EECS 230
ENGINEERING ELECTROMAGNETICS
Leland Pierce

Transmission Lines 4

University of Michigan - Fall 2024

Chapter 2 Overview

What is a transmission line?

Why study transmission lines?

model

diffeq

solve diffeq

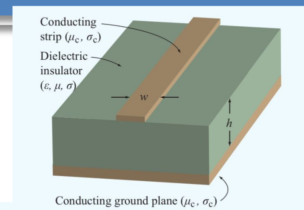
wave propagation

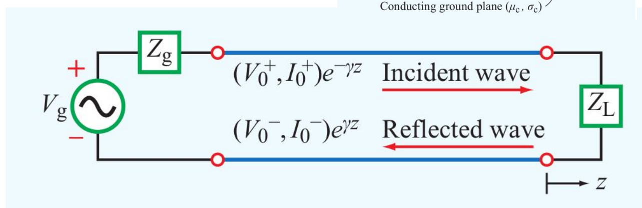
lossless xmission lir

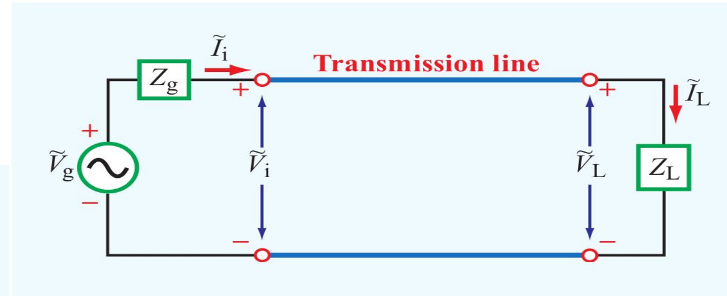
- microstrip lines
- reflections
- standing waves
- impedance
- short, open
- matching
- power flow

smith chart

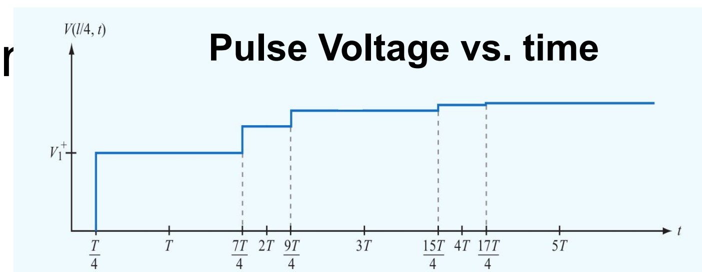
transients

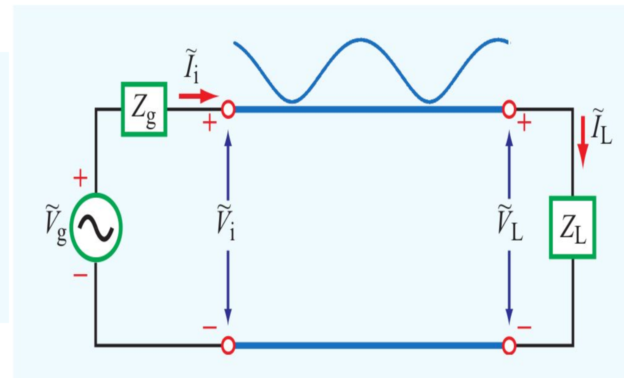




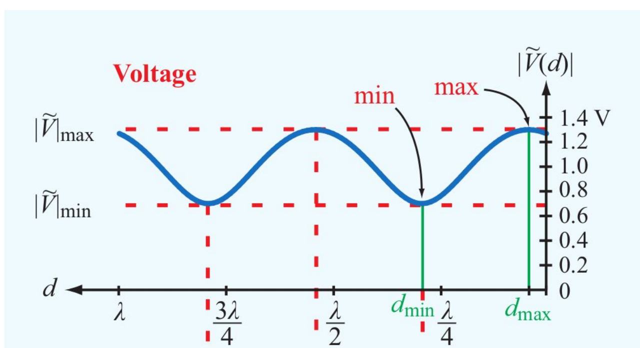


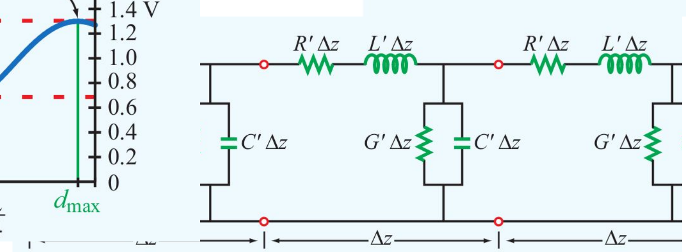
Typical High-Frequency Circuit

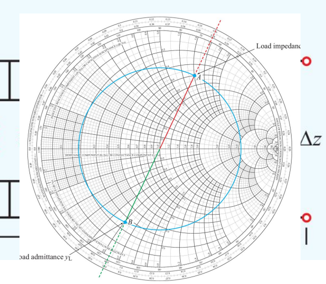




Waves on line: old methods don't work







Today's Lecture Coverage

Review Sections 2-1 through 2-6 of the book:

2-1: What is a transmission line?
Why study transmission lines?
2-2: Lumped-Element Model
2-3: Governing Differential Eqns
2-4: Solve the Differential Equations
Properties of the solution: wave propagation
2-5: Lossless Microstrip Line
2-6: Lossless Transmission Lines

Section 2-7 of the book:

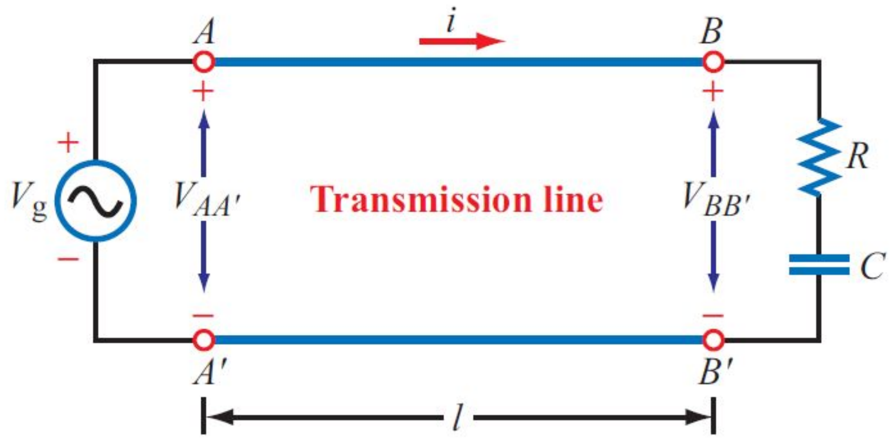
2-7: Lossless Transmission Lines: Wave Impedance

Chapter 2 Review

- A transmission line connects a generator to a load.

Chapter 2 Review

Phase Delay due to length of transmission line:



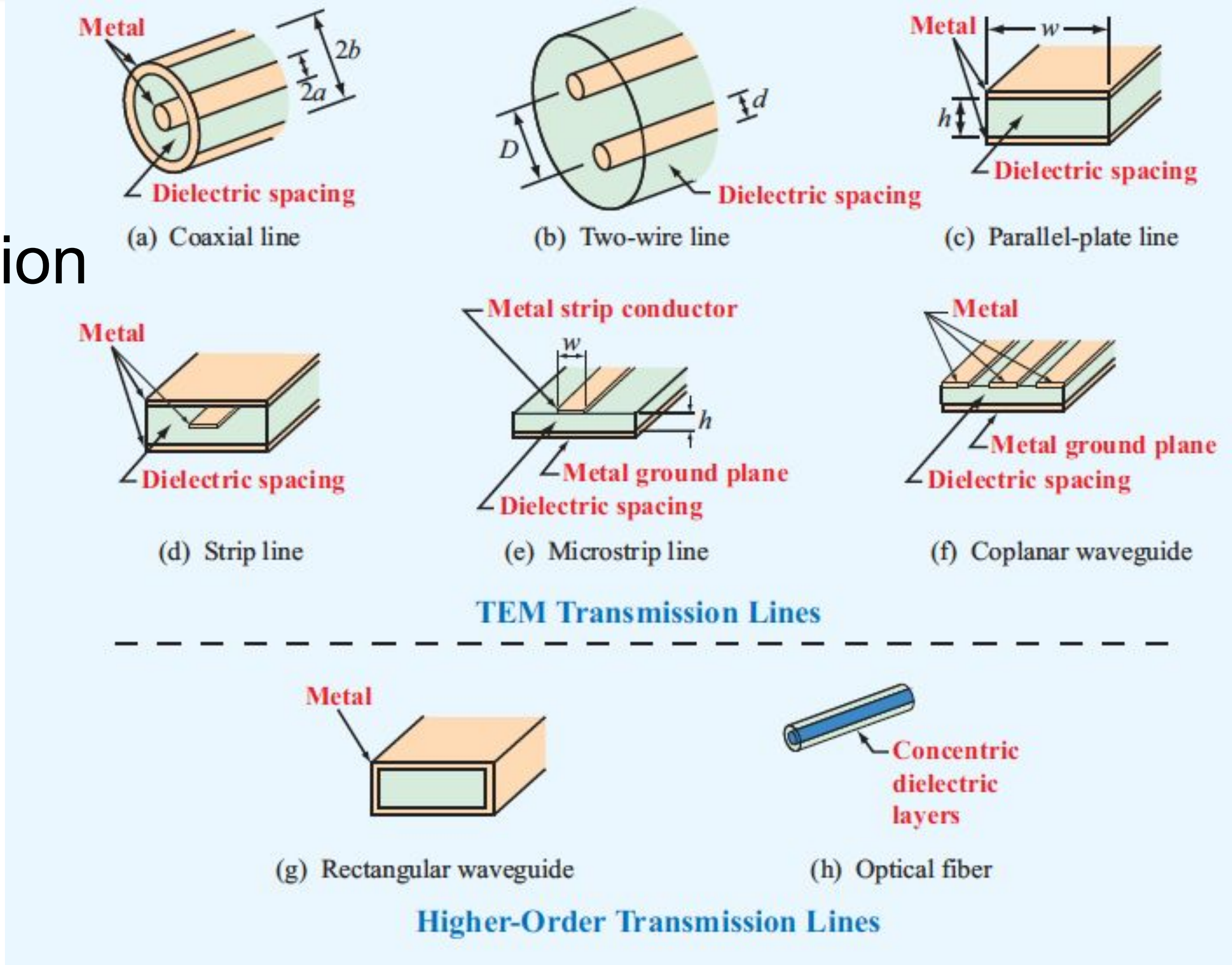
$$\phi_0 = \frac{\omega l}{c} = \frac{2\pi f l}{c} = 2\pi \frac{l}{\lambda} \quad \text{radians.}$$

$l/\lambda \lesssim 0.01$: Can ignore transmission-line effects

$l/\lambda \gtrsim 0.01$: Must deal with phase shift, and other effects...

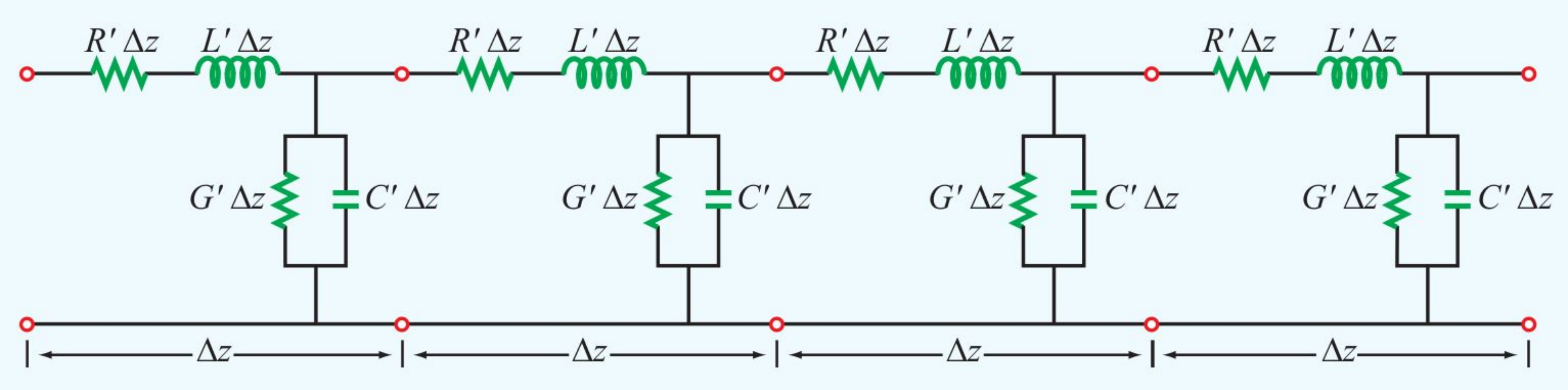
Chapter 2 Review

Different geometries for transmission lines



Chapter 2 Review

Lumped-Element Model:



All parameters are "per unit length":

R': Combined Resistance of BOTH conductors: □/m

L': Combined Inductance of BOTH conductors, H/m

G': Conductance of insulation
between inner and outer conductor, S/m

C': Capacitance
between inner and outer conductors, F/m

Chapter 2 Review

Lumped-Element Values: geometry/materials/freq

Table 2-1 **Transmission-line parameters R', L', G', and C' for three types of lines.**

Parameter	Coaxial	Two-Wire	Parallel-Plate	Unit
R'	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{2R_s}{\pi d}$	$\frac{2R_s}{w}$	Ω/m
L'	$\frac{\mu}{2\pi}\ln(b/a)$	$\frac{\mu}{\pi}\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]$	$\frac{\mu h}{w}$	H/m
G'	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\pi\sigma}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\sigma w}{h}$	S/m
C'	$\frac{2\pi\varepsilon}{\ln(b/a)}$	$\frac{\pi\varepsilon}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\varepsilon w}{h}$	F/m

$$R_s = \sqrt{\pi f \mu_c / \sigma_c}$$

Chapter 2 Review

Lumped-Element Values: geometry/materials/freq

$$L'C' = \mu\epsilon$$

$$\frac{G'}{C'} = \frac{\sigma}{\epsilon}$$

This turns out to be true for all the transmission-lines we study.

