EECS 230
ENGINEERING ELECTROMAGNETICS
Leland Pierce

Transmission Lines 8

University of Michigan - Fall 2024

Announcements

post-lab2 and pre-lab3 are due this sunday night

exam1: monday, oct 7, 5-6:20pm (80 minutes)
alternate: tuesday, oct 8, 5-6:20pm rooms: TBD

email me if you want to take the alternate.
those in monday lab can take the alternate, I hope

coverage: chapters 1-2
thru today's lecture/homework

4 problems, similar to homework

3-page formula sheet on canvas (will be provided)
bring handwritten single page
bring calculator, ruler, compass, pencils

Chapter 2 Overview

What is a transmission line?

Why study transmission lines?

model

diffeq

solve diffeq

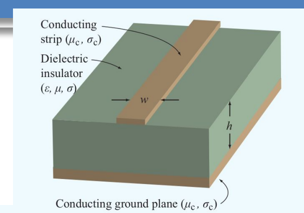
wave propagation

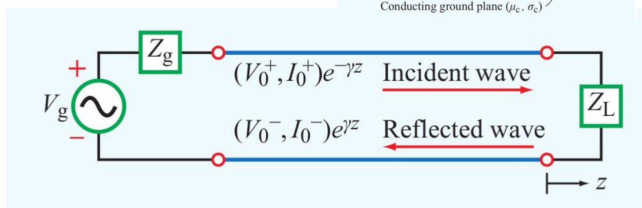
lossless xmission lir

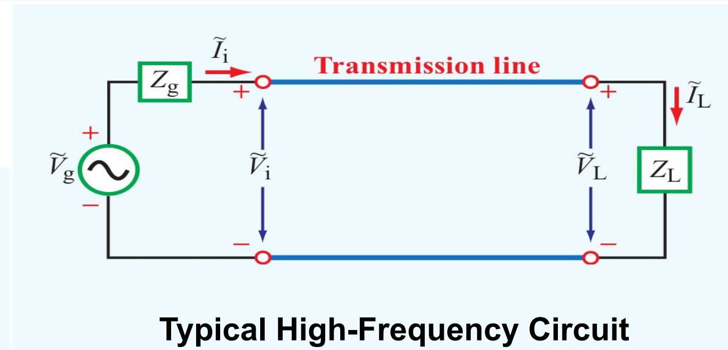
- microstrip lines
- reflections
- standing waves
- impedance
- short, open
- matching
- power flow

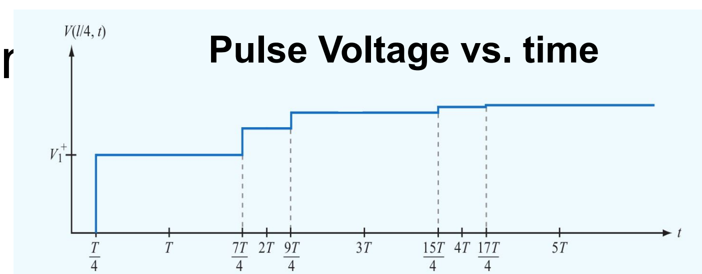
smith chart

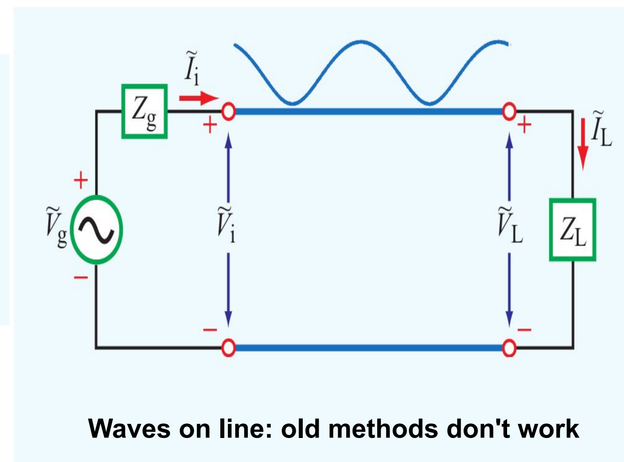
transients

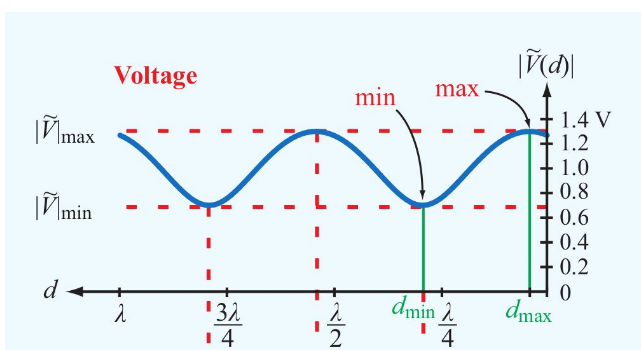


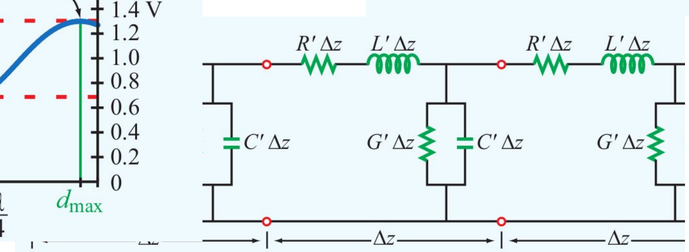


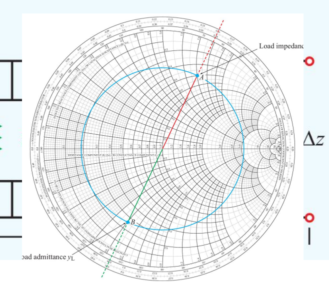












Today's Lecture Coverage

Review Sections 2-1 through 2-10 of the book:

2-1: What is a transmission line? Why study transmission lines?
2-2: Lumped-Element Model
2-3: Governing Differential Eqns
2-4: Solve the Differential Equations
Properties of the solution: wave propagation
2-5: Lossless Microstrip Line
2-6: Lossless Transmission Lines
2-7: Lossless Transmission Lines: Wave Impedance
2-8: Lossless Transmission Lines: Special Cases
2-9: Lossless Transmission Lines: Power Flow
2-10: The Smith Chart
2-11: Matching using the Smith Chart

Sections 2-11, 2-12 of the book:

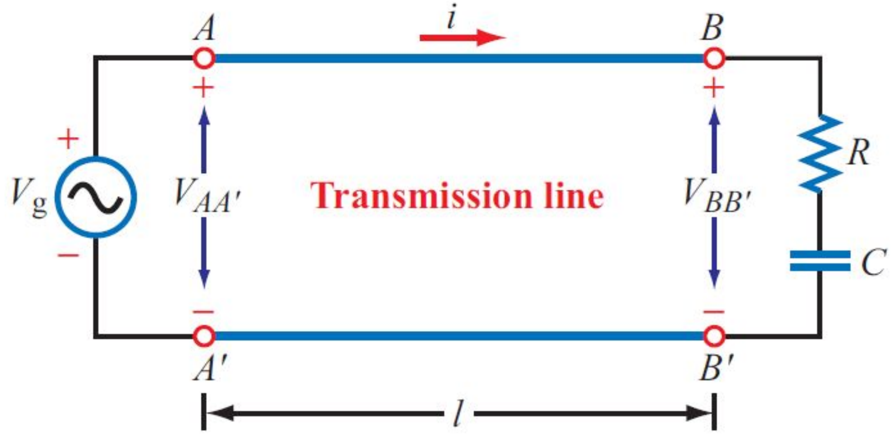
2-11: Matching using the Smith Chart
2-12: Transients on Transmission Lines

Chapter 2 Review

- A transmission line connects a generator to a load.

Chapter 2 Review

Phase Delay due to length of transmission line:



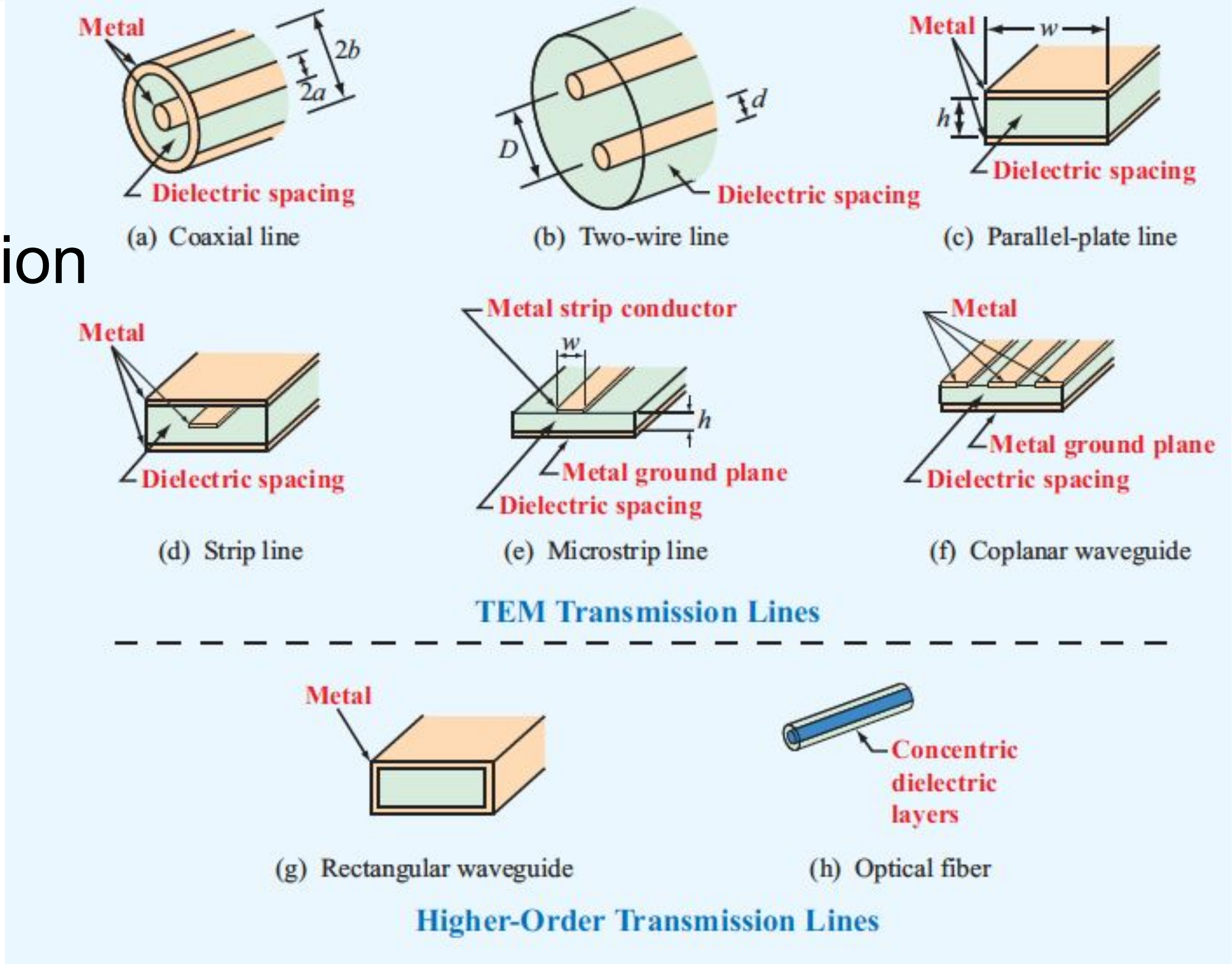
$$\phi_0 = \frac{\omega l}{c} = \frac{2\pi f l}{c} = 2\pi \frac{l}{\lambda} \quad \text{radians.}$$

$l/\lambda \lesssim 0.01$: Can ignore transmission-line effects

$l/\lambda \gtrsim 0.01$: Must deal with phase shift, and other effects...

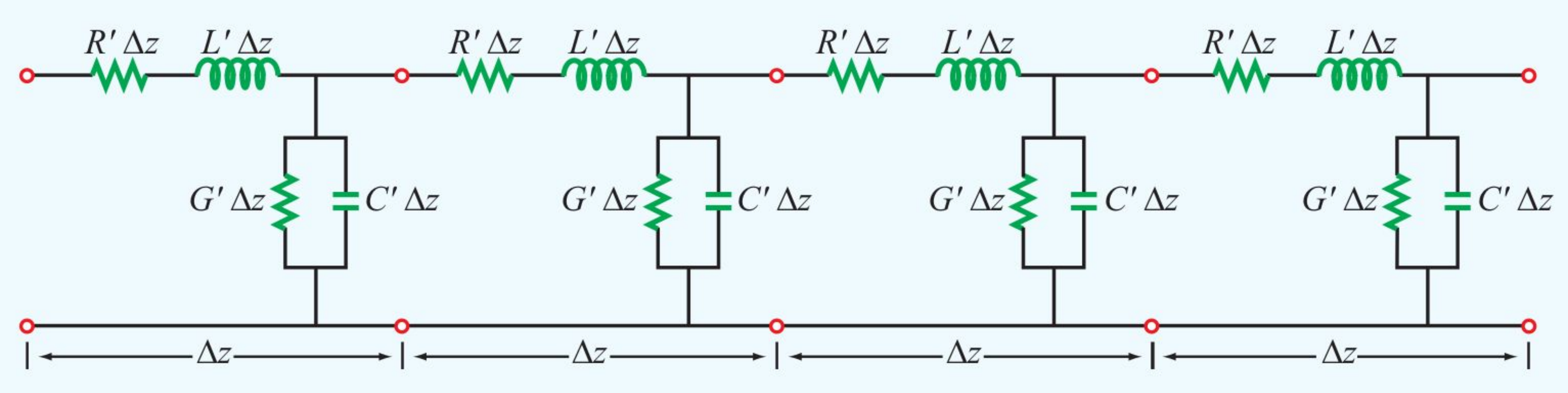
Chapter 2 Review

Different geometries for transmission lines



Chapter 2 Review

Lumped-Element Model:



All parameters are "per unit length":

R': Combined Resistance of BOTH conductors: □/m

L': Combined Inductance of BOTH conductors, H/m

G': Conductance of insulation
between inner and outer conductor, S/m

C': Capacitance
between inner and outer conductors, F/m

Chapter 2 Review

Lumped-Element Values: geometry/materials/freq

Table 2-1 **Transmission-line parameters R', L', G', and C' for three types of lines.**

Parameter	Coaxial	Two-Wire	Parallel-Plate	Unit
R'	$\dfrac{R_s}{2\pi}\left(\dfrac{1}{a}+\dfrac{1}{b}\right)$	$\dfrac{2R_s}{\pi d}$	$\dfrac{2R_s}{w}$	Ω/m
L'	$\dfrac{\mu}{2\pi}\ln(b/a)$	$\dfrac{\mu}{\pi}\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]$	$\dfrac{\mu h}{w}$	H/m
G'	$\dfrac{2\pi\sigma}{\ln(b/a)}$	$\dfrac{\pi\sigma}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\dfrac{\sigma w}{h}$	S/m
C'	$\dfrac{2\pi\varepsilon}{\ln(b/a)}$	$\dfrac{\pi\varepsilon}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\dfrac{\varepsilon w}{h}$	F/m

$$R_s = \sqrt{\pi f \mu_c / \sigma_c}$$

Chapter 2 Review

Transmission-line governing Differential Equations

$$-\frac{d\widetilde{V}(z)}{dz} = (R' + j\omega L')\,\tilde{I}(z),$$

$$-\frac{d\tilde{I}(z)}{dz} = (G' + j\omega C')\,\widetilde{V}(z).$$

(telegrapher's equations in phasor form)

Chapter 2 Review

Transmission-line governing Differential Equation for *V*:

$$\frac{d^2\widetilde{V}(z)}{dz^2} - \gamma^2\widetilde{V}(z) = 0,$$

(wave equation for $\widetilde{V}(z)$)

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}.$$

(propagation constant)

Chapter 2 Review

Complex propagation constant:

$$\gamma = \alpha + j\beta$$

Where:

γ: Units of 1/m

α: Attenuation constant, units of Np/m (>0 in this class)

β: Phase constant, units of rad/m

Np and radians are both "dimensionless", and are the conventional "units" for these parameters.

Chapter 2 Review

Form of the solution: traveling waves, going in both directions:

$$\widetilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \qquad \text{(V)},$$

$$\widetilde{I}(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z} \qquad \text{(A)}.$$

$$Z_0 = \frac{R' + j\omega L'}{\gamma} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \qquad (\Omega),$$

Chapter 2 Review

Solution in time-domain

$$v(z,t) = |V_0^+| e^{-\alpha z} \cos(\omega t - \beta z + \phi^+) + |V_0^-| e^{+\alpha z} \cos(\omega t + \beta z + \phi^-)$$

Remaining unknowns are determined via specification of source and load.

Chapter 2 Review

- The wave equation for a general Transmission Line.

$$\frac{d^2 \tilde{V}(z)}{dz^2} - \gamma^2 \tilde{V}(z) = 0, \qquad (2.21)$$

- General solution of the wave equation
  - *It involves both incident and reflected waves*

$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \qquad (\text{V}),$$

$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \qquad (\text{A}).$$

Chapter 2 Review

- Useful Relations for lossless Transmission Lines:

$$\alpha = 0 \qquad \text{(lossless line)},$$
$$\beta = \omega\sqrt{L'C'} \qquad \text{(lossless line)}. \qquad (2.45)$$

$$\beta = \omega\sqrt{\mu\varepsilon} \qquad \text{(rad/m)}, \qquad (2.49)$$

$$u_p = \frac{1}{\sqrt{\mu_0\epsilon_0}}\frac{1}{\sqrt{\epsilon_r}} = \frac{c}{\sqrt{\epsilon_r}}$$

$$\lambda = \frac{u_p}{f} = \frac{c}{f}\frac{1}{\sqrt{\varepsilon_r}} = \frac{\lambda_0}{\sqrt{\varepsilon_r}}$$

$$Z_0 = \sqrt{\frac{L'}{C'}} \qquad \text{(lossless line)}, \qquad (2.46)$$

(REAL)

Chapter 2 Review

- Voltage reflection coefficient due to load:

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1}$$

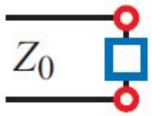
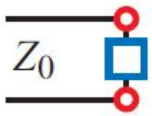
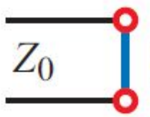
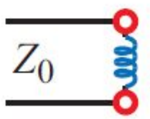
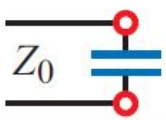
$$= \frac{z_L - 1}{z_L + 1}$$

- Load impedance in terms of Γ:

$$Z_L = \frac{1 + \Gamma}{1 - \Gamma} Z_0$$

Chapter 2 Review

Reflection Coefficient $\Gamma = |\Gamma| e^{j\theta_r}$

Load	$\lvert\Gamma\rvert$	θ_r
 $Z_L = (r + jx)Z_0$	$\left[\frac{(r-1)^2 + x^2}{(r+1)^2 + x^2}\right]^{1/2}$	$\tan^{-1}\left(\frac{x}{r-1}\right) - \tan^{-1}\left(\frac{x}{r+1}\right)$
 Z_0	0 (no reflection)	irrelevant
 (short)	1	$\pm 180°$ (phase opposition)
 (open)	1	0 (in-phase)
 $jX = j\omega L$	1	$\pm 180° - 2\tan^{-1} x$
 $jX = \frac{-j}{\omega C}$	1	$\pm 180° + 2\tan^{-1} x$

$$z_L = Z_L / Z_0 = (R + jX)/Z_0 = r + jx$$

Chapter 2 Review

- Concept of standing wave

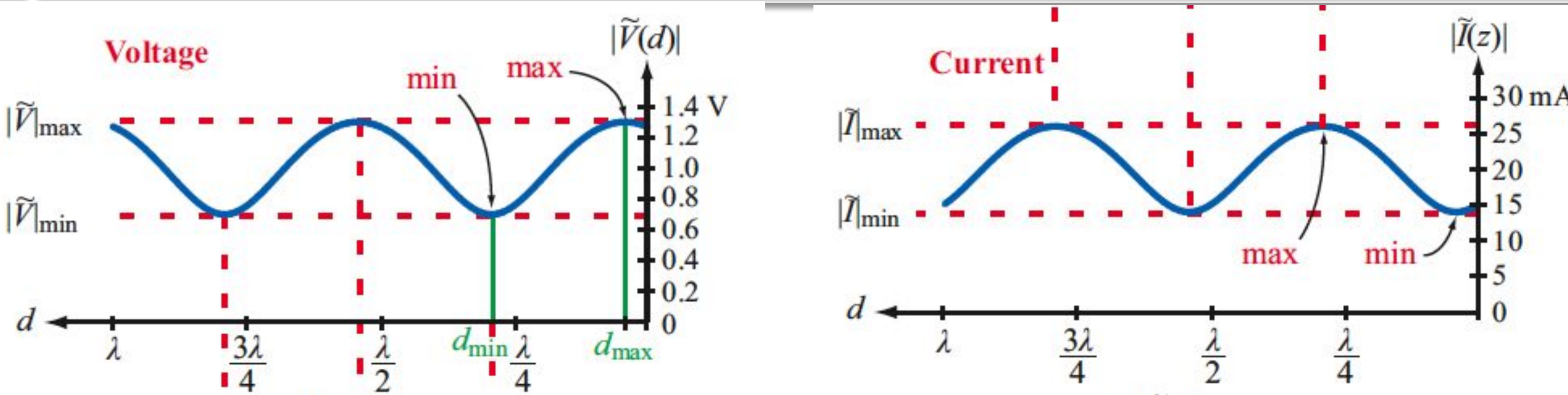
$$\widetilde{V}(z) = V_0^+(e^{-j\beta z} + \Gamma e^{j\beta z}),$$

$$\widetilde{I}(z) = \frac{V_0^+}{Z_0}(e^{-j\beta z} - \Gamma e^{j\beta z}).$$

  - Voltage (current) magnitudes at any point on line:

$$|\widetilde{V}(d)| = |V_0^+|\left[1 + |\Gamma|^2 + 2|\Gamma|\cos(2\beta d - \theta_r)\right]^{1/2}. \quad (2.66)$$

Chapter 2 Review



(a) $|\widetilde{V}(d)|$ versus d

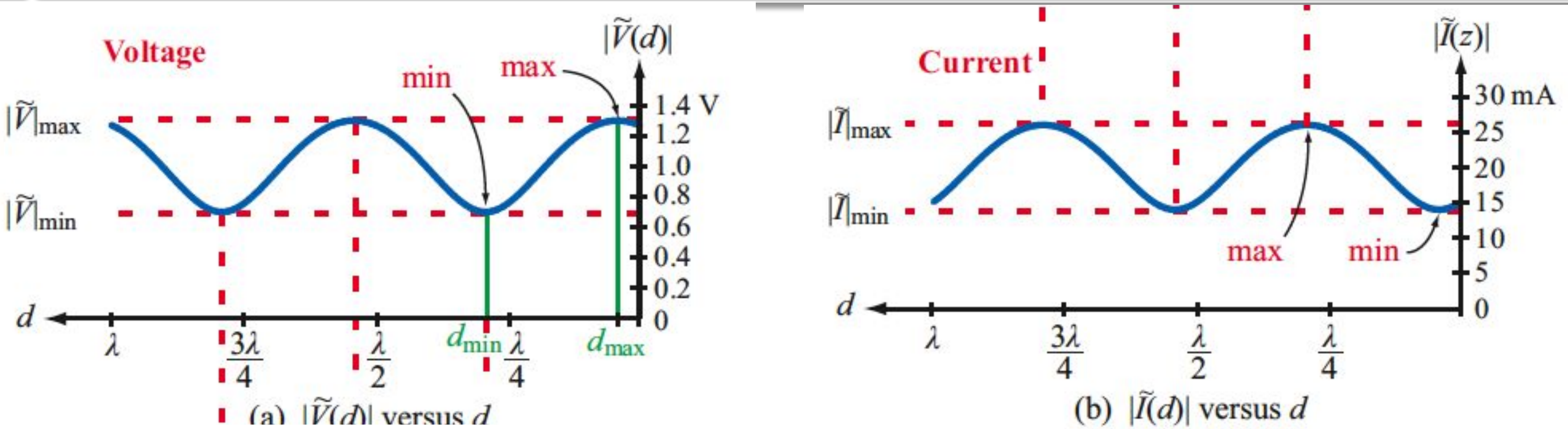
(b) $|\widetilde{I}(d)|$ versus d

- Location of minima / maxima

$$d_{max} = \frac{\theta_r + 2n\pi}{2\beta} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2}, \quad \begin{cases} n = 1, 2, \ldots & \text{if } \theta_r < 0, \\ n = 0, 1, 2, \ldots & \text{if } \theta_r \geq 0, \end{cases} \qquad (2.70)$$

Value of V_{max}: $|\widetilde{V}(d)| = |\widetilde{V}|_{max} = |V_0^+|[1 + |\Gamma|],$

Chapter 2 Review



- Spatial period of standing wave: $\frac{\lambda}{2}$

- Standing wave ratio S:

$$S = \frac{|\widetilde{V}|_{max}}{|\widetilde{V}|_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Chapter 2 Review

Wave Impedance:

