EECS 230
ENGINEERING ELECTROMAGNETICS
Leland Pierce

Electrostatics 4

University of Michigan - Fall 2024

Chapter 4 Overview

Maxwell's Equations
- Electrostatics
- Magnetostatics

Charge density
Current density
Electric field from charges
Gauss's Law
Electric Scalar Potential Field
- Dipole Field
- Poisson's eqn

Conductors
- current
- resistance
- joule's law

Dielectrics
- polarization

Boundary Conditions
Capacitance
Potential Energy
Image method

$$\nabla \cdot \mathbf{D} = \rho_v,$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$
$$\nabla \cdot \mathbf{B} = 0,$$
$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}.$$

$$\mathbf{E} = \int_{v'} d\mathbf{E} = \frac{1}{4\pi\varepsilon} \int_{v'} \hat{\mathbf{R}}' \frac{\rho_v \, dv'}{R'^2}.$$

(volume distribution)

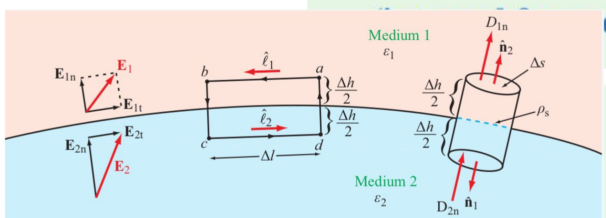
$$\nabla \cdot \mathbf{D} = \rho_v,$$

(differential form of Gauss's law)

$$\nabla \cdot \mathbf{D} = \rho_v,$$
$$\nabla \times \mathbf{E} = 0.$$

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q.$$

of Gauss's law)



$$V = -\int_{\infty}^{P} \mathbf{E} \cdot d\mathbf{l}$$

$$\nabla \cdot \mathbf{B} = 0,$$
$$\nabla \times \mathbf{H} = \mathbf{J}.$$

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon} \qquad \mathbf{E} = -\nabla V.$$

$$Q = \int_v \rho_v \, dv \qquad \text{(C)}.$$

$$\mathbf{J} = \sigma \mathbf{E} \quad (\text{A/m}^2) \quad \text{(Ohm's law)},$$

$$\mathbf{J} = \rho_v \mathbf{u} \qquad (\text{A/m}^2)$$

$$R = \frac{V}{I} = \frac{-\int_l \mathbf{E} \cdot d\mathbf{l}}{\int_S \mathbf{J} \cdot d\mathbf{s}} = \frac{-\int_l \mathbf{E} \cdot d\mathbf{l}}{\int_S \sigma \mathbf{E} \cdot d\mathbf{s}}.$$

$$P = \int_v \mathbf{E} \cdot \mathbf{J} \, dv \quad \text{(W)}$$

$$I = \int_S \mathbf{J} \cdot d\mathbf{s} \qquad \text{(A)}.$$

Lecture Coverage

Today's lecture:

Review of Sections 4-1 through 4-6 of the book:

4-1: Maxwell's Equations

4-2: Charge and Current Distributions

4-3: Coulomb's Law

4-4: Gauss's Law

4-5: Voltage (Electric Scalar Potential)

4-6: Conductors

Sections 4-7, 4-8 of the book:

4-7: Dielectrics

4-8: Boundary Conditions

Chapter 4 Review

Maxwell's Equations:

$$\nabla \cdot \mathbf{D} = \rho_{\mathrm{V}},$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \cdot \mathbf{B} = 0,$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}.$$

Empirically derived from many measurements

$$\mathbf{D} = \varepsilon \mathbf{E}$$

$$\mathbf{B} = \mu \mathbf{H}.$$

E: Electric Field
H: Magnetic Field
J: Current Density
ρ_v: Charge Density

Chapter 4 Review

Static Conditions:

Electrostatics

$$\frac{\partial \rho_v}{\partial t} = 0$$

$$\nabla \cdot \mathbf{D} = \rho_v,$$
$$\nabla \times \mathbf{E} = 0.$$

Magnetostatics

$$\frac{\partial \mathbf{J}}{\partial t} = 0$$

$$\nabla \cdot \mathbf{B} = 0,$$
$$\nabla \times \mathbf{H} = \mathbf{J}.$$

Electric and Magnetic Fields are decoupled.

Chapter 4 Review

$$\mathbf{E} = \frac{1}{4\pi\varepsilon} \int_{\upsilon'} \hat{\mathbf{R}}' \frac{\rho_{\mathrm{v}} \, d\upsilon'}{R'^2}$$ **(volume distribution)**

$$\mathbf{E} = \frac{1}{4\pi\varepsilon} \int_{S'} \hat{\mathbf{R}}' \frac{\rho_{\mathrm{s}} \, ds'}{R'^2}$$ **(surface distribution)**

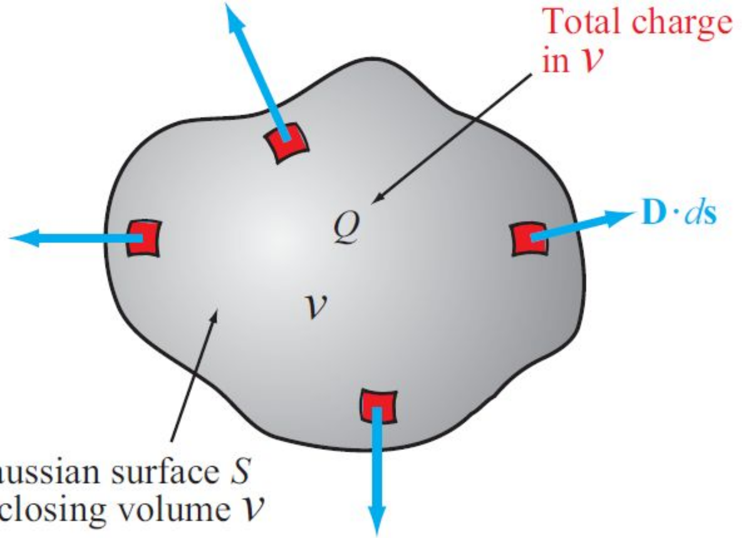
$$\mathbf{E} = \frac{1}{4\pi\varepsilon} \int_{l'} \hat{\mathbf{R}}' \frac{\rho_{\ell} \, dl'}{R'^2}$$ **(line distribution)**

Chapter 4 Review

Gauss's Law

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q \quad (4.2$$

(Integral form of Gauss's law).



or:

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = \int_{\mathcal{V}} \rho_{\mathrm{v}} \, d\mathcal{V}$$

where the closed-surface S is the boundary of $\mathcal{V}$

Chapter 4 Review

Voltage:

$$V = -\int_{\infty}^{P} \mathbf{E} \cdot d\mathbf{l} \qquad \text{(V)}. \qquad (4.43)$$

N Point Charges:

$$V = \frac{1}{4\pi\varepsilon} \sum_{i=1}^{N} \frac{q_i}{|\mathbf{R} - \mathbf{R}_i|} \qquad \text{(V)}.$$

Chapter 4 Review

$$V = \frac{1}{4\pi\varepsilon} \int_{\upsilon'} \frac{\rho_v}{R'} \, d\upsilon' \quad \text{(volume distribution)},$$

$$V = \frac{1}{4\pi\varepsilon} \int_{S'} \frac{\rho_s}{R'} \, ds' \quad \text{(surface distribution)},$$

$$V = \frac{1}{4\pi\varepsilon} \int_{l'} \frac{\rho_\ell}{R'} \, dl' \quad \text{(line distribution)}.$$

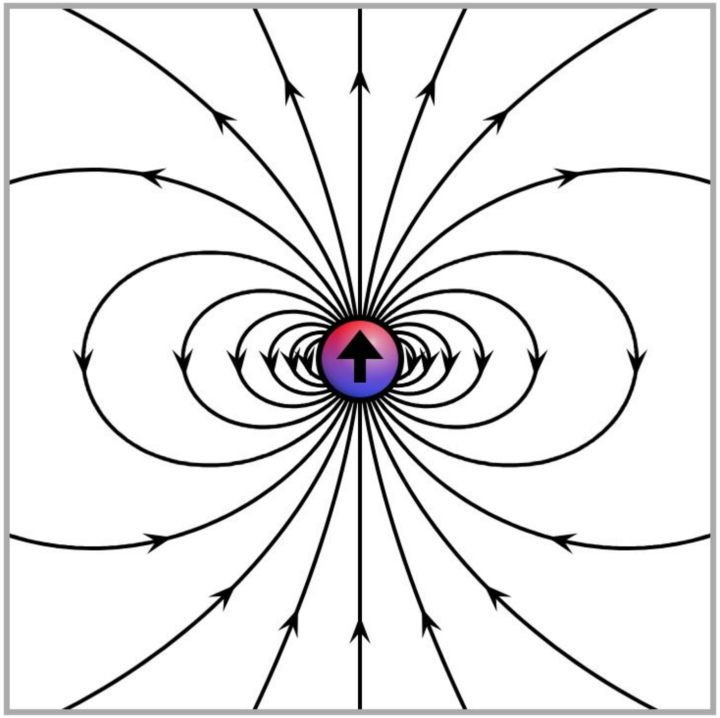
$$R' = |\mathbf{R} - \mathbf{R}_i|$$

Chapter 4 Review

$$\mathbf{E} = -\nabla V.$$

Electric Dipole:

$$\mathbf{E} = \frac{qd}{4\pi\varepsilon_0 R^3}\left(\hat{\mathbf{R}}\, 2\cos\theta + \hat{\boldsymbol{\theta}}\sin\theta\right) \quad \text{(V/m)}$$



(wikipedia.org)

Chapter 4 Review

Since:

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon} \qquad \textbf{(Poisson's equation)}$$

if ρ_v=0:

$$\nabla^2 V = 0 \qquad \textbf{(Laplace's equation)}$$

Useful for problems where V is known on boundaries.

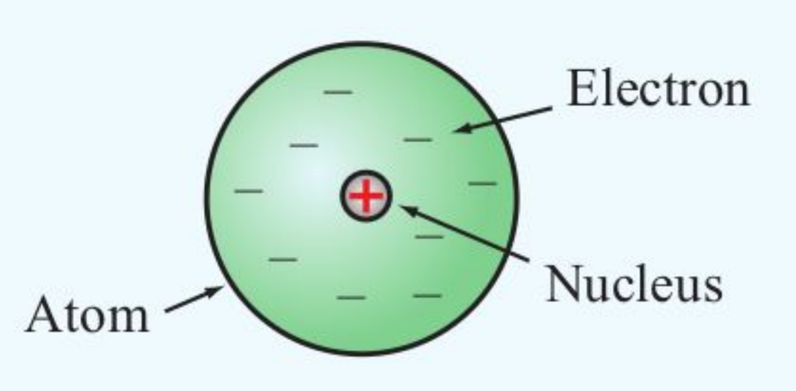
Chapter 4 Review

$$\mathbf{J} = \sigma \mathbf{E} \qquad (\text{A/m}^2) \quad (\text{Ohm's law}).$$

$$P = \int_V \mathbf{E} \cdot \mathbf{J} \, dV \qquad (\text{W}) \quad (\text{Joule's law})$$

4-7 Dielectric Materials

- Nature of insulators: electrons in the outer-most atomic shells are strongly bound to the atom.



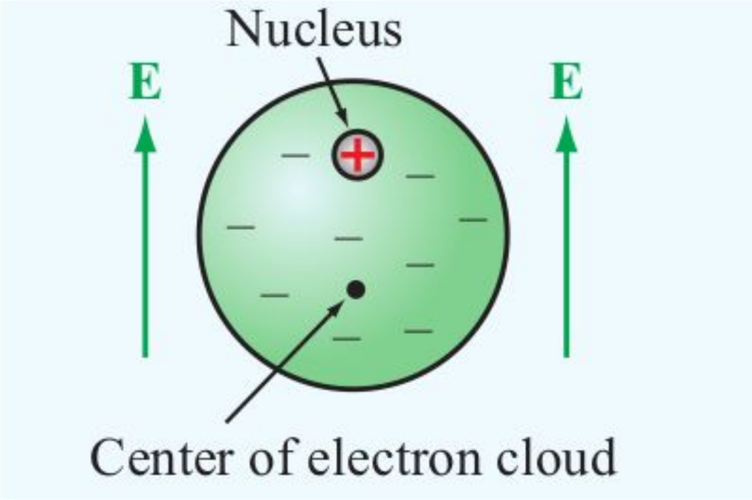
(a) External $\mathbf{E}_{ext} = 0$

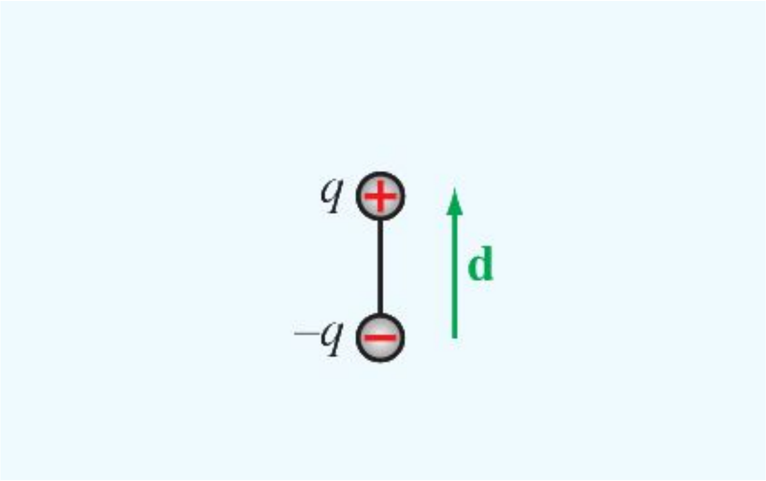
- Electron "cloud" is uniformly distributed around the nucleus

Note: this is a "classical" model of QM: only kinda true...

4-7 Dielectric Materials

- External field **E** will polarize the atoms by moving the center of the electron cloud away from nucleus.
  - Polarized structure is called a dipole
  - Induced electric field by the dipole is called the polarization field **P.**

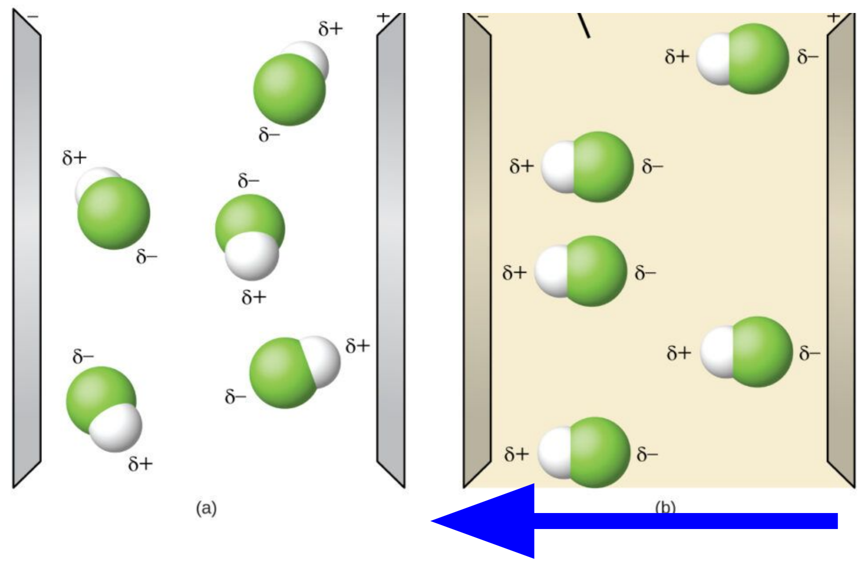




4-7 Dielectric Materials

- Some molecules already have a non-uniform charge distribution.
- Called Polar molecules
- They can be re-oriented in the presence of an external field.

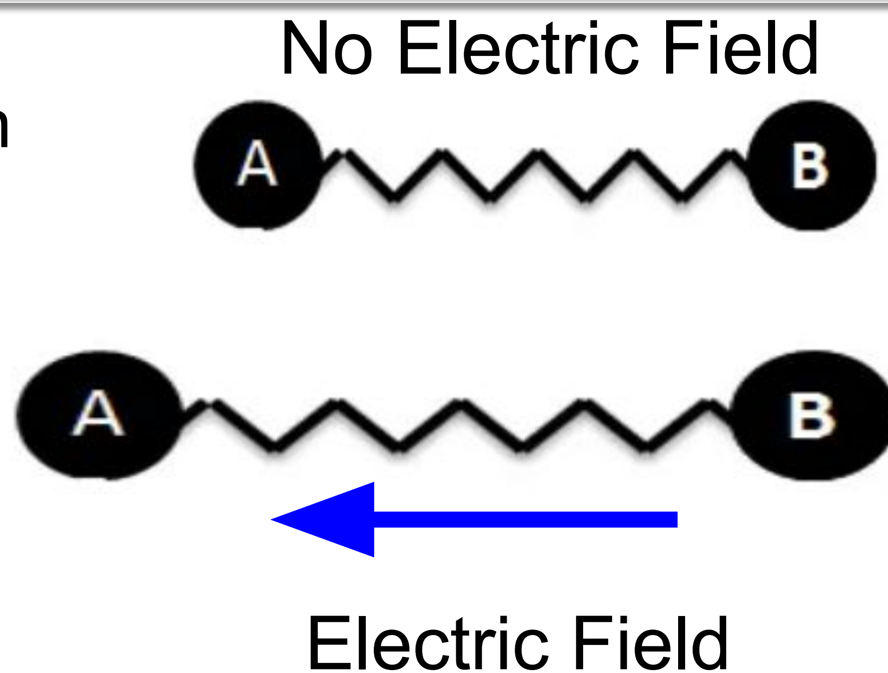
https://chem-textbook.ucalgary.ca/



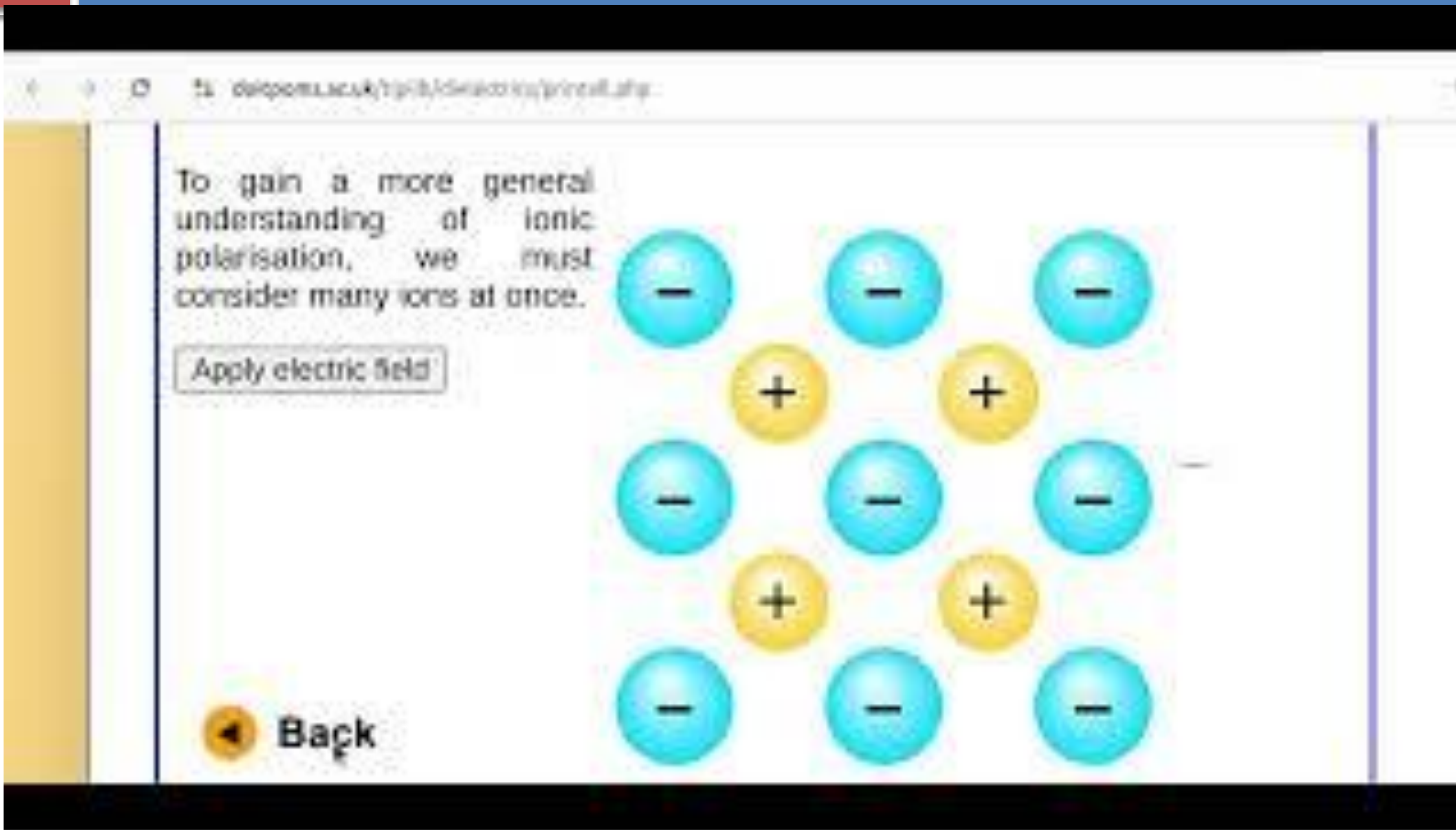
Electric Field

4-7 Dielectric Materials

- The Electric Field can even change the bond lengths between atoms.