Chapter 2 Review

Transmission-line governing Differential Equations

$$-\frac{d\widetilde{V}(z)}{dz} = (R' + j\omega L')\,\widetilde{I}(z),$$

$$-\frac{d\widetilde{I}(z)}{dz} = (G' + j\omega C')\,\widetilde{V}(z).$$

(telegrapher's equations in phasor form)

Chapter 2 Review

Transmission-line governing Differential Equation for *V*:

$$\frac{d^2\widetilde{V}(z)}{dz^2} - \gamma^2\,\widetilde{V}(z) = 0,$$

(wave equation for $\widetilde{V}(z)$)

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}\,.$$

(propagation constant)

Chapter 2 Review

Complex propagation constant:

$$\gamma = \alpha + j\beta$$

Where:

γ: Units of 1/m

α: Attenuation constant, units of Np/m (>0 in this class)

β: Phase constant, units of rad/m

Np and radians are both "dimensionless", and are the conventional "units" for these parameters.

Chapter 2 Review

Form of the solution: traveling waves, going in both directions:

$$\widetilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \qquad \text{(V)},$$

$$\widetilde{I}(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z} \qquad \text{(A)}.$$

$$Z_0 = \frac{R' + j\omega L'}{\gamma} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \qquad (\Omega),$$

Chapter 2 Review

Solution in time-domain

$$v(z,t) = |V_0^+| e^{-\alpha z} \cos(\omega t - \beta z + \phi^+) + |V_0^-| e^{+\alpha z} \cos(\omega t + \beta z + \phi^-)$$

Remaining unknowns are determined via specification of source and load.

Chapter 2 Review

Microstrip Transmission-Line Parameters:

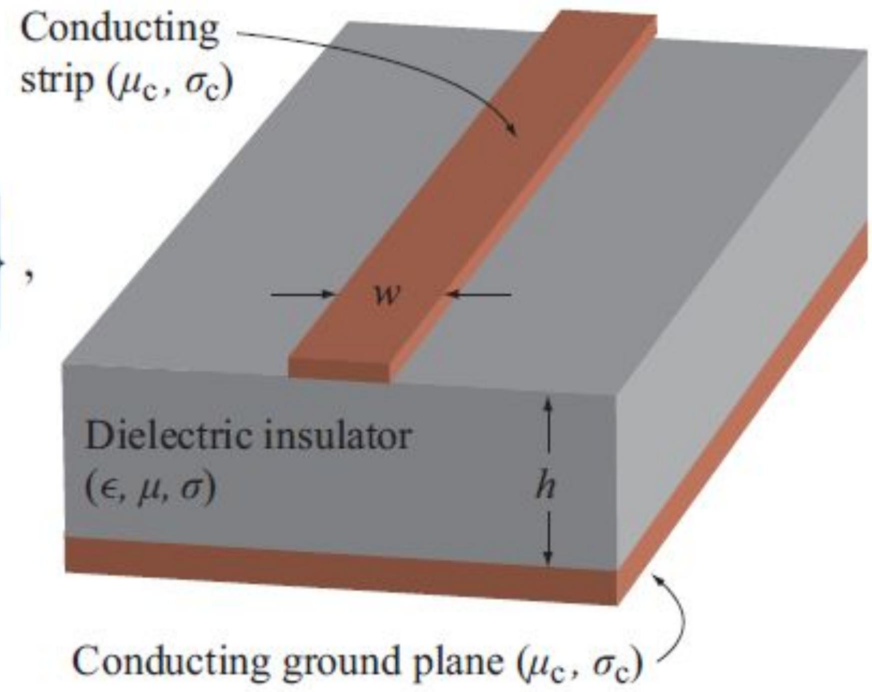
$$Z_0 = \frac{60}{\sqrt{\varepsilon_{\text{eff}}}} \ln\left\{\frac{6 + (2\pi - 6)e^{-t}}{s} + \sqrt{1 + \frac{4}{s^2}}\right\},$$

$$t = \left(\frac{30.67}{s}\right)^{0.75} \qquad s = \frac{w}{h},$$

$$\varepsilon_{\text{eff}} = \frac{\varepsilon_r + 1}{2} + \left(\frac{\varepsilon_r - 1}{2}\right)\left(1 + \frac{10}{s}\right)^{-xy},$$

$$x = 0.56\left[\frac{\varepsilon_r - 0.9}{\varepsilon_r + 3}\right]^{0.05},$$

$$y = 1 + 0.02 \ln\left(\frac{s^4 + 3.7 \times 10^{-4} s^2}{s^4 + 0.43}\right) + 0.05 \ln(1 + 1.7 \times 10^{-4} s^3).$$



Largely the result of fitting lots of data using functional forms based on intuition.

Chapter 2 Review

- The wave equation for a general Transmission Line.

$$\frac{d^2\widetilde{V}(z)}{dz^2} - \gamma^2\,\widetilde{V}(z) = 0, \qquad (2.21)$$

- General solution of the wave equation
  - *It involves both incident and reflected waves*

$$\widetilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \qquad (\text{V}),$$

$$\widetilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \qquad (\text{A}).$$

Chapter 2 Review

- Useful Relations for lossless Transmission Lines:

$$\alpha = 0 \qquad \text{(lossless line)},$$
$$\beta = \omega\sqrt{L'C'} \qquad \text{(lossless line)}. \qquad (2.45)$$

$$\beta = \omega\sqrt{\mu\varepsilon} \qquad \text{(rad/m)}, \qquad (2.49)$$

$$u_p = \frac{1}{\sqrt{\mu_0\epsilon_0}}\frac{1}{\sqrt{\epsilon_r}} = \frac{c}{\sqrt{\epsilon_r}}$$

$$\lambda = \frac{u_\text{p}}{f} = \frac{c}{f}\,\frac{1}{\sqrt{\varepsilon_\text{r}}} = \frac{\lambda_0}{\sqrt{\varepsilon_\text{r}}}$$

$$Z_0 = \sqrt{\frac{L'}{C'}} \qquad \text{(lossless line)}, \qquad (2.46)$$

(REAL)

Chapter 2 Review

- Voltage reflection coefficient due to load:

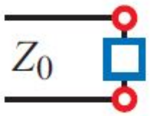
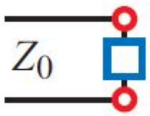
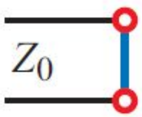
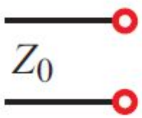
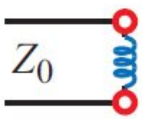
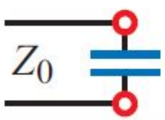
$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1} = \frac{z_L - 1}{z_L + 1}$$

- Load impedance in terms of Γ:

$$Z_L = \frac{1+\Gamma}{1-\Gamma} Z_0$$

Chapter 2 Review

Reflection Coefficient $\Gamma = |\Gamma| e^{j\theta_r}$

Load	$\lvert\Gamma\rvert$	θ_r
 $Z_L = (r + jx)Z_0$	$\left[\dfrac{(r-1)^2 + x^2}{(r+1)^2 + x^2}\right]^{1/2}$	$\tan^{-1}\left(\dfrac{x}{r-1}\right) - \tan^{-1}\left(\dfrac{x}{r+1}\right)$
 Z_0	0 (no reflection)	irrelevant
 (short)	1	$\pm 180°$ (phase opposition)
 (open)	1	0 (in-phase)
 $jX = j\omega L$	1	$\pm 180° - 2\tan^{-1} x$
 $jX = \dfrac{-j}{\omega C}$	1	$\pm 180° + 2\tan^{-1} x$

$$z_L = Z_L / Z_0 = (R + jX)/Z_0 = r + jx$$

Chapter 2 Review

- Concept of standing wave

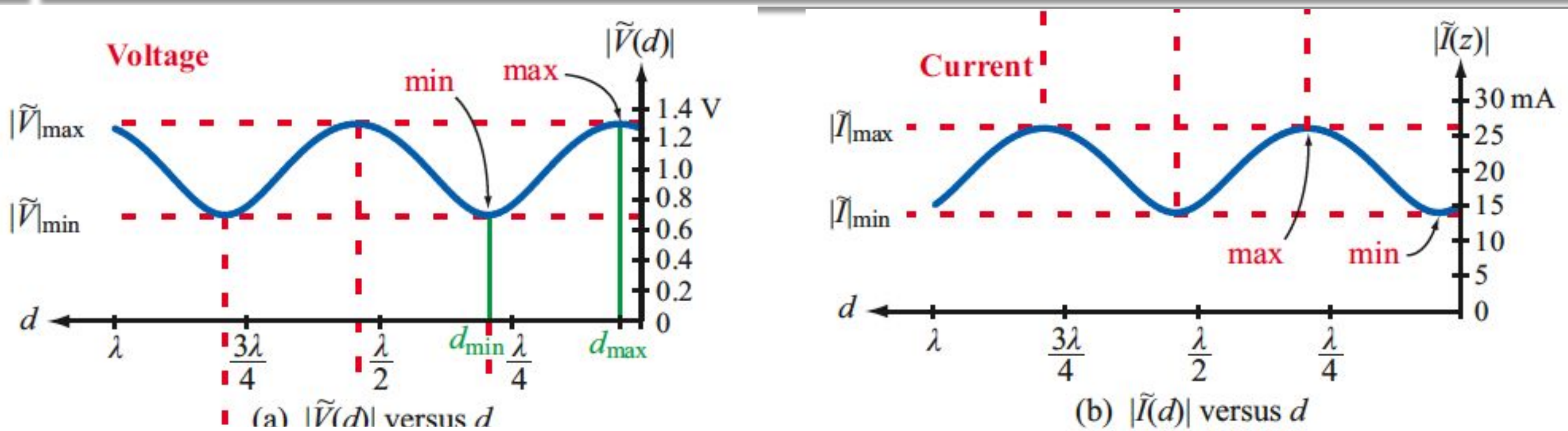
$$\tilde{V}(z) = V_0^+ (e^{-j\beta z} + \Gamma e^{j\beta z}),$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0} (e^{-j\beta z} - \Gamma e^{j\beta z}).$$

  - Voltage (current) magnitudes at any point on line:

$$|\tilde{V}(d)| = |V_0^+| \left[1 + |\Gamma|^2 + 2|\Gamma| \cos(2\beta d - \theta_r) \right]^{1/2}. \quad (2.66)$$

Chapter 2 Review

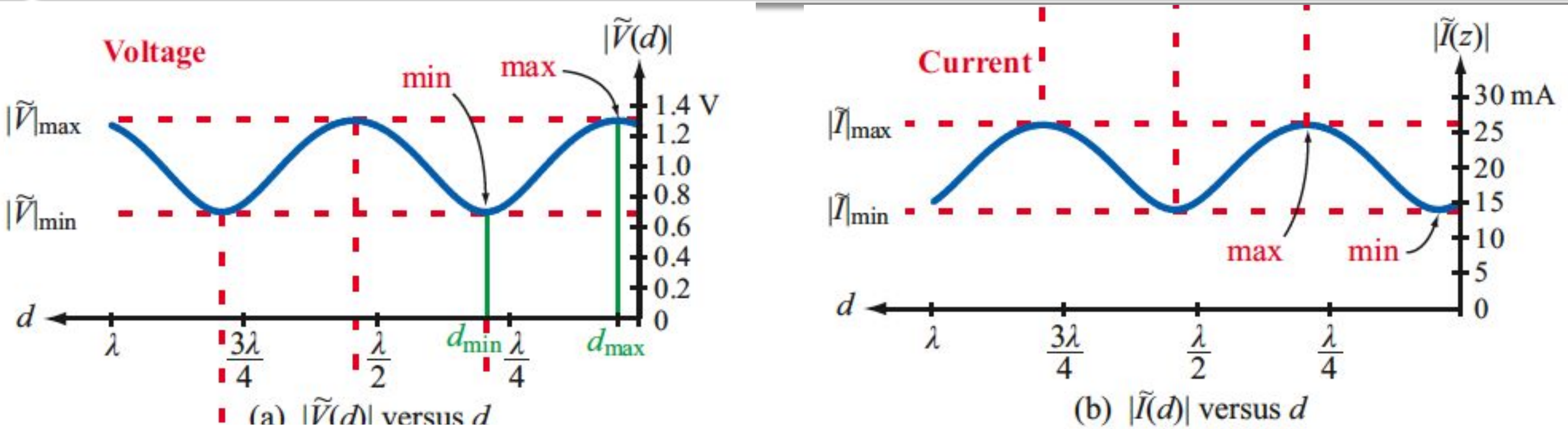


- Location of minima / maxima

$$d_{max} = \frac{\theta_r + 2n\pi}{2\beta} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2}, \quad \begin{cases} n = 1, 2, \ldots & \text{if } \theta_r < 0, \\ n = 0, 1, 2, \ldots & \text{if } \theta_r \geq 0, \end{cases} \qquad (2.70)$$