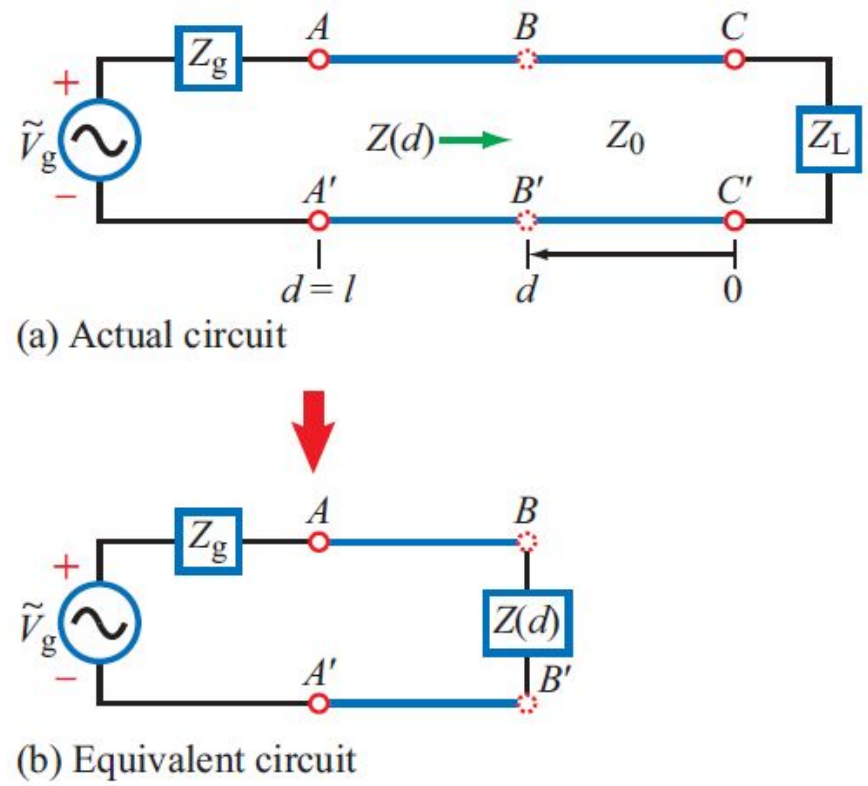
At a distance *d* from the load:

$$Z(d) = \frac{\tilde{V}(d)}{\tilde{I}(d)}$$

$$= \frac{V_0^+[e^{j\beta d} + \Gamma e^{-j\beta d}]}{V_0^+[e^{j\beta d} - \Gamma e^{-j\beta d}]} Z_0$$

$$= Z_0 \left[\frac{1 + \Gamma e^{-j2\beta d}}{1 - \Gamma e^{-j2\beta d}}\right]$$

$$= Z_0 \left[\frac{1 + \Gamma_d}{1 - \Gamma_d}\right] \qquad (\Omega),$$

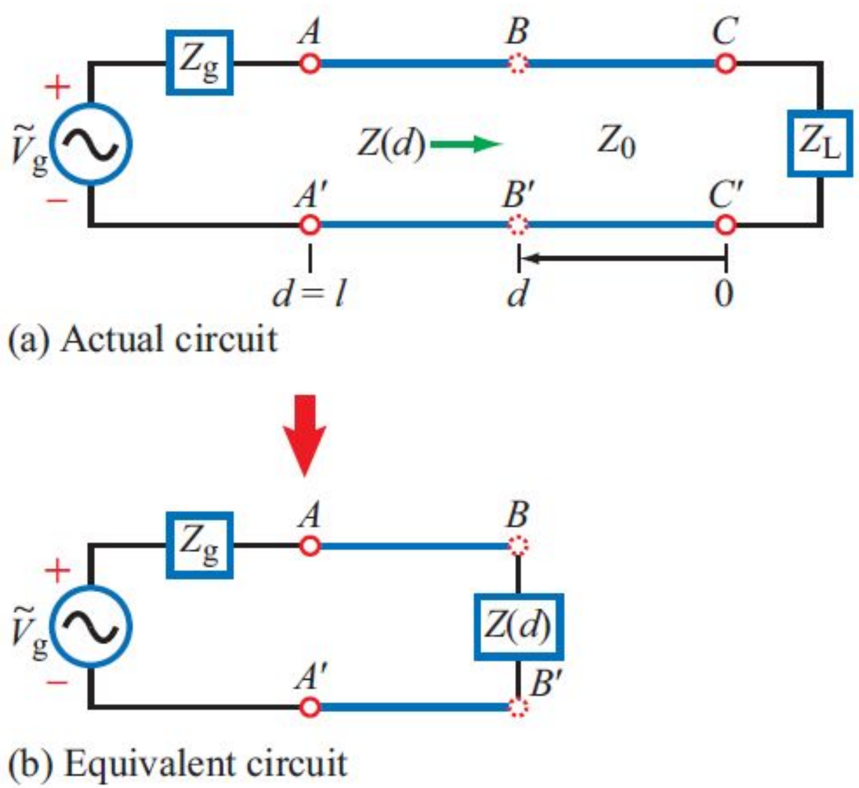


Chapter 2 Review

Define the phase-shifted voltage reflection coefficient:

$$\Gamma_d = \Gamma e^{-j2\beta d} = |\Gamma| e^{j\theta_r} e^{-j2\beta d}$$

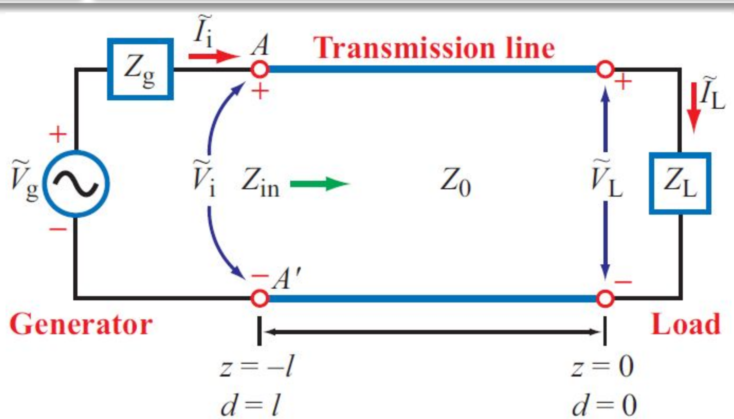
$$= |\Gamma| e^{j(\theta_r - 2\beta d)}$$

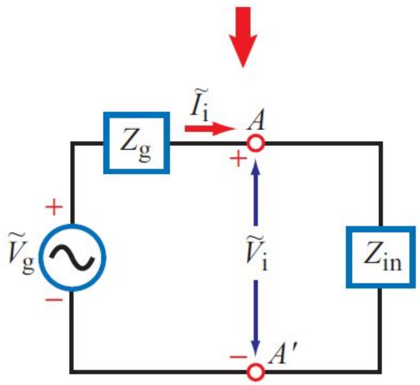


Z(d) is different than Z_0:
Ratio of Total Voltage and Current

Recall: $Z_0 = \dfrac{V_0^+}{I_0^+}$

Chapter 2 Review





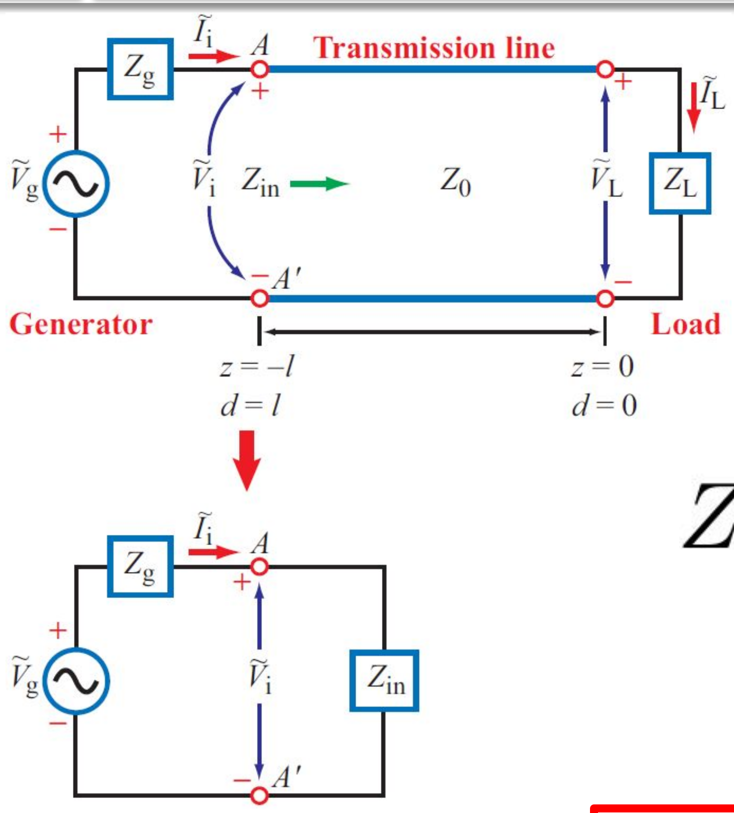
Input impedance:
impedance of the transmission line at *d=l:*

$$Z_{\text{in}} = Z(d = l) = \frac{\widetilde{V}(d = l)}{\widetilde{I}(d = l)}$$

Note that Z_{in} is different from the Characteristic Impedance, and is different from the Load Impedance:

$$Z_{\text{in}} \neq Z_0 \qquad Z_{\text{in}} \neq Z_L$$

Chapter 2 Review



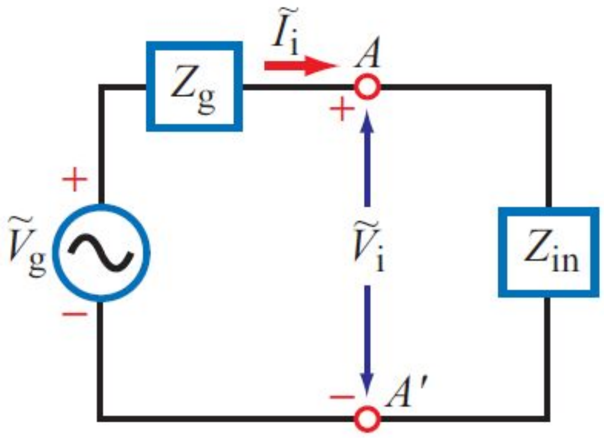
Input impedance:

impedance of the transmission line at *d=l:*

$$Z_{in} = Z(d = l) = \frac{\tilde{V}(d = l)}{\tilde{I}(d = l)}$$

$$Z_{in} = Z_0 \left[\frac{z_L + j\tan(\beta l)}{1 + jz_L \tan(\beta l)} \right]$$

Chapter 2 Review



Voltage Amplitude:

$$V_0^+ = \left(\frac{\tilde{V}_g Z_{in}}{Z_g + Z_{in}}\right)\left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right).$$

We used the boundary conditions (Z_L, Z_g , V_g, l) to solve for the 2 unknowns.

This completes the solution of the transmission line differential equation.

Chapter 2 Review

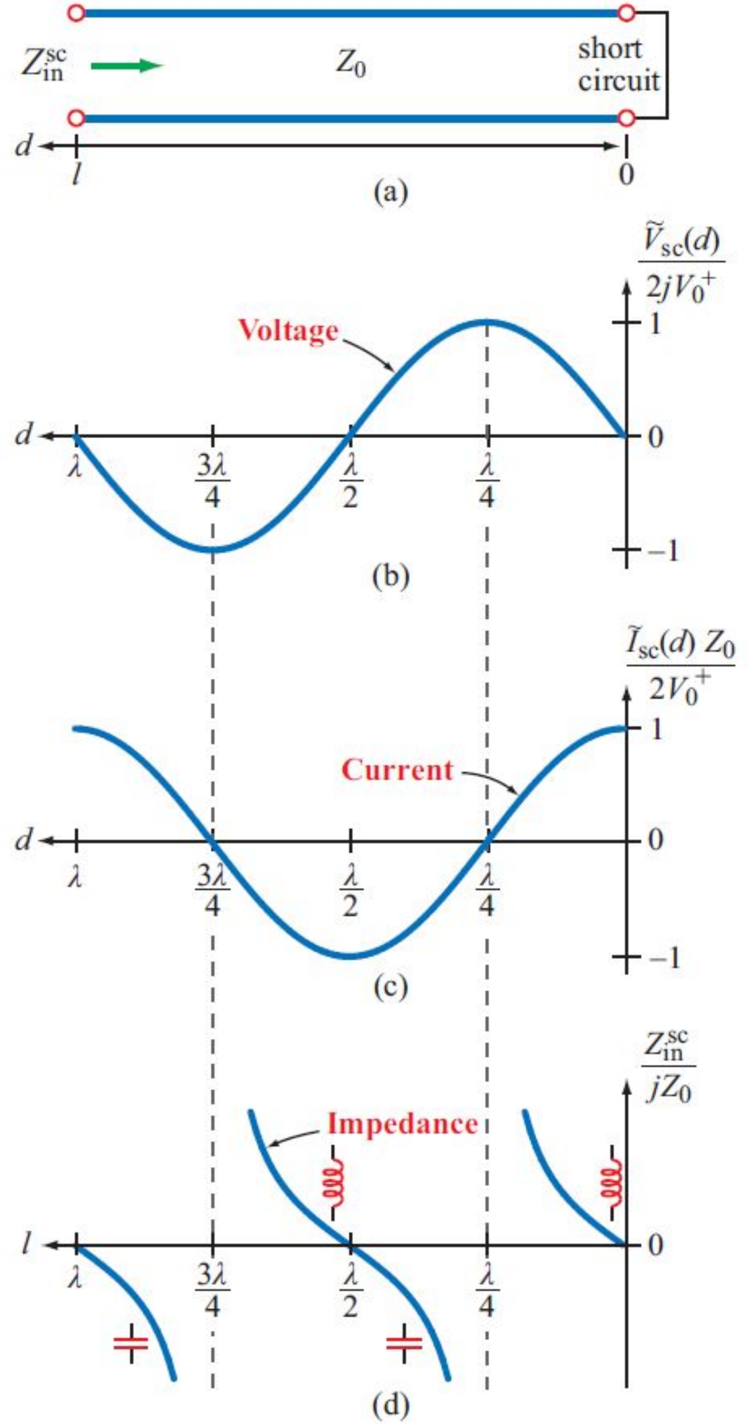
For the short-circuited line:

$$\Gamma = -1$$

$$\widetilde{V}_{sc}(d) = 2jV_0^+ \sin \beta d,$$

$$\widetilde{I}_{sc}(d) = \frac{2V_0^+}{Z_0} \cos \beta d,$$

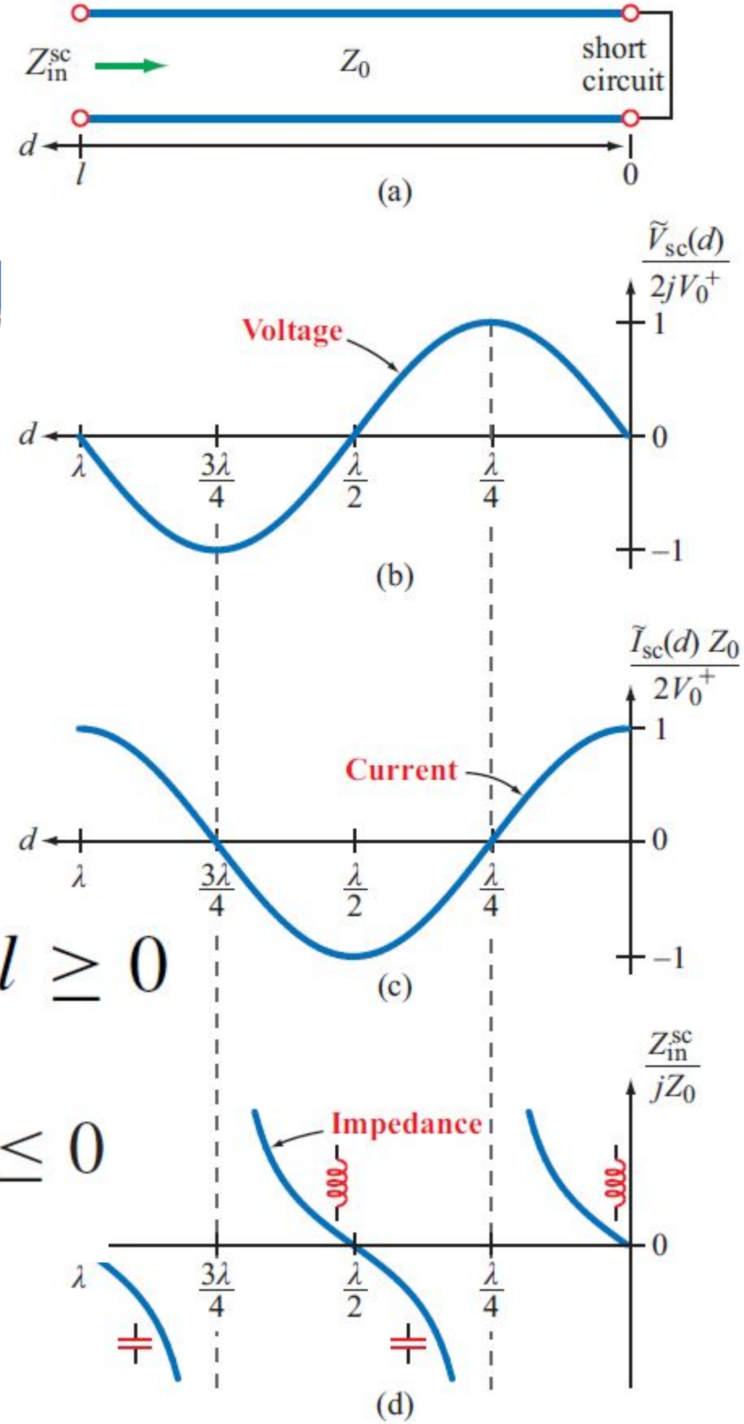
$$Z_{sc}(d) = \frac{\widetilde{V}_{sc}(d)}{\widetilde{I}_{sc}(d)} = jZ_0 \tan \beta d.$$



Chapter 2 Review

At its input, the short-circuited line **appears** like an inductor or a capacitor depending on the sign of

$$\tan \beta d$$



$$j\omega L_{eq} = jZ_0 \tan \beta l, \qquad \text{if } \tan \beta l \geq 0$$

$$\frac{1}{j\omega C_{eq}} = jZ_0 \tan \beta l, \qquad \text{if } \tan \beta l \leq 0$$

Chapter 2 Review

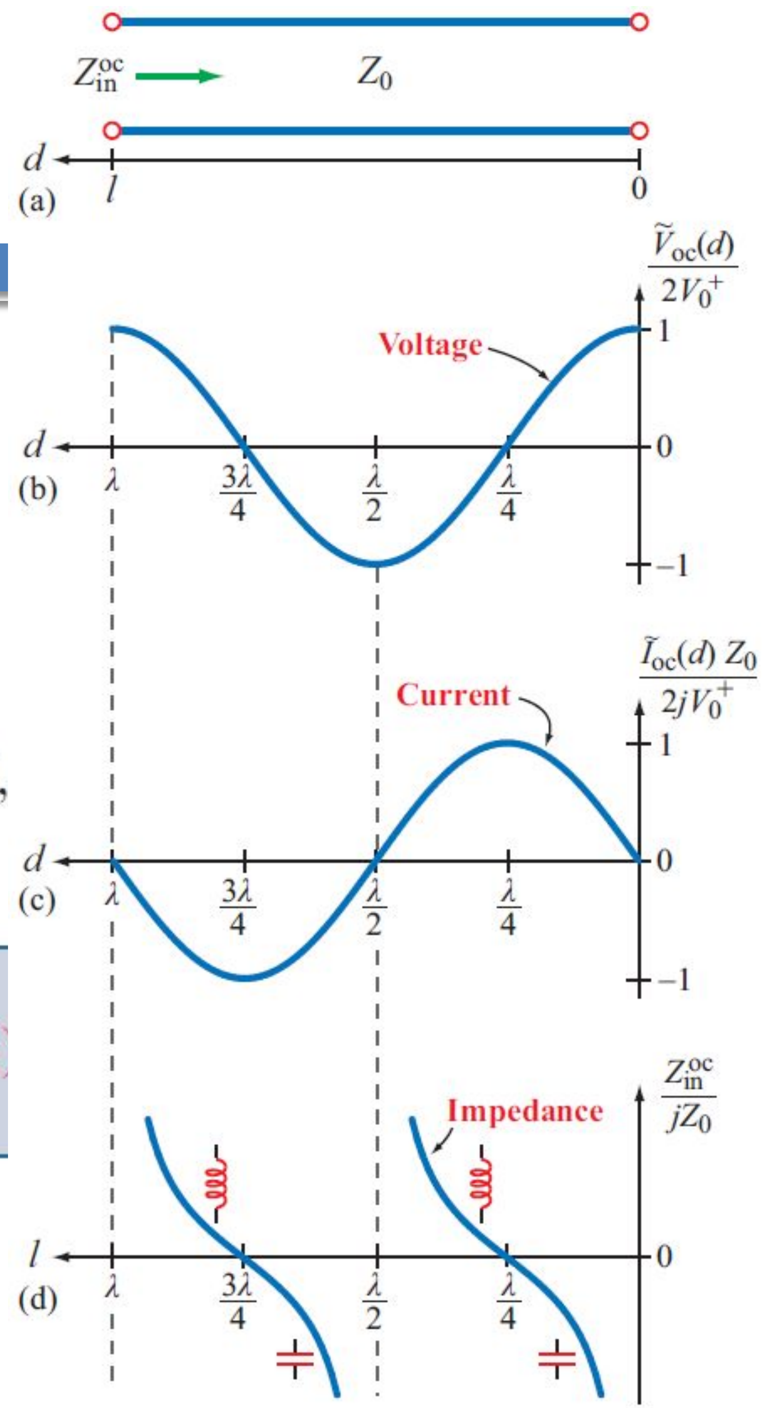
Open-circuited Line:

$$\Gamma = 1$$

$$\widetilde{V}_{oc}(d) = V_0^+[e^{j\beta d} + e^{-j\beta d}] = 2V_0^+ \cos \beta d,$$

$$\widetilde{I}_{oc}(d) = \frac{V_0^+}{Z_0}[e^{j\beta d} - e^{-j\beta d}] = \frac{2jV_0^+}{Z_0} \sin \beta d,$$

$$Z_{in}^{oc} = \frac{\widetilde{V}_{oc}(l)}{\widetilde{I}_{oc}(l)} = -jZ_0 \cot \beta l. \qquad (2.93)$$



Chapter 2 Review

Short-Circuit/Open-Circuit Method:

- Given length l
- Measure Z_{in} twice:
  - when terminated in a short
  - when terminated in an open

Use both: get Z_0, β:

$$Z_{\text{in}}^{\text{sc}} = \frac{\widetilde{V}_{\text{sc}}(l)}{\widetilde{I}_{\text{sc}}(l)} = jZ_0 \tan \beta l.$$

$$Z_{\text{in}}^{\text{oc}} = \frac{\widetilde{V}_{\text{oc}}(l)}{\widetilde{I}_{\text{oc}}(l)} = -jZ_0 \cot \beta l.$$

$$Z_0 = +\sqrt{Z_{\text{in}}^{\text{sc}} \, Z_{\text{in}}^{\text{oc}}} \, ,$$

$$\tan \beta l = \sqrt{\frac{-Z_{\text{in}}^{\text{sc}}}{Z_{\text{in}}^{\text{oc}}}} \, .$$

Chapter 2 Review

Half-Wavelength Line:

$$Z_{\text{in}} = Z_0 \left(\frac{z_L + j\tan(\beta l)}{1 + jz_L \tan(\beta l)} \right)$$

If $l = n\lambda/2$, where n is an integer,

$$\tan \beta l = \tan\left[(2\pi/\lambda)\,(n\lambda/2)\right] = \tan n\pi = 0.$$

Consequently, Eq. (2.79) reduces to

$$Z_{\text{in}} = Z_{\text{L}}, \qquad \text{for } l = n\lambda/2, \qquad (2.96)$$

which means that a half-wavelength line (or any integer multiple of $\lambda/2$) does not modify the load impedance.