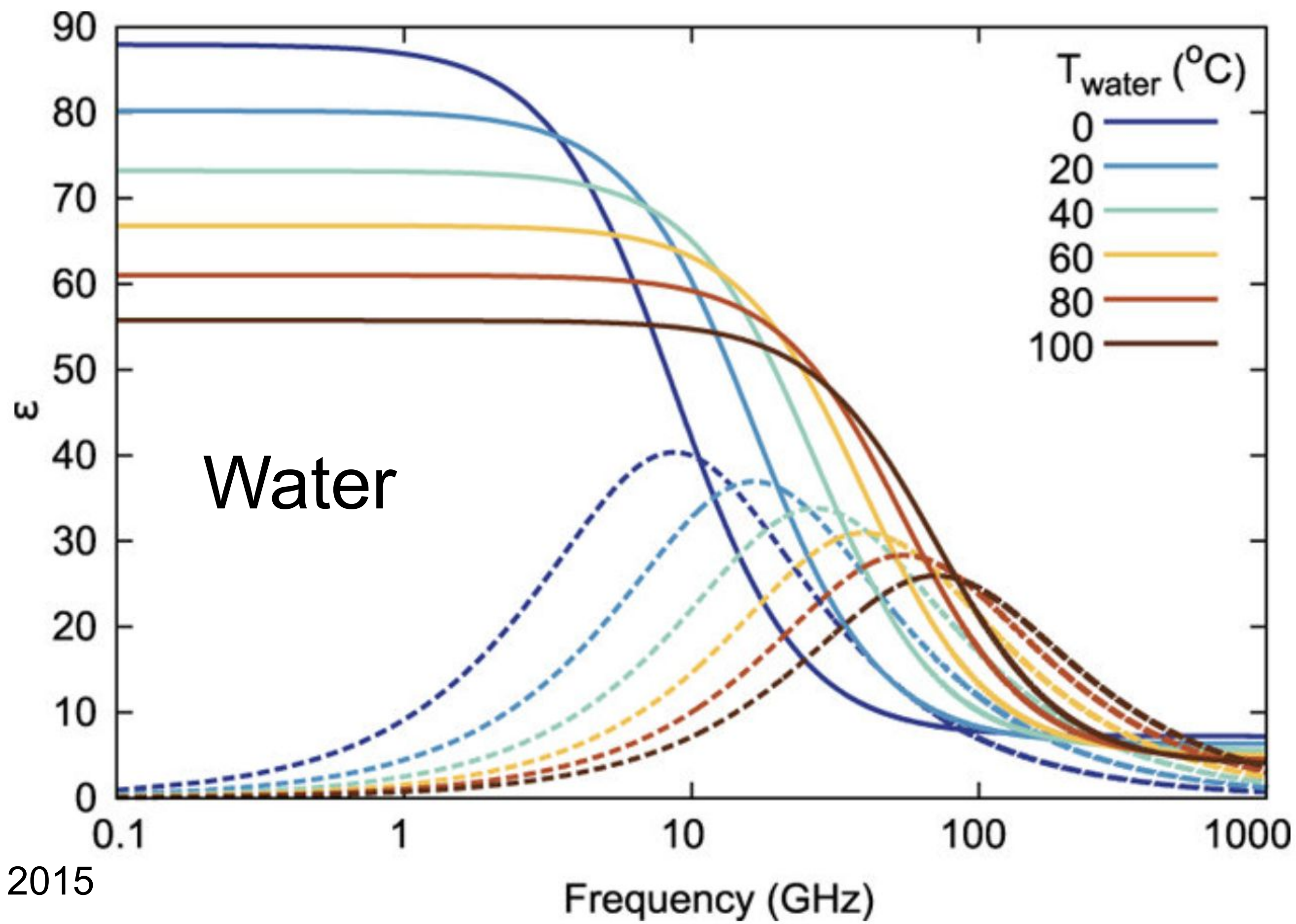


4-7 Dielectric Materials



4-7 Dielectric Materials

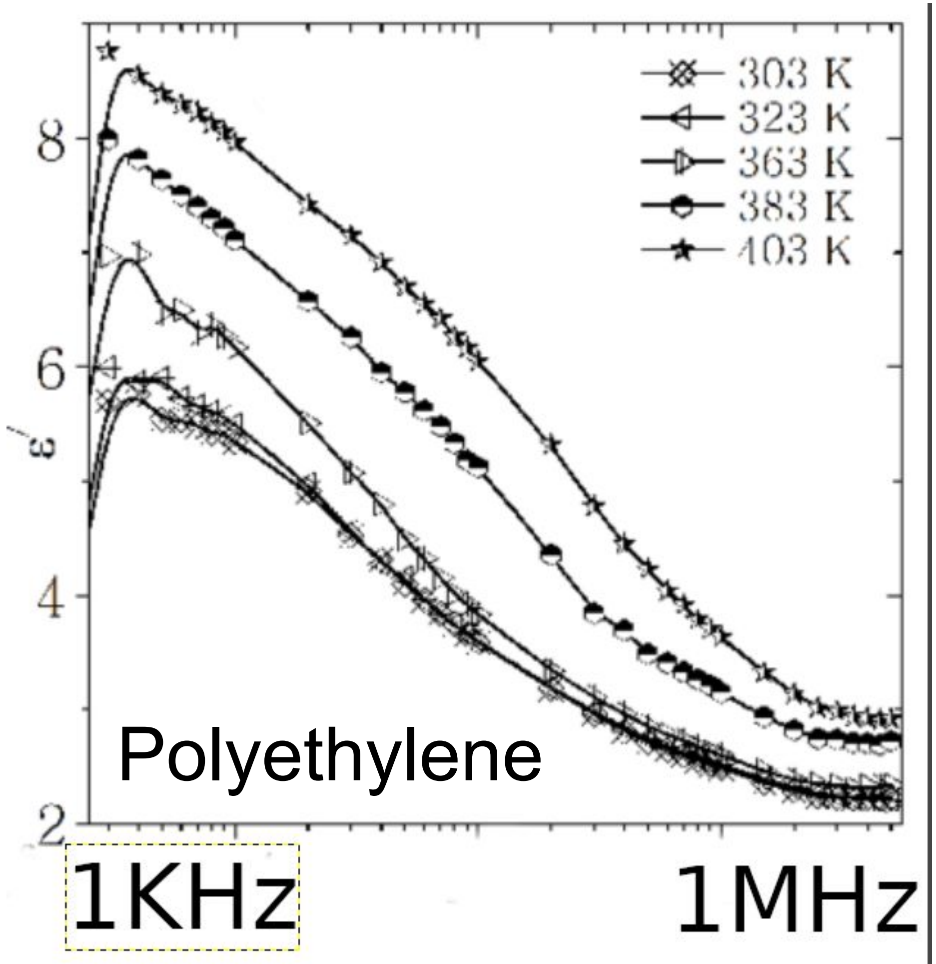
Depends on Frequency and Temperature



Andryieuski, 2015

4-7 Dielectric Materials

Depends on Frequency and Temperature

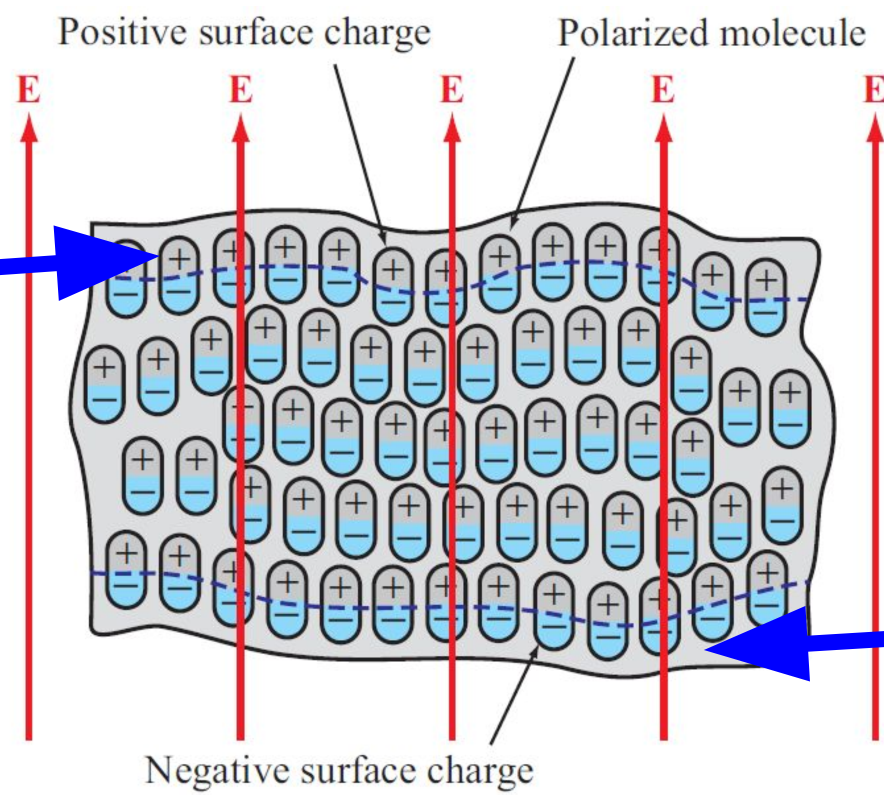


Ahmed, 2012

4-7 Dielectric Materials

Polarized material, showing polarized molecules

Positive surface charge on one side



Negative surface charge on other side

4-7 Dielectric Materials

- Polar v.s. Nonpolar molecules
  - Nonpolar material: does not possess **permanent** dipole moment, Only generated once an external field is applied.
  - Polar material (e.g. NaCl) have **permanent** dipole moments, but are randomly oriented without the presence of an external field.

4-7 Polarization Field

In free space: $\mathbf{D} = \varepsilon_0 \mathbf{E}$

In a dielectric: $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$

P = electric flux density induced by **E**

$$\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E}, \tag{4.84}$$

where χ_e is called the ***electric susceptibility*** of the material.

$$\begin{aligned} \mathbf{D} &= \varepsilon_0 \mathbf{E} + \varepsilon_0 \chi_e \mathbf{E} \\ &= \varepsilon_0 (1 + \chi_e) \mathbf{E} = \varepsilon \mathbf{E}, \end{aligned} \tag{4.85}$$

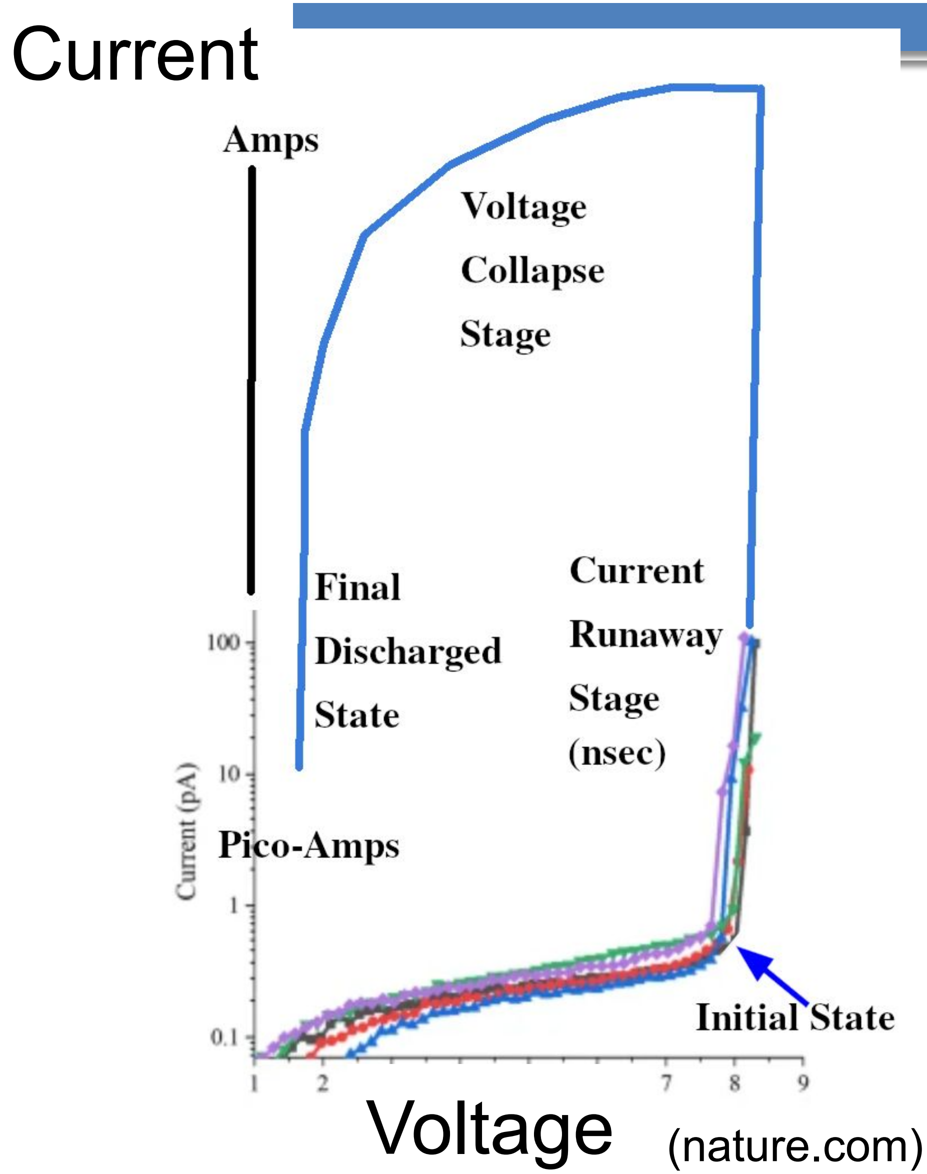
4-7 Electric Breakdown

Dielectric Strength:
Magnitude of **E** such that current arcs through the material

Material	Relative Permittivity, ε_r	Dielectric Strength, E_{ds} (MV/m)
Air (at sea level)	1.0006	3
Petroleum oil	2.1	12
Polystyrene	2.6	20
Glass	4.5–10	25–40
Quartz	3.8–5	30
Bakelite	5	20
Mica	5.4–6	200

Note: $\varepsilon = \varepsilon_r \varepsilon_0$ and $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m.

4-7 Electric Breakdown



Initial State:
High voltage, low current

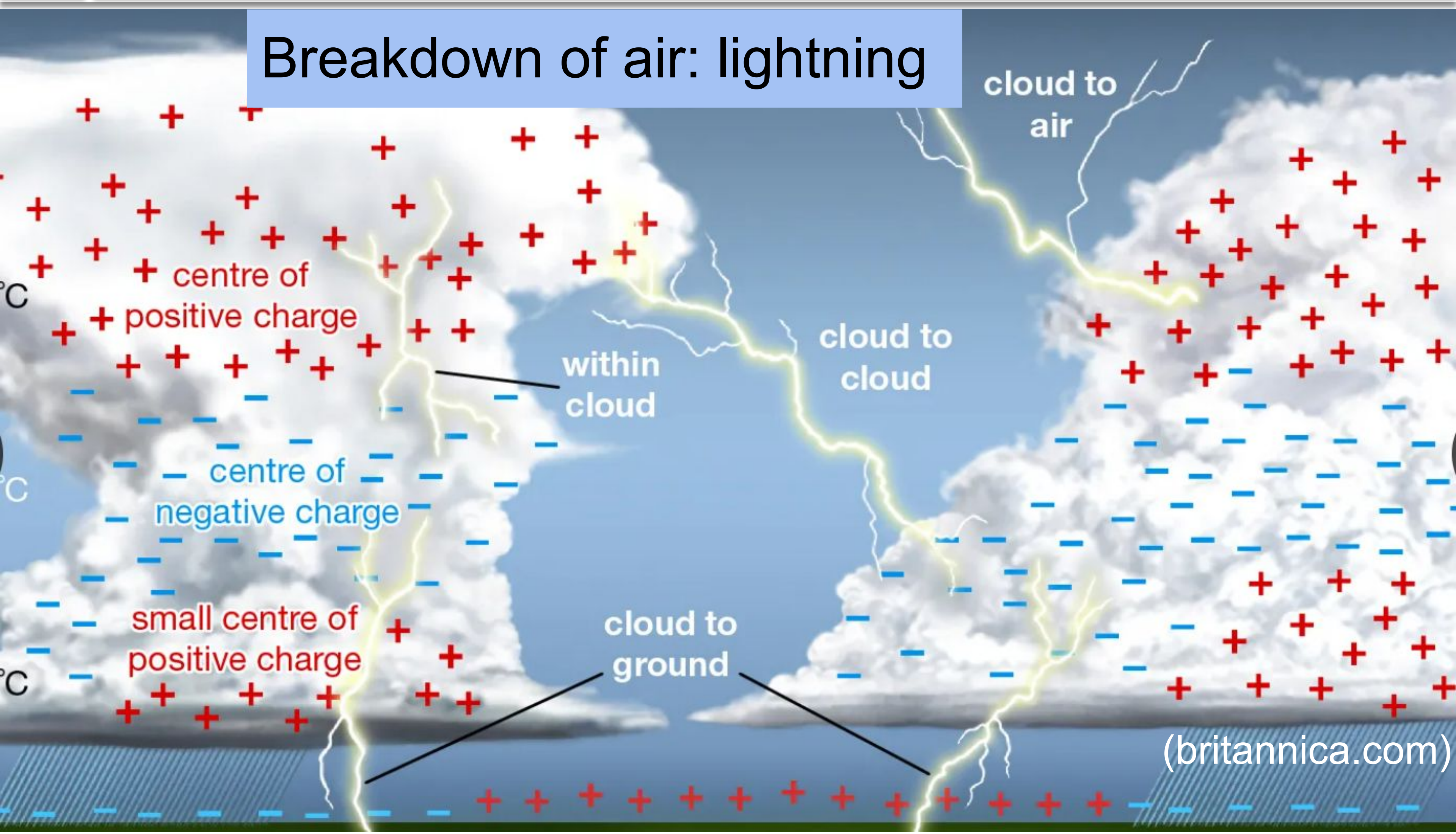
Runaway Current:
High voltage, high current occurs in nsec

Voltage Collapse:
voltage discharged
current decays

Discharged:
low voltage, low current

4-7 Electric Breakdown: examples

Breakdown of air: lightning



(britannica.com)

4-7 Electric Breakdown: examples

(amazon.com)

Spark Plugs in cars are designed to cause breakdown

The spark ignites the gasoline,

the expanding gas pushes on the pistons,

rotating the drive shaft,

moving the wheels

4-7 Electric Breakdown: examples


(pottselectric.com)

Often, plugging into the grid will cause an arc between the receptacle and the plug.

This is due to the breakdown of air between the contacts.

