EECS 230
ENGINEERING ELECTROMAGNETICS
Leland Pierce

Transmission Lines 7

University of Michigan - Fall 2024

Chapter 2 Overview

What is a transmission line?

Why study transmission lines?

model

diffeq

solve diffeq

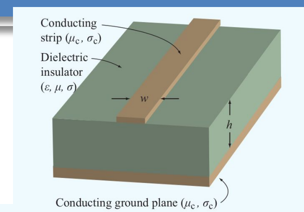
wave propagation

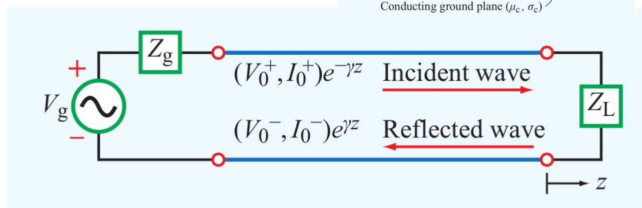
lossless xmission li

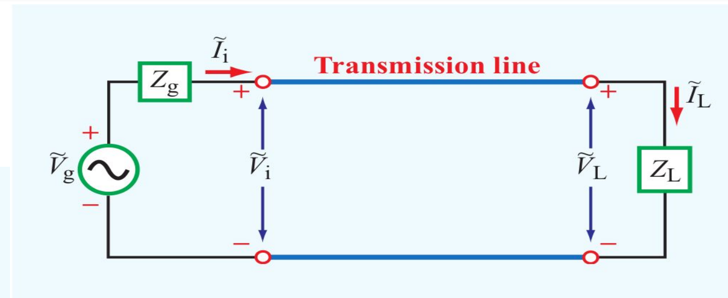
- microstrip lines
- reflections
- standing waves
- impedance
- short, open
- matching
- power flow

smith chart

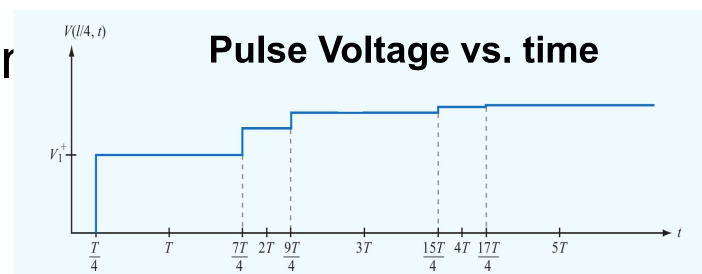
transients

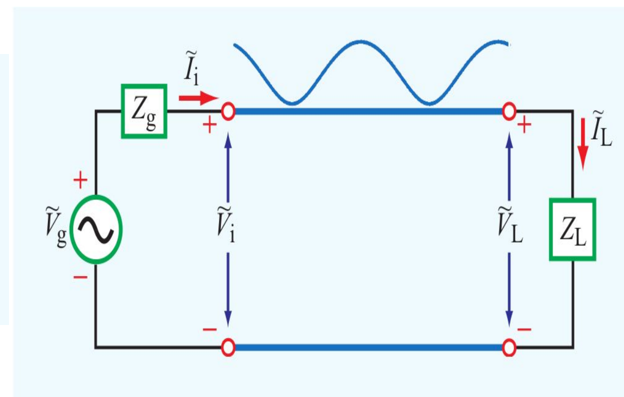




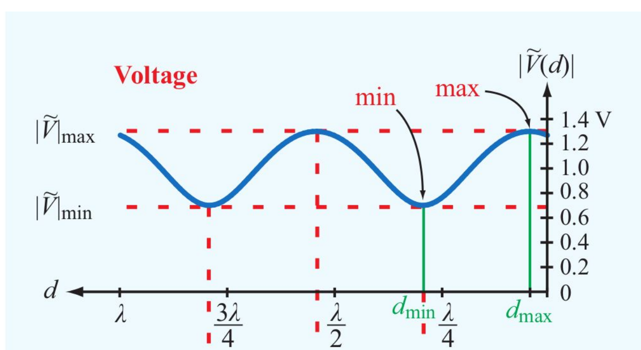


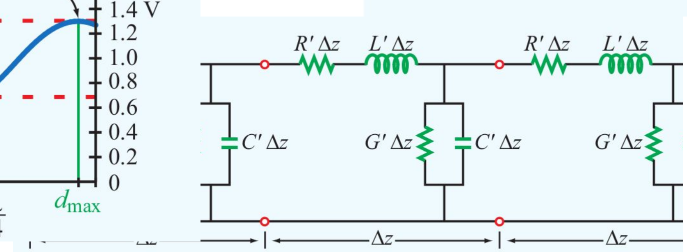
Typical High-Frequency Circuit

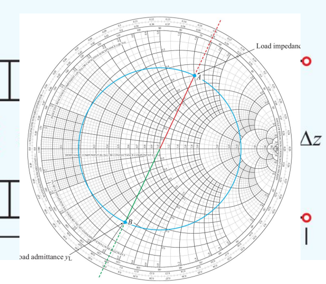




Waves on line: old methods don't work







Today's Lecture Coverage

Review Sections 2-1 through 2-10 of the book:

2-1: What is a transmission line? Why study transmission lines?
2-2: Lumped-Element Model
2-3: Governing Differential Eqns
2-4: Solve the Differential Equations
Properties of the solution: wave propagation
2-5: Lossless Microstrip Line
2-6: Lossless Transmission Lines
2-7: Lossless Transmission Lines: Wave Impedance
2-8: Lossless Transmission Lines: Special Cases
2-9: Lossless Transmission Lines: Power Flow
2-10: The Smith Chart

Section 2-11 of the book:

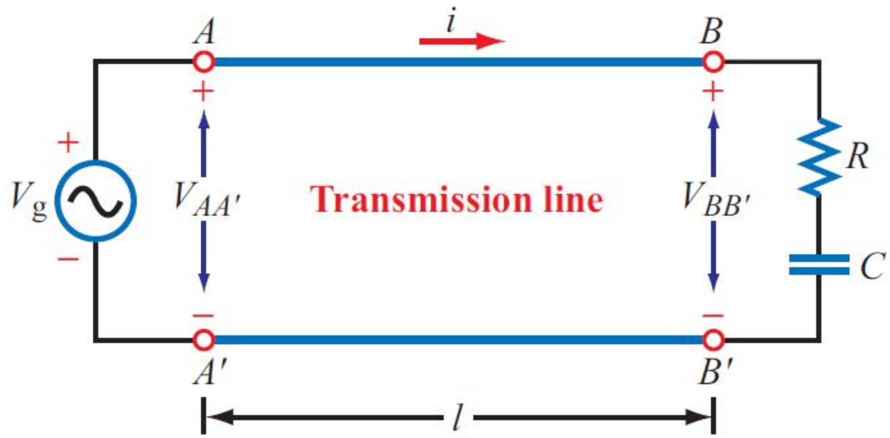
2-11: Matching using the Smith Chart

Chapter 2 Review

- A transmission line connects a generator to a load.

Chapter 2 Review

Phase Delay due to length of transmission line:



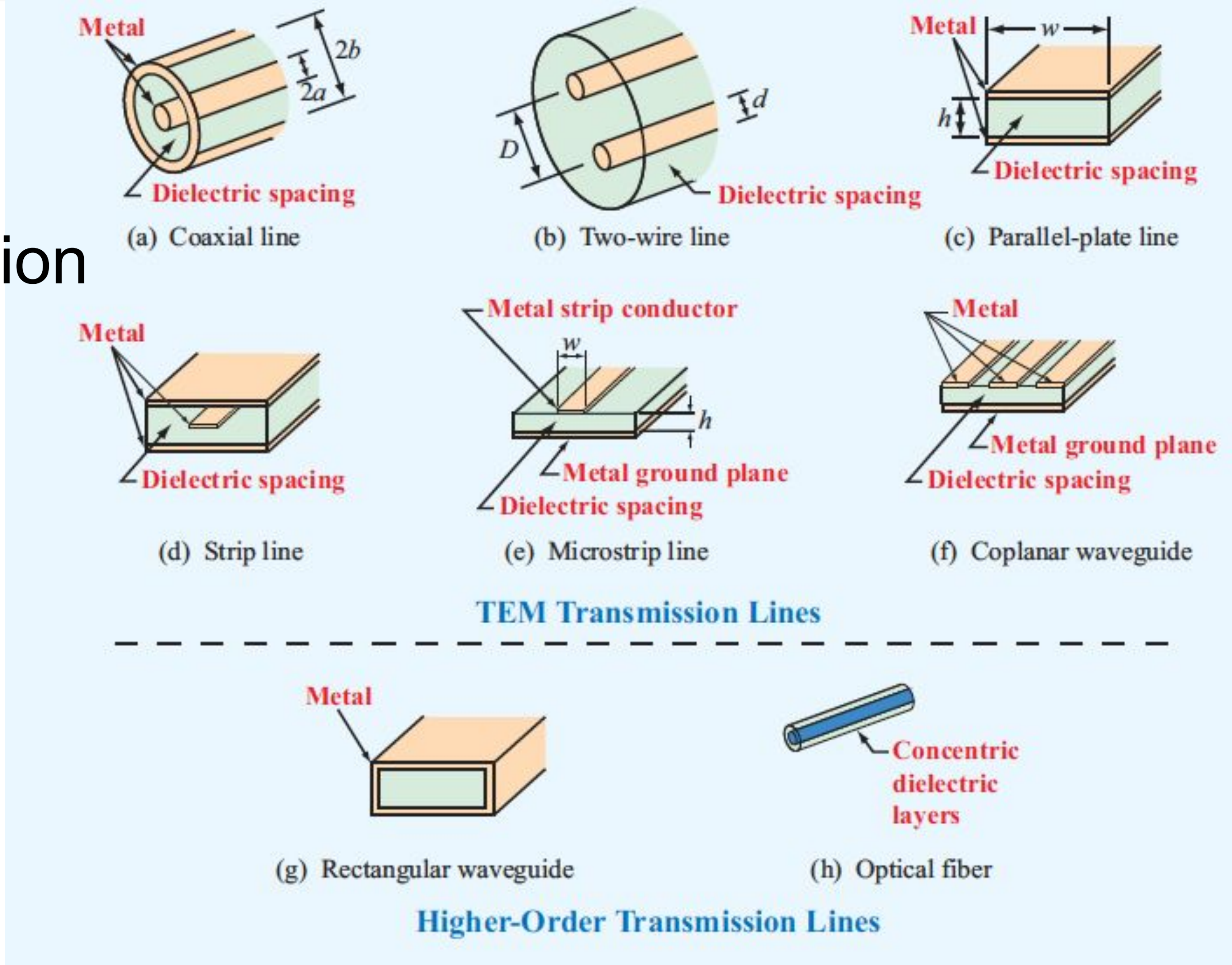
$$\phi_0 = \frac{\omega l}{c} = \frac{2\pi f l}{c} = 2\pi \frac{l}{\lambda} \quad \text{radians.}$$

$l/\lambda \lesssim 0.01$: Can ignore transmission-line effects

$l/\lambda \gtrsim 0.01$: Must deal with phase shift, and other effects...

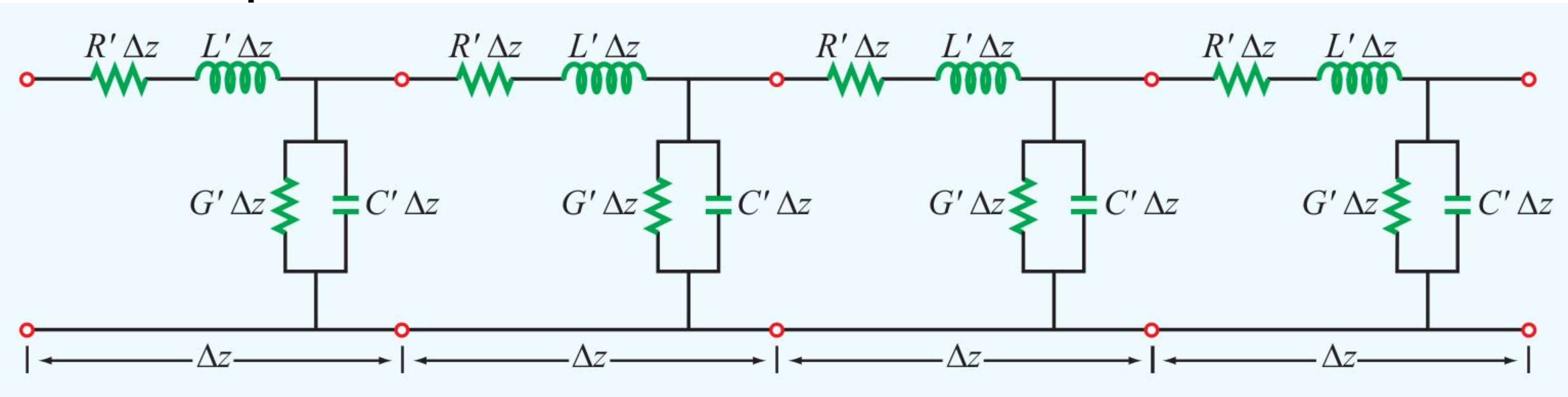
Chapter 2 Review

Different geometries for transmission lines



Chapter 2 Review

Lumped-Element Model:



All parameters are "per unit length":

R': Combined Resistance of BOTH conductors: □/m

L': Combined Inductance of BOTH conductors, H/m

G': Conductance of insulation
between inner and outer conductor, S/m

C': Capacitance
between inner and outer conductors, F/m

Chapter 2 Review

Lumped-Element Values: geometry/materials/freq

Table 2-1 **Transmission-line parameters R', L', G', and C' for three types of lines.**

Parameter	Coaxial	Two-Wire	Parallel-Plate	Unit
R'	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{2R_s}{\pi d}$	$\frac{2R_s}{w}$	Ω/m
L'	$\frac{\mu}{2\pi}\ln(b/a)$	$\frac{\mu}{\pi}\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]$	$\frac{\mu h}{w}$	H/m
G'	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\pi\sigma}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\sigma w}{h}$	S/m
C'	$\frac{2\pi\varepsilon}{\ln(b/a)}$	$\frac{\pi\varepsilon}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\varepsilon w}{h}$	F/m

$$R_s = \sqrt{\pi f \mu_c/\sigma_c}$$

Chapter 2 Review

Transmission-line governing Differential Equations

$$-\frac{d\tilde{V}(z)}{dz} = (R' + j\omega L')\,\tilde{I}(z),$$

$$-\frac{d\tilde{I}(z)}{dz} = (G' + j\omega C')\,\tilde{V}(z).$$

(telegrapher's equations in phasor form)

Chapter 2 Review

Transmission-line governing Differential Equation for *V*:

$$\frac{d^2\widetilde{V}(z)}{dz^2} - \gamma^2\,\widetilde{V}(z) = 0,$$

(wave equation for $\widetilde{V}(z)$)

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}\,.$$

(propagation constant)

Chapter 2 Review

Complex propagation constant:

$$\gamma = \alpha + j\beta$$

Where:

γ: Units of 1/m

α: Attenuation constant, units of Np/m (>0 in this class)

β: Phase constant, units of rad/m

Np and radians are both "dimensionless", and are the conventional "units" for these parameters.

Chapter 2 Review

Form of the solution: traveling waves, going in both directions:

$$\widetilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \qquad \text{(V)},$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z} \qquad \text{(A)}.$$

$$Z_0 = \frac{R' + j\omega L'}{\gamma} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \qquad (\Omega),$$

Chapter 2 Review

Solution in time-domain

$$\upsilon(z,t) = |V_0^+|e^{-\alpha z}\cos(\omega t - \beta z + \phi^+) + |V_0^-|e^{+\alpha z}\cos(\omega t + \beta z + \phi^-)$$

Remaining unknowns are determined via specification of source and load.

Chapter 2 Review

- The wave equation for a general Transmission Line.

$$\frac{d^2\widetilde{V}(z)}{dz^2} - \gamma^2\,\widetilde{V}(z) = 0, \qquad (2.21)$$

- General solution of the wave equation
  - *It involves both incident and reflected waves*

$$\widetilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \qquad \text{(V)},$$

$$\widetilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \qquad \text{(A)}.$$

Chapter 2 Review

- Useful Relations for lossless Transmission Lines:

$$\alpha = 0 \qquad \text{(lossless line)},$$

$$\beta = \omega\sqrt{L'C'} \qquad \text{(lossless line)}. \qquad (2.45)$$

$$\beta = \omega\sqrt{\mu\varepsilon} \qquad \text{(rad/m)}, \qquad (2.49)$$

$$u_p = \frac{1}{\sqrt{\mu_0\epsilon_0}}\frac{1}{\sqrt{\epsilon_r}} = \frac{c}{\sqrt{\epsilon_r}}$$

$$\lambda = \frac{u_p}{f} = \frac{c}{f}\,\frac{1}{\sqrt{\varepsilon_r}} = \frac{\lambda_0}{\sqrt{\varepsilon_r}}$$

$$Z_0 = \sqrt{\frac{L'}{C'}} \qquad \text{(lossless line)}, \qquad (2.46)$$

(REAL)

Chapter 2 Review

- Voltage reflection coefficient due to load:

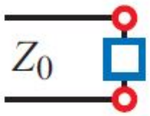
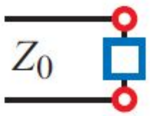
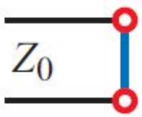
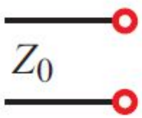
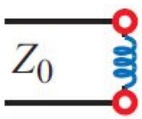
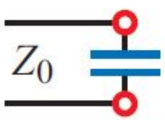
$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1} = \frac{z_L - 1}{z_L + 1}$$

- Load impedance in terms of Γ:

$$Z_L = \frac{1+\Gamma}{1-\Gamma} Z_0$$

Chapter 2 Review

Reflection Coefficient $\Gamma = |\Gamma| e^{j\theta_r}$

Load	$\lvert\Gamma\rvert$	θ_r
 $Z_L = (r + jx)Z_0$	$\left[\frac{(r-1)^2 + x^2}{(r+1)^2 + x^2}\right]^{1/2}$	$\tan^{-1}\left(\frac{x}{r-1}\right) - \tan^{-1}\left(\frac{x}{r+1}\right)$
 Z_0	0 (no reflection)	irrelevant
 (short)	1	$\pm 180°$ (phase opposition)
 (open)	1	0 (in-phase)
 $jX = j\omega L$	1	$\pm 180° - 2\tan^{-1} x$
 $jX = \frac{-j}{\omega C}$	1	$\pm 180° + 2\tan^{-1} x$

$$z_L = Z_L/Z_0 = (R + jX)/Z_0 = r + jx$$

Chapter 2 Review

- Concept of standing wave

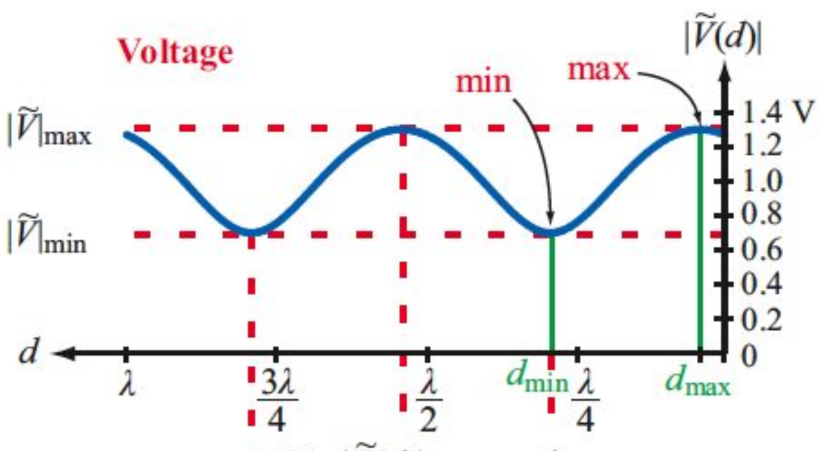
$$\widetilde{V}(z) = V_0^+(e^{-j\beta z} + \Gamma e^{j\beta z}),$$

$$\widetilde{I}(z) = \frac{V_0^+}{Z_0}(e^{-j\beta z} - \Gamma e^{j\beta z}).$$

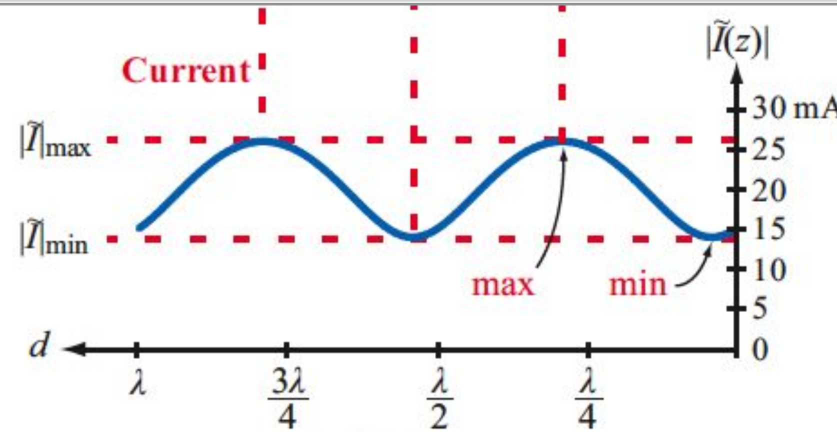
  - Voltage (current) magnitudes at any point on line:

$$|\widetilde{V}(d)| = |V_0^+|\left[1 + |\Gamma|^2 + 2|\Gamma|\cos(2\beta d - \theta_r)\right]^{1/2}. \quad (2.66)$$

Chapter 2 Review



(a) $|\tilde{V}(d)|$ versus d



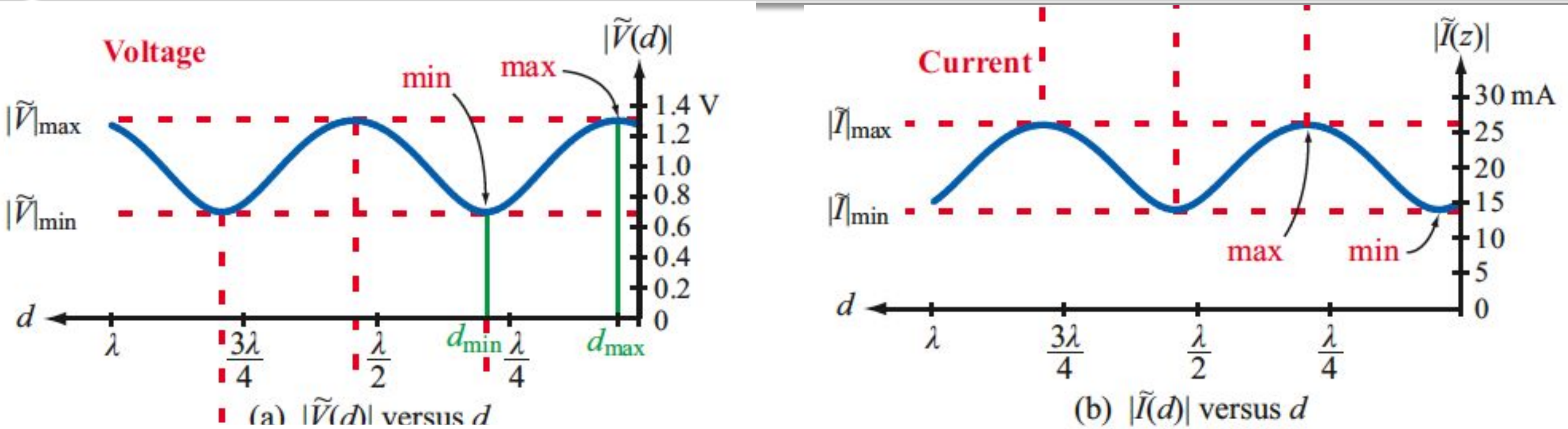
(b) $|\tilde{I}(d)|$ versus d

- Location of minima / maxima

$$d_{max} = \frac{\theta_r + 2n\pi}{2\beta} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2}, \quad \begin{cases} n = 1, 2, \ldots & \text{if } \theta_r < 0, \\ n = 0, 1, 2, \ldots & \text{if } \theta_r \geq 0, \end{cases} \qquad (2.70)$$

Value of V_{max}: $|\tilde{V}(d)| = |\tilde{V}|_{max} = |V_0^+|[1 + |\Gamma|],$

Chapter 2 Review



- Spatial period of standing wave: $\frac{\lambda}{2}$

- Standing wave ratio S:

$$S = \frac{|\tilde{V}|_{\max}}{|\tilde{V}|_{\min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Chapter 2 Review

Wave Impedance:

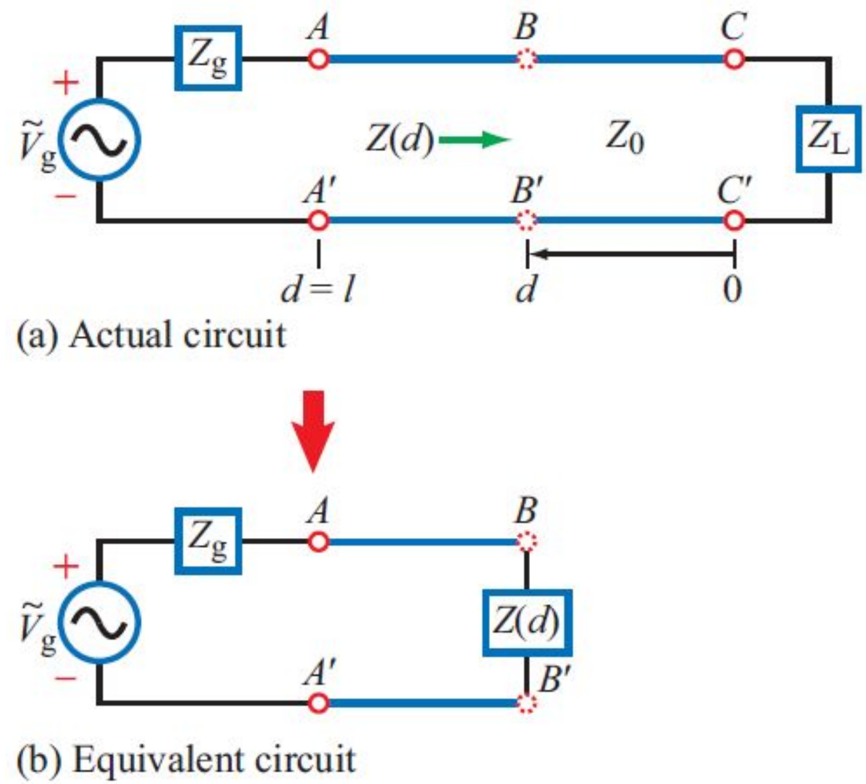
At a distance *d* from the load:

$$Z(d) = \frac{\tilde{V}(d)}{\tilde{I}(d)}$$

$$= \frac{V_0^+[e^{j\beta d} + \Gamma e^{-j\beta d}]}{V_0^+[e^{j\beta d} - \Gamma e^{-j\beta d}]} Z_0$$

$$= Z_0 \left[\frac{1 + \Gamma e^{-j2\beta d}}{1 - \Gamma e^{-j2\beta d}} \right]$$

$$= Z_0 \left[\frac{1 + \Gamma_d}{1 - \Gamma_d} \right] \qquad (\Omega),$$

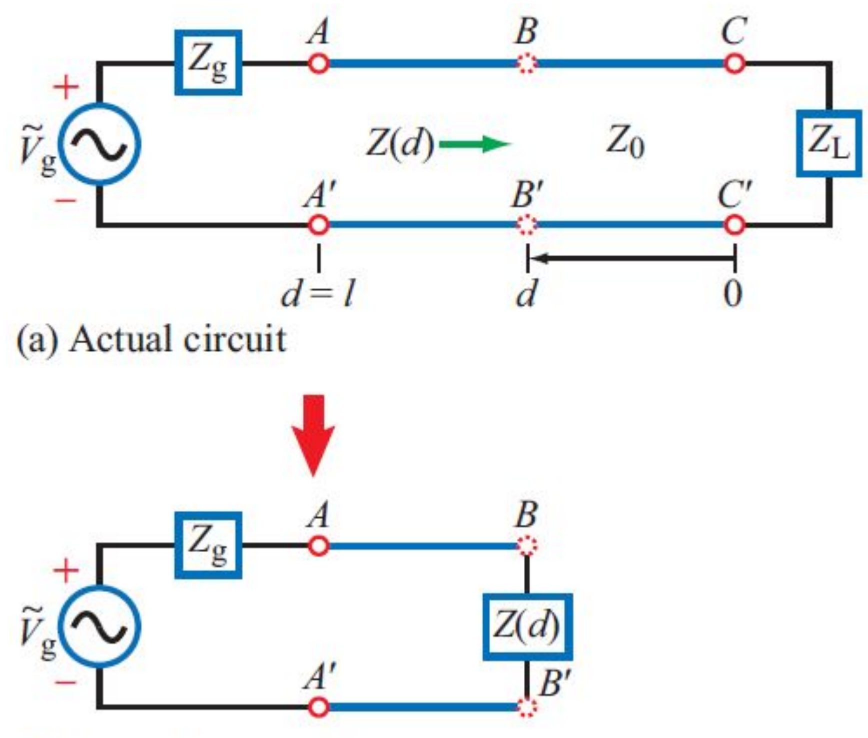


Chapter 2 Review

Define the phase-shifted voltage reflection coefficient:

$$\Gamma_d = \Gamma e^{-j2\beta d} = |\Gamma| e^{j\theta_r} e^{-j2\beta d}$$

$$= |\Gamma| e^{j(\theta_r - 2\beta d)}$$



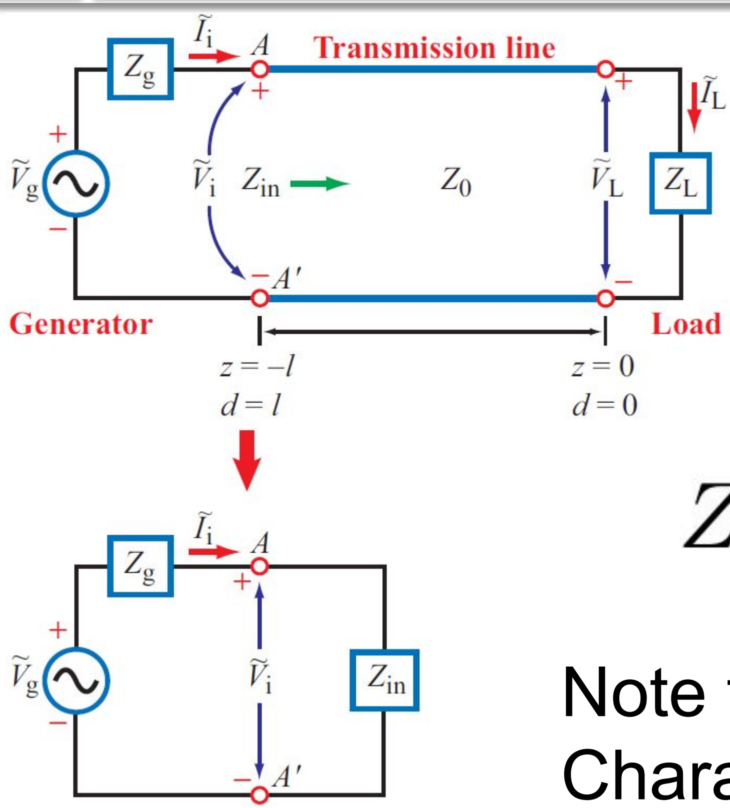
(a) Actual circuit

(b) Equivalent circuit

Z(d) is different than Z_0:
Ratio of Total Voltage and Current

Recall: $Z_0 = \dfrac{V_0^+}{I_0^+}$

Chapter 2 Review



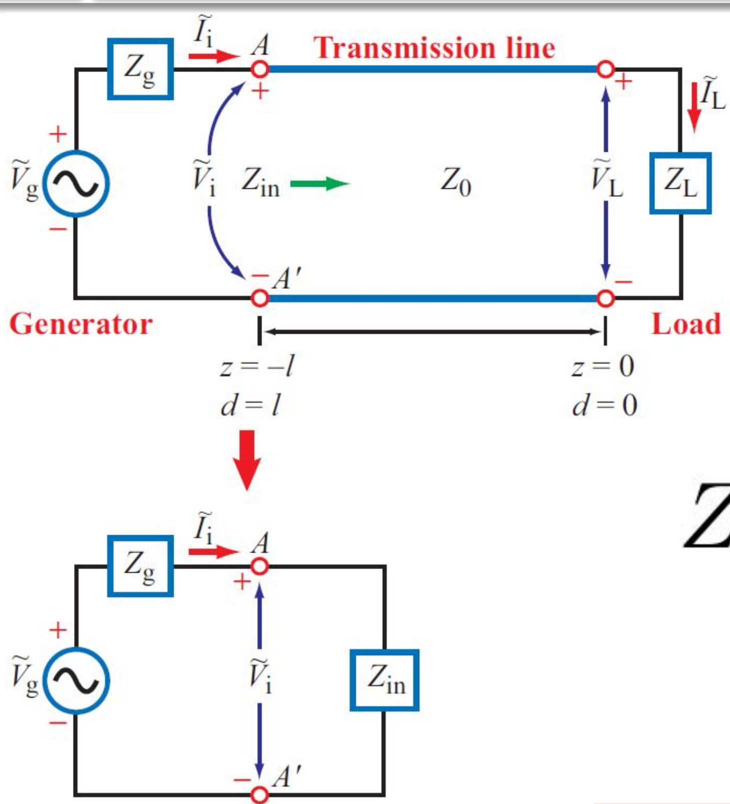
Input impedance:
impedance of the transmission line at *d=l:*

$$Z_{\text{in}} = Z(d=l) = \frac{\widetilde{V}(d=l)}{\widetilde{I}(d=l)}$$

Note that Z_{in} is different from the Characteristic Impedance, and is different from the Load Impedance:

$$Z_{\text{in}} \neq Z_0 \qquad\qquad Z_{\text{in}} \neq Z_L$$

Chapter 2 Review



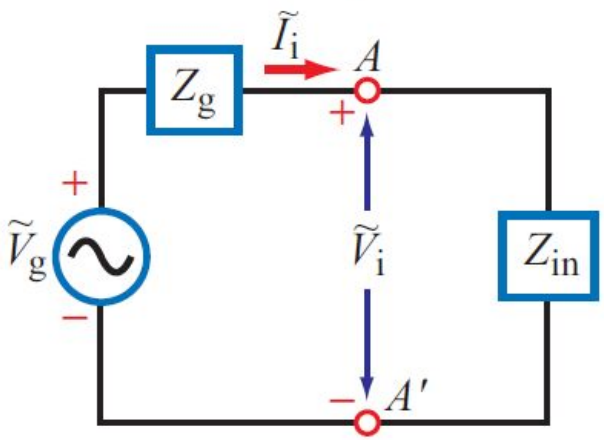
Input impedance:

impedance of the transmission line at *d=l:*

$$Z_{in} = Z(d = l) = \frac{\widetilde{V}(d = l)}{\widetilde{I}(d = l)}$$

$$Z_{in} = Z_0 \left[\frac{z_L + j\tan(\beta l)}{1 + jz_L \tan(\beta l)} \right]$$

Chapter 2 Review



Voltage Amplitude:

$$V_0^+ = \left(\frac{\tilde{V}_g Z_{in}}{Z_g + Z_{in}}\right)\left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right).$$

We used the boundary conditions (Z_L, Z_g , V_g, l) to solve for the 2 unknowns.