Chapter 2 Review

Quarter-Wavelength Line:

$$Z_{\text{in}} = Z_0 \left(\frac{z_L + j\tan(\beta l)}{1 + jz_L\tan(\beta l)} \right)$$

For $l = \lambda/4$, $\beta l = (2\pi/\lambda)(\lambda/4) = \pi/2$

So, as $\beta l \to \pi/2$, $\tan(\beta l) \to \infty$

$$\lim_{\beta l \to \pi/2} Z_{\text{in}} = Z_0 \left(\frac{j\tan(\beta l)}{jz_L\tan(\beta l)} \right) = \frac{Z_0^2}{Z_L}$$

$$Z_{\text{in}} = \frac{Z_0^2}{Z_L}, \qquad \text{for } l = \lambda/4 + n\lambda/2.$$

Chapter 2 Review

Instantaneous Power Flow:

$$P(d,t) = P^{\mathrm{i}}(d,t) + P^{\mathrm{r}}(d,t)$$

The 2 terms are the Incident and Reflected power:

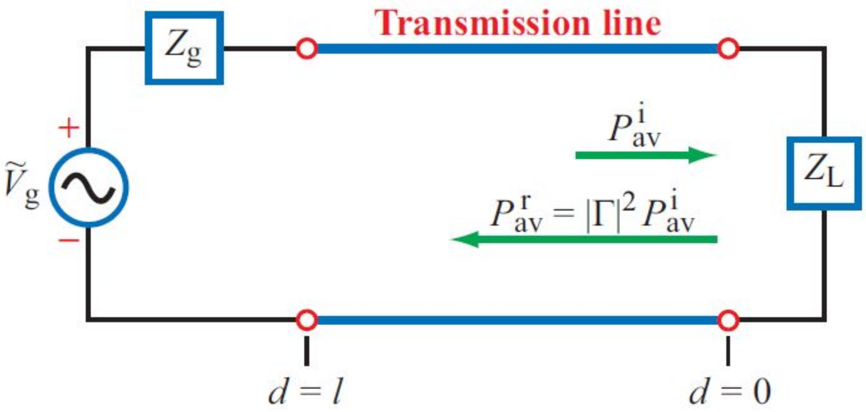
$$P^{\mathrm{i}}(d,t) = \frac{|V_0^+|^2}{2Z_0}[1 + \cos(2\omega t + 2\beta d + 2\phi^+)],$$

$$P^{\mathrm{r}}(d,t) = -|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0}[1 + \cos(2\omega t - 2\beta d + 2\phi^+ + 2\theta_{\mathrm{r}})].$$

oscillating at TWICE the frequency of V or I

Chapter 2 Review

Average Power Flow:



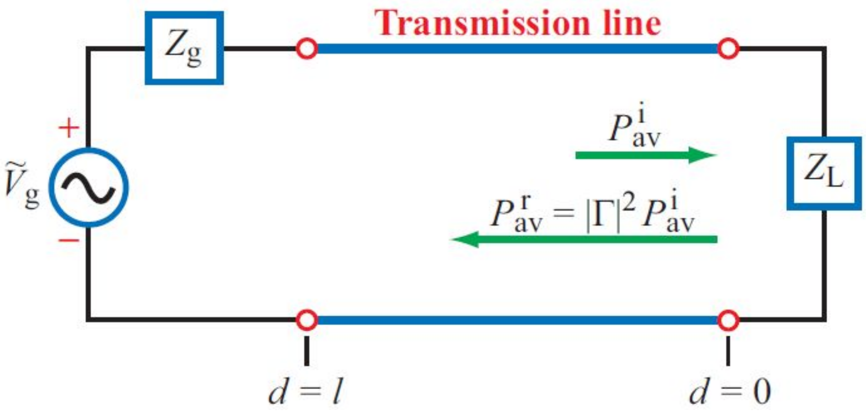
For the incident power, average over one period:

$$P^{i}_{\text{avg}} = \frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} \frac{|V_0^+|^2}{2Z_0} \left[1 + \cos(2\omega t + 2\beta d + 2\phi^+)\right] \mathrm{d}t$$

$$\omega = \frac{2\pi}{T}, \qquad hence \quad T = \frac{2\pi}{\omega}$$

Chapter 2 Review

Average Power Flow:



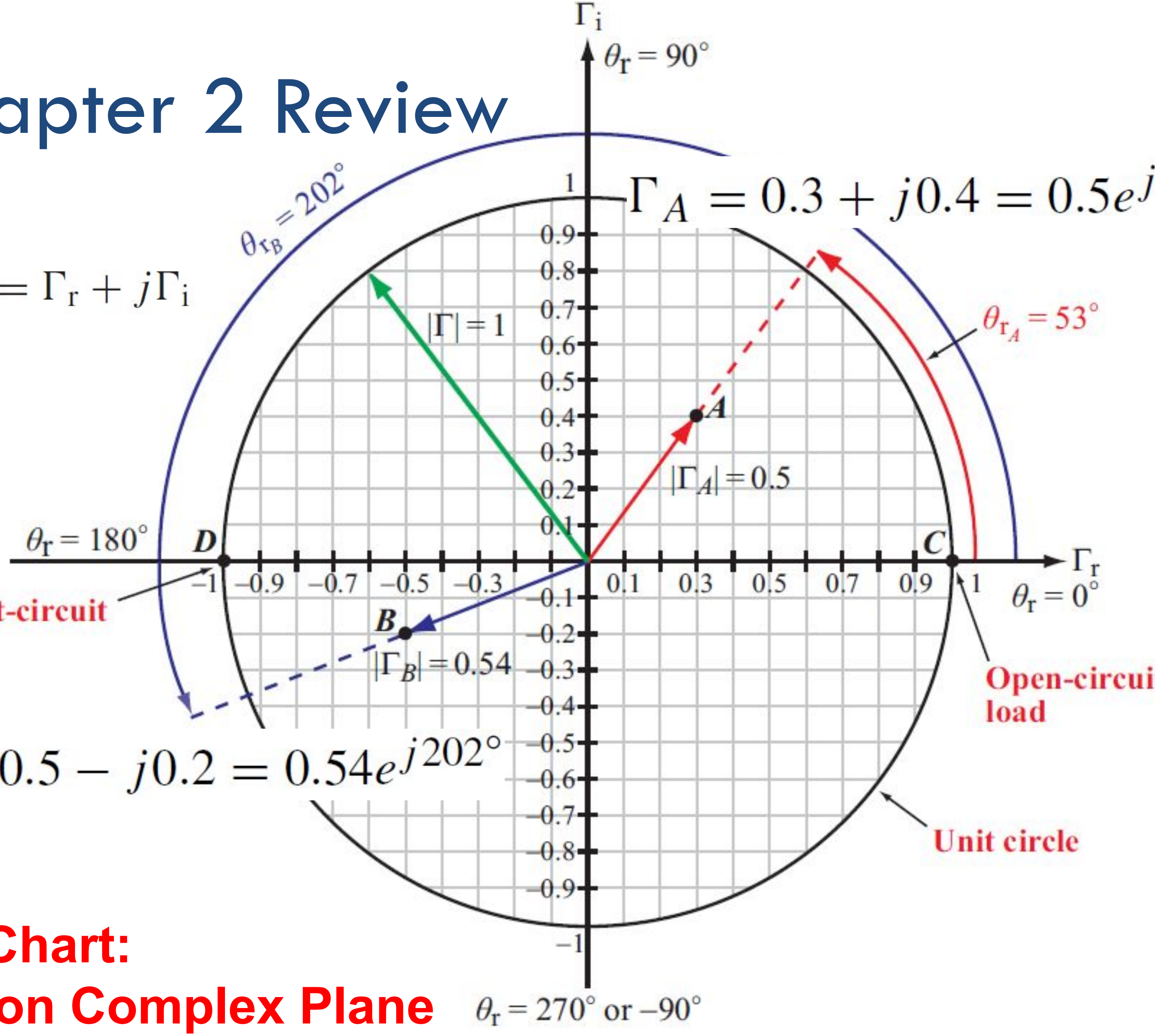
$$P^{i}_{\text{avg}} = \frac{|V_0^+|^2}{2Z_0}$$

$$P^{r}_{\text{avg}} = -|\Gamma|^2 P^{i}_{\text{avg}}$$

Chapter 2 Review

$$\Gamma = |\Gamma| e^{j\theta_r} = \Gamma_r + j\Gamma_i$$

$$\Gamma_A = 0.3 + j0.4 = 0.5e^{j53}$$

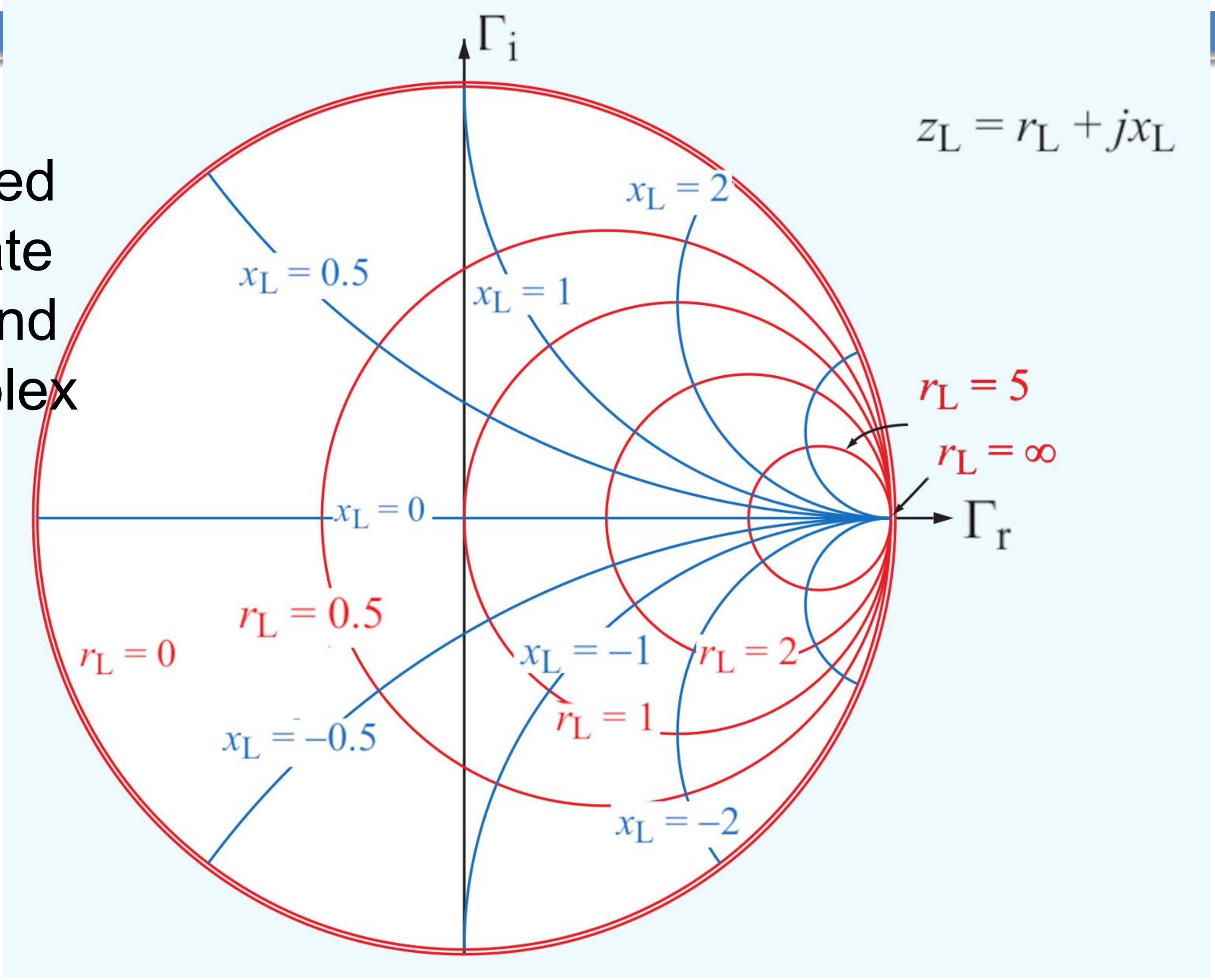


$$\Gamma_B = -0.5 - j0.2 = 0.54e^{j202°}$$

Smith Chart:
Based on Complex Plane

Chapter 2 Review

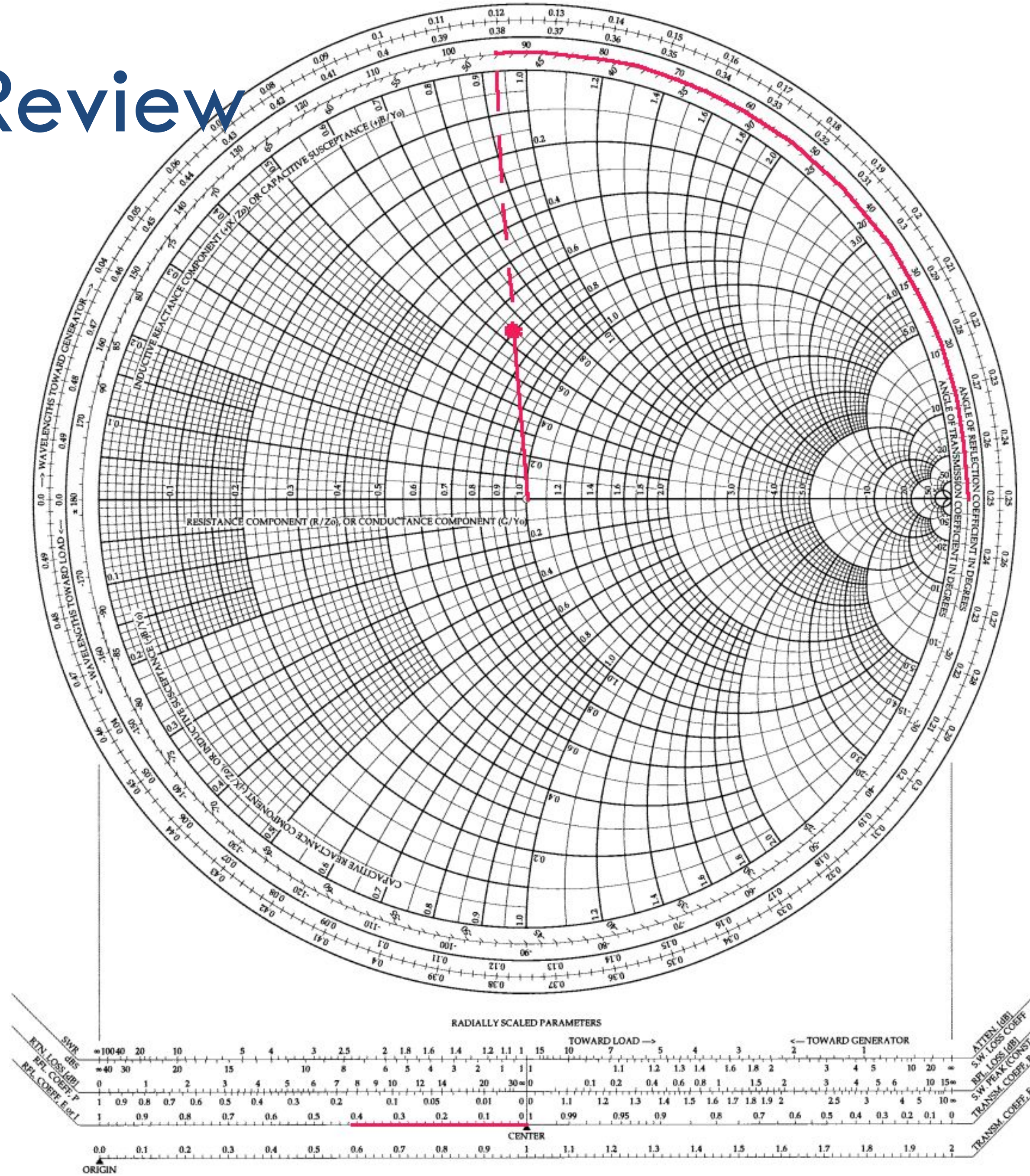
Superimposed are coordinate axes for r_L and x_L: the complex load: z_L



Chapter 2 Review

If plotted z_L:
Obtain values for Γ_L from the angle scale and magnitude scale:

$0.41, 94^{\circ}$

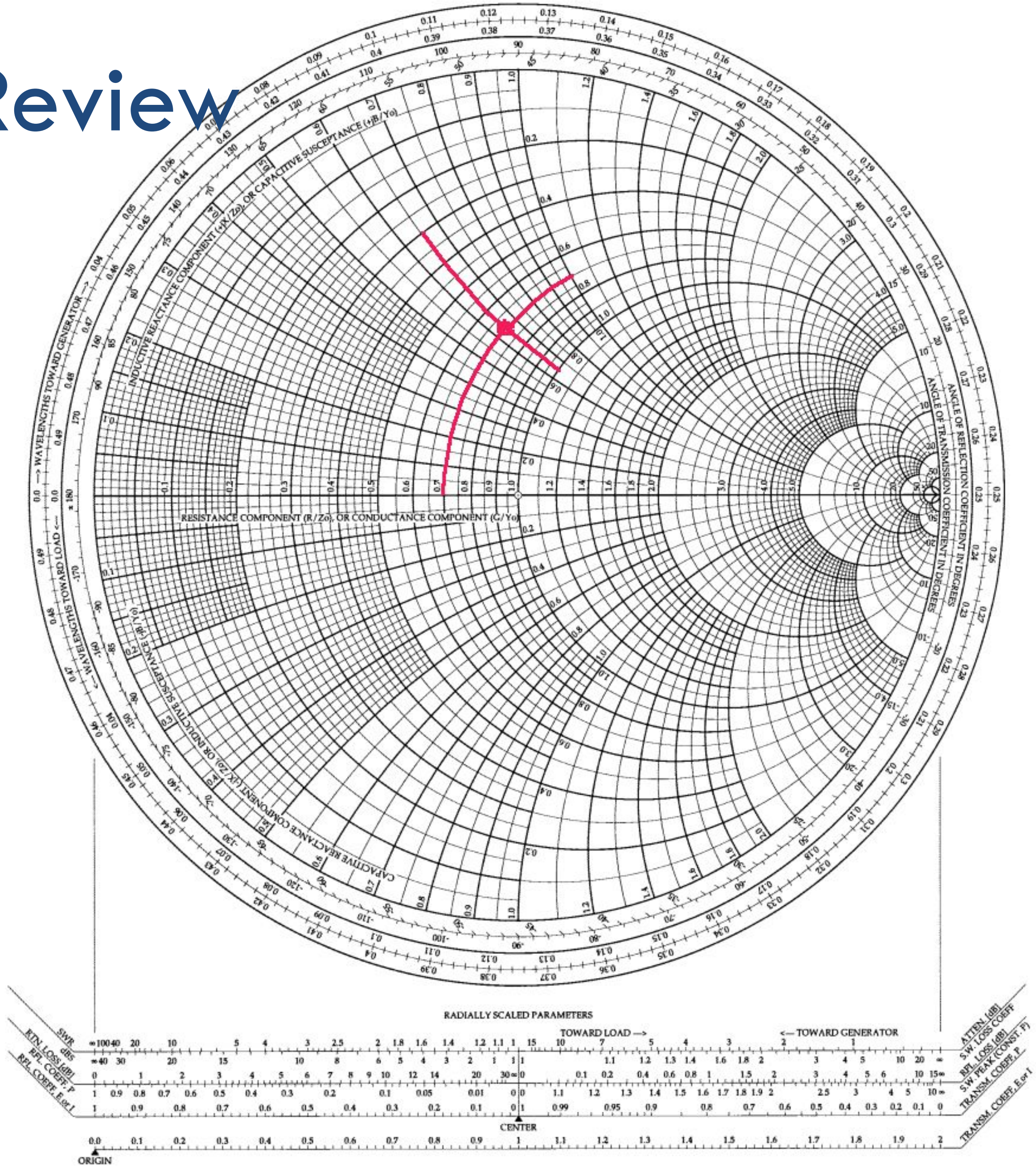


Chapter 2 Review

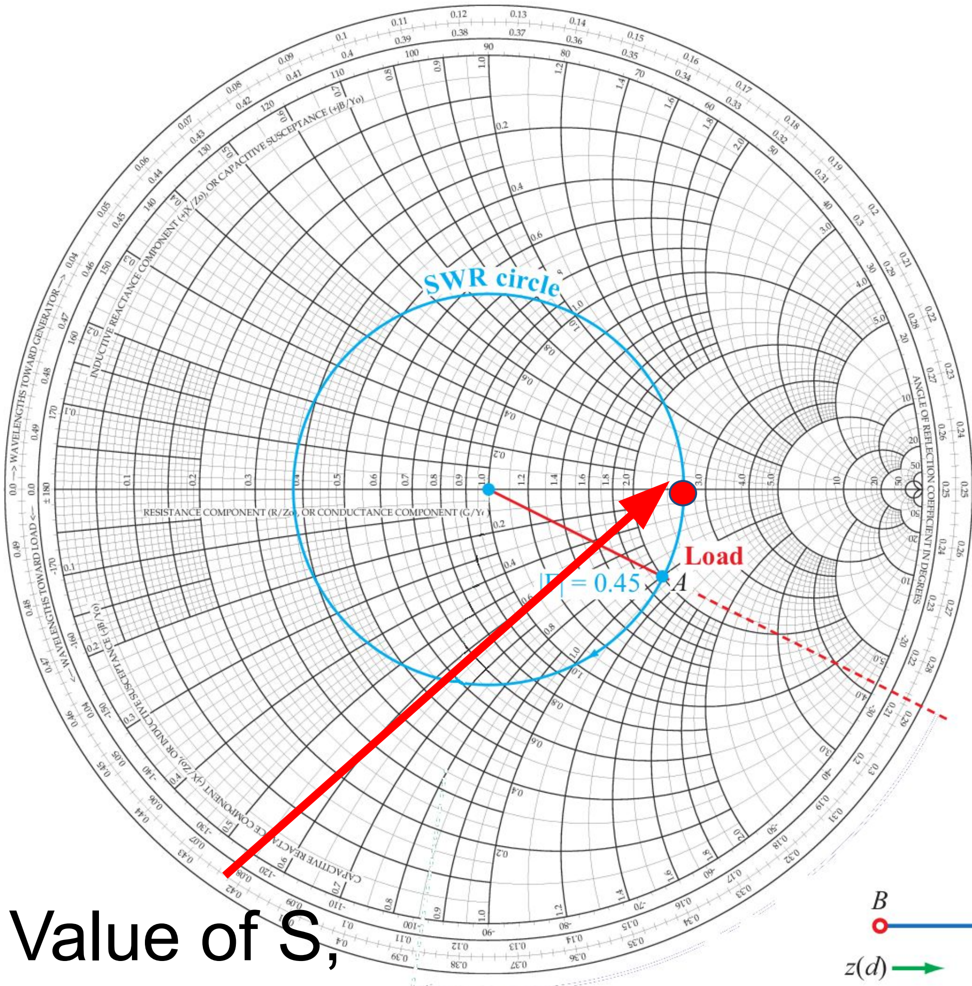
If plotted Γ_L:
Obtain values for r_L, x_L from the nearest circles:

$z_L = (0.7, 0.65)$

To get Z_L:
Multiply z_L by Z_0



Chapter 2 Review



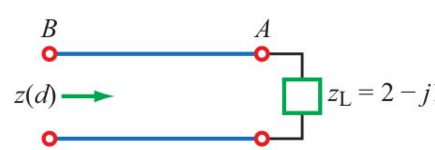
All points on a circle centered at the origin have the same $|\Gamma|$.

Since

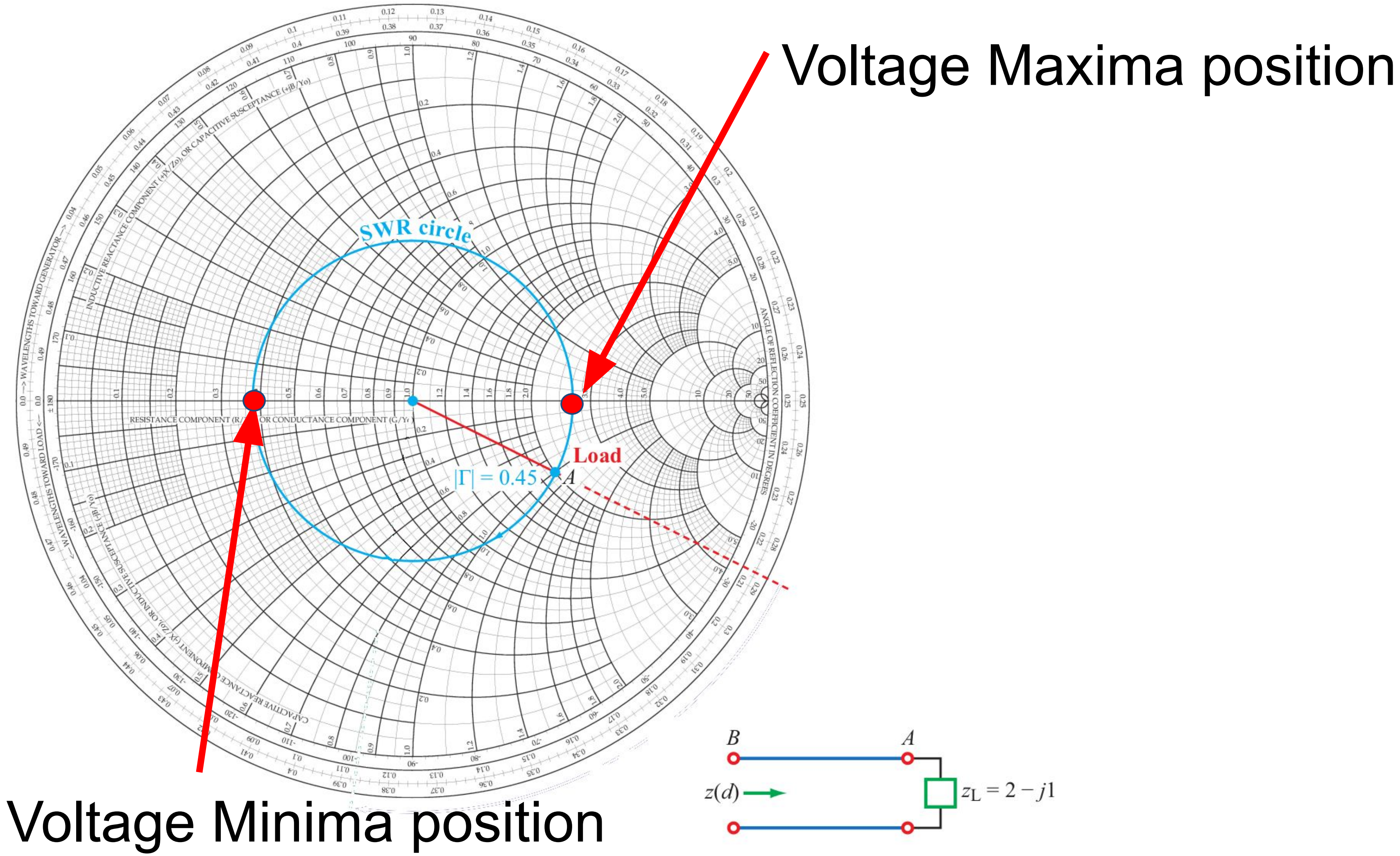
$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

this is a also a circle of constant S (VSWR).