Value of V_{max}: $|\tilde{V}(d)| = |\tilde{V}|_{max} = |V_0^+|[1 + |\Gamma|]$,

Chapter 2 Review



- Spatial period of standing wave: $\frac{\lambda}{2}$

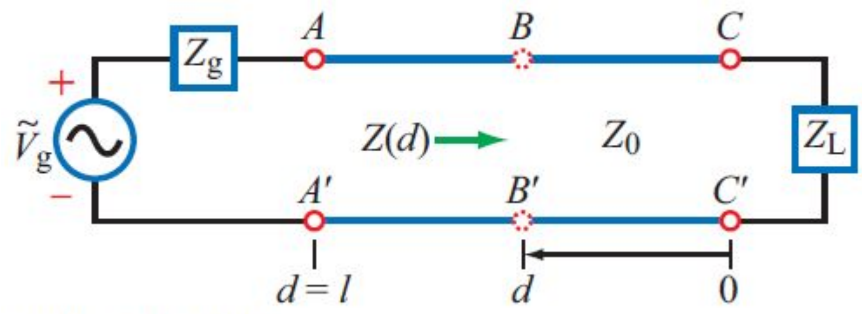
- Standing wave ratio S:

$$S = \frac{|\widetilde{V}|_{max}}{|\widetilde{V}|_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

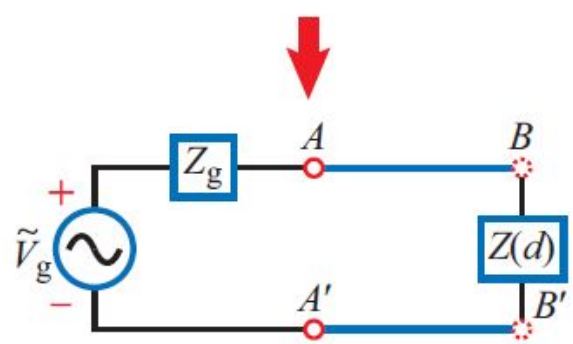
2-7 Wave Impedance

At a distance *d* from the load:

$$
\begin{aligned}
Z(d) &= \frac{\widetilde{V}(d)}{\widetilde{I}(d)} \\
&= \frac{V_0^+[e^{j\beta d} + \Gamma e^{-j\beta d}]}{V_0^+[e^{j\beta d} - \Gamma e^{-j\beta d}]} Z_0 \\
&= Z_0 \left[\frac{1 + \Gamma e^{-j2\beta d}}{1 - \Gamma e^{-j2\beta d}}\right] \\
&= Z_0 \left[\frac{1 + \Gamma_d}{1 - \Gamma_d}\right] \qquad (\Omega),
\end{aligned}
$$



(a) Actual circuit



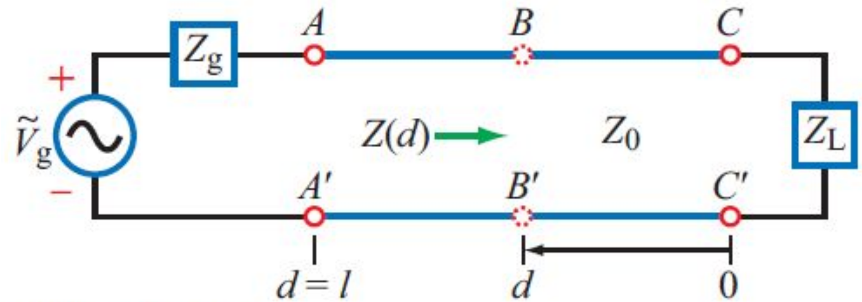
(b) Equivalent circuit

2-7 Wave Impedance

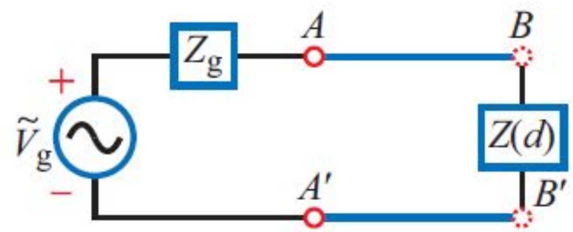
Define the phase-shifted voltage reflection coefficient:

$$\Gamma_d = \Gamma e^{-j2\beta d} = |\Gamma| e^{j\theta_r} e^{-j2\beta d}$$

$$= |\Gamma| e^{j(\theta_r - 2\beta d)}$$



(a) Actual circuit



(b) Equivalent circuit

Z(d) is different than Z_0:
Ratio of Total Voltage and Current

Recall: $Z_0 = \dfrac{V_0^+}{I_0^+}$

The ratio for the forward-going wave only.

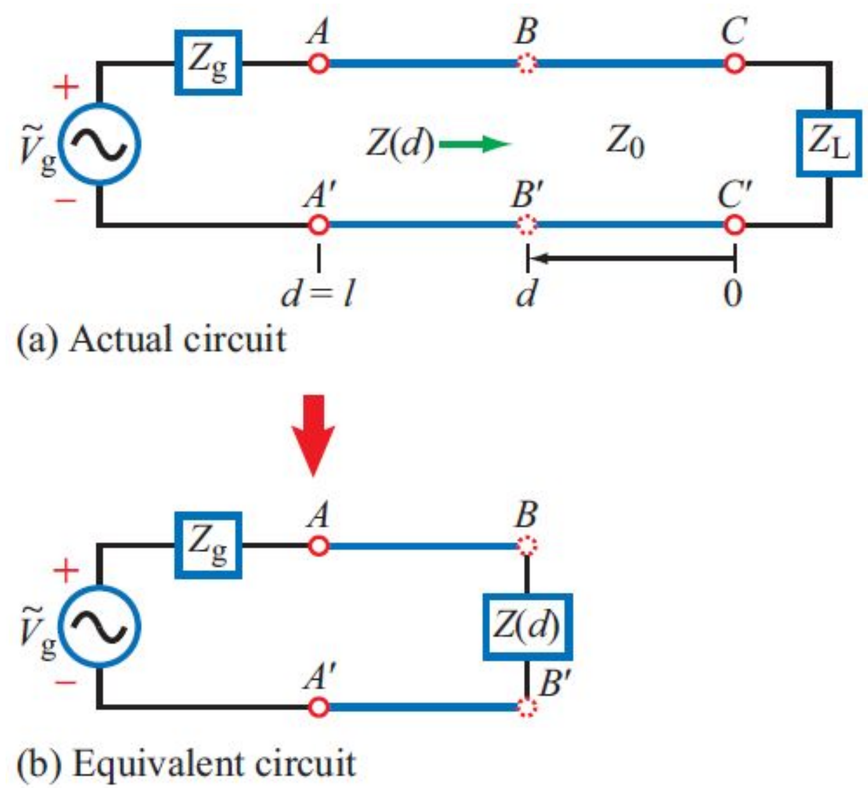
2-7 Wave Impedance

Reflection coefficient at $d=l$:

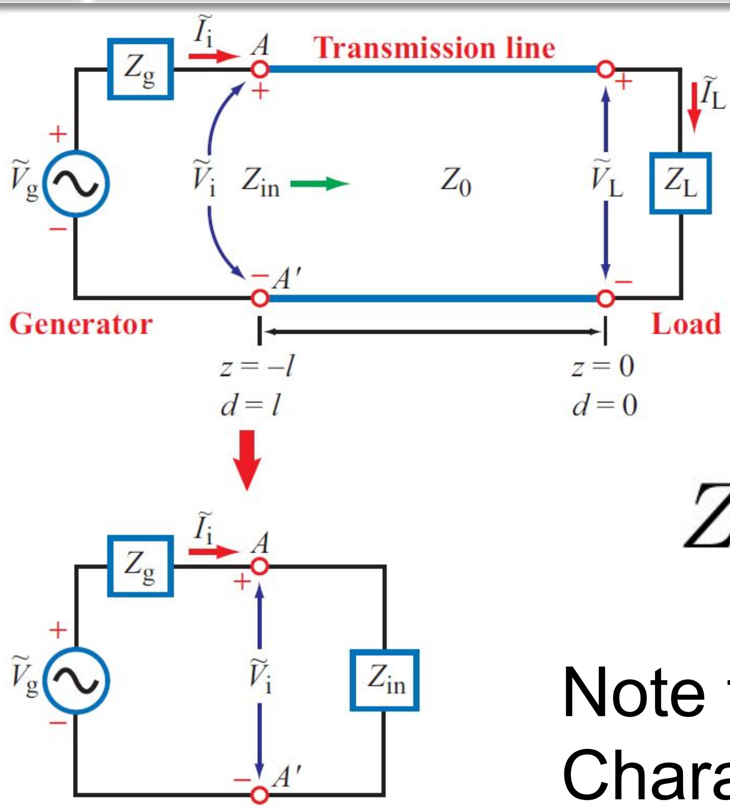
$$\Gamma_l = \Gamma e^{-j2\beta l} = |\Gamma| e^{j(\theta_r - 2\beta l)}.$$

Impedance at z=l:

$$Z_{in} = Z(d=l) = Z_0 \left[\frac{1+\Gamma_l}{1-\Gamma_l} \right]$$



2-7 Input Impedance



Alternative expression:

Input impedance:

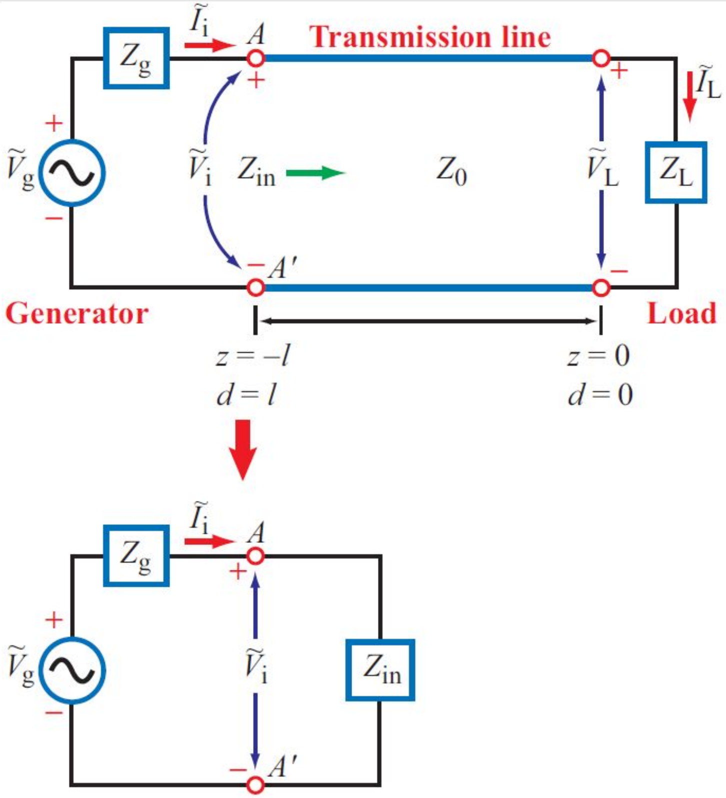
impedance of the transmission line at *d*=*l:*

$$Z_{in} = Z(d = l) = \frac{\widetilde{V}(d = l)}{\widetilde{I}(d = l)}$$

Note that Z_{in} is different from the Characteristic Impedance, and is different from the Load Impedance:

$$Z_{in} \neq Z_0 \qquad Z_{in} \neq Z_L$$

2-7 Input Impedance

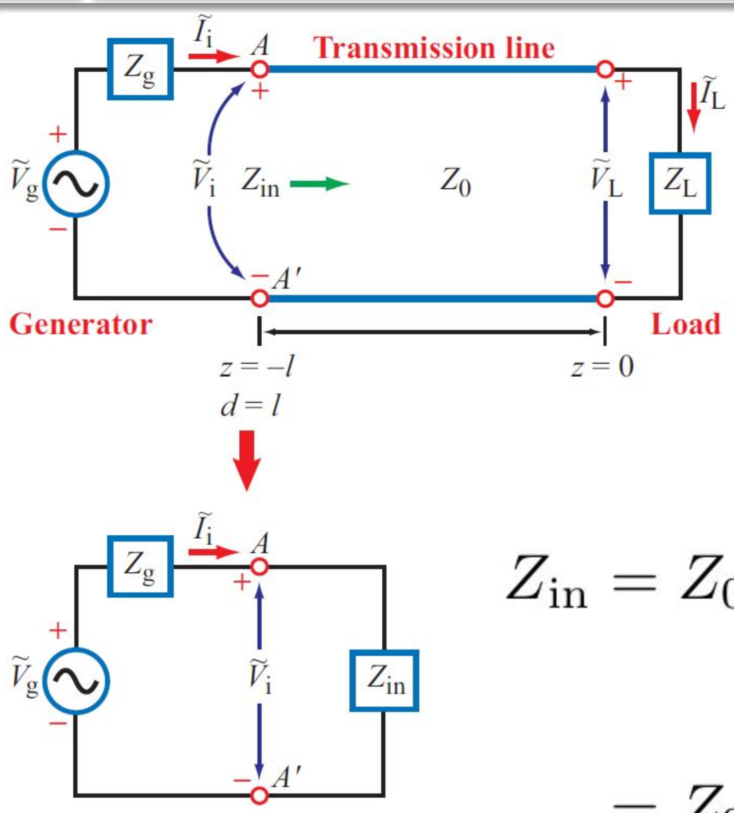


Use voltage and current at *d*=*l*:

$$Z_{\text{in}} = \frac{V_0^+ \left[e^{+j\beta l} + \Gamma_L e^{-j\beta l}\right]}{\frac{V_0^+}{Z_0}\left[e^{+j\beta l} - \Gamma_L e^{-j\beta l}\right]}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

