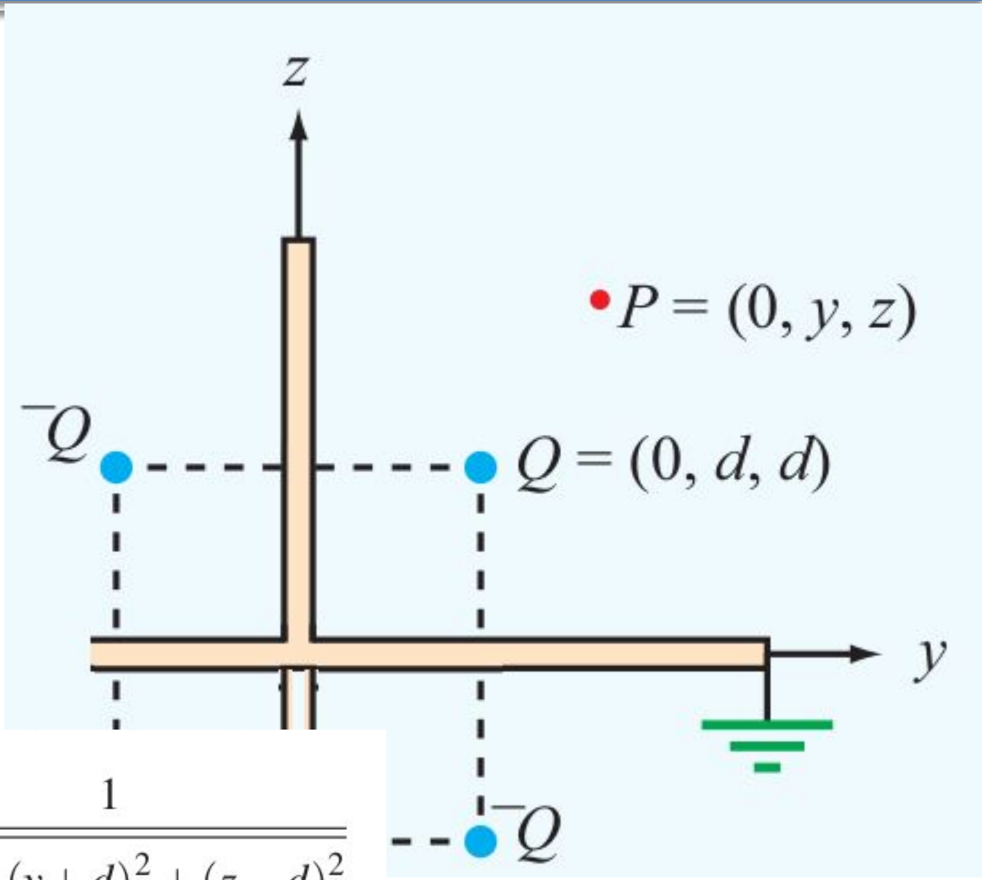


Image Method

Positive Charge near grounded inside corner:

Sum the Voltages from each of the 4 charges:



$$V(x,y,z) =$$

$$\frac{Q}{4\pi\varepsilon}\left(\frac{1}{\sqrt{x^2+(y-d)^2+(z-d)^2}}-\frac{1}{\sqrt{x^2+(y+d)^2+(z-d)^2}}\right.$$

$$\left.+\frac{1}{\sqrt{x^2+(y+d)^2+(z+d)^2}}-\frac{1}{\sqrt{x^2+(y-d)^2+(z+d)^2}}\right)$$

Image Method

Positive Charge near grounded inside corner:

$$\mathbf{E} = -\nabla V$$

$$= \frac{-Q}{4\pi\varepsilon}\left(\nabla\frac{1}{\sqrt{x^2+(y-d)^2+(z-d)^2}} - \nabla\frac{1}{\sqrt{x^2+(y+d)^2+(z-d)^2}}\right.$$

$$\left. + \nabla\frac{1}{\sqrt{x^2+(y+d)^2+(z+d)^2}} - \nabla\frac{1}{\sqrt{x^2+(y-d)^2+(z+d)^2}}\right)$$

Image Method

Positive Charge near grounded inside corner:

example of the gradient applied to one term:

$$-\left(\hat{\mathbf{x}}\frac{\partial}{\partial x}+\hat{\mathbf{y}}\frac{\partial}{\partial y}+\hat{\mathbf{z}}\frac{\partial}{\partial z}\right)\left(x^2+(y-d)^2+(z-d)^2\right)^{-1/2}$$

$$\frac{1}{2}\left(\hat{\mathbf{x}}2x+\hat{\mathbf{y}}2(y-d)+\hat{\mathbf{z}}2(z-d)\right)\left(x^2+(y-d)^2+(z-d)^2\right)^{-3/2}$$

$$\left(\hat{\mathbf{x}}x+\hat{\mathbf{y}}(y-d)+\hat{\mathbf{z}}(z-d)\right)\left(x^2+(y-d)^2+(z-d)^2\right)^{-3/2}$$