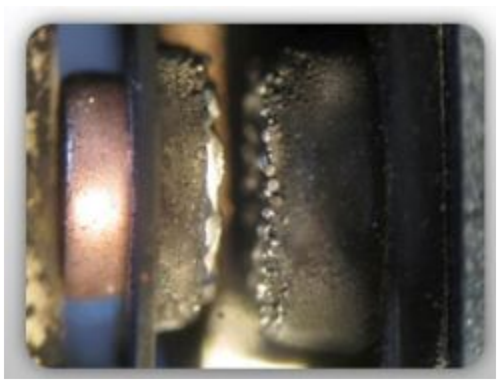
Usually not a problem.

Can be a problem if using a relay...

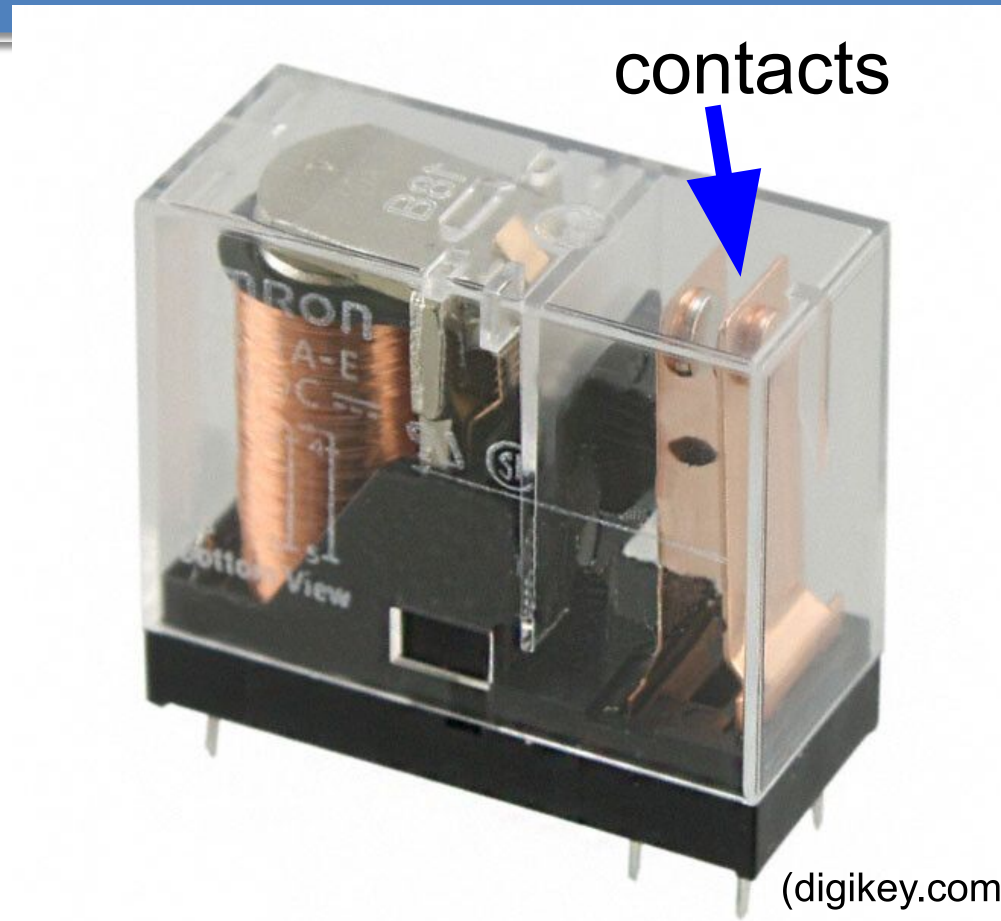
4-7 Electric Breakdown: examples

High-current relay

Worn contacts:
little to no current flow:



(electronic-components.com.au)



(digikey.com

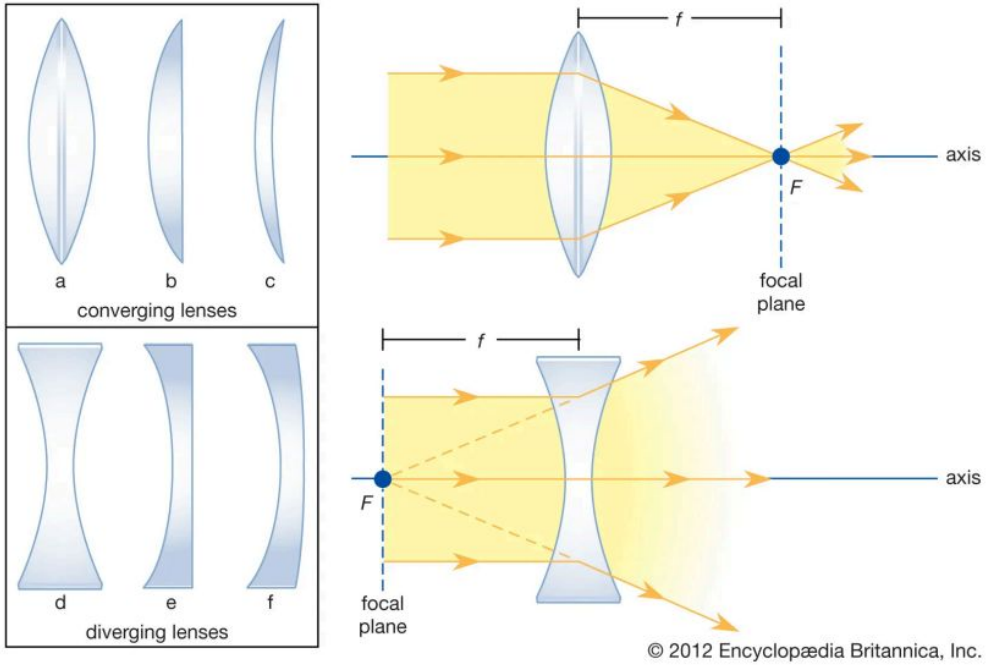
Can get arc-suppression circuits to put in parallel with the contacts to keep them working longer.

4-8 Dielectric Boundary Conditions

Because **D** = ε **E**:

We expect the fields to be different in different materials.

We also expect to see an abrupt change across a boundary between different materials.

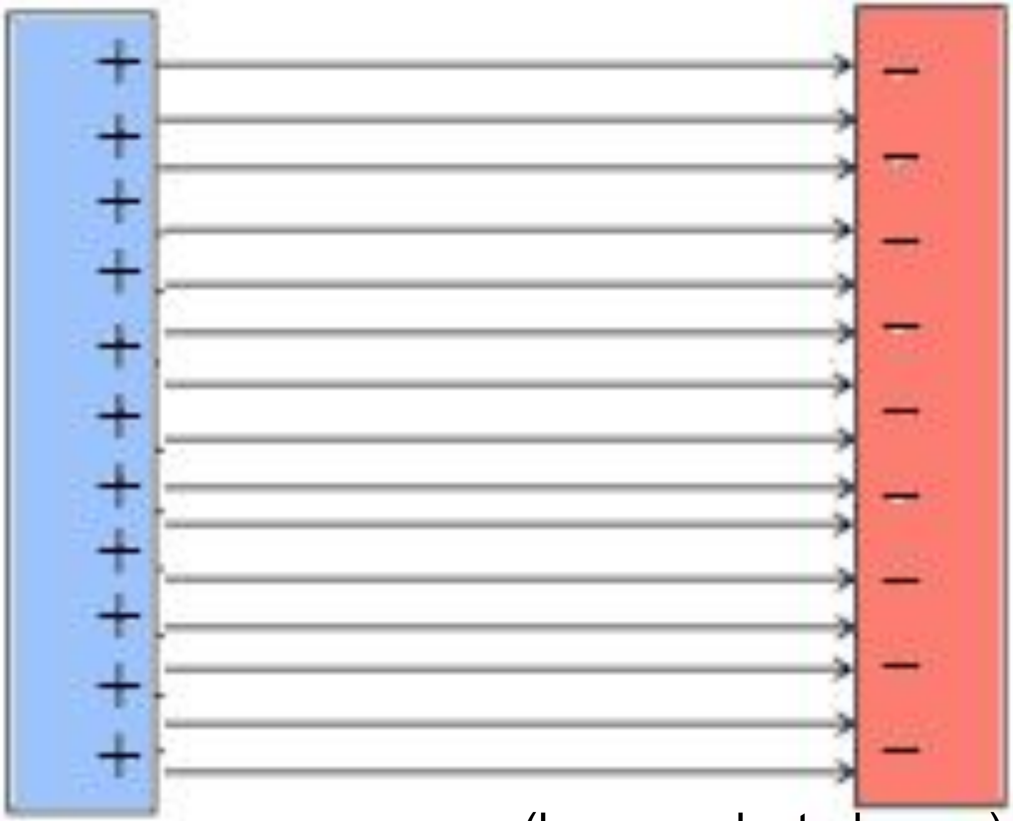


Example: fields are changed after passing through a lens

4-8 Dielectric Boundary Conditions

Scenario:

We have a static electric field in free space.



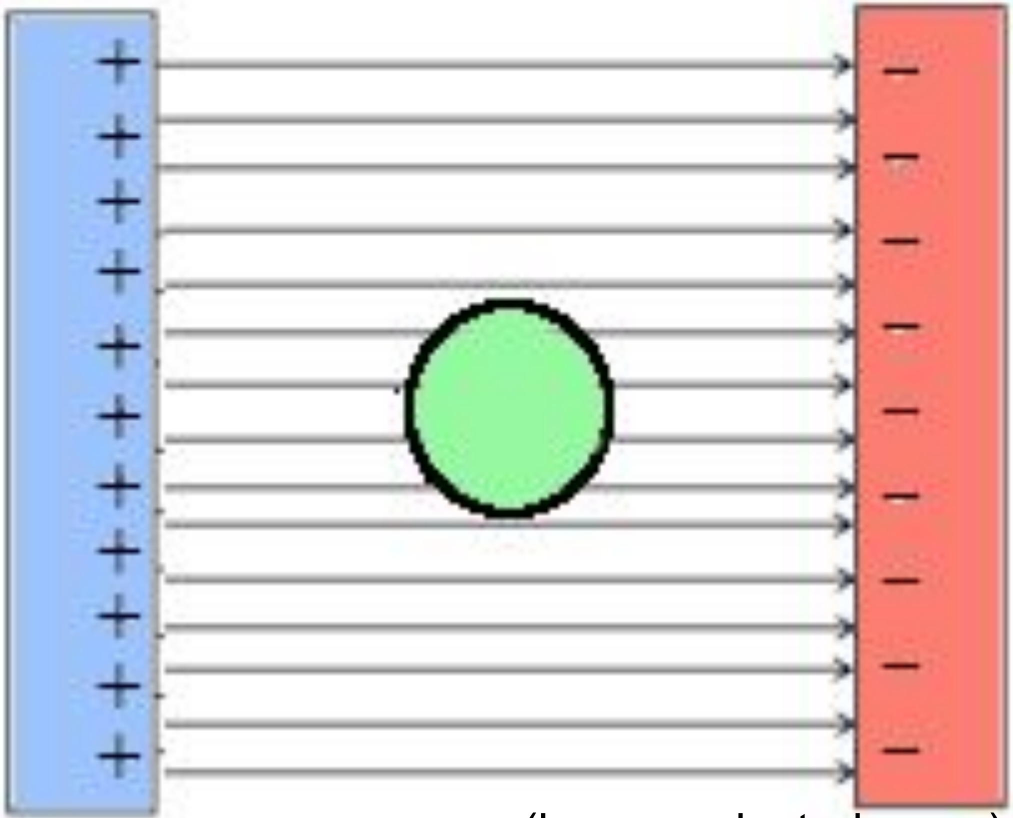
(homework.study.com)

4-8 Dielectric Boundary Conditions

Scenario:

We have a static electric field in free space.

Move an object into this field.



(homework.study.com)

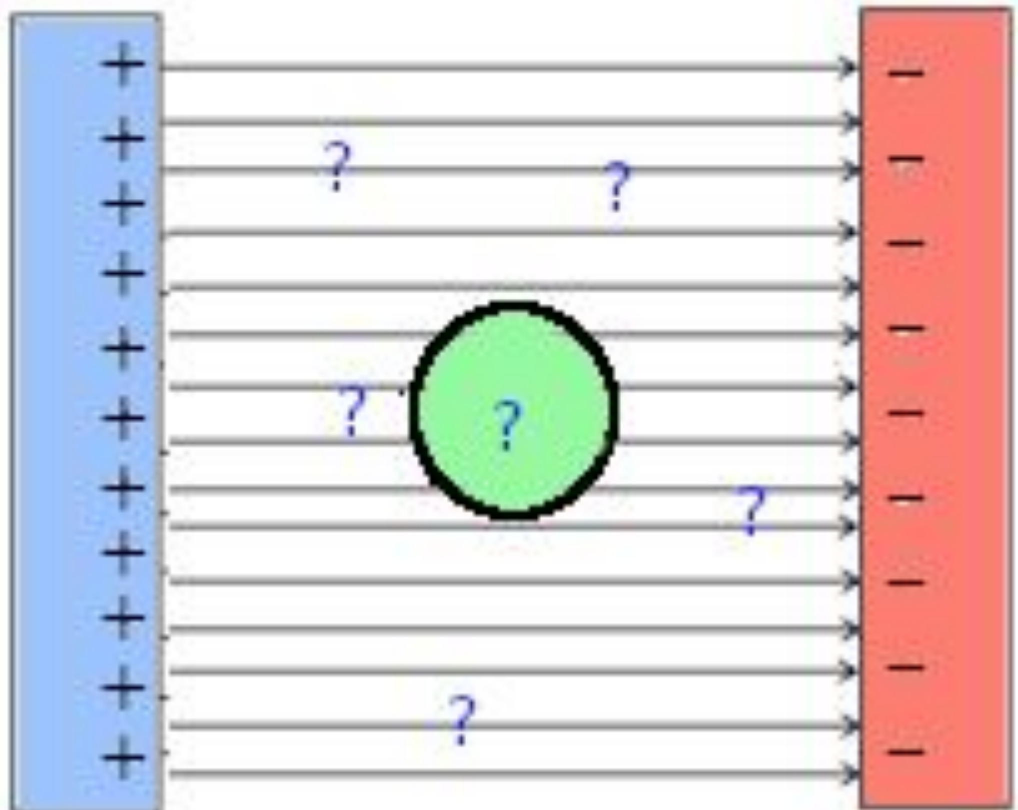
4-8 Dielectric Boundary Conditions

Scenario:

We have a static electric field in free space.

Move an object into this field.

What are the fields inside the object?



(homework.study.com)

4-8 Dielectric Boundary Conditions

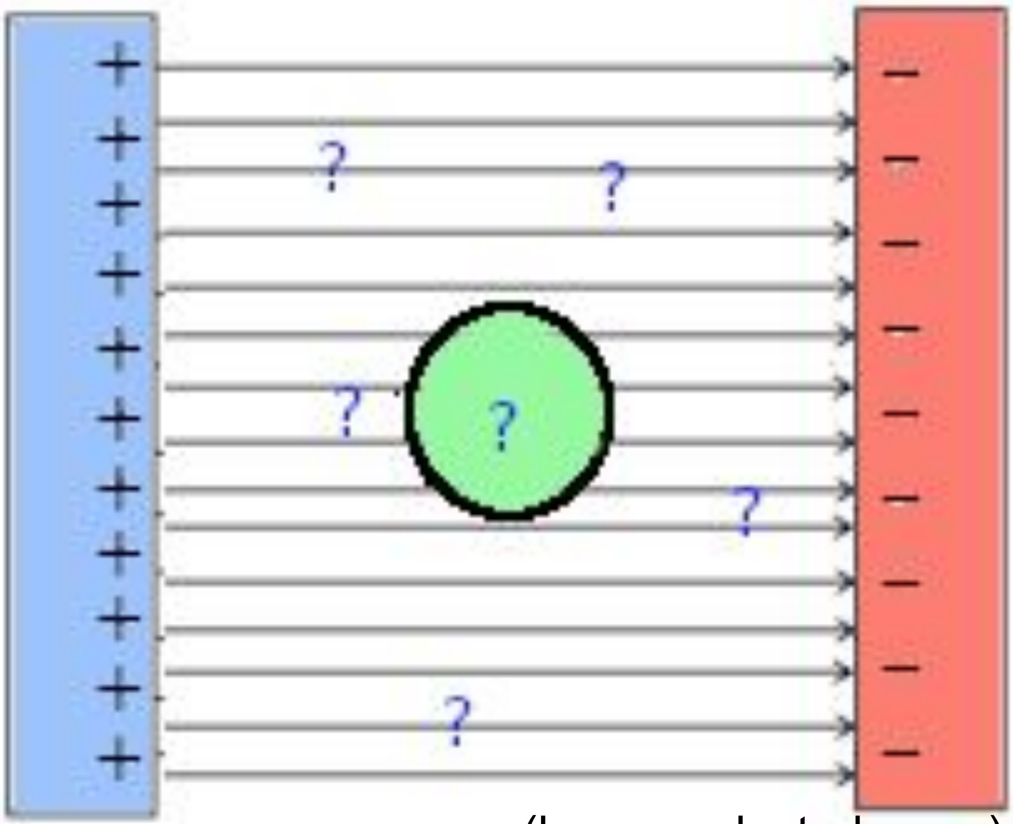
Scenario:

We have a static electric field in free space.

Move an object into this field.

What are the fields inside the object?

How do the fields outside change?



(homework.study.com)

4-8 Dielectric Boundary Conditions

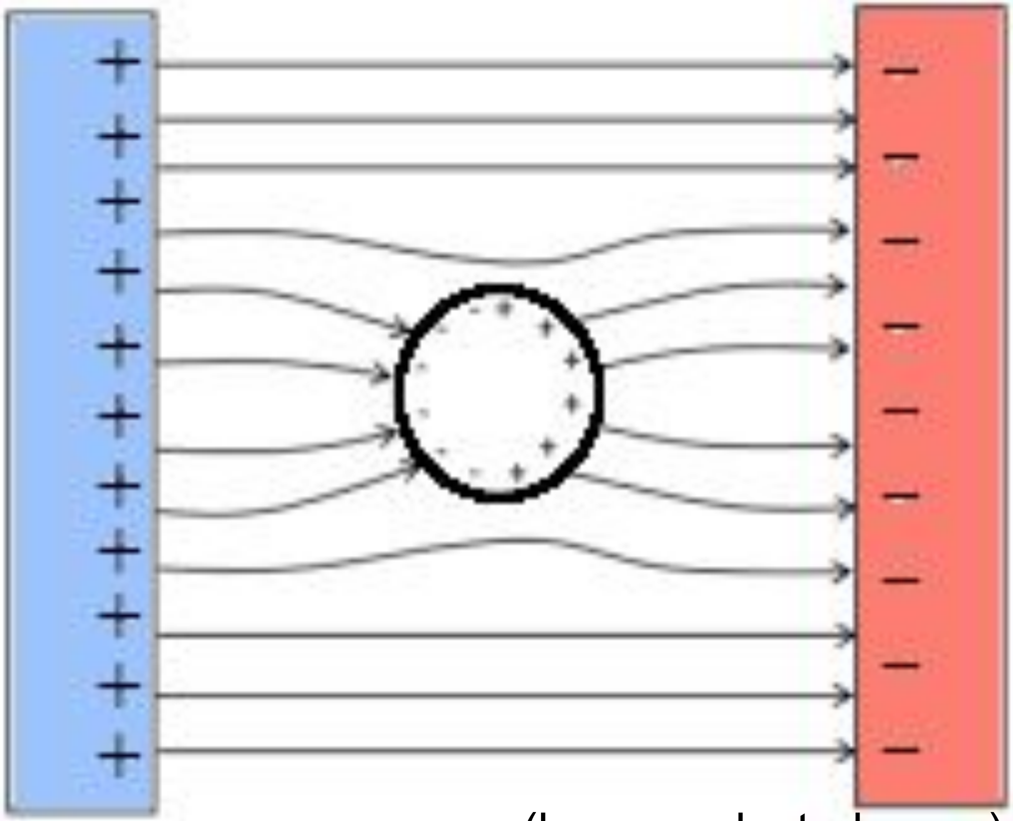
Scenario:

We have a static electric field in free space.