2-7 Input Impedance



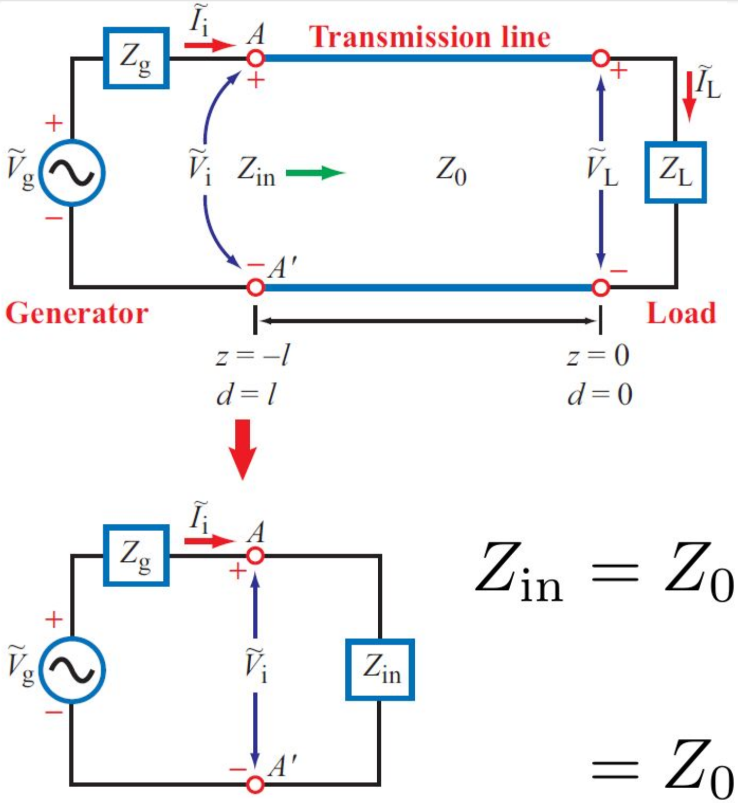
Use expression for Γ:

$$Z_{in} = Z_0 \left[\frac{e^{+j\beta l} + \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) e^{-j\beta l}}{e^{+j\beta l} - \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) e^{-j\beta l}} \right]$$

$$= Z_0 \left[\frac{(Z_L + Z_0)e^{+j\beta l} + (Z_L - Z_0)e^{-j\beta l}}{(Z_L + Z_0)e^{+j\beta l} - (Z_L - Z_0)e^{-j\beta l}} \right]$$

$$= Z_0 \left[\frac{Z_L(e^{+j\beta l} + e^{-j\beta l}) + Z_0(e^{+j\beta l} - e^{-j\beta l})}{Z_L(e^{+j\beta l} - e^{-j\beta l}) + Z_0(e^{+j\beta l} + e^{-j\beta l})} \right]$$

2-7 Input Impedance



Use expressions for sin and cos:

$$e^{+j\beta l} + e^{-j\beta l} = 2\cos(\beta l)$$

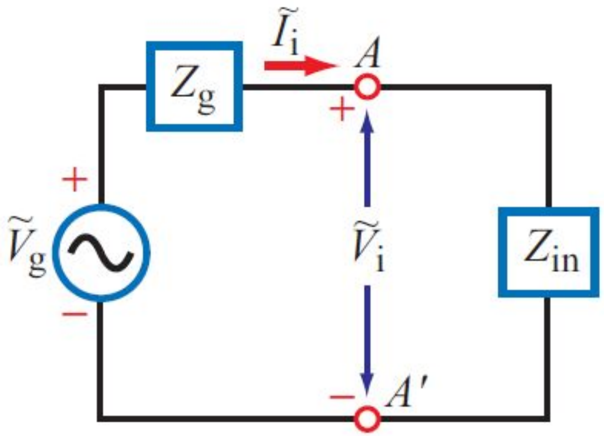
$$e^{+j\beta l} - e^{-j\beta l} = 2j\sin(\beta l)$$

$$Z_{\text{in}} = Z_0 \left[\frac{Z_L \cos(\beta l) + jZ_0 \sin(\beta l)}{jZ_L \sin(\beta l) + Z_0 \cos(\beta l)} \right]$$

$$= Z_0 \left[\frac{z_L \cos(\beta l) + j\sin(\beta l)}{\cos(\beta l) + jz_L \sin(\beta l)} \right]$$

$$Z_{\text{in}} = Z_0 \left[\frac{z_L + j\tan(\beta l)}{1 + jz_L \tan(\beta l)} \right]$$

2-7 Voltage Amplitude



Finally, can solve for the amplitude of the voltage. Voltage division:

$$\tilde{V}_i = \tilde{I}_i Z_{in} = \frac{\tilde{V}_g Z_{in}}{Z_g + Z_{in}},$$

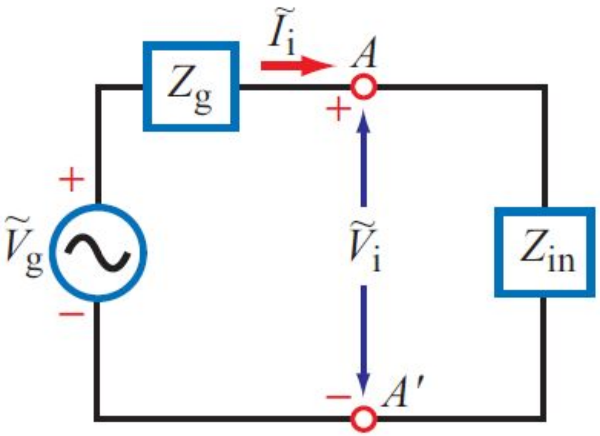
And from the transmission line solution:

$$\tilde{V}(z) = V_0^+(e^{-j\beta z} + \Gamma e^{j\beta z})$$

at $z=-l$:

$$\tilde{V}_i = \tilde{V}(-l) = V_0^+[e^{j\beta l} + \Gamma e^{-j\beta l}].$$

2-7 Voltage Amplitude

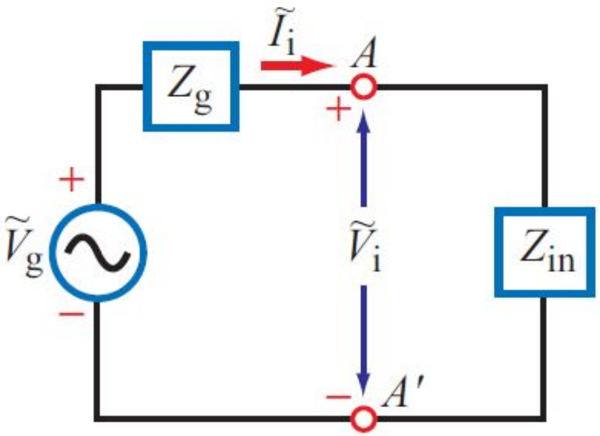


Equate these 2 expressions.
Solve for the Voltage Amplitude:

$$\frac{\widetilde{V}_g Z_{in}}{Z_g + Z_{in}} = V_0^+ \left[e^{j\beta l} + \Gamma e^{-j\beta l} \right]$$

$$\frac{\widetilde{V}_g Z_{in}}{Z_g + Z_{in}} \frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}} = V_0^+$$

2-7 Voltage Amplitude



Equate these 2 expressions.
Solve for the Voltage Amplitude:

$$V_0^+ = \left(\frac{\tilde{V}_g Z_{in}}{Z_g + Z_{in}} \right) \left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}} \right).$$

We used the boundary conditions (Z_L, Z_g , V_g, l) to solve for the 2 remaining unknowns: V_0^+, V_0^-

This completes the solution of the transmission line differential equation.

Example 2-7

Given: Lossless transmission line with:

$f = 1.05$ GHz

$Z_g = 10\ \Omega$

$V_g = 10V \sin(\omega t + 30^o)$

$Z_L = 100 + j\,50\ \Omega$

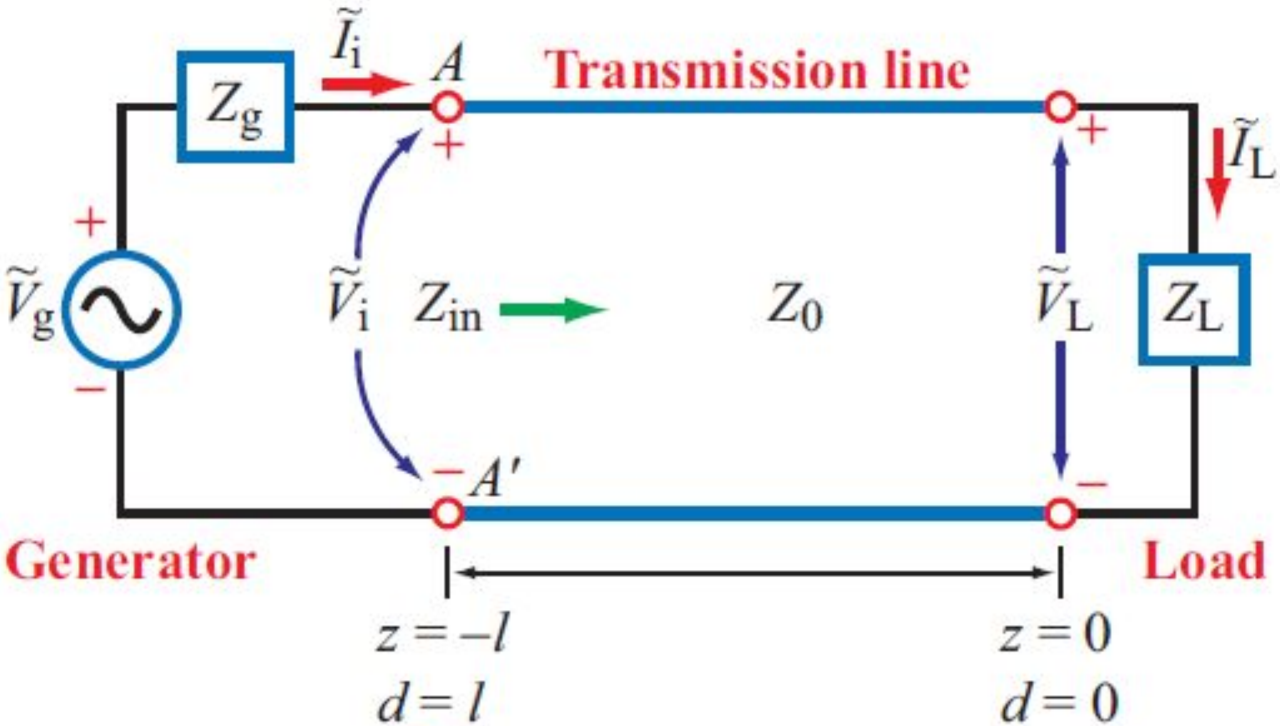
$Z_0 = 50\ \Omega$

$l = 67$ cm

$u_p = 0.7c$

Find: $v(d,t)$, $i(d,t)$ on the line.

Example 2-7



Z_g
$\tilde{I}_i$
A
Transmission line
+
$\tilde{I}_L$
+
+
$\tilde{V}_g$
$\tilde{V}_i$
Z_{in}
Z_0
$\tilde{V}_L$
Z_L
−
− A′
−
Generator
Load
$z = -l$
$d = l$
$z = 0$
$d = 0$

Example 2-7

Know:

$$\widetilde{V}(d) = V_0^+(e^{j\beta d} + \Gamma e^{-j\beta d})$$

$$\widetilde{I}(d) = \frac{V_0^+}{Z_0}(e^{j\beta d} - \Gamma e^{-j\beta d})$$

$$V_0^+ = \left(\frac{\widetilde{V}_g Z_{in}}{Z_g + Z_{in}}\right)\left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right). \quad (2.82)$$

$$Z_{in} = Z(l) = Z_0\left[\frac{1+\Gamma_l}{1-\Gamma_l}\right].$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \quad \beta = \frac{2\pi}{\lambda} \quad u_p = f\lambda \quad \Gamma_l = |\Gamma| e^{j(\theta_r - 2\beta l)}$$

Example 2-7

Solution:

$$u_{\mathrm{p}} = \lambda f$$

$$\lambda = \frac{u_{\mathrm{p}}}{f}$$

$$\lambda = \frac{(0.7)3 \times 10^8 \text{ m/sec}}{1.05 \times 10^9 \text{ Hz}}$$

$$\lambda = 0.2\,\text{m}$$

Example 2-7

Solution:

$$\beta l = \frac{2\pi}{\lambda} l$$

$$\beta l = \frac{2\pi}{0.2\,\mathrm{m}} (0.67\,\mathrm{m})$$

$$\beta l = 6.7\pi$$

Note that since this is used only in the phase terms, can remove multiples of 2π

$$\beta l = 0.7\pi = 2.2\,\mathrm{rad} = 126^\circ$$

$$2\beta l = 252^\circ$$

Example 2-7

Solution:

$$\Gamma = \frac{Z_{\mathrm{L}} - Z_0}{Z_{\mathrm{L}} + Z_0}$$

$$\Gamma = \frac{100 + j50 - 50}{100 + j50 + 50}$$

$$\Gamma = \frac{50 + j50}{150 + j50}$$

$$\Gamma = \frac{\sqrt{50^2 + 50^2} e^{j \tan^{-1}(1)}}{\sqrt{150^2 + 50^2} e^{j \tan^{-1}(50/150)}}$$