This completes the solution of the transmission line differential equation.

Chapter 2 Review

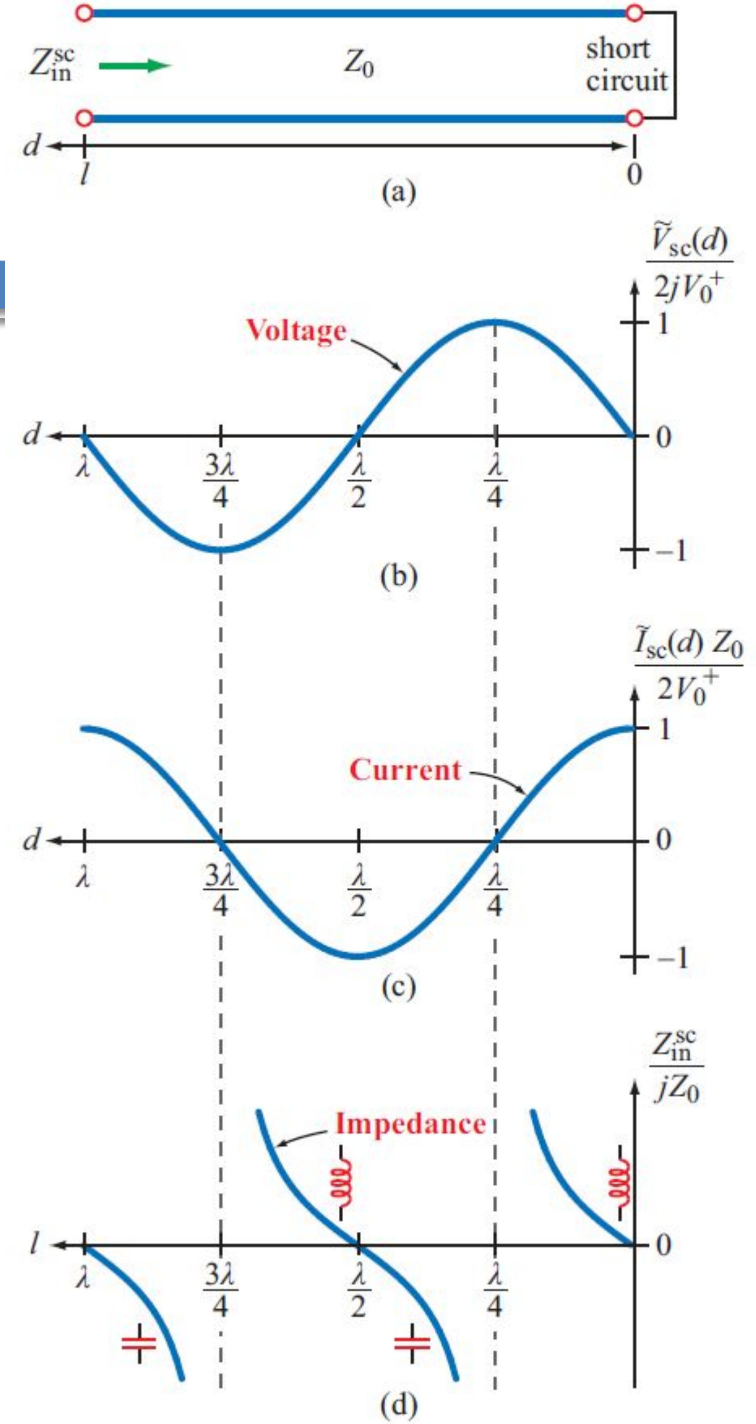
For the short-circuited line:

$$\Gamma = -1$$

$$\widetilde{V}_{sc}(d) = 2jV_0^+ \sin \beta d,$$

$$\widetilde{I}_{sc}(d) = \frac{2V_0^+}{Z_0} \cos \beta d,$$

$$Z_{sc}(d) = \frac{\widetilde{V}_{sc}(d)}{\widetilde{I}_{sc}(d)} = jZ_0 \tan \beta d.$$



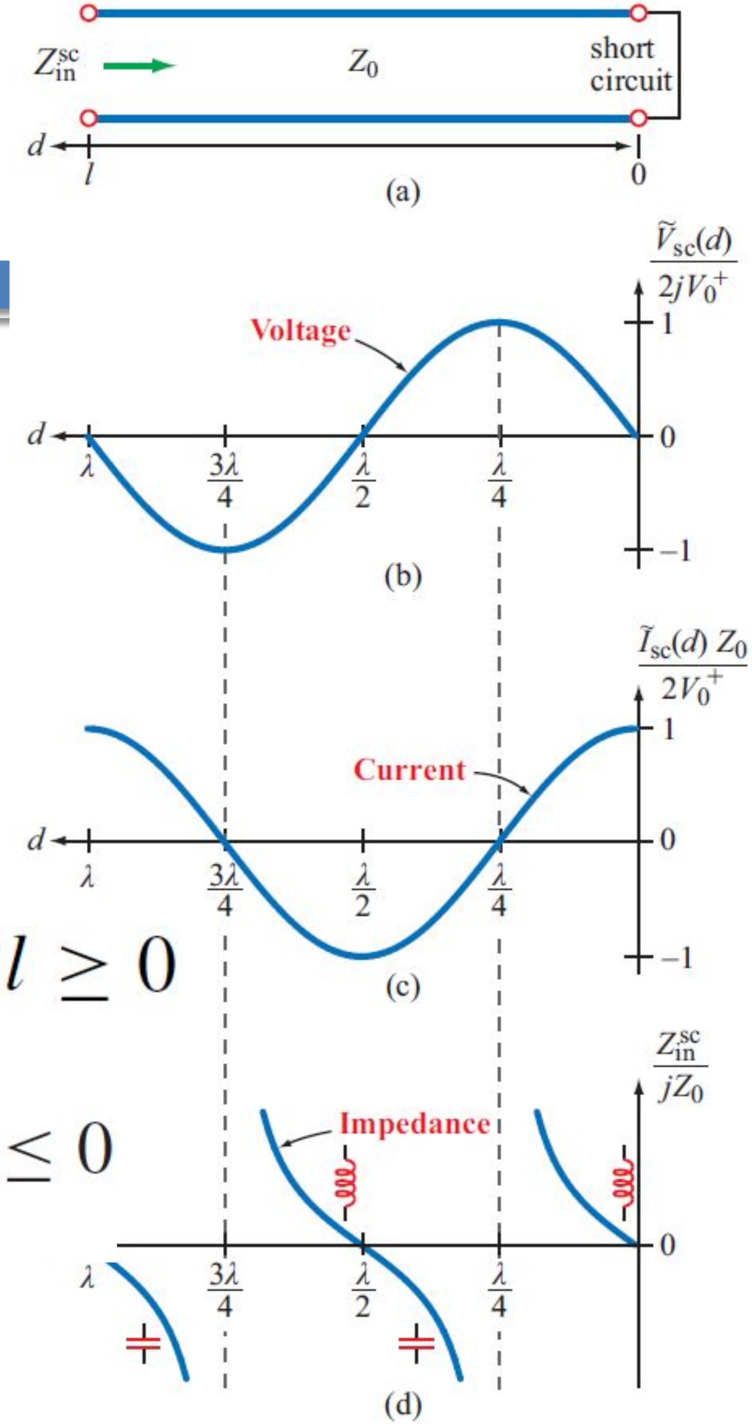
Chapter 2 Review

At its input, the short-circuited line **appears** like an inductor or a capacitor depending on the sign of

$$\tan \beta d$$

$$j\omega L_{\text{eq}} = jZ_0 \tan \beta l, \qquad \text{if } \tan \beta l \geq 0$$

$$\frac{1}{j\omega C_{\text{eq}}} = jZ_0 \tan \beta l, \qquad \text{if } \tan \beta l \leq 0$$



Chapter 2 Review

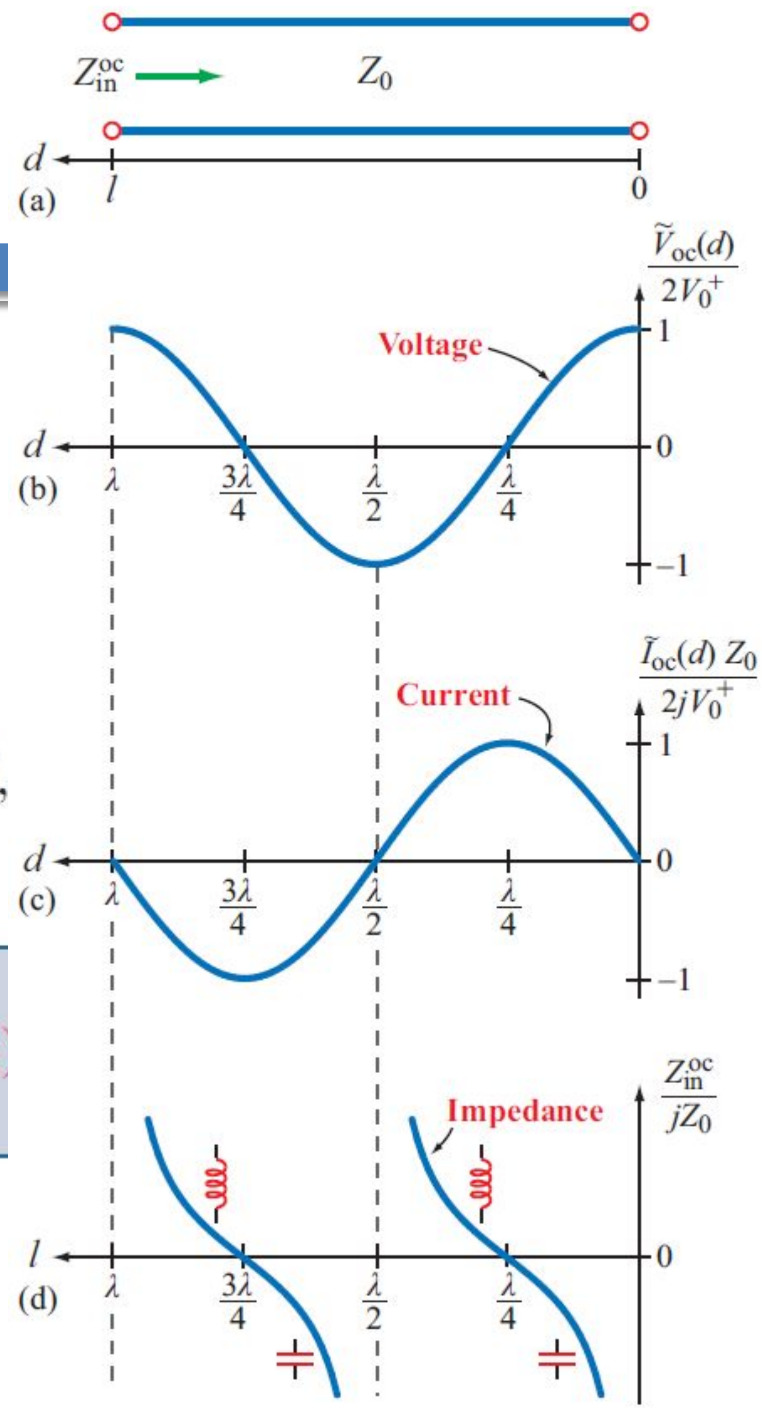
Open-circuited Line:

$$\Gamma = 1$$

$$\widetilde{V}_{oc}(d) = V_0^+[e^{j\beta d} + e^{-j\beta d}] = 2V_0^+ \cos \beta d,$$

$$\widetilde{I}_{oc}(d) = \frac{V_0^+}{Z_0}[e^{j\beta d} - e^{-j\beta d}] = \frac{2jV_0^+}{Z_0} \sin \beta d,$$

$$Z_{in}^{oc} = \frac{\widetilde{V}_{oc}(l)}{\widetilde{I}_{oc}(l)} = -jZ_0 \cot \beta l. \quad (2.93)$$



Chapter 2 Review

Short-Circuit/Open-Circuit Method:

- Given length *l*
- Measure Z_{in} twice:
  - when terminated in a short
  - when terminated in an open

Use both: get Z_0, β:

$$Z_{in}^{sc} = \frac{\widetilde{V}_{sc}(l)}{\widetilde{I}_{sc}(l)} = jZ_0 \tan\beta l.$$

$$Z_{in}^{oc} = \frac{\widetilde{V}_{oc}(l)}{\widetilde{I}_{oc}(l)} = -jZ_0 \cot\beta l.$$

$$Z_0 = +\sqrt{Z_{in}^{sc}\, Z_{in}^{oc}}\,,$$

$$\tan\beta l = \sqrt{\frac{-Z_{in}^{sc}}{Z_{in}^{oc}}}\,.$$

Chapter 2 Review

Half-Wavelength Line:

$$Z_{\text{in}} = Z_0 \left(\frac{z_L + j\tan(\beta l)}{1 + j z_L \tan(\beta l)} \right)$$

If $l = n\lambda/2$, where n is an integer,

$$\tan \beta l = \tan[(2\pi/\lambda)(n\lambda/2)] = \tan n\pi = 0.$$

Consequently, Eq. (2.79) reduces to

$$Z_{\text{in}} = Z_{\text{L}}, \qquad \text{for } l = n\lambda/2, \qquad (2.96)$$

which means that a half-wavelength line (or any integer multiple of $\lambda/2$) does not modify the load impedance.

Chapter 2 Review

Quarter-Wavelength Line:

$$Z_{\text{in}} = Z_0 \left(\frac{z_L + j\tan(\beta l)}{1 + j z_L \tan(\beta l)} \right)$$

For $l = \lambda/4$, $\beta l = (2\pi/\lambda)(\lambda/4) = \pi/2$

So, as $\beta l \to \pi/2$, $\tan(\beta l) \to \infty$

$$\lim_{\beta l \to \pi/2} Z_{\text{in}} = Z_0 \left(\frac{j\tan(\beta l)}{j z_L \tan(\beta l)} \right) = \frac{Z_0^2}{Z_L}$$

$$Z_{\text{in}} = \frac{Z_0^2}{Z_L}, \qquad \text{for } l = \lambda/4 + n\lambda/2.$$

Chapter 2 Review

Instantaneous Power Flow:

$$P(d,t) = P^{\mathrm{i}}(d,t) + P^{\mathrm{r}}(d,t)$$

The 2 terms are the Incident and Reflected power:

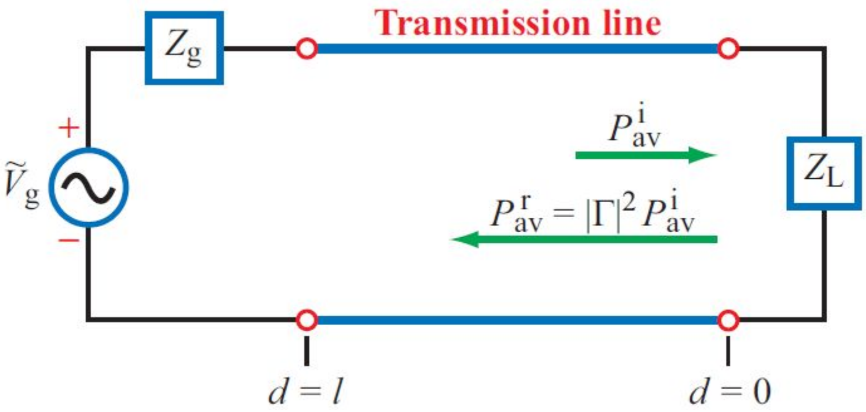
$$P^{\mathrm{i}}(d,t) = \frac{|V_0^+|^2}{2Z_0}[1 + \cos(2\omega t + 2\beta d + 2\phi^+)],$$

$$P^{\mathrm{r}}(d,t) = -|\Gamma|^2 \frac{|V_0^+|^2}{2Z_0}[1 + \cos(2\omega t - 2\beta d + 2\phi^+ + 2\theta_{\mathrm{r}})].$$

oscillating at TWICE the frequency of V or I

Chapter 2 Review

Average Power Flow:



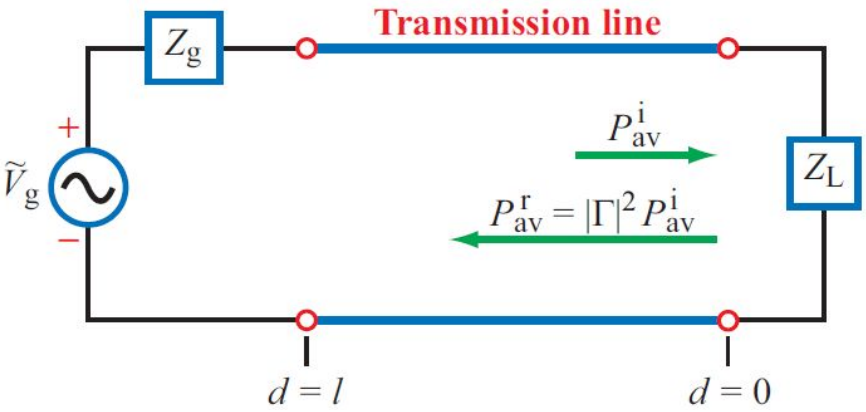
For the incident power, average over one period:

$$P^i_{\text{avg}} = \frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} \frac{|V_0^+|^2}{2Z_0} \left[1 + \cos(2\omega t + 2\beta d + 2\phi^+)\right] \text{d}t$$

$$\omega = \frac{2\pi}{T}, \qquad hence \quad T = \frac{2\pi}{\omega}$$

Chapter 2 Review

Average Power Flow:



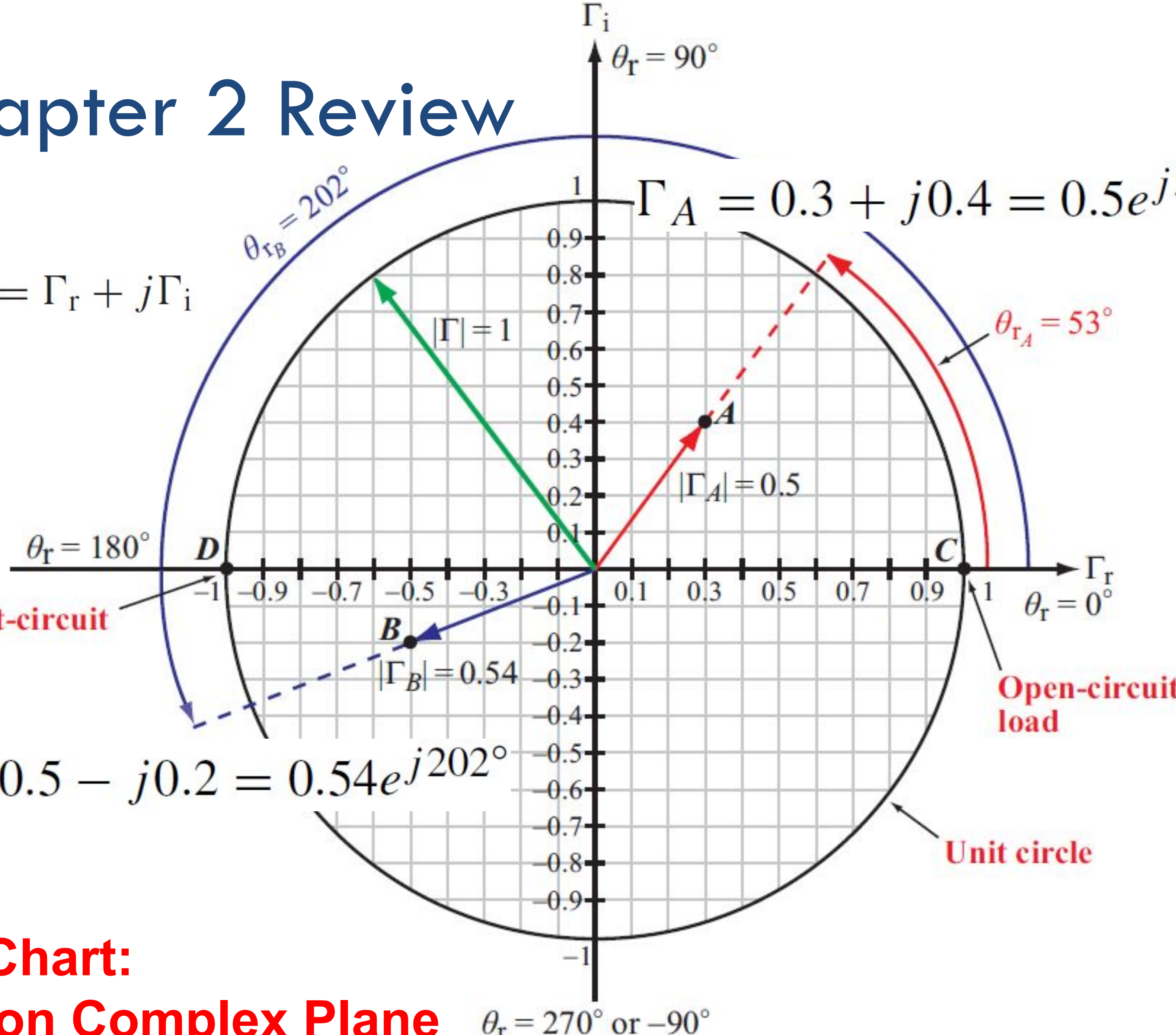
$$P^i_{\text{avg}} = \frac{|V_0^+|^2}{2Z_0}$$

$$P^r_{\text{avg}} = -|\Gamma|^2 P^i_{\text{avg}}$$

Chapter 2 Review

$$\Gamma = |\Gamma| e^{j\theta_r} = \Gamma_r + j\Gamma_i$$

$$\Gamma_A = 0.3 + j0.4 = 0.5e^{j53^\circ}$$

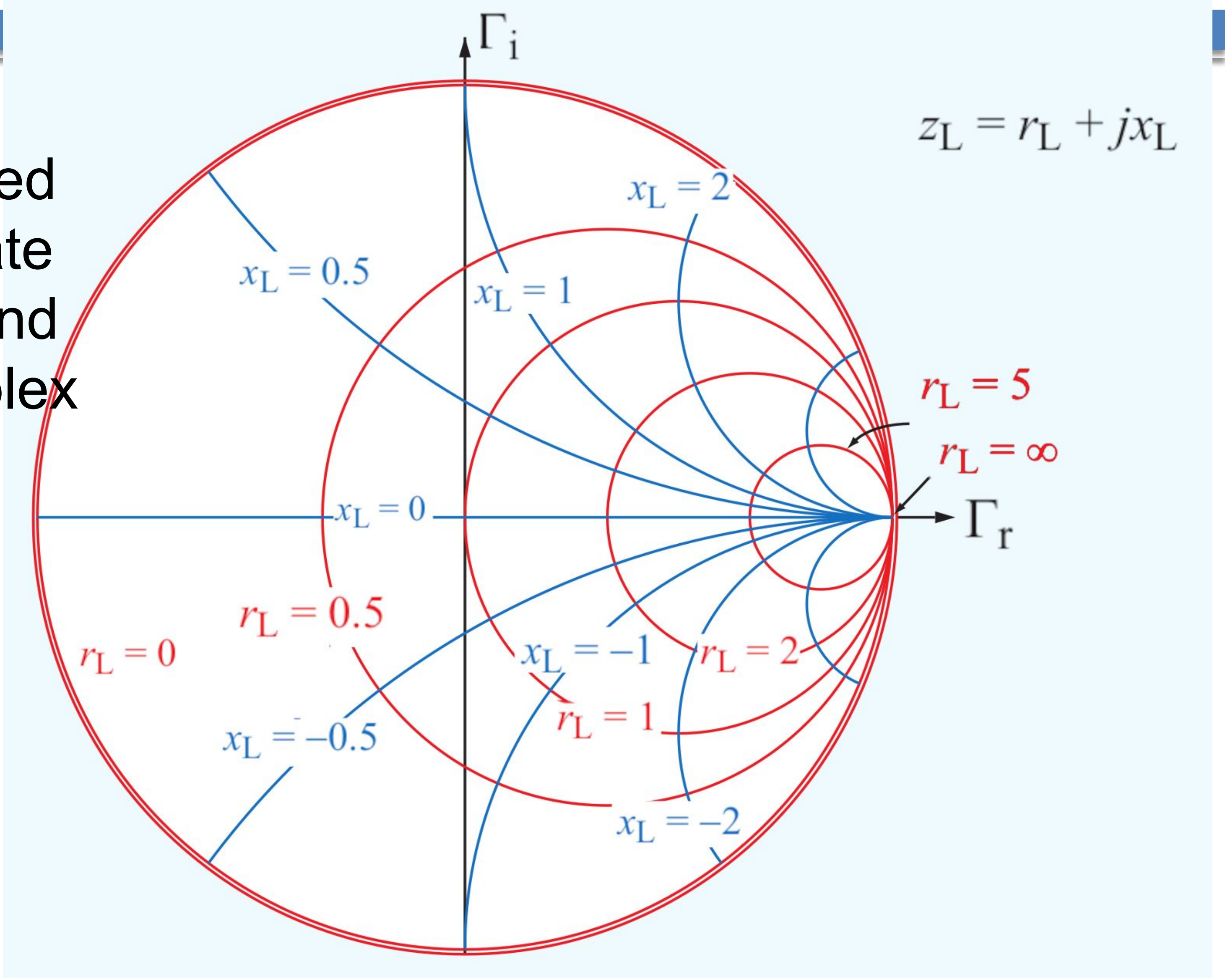


$$\Gamma_B = -0.5 - j0.2 = 0.54e^{j202^\circ}$$

Smith Chart:
Based on Complex Plane

Chapter 2 Review

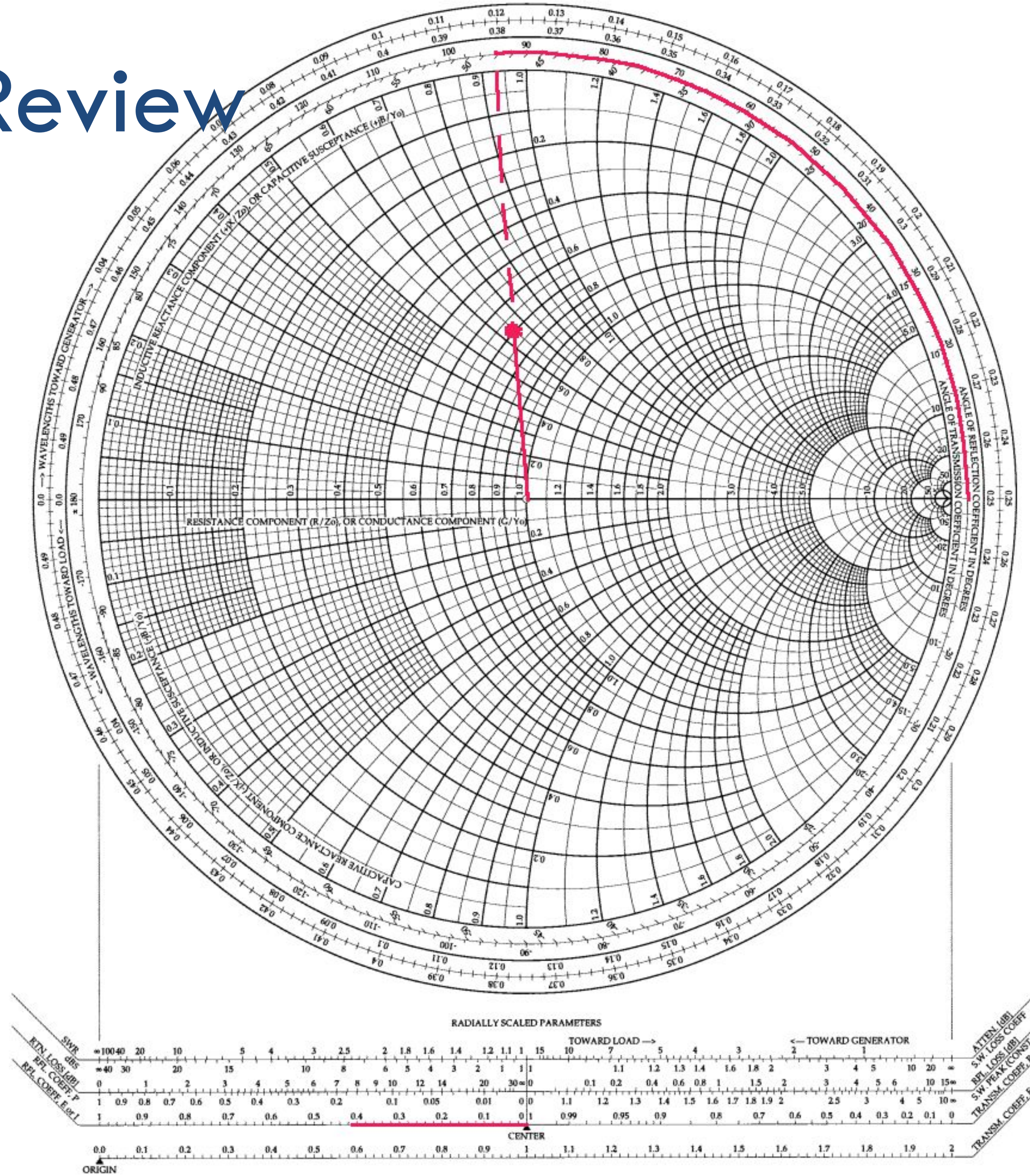
Superimposed are coordinate axes for r_L and x_L: the complex load: z_L



Chapter 2 Review

If plotted z_L:
Obtain values for Γ_L from the angle scale and magnitude scale:

0.41, 94°

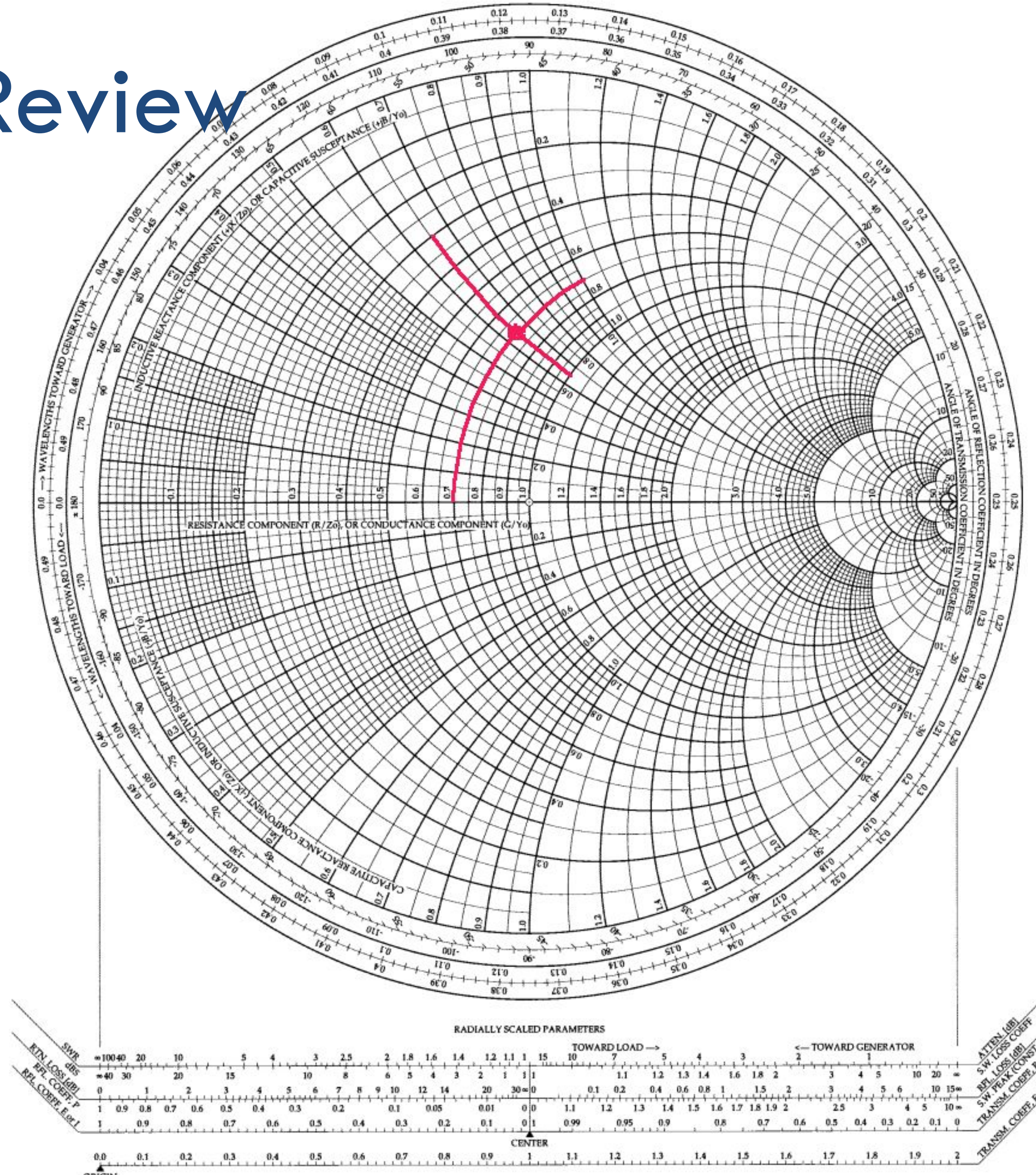


Chapter 2 Review

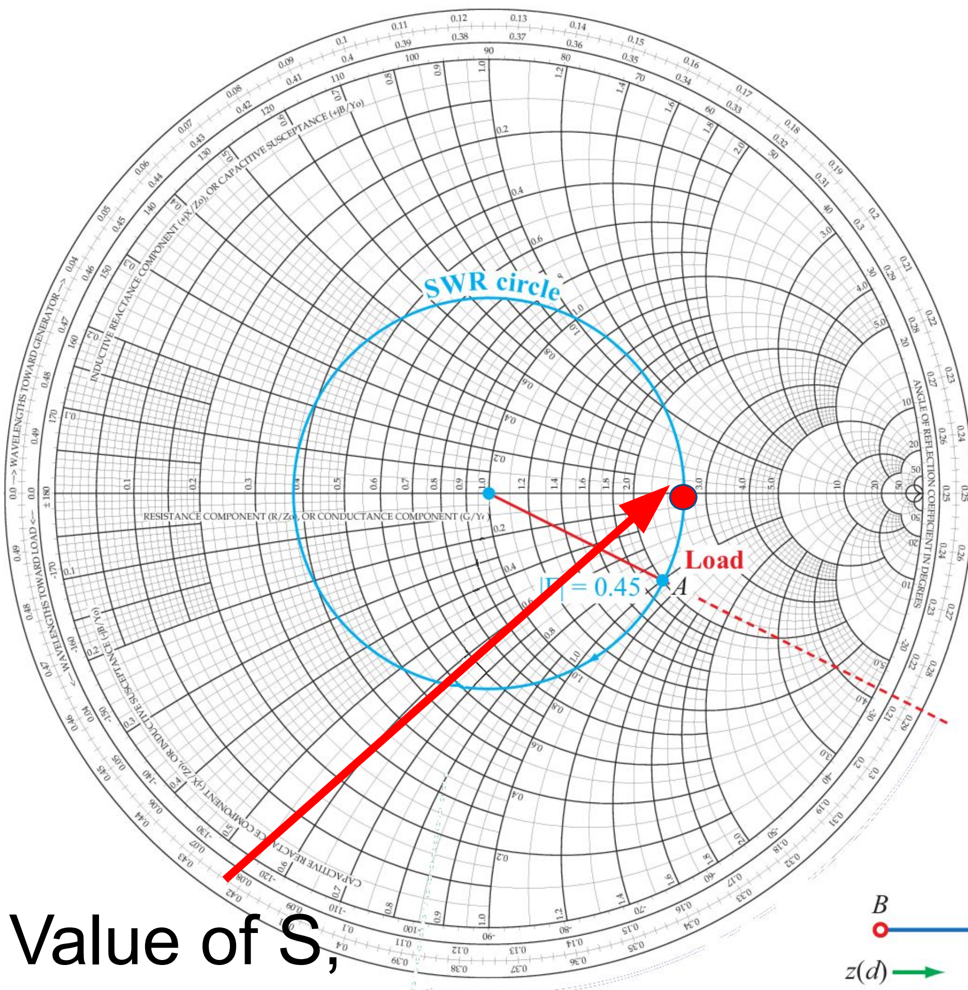
If plotted Γ_L:
Obtain values for r_L, x_L from the nearest circles:

$z_L = (0.7,0.65)$

To get Z_L:
Multiply z_L by Z_0



Chapter 2 Review



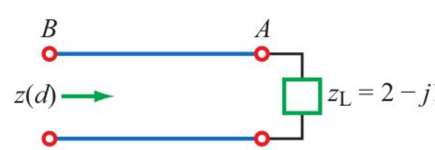
All points on a circle centered at the origin have the same $|\Gamma|$.

Since

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

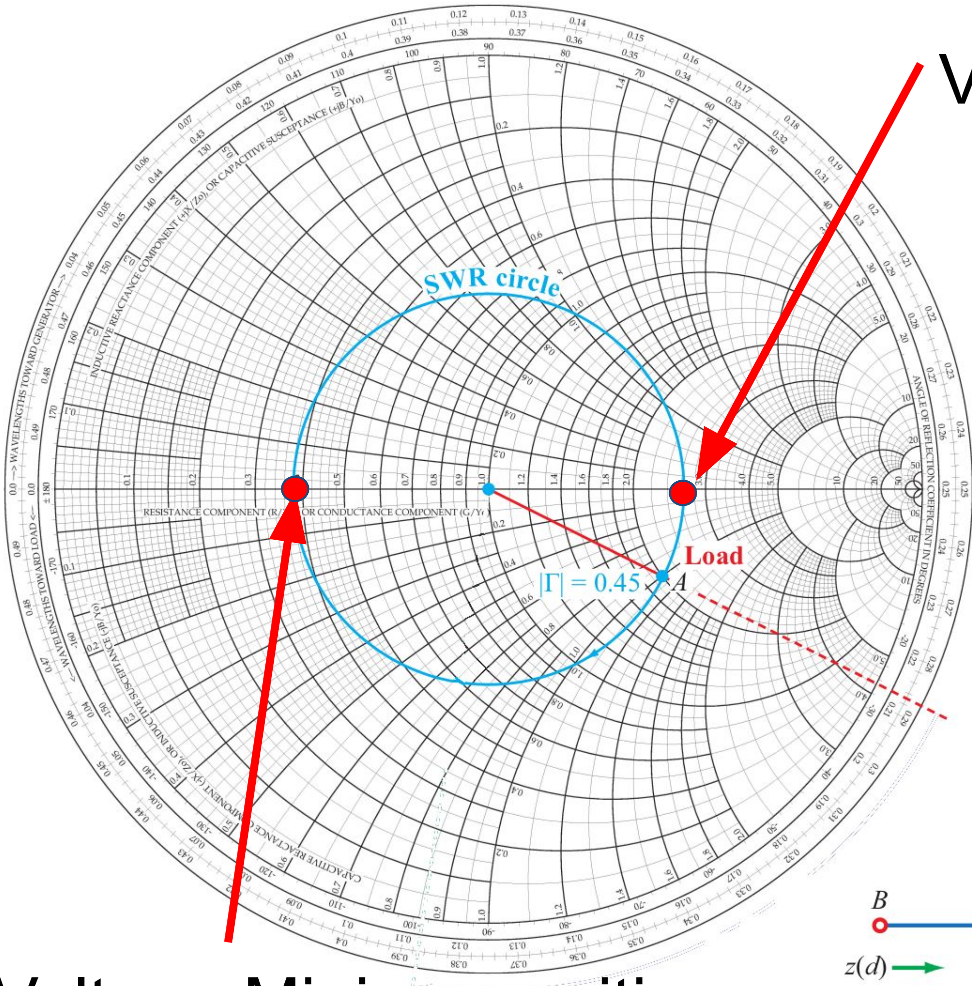
this is a also a circle of constant S (VSWR).

Value of S, from r_L scale

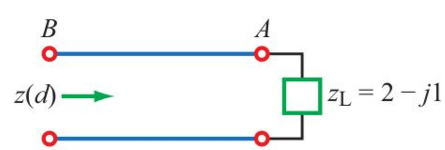


Chapter 2 Review

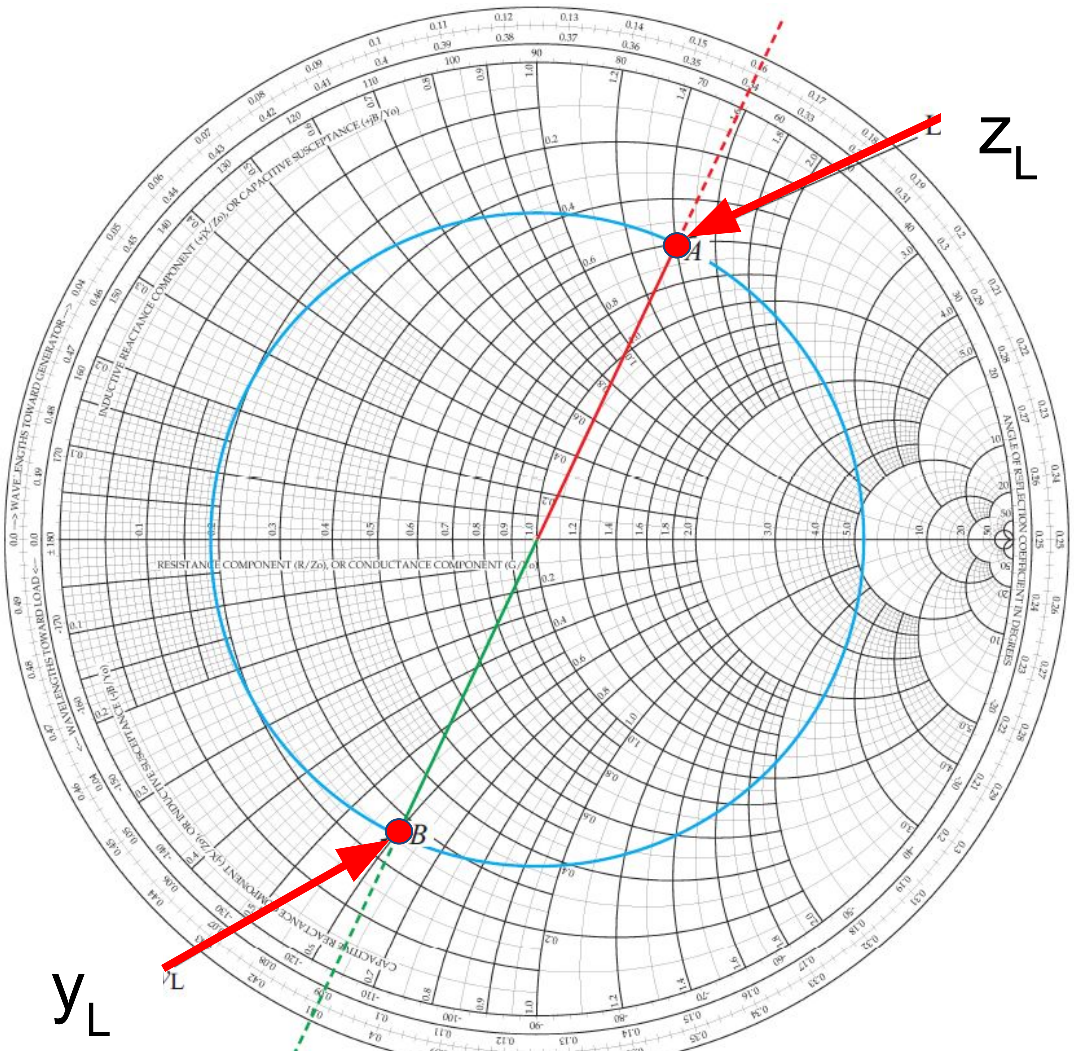
Voltage Maxima position



Voltage Minima position



Chapter 2 Review



y and z are inverses of each other.

Occur $\lambda/4$ apart on the Smith Chart.

2-11 Impedance Matching

Often, one wants to maximize the power delivered to a load.

In EECS 215 you learned to have the load resistor equal to the source resistance.

In this class it's similar:
For a lossless transmission-line:
The load resistance should be equal to the transmission-line characteristic impedance:

$$Z_L = Z_0$$

2-11 Impedance Matching

Consequences:

$$\Gamma = (Z_L - Z_0) / (Z_L + Z_0) = 0$$

no reflected wave

$$P_{av\_inc} = |V_0^+|^2/(2\, Z_0)$$

$$P_{av\_refl} = -|\Gamma|^2\, P_{av\_inc} = 0$$

So, often the load is complex, and so matching it to the transmission-line is important.

2-11 Impedance Matching

Common Scenario:

The load was optimized with other issues in mind

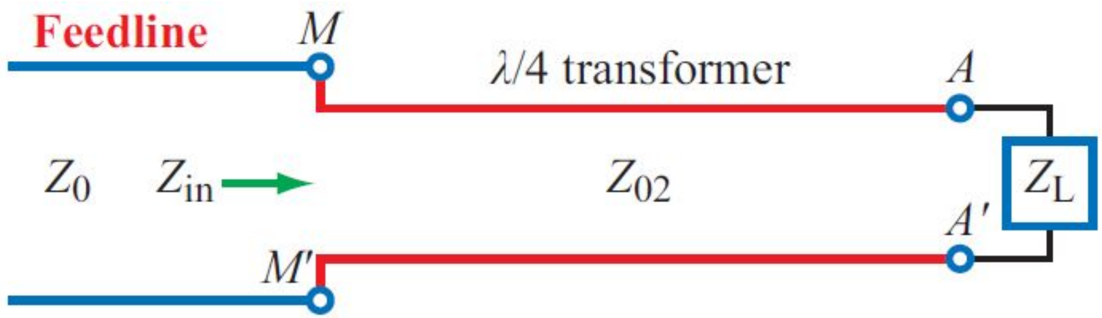
The load is not matched to the transmission-line

We need to match it... somehow.

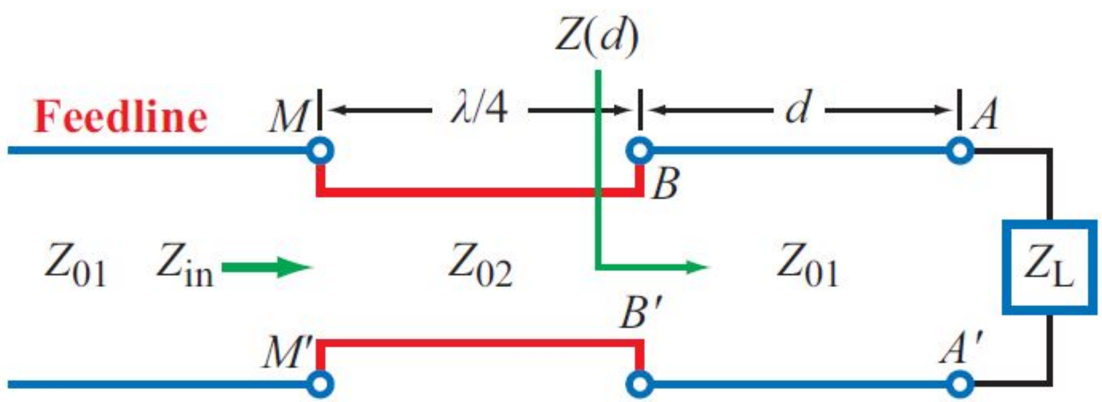
Can add something in series or parallel to modify the *equivalent impedance* to be = Z_0

2-11 Impedance Matching

Example Matching Networks



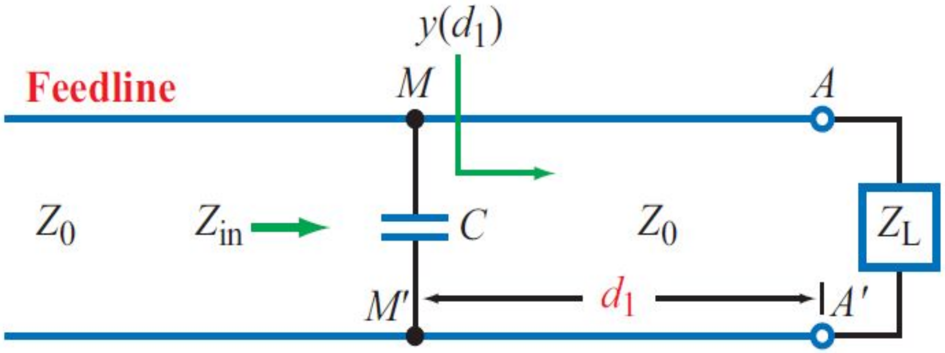
(a) In-series $\lambda/4$ transformer inserted at AA'



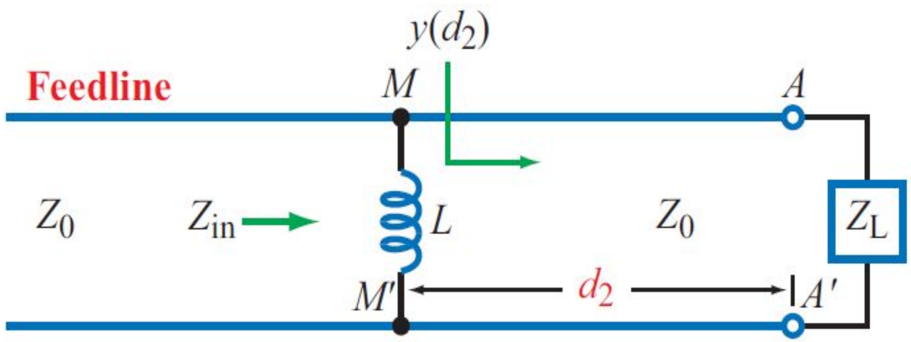
(b) In-series $\lambda/4$ transformer inserted at $d = d_{max}$ or $d = d_{min}$

2-11 Impedance Matching

Example Matching Networks



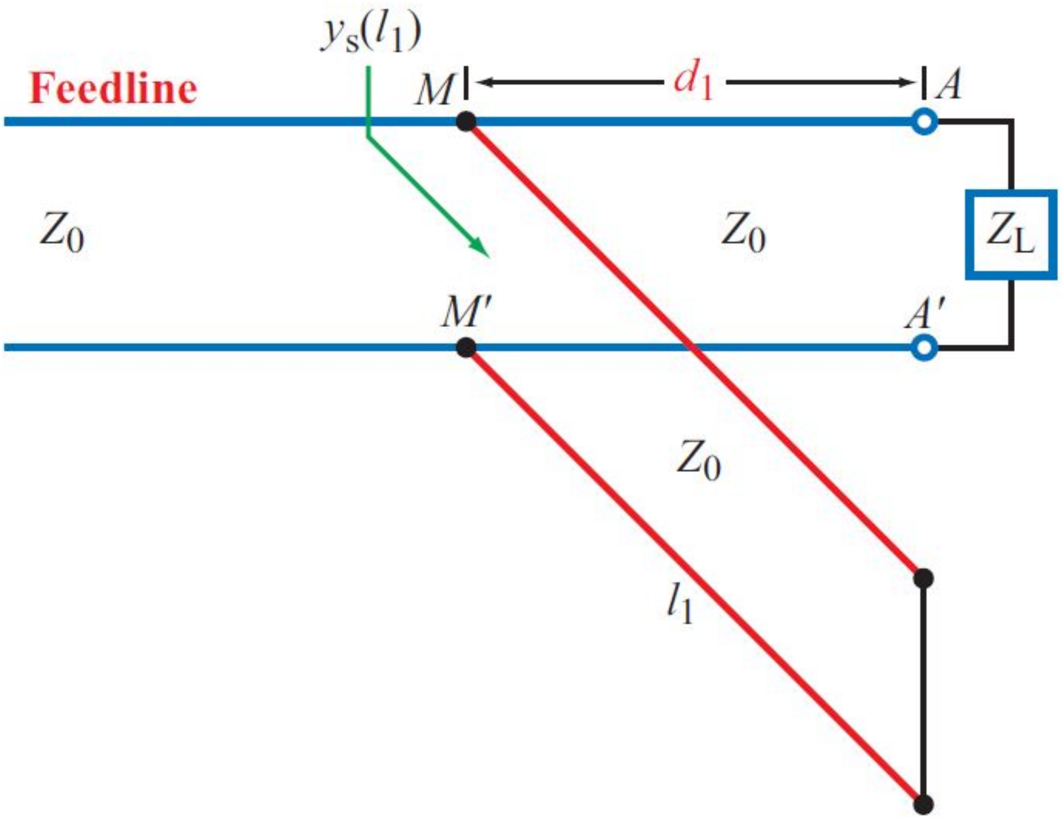
(c) In-parallel insertion of capacitor at distance d_1



(d) In-parallel insertion of inductor at distance d_2

2-11 Impedance Matching

Example Matching Networks



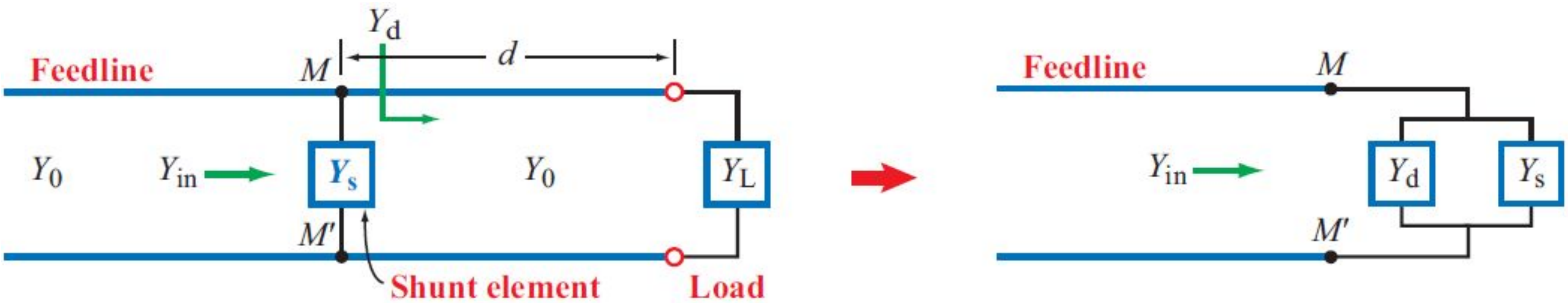
(e) In-parallel insertion of a short-circuited stub

In General:

Matching Networks may consist of:
* lumped elements, or:
* TL sections,

Placed in
* series or
* parallel.

2-11 Impedance Matching



Lumped-Element matching using admittance (*Y*=1/*Z*)

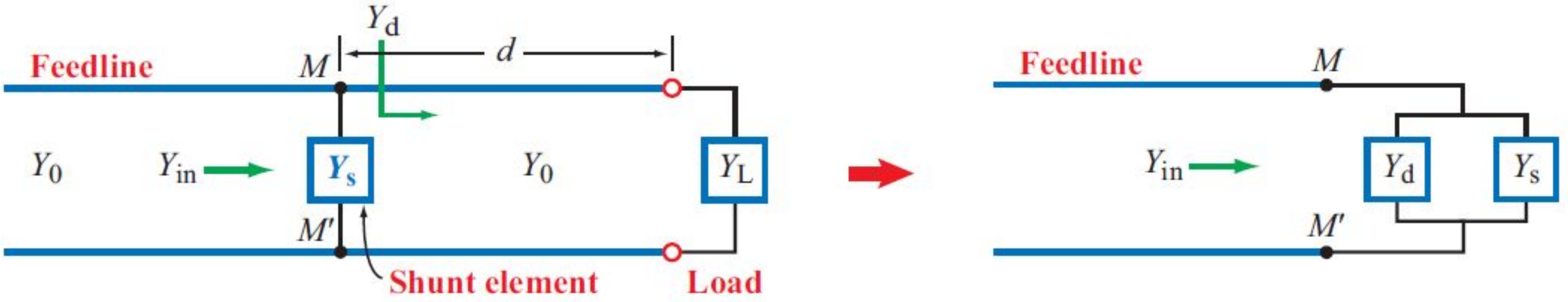
Y_L transformed to Y_d by the length of transmission line

$$Y_{in} = Y_d + Y_s$$

$$Y_{in} = (G_d + jB_d) + jB_s$$
$$= G_d + j(B_d + B_s).$$

Y_S only needs to be reactive,
to cancel
the reactive part of Y_d

2-11 Impedance Matching



Lumped-Element matching using admittance

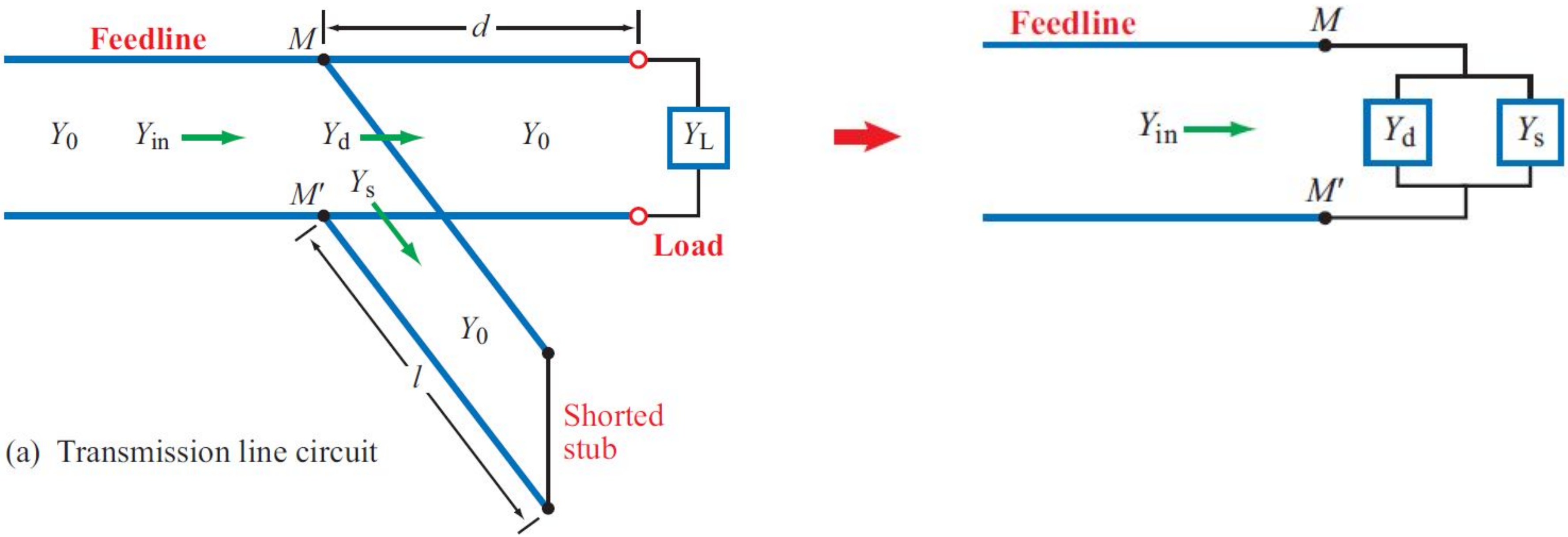
Normalized:

$$y_{in} = g_d + j(b_d + b_s).$$

$$g_d = 1 \quad \text{(real-part condition)},$$

$$b_s = -b_d \quad \text{(imaginary-part condition)}.$$

2-11 Impedance Matching

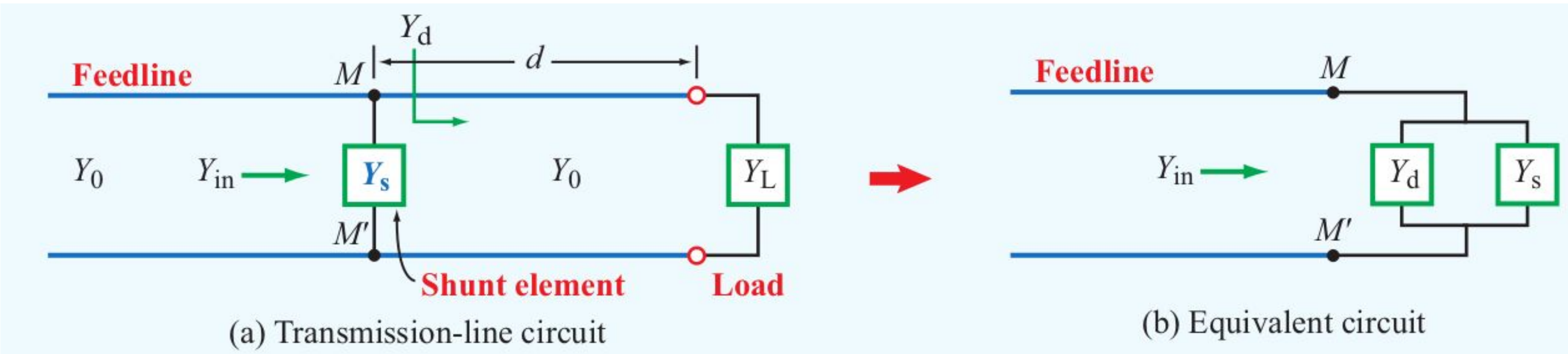


Single-stub Matching:

Can use the same process as we did for lumped-element matching.

Except: transform y_s to a length of shorted transmission line instead of a lumped element.