Move an object into this field.

What are the fields inside the object?

How do the fields outside change?

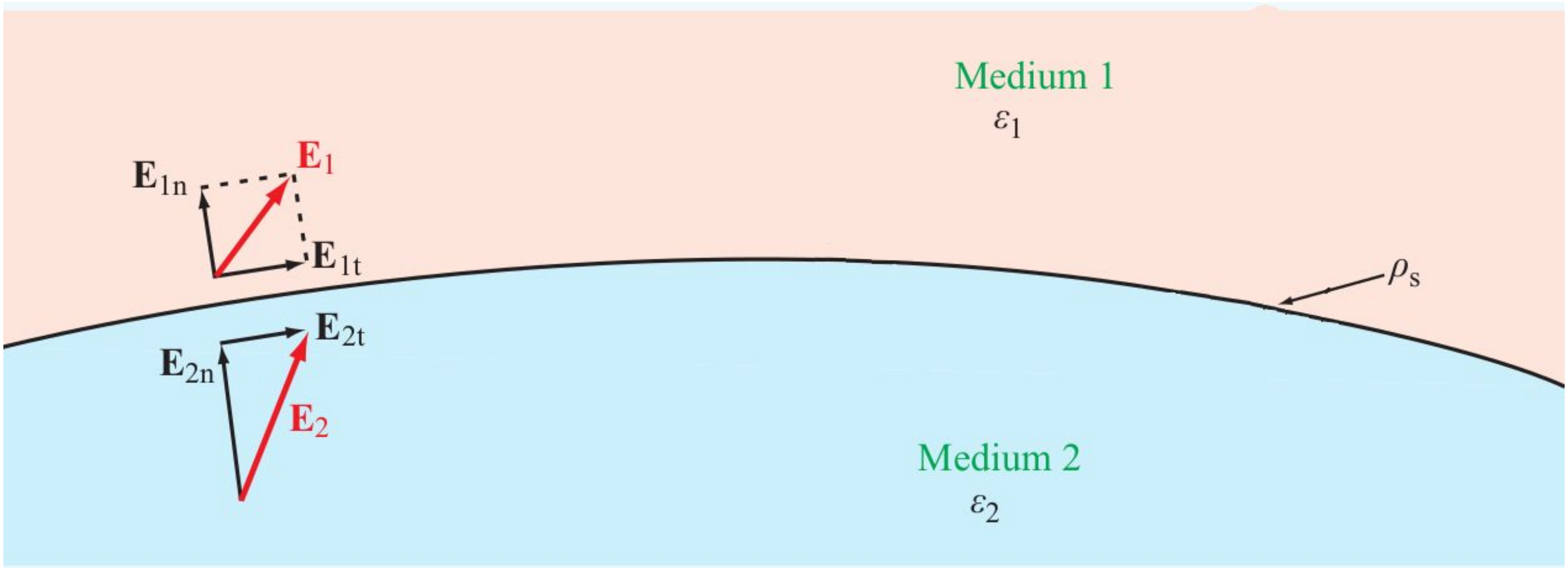


(homework.study.com)

Possible Resulting Fields

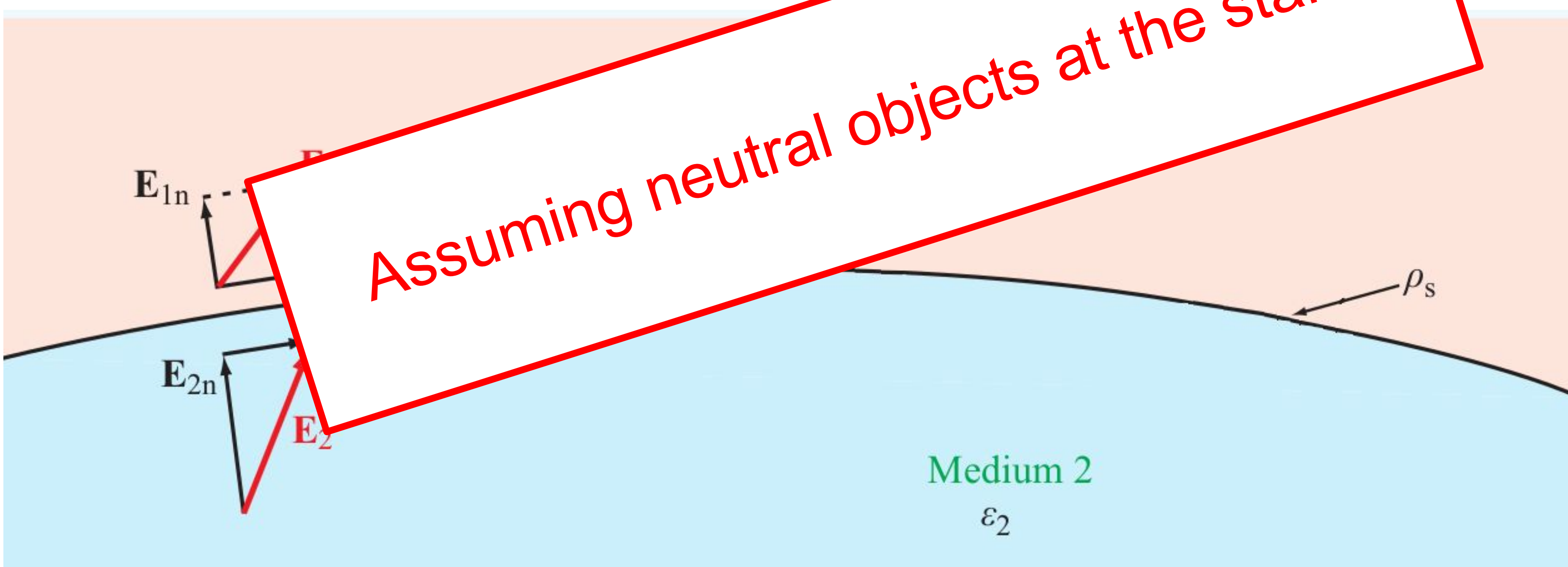
4-8 Dielectric Boundary Conditions

- How do the fields (**E, D, J**) change across a boundary?
- Boundary defined by: different materials: ε
- ...and possibly a surface charge



4-8 Dielectric Boundary Conditions

- How do the fields (**E, D, J**) change across a boundary?
- Boundary defined by: different materials: ε
- ...and possibly a surface charge



Assuming neutral objects at the start

4-8 Dielectric Boundary Conditions

From math we know:

$$\iiint_{\mathcal{V}} \nabla \times \mathbf{F}\, d\mathcal{V} = -\oiint_{S} \mathbf{F} \times d\mathbf{s}$$

From Maxwell's equations (electrostatics):

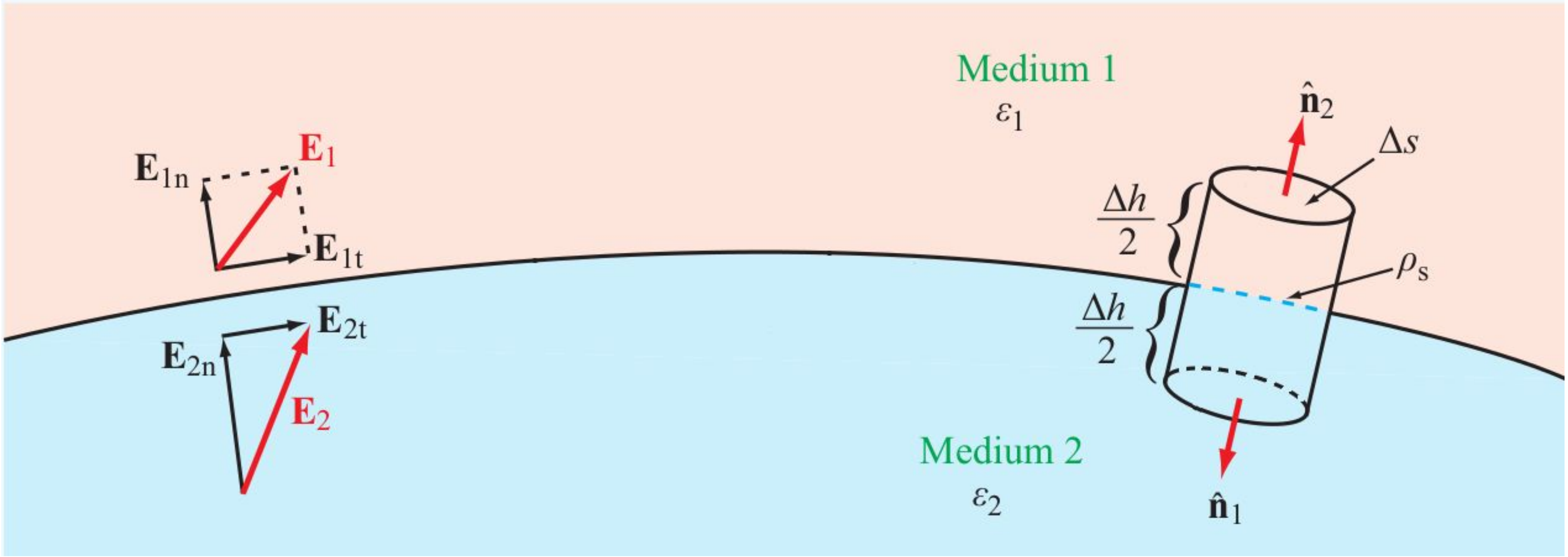
$$\nabla \times \mathbf{E} = 0.$$

Plug in:

$$\oiint_{S} \mathbf{E} \times d\mathbf{s} = 0$$

4-8 Dielectric Boundary Conditions

Choose the surface to be a box that spans the boundary:



In the limit as $\Delta h \rightarrow 0$, we are left with the two end-caps.
The "vector" surface d**s** uses the outward normal:

in medium 2: $\hat{\mathbf{n}}_1$

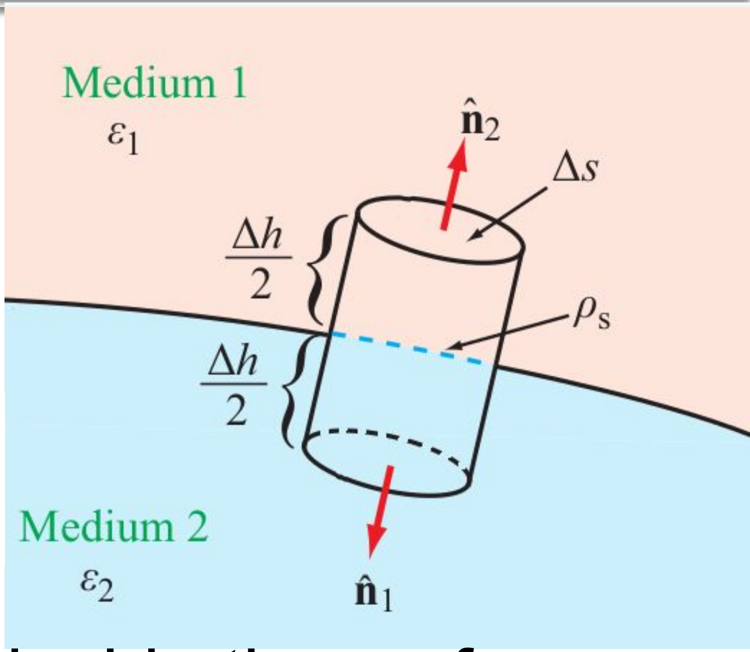
in medium 1: $\hat{\mathbf{n}}_2$

4-8 Dielectric Boundary Conditions

$$\oiint_S \mathbf{E} \times d\mathbf{s} = \oiint_S \mathbf{E}_1 \times \hat{\mathbf{n}}_2 ds + \oiint_S \mathbf{E}_2 \times \hat{\mathbf{n}}_1 ds$$

Since $\hat{\mathbf{n}}_1 = -\hat{\mathbf{n}}_2$:

$$\oiint_S \mathbf{E} \times d\mathbf{s} = -\oiint_S \mathbf{E}_1 \times \hat{\mathbf{n}}_1 ds + \oiint_S \mathbf{E}_2 \times \hat{\mathbf{n}}_1 ds$$



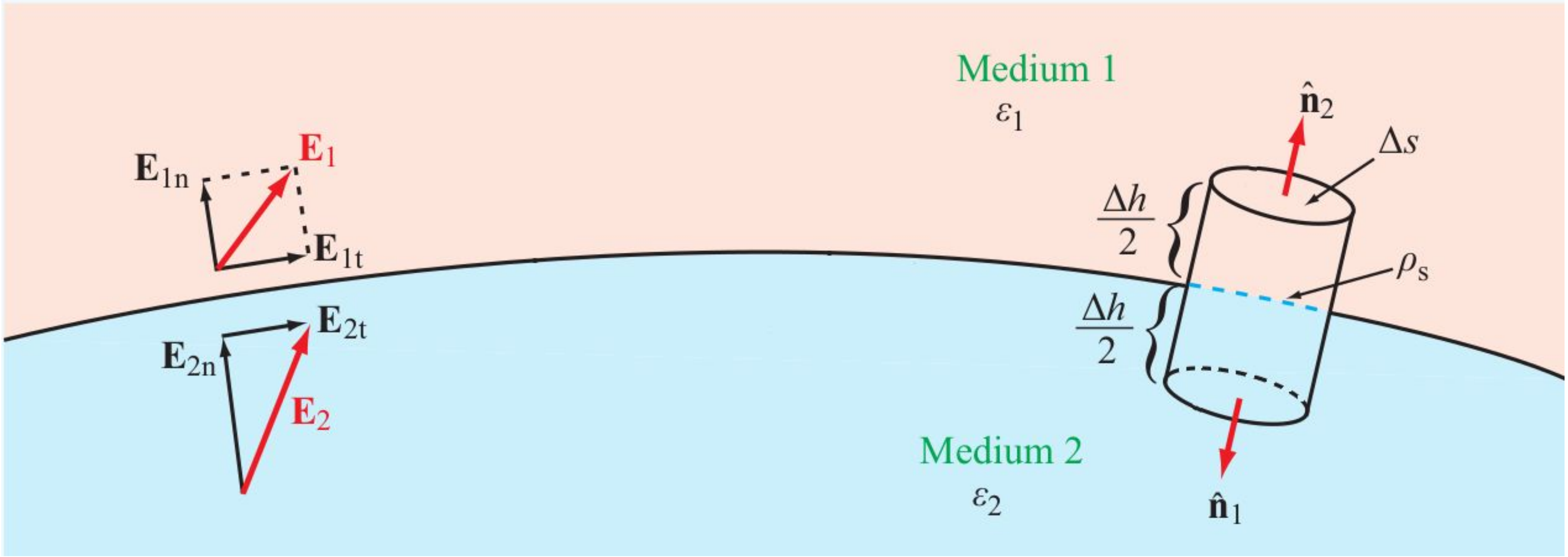
Very small: so nothing changing inside the surface:

$$\oiint_S \mathbf{E} \times d\mathbf{s} = -\mathbf{E}_1 \times \hat{\mathbf{n}} \Delta s + \mathbf{E}_2 \times \hat{\mathbf{n}} \Delta s = 0$$

$$(\mathbf{E}_1 - \mathbf{E}_2) \times \hat{\mathbf{n}} = 0$$

where: $\mathbf{E} \times \hat{\mathbf{n}} = \mathbf{E}_{\text{tangential}}$

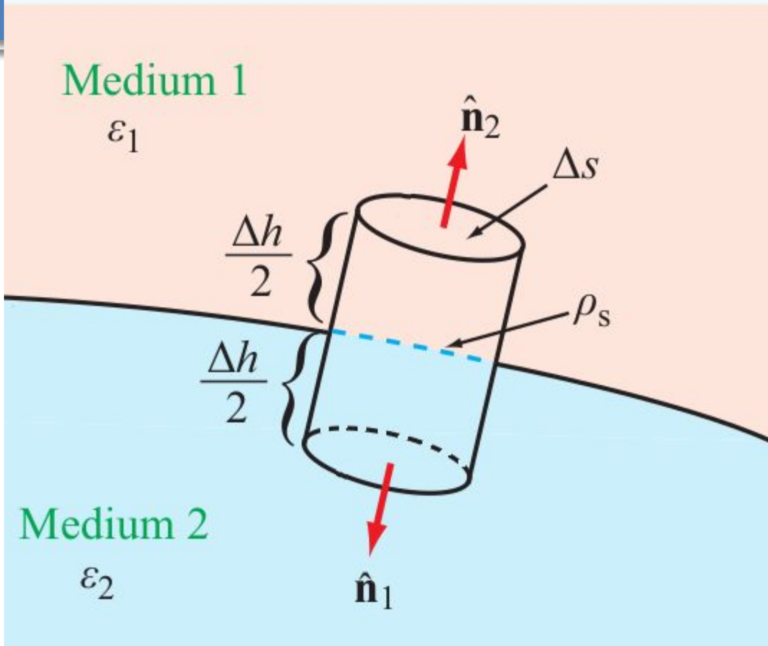
4-8 Dielectric Boundary Conditions



$$(\mathbf{E}_1 - \mathbf{E}_2) \times \hat{\mathbf{n}} = 0$$

in words: Tangential **E** is continuous

4-8 Dielectric Boundary Conditions



Apply Gauss' Law:

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q$$

For the cylindrical surface,

in the limit as $\Delta h \to 0$:

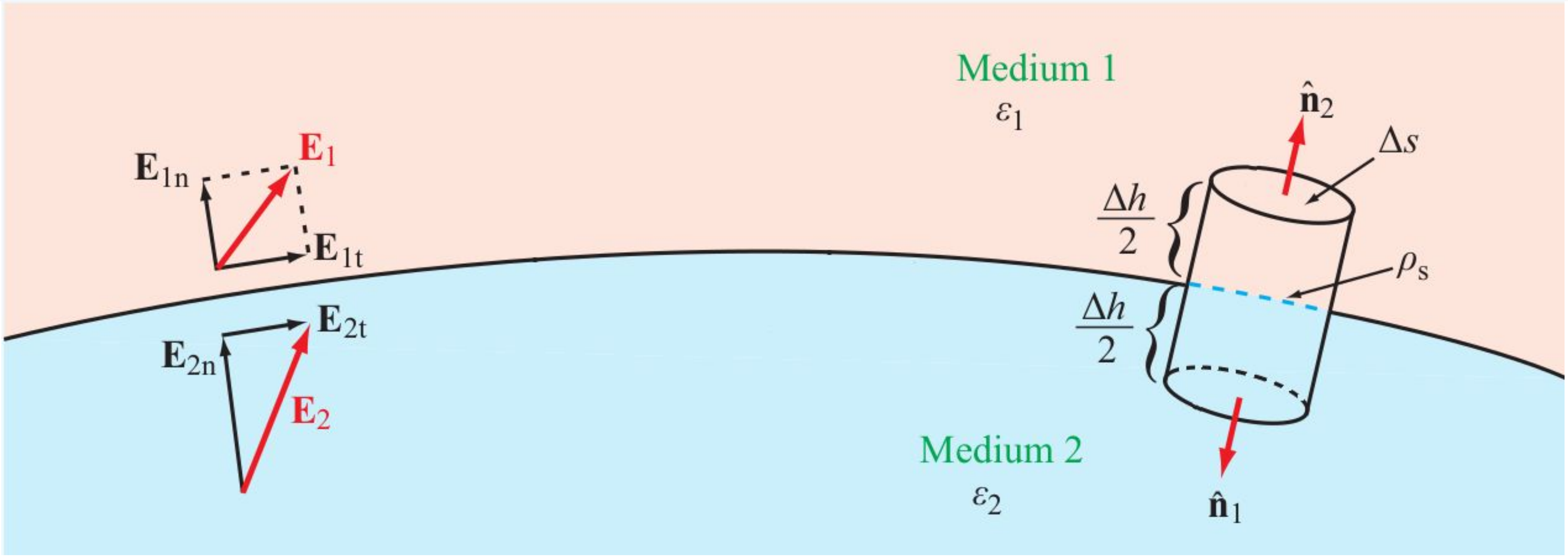
$$\oint_S \mathbf{D} \cdot d\mathbf{s} = \int_{\text{top}} \mathbf{D}_1 \cdot \hat{\mathbf{n}}_2 \, ds + \int_{\text{bottom}} \mathbf{D}_2 \cdot \hat{\mathbf{n}}_1 \, ds = \rho_s \Delta s,$$

$$\hat{\mathbf{n}}_2 \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_s \qquad (\text{C/m}^2).$$

$$D_{1n} - D_{2n} = \rho_s \qquad (\text{C/m}^2).$$

Normal components of **D** change by the surface charge density.