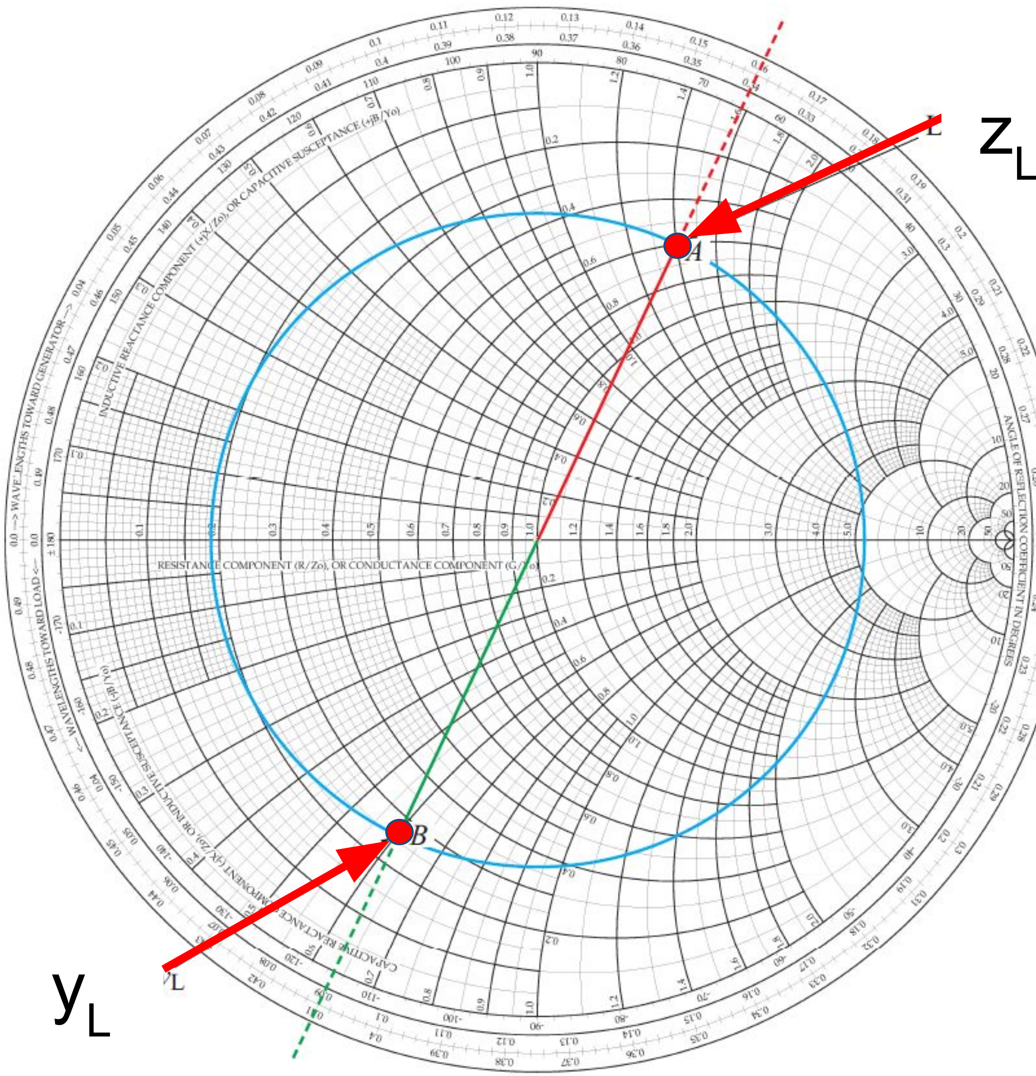
Value of S, from r_L scale



Chapter 2 Review



Chapter 2 Review

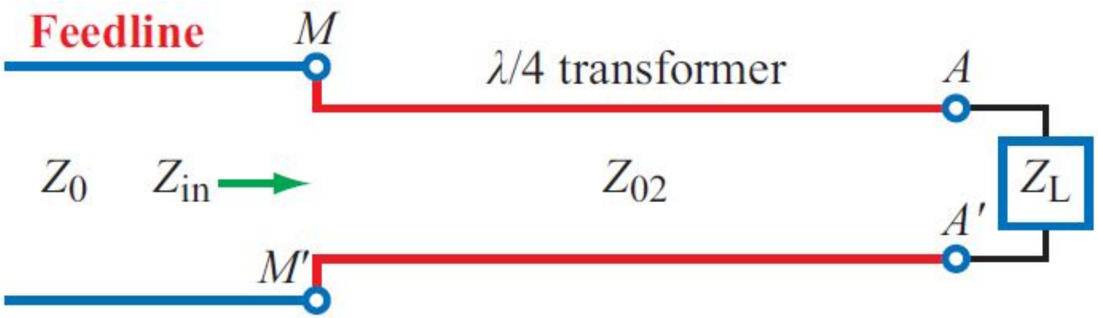


y and z are inverses of each other.

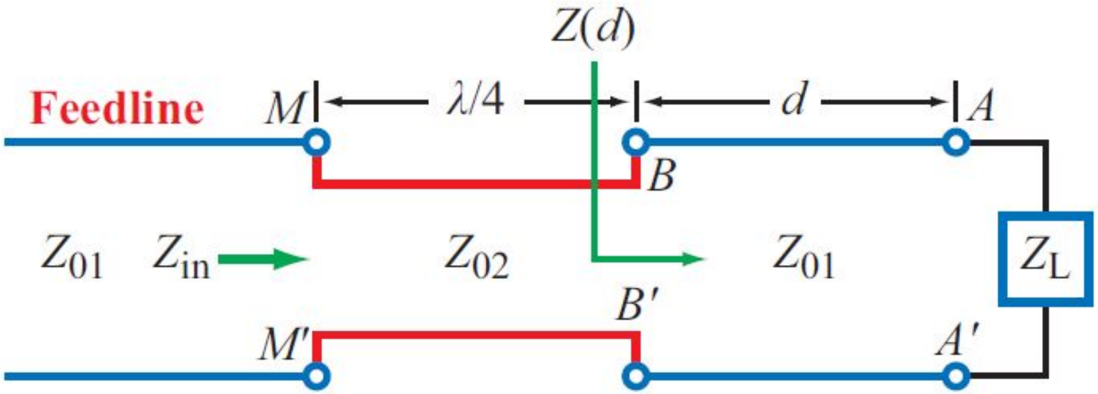
Occur $\lambda/4$ apart on the Smith Chart.

Chapter 2 Review

Example Matching Networks



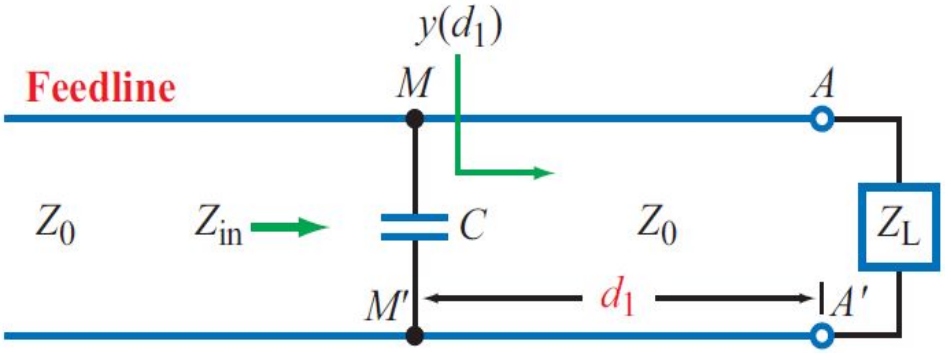
(a) In-series $\lambda/4$ transformer inserted at AA'



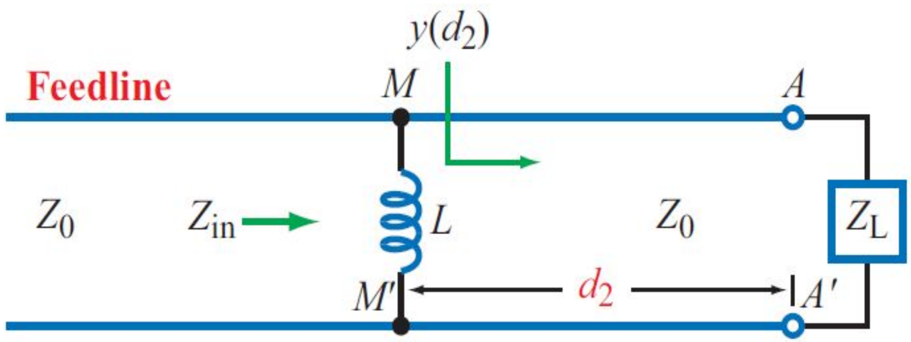
(b) In-series $\lambda/4$ transformer inserted at $d = d_{max}$ or $d = d_{min}$

Chapter 2 Review

Example Matching Networks



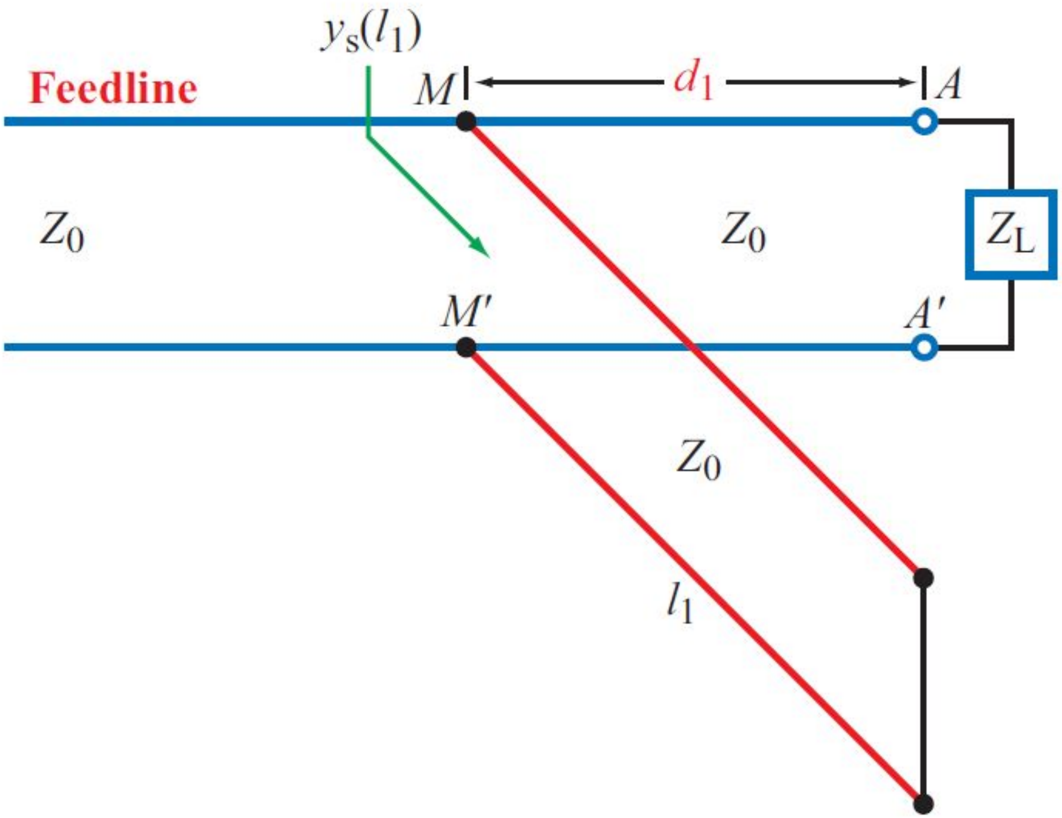
(c) In-parallel insertion of capacitor at distance d_1



(d) In-parallel insertion of inductor at distance d_2

Chapter 2 Review

Example Matching Networks



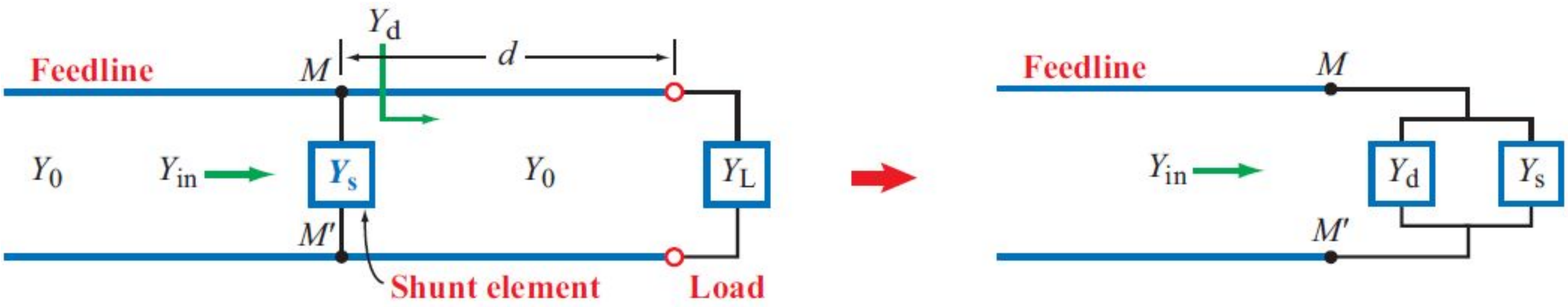
(e) In-parallel insertion of a short-circuited stub

In General:

Matching Networks may consist of:
* lumped elements, or:
* TL sections,

Placed in
* series or
* parallel.

Chapter 2 Review



Lumped-Element matching using admittance

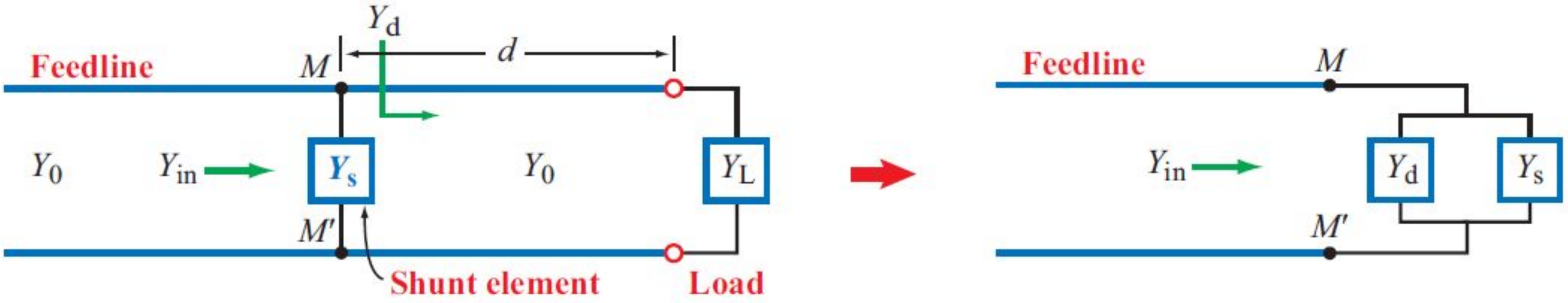
Y_L transformed to Y_d by the length of transmission line

$$Y_{in} = Y_d + Y_s$$

$$Y_{in} = (G_d + jB_d) + jB_s$$
$$= G_d + j(B_d + B_s).$$

Y_S only needs to be reactive, to cancel the reactive part of Y_d

Chapter 2 Review



Lumped-Element matching using admittance

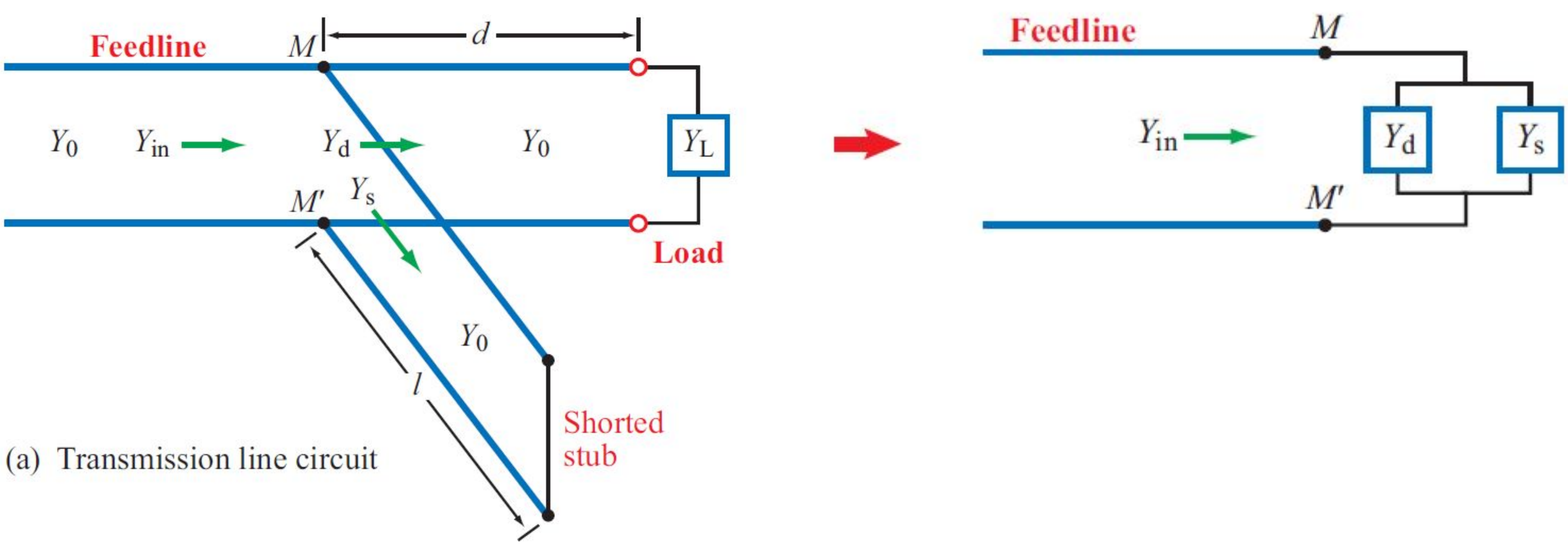
Normalized:

$$y_{in} = g_d + j(b_d + b_s).$$

$$g_d = 1 \quad \text{(real-part condition)},$$

$$b_s = -b_d \quad \text{(imaginary-part condition)}.$$

Chapter 2 Review

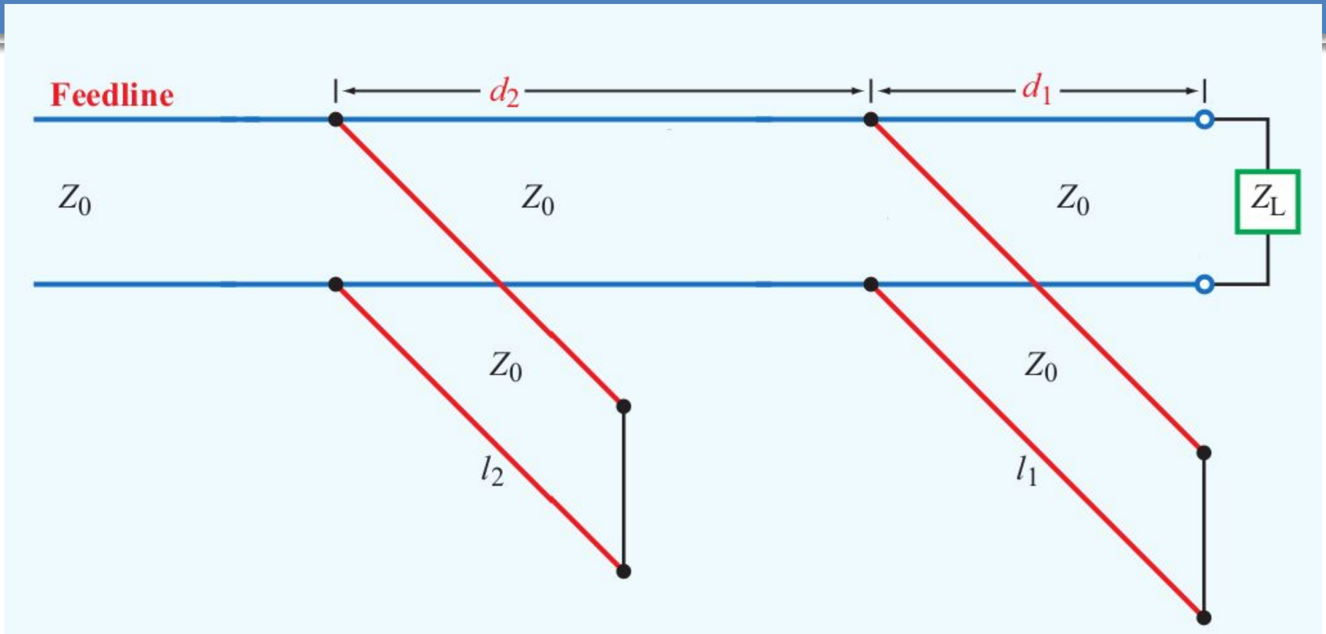


Single-stub Matching:

Can use the same process as before.

Except: transform y_s to a length of shorted transmission line instead of a lumped element.

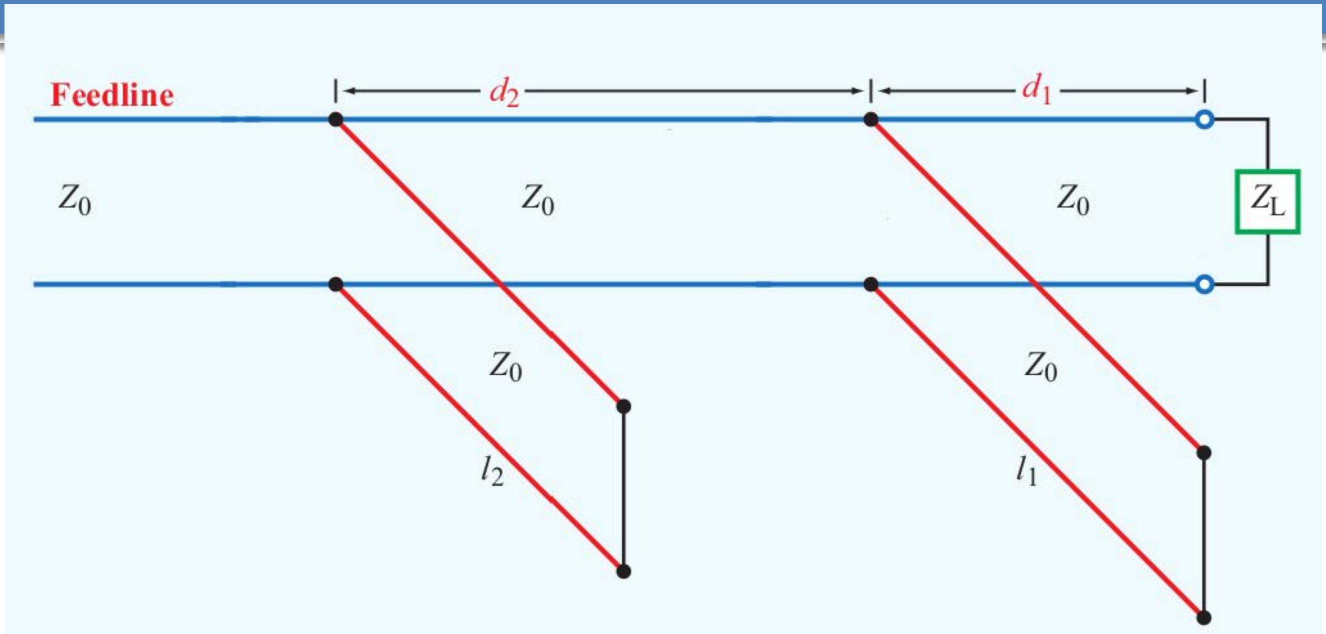
2-Stub Matching



We are given 2 places along a transmission line where we add shorted stubs: d_1, d_2

What lengths, l_1, l_2, should we use for each stub to match the load?