Example 2-7

Solution:

$$\Gamma = \frac{\sqrt{50^2 + 50^2} e^{j \tan^{-1}(1)}}{\sqrt{150^2 + 50^2} e^{j \tan^{-1}(50/150)}}$$

$$\Gamma = \frac{70.7 e^{j45^\circ}}{158.1 e^{j18.4^\circ}}$$

$$\Gamma = 0.447 e^{j(45^\circ - 18.4^\circ)}$$

$$\Gamma = 0.447 e^{j26.6^\circ}$$

Example 2-7

Solution:

$$Z_{\text{in}} = Z_0 \left(\frac{1 + \Gamma_l}{1 - \Gamma_l} \right)$$

$$\Gamma_l = \Gamma e^{-j2\beta l}$$

$$\Gamma_l = 0.447 e^{j26.6^\circ} e^{-j252^\circ}$$

$$\Gamma_l = 0.447 e^{-j225.4^\circ}$$

$$\Gamma_l = 0.447 \cos(-225.4^\circ) + j0.447 \sin(-225.4^\circ)$$

$$\Gamma_l = -0.314 + j0.318$$

Example 2-7

Solution:

$$Z_{\text{in}} = Z_0 \left(\frac{1+\Gamma_l}{1-\Gamma_l} \right)$$

$$Z_{\text{in}} = (50\,\Omega) \left(\frac{1+(-0.314+j0.318)}{1-(-0.314+j0.318)} \right)$$

$$Z_{\text{in}} = (50\,\Omega) \left(\frac{0.686+j0.318}{1.314-j0.318} \right)$$

$$Z_{\text{in}} = (50\,\Omega) \left(\frac{\sqrt{0.686^2+0.318^2} e^{j\tan^{-1}(0.318/0.686)}}{\sqrt{1.314^2+0.318^2} e^{j\tan^{-1}(-0.318/1.314)}} \right)$$

Example 2-7

Solution:

$$Z_{\text{in}} = (50\,\Omega)\left(\frac{\sqrt{0.686^2+0.318^2}e^{j\tan^{-1}(0.318/0.686)}}{\sqrt{1.314^2+0.318^2}e^{j\tan^{-1}(-0.318/1.314)}}\right)$$

$$Z_{\text{in}} = (50\,\Omega)\left(\frac{0.756e^{j24.87^\circ}}{1.352e^{-j13.6^\circ}}\right)$$

$$Z_{\text{in}} = (50\,\Omega)0.559e^{j38.47^\circ}$$

$$Z_{\text{in}} = (27.95\,\Omega)\cos(38.47^\circ) + j(27.95\,\Omega)\sin(38.47^\circ)$$

$$Z_{\text{in}} = 21.88 + j17.39\,\Omega$$

Example 2-7

Solution:

$$\nu_g(t) = 10\,\mathrm{V}\sin(\omega t + 30^\circ)$$

$$\nu_g(t) = 10\,\mathrm{V}\cos(90^\circ - (\omega t + 30^\circ))$$

$$\nu_g(t) = 10\,\mathrm{V}\cos(-\omega t + 60^\circ)$$

$$\nu_g(t) = 10\,\mathrm{V}\cos(\omega t - 60^\circ)$$

$$\widetilde{V}_g = 10e^{-j60^\circ}\ \mathrm{V}$$

Example 2-7

Solution:

$$V_0^+ = \left[\frac{\widetilde{V}_g Z_{\text{in}}}{Z_{\text{g}} + Z_{\text{in}}}\right]\left[\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right]$$

$$Z_{\text{g}} + Z_{\text{in}} = 10 + 21.88 + j17.39$$

$$Z_{\text{g}} + Z_{\text{in}} = 31.88 + j17.39$$

$$Z_{\text{g}} + Z_{\text{in}} = \sqrt{31.88^2 + 17.39^2}e^{j\tan^{-1}(17.39/31.88)}$$

$$Z_{\text{g}} + Z_{\text{in}} = 36.31e^{j28.6^\circ}$$

Example 2-7

Solution:

$$\frac{\widetilde{V}_g Z_{\text{in}}}{Z_{\text{g}} + Z_{\text{in}}} = \frac{10e^{-j60^\circ} 27.95e^{j38.47^\circ}}{36.31e^{j28.6^\circ}}$$

$$\frac{\widetilde{V}_g Z_{\text{in}}}{Z_{\text{g}} + Z_{\text{in}}} = \frac{279.5e^{-j21.53^\circ}}{36.31e^{j28.6^\circ}}$$

$$\frac{\widetilde{V}_g Z_{\text{in}}}{Z_{\text{g}} + Z_{\text{in}}} = 7.7e^{-j50.13^\circ}$$

Example 2-7

Solution:

$$e^{j\beta l} + \Gamma e^{-j\beta l} = e^{j126^\circ} + 0.447 e^{j26.6^\circ} e^{-j126^\circ}$$

$$e^{j\beta l} + \Gamma e^{-j\beta l} = e^{j126^\circ} + 0.447 e^{-j99.4^\circ}$$

$$e^{j\beta l} + \Gamma e^{-j\beta l} = \cos(126^\circ) + j\sin(126^\circ) + 0.447\cos(-99.4^\circ) + j0.447\sin(-99.4^\circ)$$

$$e^{j\beta l} + \Gamma e^{-j\beta l} = -0.588 + j0.809 + -0.073 - j0.441$$

$$e^{j\beta l} + \Gamma e^{-j\beta l} = -0.661 + j0.368$$

Example 2-7

Solution:

$$e^{j\beta l} + \Gamma e^{-j\beta l} = -0.661 + j0.368$$

$$e^{j\beta l} + \Gamma e^{-j\beta l} = \sqrt{0.661^2 + 0.368^2} e^{j(180^\circ + \tan^{-1}(-0.368/.661))}$$

$$e^{j\beta l} + \Gamma e^{-j\beta l} = 0.757 e^{j(180^\circ - 29.1^\circ)}$$

$$e^{j\beta l} + \Gamma e^{-j\beta l} = 0.757 e^{j150.9^\circ}$$

$$\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}} = 1.32 e^{-j150.9^\circ}$$

Example 2-7

Solution:

$$V_0^+ = 7.7e^{-j50.13^\circ}\ 1.32e^{-j150.93^\circ}\ \text{V}$$

$$V_0^+ = 10.17e^{-j201.06^\circ}\ \text{V}$$

$$V_0^+ = 10.17e^{j(360^\circ - 201.06^\circ)}\ \text{V}$$

$$V_0^+ = 10.17e^{j158.94^\circ}\ \text{V}$$

Example 2-7

Solution: combining all this:

$$\widetilde{V}(d) = V_0^+ \left(e^{j\beta d} + \Gamma e^{-j\beta d} \right)$$

$$\widetilde{V}(d) = 10.17 e^{j158.94^\circ} \text{ V} \left(e^{j\beta d} + 0.447 e^{j26.6^\circ} e^{-j\beta d} \right)$$

$$\widetilde{V}(d) = 10.17 e^{j158.94^\circ} e^{j\beta d} \text{ V} + 4.55 e^{j158.94^\circ} e^{j26.6^\circ} e^{-j\beta d}$$

$$\widetilde{V}(d) = 10.17 e^{j158.94^\circ} e^{j\beta d} + 4.55 e^{j185.54^\circ} e^{-j\beta d} \text{ V}$$

Example 2-7

Solution: and in the time-domain:

$$\nu(d,t) = \Re e\left\{\widetilde{V}(d)e^{j\omega t}\right\}$$

$$\nu(d,t) = 10.17\cos(\omega t + \beta d + 158.94^\circ) + 4.55\cos(\omega t - \beta d + 185.54^\circ)\ \text{V}$$

Example 2-7

Solution: and for the current:

$$\widetilde{I}(d) = \frac{V_0^+}{Z_0}\left(e^{j\beta d} - \Gamma e^{-j\beta d}\right)$$

$$\widetilde{I}(d) = \frac{10.17e^{j158.94^\circ}}{50}\,\mathrm{A}\left(e^{j\beta d} - 0.447e^{j26.6^\circ}e^{-j\beta d}\right)$$

$$\widetilde{I}(d) = 0.2e^{j158.94^\circ}e^{j\beta d} - 0.9e^{j185.54^\circ}e^{-j\beta d}\,\mathrm{V}$$

$$i(d,t) = \Re e\left\{\widetilde{I}(d)e^{j\omega t}\right\}$$

$$i(d,t) = 0.2\cos(\omega t + \beta d + 158.94^\circ) - 0.9\cos(\omega t - \beta d + 185.54^\circ)\,\mathrm{A}$$

Example 2-7

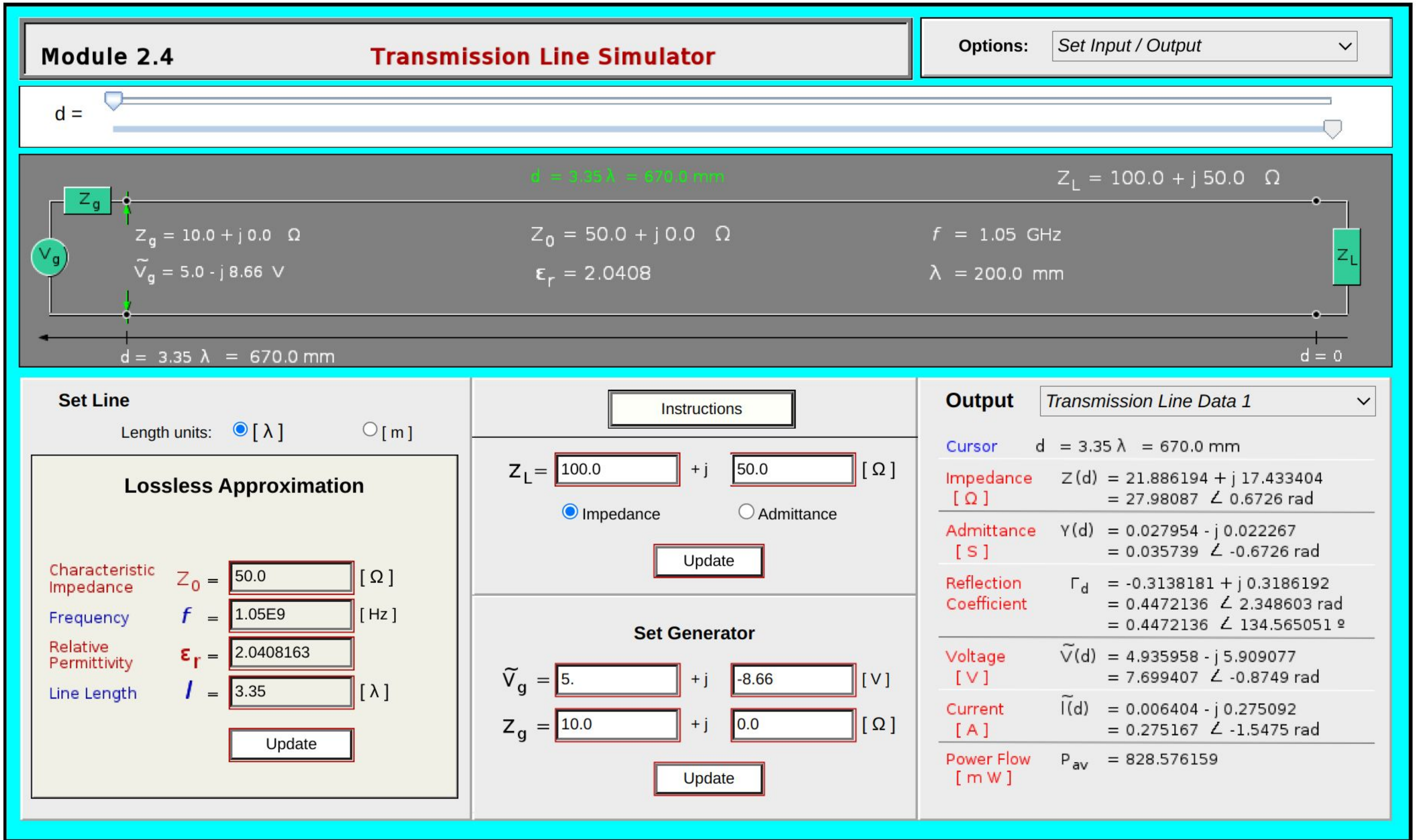
Solution:

where:

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{0.2\,\text{m}} = 31.42\,\text{rad/m}$$

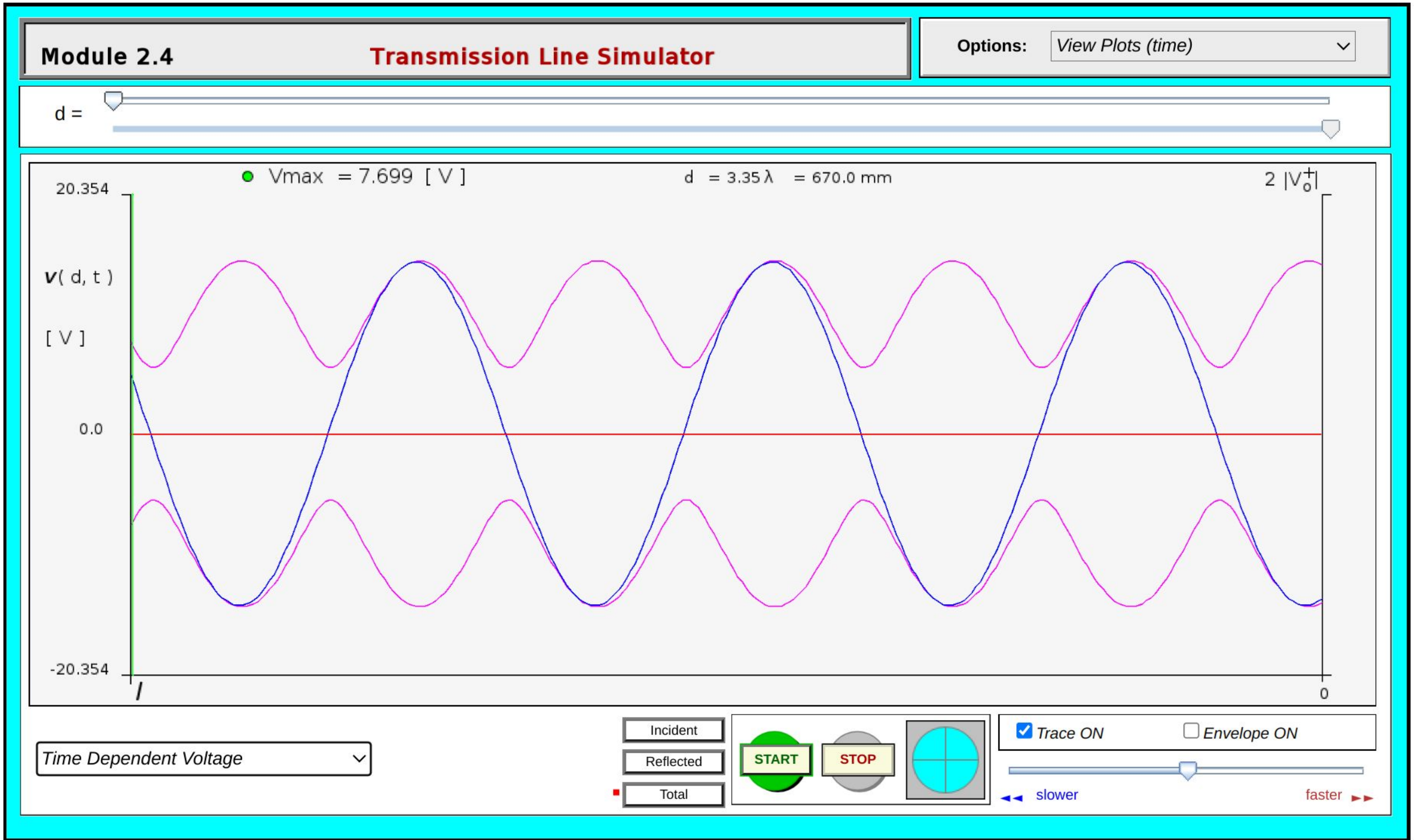
$$\omega = 2\pi f = 2\pi(1.05 \times 10^9\ \text{Hz}) = 6.6 \times 10^9\ \text{rad/sec}$$