Example 2-13 Lumped-Element Matching

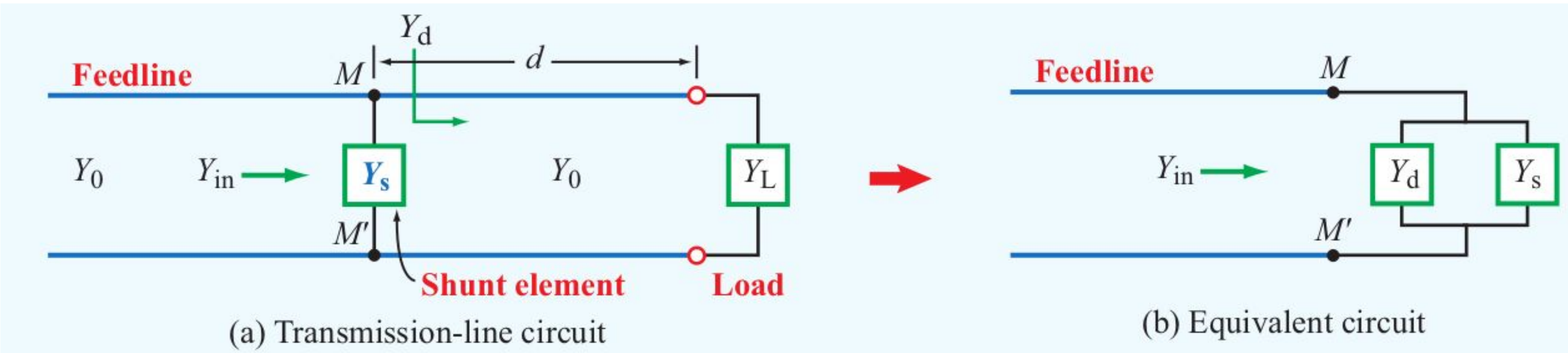


Given: Lossless transmission line, Z_0 = 50 Ω

Load: Z_L = 25 - j 50 Ω, f= 100 MHz

Find: Element *type* and *value* and *d* to match the load.

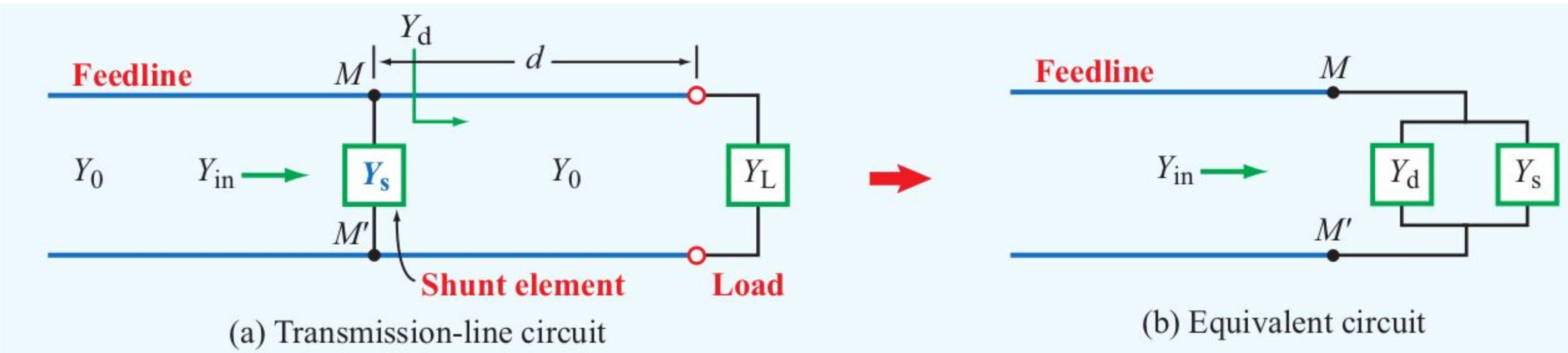
Example 2-13 Lumped-Element Matching



Strategy:

1. choose *d* to get real-part of Z_{equiv} = 50 Ω.
   will still have non-zero imag-part
2. then choose element to cancel that imag-part.
   will be an inductor or a capacitor

Example 2-13 Lumped-Element Matching



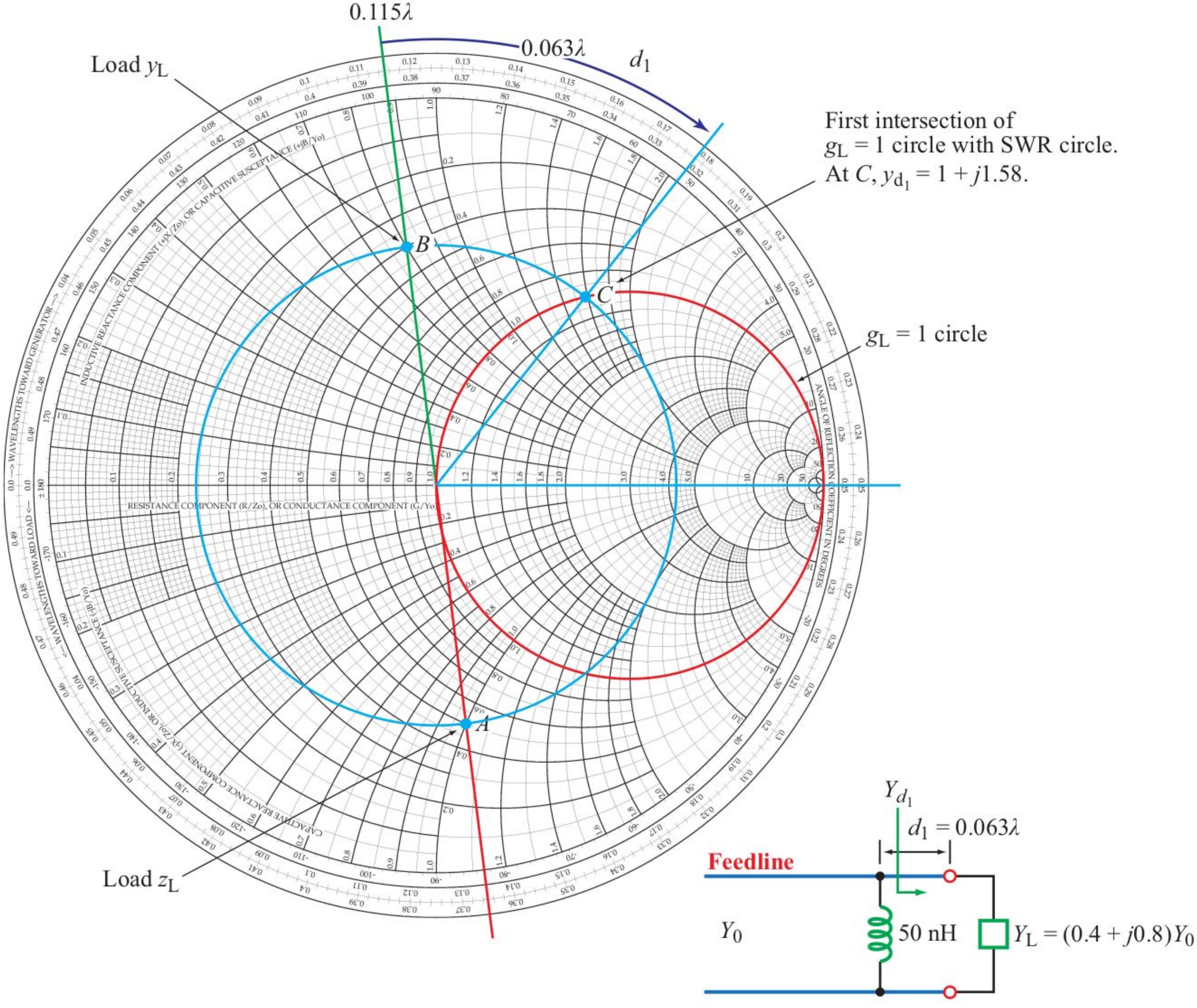
Solution:

step1: normalize the load and plot on Smith Chart

$$z_L = Z_L / Z_0 = (25 - j\,50) / 50 = 0.5 - j1$$

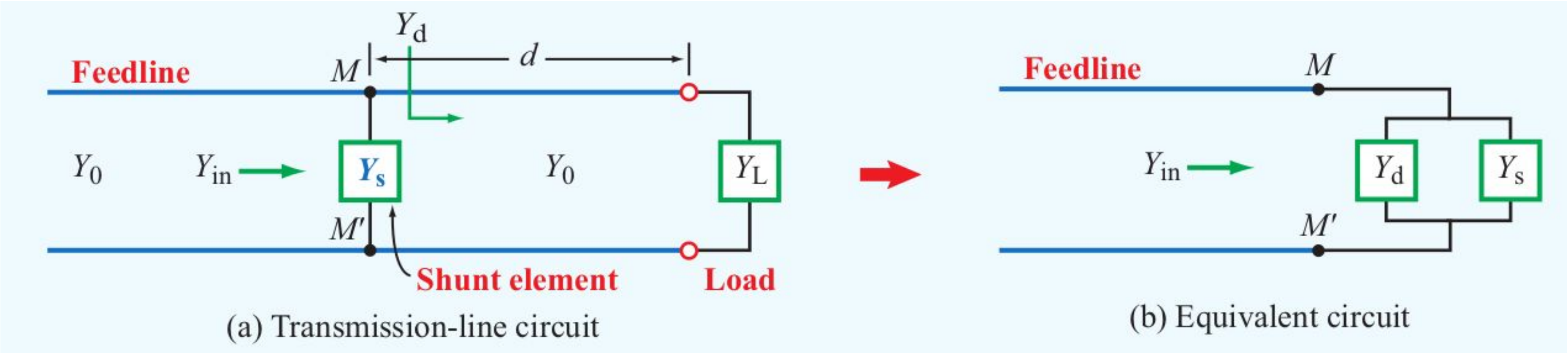
step2: draw the VSWR circle thru z_L

step3: locate y_L across the circle from z_L



First solution

Example 2-13 Lumped-Element Matching

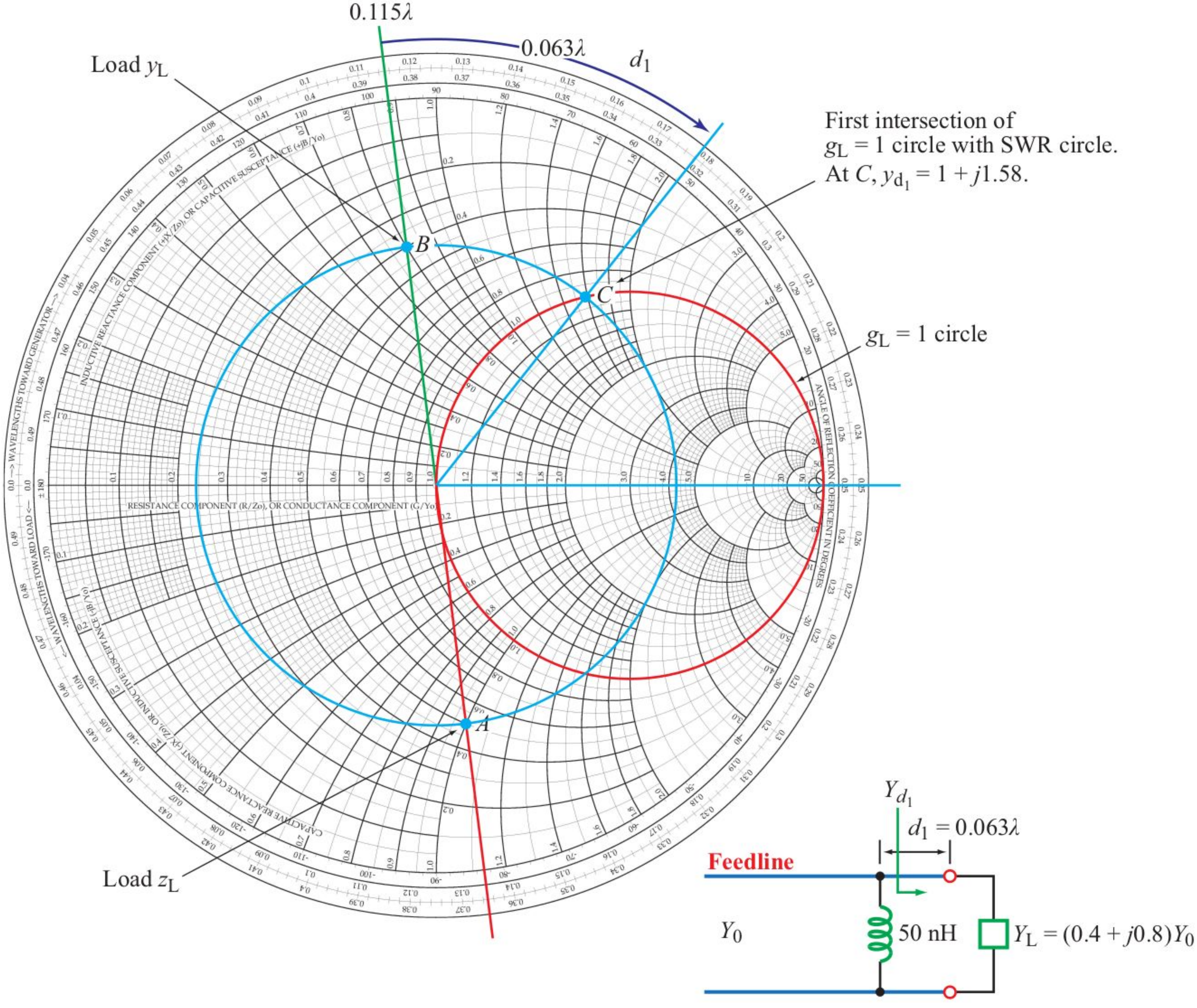


Solution:

step4: read the value of y_L from the chart:

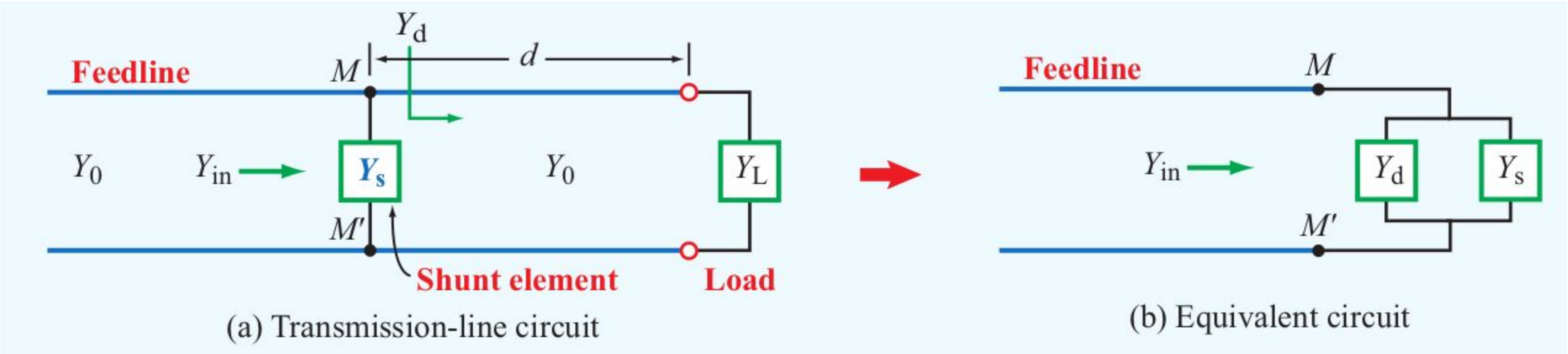
$$y_L = 0.4 + j0.8$$

step5: draw radial line thru y_L, read off WTG: 0.115λ



First solution

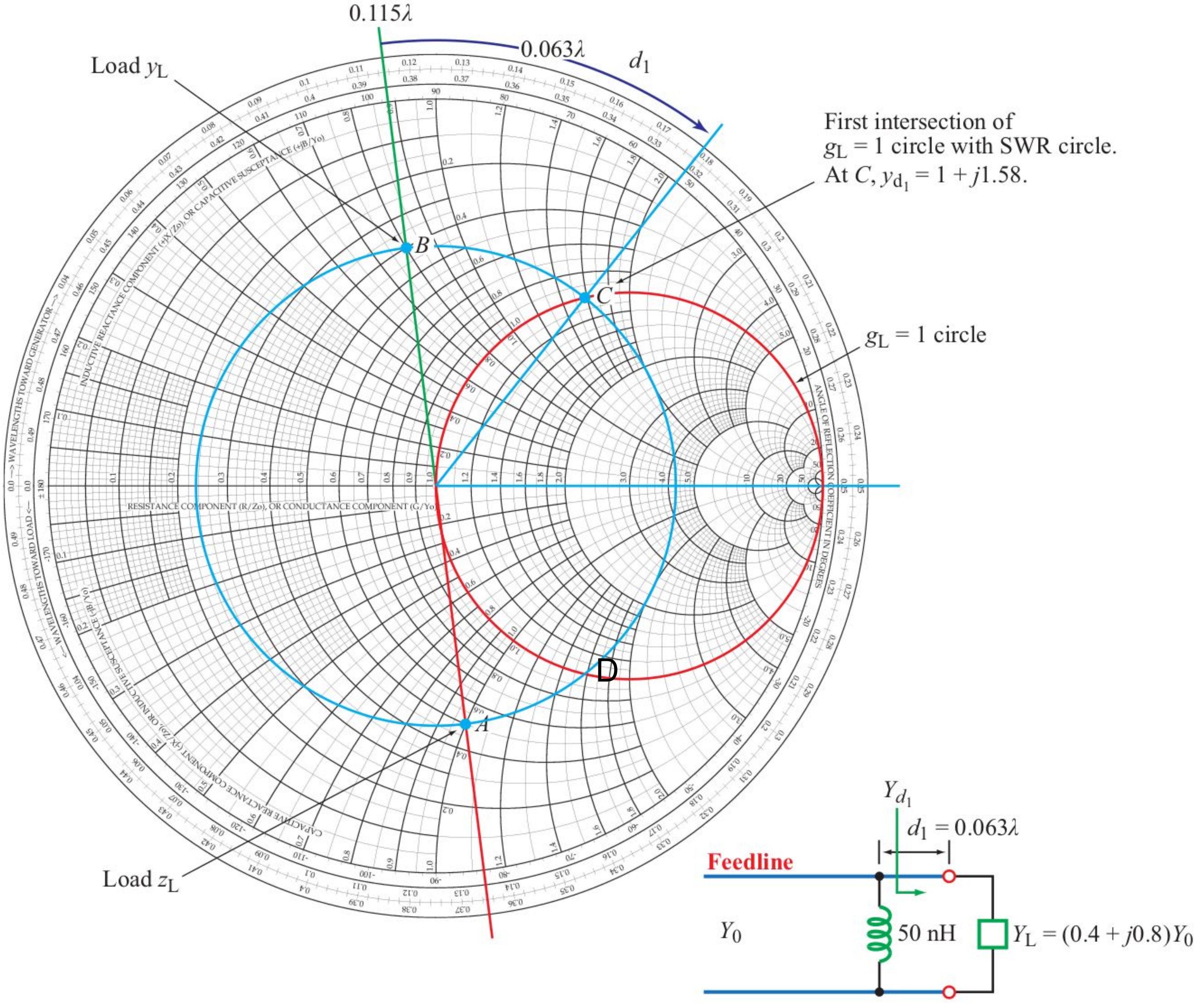
Example 2-13 Lumped-Element Matching



Solution:

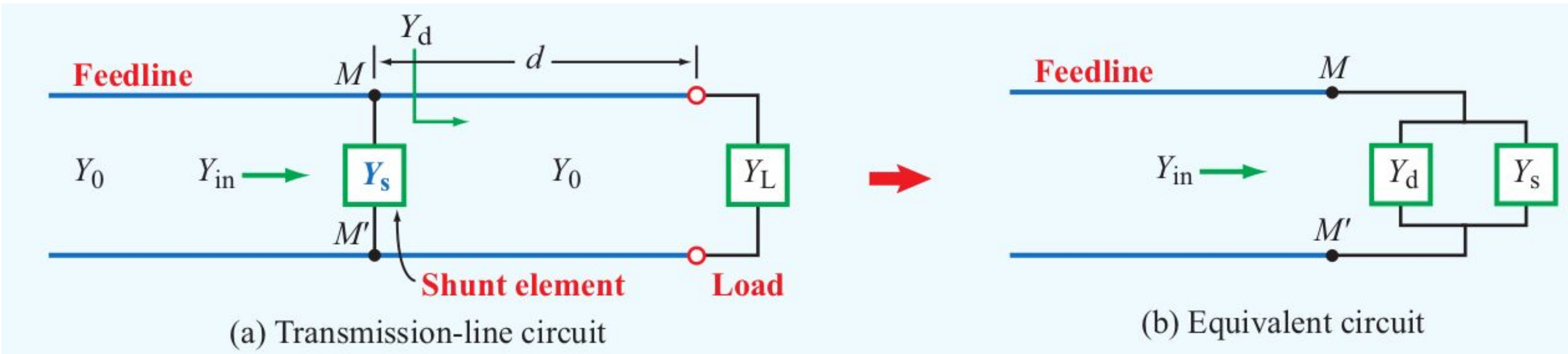
step6: move toward generator until Re.y_d = 1

note: there are 2 values of *d* where this is true: C, D



First solution

Example 2-13 Lumped-Element Matching



Solution:

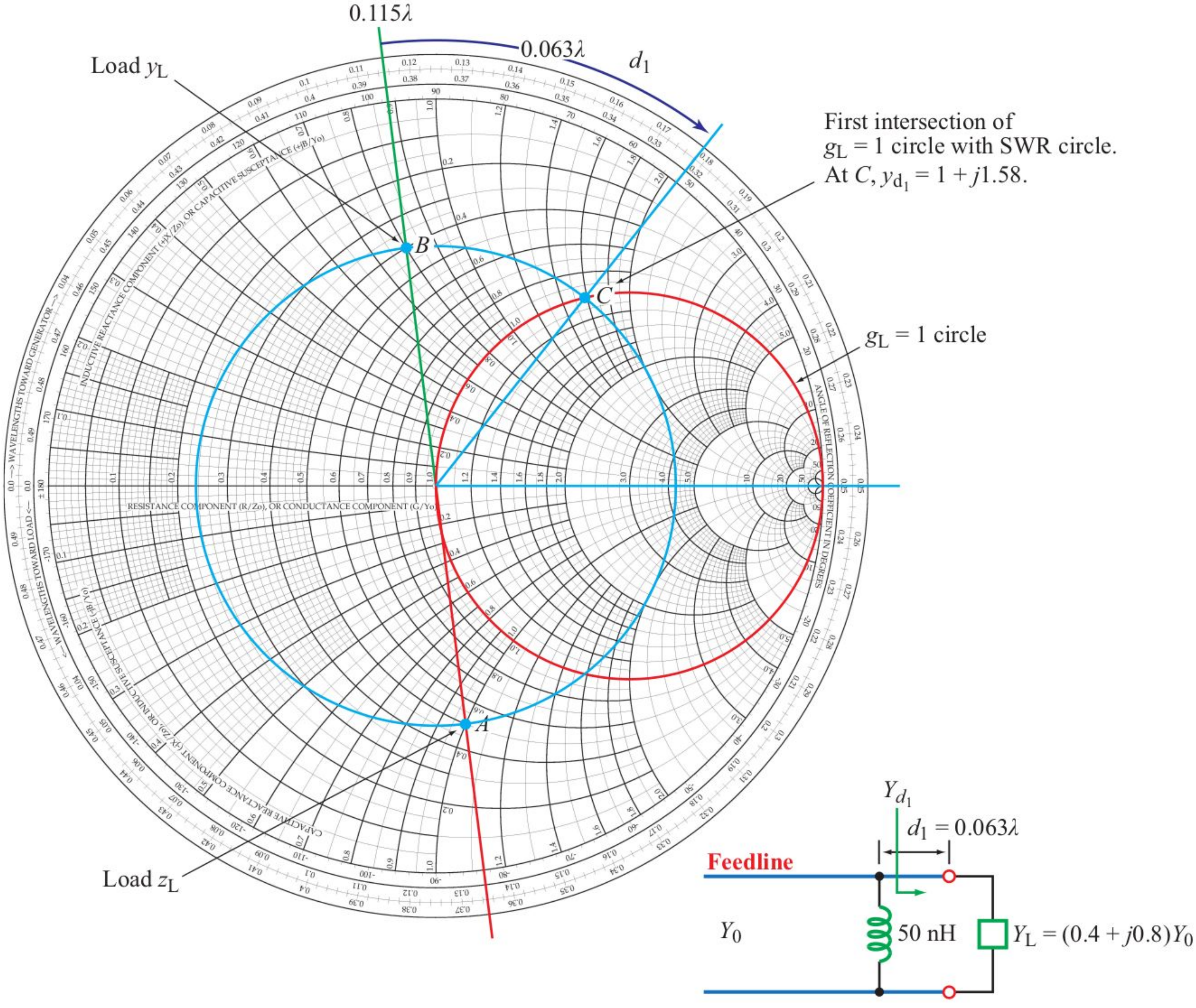
step7: For point C, read off value of y_d:

$$y_d = 1 + j\,1.58$$

step8: draw radial line thru y_d: WTG=0.178λ

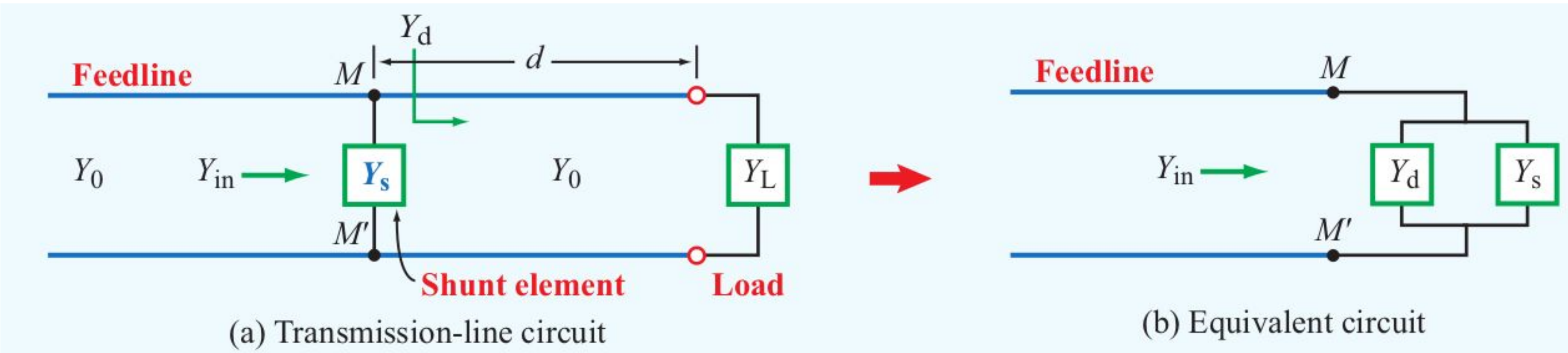
so distance moved is 0.178λ - 0.115λ

$d = 0.063\lambda$



First solution

Example 2-13 Lumped-Element Matching



Solution:

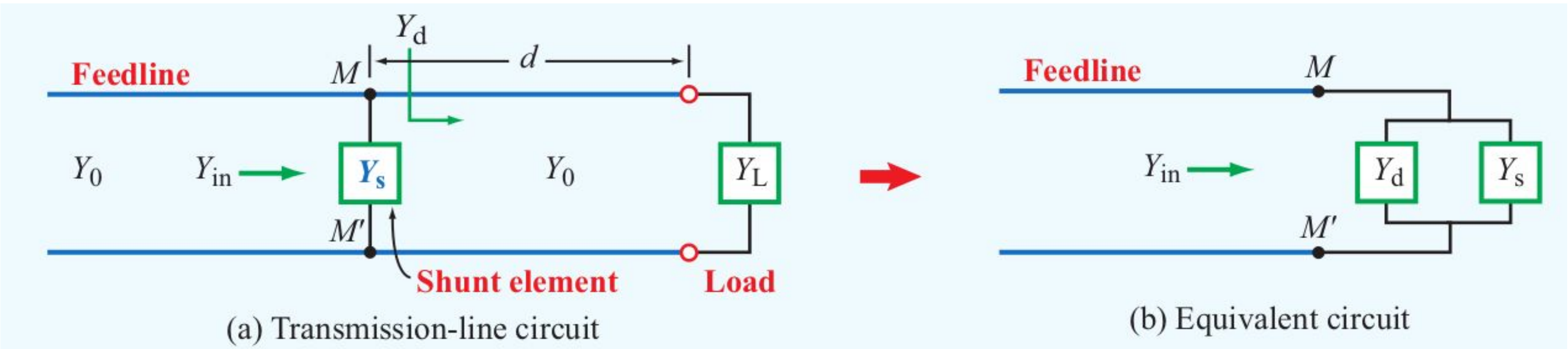
step9: Since the line "sees" the parallel combination of the *d*-offset load and the lumped element:

$$y_{in} = y_d + y_s$$

we want:

$$1 = 1 + j\,1.58 + y_s \qquad \text{or:} \qquad y_s = -j\,1.58$$

Example 2-13 Lumped-Element Matching



Solution:

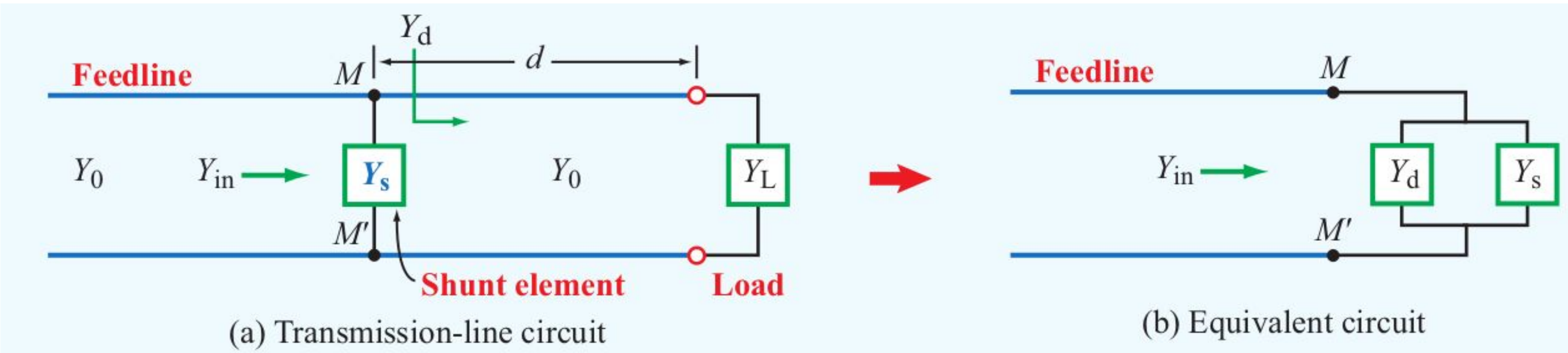
step10: Normalized impedance of lumped element is therefore:

$$z_s = 1 / y_s = 1 / -j\ 1.58 = j / 1.58 = j\ 0.633$$

so:

$$Z_s = z_s Z_0 = (j\ \ 0.633)\ 50\ \Omega = j\ 31.65\ \Omega$$

Example 2-13 Lumped-Element Matching



(a) Transmission-line circuit (b) Equivalent circuit

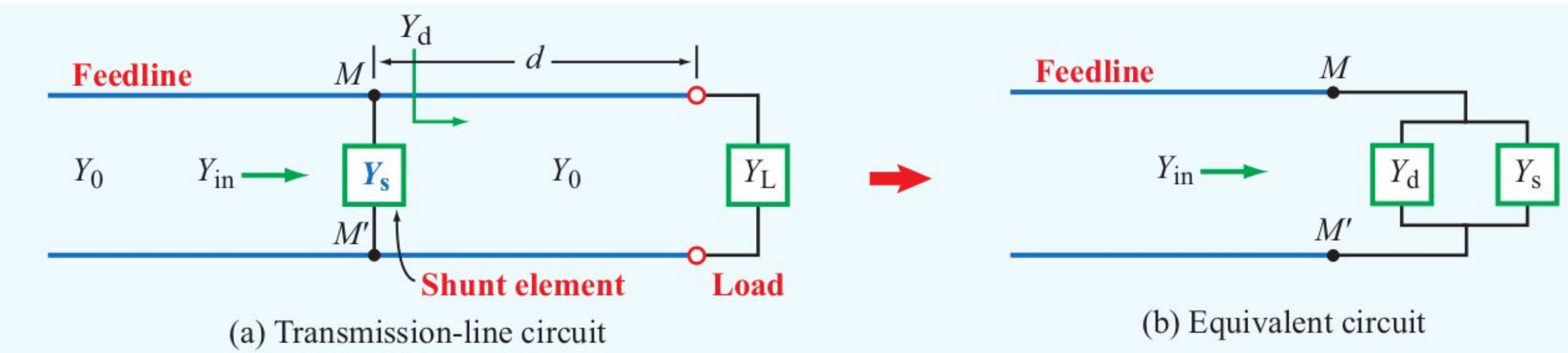
Solution:

step11: Both inductors and capacitors have imaginary impedances. How to choose?