Image Method

Positive Charge near grounded inside corner:

$$\mathbf{E} = \frac{Q}{4\pi\varepsilon}\left(\frac{\hat{\mathbf{x}}x+\hat{\mathbf{y}}(y-d)+\hat{\mathbf{z}}(z-d)}{\left(x^2+(y-d)^2+(z-d)^2\right)^{3/2}} - \frac{\hat{\mathbf{x}}x+\hat{\mathbf{y}}(y+d)+\hat{\mathbf{z}}(z-d)}{\left(x^2+(y+d)^2+(z-d)^2\right)^{3/2}} + \frac{\hat{\mathbf{x}}x+\hat{\mathbf{y}}(y+d)+\hat{\mathbf{z}}(z+d)}{\left(x^2+(y+d)^2+(z+d)^2\right)^{3/2}} - \frac{\hat{\mathbf{x}}x+\hat{\mathbf{y}}(y-d)+\hat{\mathbf{z}}(z+d)}{\left(x^2+(y-d)^2+(z+d)^2\right)^{3/2}}\right) \quad \text{(V/m)}.$$

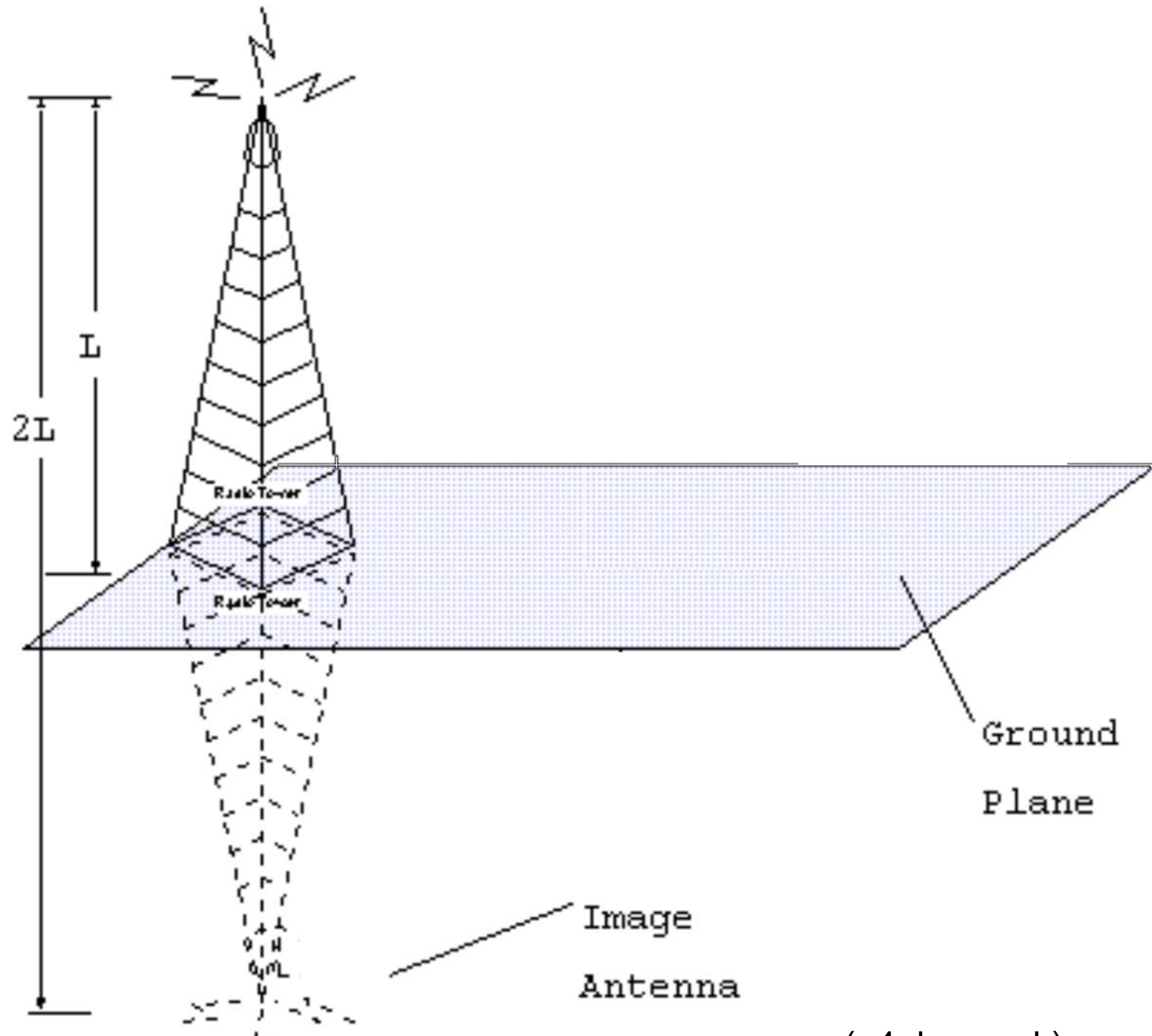
Image Method

Antennas placed near the surface of the Earth:

The Earth is similar to a flat perfect conductor.

Radio stations use this idea.

So do cars.



(g4akc.co.uk)

Homework

Homework 17 is due tomorrow at midnight.

submit to gradescope via the canvas site.

Next Time

Sections 5-1 through 5-2:

Magnetostatics:
  Magnetic Forces and Torques
H due to a steady current (Biot-Savart Law)