4-8 Dielectric Boundary Conditions



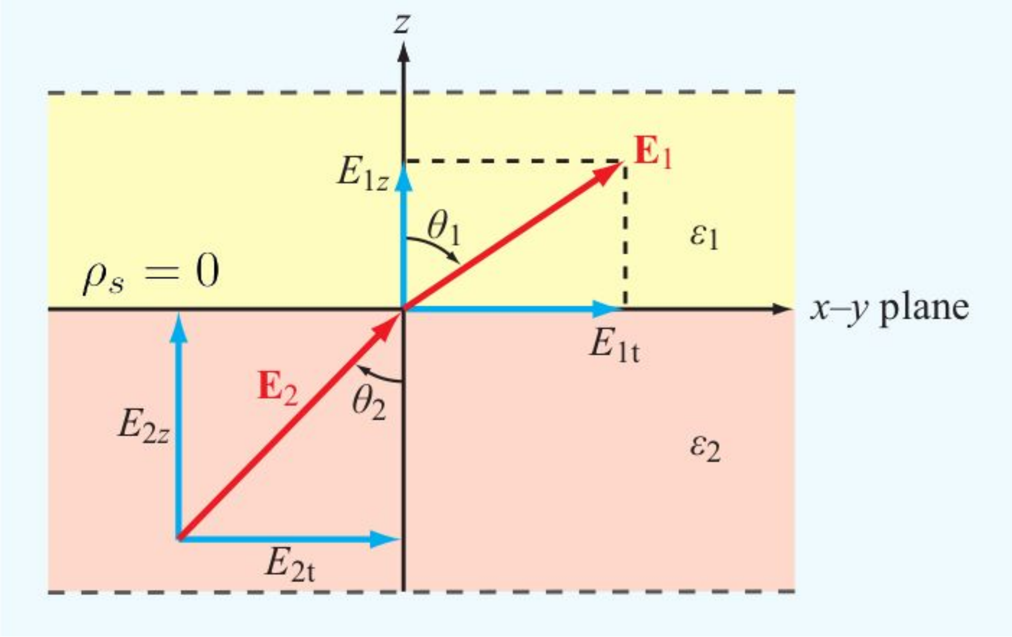
$$\mathbf{E}_{1t} = \mathbf{E}_{2t}$$

$$D_{1n} - D_{2n} = \rho_s \qquad (\mathrm{C/m^2}).$$

Tangential **E** is continuous

Normal **D** is discontinuous by the surface charge density

Example 4-13



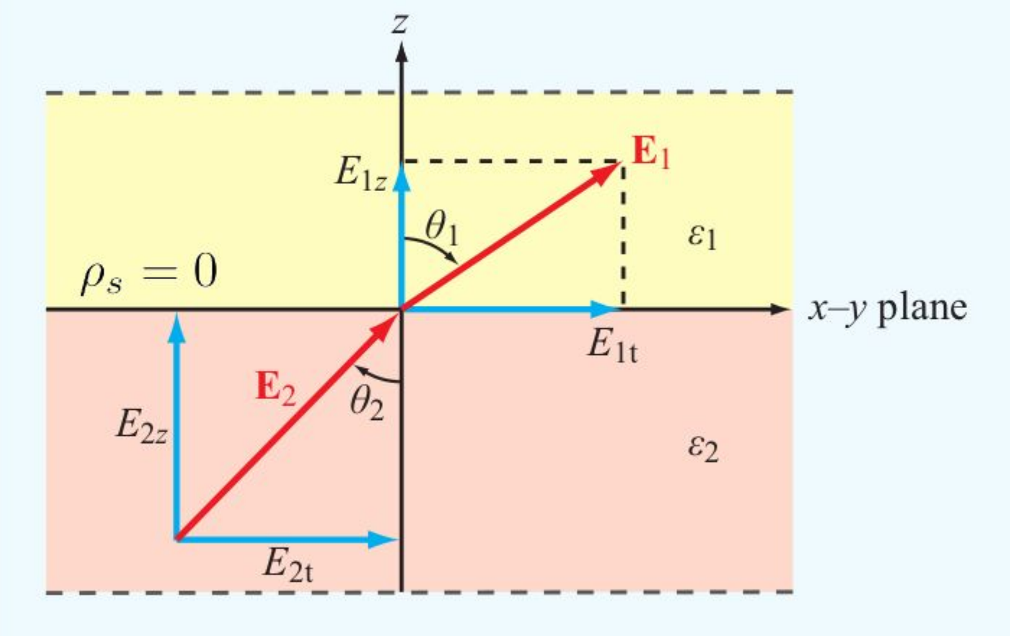
Given: $\boldsymbol{\rho}_s$ = 0, planar interface between 2 matls: ε_1, ε_2

$$\mathbf{E}_1 = \hat{\mathbf{x}}E_{1x} + \hat{\mathbf{y}}E_{1y} + \hat{\mathbf{z}}E_{1z}$$

Find: (a) $\mathbf{E}_2$
(b) θ_1, θ_2

Note that this definition of the angles is different than usual

Example 4-13



Solution:

assume $\mathbf{E}_2$ has the form:

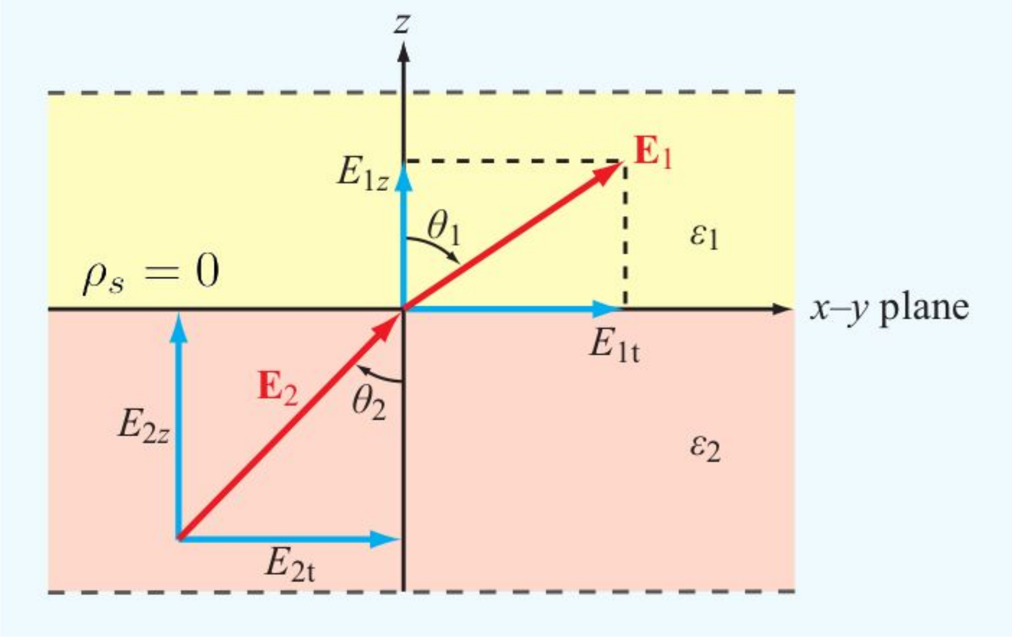
$$\mathbf{E}_2 = \hat{\mathbf{x}}E_{2x} + \hat{\mathbf{y}}E_{2y} + \hat{\mathbf{z}}E_{2z}.$$

Enforce boundary conditions:

Tangential **E** is continuous:

$$E_{2x} = E_{1x}, \qquad E_{2y} = E_{1y},$$

Example 4-13



Solution:

assume $\mathbf{E}_2$ has the form:

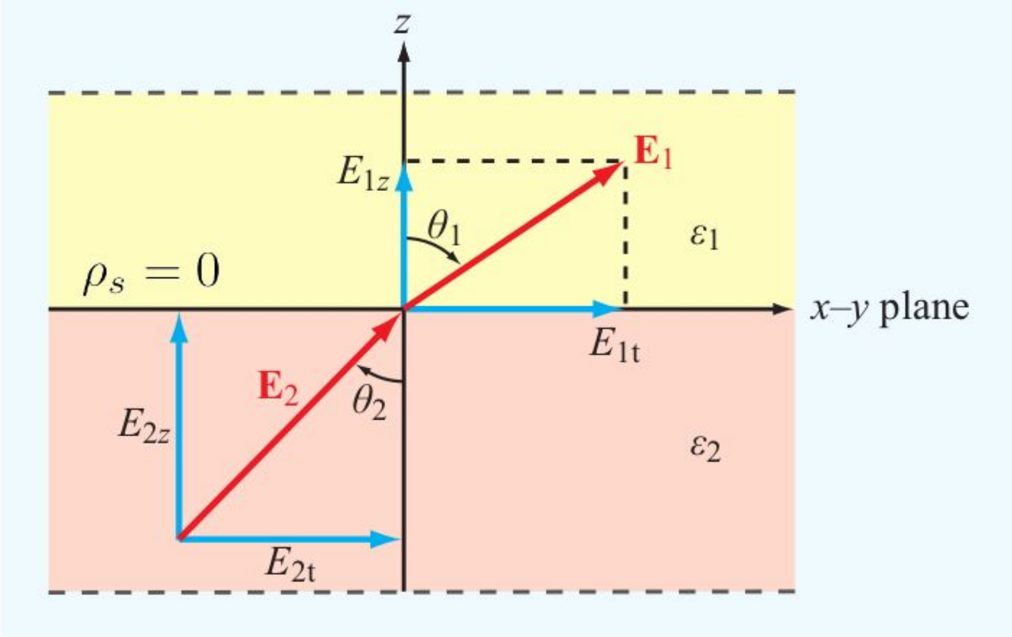
$$\mathbf{E}_2 = \hat{\mathbf{x}}E_{2x} + \hat{\mathbf{y}}E_{2y} + \hat{\mathbf{z}}E_{2z}.$$

Enforce boundary conditions:

Because ρ_s=0, Normal **D** is continuous:

$$D_{2z} = D_{1z} \qquad \text{or} \qquad \varepsilon_2 E_{2z} = \varepsilon_1 E_{1z}.$$

Example 4-13

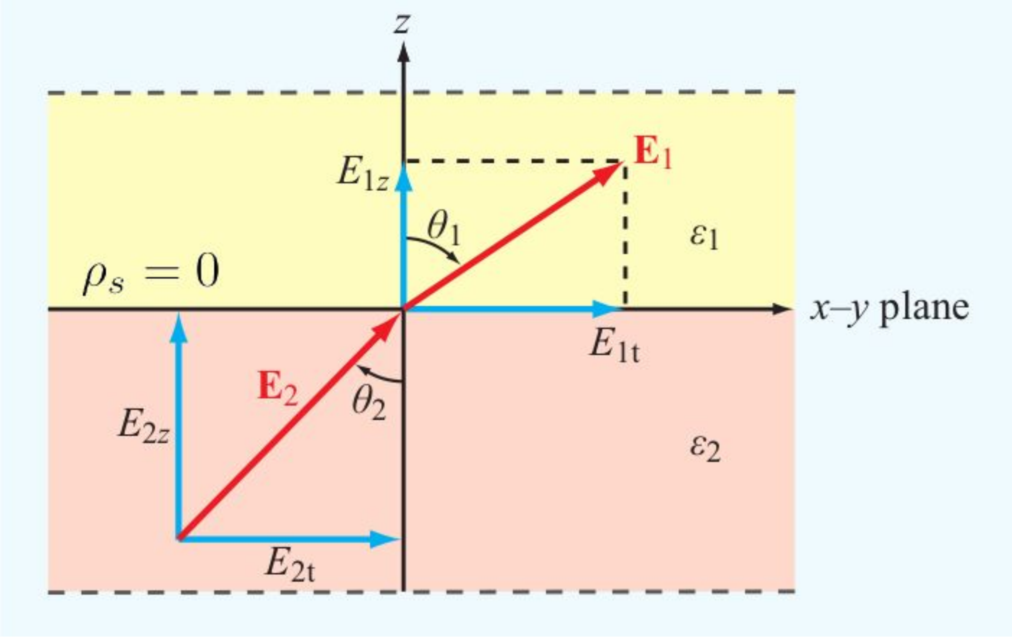


Solution:

Combine the results of the Boundary Conditions:

$$\mathbf{E}_2 = \hat{\mathbf{x}} E_{1x} + \hat{\mathbf{y}} E_{1y} + \hat{\mathbf{z}} \frac{\varepsilon_1}{\varepsilon_2} E_{1z}$$

Example 4-13



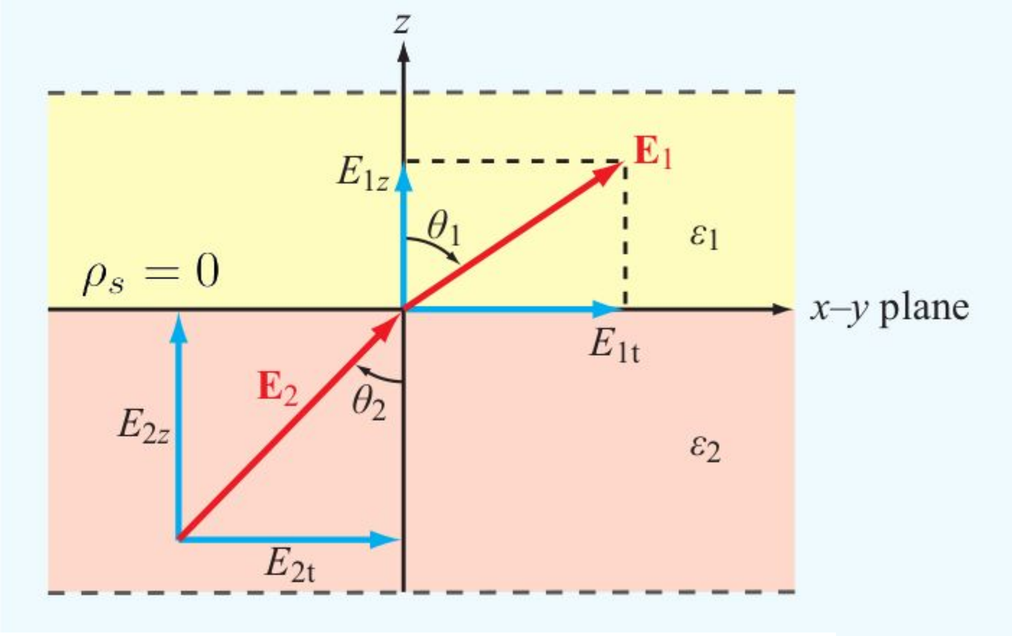
Solution:

Now: how about those angles?

We know the magnitude of the tangential components are:

$$E_{1t} = \sqrt{E_{1x}^2 + E_{1y}^2} \qquad \text{and} \qquad E_{2t} = \sqrt{E_{2x}^2 + E_{2y}^2}$$

Example 4-13

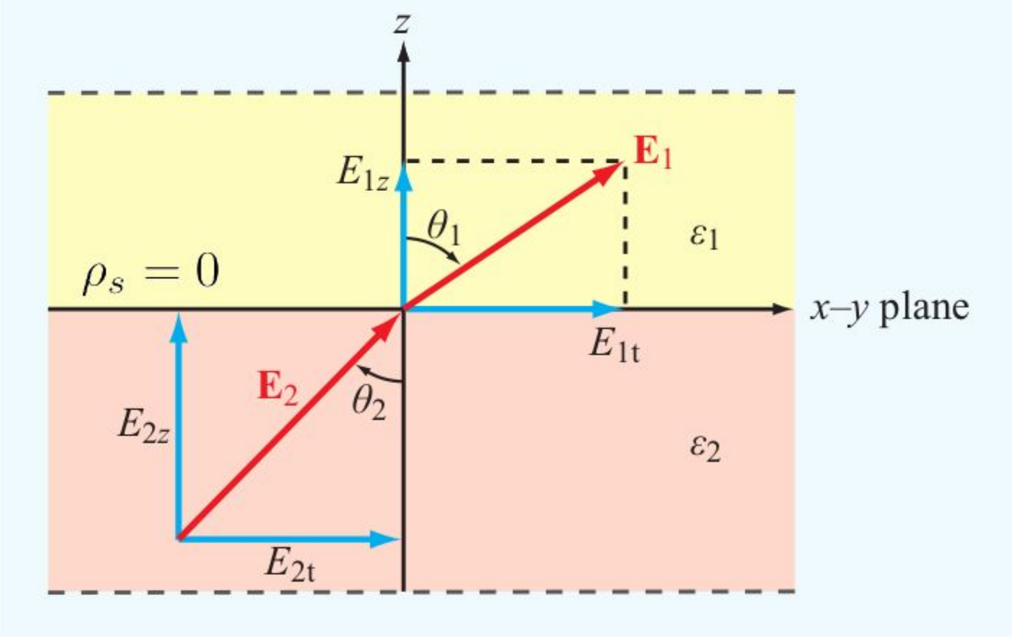


The angles θ_1 and θ_2 are then given by

$$\tan\theta_1 = \frac{E_{1t}}{E_{1z}} = \frac{\sqrt{E_{1x}^2 + E_{1y}^2}}{E_{1z}},$$

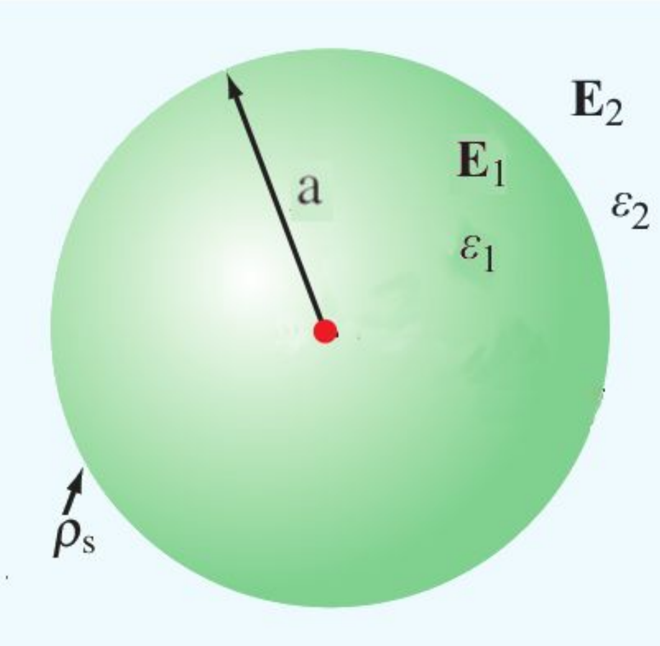
$$\tan\theta_2 = \frac{E_{2t}}{E_{2z}} = \frac{\sqrt{E_{2x}^2 + E_{2y}^2}}{E_{2z}} = \frac{\sqrt{E_{1x}^2 + E_{1y}^2}}{(\varepsilon_1/\varepsilon_2)E_{1z}},$$

Example 4-13



$$\frac{\tan\theta_2}{\tan\theta_1} = \frac{\varepsilon_2}{\varepsilon_1}$$

Example 2



Given: a dielectric sphere of radius = *a*

surface charge density $\boldsymbol{\rho}_s$

Interior field: $\mathbf{E}_1 = E_{1R}\hat{\mathbf{R}} + E_{1\theta}\hat{\boldsymbol{\theta}} + E_{1\phi}\hat{\boldsymbol{\phi}}$