2-Stub Matching

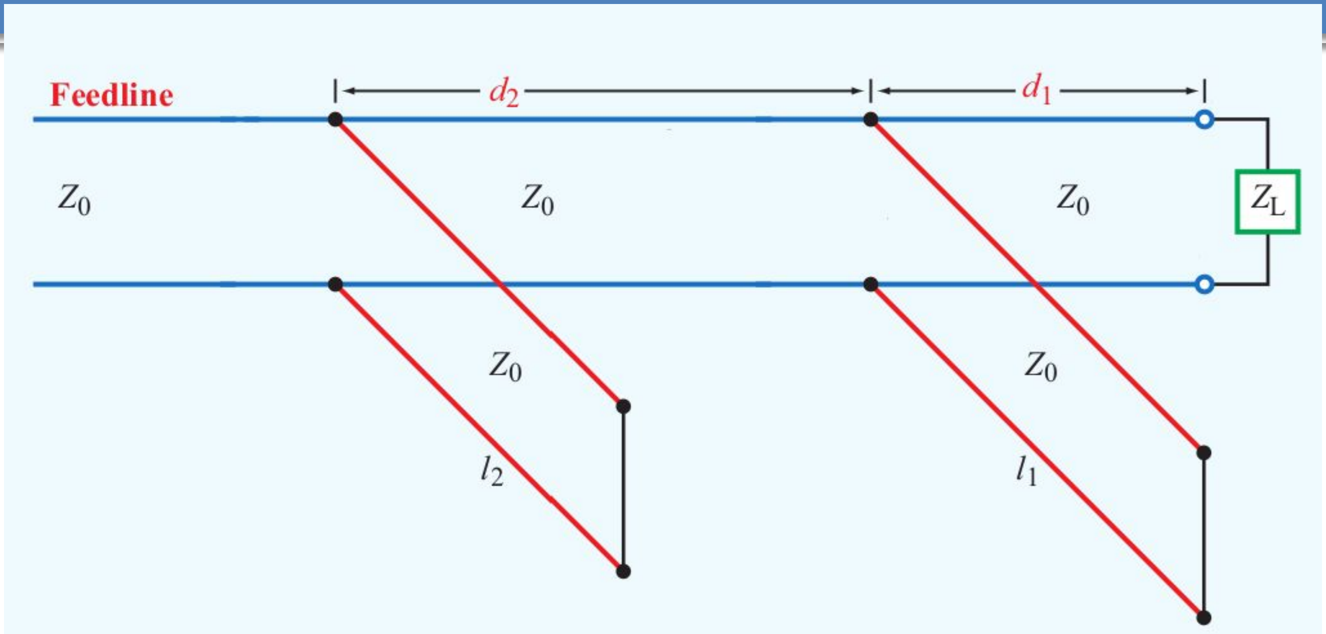


Given: $Z_0 = 50\ \Omega$, $Z_L = 100 + j\ 100\ \Omega$

$d_1 = 0.6\ \lambda$ $d_2 = 0.1\ \lambda$

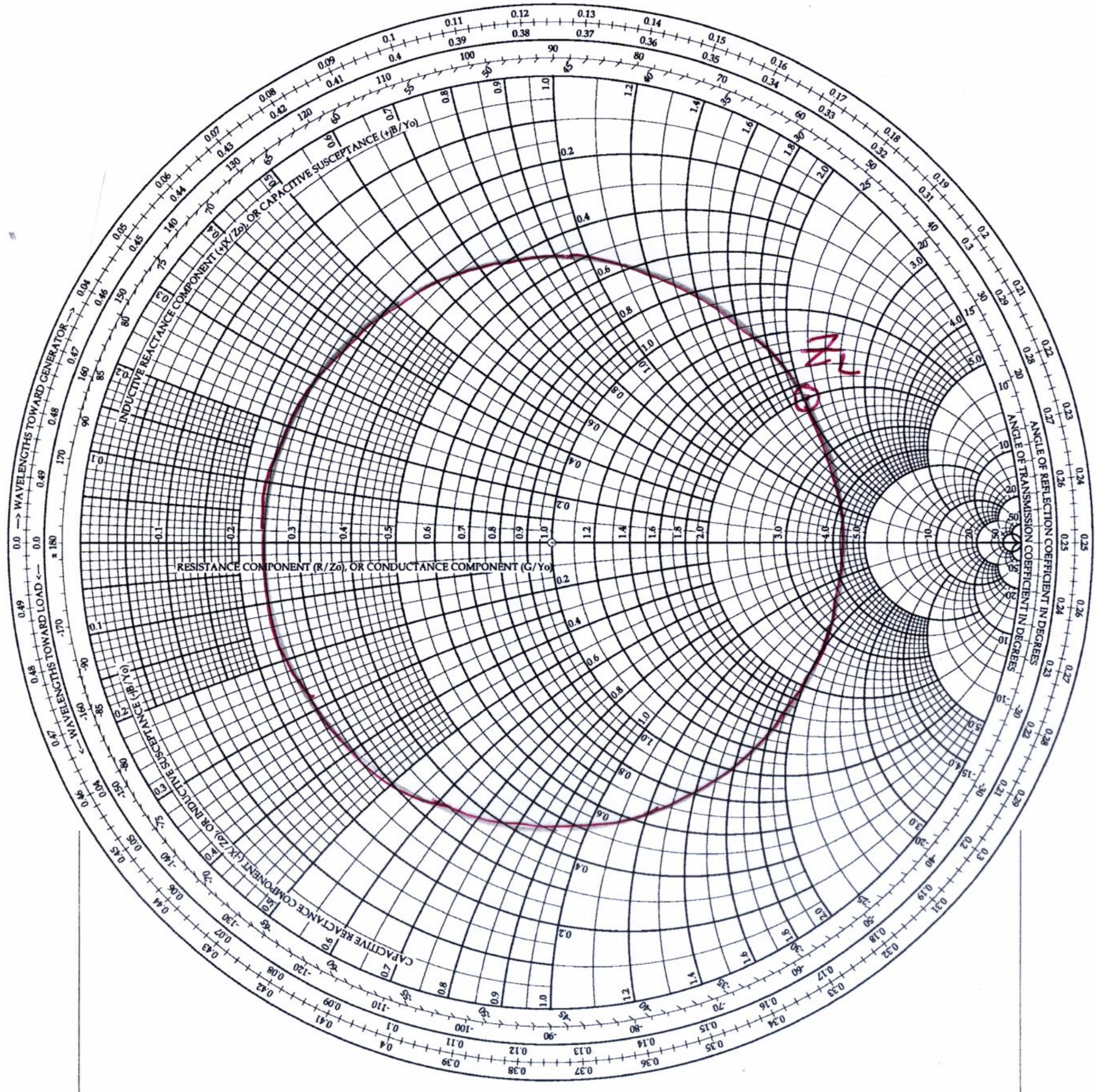
Find: l_1, l_2, to match the load.

2-Stub Matching



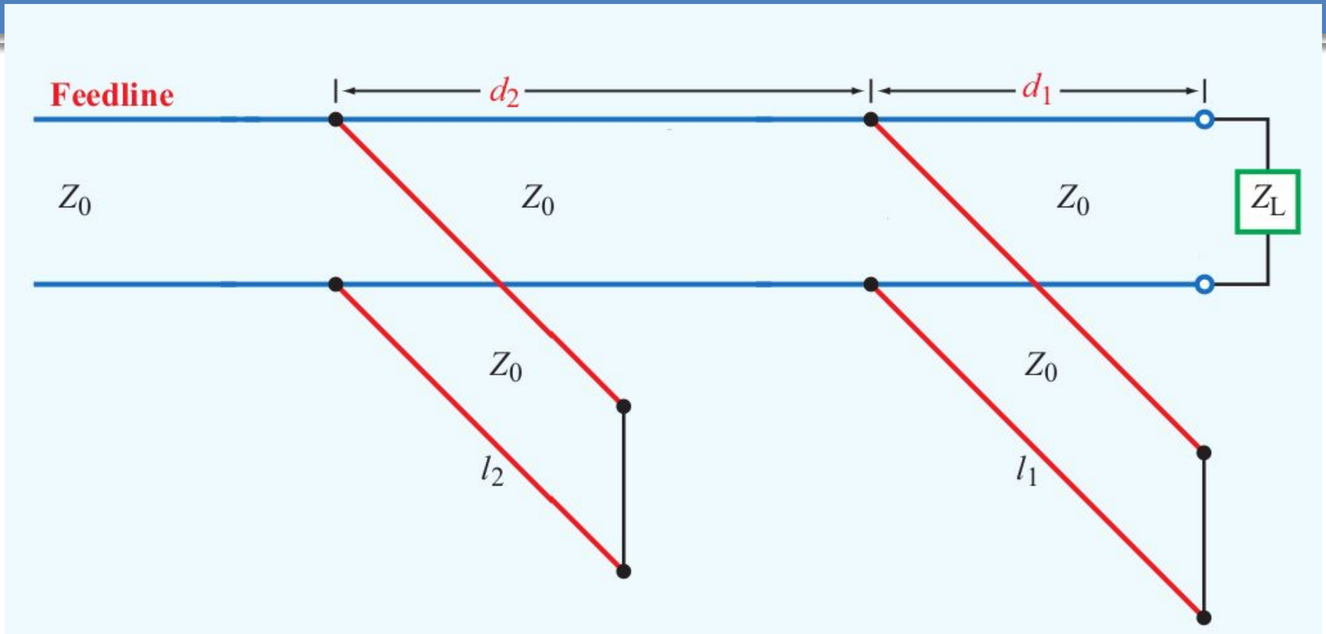
Step 1: plot $z_L = Z_L / Z_0 = 2 + j2$

Step 2: Draw VSWR circle through z_L

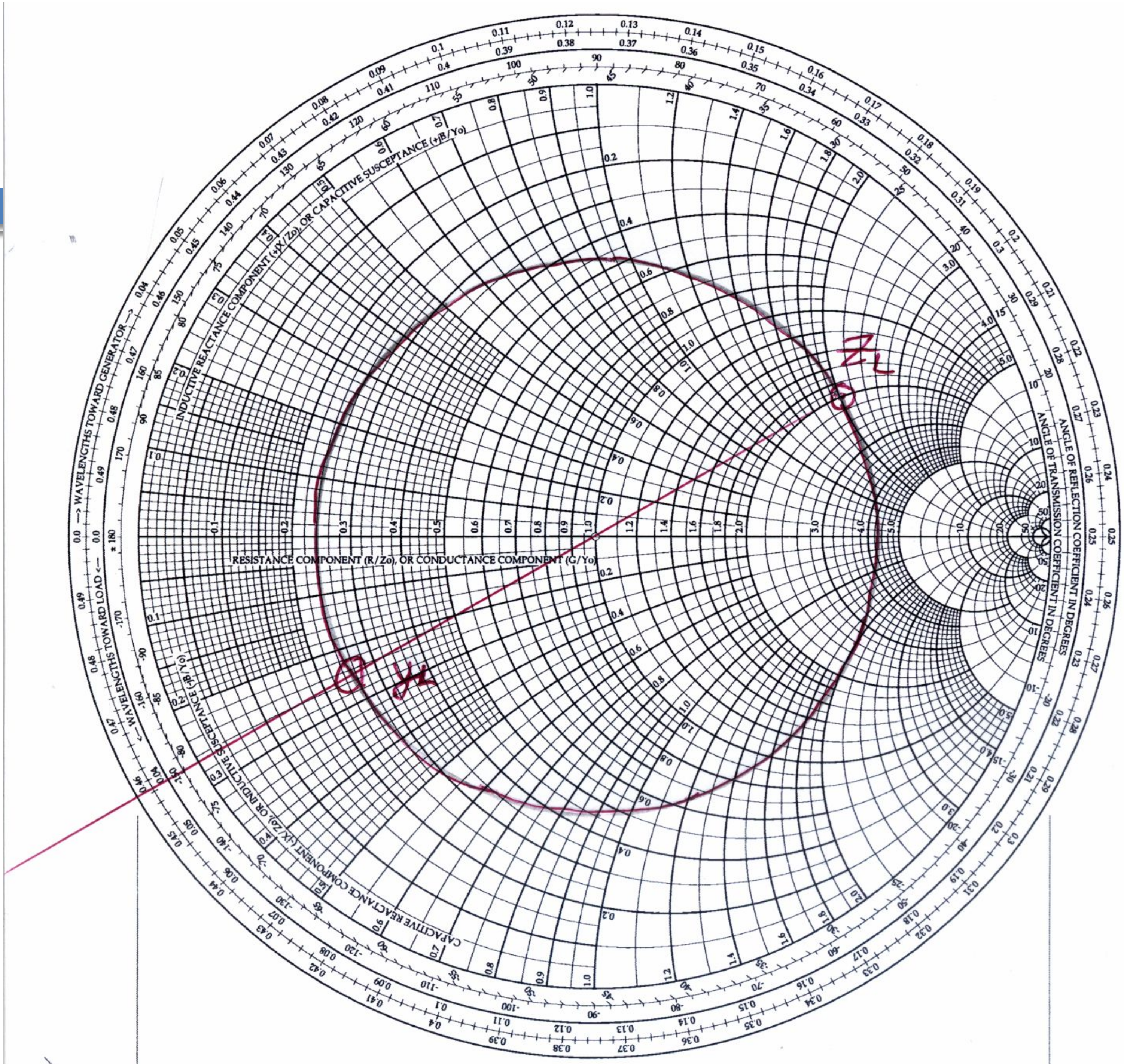


RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
—> WAVELENGTHS TOWARD GENERATOR —>
<— WAVELENGTHS TOWARD LOAD <—
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
Z_L

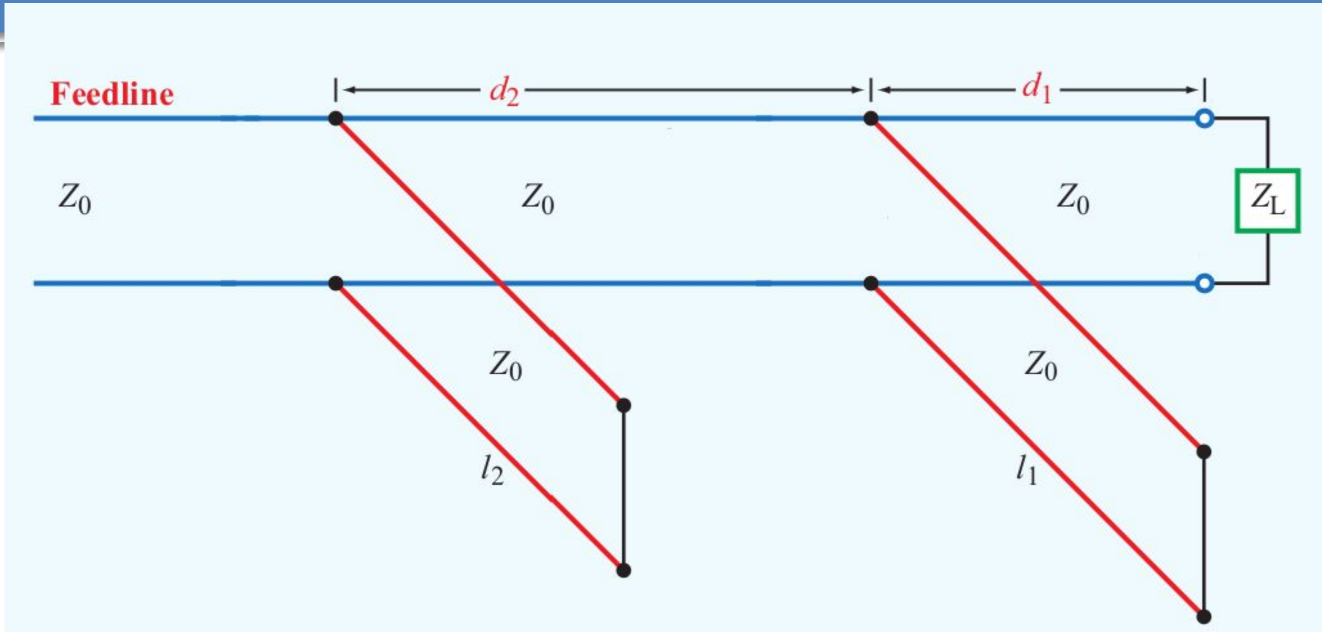
2-Stub Matching



Step 3: Find $y_L = 0.25 - j\,0.25$


RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo) OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
—> WAVELENGTHS TOWARD GENERATOR —>
<— WAVELENGTHS TOWARD LOAD <—
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
ZL
YL

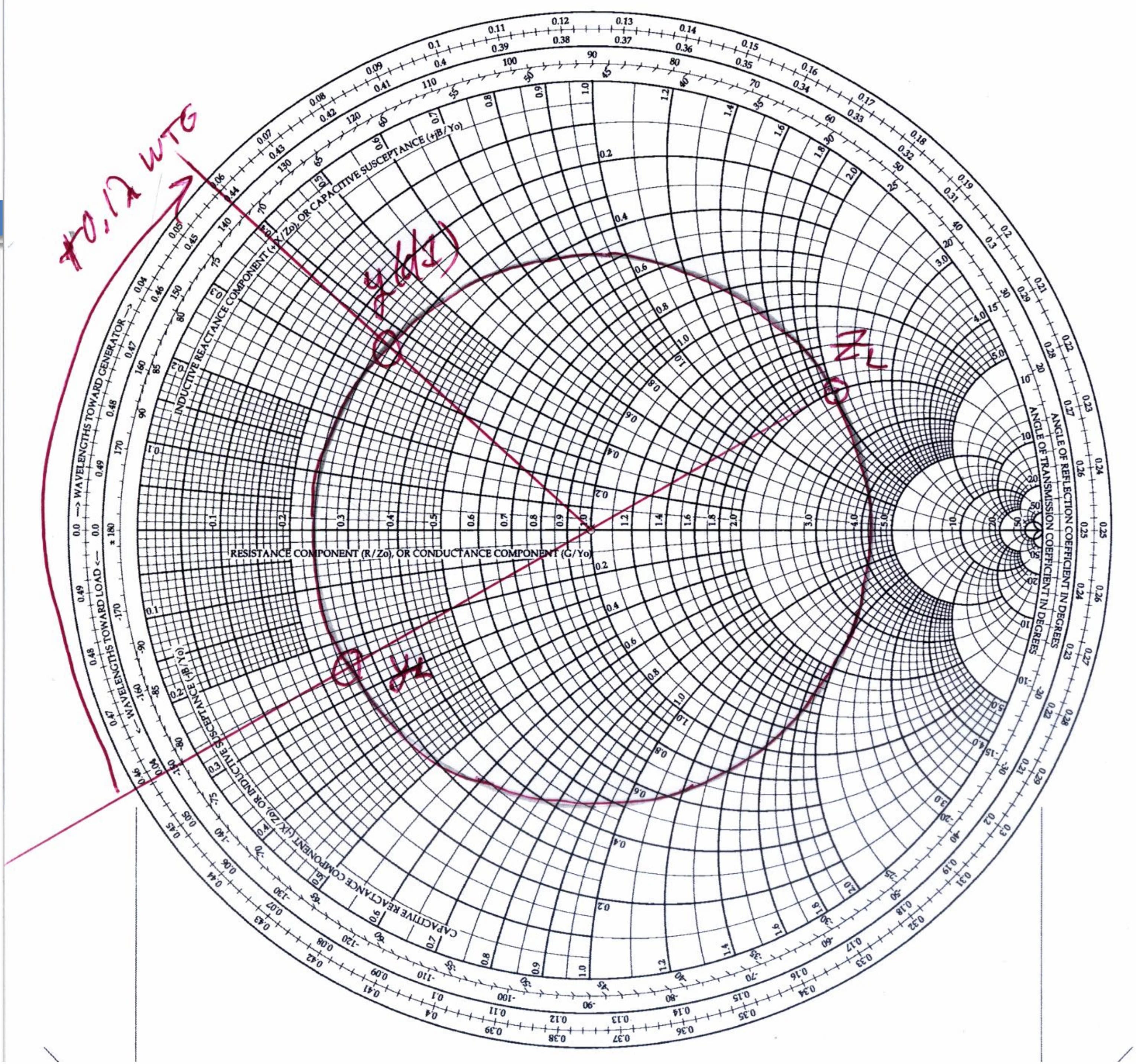
2-Stub Matching



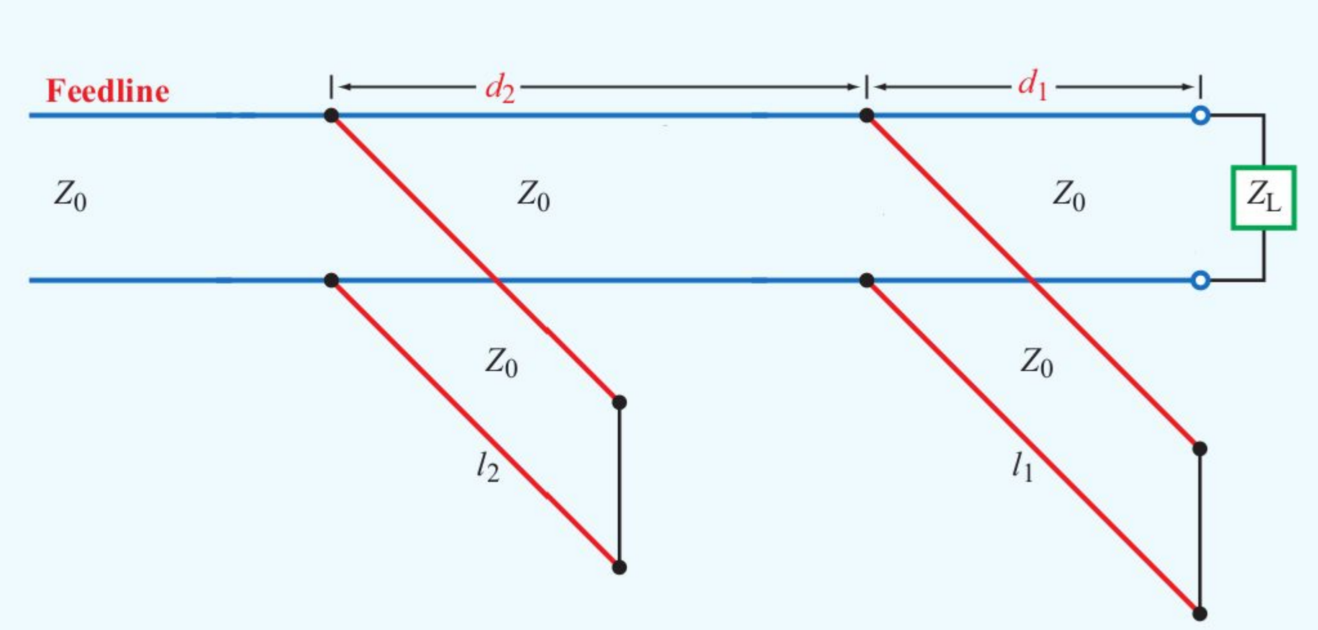
Step 4: Find y(d1) by rotating toward generator:

to rotate 0.6λ, can rotate 0.1λ from 0.4585λ to: +0.0585λ

get: y(d1) = 0.28 + j 0.36


+0.12λ WTG
y(d1)
ZL
yL
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
—> WAVELENGTHS TOWARD GENERATOR —>
<— WAVELENGTHS TOWARD LOAD <—
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES

2-Stub Matching



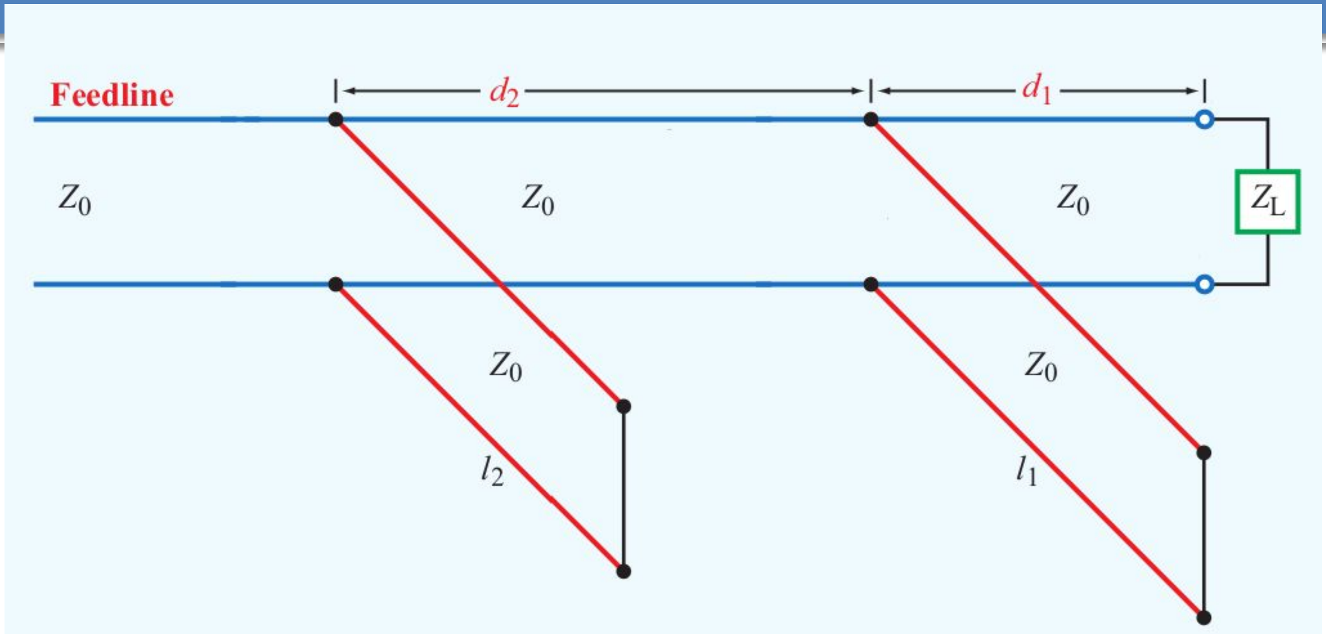
Step 5: Add the shorted stub.

Results in a new y(d1+stub)

with a different imaginary part

What imaginary part do we want?