Since: $Z_C = 1/(j\omega C) = -j/(\omega C)$ and $Z_L = j\omega L$

We must choose an inductor since we need a positive imaginary part.

Example 2-13 Lumped-Element Matching



(a) Transmission-line circuit

(b) Equivalent circuit

Solution:

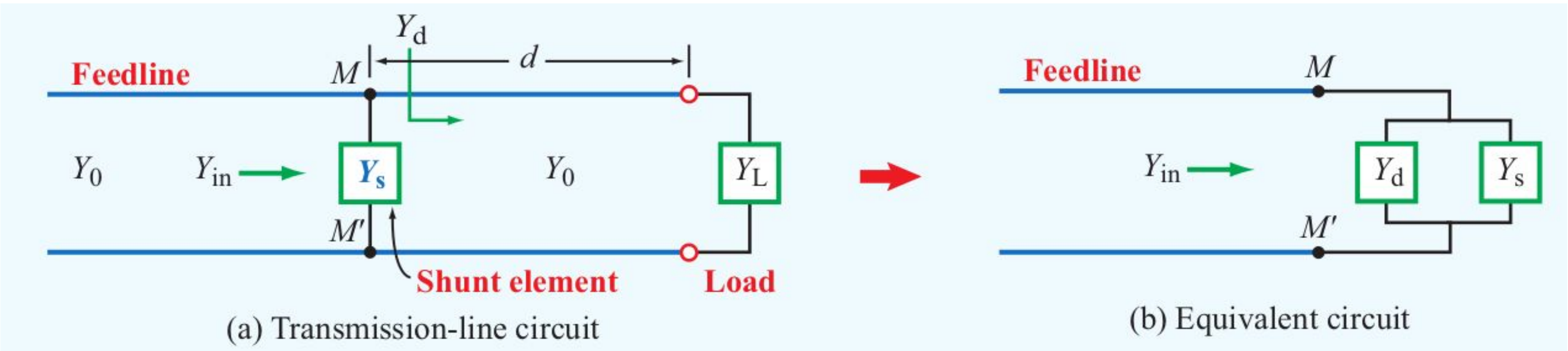
step11: Choose an inductor: $Z_L = j\omega L$

$$Z_s = j\,31.65\ \Omega = j\omega L$$

so:

$$L = 31.65\Omega\ /\ \omega = 31.65\Omega\ /\ (2\pi f)$$

Example 2-13 Lumped-Element Matching



Solution:

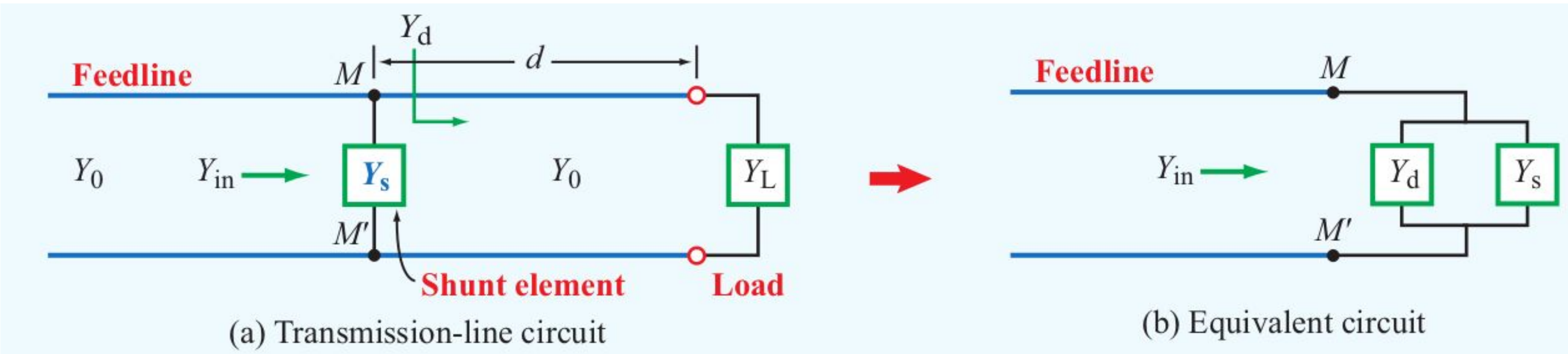
step11: $L = 31.65\Omega / (2\pi f)$

$L = 31.65\Omega / (2\pi\,(100X10^6\ \text{Hz})\,)$

$L = 5x10^{-8}$ H

$L = 50$ nH

Example 2-13 Lumped-Element Matching



Solution: Do again, choosing point D:

step7: For point D, read off value of y_d:

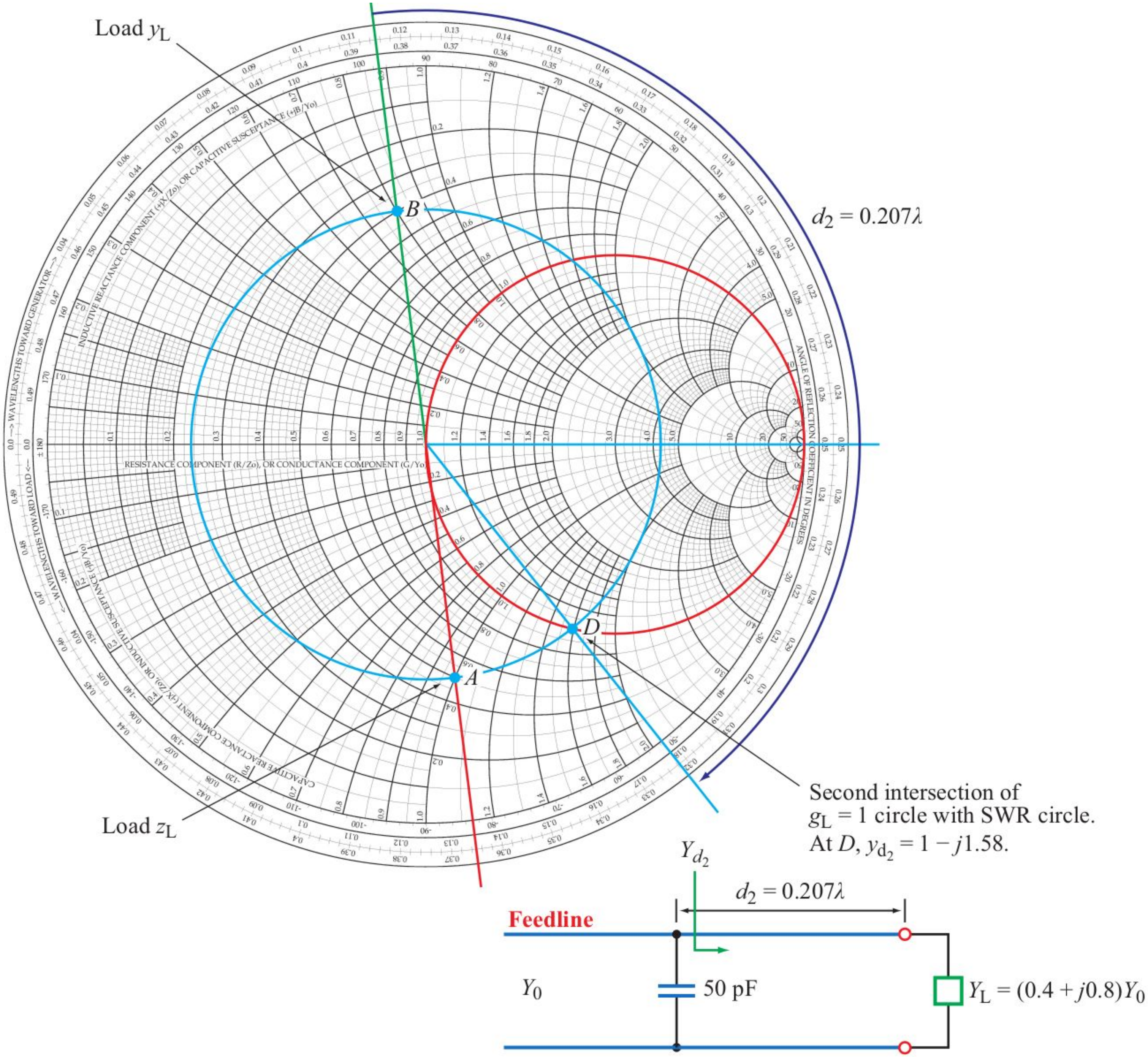
$$y_d = 1 - j\,1.58$$

step8: draw radial line thru y_d: WTG=0.322λ

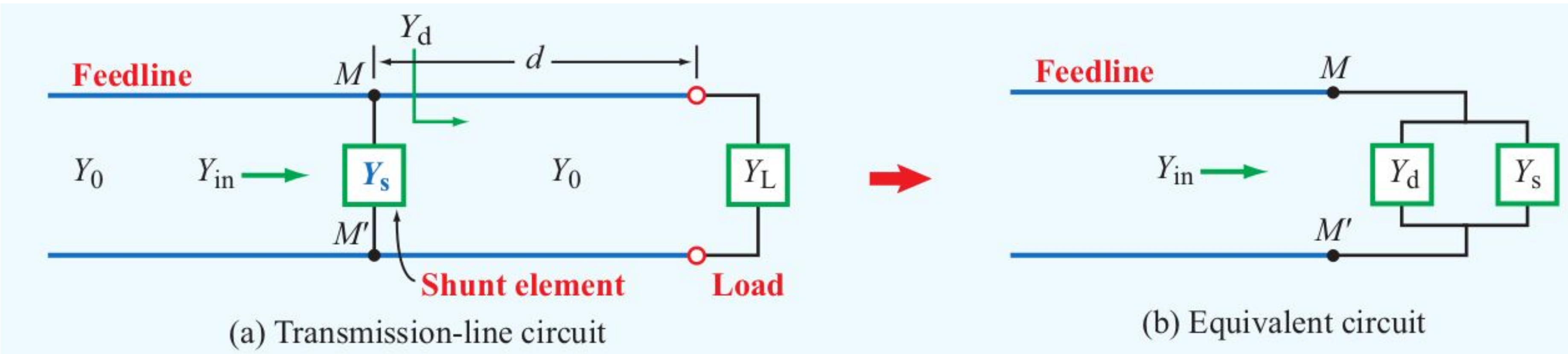
so distance moved is 0.322λ - 0.115λ

d = 0.207λ

E M


Load y_L
B
$d_2 = 0.207\lambda$
D
A
Load z_L
Second intersection of
$g_L = 1$ circle with SWR circle.
At D, $y_{d_2} = 1 - j1.58$.
Y_{d_2}
$d_2 = 0.207\lambda$
Feedline
Y_0
50 pF
$Y_L = (0.4 + j0.8)Y_0$

Example 2-13 Lumped-Element Matching



Solution:

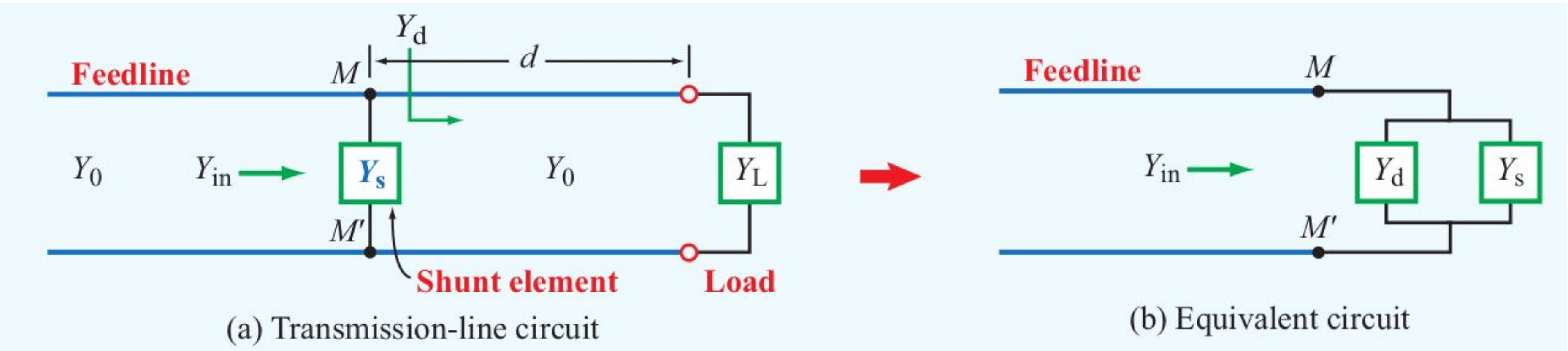
step9: Since the line sees the parallel combination of the *d*-offset load and the lumped element:

$$y_{in} = y_d + y_s$$

we want:

$$1 = 1 - j\,1.58 + y_s \qquad \text{or:} \qquad y_s = +j\,1.58$$

Example 2-13 Lumped-Element Matching



Solution:

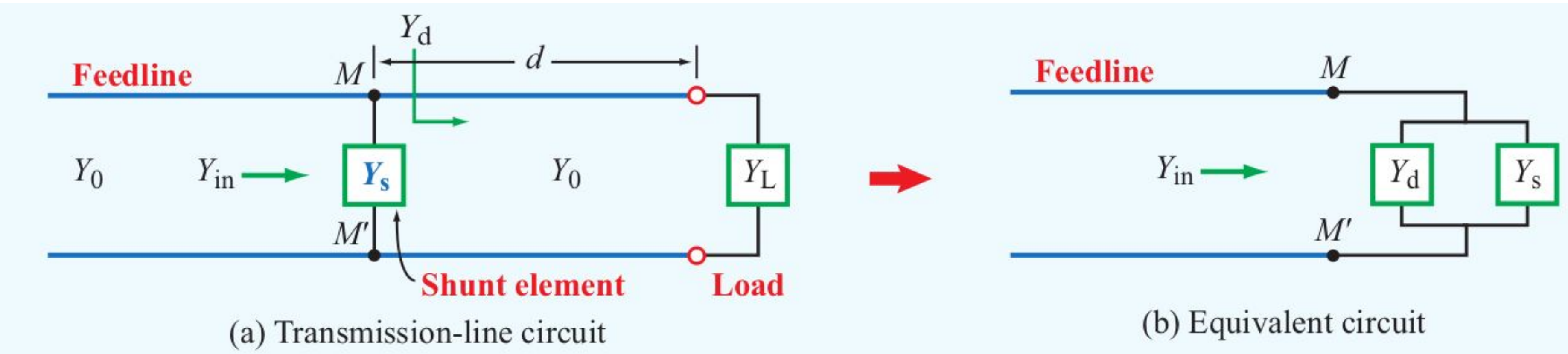
step10: Normalized impedance of lumped element is therefore:

$$z_s = 1 / y_s = 1 / j\ 1.58 = j / 1.58 = -j\ 0.633$$

so:

$$Z_s = z_s Z_0 = (\ -j\ \ 0.633)\ 50\ \Omega = -j\ 31.65\ \Omega$$

Example 2-13 Lumped-Element Matching



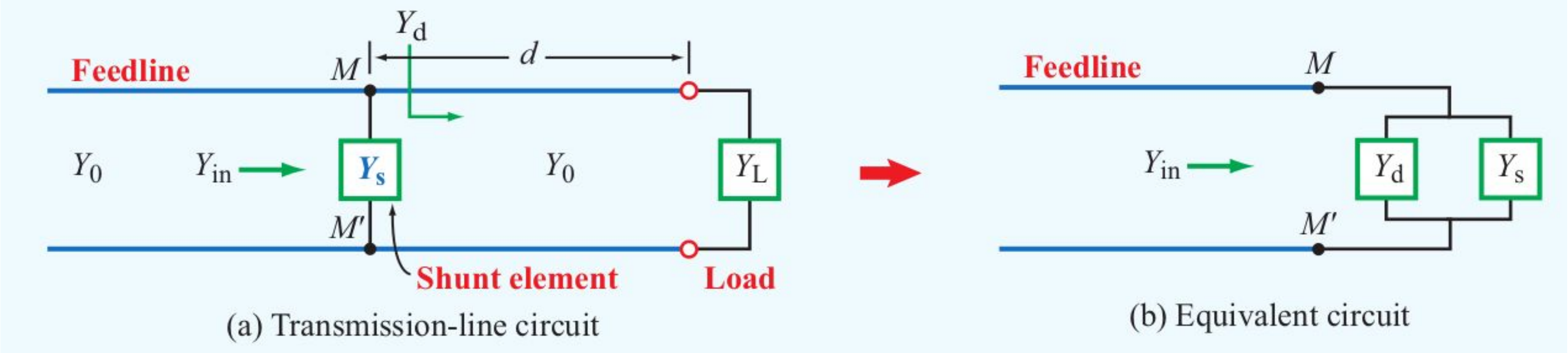
Solution:

step11: Both inductors and capacitors have imaginary impedances. How to choose?

Since: $Z_C = 1/(j\omega C) = -j/(\omega C)$ and $Z_L = j\omega L$

We must choose an capacitor since we need a negative imaginary part.

Example 2-13 Lumped-Element Matching



Solution:

step11: Choose a capacitor: $Z_c = -j / (\omega C)$

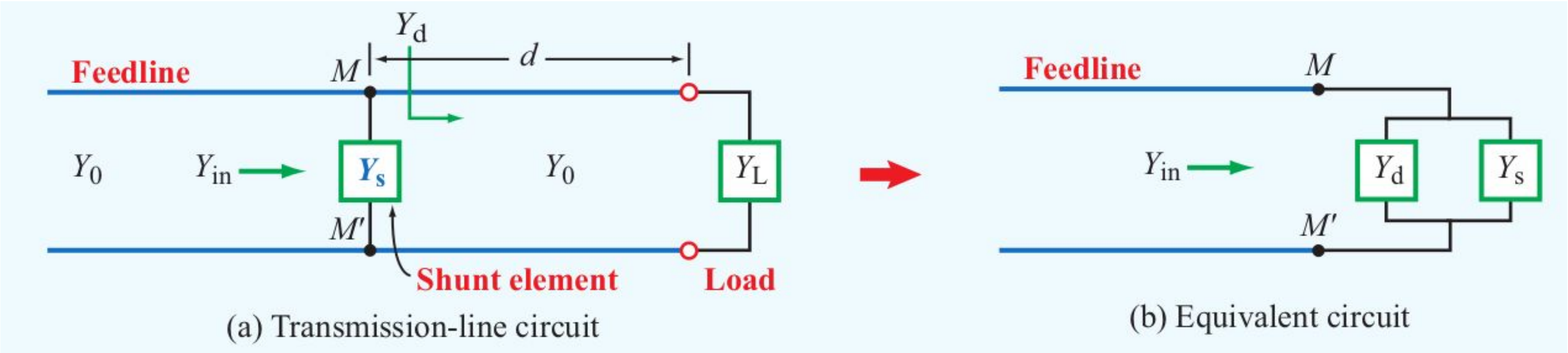
$$Z_s = -j\,31.65\ \Omega = -j / \omega C$$

so:

$$C = 1 / (\ \omega\ 31.65\ \Omega)$$

$$= 1 / (2\pi\ (100 \times 10^6 \text{Hz})\ \ (31.65\ \Omega))$$

Example 2-13 Lumped-Element Matching



(a) Transmission-line circuit (b) Equivalent circuit

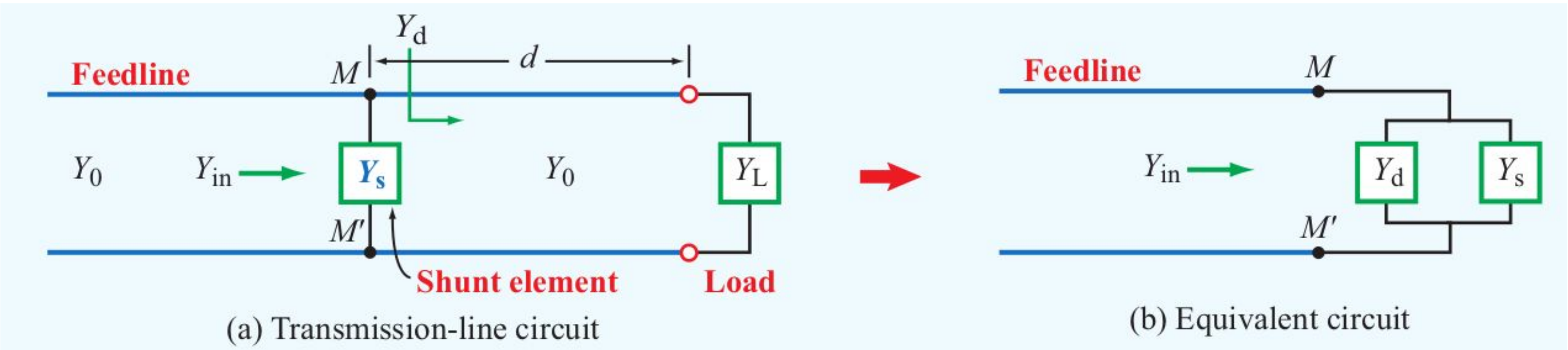
Solution:

step11: $C = 1 / (2\pi\ (100X10^6 Hz)\ (31.65\ \Omega))$

$C = 50x10^{-12}\ F$

$C = 50\ pF$

Example 2-13 Lumped-Element Matching



Summary:

at a distance from load of:

$d = 0.207\lambda$

place a capacitor in parallel with value:

$C = 50$ pF

OR:

at a distance from load of:

$d = 0.063\lambda$

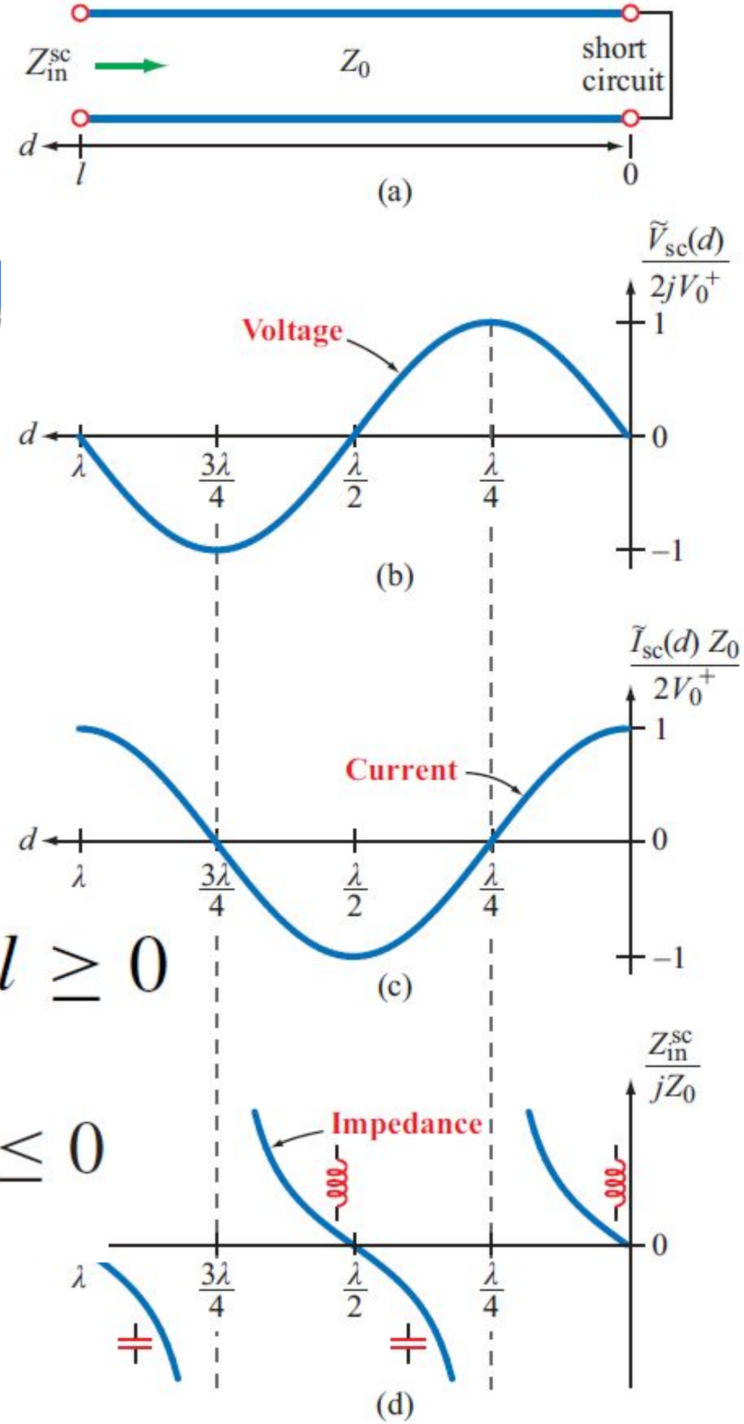
place an inductor in parallel, with value:

$L = 50$ nH

2-11 Impedance Matching

Recall from before that a shorted transmission line can look like an inductor or a capacitor by just changing its length.

Chapter 2 Review



At its input, the short-circuited line **appears** like an inductor or a capacitor depending on the sign of

$$\tan \beta d$$

$$j\omega L_{eq} = jZ_0 \tan \beta l, \qquad \text{if } \tan \beta l \geq 0$$

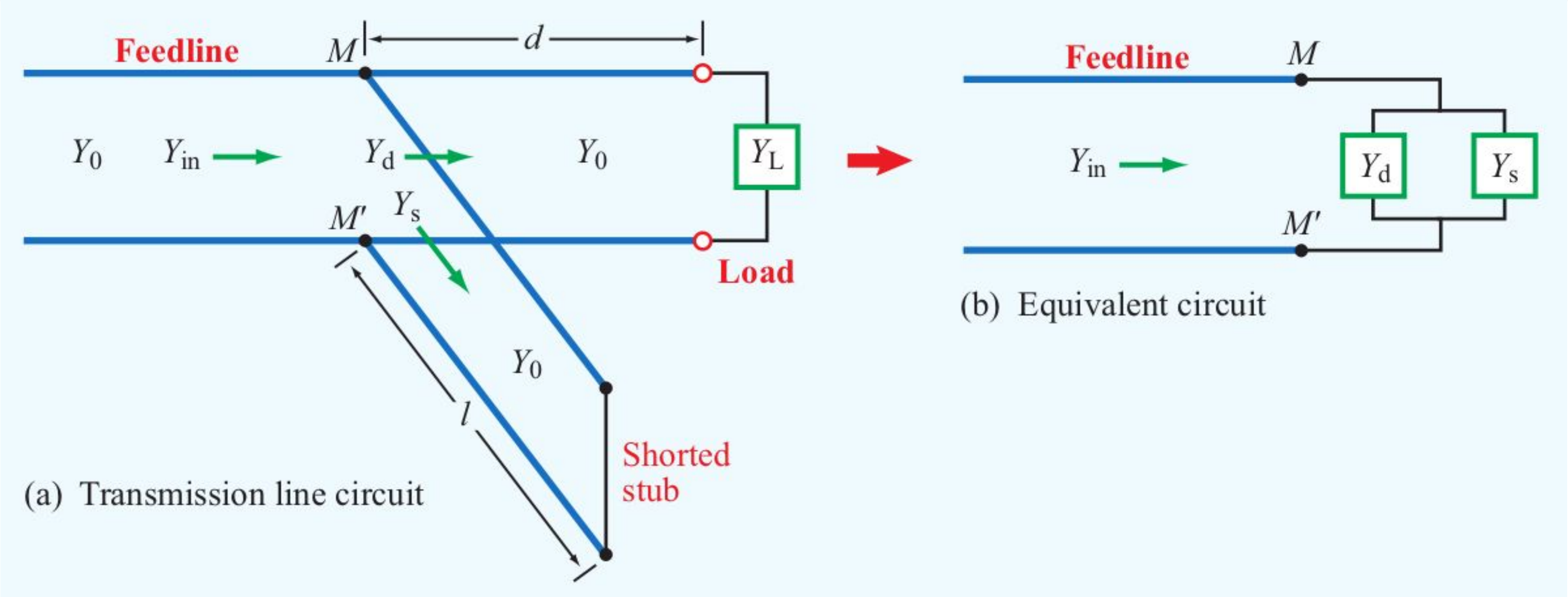
$$\frac{1}{j\omega C_{eq}} = jZ_0 \tan \beta l, \qquad \text{if } \tan \beta l \leq 0$$

2-11 Impedance Matching

So let's re-do the matching problem from before, but use a shorted transmission-line piece instead of lumped-element.

This is called "Single-Stub Matching"

Example 2-14 Single-Stub Matching

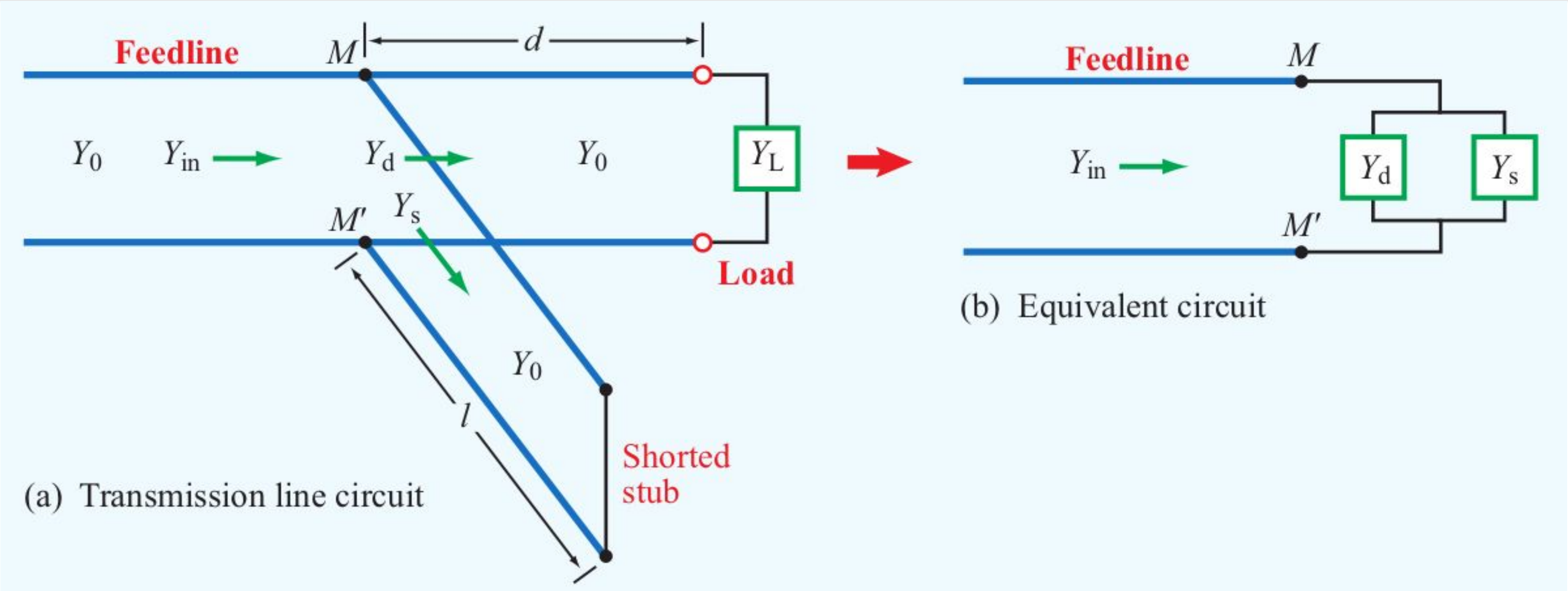


From Example 2-13 we had 2 solutions:

1. $d_1 = 0.063\lambda$, and $y_{s_1} = jb_{s_1} = -j1.58$,

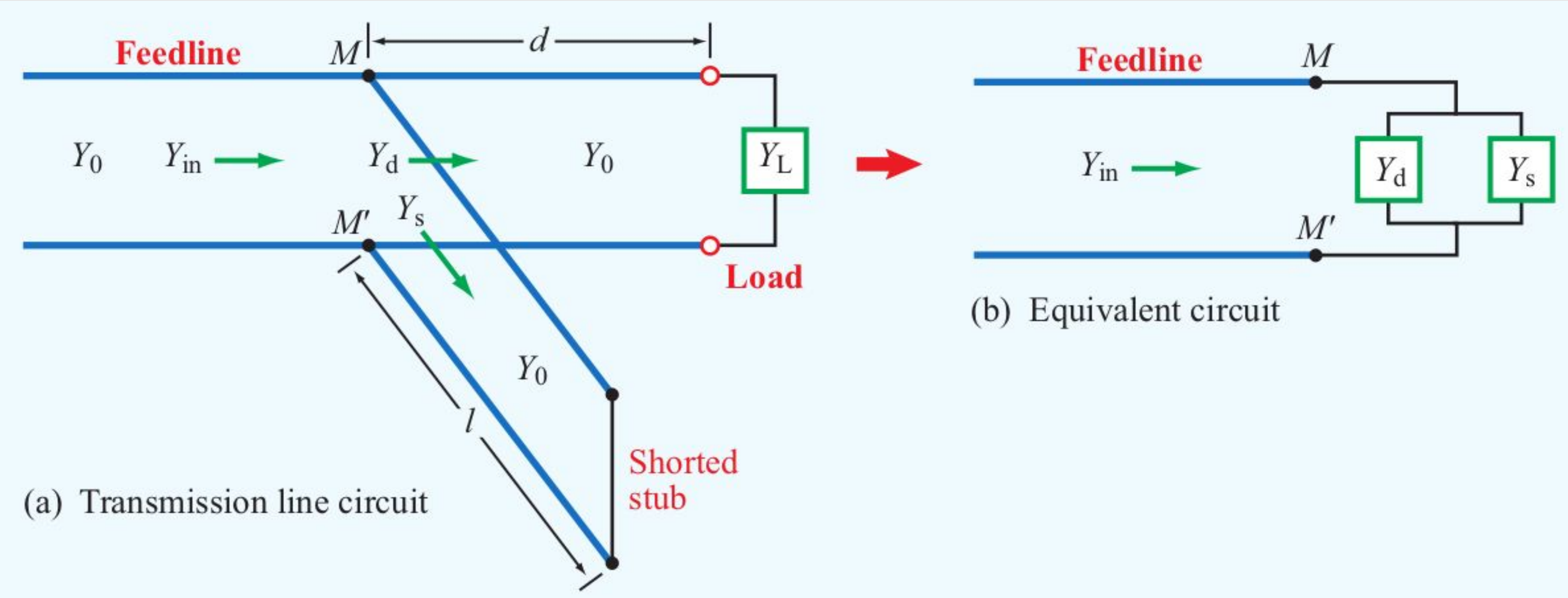
2. $d_2 = 0.207\lambda$, and $y_{s_2} = jb_{s_2} = j1.58$.