Interior: ε_1, Exterior: ε_2

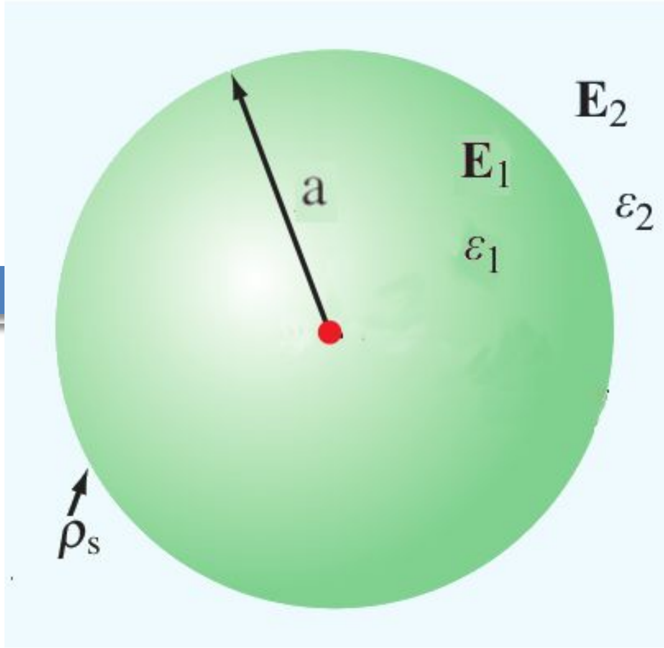
Find: Exterior electric field: $\mathbf{E}_2$

Solution: We know: tangential **E** is continuous:

$$E_{2\theta}\hat{\boldsymbol{\theta}} = E_{1\theta}\hat{\boldsymbol{\theta}}$$

$$E_{2\phi}\hat{\boldsymbol{\phi}} = E_{1\phi}\hat{\boldsymbol{\phi}}$$

Example 2



Solution:

$$E_{2\theta}\hat{\boldsymbol{\theta}} = E_{1\theta}\hat{\boldsymbol{\theta}}$$

$$E_{2\phi}\hat{\boldsymbol{\phi}} = E_{1\phi}\hat{\boldsymbol{\phi}}$$

We know: normal **D** is discontinuous:

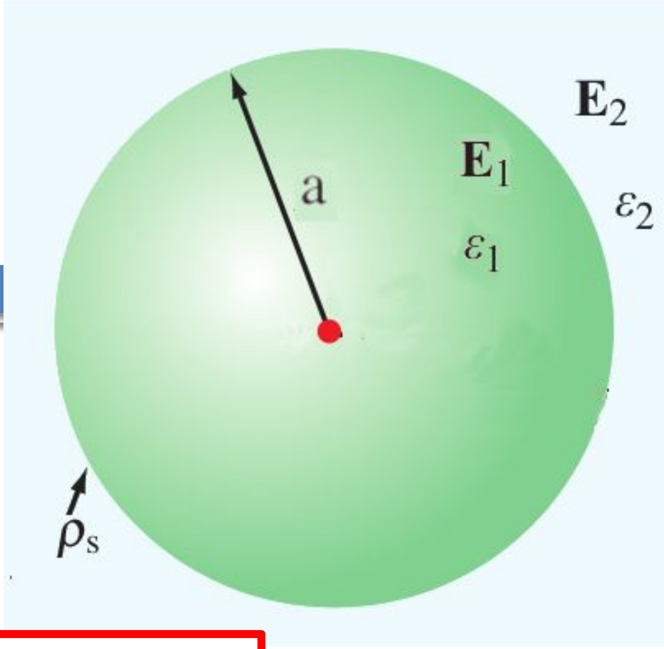
$$D_{1\mathrm{n}} - D_{2\mathrm{n}} = \rho_{\mathrm{s}}$$

so:

$$\varepsilon_1 E_{1R} - \varepsilon_2 E_{2R} = \rho_s$$

$$E_{2R} = \frac{\varepsilon_1 E_{1R} - \rho_s}{\varepsilon_2}$$

Example 2

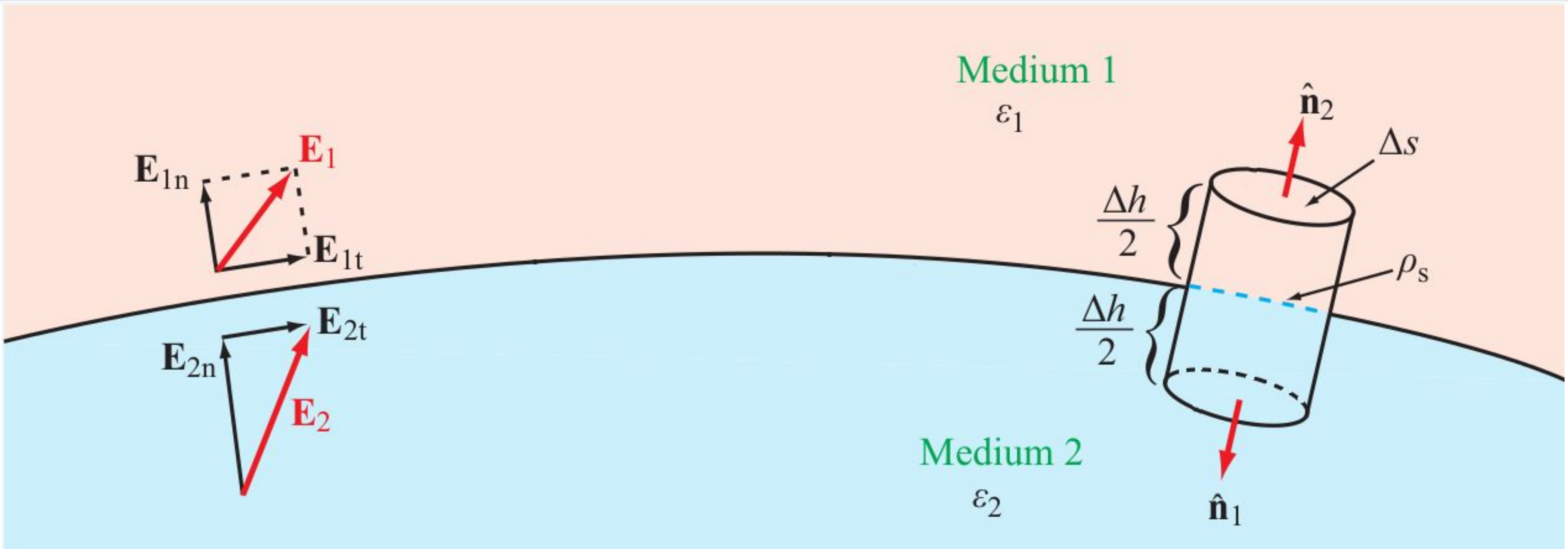


Solution:

so:

$$\mathbf{E}_2 = \frac{\varepsilon_1 E_{1R} - \rho_s}{\varepsilon_2}\hat{\mathbf{R}} + E_{1\theta}\hat{\boldsymbol{\theta}} + E_{1\phi}\hat{\boldsymbol{\phi}}$$

4-8 Dielectric Boundary Conditions



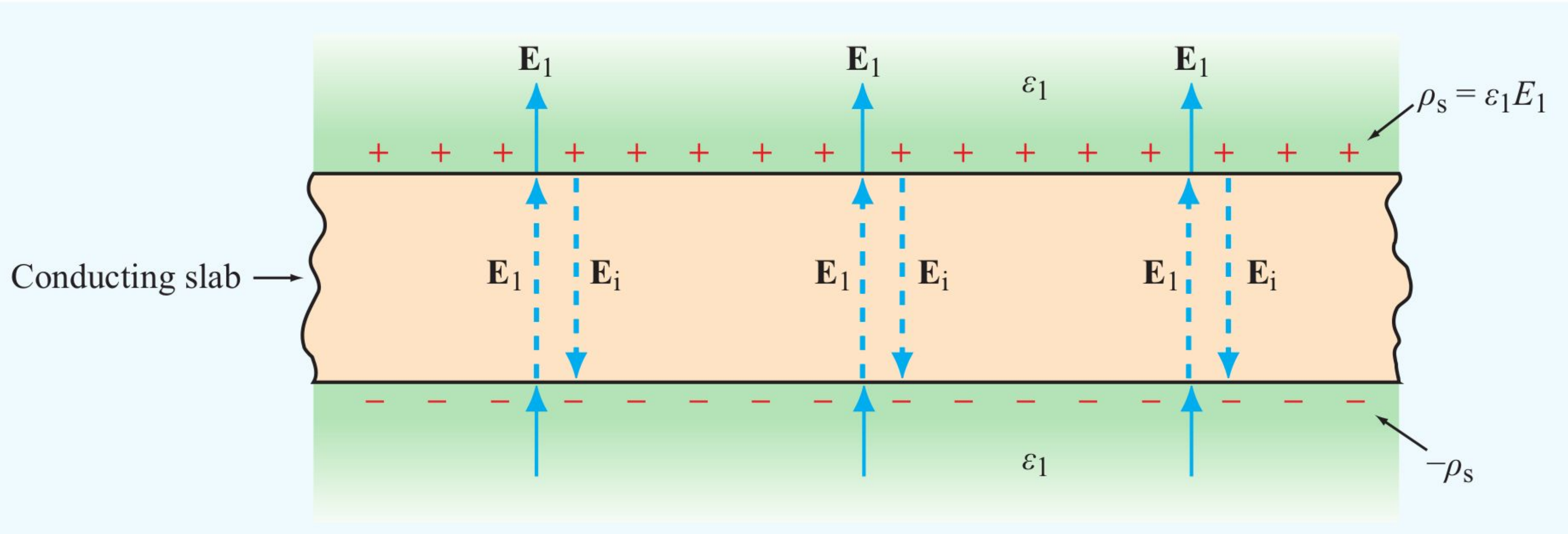
$$\mathbf{E}_{1t} = \mathbf{E}_{2t}$$

$$D_{1n} - D_{2n} = \rho_s \qquad (\text{C/m}^2).$$

Tangential **E** is continuous

Normal **D** is discontinuous by the surface charge density

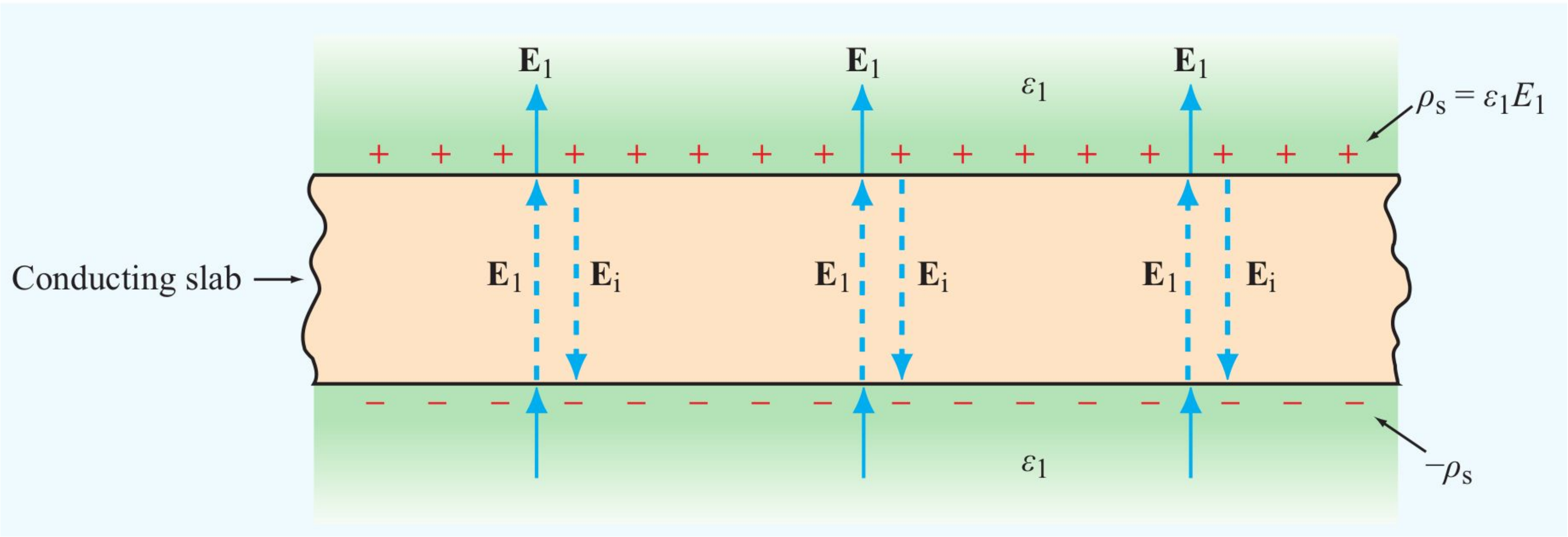
4-8 Dielectric/Perfect Conductor BC's



Net electric field inside a perfect conductor is zero

$$\mathbf{E}_2 = \mathbf{D}_2 = 0,$$

4-8 Dielectric/Perfect Conductor BC's

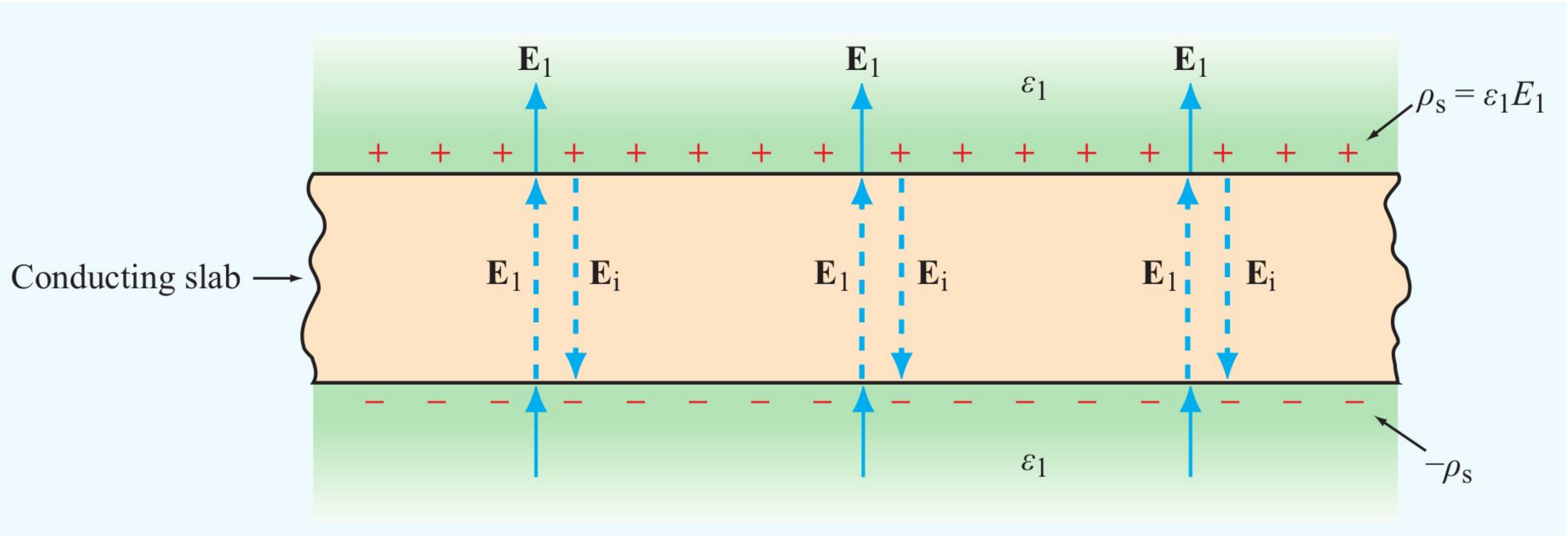


$$\mathbf{E}_2 = \mathbf{D}_2 = 0,$$

Tangential **E** is continuous, so:

$$E_{1t} = D_{1t} = 0,$$

4-8 Dielectric/Perfect Conductor BC's

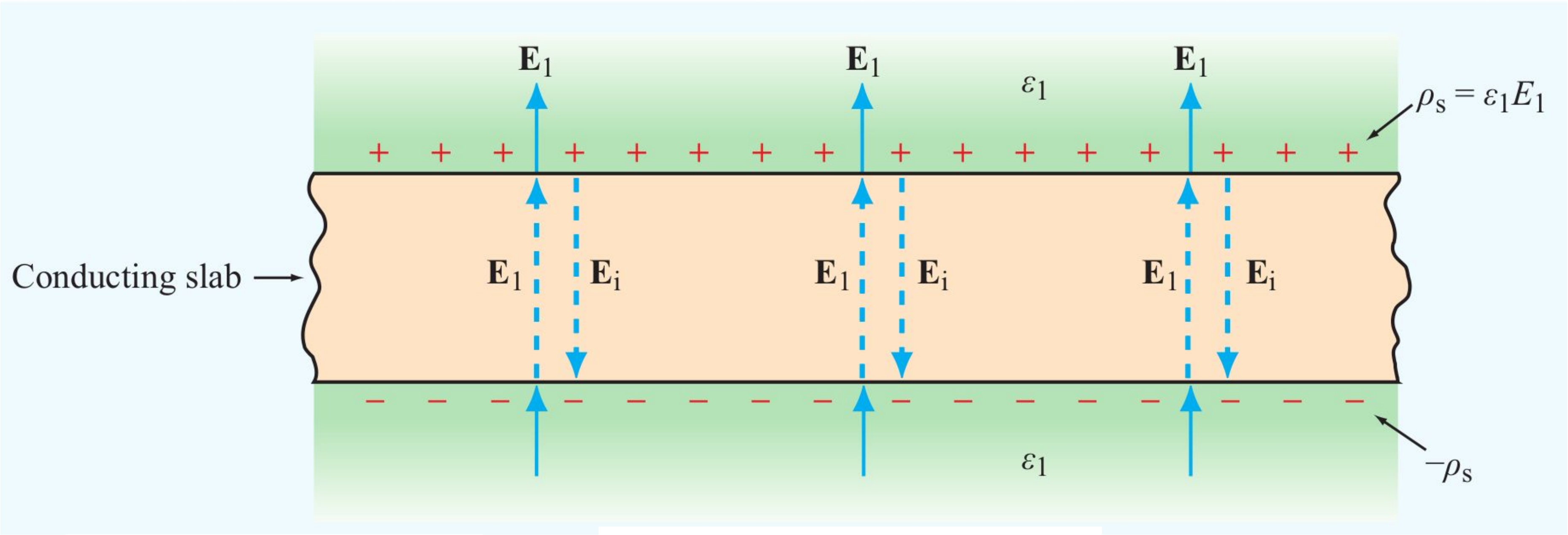


$$\mathbf{E}_2 = \mathbf{D}_2 = 0,$$

Normal **D**: $D_{1n} - D_{2n} = \rho_s$

$$D_{1n} = \varepsilon_1 E_{1n} = \rho_s$$

4-8 Dielectric/Perfect Conductor BC's



$$E_{1t} = D_{1t} = 0, \qquad D_{1n} = \varepsilon_1 E_{1n} = \rho_s$$

$$\mathbf{D}_1 = \varepsilon_1 \mathbf{E}_1 = \hat{\mathbf{n}}\rho_s,$$

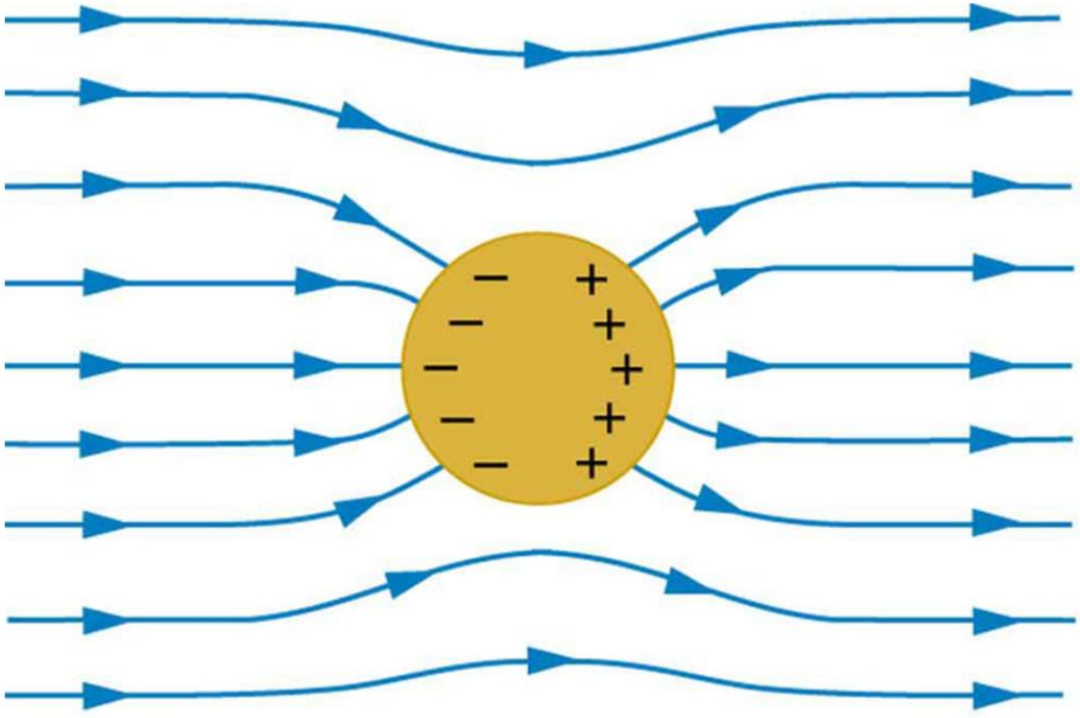
(at conductor surface)