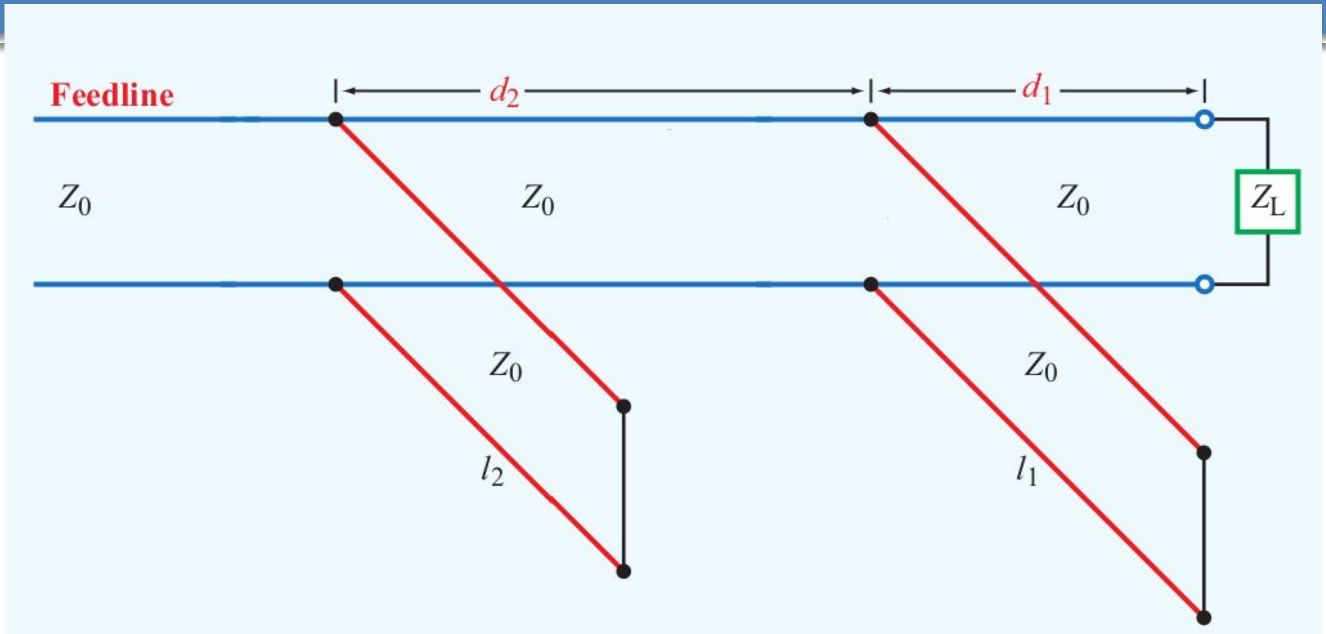
2-Stub Matching



Step 5: need y(d1+stub) such that y(d2) will have a real part of 1.

So when y(d1+stub) gets rotated on its VSWR circle it ends up on the y=1 circle.

2-Stub Matching



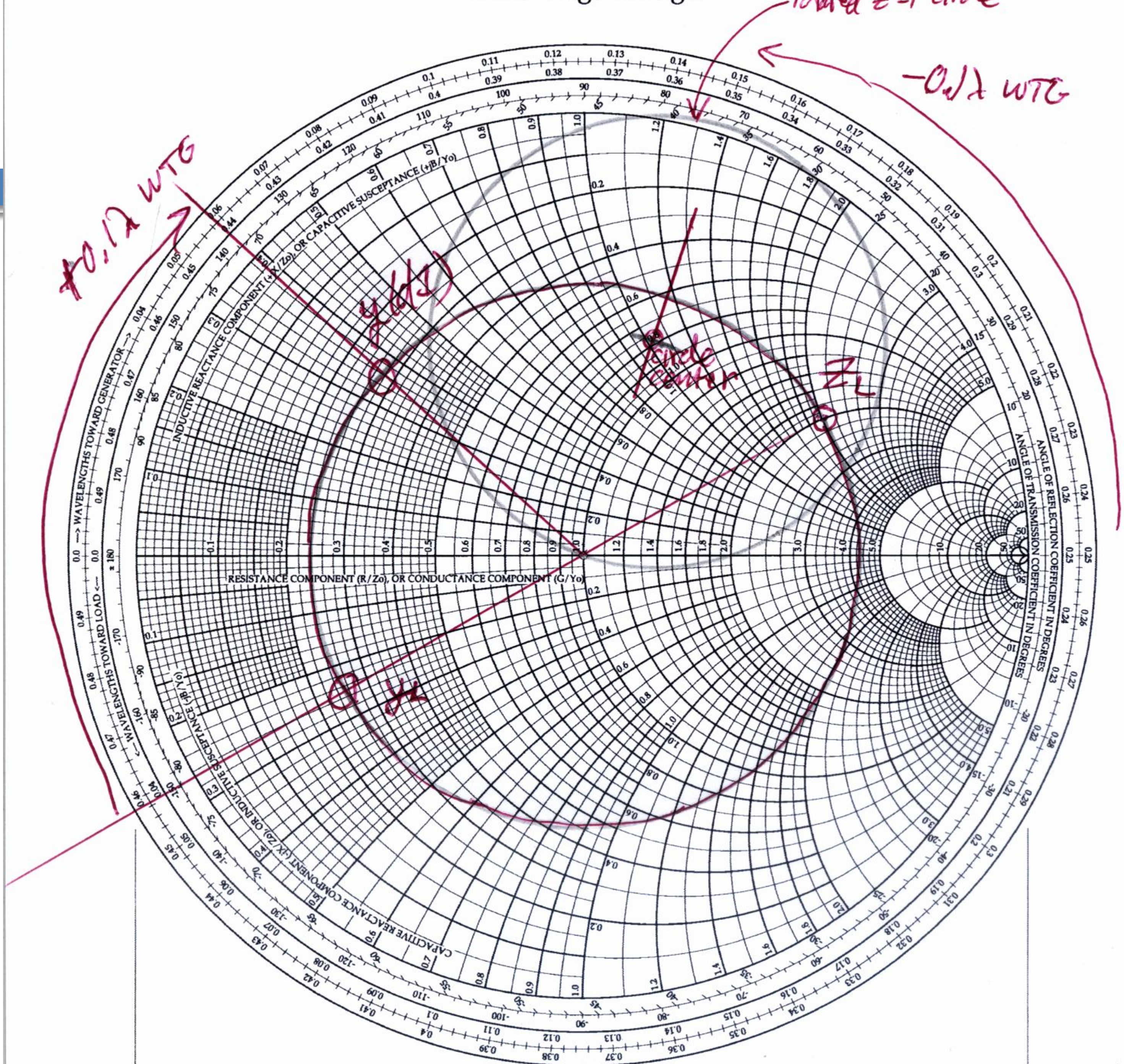
Step 5a: need to rotate the y=1 circle towards the load by d2=0.1λ

from WTG=0.25 to 0.15λ

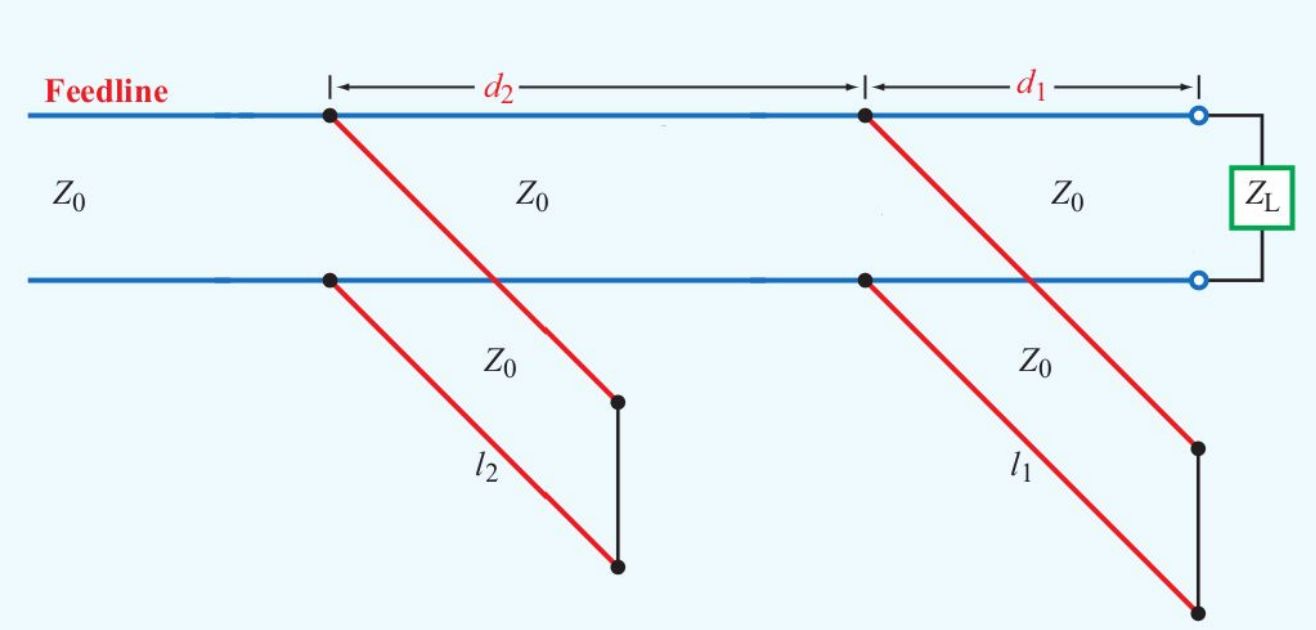
draw circle centered at |Γ|=0.5

on this radial line

Black Magic Design



2-Stub Matching

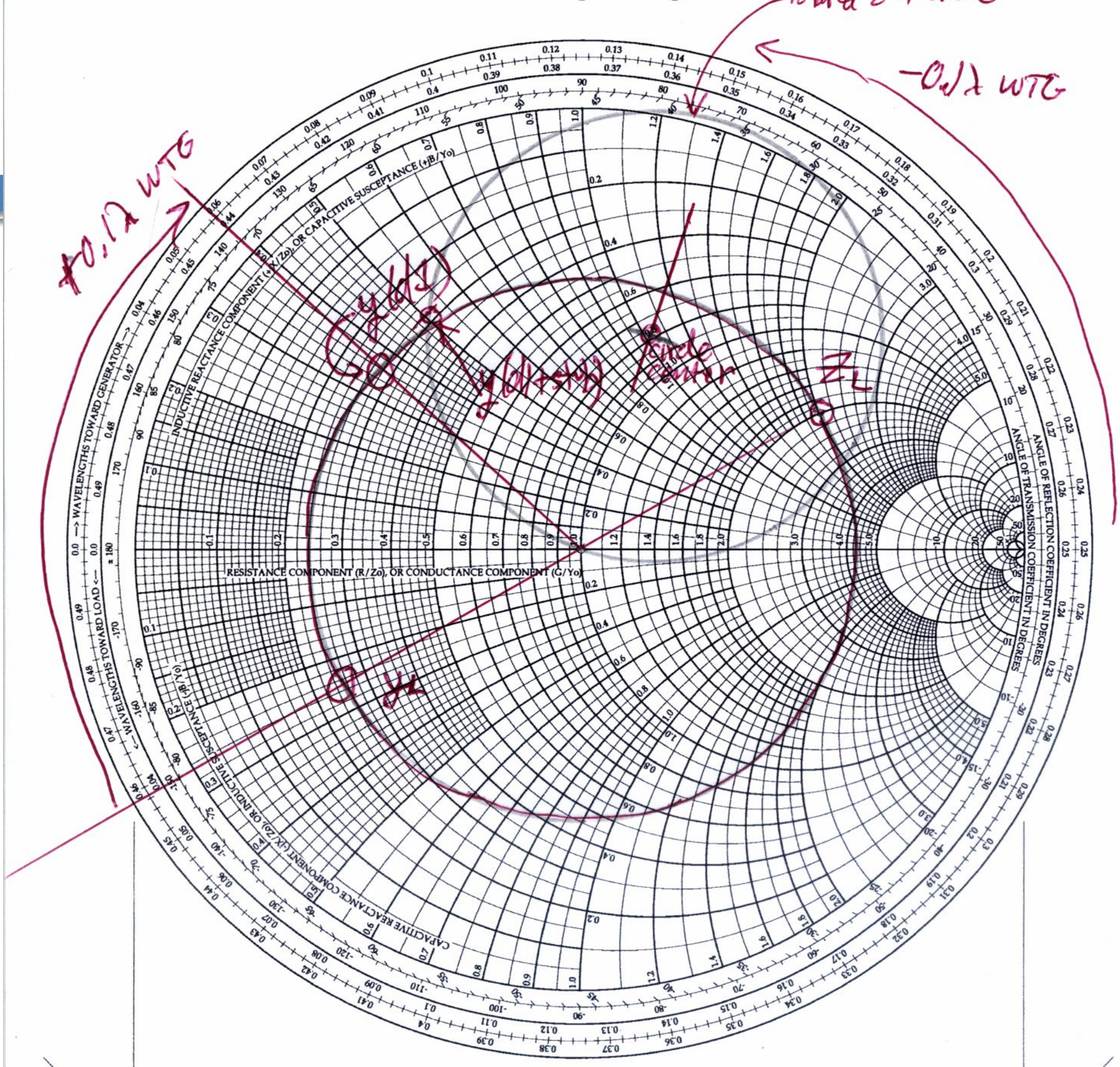


Step 5b: choose 1st stub length so we end up on this rotated circle.

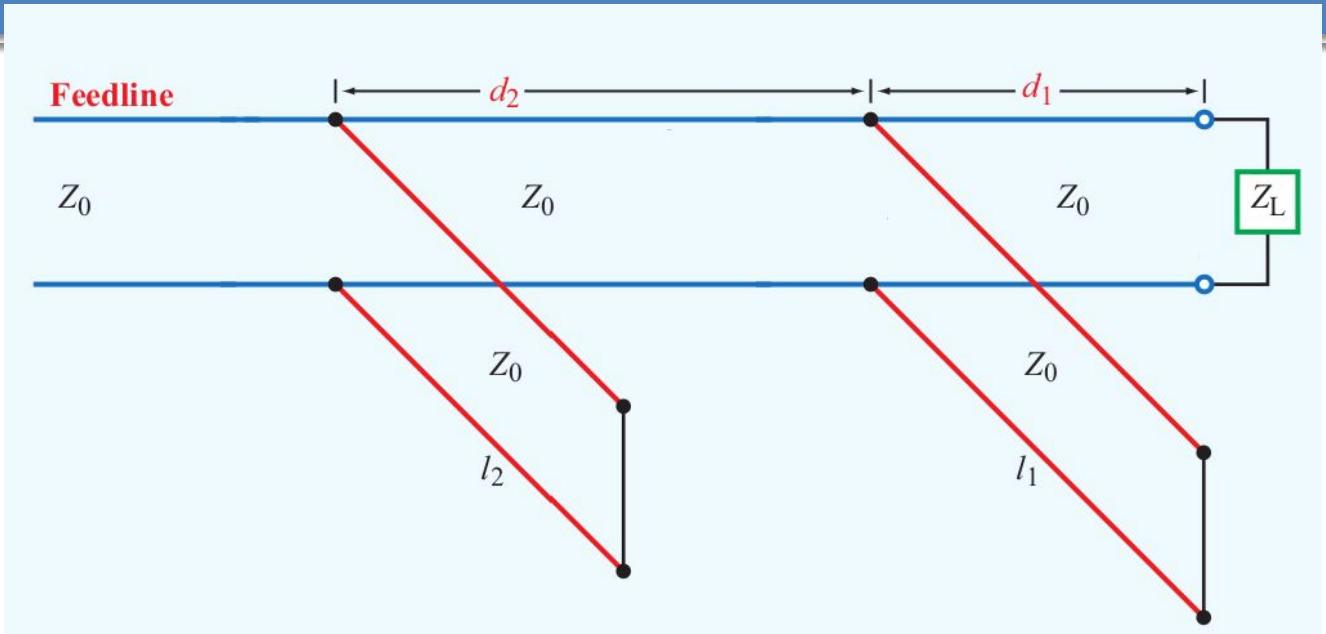
y(real part) does not change

y(d1+stub) = 0.28 + j 0.51

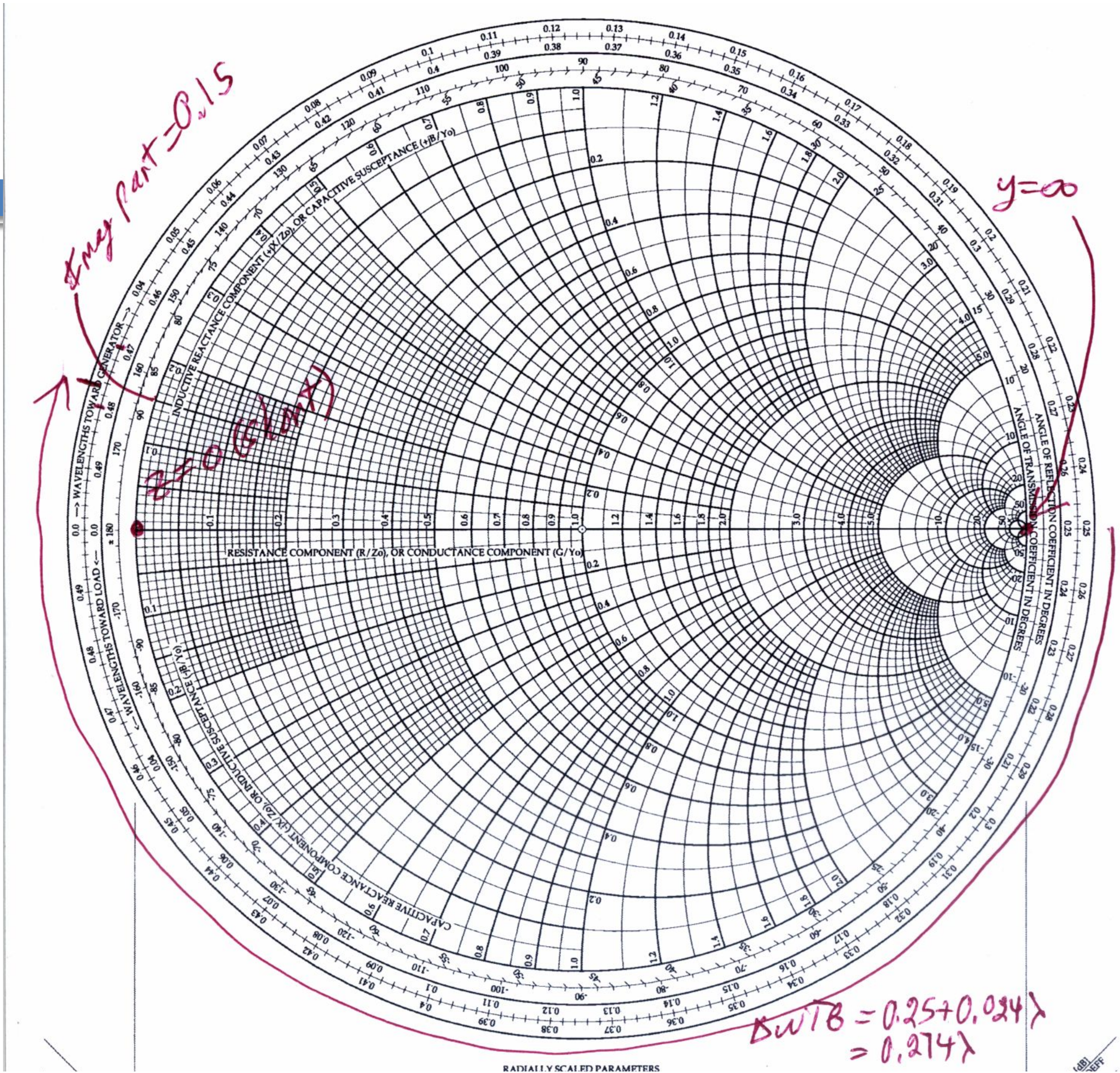
Black Magic Design



2-Stub Matching

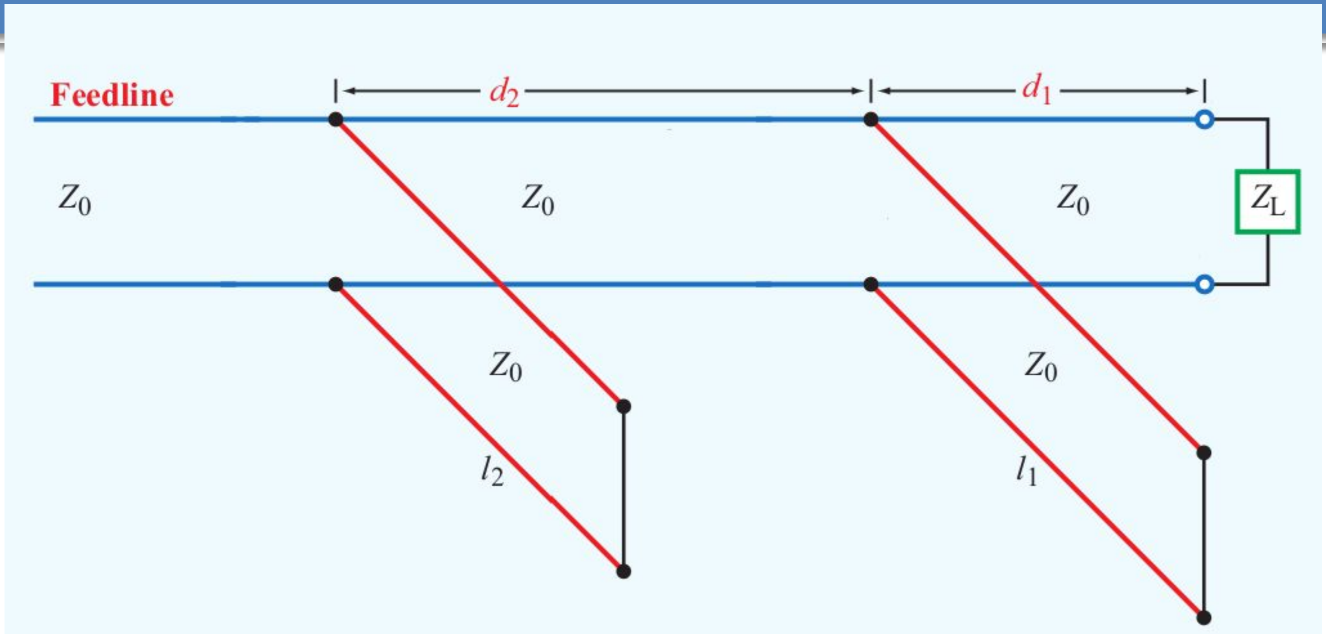


Step 6: Use another smith chart to find the stub length to make this change in the imaginary part.
delta-imag -part = 0.51 - 0.36 = 0.15
gives length of stub1 as 0.274λ



Imag part = 0.15
z = 0 (short)
y = ∞
ΔWTB = 0.25+0.024λ
= 0.274λ
0.0 —> WAVELENGTHS TOWARD GENERATOR —>
<— WAVELENGTHS TOWARD LOAD <—
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
RADIALLY SCALED PARAMETERS