Example 2-7



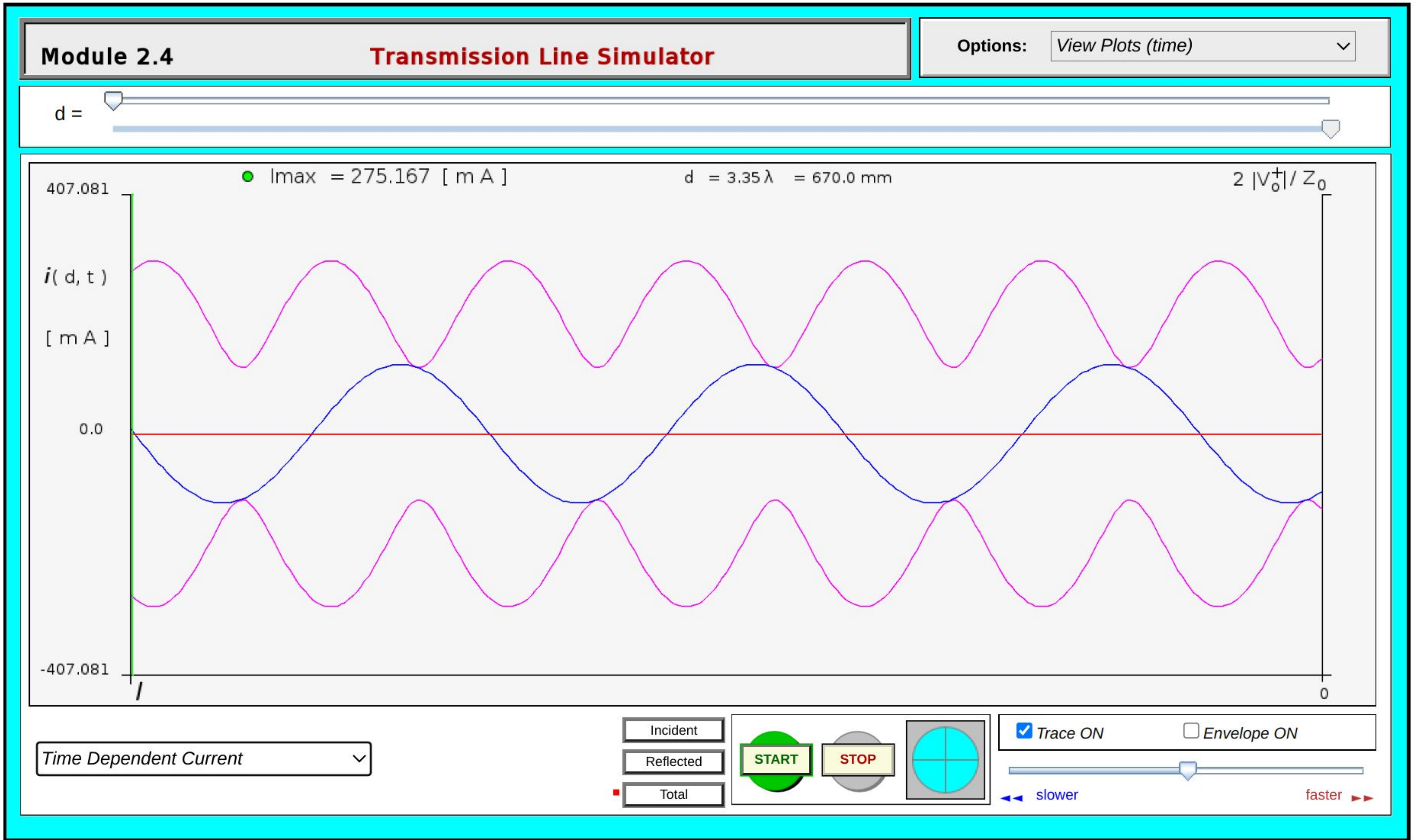
Module 2.4
Transmission Line Simulator
Options: Set Input / Output
d =
d = 3.35 λ = 670.0 mm
Z_L = 100.0 + j 50.0 Ω
Z_g
V_g
Z_g = 10.0 + j 0.0 Ω
V_g = 5.0 - j 8.66 V
Z_0 = 50.0 + j 0.0 Ω
ε_r = 2.0408
f = 1.05 GHz
λ = 200.0 mm
Z_L
d = 3.35 λ = 670.0 mm
d = 0
Set Line
Length units: [λ] [m]
Lossless Approximation
Characteristic Impedance Z_0 = 50.0 [Ω]
Frequency f = 1.05E9 [Hz]
Relative Permittivity ε_r = 2.0408163
Line Length l = 3.35 [λ]
Update
Instructions
Z_L = 100.0 + j 50.0 [Ω]
Impedance
Admittance
Update
Set Generator
V_g = 5. + j -8.66 [V]
Z_g = 10.0 + j 0.0 [Ω]
Update
Output
Transmission Line Data 1
Cursor d = 3.35 λ = 670.0 mm
Impedance [Ω] Z(d) = 21.886194 + j 17.433404
= 27.98087 ∠ 0.6726 rad
Admittance [S] Y(d) = 0.027954 - j 0.022267
= 0.035739 ∠ -0.6726 rad
Reflection Coefficient Γ_d = -0.3138181 + j 0.3186192
= 0.4472136 ∠ 2.348603 rad
= 0.4472136 ∠ 134.565051 º
Voltage [V] V(d) = 4.935958 - j 5.909077
= 7.699407 ∠ -0.8749 rad
Current [A] I(d) = 0.006404 - j 0.275092
= 0.275167 ∠ -1.5475 rad
Power Flow [m W] P_av = 828.576159

Example 2-7



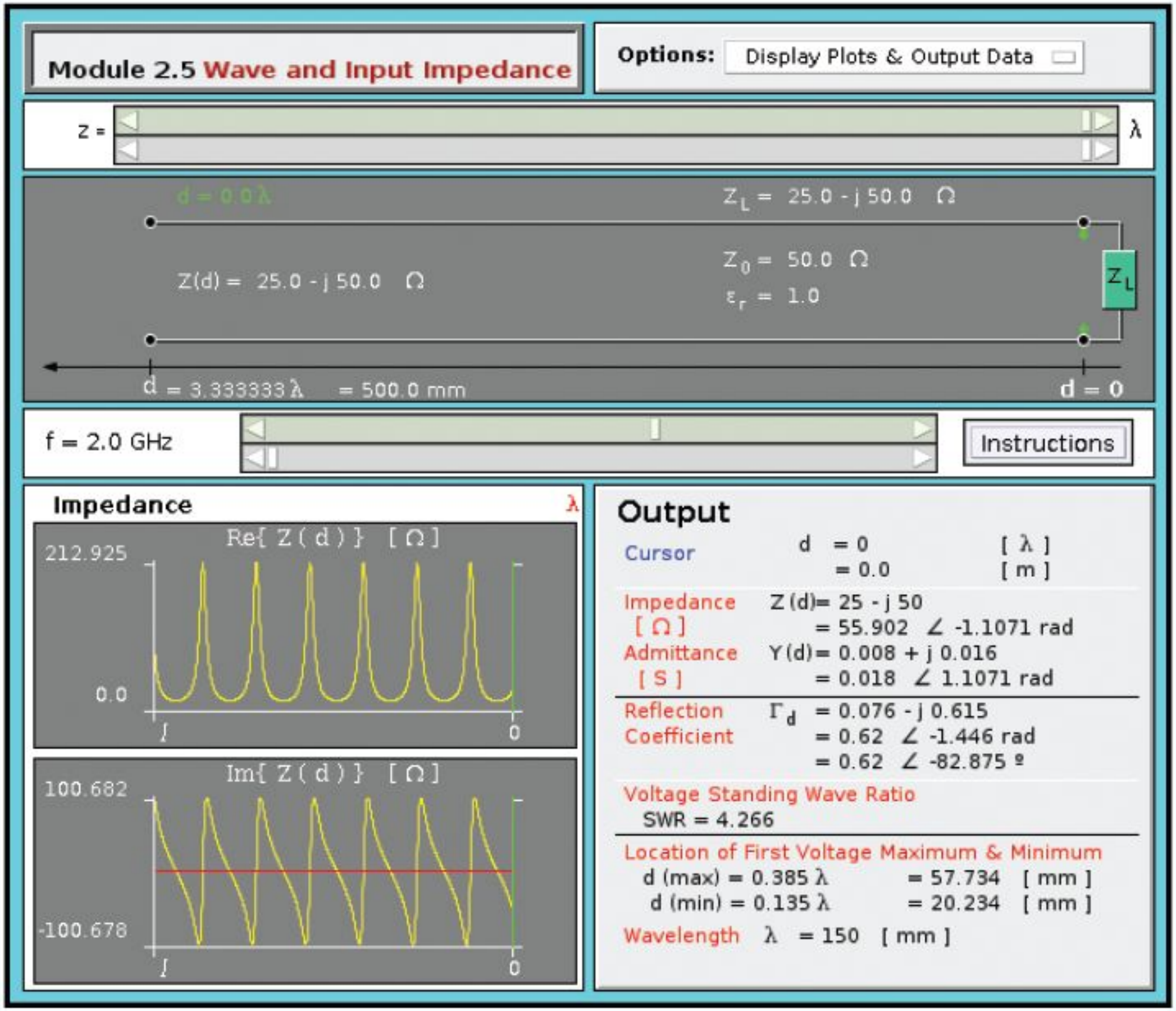
Module 2.4
Transmission Line Simulator
Options:
View Plots (time)
d =
Vmax = 7.699 [V]
d = 3.35 λ = 670.0 mm
20.354
2 |V₀⁺|
v(d, t)
[V]
0.0
-20.354
l
0
Time Dependent Voltage
Incident
Reflected
Total
START
STOP
Trace ON
Envelope ON
slower
faster

Example 2-7

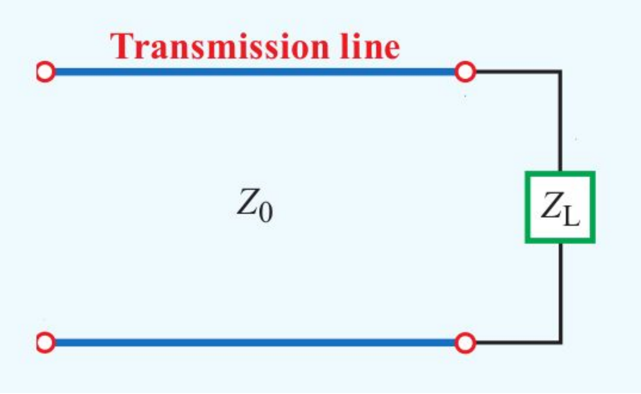


Module 2.4
Transmission Line Simulator
Options:
View Plots (time)
d =
Imax = 275.167 [m A]
d = 3.35 λ = 670.0 mm
2 |V+o|/ Zo
407.081
i(d, t)
[m A]
0.0
-407.081
l
0
Time Dependent Current
Incident
Reflected
Total
START
STOP
Trace ON
Envelope ON
slower
faster

Module 2.5 Wave and Input Impedance The wave impedance, $Z(d) = \tilde{V}(d)/\tilde{I}(d)$, exhibits a cyclical pattern as a function of position along the line. This module displays plots of the real and imaginary parts of $Z(d)$, specifies the locations of the voltage maximum and minimum nearest to the load, and provides other related information.