E is always normal to a perfect conductor's surface

4-8 Dielectric/Perfect Conductor BC's

E is always normal to a perfect conductor's surface

Field lines near a perfectly-conducting sphere:



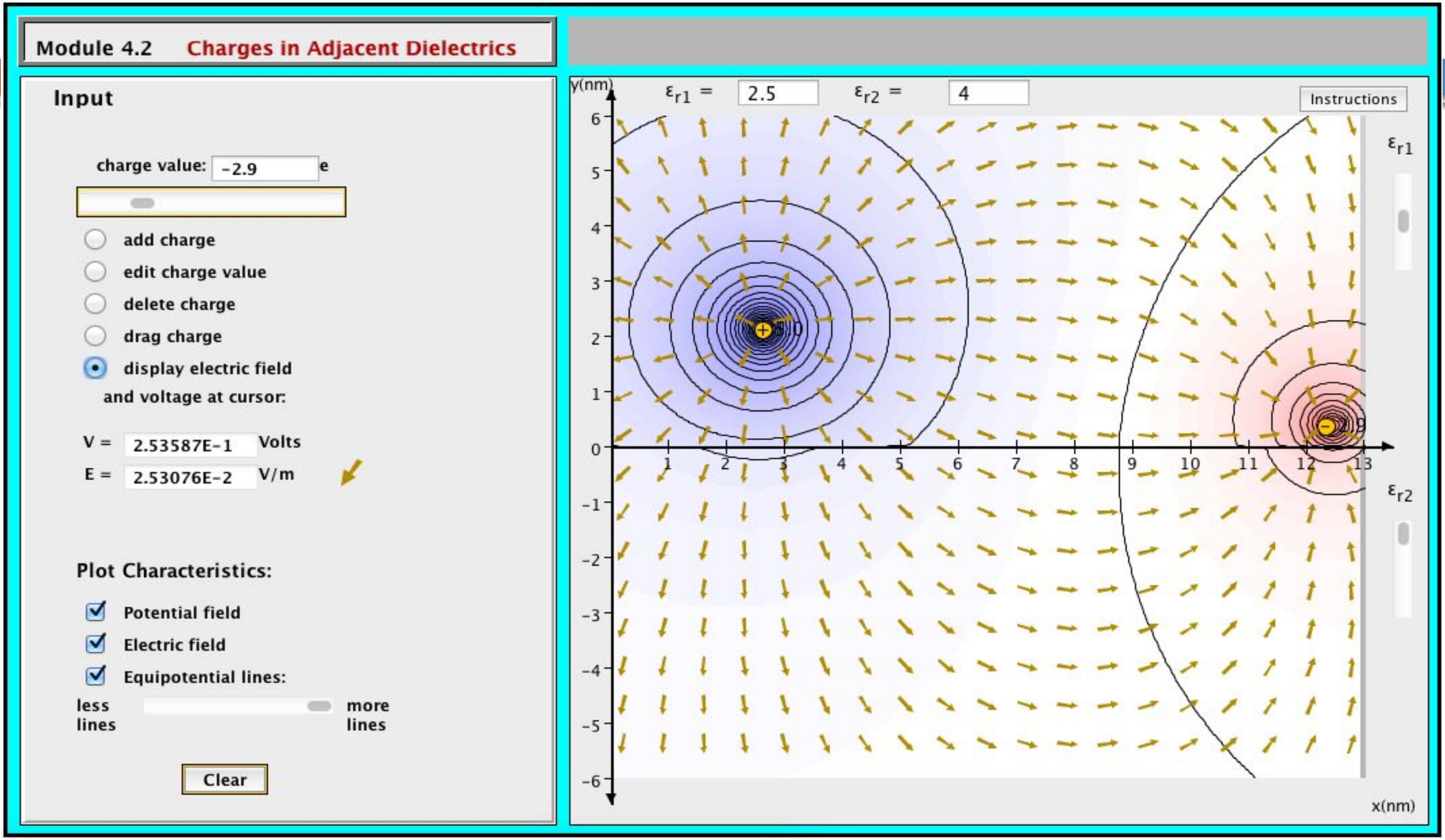
(lumenlearning.com)

4-8 Summary of Boundary Conditions

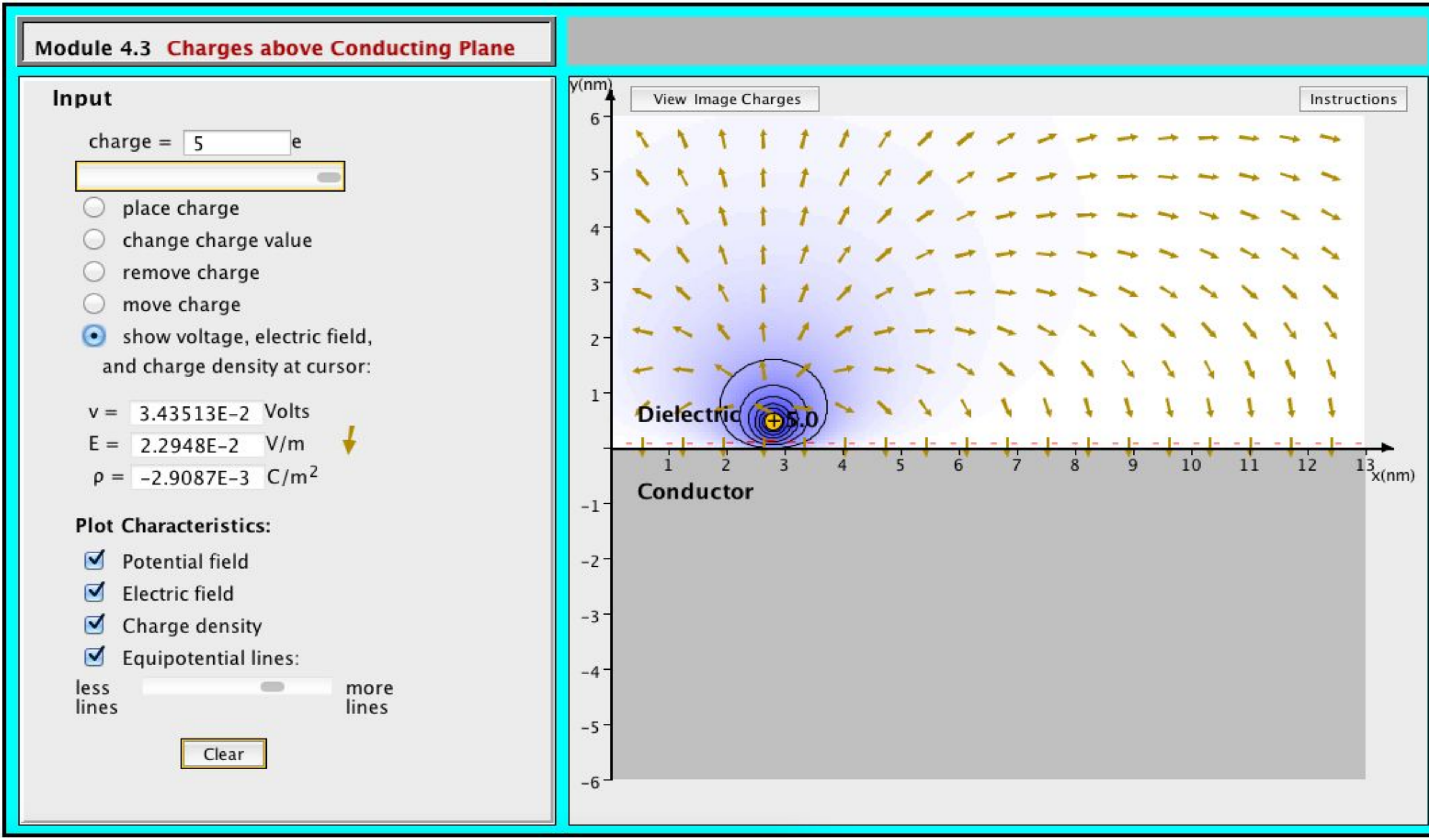
Field Component	Any Two Media	Medium 1 Dielectric ε_1	Medium 2 Conductor
Tangential E	$\mathbf{E}_{1t} = \mathbf{E}_{2t}$	$\mathbf{E}_{1t} = \mathbf{E}_{2t} = 0$	
Tangential D	$\mathbf{D}_{1t}/\varepsilon_1 = \mathbf{D}_{2t}/\varepsilon_2$	$\mathbf{D}_{1t} = \mathbf{D}_{2t} = 0$	
Normal E	$\varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = \rho_s$	$E_{1n} = \rho_s/\varepsilon_1$	$E_{2n} = 0$
Normal D	$D_{1n} - D_{2n} = \rho_s$	$D_{1n} = \rho_s$	$D_{2n} = 0$

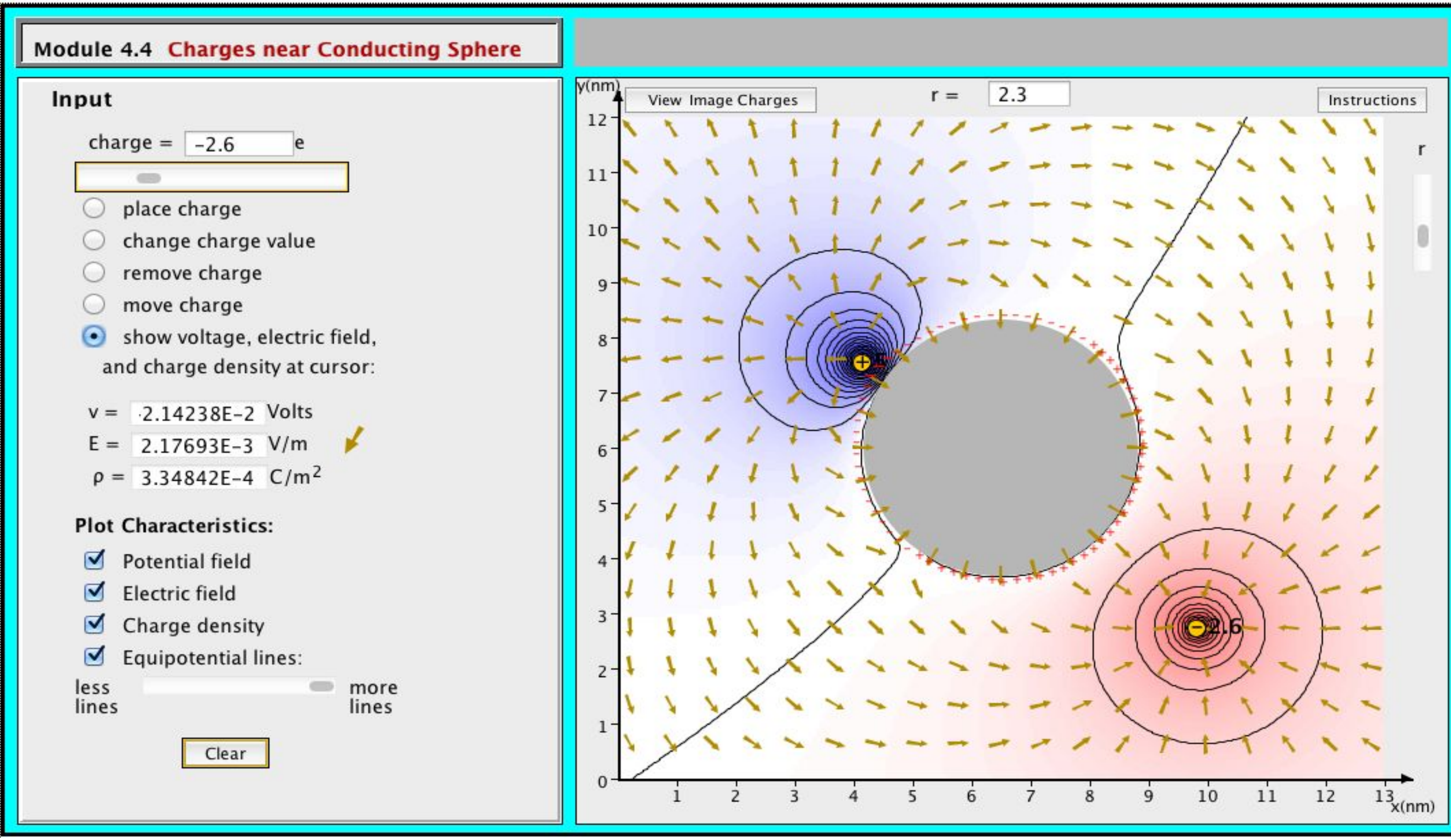
Notes: (1) ρ_s is the surface charge density at the boundary; (2) normal components of $\mathbf{E}_1$, $\mathbf{D}_1$, $\mathbf{E}_2$, and $\mathbf{D}_2$ are along $\hat{\mathbf{n}}_2$, the outward normal unit vector of medium 2.

Remember $\mathbf{E} = 0$ in a good conductor



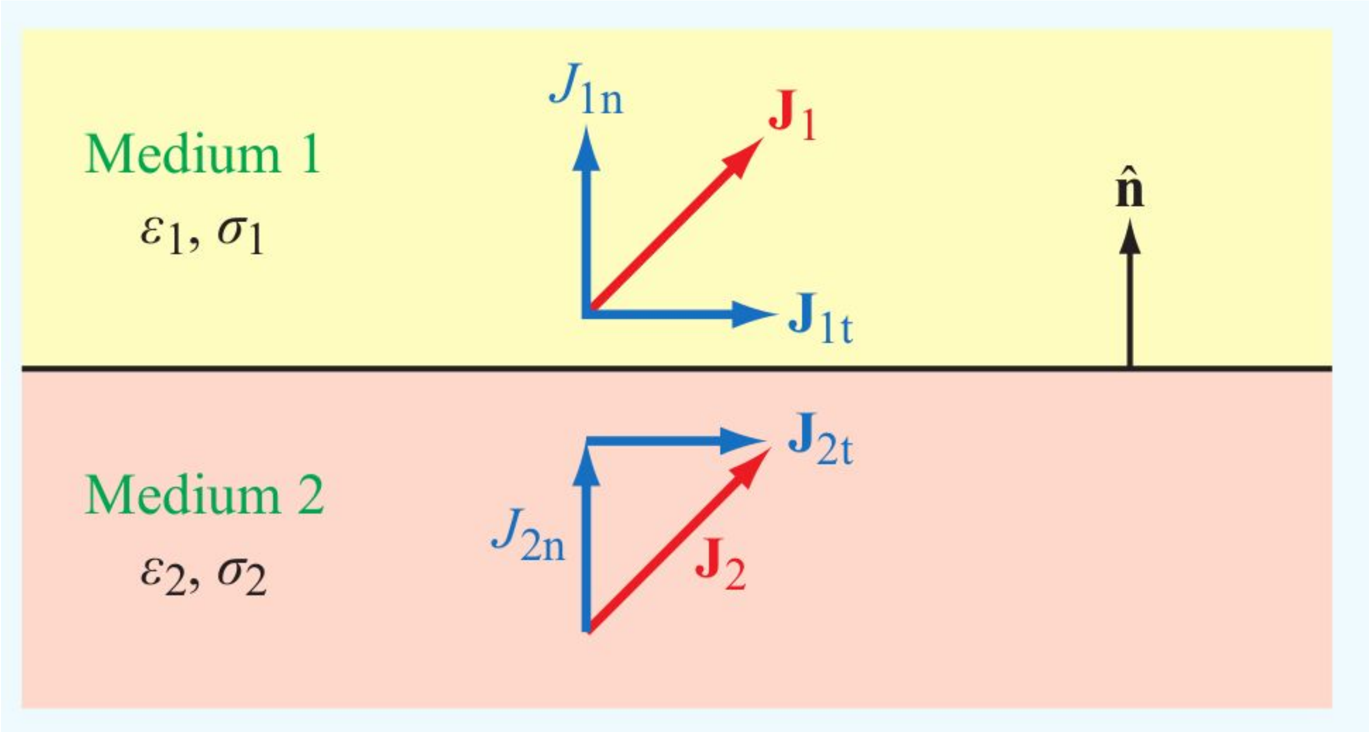
Module 4.2
Charges in Adjacent Dielectrics
Input
charge value: -2.9 e
add charge
edit charge value
delete charge
drag charge
display electric field
and voltage at cursor:
V = 2.53587E-1 Volts
E = 2.53076E-2 V/m
Plot Characteristics:
Potential field
Electric field
Equipotential lines:
less lines
more lines
Clear
y(nm)
ε r1 = 2.5
ε r2 = 4
Instructions
ε r1
ε r2
x(nm)


Module 4.3 Charges above Conducting Plane
Input
charge = 5 e
place charge
change charge value
remove charge
move charge
show voltage, electric field, and charge density at cursor:
v = 3.43513E-2 Volts
E = 2.2948E-2 V/m
ρ = -2.9087E-3 C/m2
Plot Characteristics:
Potential field
Electric field
Charge density
Equipotential lines:
less lines
more lines
Clear
View Image Charges
Instructions
y(nm)
x(nm)
Dielectric
5.0
Conductor


Module 4.4 Charges near Conducting Sphere
Input
charge = -2.6 e
place charge
change charge value
remove charge
move charge
show voltage, electric field,
and charge density at cursor:
v = 2.14238E-2 Volts
E = 2.17693E-3 V/m
ρ = 3.34842E-4 C/m2
Plot Characteristics:
Potential field
Electric field
Charge density
Equipotential lines:
less lines
more lines
Clear
View Image Charges
r = 2.3
Instructions
y(nm)
x(nm)
r
-2.6