Example 2-14 Single-Stub Matching



Need to create a stub of some length, such that it has whatever admittance we need.
Using the Smith Chart.

Example 2-14 Single-Stub Matching

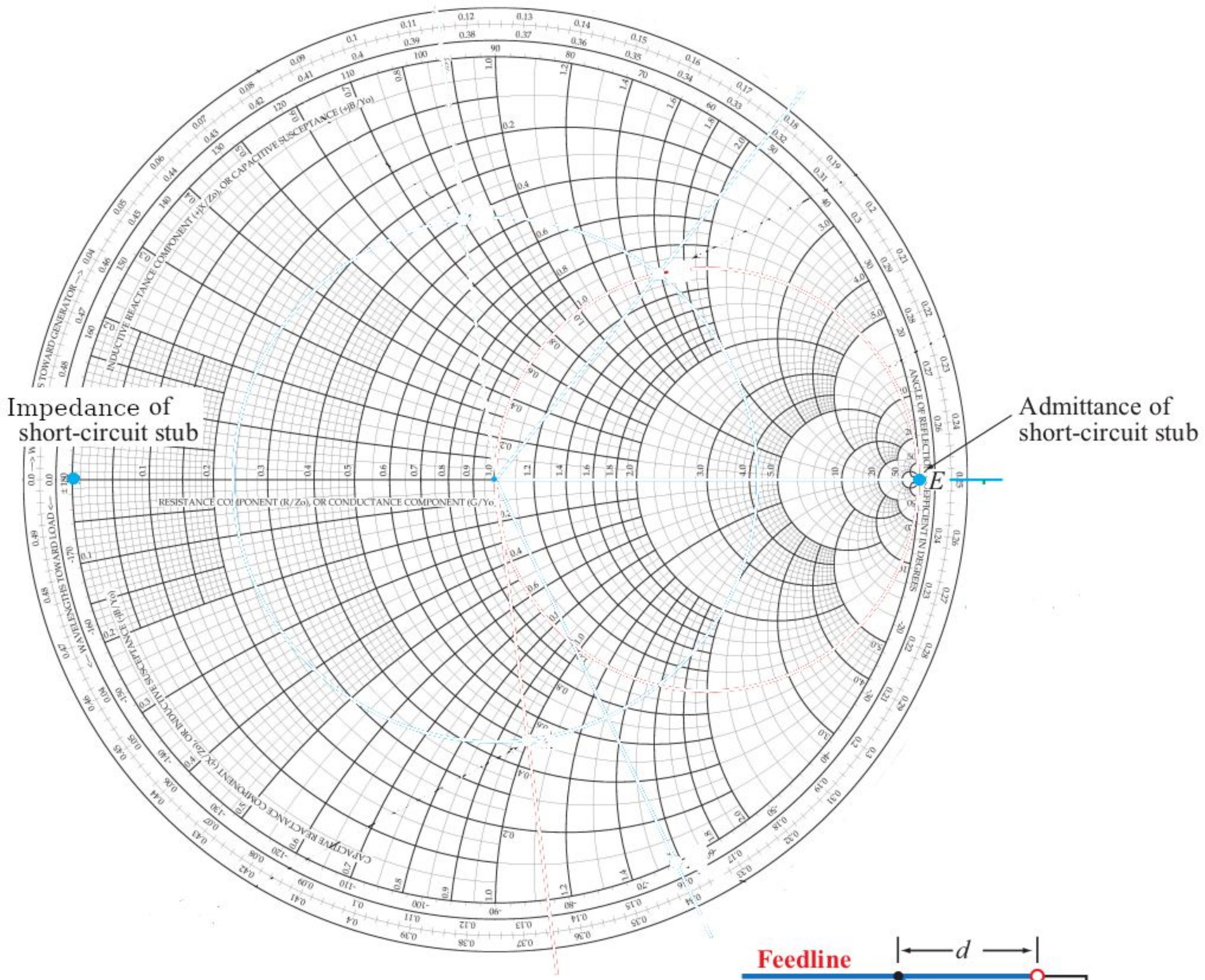


step1: stub load impedance is 0

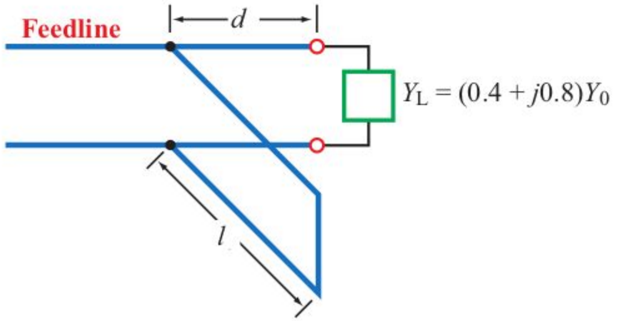
hence stub load admittance is ∞

locate these on the Smith Chart:

0 impedance is real, far to the left

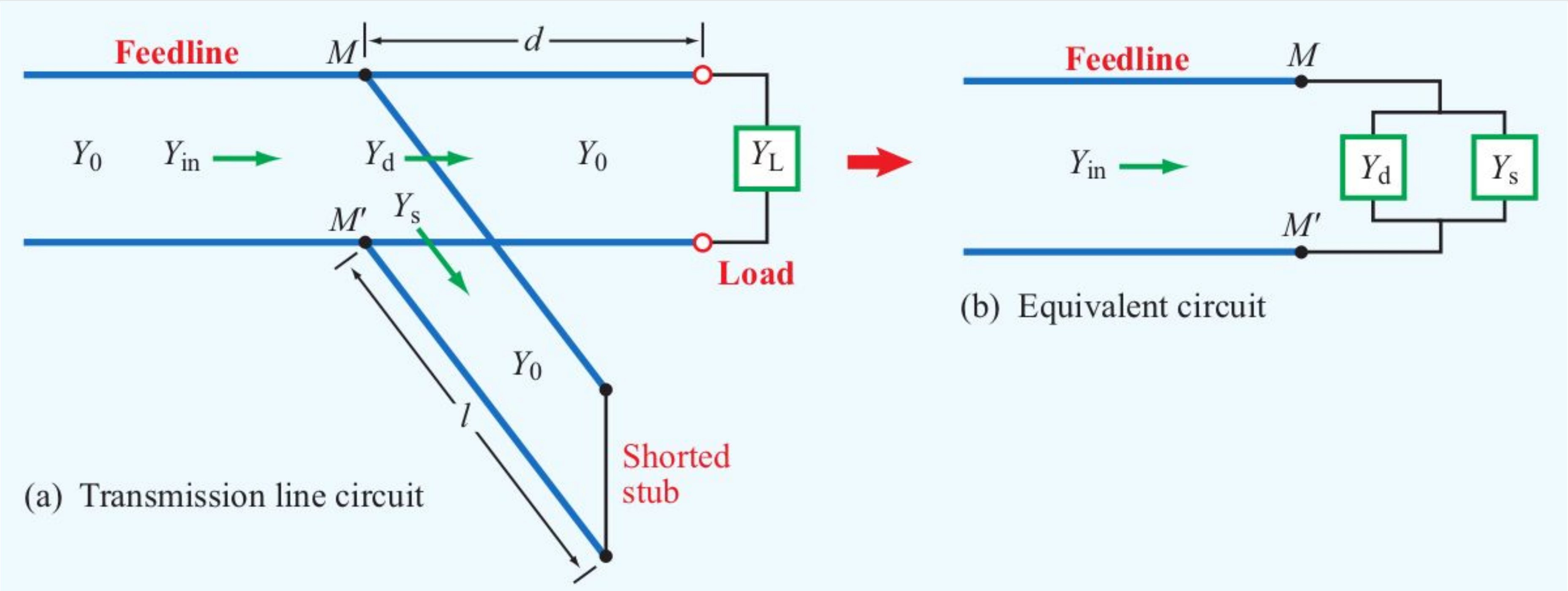


Impedance of short-circuit stub
Admittance of short-circuit stub
E
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD

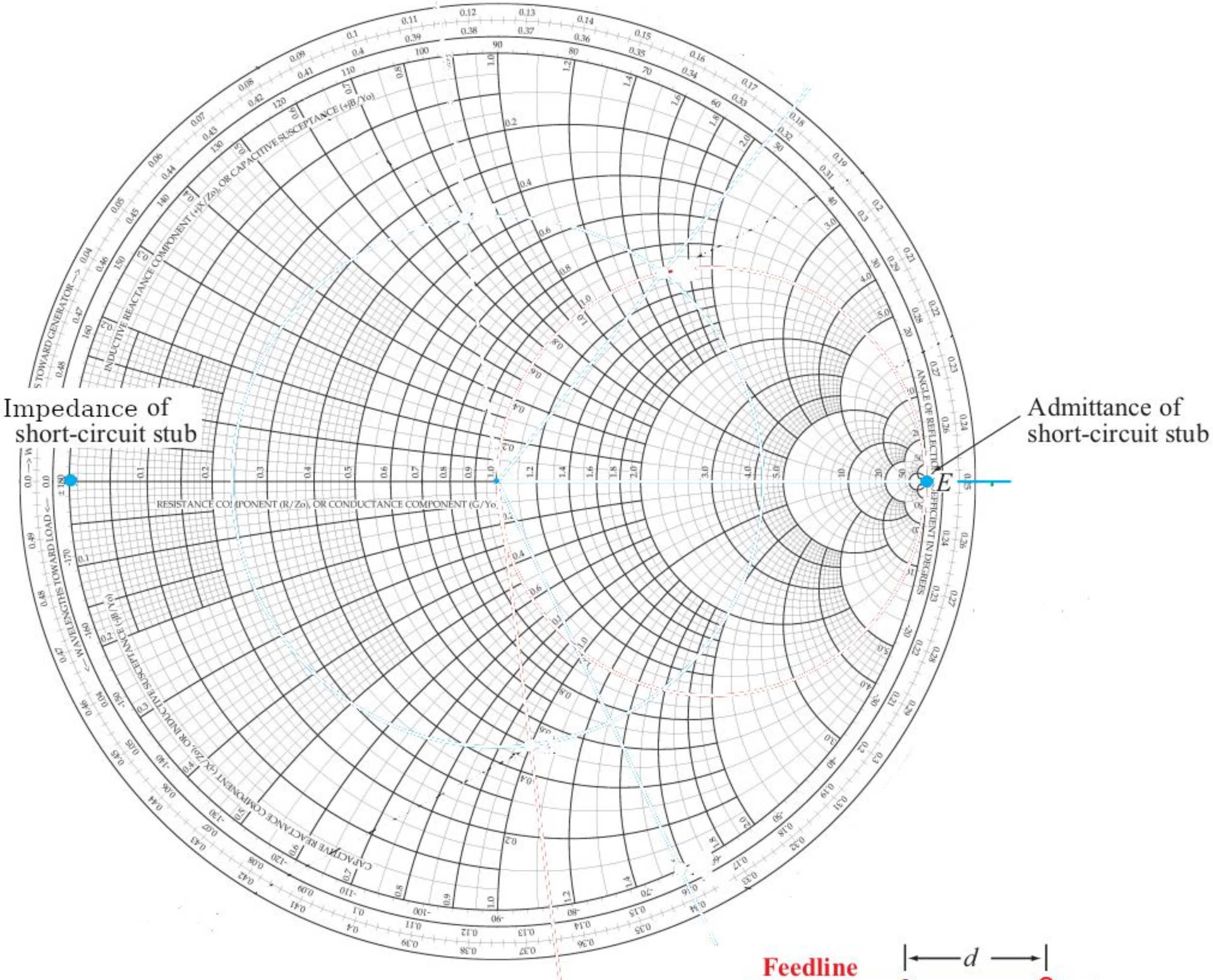


Feedline
d
l
$Y_L = (0.4 + j0.8)Y_0$

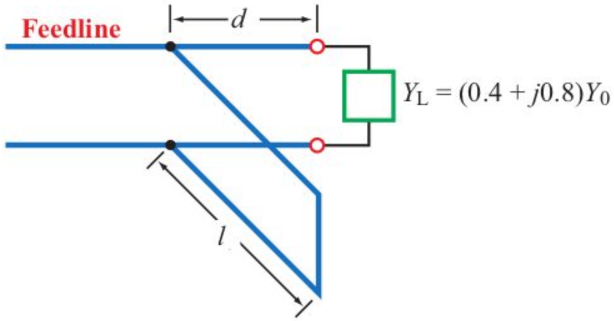
Example 2-14 Single-Stub Matching



step2: ∞ admittance is real, all the way to the right.

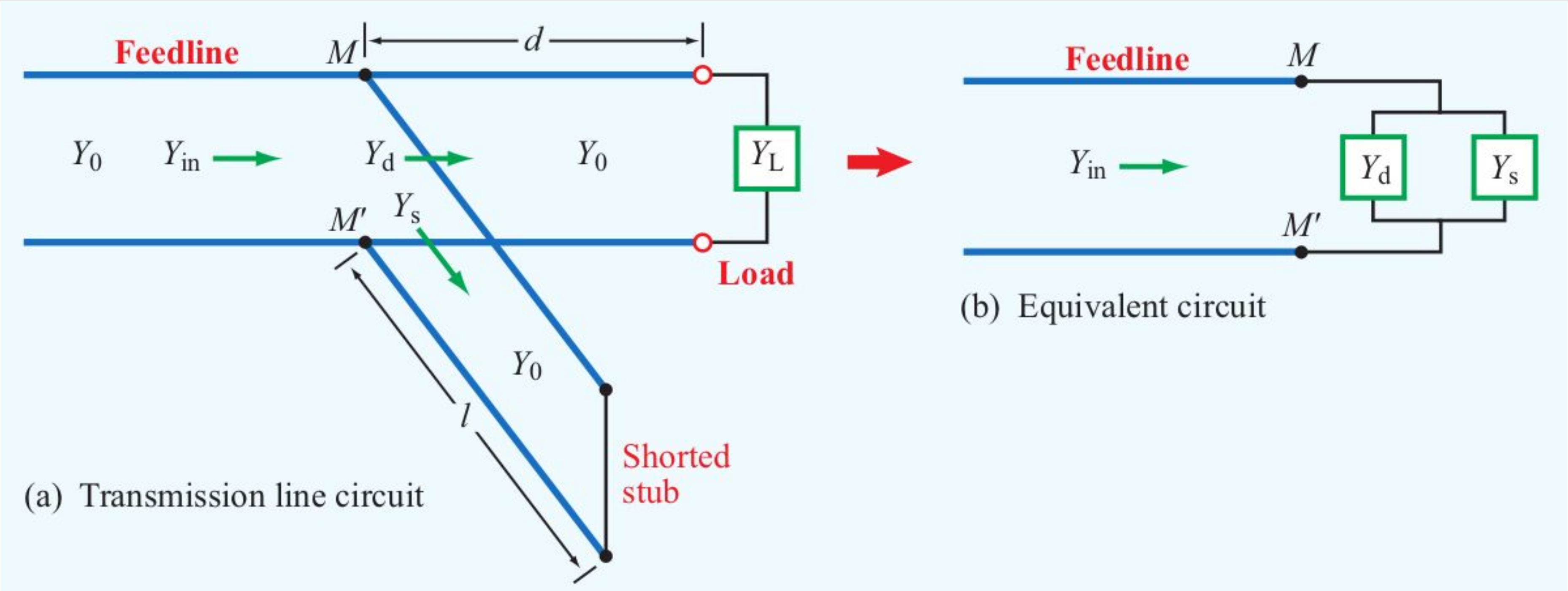


Impedance of short-circuit stub
Admittance of short-circuit stub
E



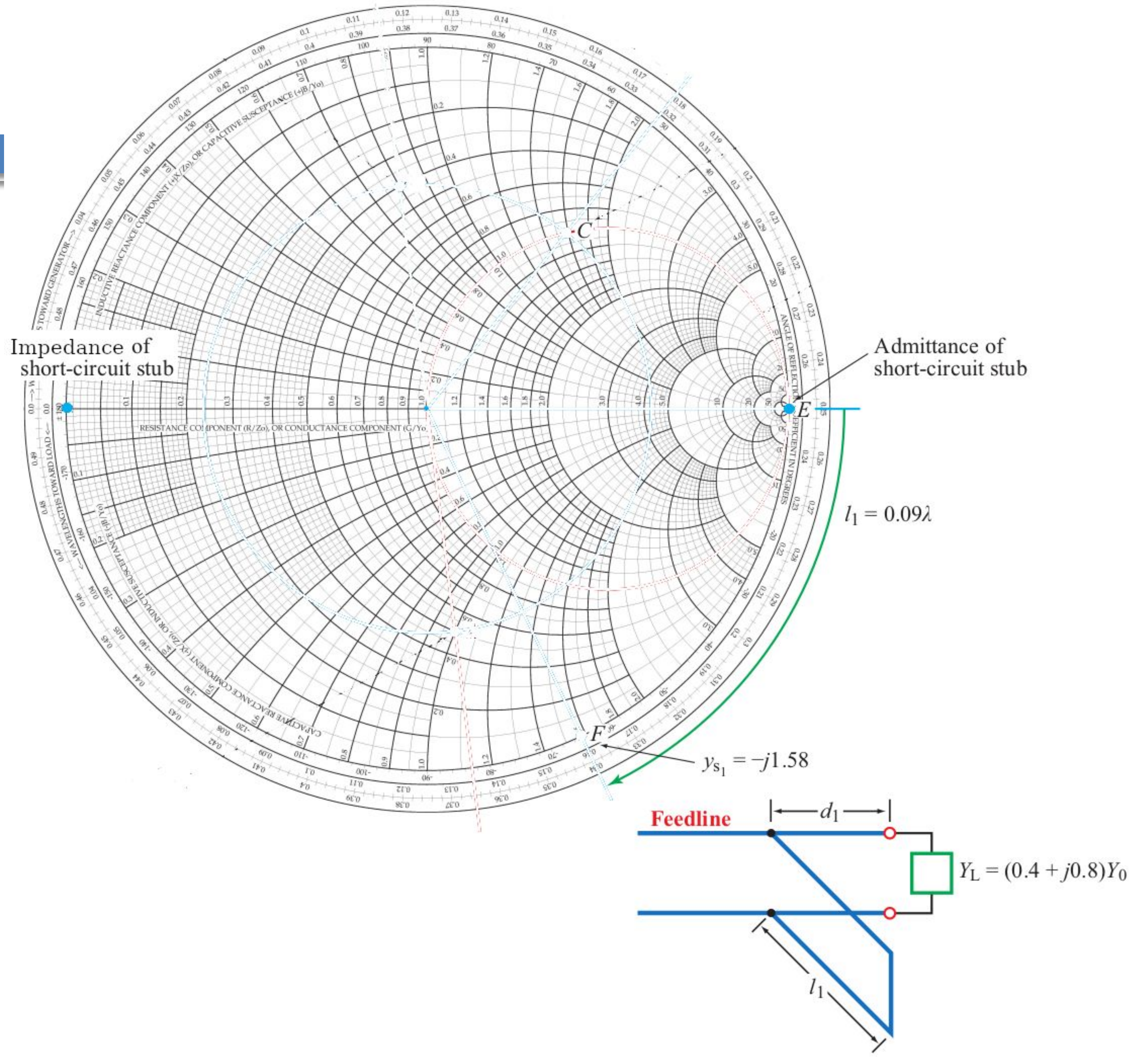
Feedline
d
l
$Y_L = (0.4 + j0.8)Y_0$

Example 2-14 Single-Stub Matching

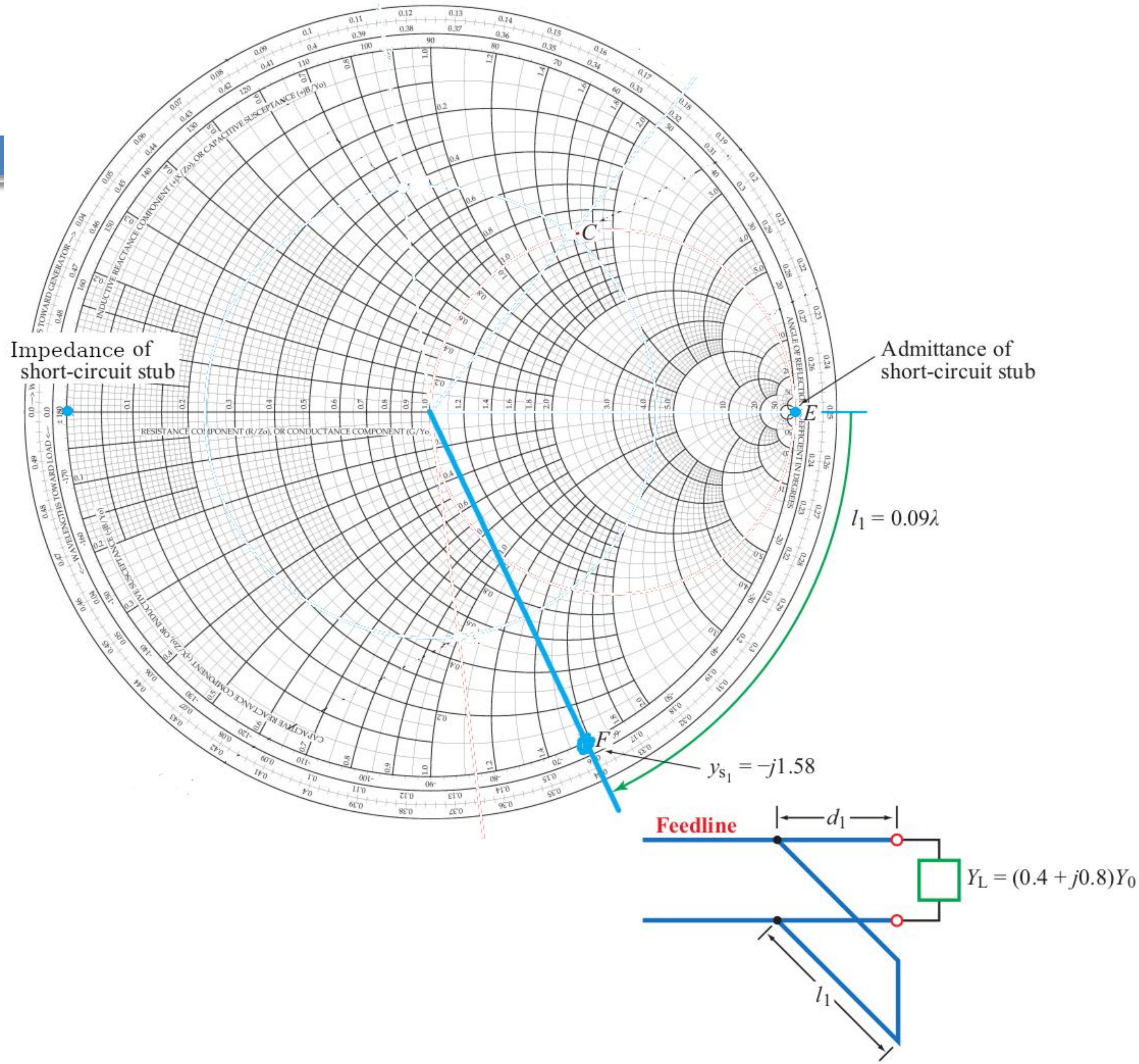


step3: find stub length: rotate toward generator until get desired admittance of -j1.58

step4: draw radial line to get WTG: 0.34λ

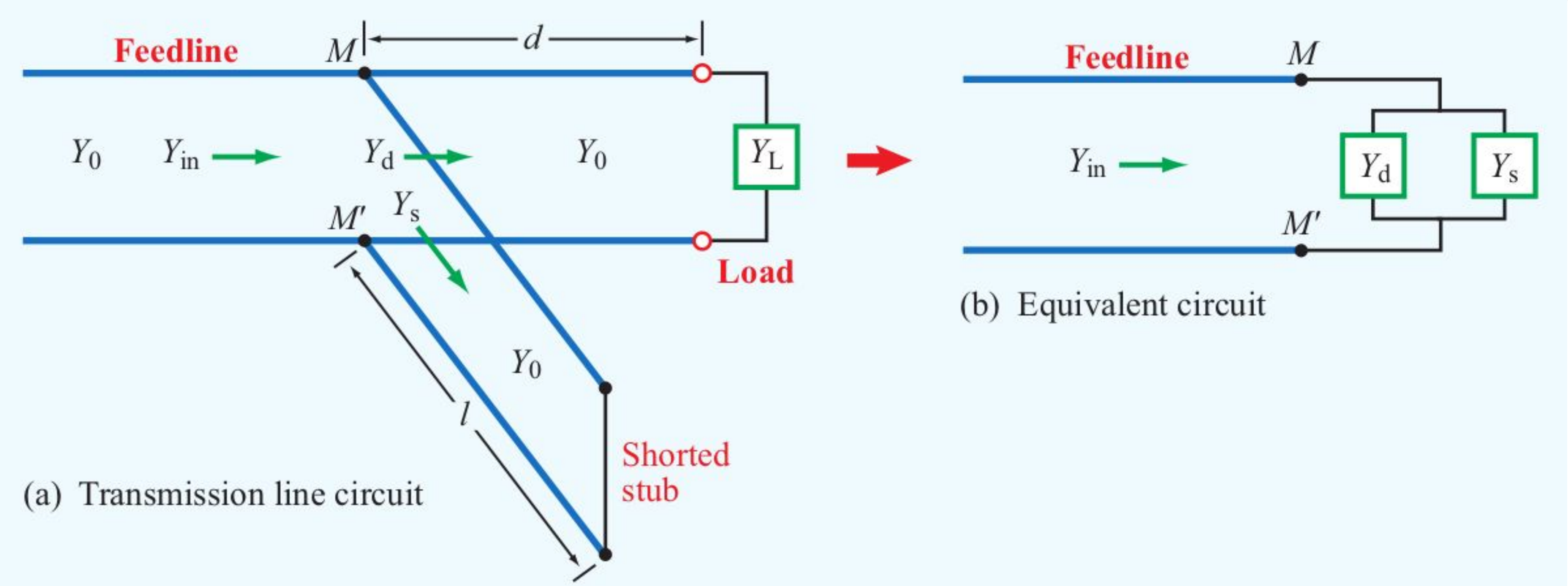


Impedance of short-circuit stub
Admittance of short-circuit stub
C
E
F
$l_1 = 0.09\lambda$
$y_{s_1} = -j1.58$
Feedline
d_1
$Y_L = (0.4 + j0.8)Y_0$
l_1



Impedance of short-circuit stub
Admittance of short-circuit stub
C
E
F
$l_1 = 0.09\lambda$
$y_{s_1} = -j1.58$
Feedline
d_1
$Y_L = (0.4 + j0.8)Y_0$
l_1

Example 2-14 Single-Stub Matching

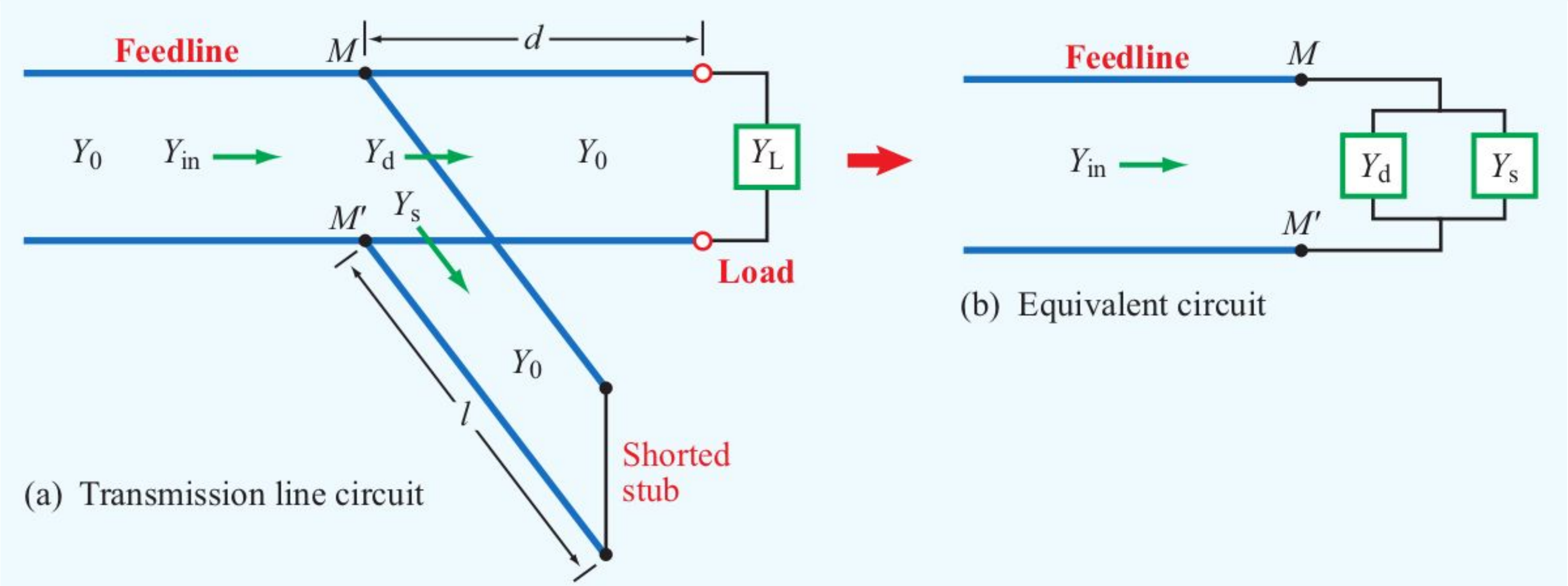


step5: find stub length: $0.34\lambda - 0.25\lambda = 0.09\lambda$

done for when use point C.