Example 2



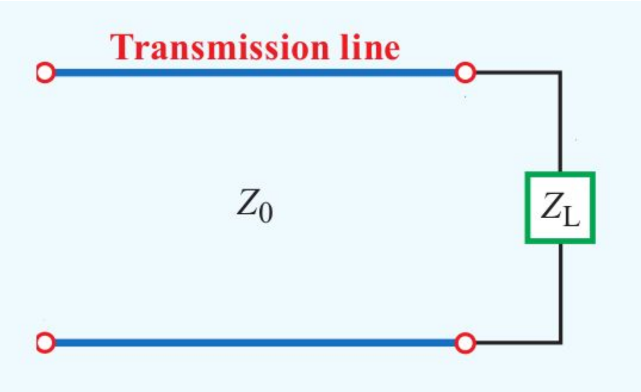
Given: Lossless transmission-line

$Z_0 = 50\ \Omega,\ l = 0.4\ \lambda$

$Z_L = 30 + j60\ \Omega$

Find: Γ, S, Z_{in}

Example 2



Solution:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma = \frac{30 + j60 - 50}{30 + j60 + 50}$$

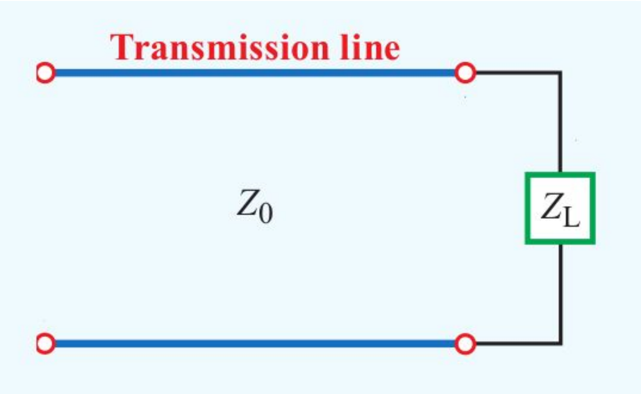
$$\Gamma = \frac{-20 + j60}{80 + j60}$$

$$\Gamma = \frac{\sqrt{20^2 + 60^2} e^{j(180^\circ + tan^{-1}(-60/20))}}{\sqrt{80^2 + 60^2} e^{j \tan^{-1}(60/80)}}$$

$$\Gamma = \frac{63.25 e^{j(180^\circ - 71.6^\circ)}}{100 e^{j36.9}}$$

Example 2

Solution:



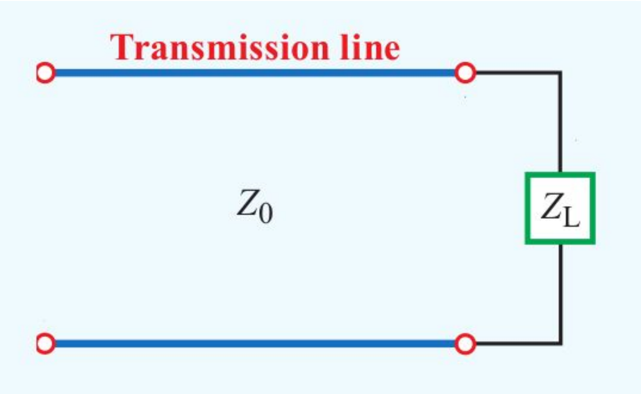
$$\Gamma = \frac{63.25e^{j(180° - 71.6°)}}{100e^{j36.9}}$$

$$\Gamma = 0.6325e^{j(180° - 71.6° - 36.9°)}$$

$$\boxed{\Gamma = 0.63e^{j71.5°}}$$

Example 2

Solution:



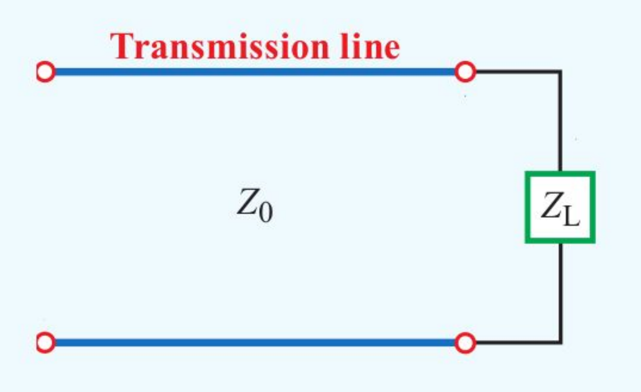
$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$S = \frac{1 + 0.63}{1 - 0.63}$$

$$\boxed{S = 4.4}$$

Example 2

Solution:



$$Z_{\text{in}} = Z_0 \left(\frac{1 + \Gamma_l}{1 - \Gamma_l} \right)$$

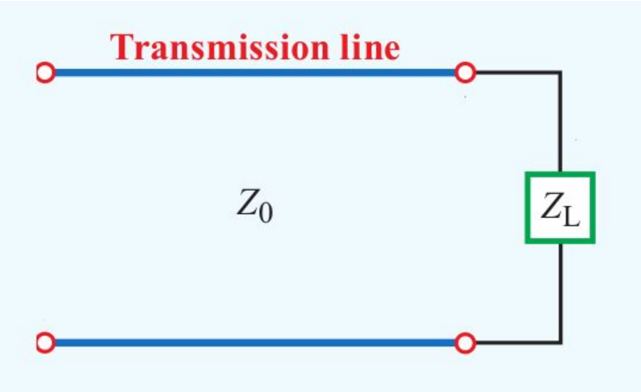
$$\Gamma_l = \Gamma e^{-j2\beta l}$$

$$\beta l = 2\pi l / \lambda$$

$$\beta l = 2\pi(0.4)$$

$$\beta l = 2.5 \, \text{rad}$$

Example 2



Solution:

$$\Gamma_l = \Gamma e^{-j2\beta l}$$

$$\Gamma_l = 0.63 e^{j71.5^\circ} e^{-j2(2.5\,\mathrm{rad})}$$

$$\Gamma_l = 0.63 e^{j(71.5^\circ - 5\,\mathrm{rad})}$$

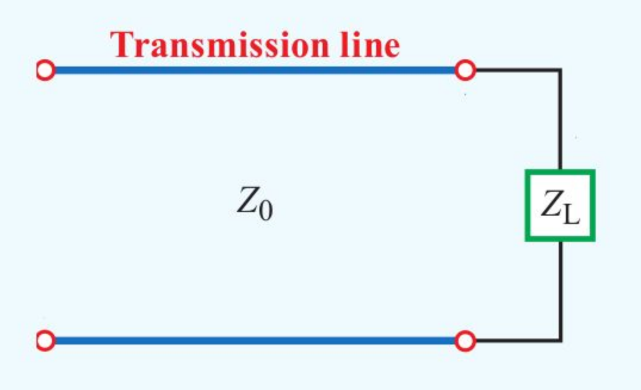
$$\Gamma_l = 0.63 e^{j(71.5^\circ - 286.5^\circ)}$$

$$\Gamma_l = 0.63 e^{-j215.5^\circ}$$

$$\Gamma_l = 0.63\cos(-215^\circ) + 0.63\sin(-215^\circ)$$

$$\Gamma_l = -0.516 + j0.36$$

Example 2



Solution:

$$Z_{\text{in}} = Z_0 \left(\frac{1 + \Gamma_l}{1 - \Gamma_l} \right)$$

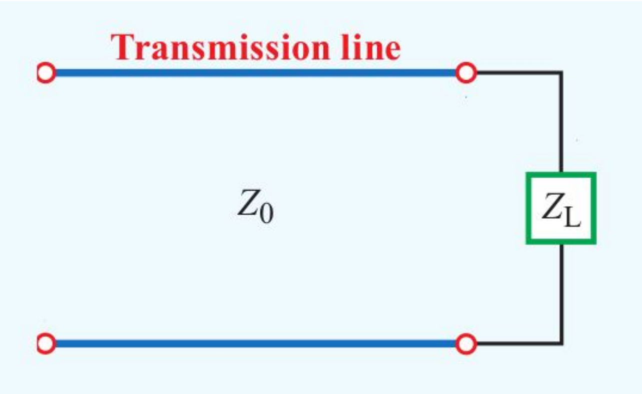
$$Z_{\text{in}} = (50\,\Omega) \left(\frac{1 + (-0.516 + j0.36)}{1 - (-0.516 + j0.36)} \right)$$

$$Z_{\text{in}} = (50\,\Omega) \left(\frac{0.484 + j0.36}{1.516 - j0.36} \right)$$

$$Z_{\text{in}} = (50\,\Omega) \left(\frac{\sqrt{0.484^2 + 0.36^2} e^{j \tan^{-1}(0.36/0.484)}}{\sqrt{1.516^2 + 0.36^2} e^{j \tan^{-1}(-0.36/1.516)}} \right)$$

Example 2

Solution:



$$Z_{\text{in}} = (50\,\Omega)\left(\frac{\sqrt{0.484^2+0.36^2}e^{j\tan^{-1}(0.36/0.484)}}{\sqrt{1.516^2+0.36^2}e^{j\tan^{-1}(-0.36/1.516)}}\right)$$

$$Z_{\text{in}} = (50\,\Omega)\left(\frac{0.6e^{j36.6^\circ}}{1.56e^{-j13.4^\circ}}\right)$$

$$Z_{\text{in}} = (50\,\Omega)0.4e^{j(36.6^\circ+13.4^\circ)}$$

$$\boxed{Z_{\text{in}} = 20e^{j50^\circ}\;\Omega}$$

Example 3

Given: Lossless transmission-line

$V_g = 10\text{V} \cos(\omega t)$

$\omega = 1\text{X}10^9$ rad/sec

$Z_g = 50\ \Omega$

$Z_0 = 50\ \Omega$

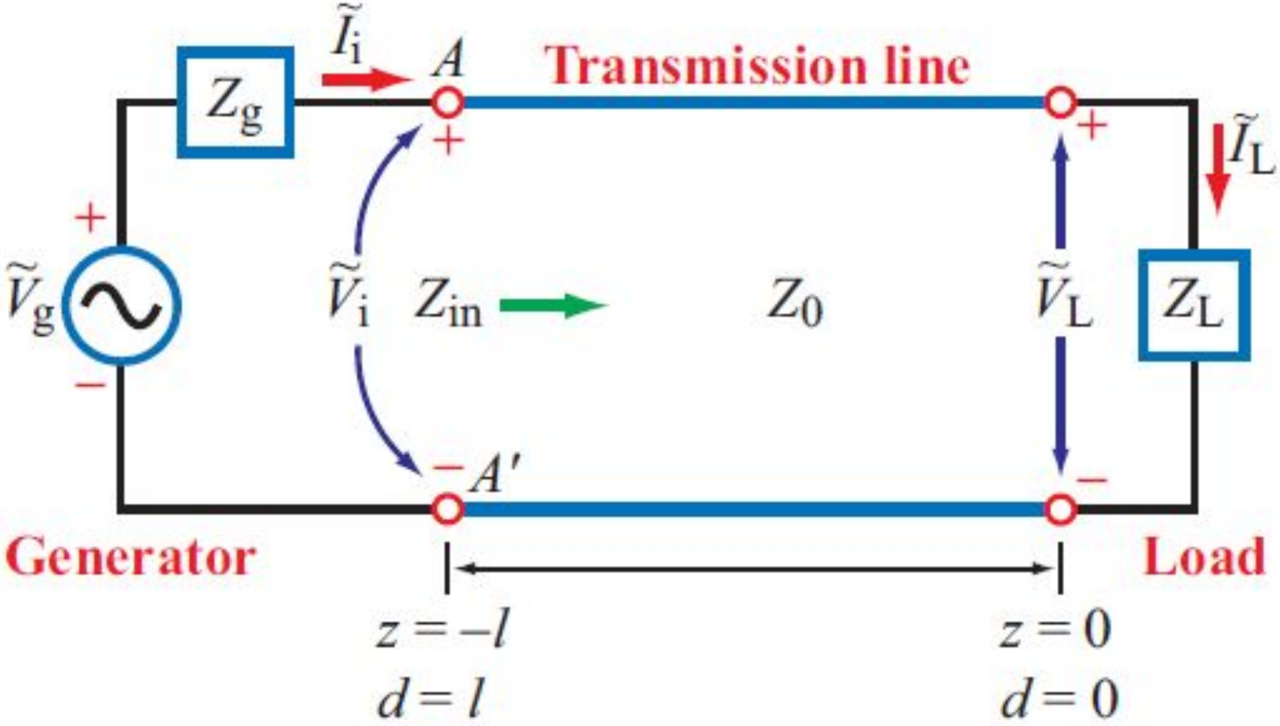
$\varepsilon_r = 3$

$l = 10$ cm

$Z_L = 50 + j\,150\ \Omega$

Find: Γ, Z_{in}, $\widetilde{V}_i$

Example 3



$\tilde{I}_i$
A
Transmission line
Z_g
+
$\tilde{I}_L$
+
+
$\tilde{V}_g$
$\tilde{V}_i$
Z_{in}
Z_0
$\tilde{V}_L$
Z_L
−
− A′
−
Generator
Load
$z = -l$
$d = l$
$z = 0$
$d = 0$

Example 3

Know:

$$\widetilde{V}_i = \left(\frac{\widetilde{V}_g Z_{in}}{Z_g + Z_{in}} \right)$$

$$Z_{in} = Z(l) = Z_0 \left[\frac{1 + \Gamma_l}{1 - \Gamma_l} \right].$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\lambda = \frac{u_p}{f} = \frac{c}{f} \frac{1}{\sqrt{\varepsilon_r}} = \frac{\lambda_0}{\sqrt{\varepsilon_r}}$$

$$\beta = \frac{2\pi}{\lambda} \qquad u_p = f\lambda \qquad \Gamma_l = |\Gamma| e^{j(\theta_r - 2\beta l)}$$