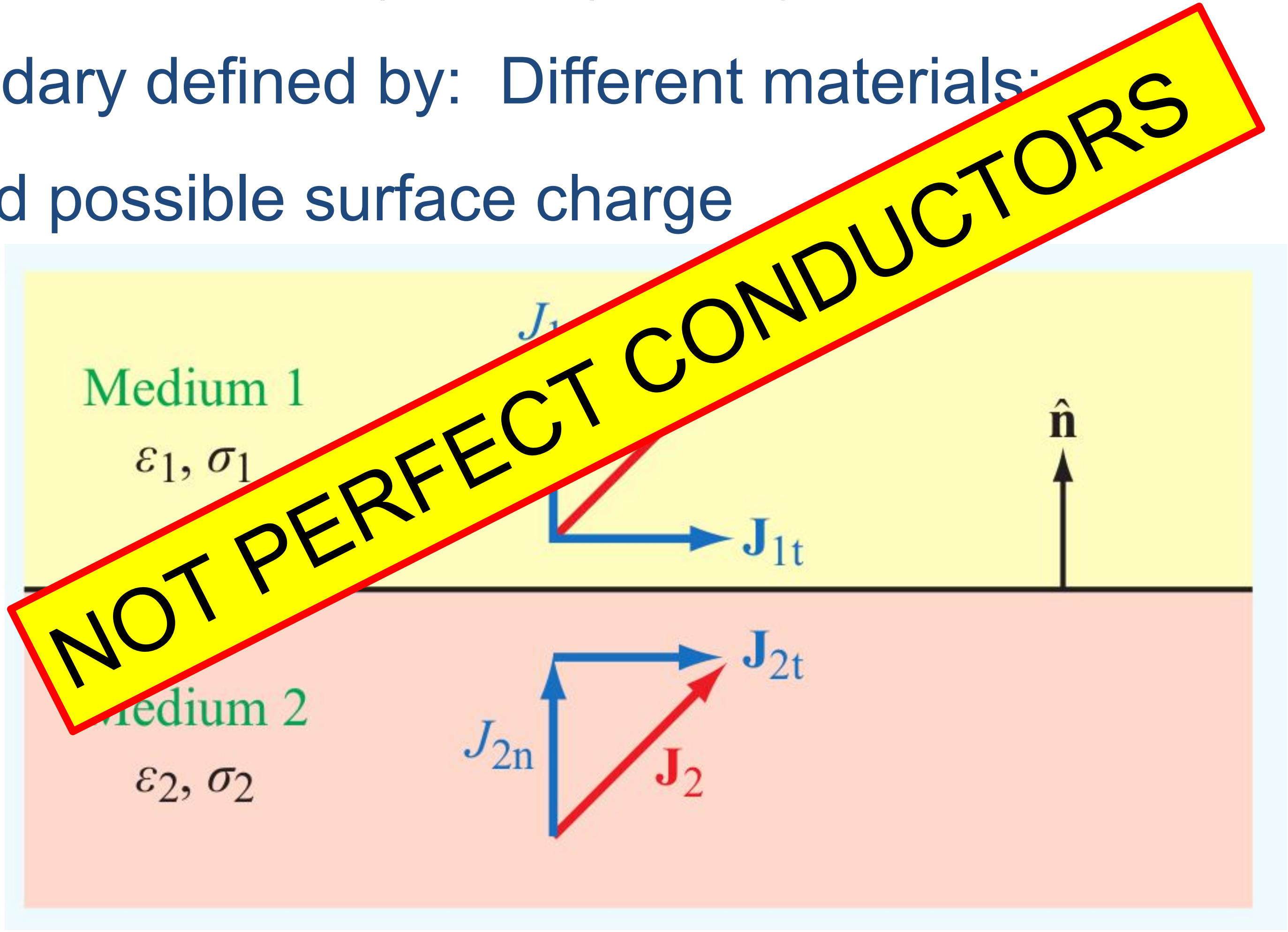
4-8 Conductor/Conductor BC's

- How do the fields (**E**, **D**, **J**) change across a boundary?
- Boundary defined by: Different materials: ε, σ
- ... and possible surface charge



4-8 Conductor/Conductor BC's

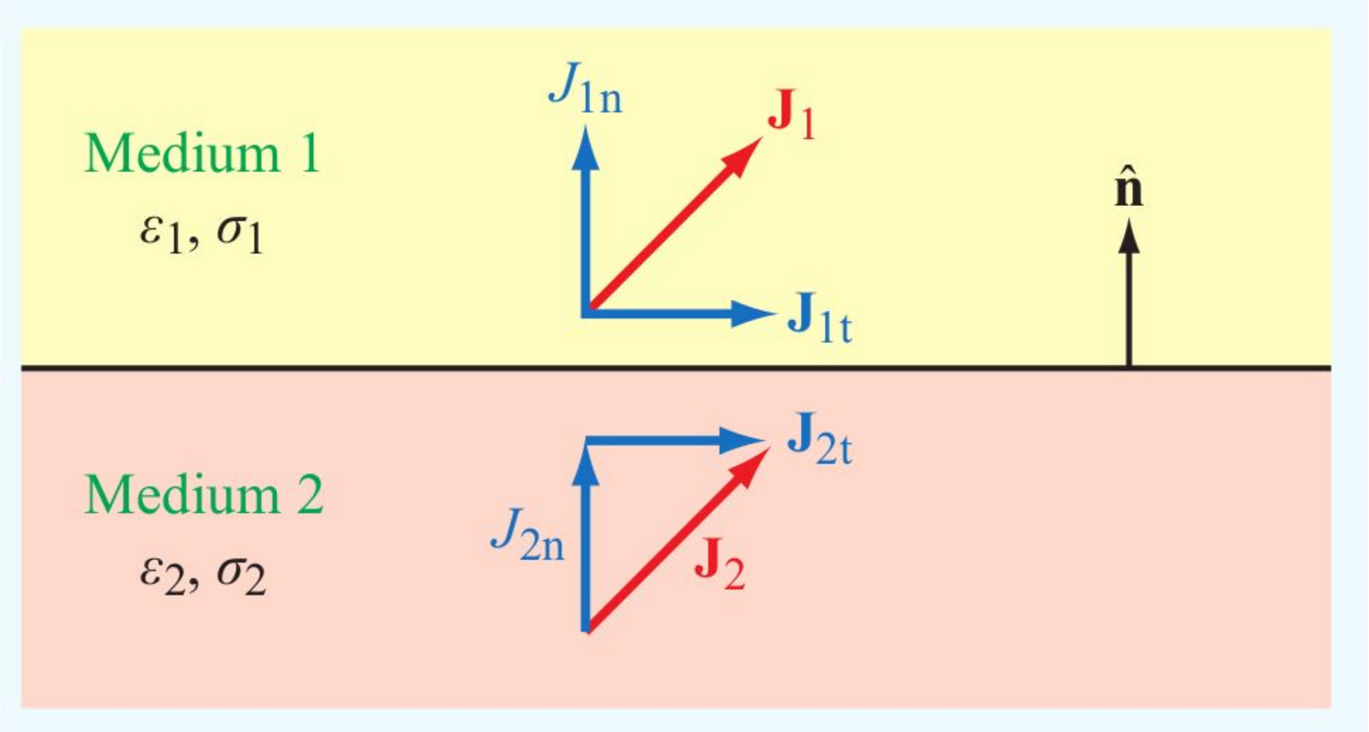
- How do the fields (**E, D, J**) change across a boundary?
- Boundary defined by: Different materials;
- ... and possible surface charge



4-8 Conductor/Conductor BC's

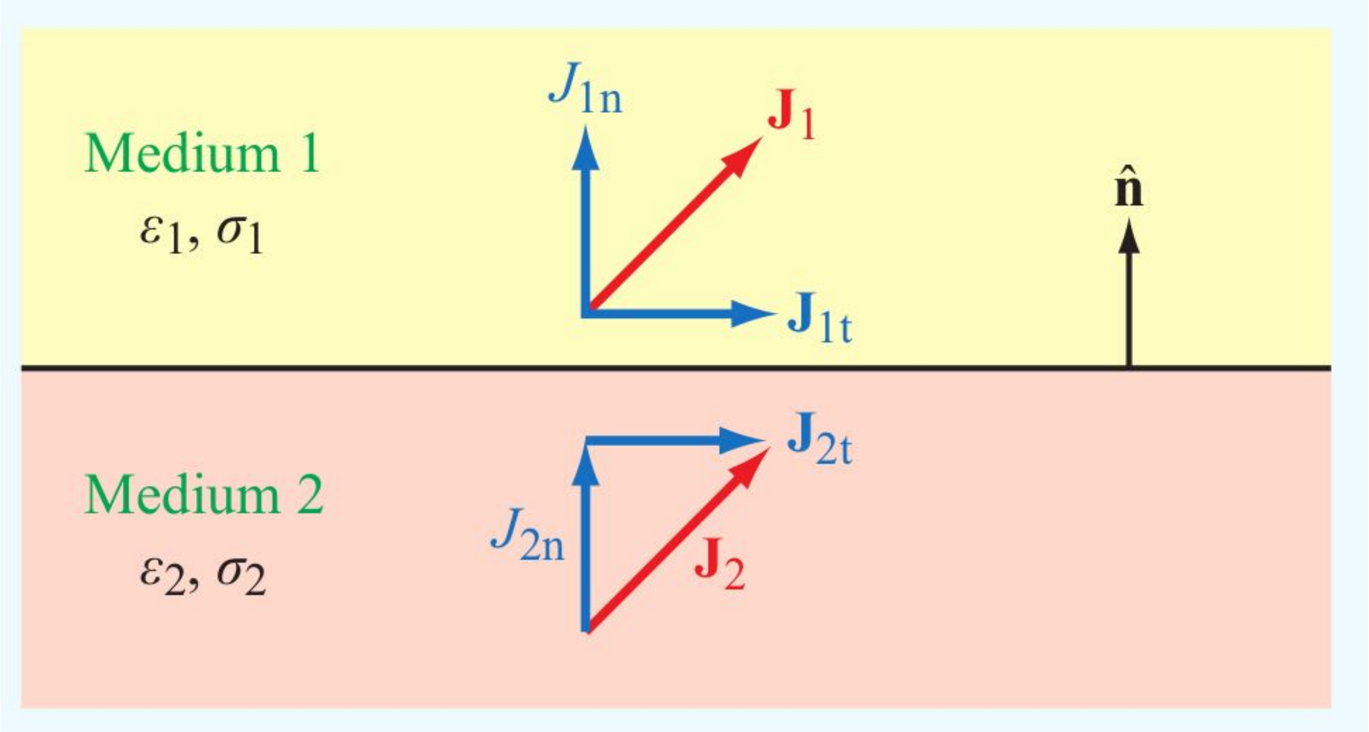
as before: $\mathbf{E}_{1t} = \mathbf{E}_{2t}, \qquad \varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = \rho_s$

new: $\mathbf{J}_1 = \sigma_1 \mathbf{E}_1$ and $\mathbf{J}_2 = \sigma_2 \mathbf{E}_2$.



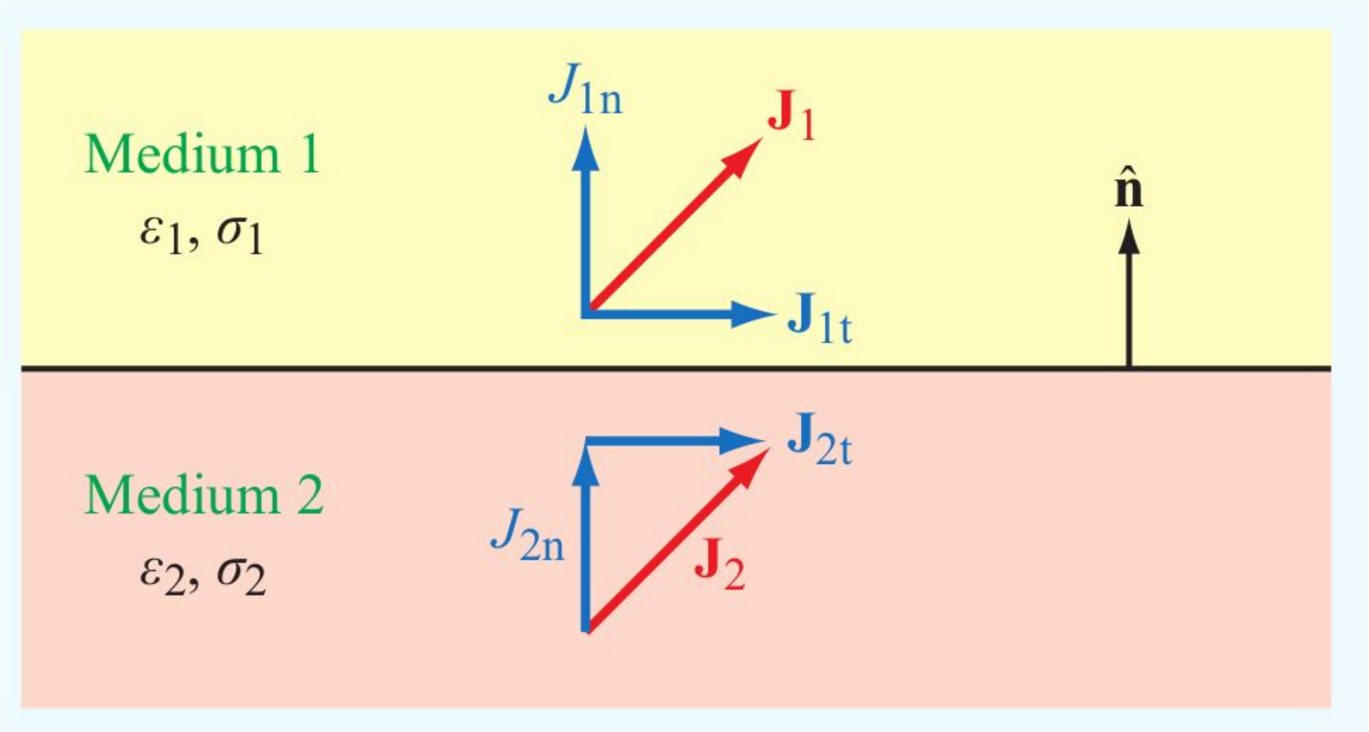
4-8 Conductor/Conductor BC's

$$\frac{\mathbf{J}_{1t}}{\sigma_1} = \frac{\mathbf{J}_{2t}}{\sigma_2}, \qquad \varepsilon_1 \frac{J_{1n}}{\sigma_1} - \varepsilon_2 \frac{J_{2n}}{\sigma_2} = \rho_s$$



4-8 Conductor/Conductor BC's

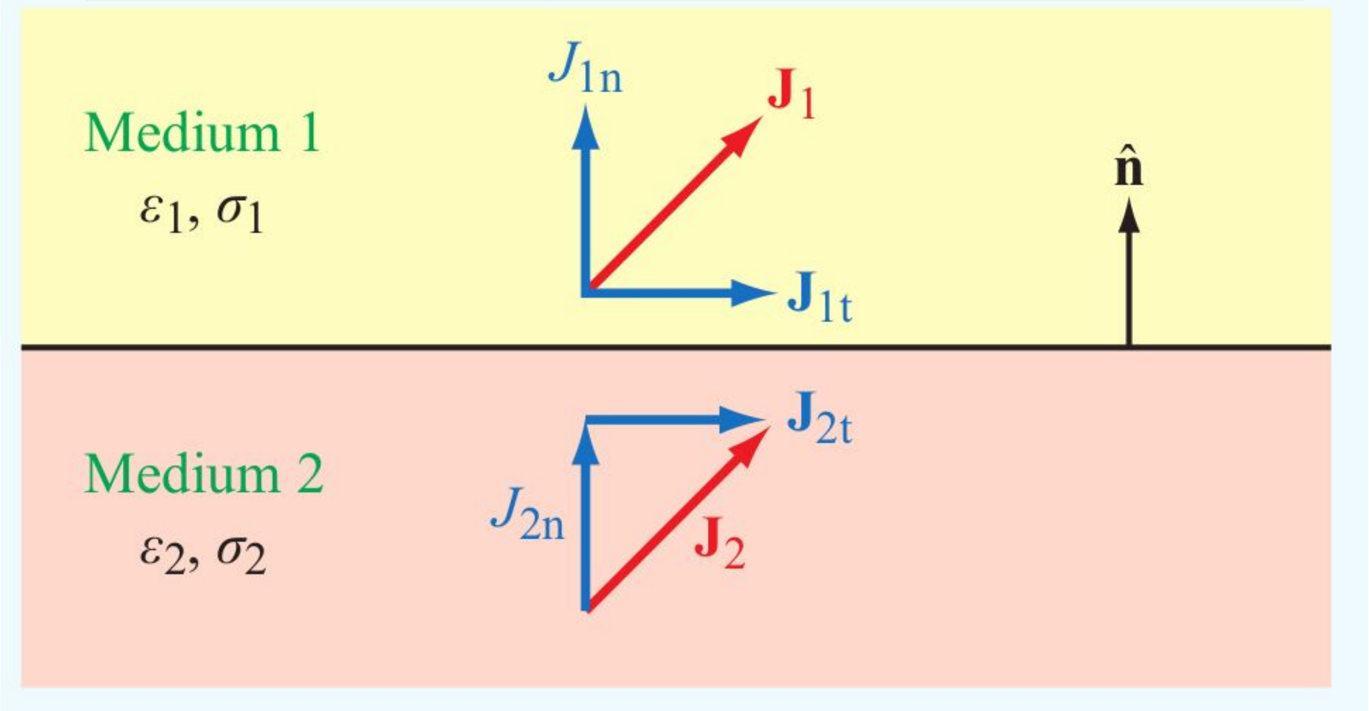
If $J_{1n} \neq J_{2n}$, a different amount of charge arrives at the boundary than leaves it. So $\boldsymbol{\rho}_s$ would change with time. Not allowed in electrostatics.



4-8 Conductor/Conductor BC's

Hence $J_{1n} = J_{2n}$:

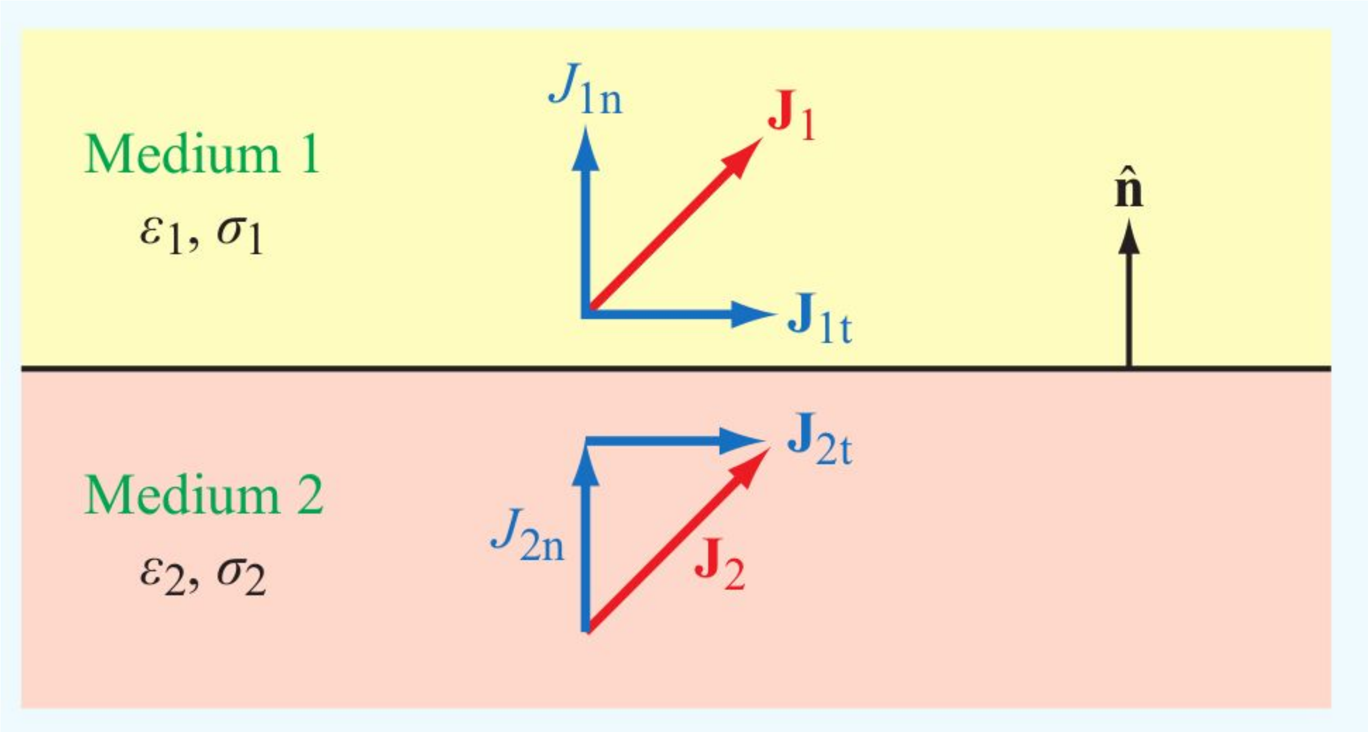
$$J_{1n}\left(\frac{\varepsilon_1}{\sigma_1} - \frac{\varepsilon_2}{\sigma_2}\right) = \rho_s \quad \text{(electrostatics)}.$$



4-8 Conductor/Conductor BC's

and $\sigma_1 E_{1n} = \sigma_2 E_{2n}$

$$E_{1n} = (\sigma_2/\sigma_1)E_{2n}$$



Homework

Homework 15 is due tomorrow at midnight.

submit to gradescope via the canvas site.

Next Time

Sections 4-9, 4-10:

Capacitance
Electrostatic Potential Energy