NEXT: point D

Example 2-14 Single-Stub Matching

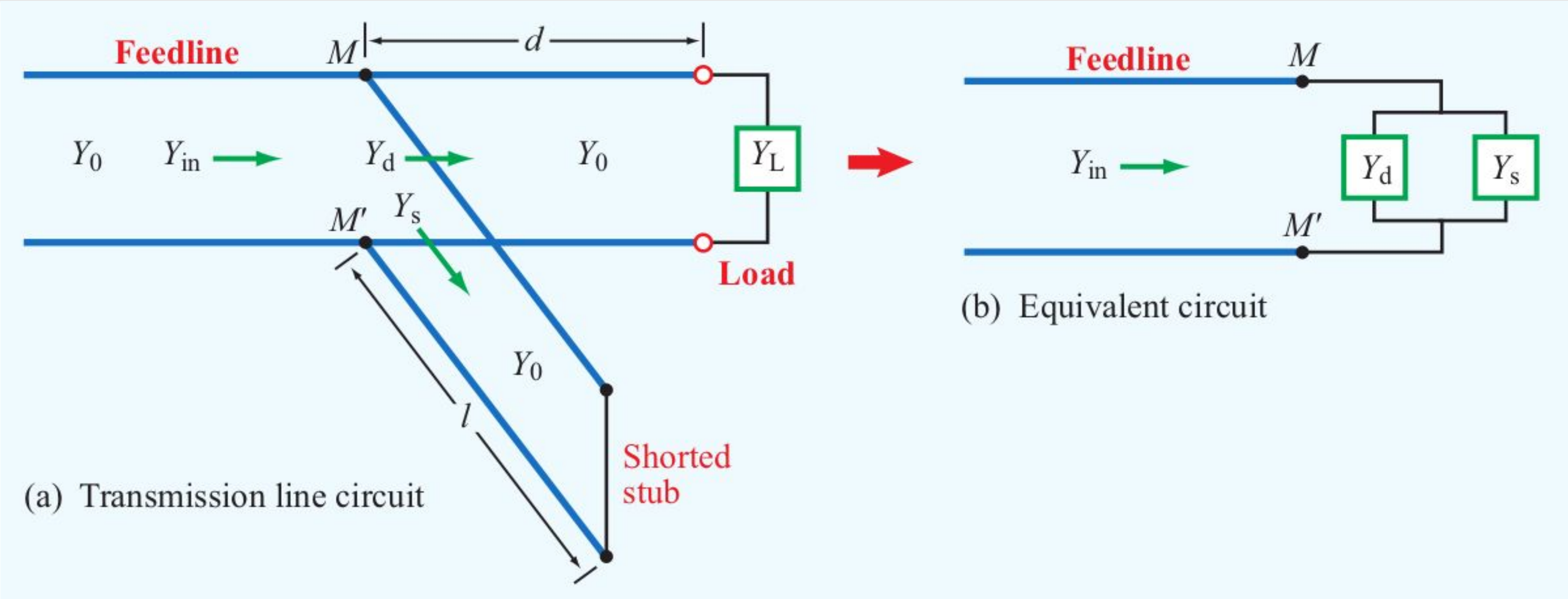


(a) Transmission line circuit

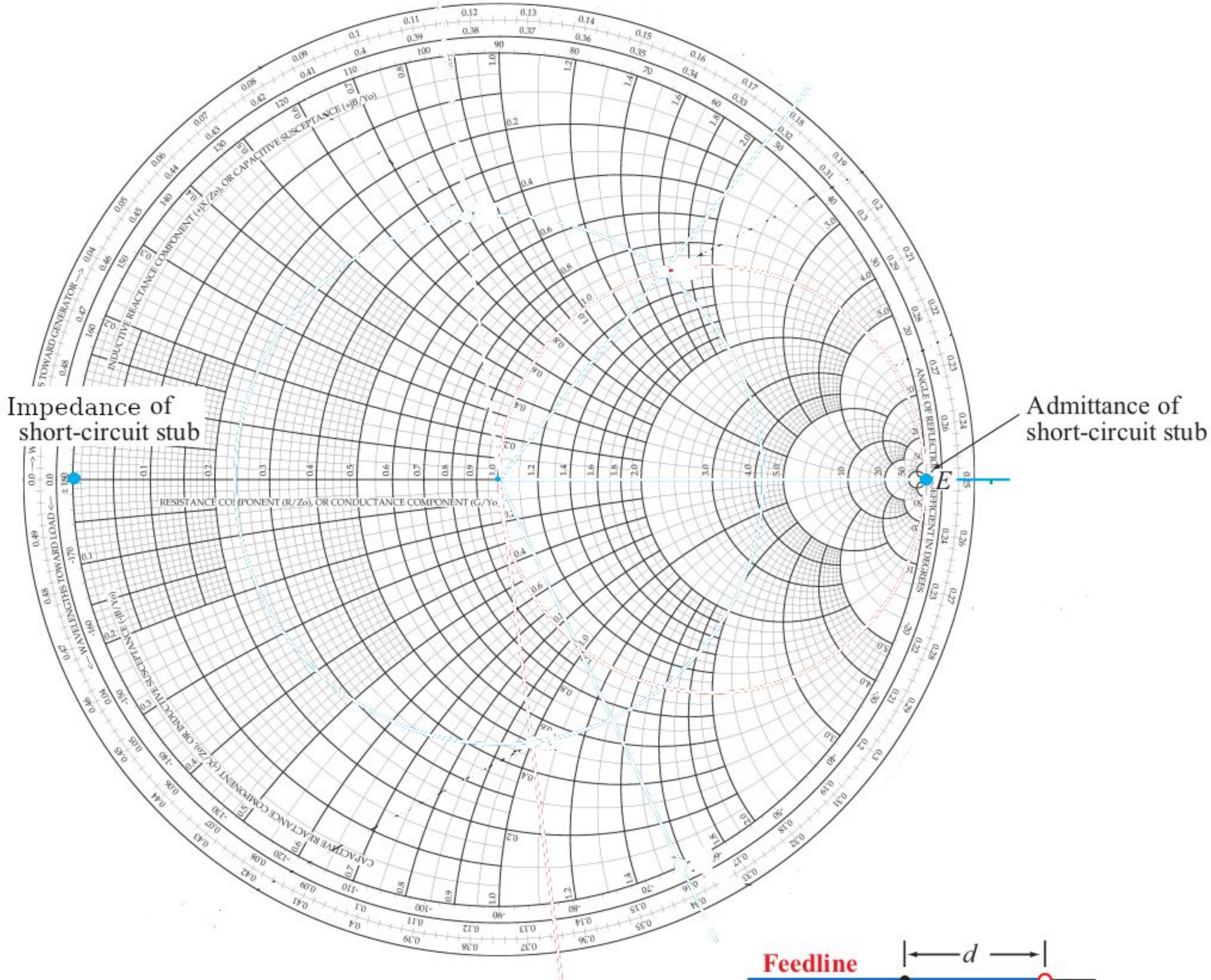
(b) Equivalent circuit

step1: stub load impedance is 0
hence stub load admittance is ∞
locate these on the Smith Chart:
0 impedance is real, far to the left

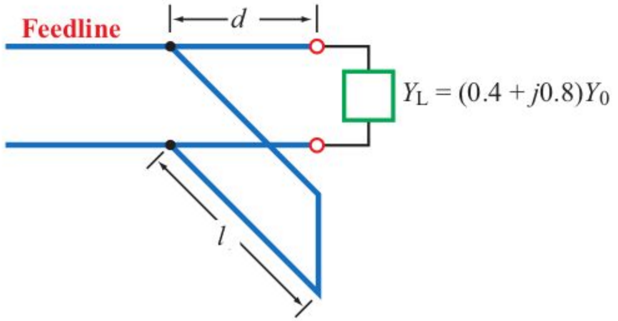
Example 2-14 Single-Stub Matching



step2: ∞ admittance is real, all the way to the right.

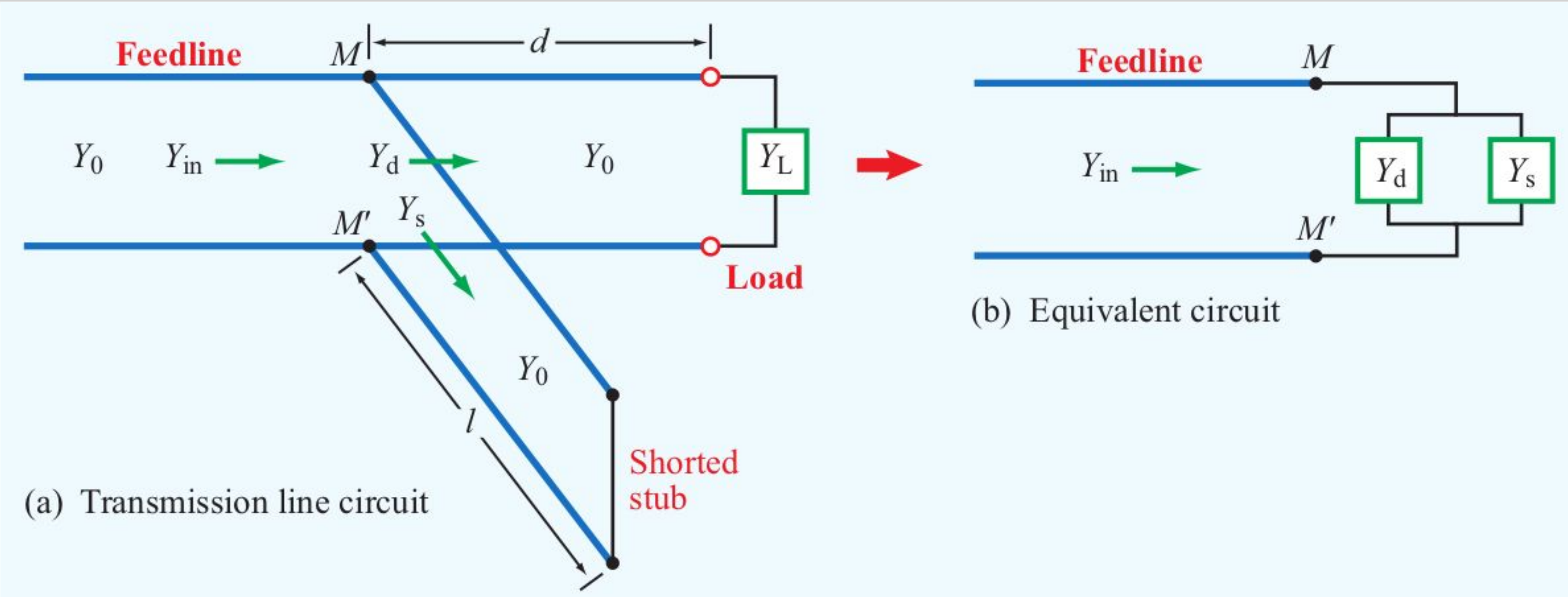


Impedance of short-circuit stub
Admittance of short-circuit stub
E



Feedline
d
l
$Y_L = (0.4 + j0.8)Y_0$

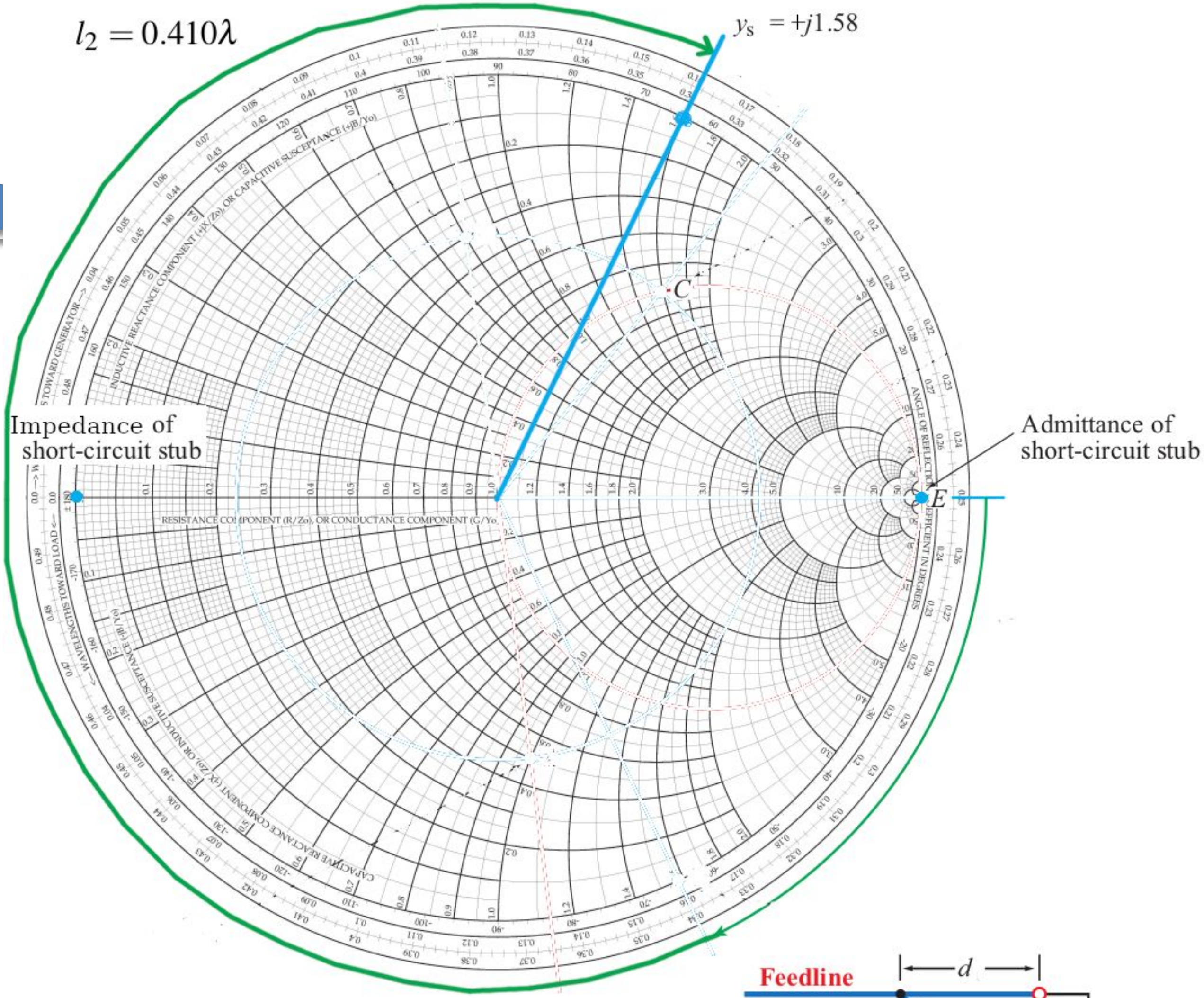
Example 2-14 Single-Stub Matching

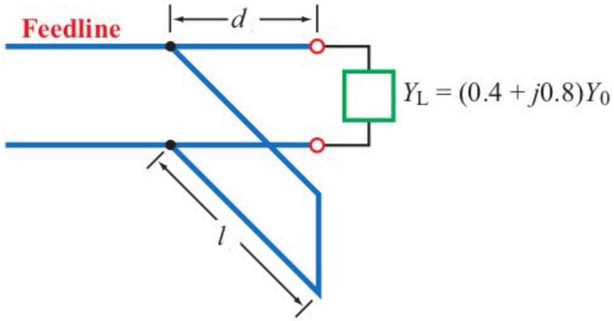


step3: find stub length: rotate toward generator until get desired admittance of +j1.58

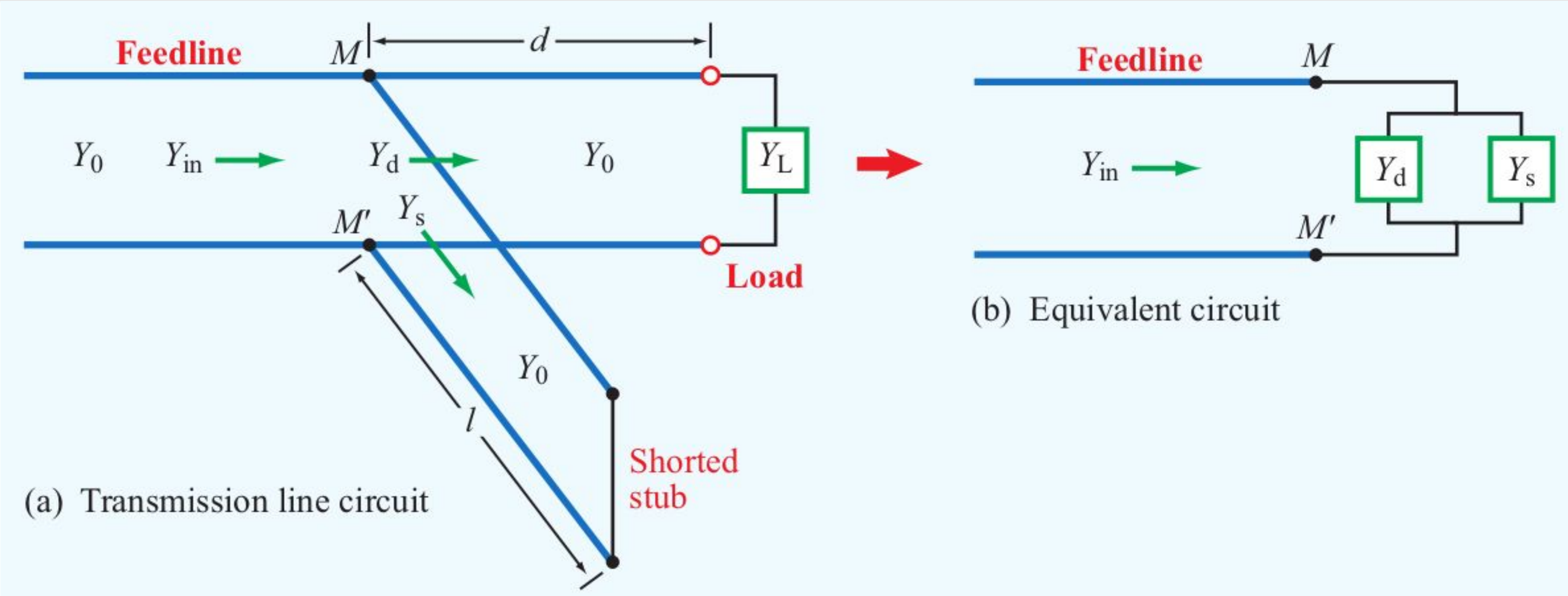
step4: draw radial line to get WTG: 0.16λ

$l_2 = 0.410\lambda$





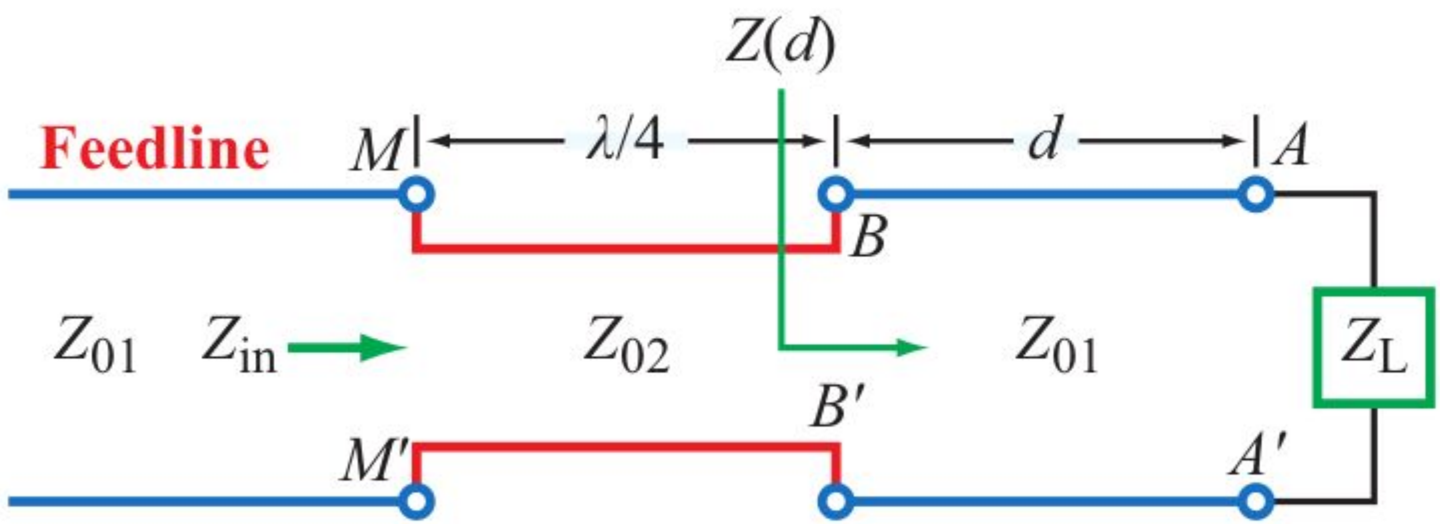
Example 2-14 Single-Stub Matching



step5: find stub length: $0.16\lambda + 0.25\lambda = 0.41\lambda$

Done matching at point D.

Example 2-15 $\lambda/4$ Matching

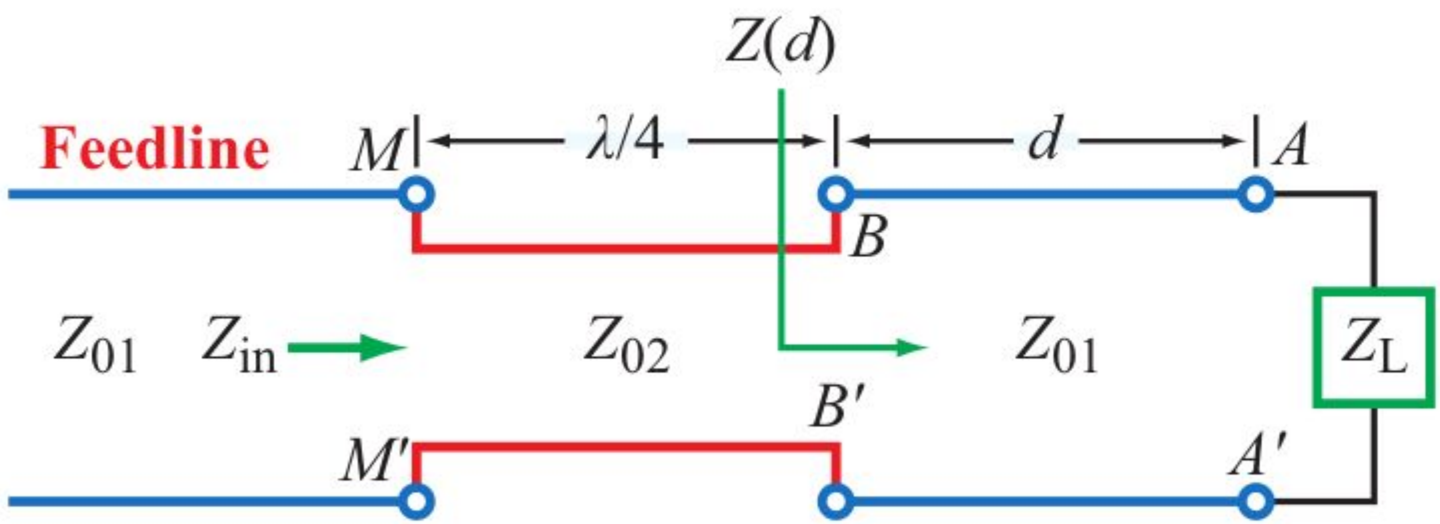


Given: A lossless transmission line with $Z_{01} = 50\Omega$

$Z_L = 100 + j100\ \Omega$

Find: distance, d, and characteristic impedance, Z_{02}, of a lossless $\lambda/4$-line to match the load.

Example 2-15 $\lambda/4$ Matching



Solution: Recall that for a $\lambda/4$-line:

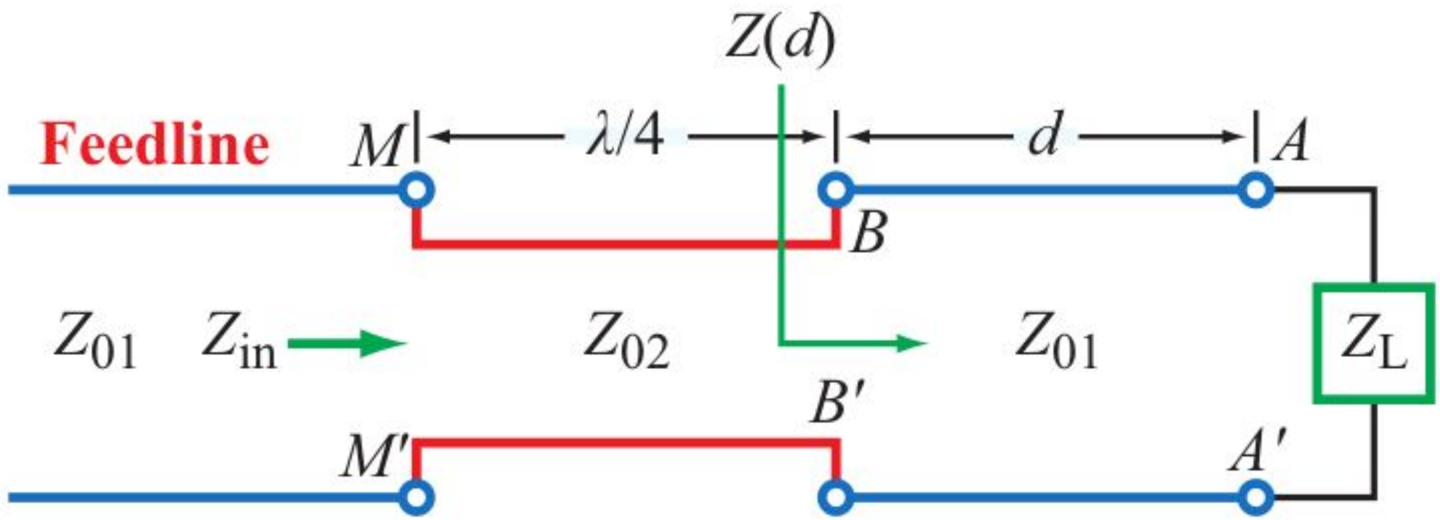
$$Z_{in}\, Z(d) = (Z_{02})^2$$

in our case:

$$Z_{in} = 50\ \Omega \quad \text{(for matching)}$$

so Z(d) must be real.

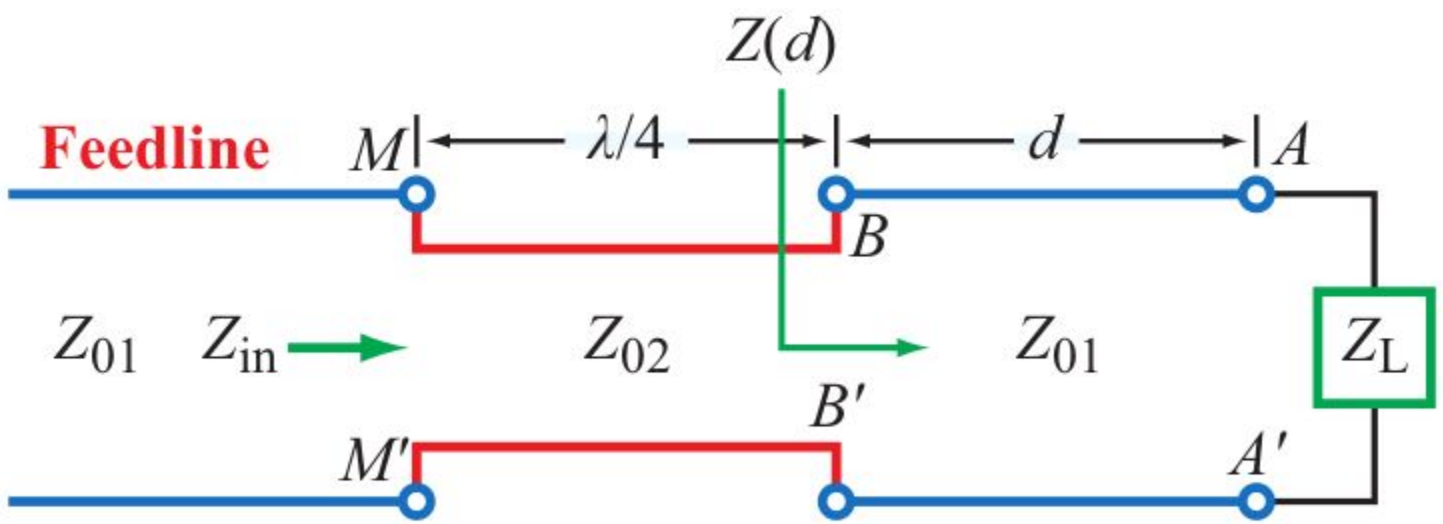
Example 2-15 $\lambda/4$ Matching



Process: 1. Figure out distance d to get $Z(d)$ real,

2. Use equation to get Z_{02}.

Example 2-15 λ/4 Matching

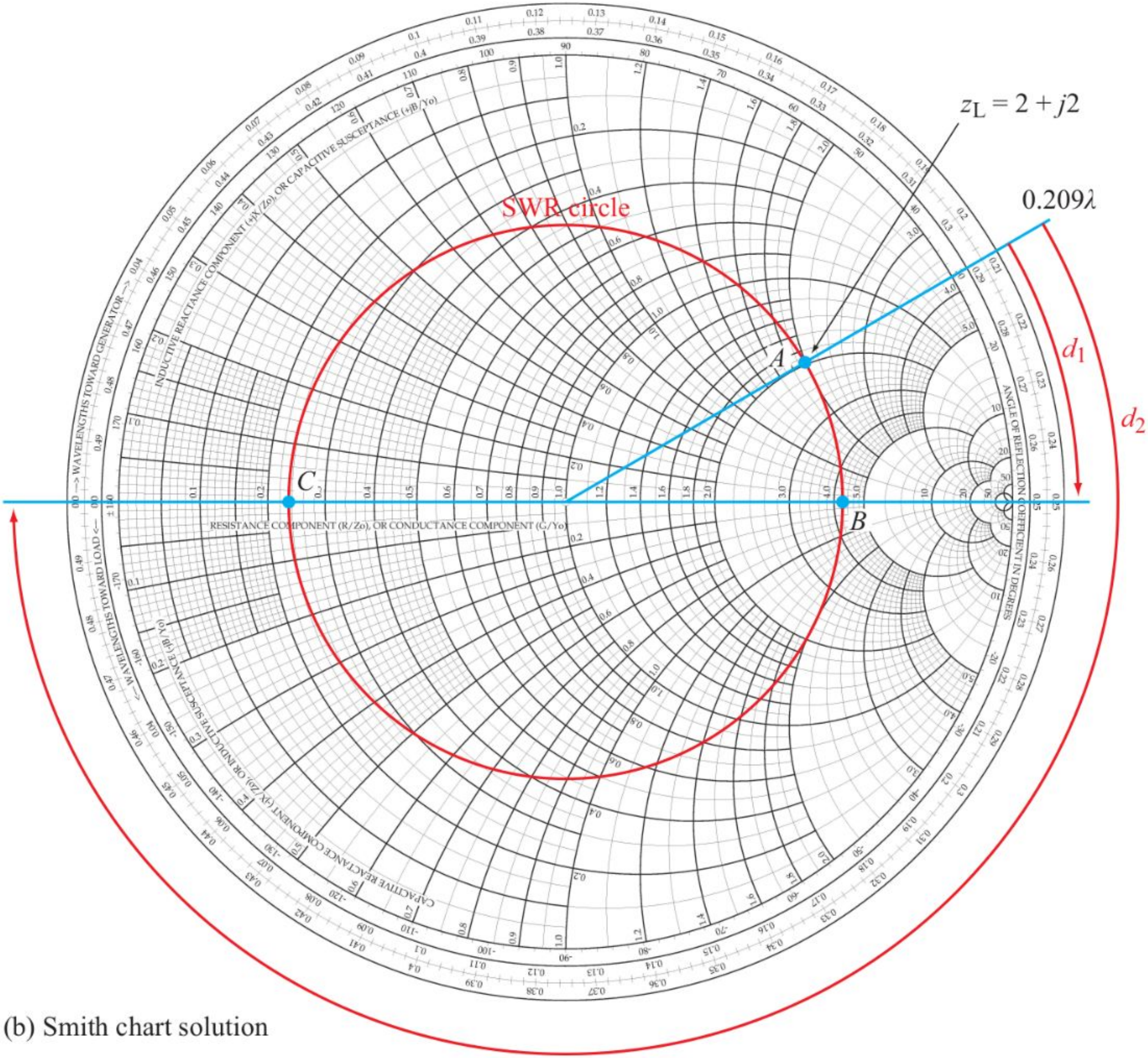


Solution:

step1: calculate z_L and plot on Smith Chart:

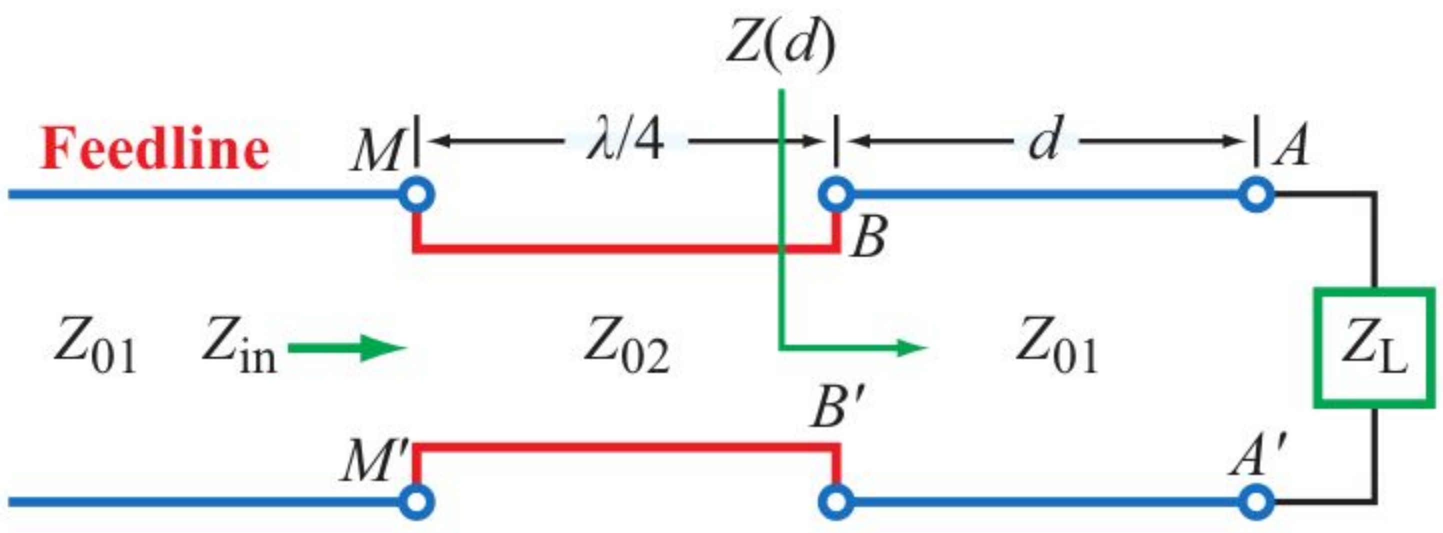
$$z_L = (100 + j100\ \Omega) / 50\ \Omega$$

$$z_L = 2 + j2$$



(b) Smith chart solution

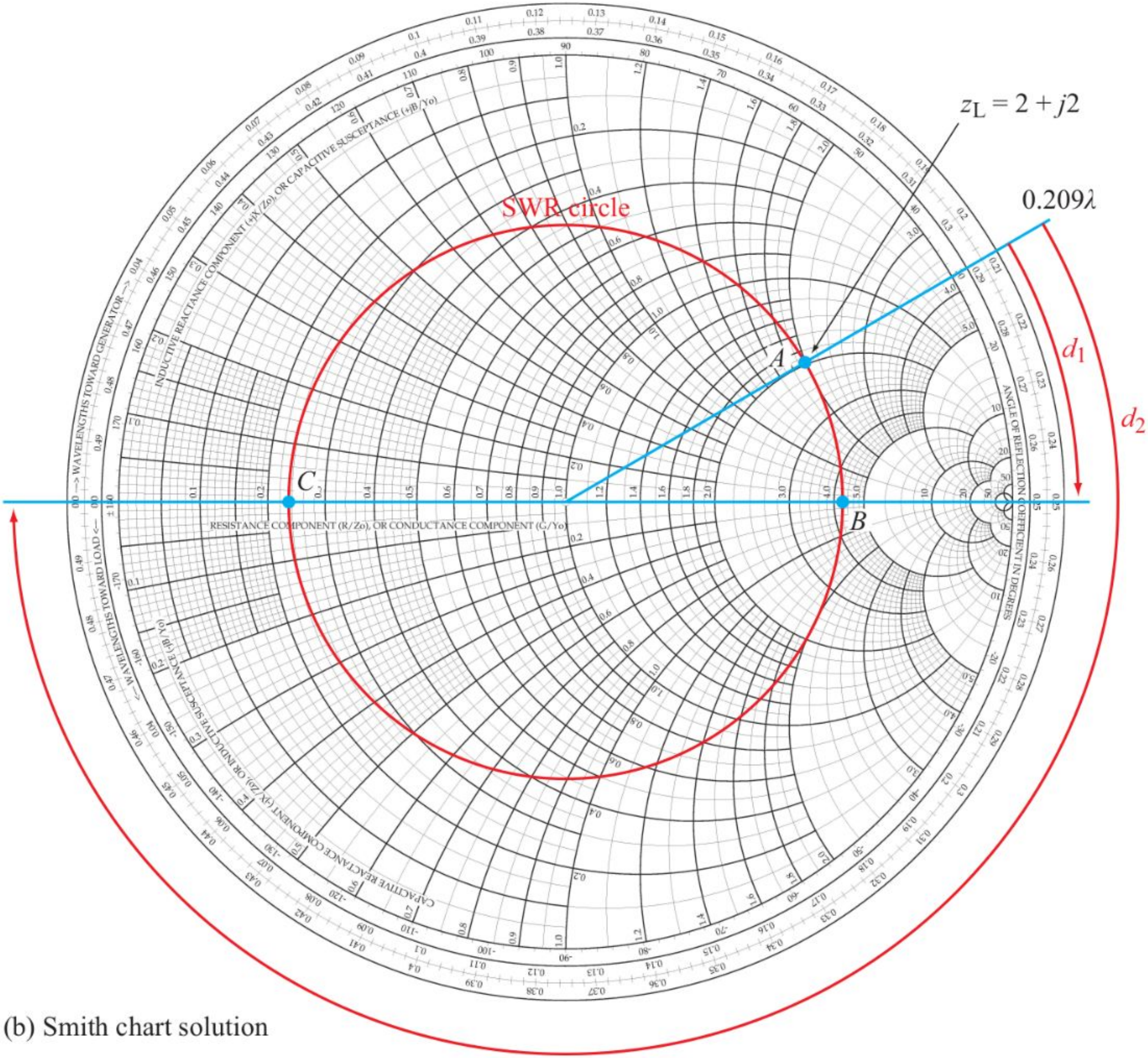
Example 2-15 $\lambda/4$ Matching



Solution:

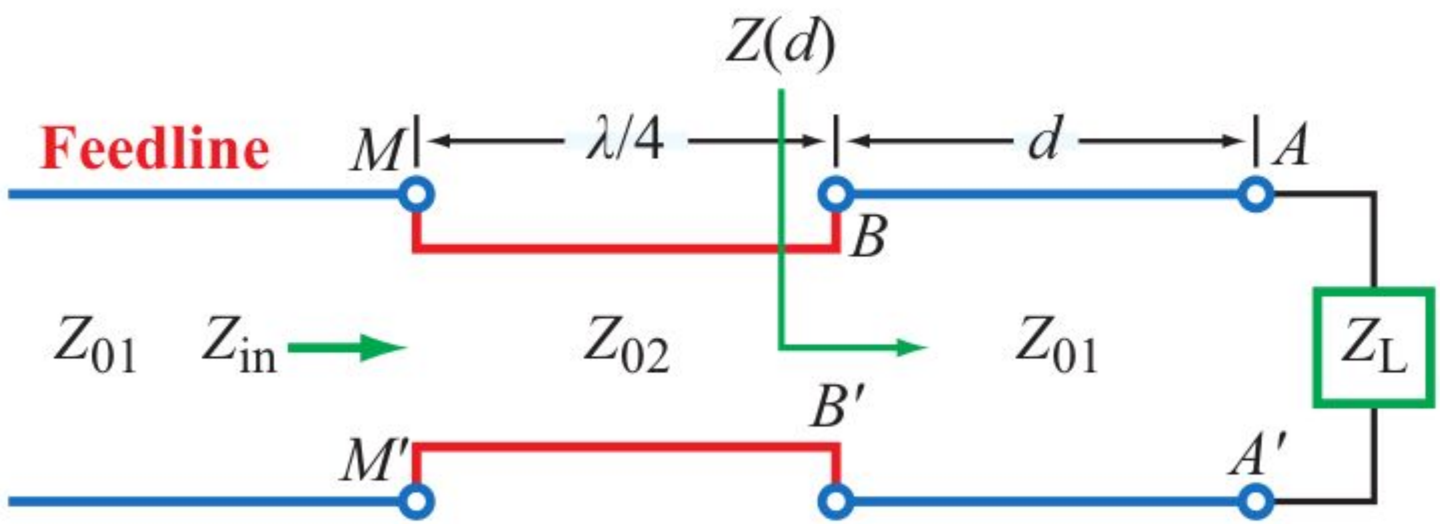
step2: draw vswr circle through this point.

step3: draw radial line, read off WTG: 0.209λ



(b) Smith chart solution

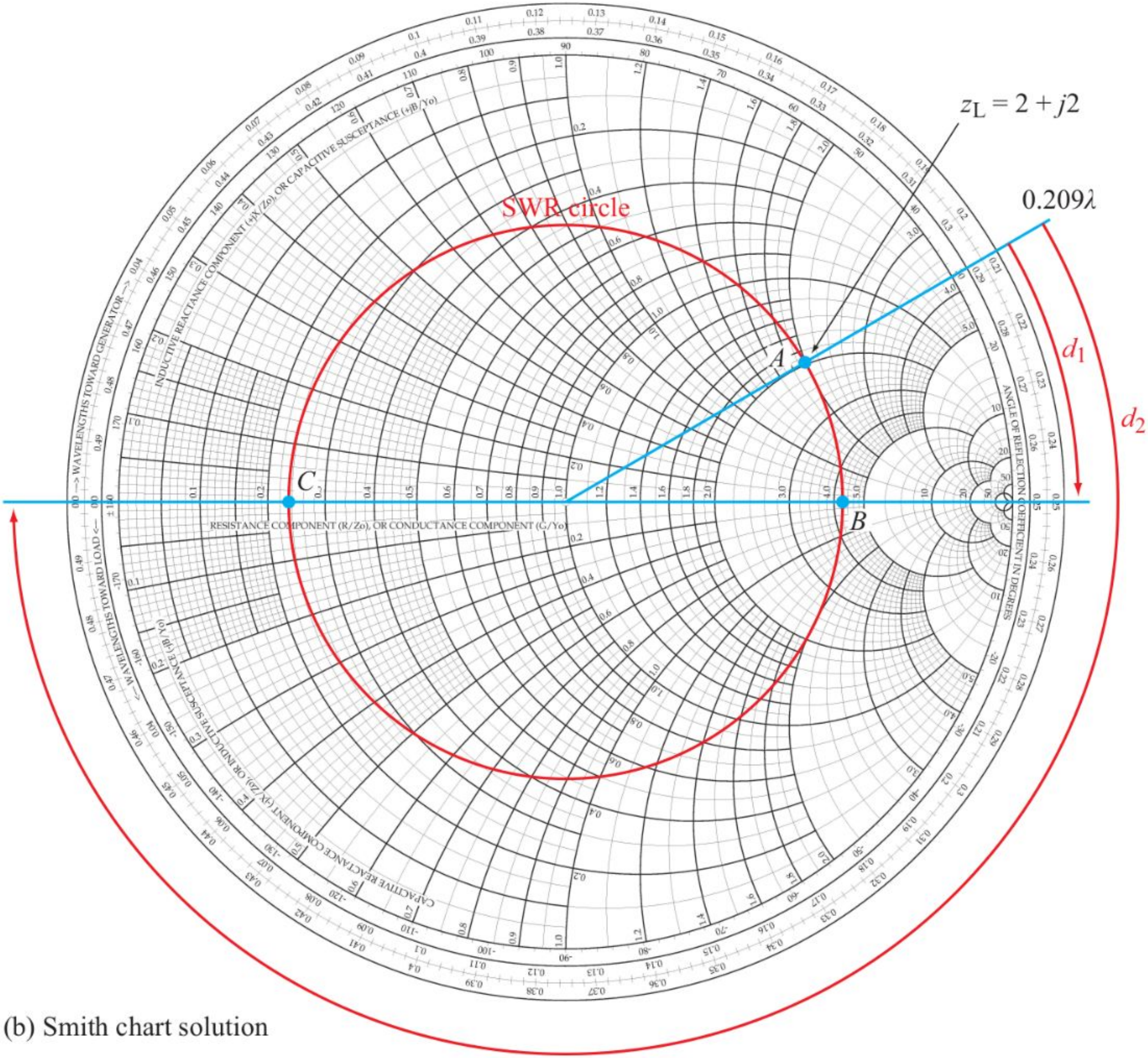
Example 2-15 $\lambda/4$ Matching



Solution:

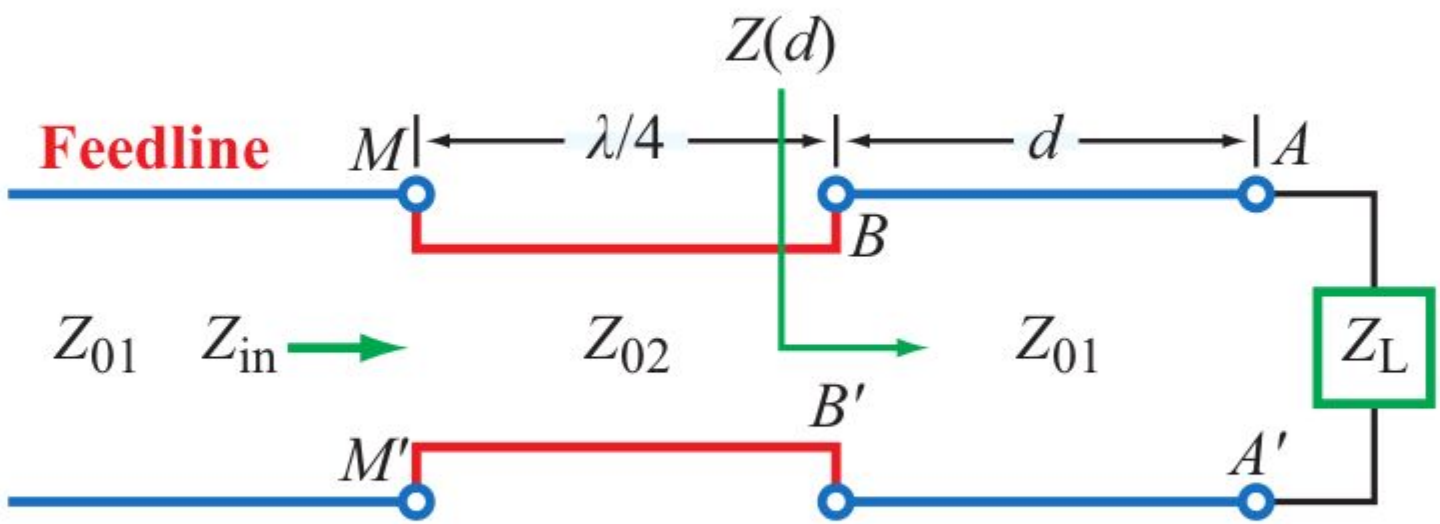
step4: move towards generator until get a real impedance:

1. first point is on positive real axis
2. second point is on negative real axis



(b) Smith chart solution

Example 2-15 λ/4 Matching



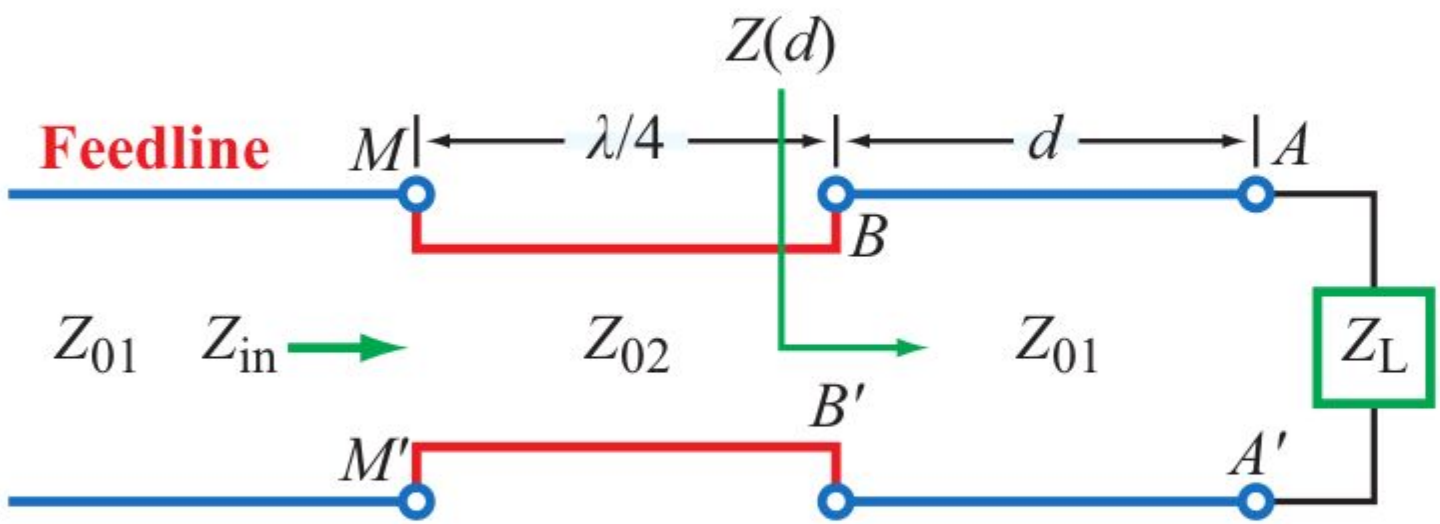
Solution:

step5: For first point:

$d = 0.25\lambda - 0.209\lambda$ so: $d = 0.041\lambda$

step6: read off z(d) = 4.27

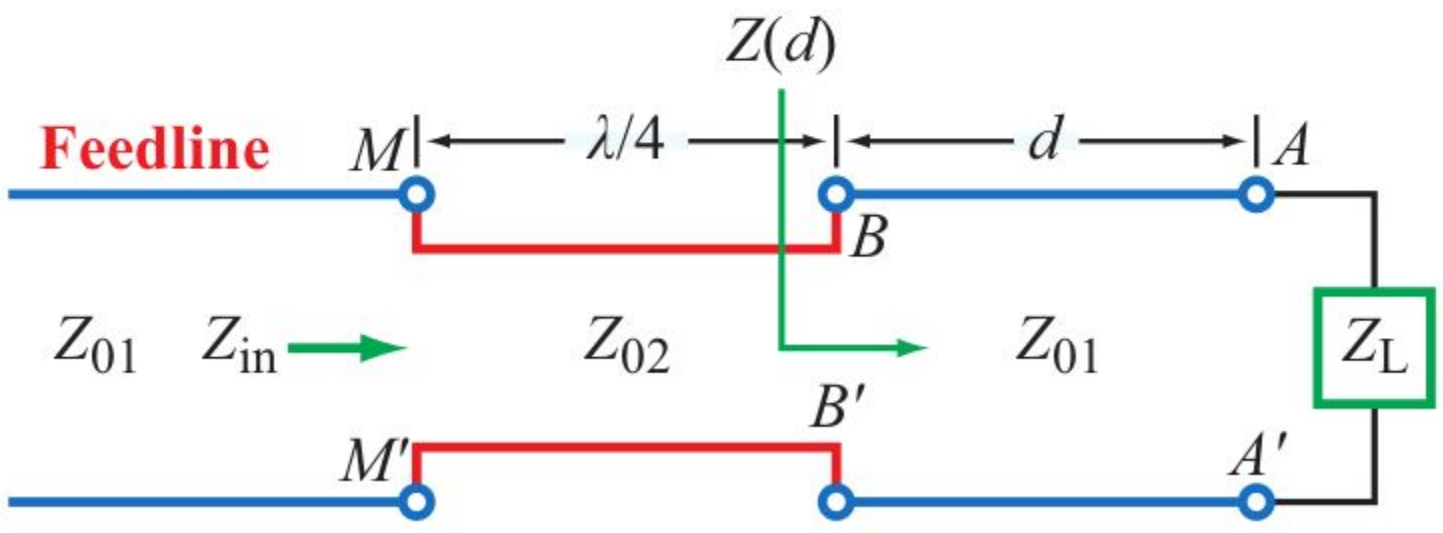
Example 2-15 $\lambda/4$ Matching



Solution:

step7: $Z(d) = Z_0\, z(d) = (50\ \Omega)\ 4.27 = 213.5\ \Omega$

Example 2-15 $\lambda/4$ Matching



Solution:

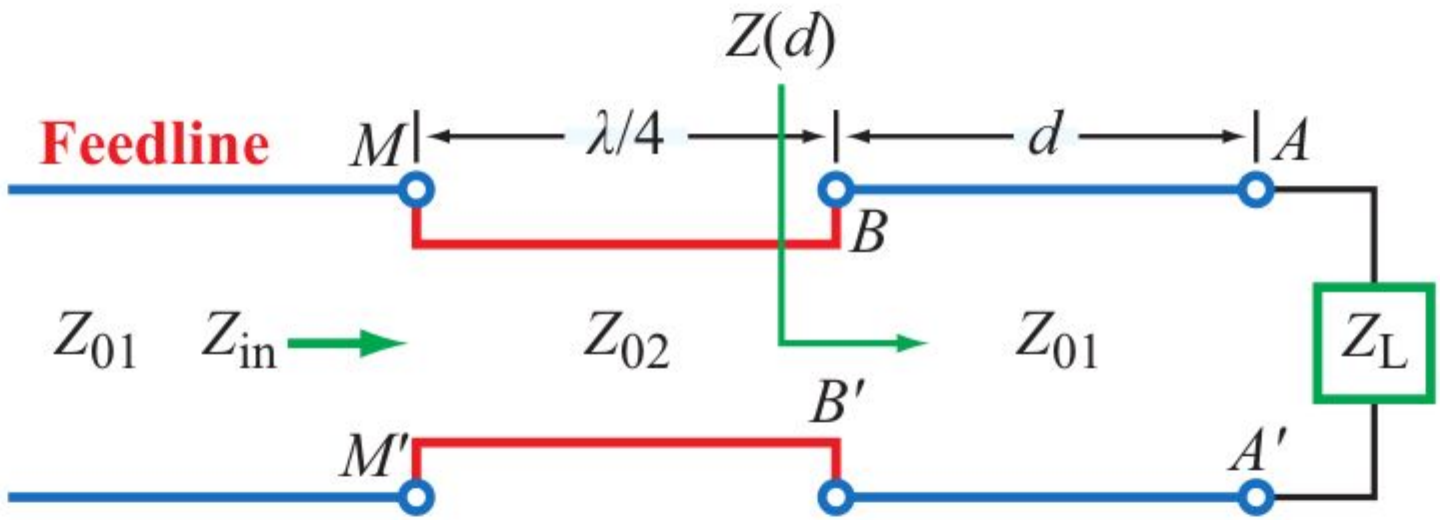
step8: calculate Z_{02}:

$$Z_{02} = \sqrt{Z(d) Z_{01}}$$

$$Z_{02} = \sqrt{(213.5\,\Omega)\,(50\,\Omega)}$$

$$\boxed{Z_{02} = 103.3\,\Omega}$$

Example 2-15 $\lambda/4$ Matching

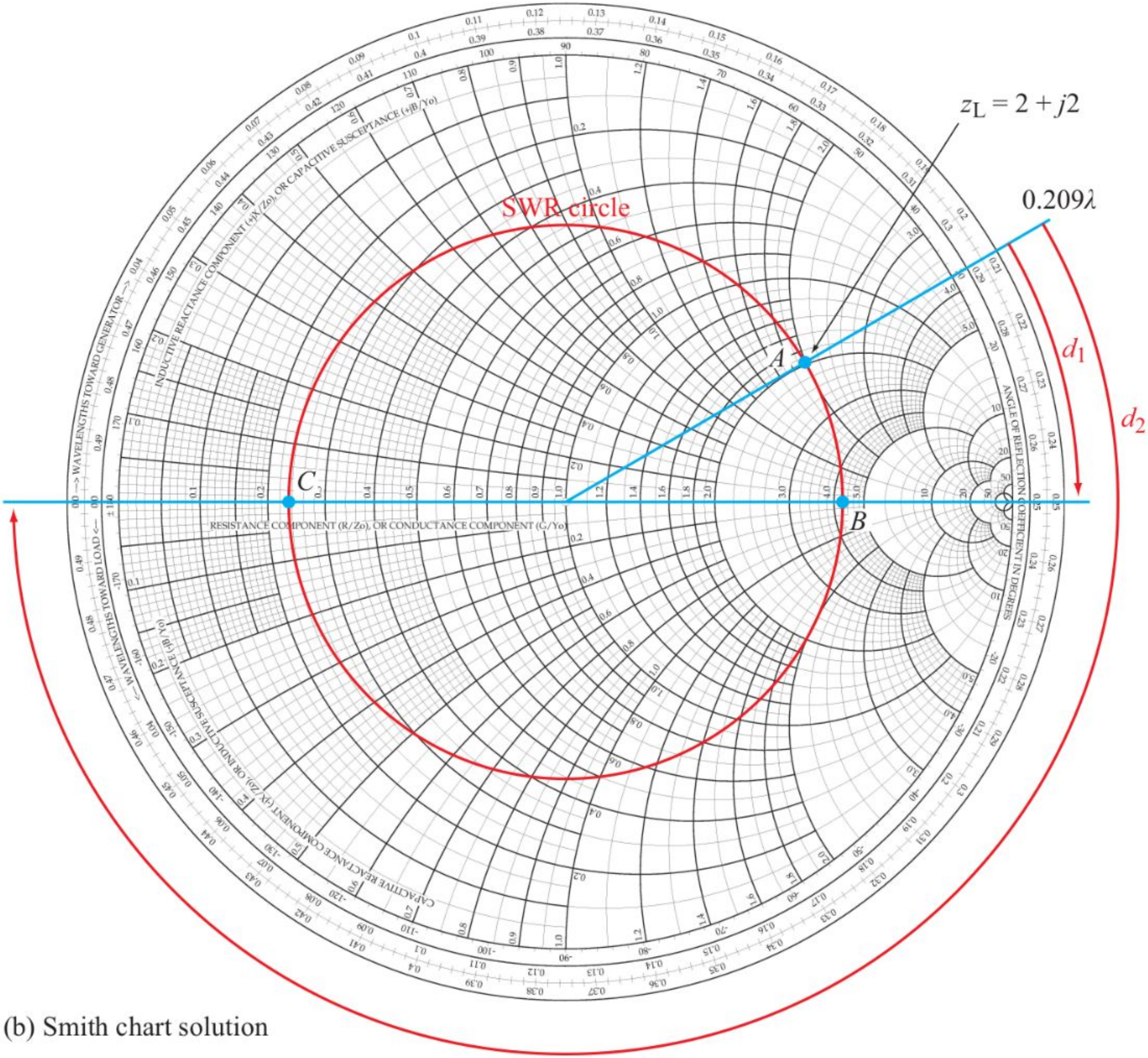


Solution:

step5: For second point:

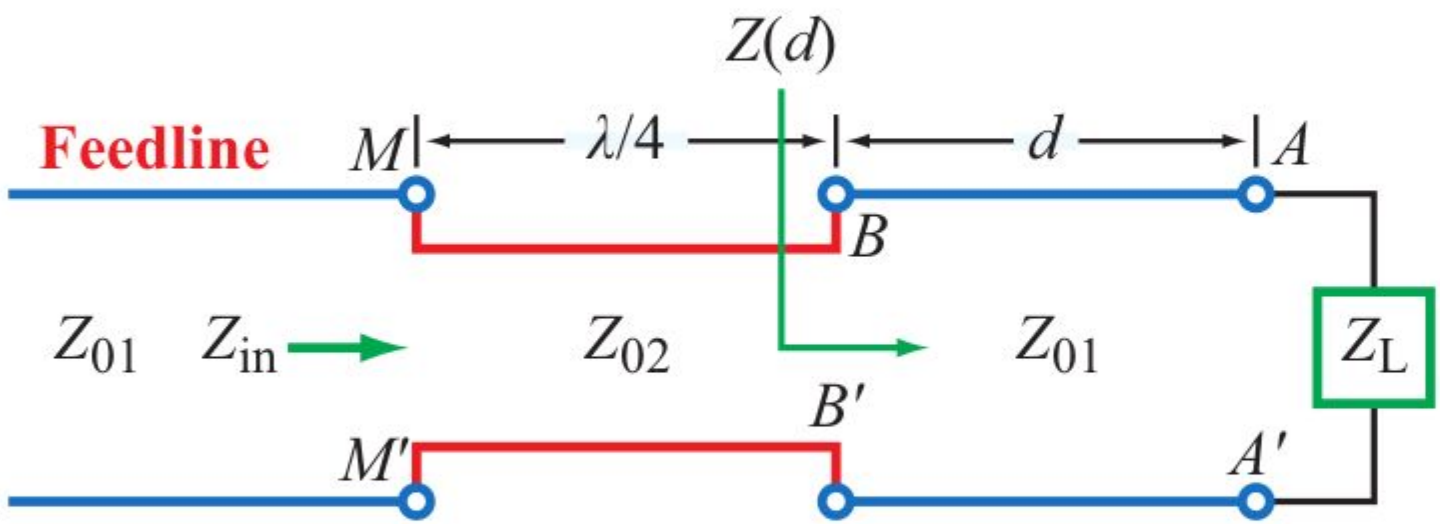
$d = 0.25\lambda + (0.25\lambda - 0.209\lambda)$ so: $d = 0.291\lambda$

step6: read off $z(d) = 0.23$



(b) Smith chart solution

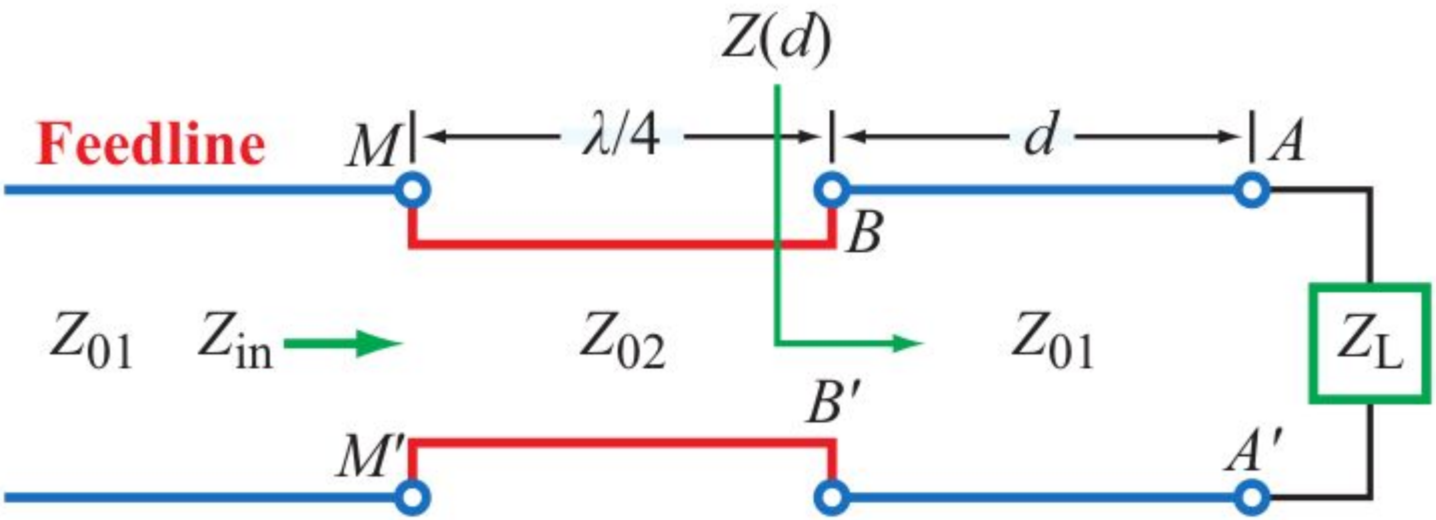
Example 2-15 $\lambda/4$ Matching



Solution:

step7: $Z(d) = Z_0\, z(d) = (50\ \Omega)\ 0.23 = 11.5\ \Omega$

Example 2-15 $\lambda/4$ Matching



Solution:

step8: calculate Z_{02}:

$$Z_{02} = \sqrt{Z(d)\,Z_{01}}$$

$$Z_{02} = \sqrt{(11.5\,\Omega)\,(50\,\Omega)}$$

$$\boxed{Z_{02} = 24.2\,\Omega}$$

Homework

Homework 9 is due tomorrow at midnight.

submit to gradescope via the canvas site.

Next Time

Sections 2-11, 2-12:

The Smith Chart: Impedance Matching

Transients on Transmission Lines