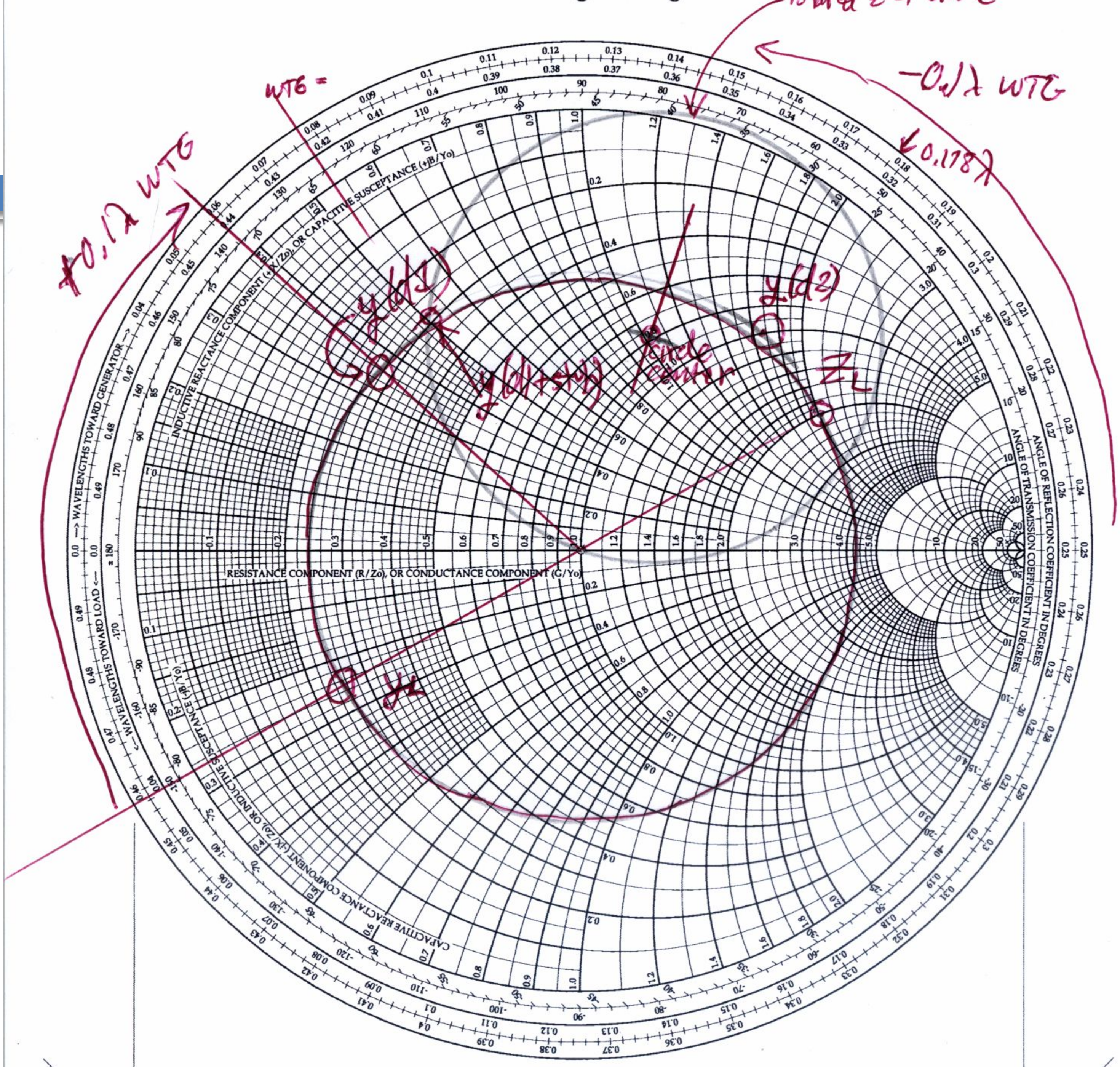
2-Stub Matching



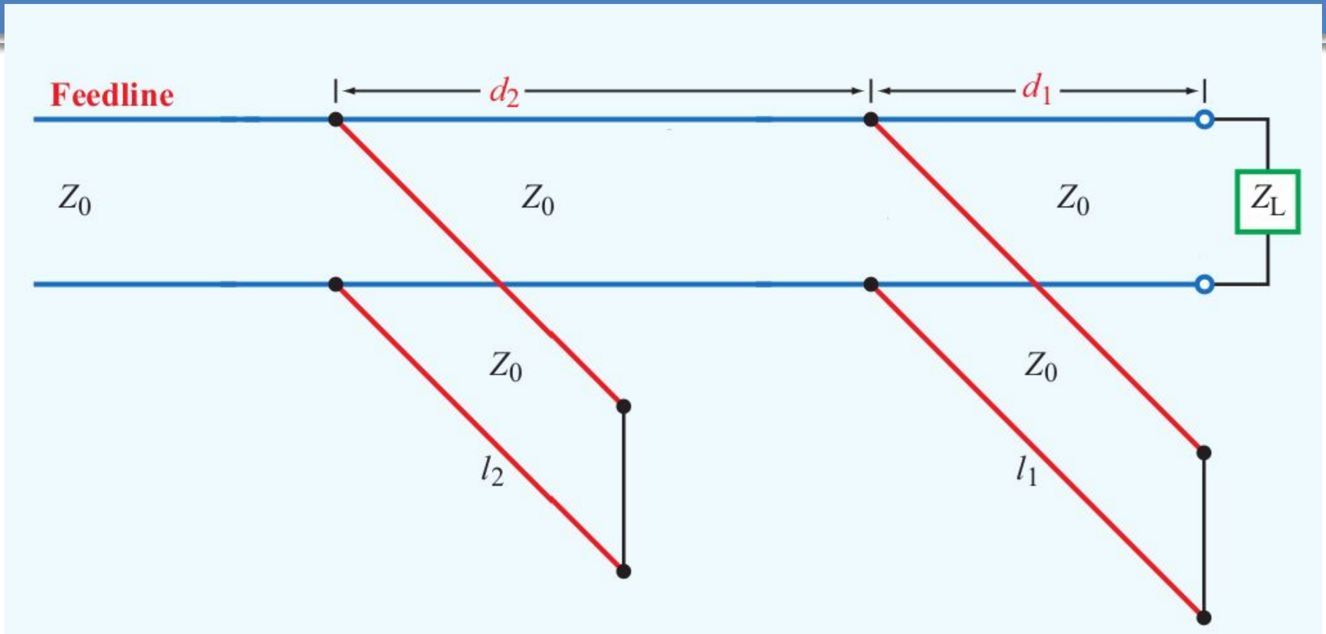
Step 7: rotate by d2 toward generator to get y(d2).
(along constant-|Γ| circle)

from 0.078λ + 0.1λ to: 0.178λ
y(d2) = 1 +j1.65 (on the y=1 circle)

Black Magic Design

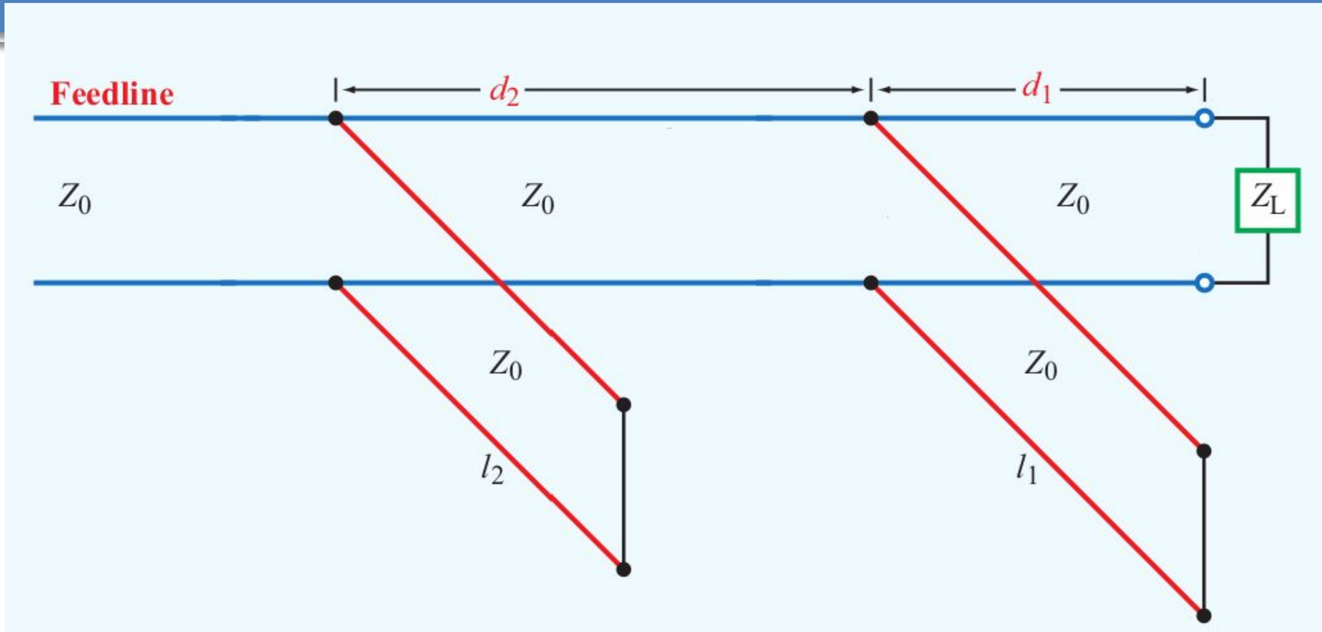


2-Stub Matching



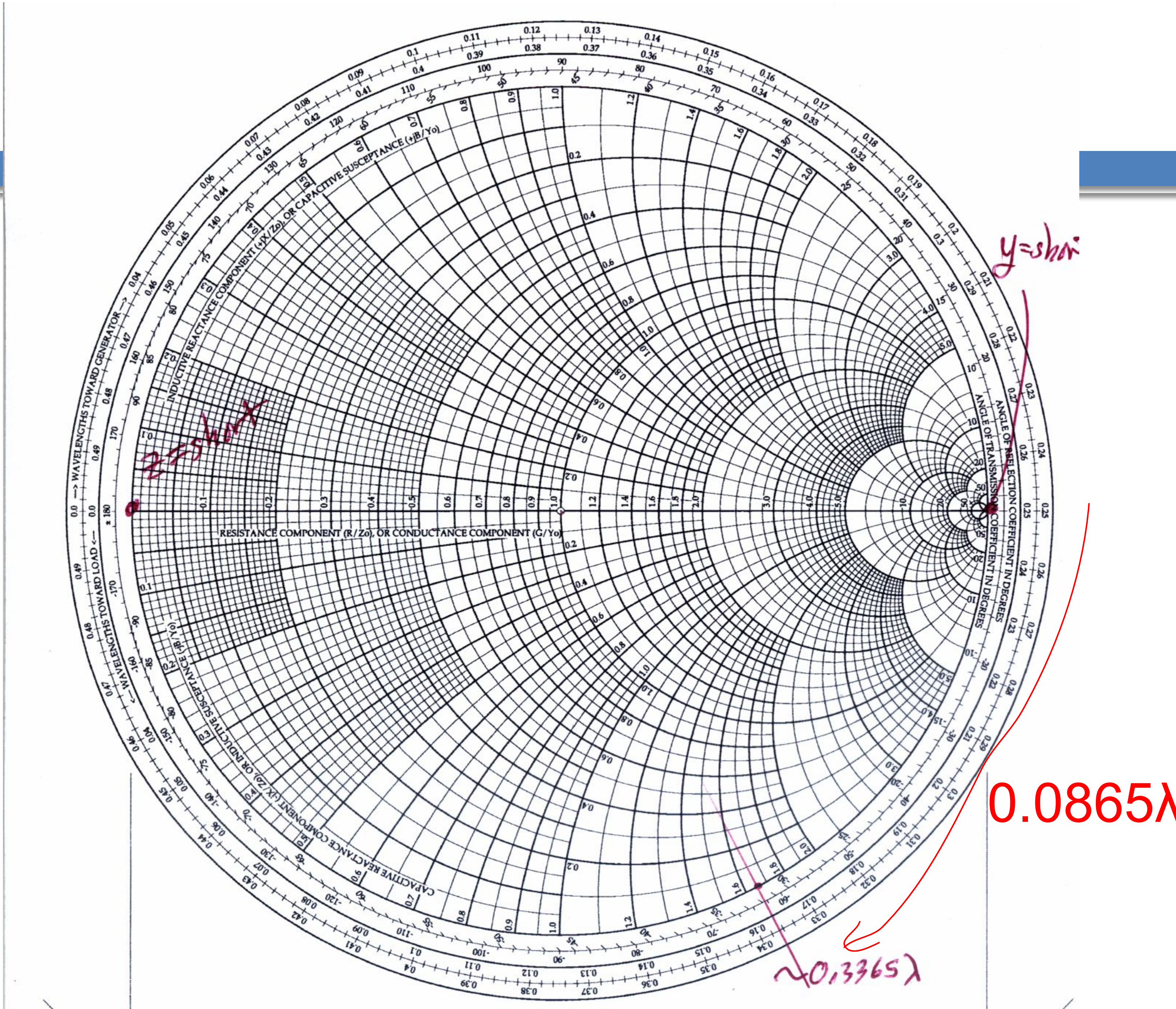
Step 8: Find change in imaginary part needed by stub2 so we end up at origin (z=1+j0).   = -j1.65

2-Stub Matching

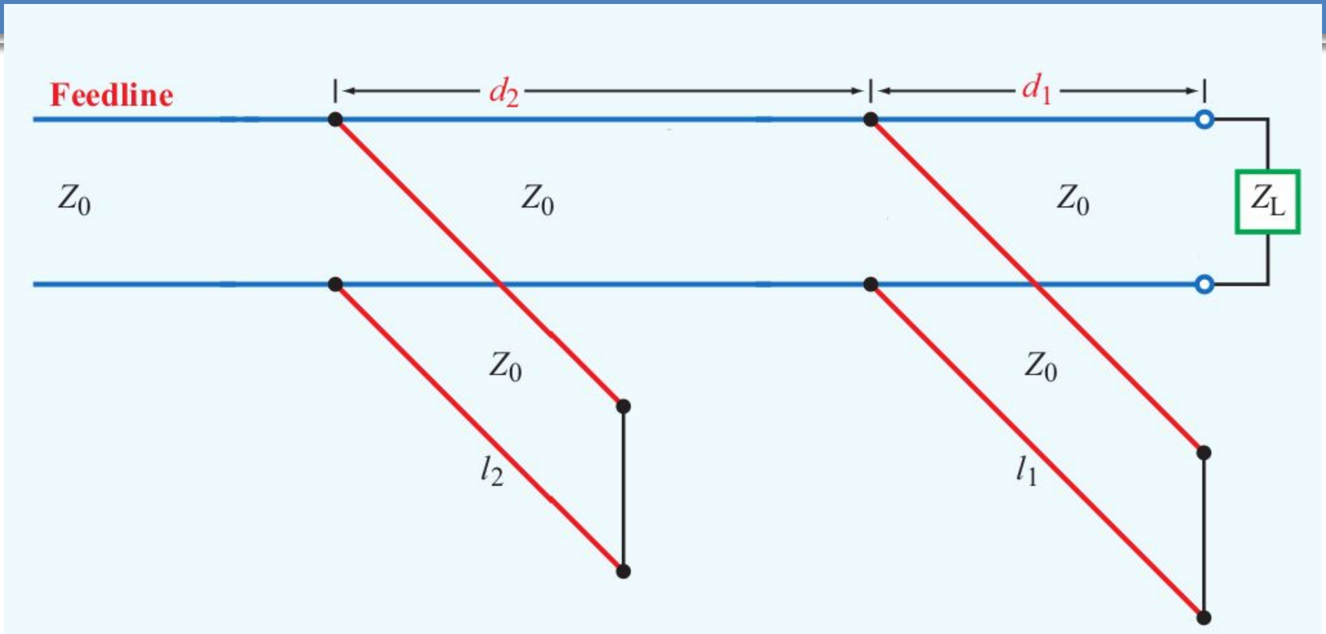


Step 9: Use another Smith Chart to find the length of the stub needed for that.

$$0.3365 - 0.25 = 0.0865\lambda$$


WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES
z=short
y=shor
0.0865λ
~0.3365λ

2-Stub Matching



Step 10: Summary:

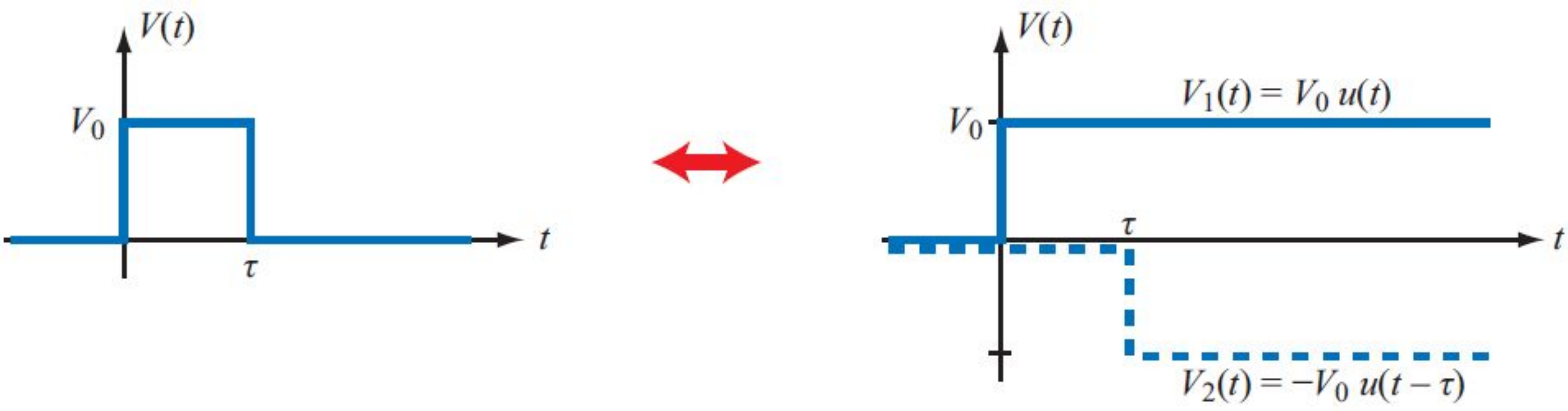
stub1 at d=0.6λ, length = 0.274λ

stub2 at d=0.1λ, length = 0.0865λ

Transients

Change to operate in the time domain (instead of phasor domain)

Instead of sinusoidal sources, use rectangular pulses:



(a) Pulse of duration τ

(b) $V(t) = V_1(t) + V_2(t)$

Rectangular pulse is equivalent to the sum of two step functions

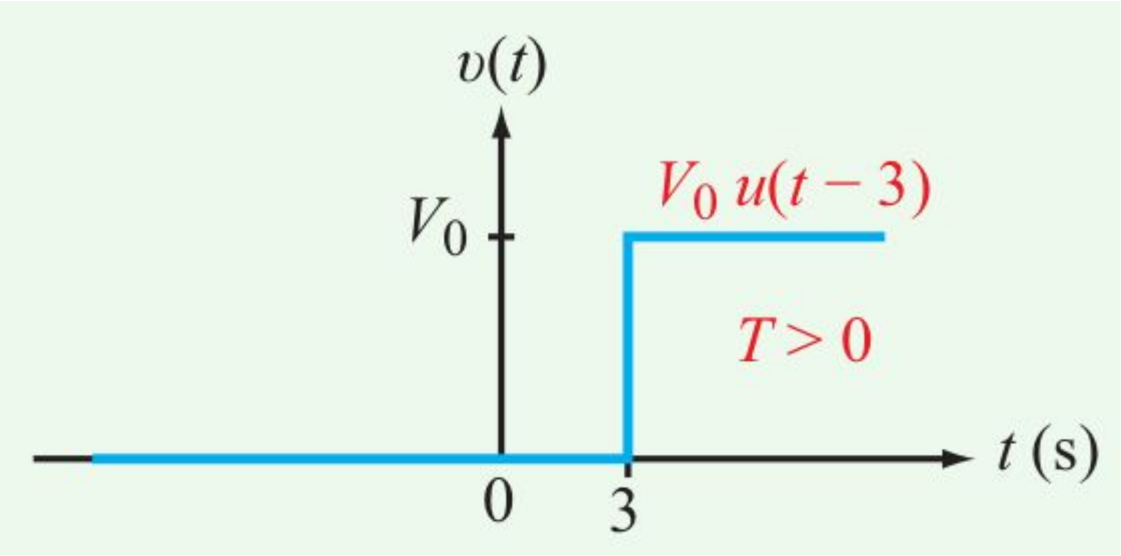
Transients

A General Step Function:

$$u(t) = A\, u(t - T)$$

A = amplitude

T = turn-on time



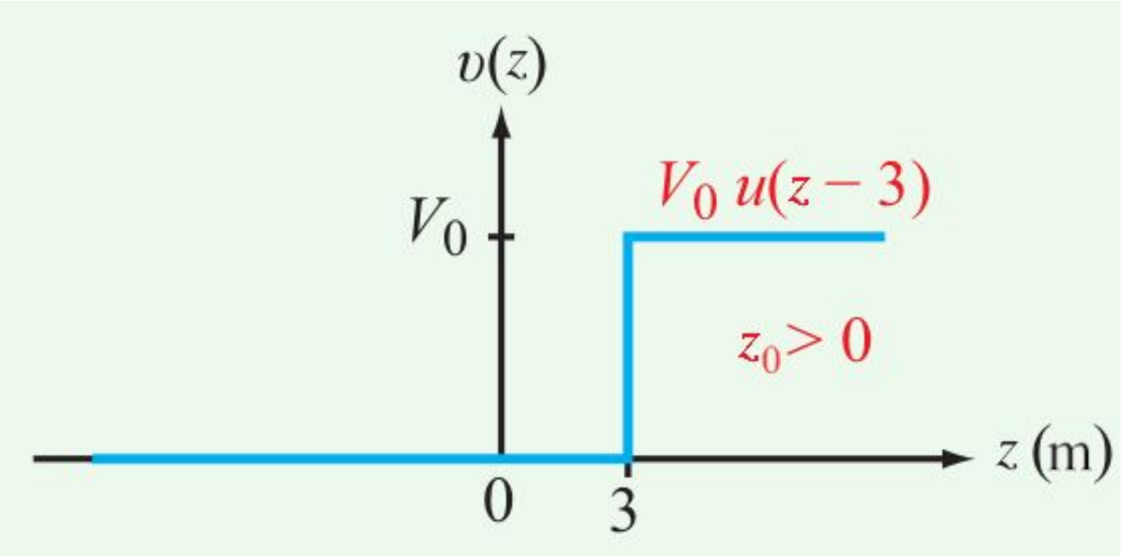
Transients

As a function of distance: (at some given time)

$$u(z) = A\, u\,(z - z_0)$$

A = amplitude

z_0 = turn-on distance



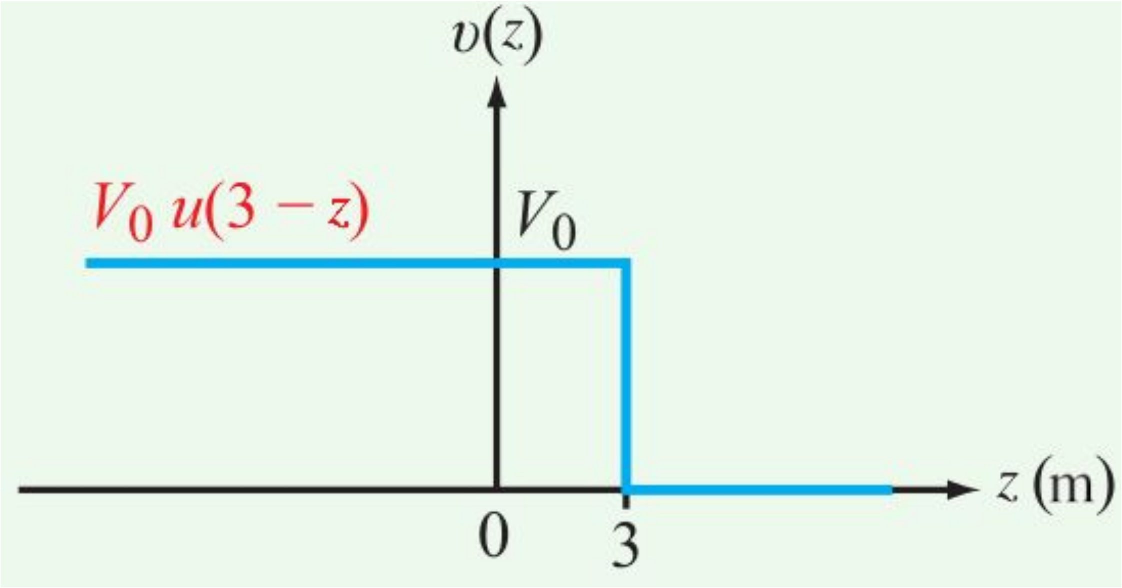
Transients

Reflected version:

$$u(z) = A\,u(z_0 - z)$$

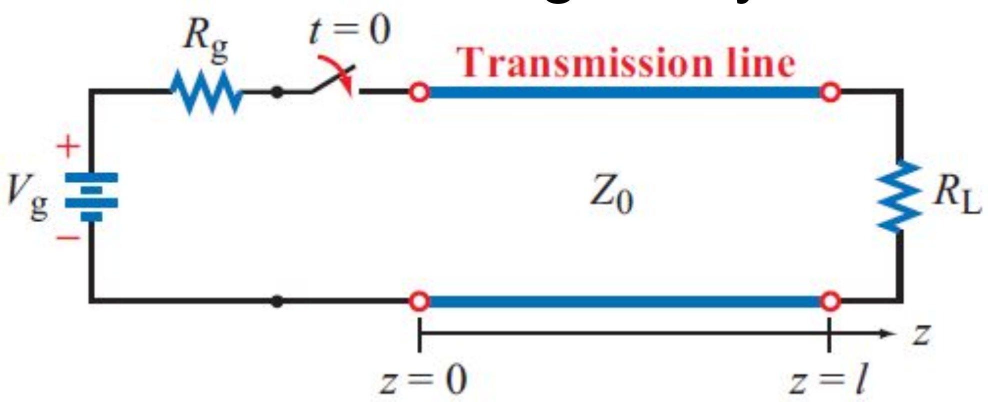
A = amplitude

z_0 = turn-off distance



Transients

Need to keep track of voltage vs. time including every reflection.

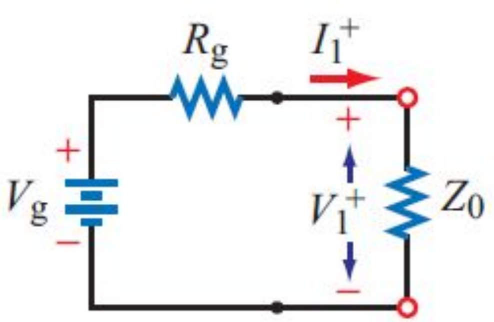


(a) Transmission-line circuit

Initial current and voltage

$$I_1^+ = \frac{V_g}{R_g + Z_0},$$

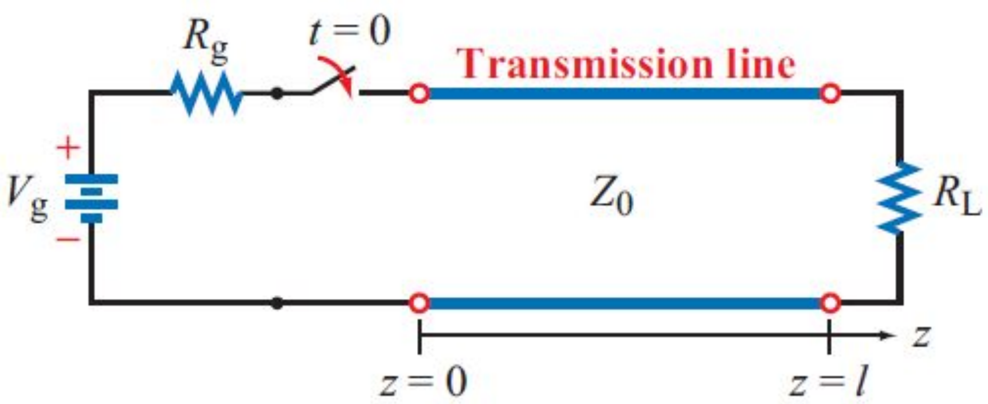
$$V_1^+ = I_1^+ Z_0 = \frac{V_g Z_0}{R_g + Z_0}$$



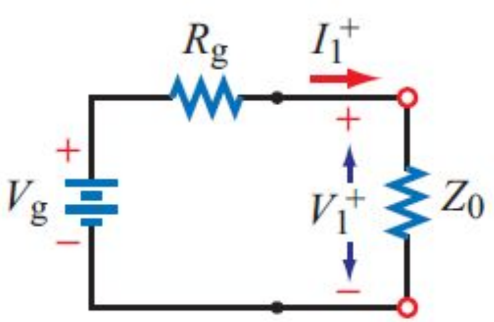
(b) Equivalent circuit at $t = 0^+$

At $t=0^+$ the equiv ckt does not "see" R_L yet, only the transmission line.

Transient Response



(a) Transmission-line circuit



(b) Equivalent circuit at $t = 0^+$

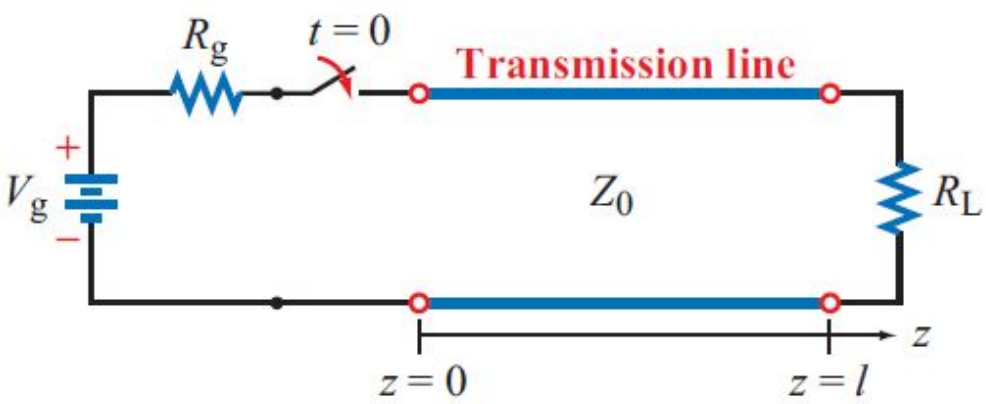
Reflection at the load

$$V_1^- = \Gamma_L V_1^+,$$

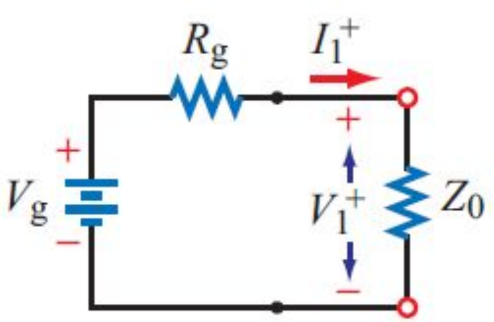
Load reflection coefficient

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0}$$

Transient Response



(a) Transmission-line circuit



(b) Equivalent circuit at $t = 0^+$

Second transient

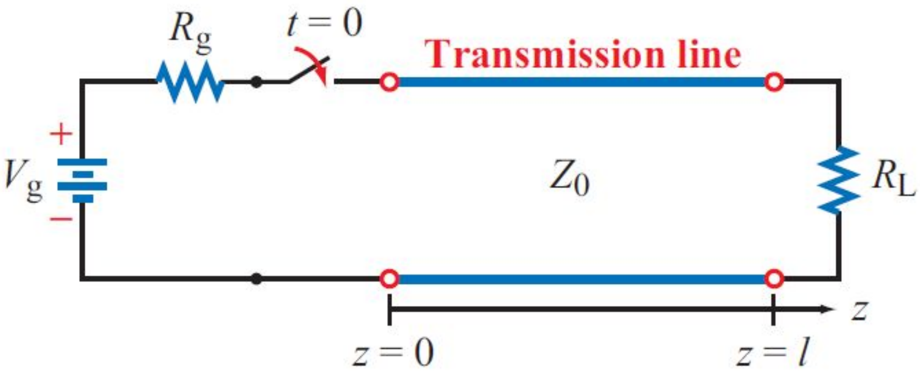
$$V_2^+ = \Gamma_g V_1^- = \Gamma_g \Gamma_L V_1^+$$

Generator reflection coefficient

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0}$$

This continues with every reflection...

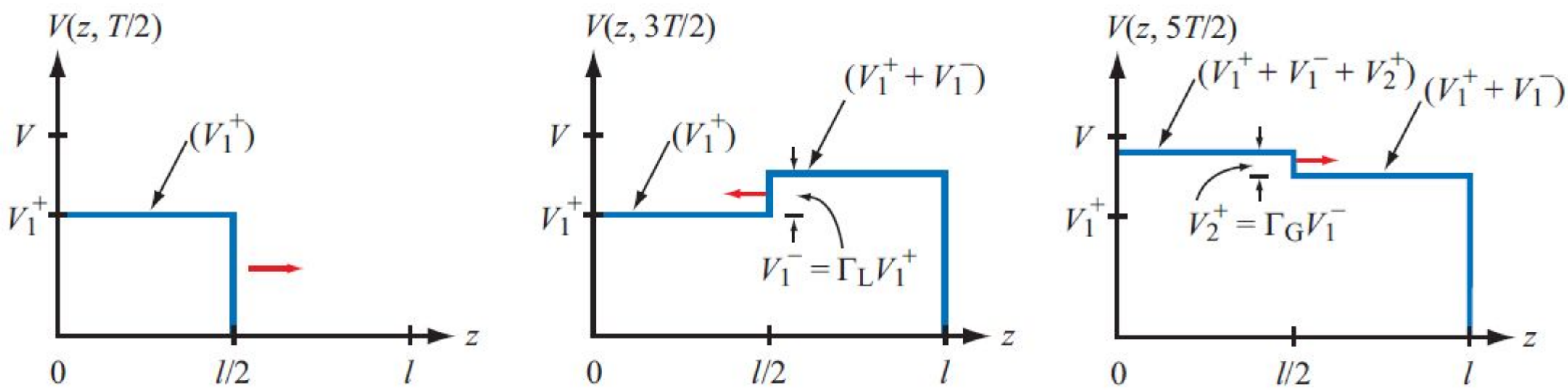
Voltage Pulse



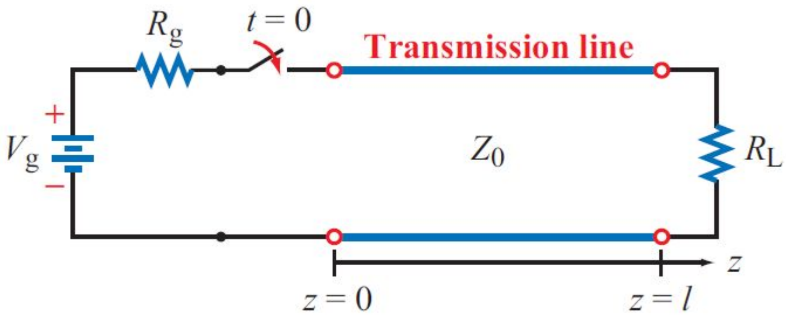
(a) Transmission-line circuit

$T = \ell / u_p$ is the time it takes the wave to travel the full length of the line

$R_g = 4Z_0$ and $R_L = 2Z_0$. The corresponding reflection coefficients are $\Gamma_L = 1/3$ and $\Gamma_g = 3/5$.



(a) $V(z)$ at $t = T/2$ (b) $V(z)$ at $t = 3T/2$ (c) $V(z)$ at $t = 5T/2$



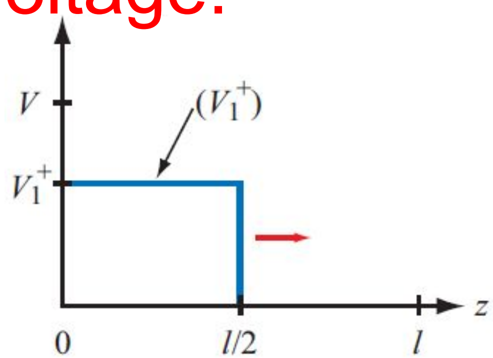
(a) Transmission-line circuit

Voltage and Current Pulses

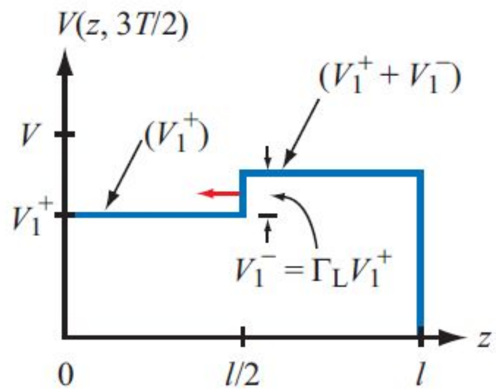
Reflection coefficient for current is the negative of that for voltage

$$I_1^- = -\Gamma_L I_1^+,$$
$$I_2^+ = -\Gamma_g I_1^- = \Gamma_g \Gamma_L I_1^+$$

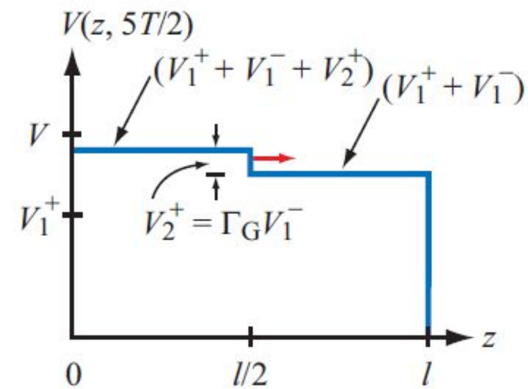
Voltage:



(a) $V(z)$ at $t = T/2$

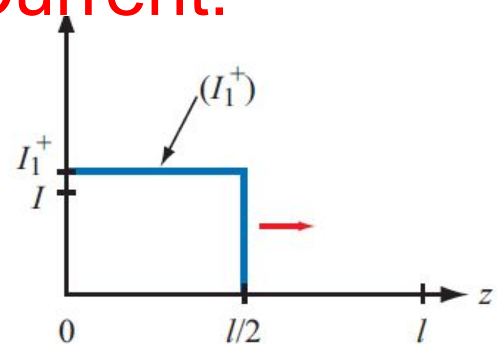


(b) $V(z)$ at $t = 3T/2$

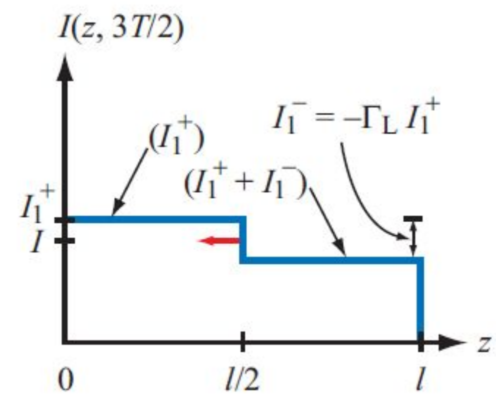


(c) $V(z)$ at $t = 5T/2$

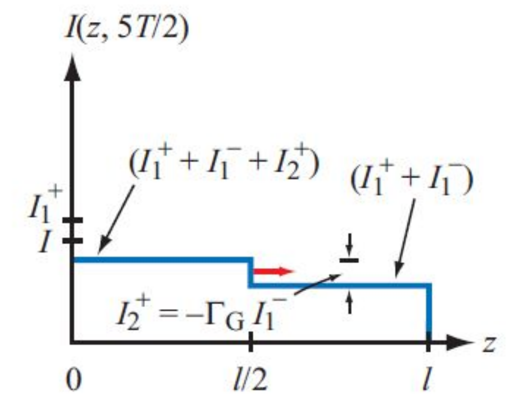
Current:



(d) $I(z)$ at $t = T/2$



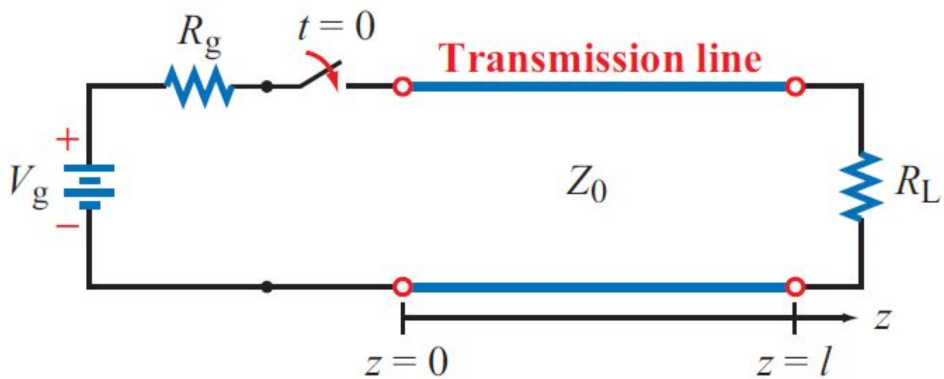
(e) $I(z)$ at $t = 3T/2$



(f) $I(z)$ at $t = 5T/2$

Transient Response

Steady State Response



(a) Transmission-line circuit

$$
\begin{aligned}
V_\infty &= V_1^+ + V_1^- + V_2^+ + V_2^- + V_3^+ + V_3^- + \cdots \\
&= V_1^+[1 + \Gamma_L + \Gamma_L\Gamma_g + \Gamma_L^2\Gamma_g + \Gamma_L^2\Gamma_g^2 + \Gamma_L^3\Gamma_g^2 + \\
&= V_1^+[(1 + \Gamma_L)(1 + \Gamma_L\Gamma_g + \Gamma_L^2\Gamma_g^2 + \cdots)] \\
&= V_1^+(1 + \Gamma_L)[1 + x + x^2 + \cdots],
\end{aligned}
$$

$$x = \Gamma_L\Gamma_g.$$

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots \qquad \text{for } |x| < 1.$$

$$V_\infty = V_1^+ \frac{1 + \Gamma_L}{1 - \Gamma_L\Gamma_g}.$$

Steady State Response

$$V_\infty = V_1^+ \frac{1+\Gamma_L}{1-\Gamma_L\Gamma_G}$$

$$= \left(\frac{V_g Z_0}{R_g + Z_0}\right) \frac{1 + \frac{R_L - Z_0}{R_L + Z_0}}{1 - \frac{R_L - Z_0}{R_L + Z_0}\frac{R_g - Z_0}{R_g + Z_0}}$$

multiply by: $\dfrac{R_L + Z_0}{R_L + Z_0}$

$$= \left(\frac{V_g Z_0}{R_g + Z_0}\right) \frac{R_L + Z_0 + R_L - Z_0}{R_L + Z_0 - (R_L - Z_0)\frac{R_g - Z_0}{R_g + Z_0}}$$

Steady State Response

$$= \left(\frac{V_g Z_0}{R_g + Z_0}\right) \frac{R_L + Z_0 + R_L - Z_0}{R_L + Z_0 - (R_L - Z_0)\frac{R_g - Z_0}{R_g + Z_0}}$$

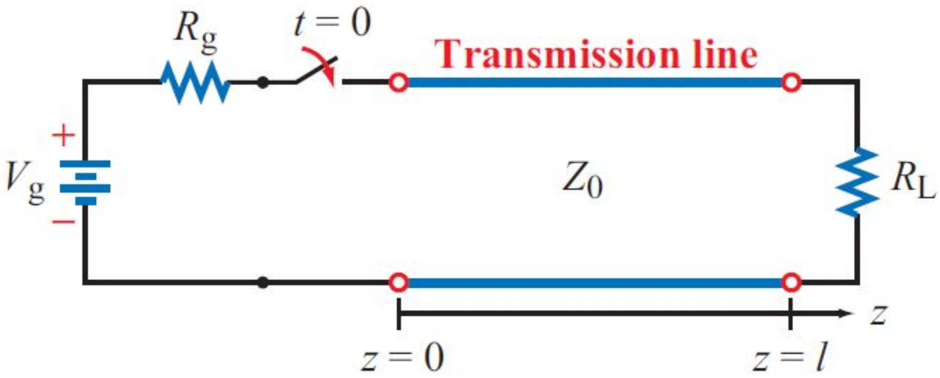
$$= (V_g Z_0) \frac{2\, R_L}{(R_g + Z_0)(R_L + Z_0) - (R_L - Z_0)(R_g - Z_0)}$$

$$= V_g \frac{2\, R_L Z_0}{R_g R_L + R_g Z_0 + R_L Z_0 + Z_0^2 - R_L R_g + R_L Z_0 + R_g Z_0 - Z_0^2}$$

$$= V_g \frac{2\, R_L Z_0}{2 R_g Z_0 + 2 R_L Z_0}$$

$$V_\infty = \frac{V_\mathrm{g} R_\mathrm{L}}{R_\mathrm{g} + R_\mathrm{L}}$$

Steady State Response



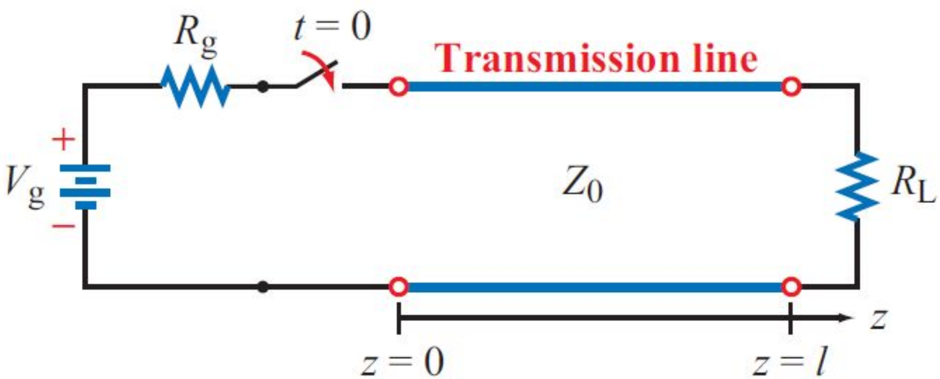
(a) Transmission-line circuit

Result is as expected from EECS 215:

$$V_\infty = \frac{V_g R_L}{R_g + R_L}$$

$$I_\infty = \frac{V_\infty}{R_L} = \frac{V_g}{R_g + R_L}$$

Steady State Response



(a) Transmission-line circuit

Result is as expecte… …0:

V_∞

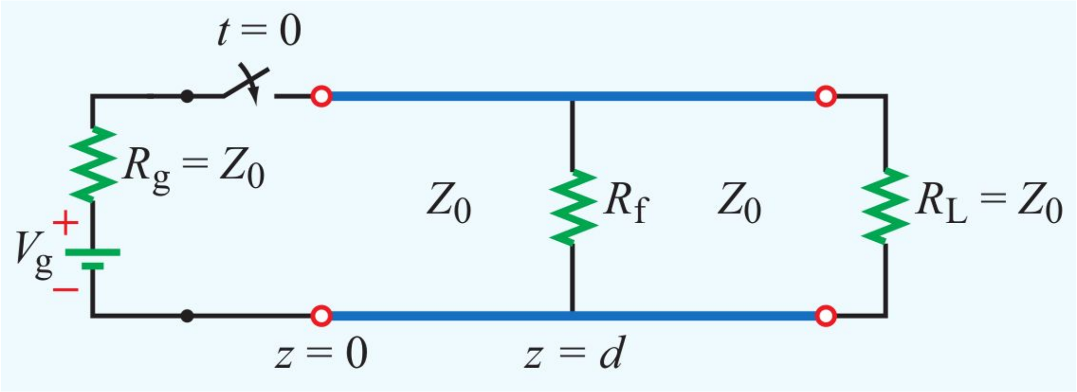
$$I_\infty = \frac{V_\infty}{R_L} = \frac{V_g}{R_g + R_L}$$

True for the Steady-State Response Only: t=∞ and DC source

Time-Domain Reflectometry

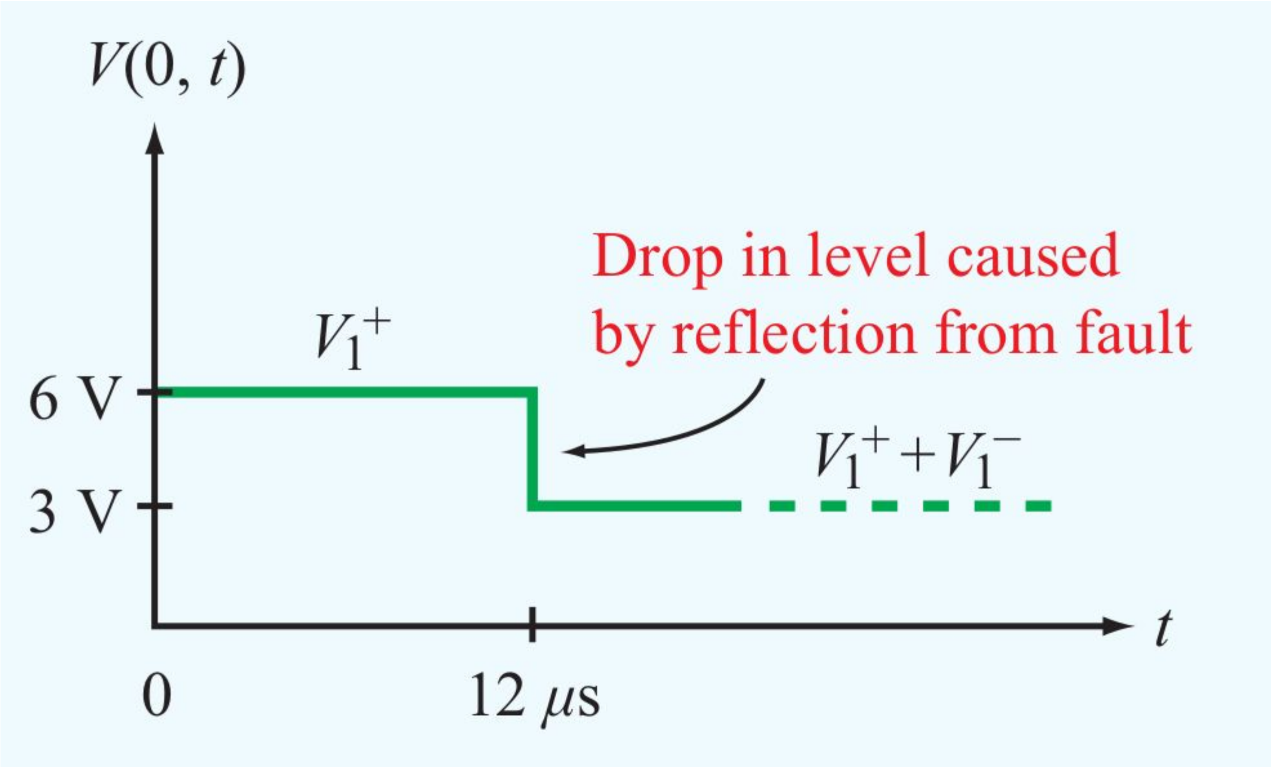
Given: A damaged undersea cable exhibits an "effective" resistance at the fault location.

Model of damaged cable:



Time-Domain Reflectometry

Send a pulse down the cable, get back:



With Z_0=75Ω, insulation: ε_r=2.1

Time-Domain Reflectometry

Find: (a) V_g
(b) d (fault distance from generator)
(c) R_f

Solution:

(a) Since the line was matched before the fault occurred:

$R_g = R_L = Z_0$

and the plot shows that: $V_1^+ = 6\text{ V}$

Time-Domain Reflectometry

We know:

$$V_1^+ = \frac{V_g Z_0}{R_g + Z_0} = \frac{V_g Z_0}{2Z_0} = \frac{V_g}{2}$$

so:

$$V_g = 2V_1^+ = 12 \text{ V}.$$

(b) to get *d*, need u_p

$$u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{2.1}} = 2.07 \times 10^8 \text{ m/s}$$

Time-Domain Reflectometry

Expected round-trip time-delay of a fault at *d*:

$$\Delta t = \frac{2d}{u_p}.$$

from the plot, Δt = 12μsec, so:

$$d = \frac{\Delta t}{2} u_p = \frac{12 \times 10^{-6}}{2} \times 2.07 \times 10^8 = \boxed{1,242 \text{ m}}$$

Time-Domain Reflectometry

From the plot:

$$V_1^- = \Gamma_f V_1^+ = -3 \text{ V},$$

so:

$$\Gamma_f = \frac{-3}{6} = -0.5$$

where:

$$\Gamma_f = \frac{R_{Lf} - Z_0}{R_{Lf} + Z_0}$$

and R_{Lf} is NOT R_f, but the parallel combination of R_f and Z_0:

$$\frac{1}{R_{Lf}} = \frac{1}{R_f} + \frac{1}{Z_0}$$

Time-Domain Reflectometry

Since:

$$R_{Lf} = Z_0 \frac{1+\Gamma_f}{1-\Gamma_f}$$

$$R_{Lf} = 75\,\Omega \frac{1+(-0.5)}{1-(-0.5)}$$

$$R_{Lf} = 25\,\Omega$$

then:

$$\frac{1}{R_f} = \frac{1}{R_{Lf}} - \frac{1}{Z_0}$$

$$R_f = \frac{R_{Lf} Z_0}{Z_0 - R_{Lf}} = \frac{(25\,\Omega)(75\,\Omega)}{75\,\Omega - 25\,\Omega} = \boxed{37.5\,\Omega}$$

Homework

Homework 10 is due tomorrow at midnight.

submit to gradescope via the canvas site.

Next Time

Chapter 3:

Review of Vector Calculus, gradient, divergence, curl

Review of the 3 major coordinate systems:
rectangular, cylindrical, spherical
how to use them
how to convert between them