Example 3

Solution:

$$\lambda = \frac{c}{f}\frac{1}{\sqrt{\epsilon_r}}$$

$$\lambda = \frac{c}{\omega/(2\pi)}\frac{1}{\sqrt{\epsilon_r}}$$

$$\lambda = \frac{3\times 10^8\ \text{m/sec}}{1\times 10^9\ \text{rad/sec}/(2\pi)}\frac{1}{\sqrt{3}}$$

$$\lambda = 1.09\,\text{m}$$

Example 3

Solution:

$$\beta l = \frac{2\pi}{\lambda} l$$

$$\beta l = \frac{2\pi}{1.09\,\mathrm{m}} 0.1\,\mathrm{m}$$

$$\beta l = 0.58\,\mathrm{rad} = 33.23^\circ$$

$$2\beta l = 1.16\,\mathrm{rad} = 66.46^\circ$$

Example 3

Solution:

$$\Gamma = \frac{Z_{\mathrm{L}} - Z_0}{Z_{\mathrm{L}} + Z_0}$$

$$\Gamma = \frac{50 + j150 - 50}{50 + j150 + 50}$$

$$\Gamma = \frac{j150}{100 + j150}$$

$$\Gamma = \frac{150e^{j90^\circ}}{\sqrt{100^2 + 150^2}e^{j\tan^{-1}(150/100)}}$$

$$\Gamma = \frac{150e^{j90^\circ}}{180.3e^{j56.3^\circ}}$$

Example 3

Solution:

$$\Gamma = \frac{150e^{j90^\circ}}{180.3e^{j56.3^\circ}}$$

$$\Gamma = 0.83e^{j(90^\circ - 56.3^\circ)}$$

$$\boxed{\Gamma = 0.83e^{j33.7^\circ}}$$

Example 3

Solution:

$$Z_{\text{in}} = Z_0 \left(\frac{1 + \Gamma_l}{1 - \Gamma_l} \right)$$

$$\Gamma_l = \Gamma e^{-j2\beta l}$$

$$\Gamma_l = 0.83 e^{j33.7^\circ} e^{-j66.46^\circ}$$

$$\Gamma_l = 0.83 e^{-j32.76^\circ}$$

$$\Gamma_l = 0.83 \cos(-32.76^\circ) + j0.83 \sin(-32.76^\circ)$$

$$\Gamma_l = 0.7 - j0.45$$

Example 3

Solution:

$$Z_{\text{in}} = (50\,\Omega)\left(\frac{1 + (0.7 - j0.45)}{1 - (0.7 - j0.45)}\right)$$

$$Z_{\text{in}} = (50\,\Omega)\left(\frac{1.7 - j0.45}{0.3 + j0.45}\right)$$

$$Z_{\text{in}} = (50\,\Omega)\left(\frac{\sqrt{1.7^2 + 0.45^2}e^{j\tan^{-1}(-0.45/1.7)}}{\sqrt{0.3^2 + 0.45^2}e^{j\tan^{-1}(0.45/0.3)}}\right)$$

$$Z_{\text{in}} = (50\,\Omega)\left(\frac{1.76e^{-j14.8^\circ}}{0.54e^{j56.3^\circ}}\right)$$

Example 3

Solution:

$$Z_{\text{in}} = (50\,\Omega)\left(\frac{1.76e^{-j14.8^\circ}}{0.54e^{j56.3^\circ}}\right)$$

$$Z_{\text{in}} = (50\,\Omega)3.26e^{j(-14.8^\circ - 56.3^\circ)}$$

$$Z_{\text{in}} = (163\,\Omega)e^{-j71.1^\circ}$$

$$Z_{\text{in}} = (163\,\Omega)\cos(-71.1^\circ) + j(163\,\Omega)\sin(-71.1^\circ)$$

$$\boxed{Z_{\text{in}} = 52.8 - j154\,\Omega}$$

Example 3

Solution:

$$\nu_g(t) = 10\,\mathrm{V}\cos(\omega t)$$

$$\widetilde{V}_g = 10\,\mathrm{V}$$

Example 3

Solution:

$$\widetilde{V}_i = \frac{\widetilde{V}_g Z_{\text{in}}}{Z_{\text{g}} + Z_{\text{in}}}$$

$$\widetilde{V}_i = \frac{(10\,\text{V})(163\,\Omega e^{-j71.1^\circ})}{50. + 52.8 - j154\,\Omega}$$

$$Z_{\text{g}} + Z_{\text{in}} = 50. + 52.8 - j154\,\Omega$$

$$Z_{\text{g}} + Z_{\text{in}} = 102.8 - j154\,\Omega$$

$$Z_{\text{g}} + Z_{\text{in}} = \sqrt{102.8^2 + 154^2} e^{j\tan^{-1}(-154/102.8)}\ \Omega$$

$$Z_{\text{g}} + Z_{\text{in}} = 185.2e^{-j56.3^\circ}\ \Omega$$

Example 3

Solution:

$$\widetilde{V}_i = \frac{\widetilde{V}_g Z_{\text{in}}}{Z_{\text{g}} + Z_{\text{in}}}$$

$$\widetilde{V}_i = \frac{(10\,\text{V})(163\,\Omega e^{-j71.1^\circ})}{185.2e^{-j56.3^\circ}\ \Omega}$$

$$\widetilde{V}_i = 8.8e^{j(-71.1^\circ + 56.3^\circ)}\ \text{V}$$

$$\boxed{\widetilde{V}_i = 8.8e^{-j14.8^\circ}\ \text{V}}$$

Example 4

Given: Lossless transmission-line

$V_g = 5\text{V} \cos(\omega t - 50^\circ)$

$\omega = 3\text{X}10^7$ rad/sec

$Z_g = 150\ \Omega$

$Z_0 = 75\ \Omega$

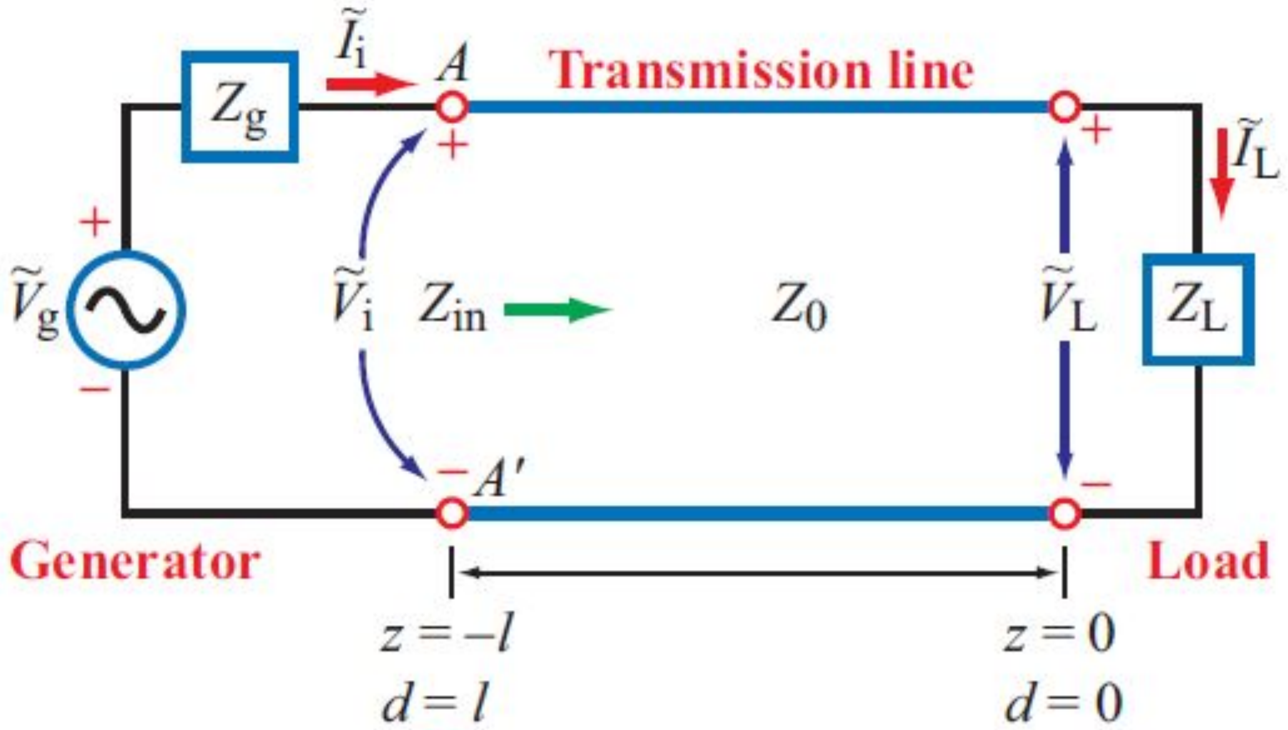
$\varepsilon_r = 4$

$l = 12$ m

$Z_L = 50 + j\,100\ \Omega$

Find: Γ, Z_{in}, $\widetilde{V}_i$

Example 4



$\tilde{I}_i$
A
Transmission line
Z_g
+
$\tilde{I}_L$
+
$\tilde{V}_g$
$\tilde{V}_i$
Z_{in}
Z_0
$\tilde{V}_L$
Z_L
−
− A′
−
Generator
Load
$z = -l$
$d = l$
$z = 0$
$d = 0$

Example 4

Know:

$$\widetilde{V}_i = \left(\frac{\widetilde{V}_g Z_{in}}{Z_g + Z_{in}} \right)$$

$$Z_{in} = Z(l) = Z_0 \left[\frac{1 + \Gamma_l}{1 - \Gamma_l} \right].$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\lambda = \frac{u_p}{f} = \frac{c}{f} \frac{1}{\sqrt{\varepsilon_r}} = \frac{\lambda_0}{\sqrt{\varepsilon_r}}$$

$$\beta = \frac{2\pi}{\lambda} \qquad u_p = f\lambda \qquad \Gamma_l = |\Gamma| e^{j(\theta_r - 2\beta l)}$$

Example 4

Solution:

$$\lambda = \frac{c}{f} \frac{1}{\sqrt{\epsilon_r}}$$

$$\lambda = \frac{c}{\omega/(2\pi)} \frac{1}{\sqrt{\epsilon_r}}$$

$$\lambda = \frac{3 \times 10^8 \text{ m/sec}}{3 \times 10^7 \text{ rad/sec}/(2\pi)} \frac{1}{\sqrt{4}}$$

$$\lambda = 31.4 \, \text{m}$$

Example 4

Solution:

$$\beta l = \frac{2\pi}{\lambda} l$$

$$\beta l = \frac{2\pi}{31.4\,\mathrm{m}} 12\,\mathrm{m}$$

$$\beta l = 2.4\,\mathrm{rad} = 137.6^\circ$$

$$2\beta l = 4.8\,\mathrm{rad} = 275.2^\circ$$

Example 4

Solution:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma = \frac{50 + j100 - 75}{50 + j100 + 75}$$

$$\Gamma = \frac{-25 + j100}{125 + j100}$$

$$\Gamma = \frac{\sqrt{25^2 + 100^2} e^{j(180^\circ + \tan^{-1}(-100/25))}}{\sqrt{125^2 + 100^2} e^{j\tan^{-1}(100/125)}}$$

$$\Gamma = \frac{103.1 e^{j104^\circ}}{160.1 e^{j38.7^\circ}}$$

Example 4

Solution:

$$\Gamma = \frac{103.1e^{j104^\circ}}{160.1e^{j38.7^\circ}}$$

$$\Gamma = 0.64e^{j(104^\circ - 38.7^\circ)}$$

$$\boxed{\Gamma = 0.64e^{j65.3^\circ}}$$

Example 4

Solution:

$$Z_{\text{in}} = Z_0 \left(\frac{1 + \Gamma_l}{1 - \Gamma_l} \right)$$

$$\Gamma_l = \Gamma e^{-j2\beta l}$$

$$\Gamma_l = 0.64 e^{j65.3^\circ} e^{-j275.2^\circ}$$

$$\Gamma_l = 0.64 e^{-j210^\circ}$$

$$\Gamma_l = 0.64 \cos(-210^\circ) + j0.64 \sin(-210^\circ)$$

$$\Gamma_l = -0.55 + j0.32$$

Example 4

Solution:

$$Z_{\text{in}} = (75\,\Omega)\left(\frac{1 + (-0.55 + j0.32)}{1 - (-0.55 + j0.32)}\right)$$

$$Z_{\text{in}} = (75\,\Omega)\left(\frac{0.45 + j0.32}{1.55 - j0.32}\right)$$

$$Z_{\text{in}} = (75\,\Omega)\left(\frac{\sqrt{0.45^2 + 0.32^2}e^{j\tan^{-1}(0.32/0.45)}}{\sqrt{1.55^2 + 0.32^2}e^{j\tan^{-1}(-0.32/1.55)}}\right)$$

$$Z_{\text{in}} = (75\,\Omega)\left(\frac{0.55e^{j35.4^\circ}}{1.58e^{-j11.7^\circ}}\right)$$

$$Z_{\text{in}} = (75\,\Omega)0.348e^{j(35.4^\circ + 11.7^\circ)}$$

Example 4

Solution:

$$Z_{\text{in}} = (75\,\Omega)0.348e^{j(35.4^\circ+11.7^\circ)}$$

$$Z_{\text{in}} = (26.1\,\Omega)e^{j47.1^\circ}$$

$$Z_{\text{in}} = (26.1\,\Omega)\cos(47.1^\circ) + j(26.1\,\Omega)\sin(47.1^\circ)$$

$$\boxed{Z_{\text{in}} = 17.8 + j19.1\,\Omega}$$

Example 4

Solution:

$$\nu_g(t) = 5\,\text{V}\cos(\omega t - 50^\circ)$$

$$\widetilde{V}_g = 5e^{-j50^\circ}\ \text{V}$$

Example 4

Solution:

$$\widetilde{V}_i = \frac{\widetilde{V}_g Z_{\text{in}}}{Z_{\text{g}} + Z_{\text{in}}}$$

$$\widetilde{V}_i = \frac{(5e^{-j50^\circ}\ \text{V})(26.1\,\Omega\, e^{j47.1^\circ})}{150. + 17.8 + j19.1\,\Omega}$$

Example 4

Solution:

$$Z_g + Z_{in} = 150. + 17.8 + j19.1\,\Omega$$

$$Z_g + Z_{in} = 167.8 + j19.1\,\Omega$$

$$Z_g + Z_{in} = \sqrt{167.8^2 + 19.1^2} e^{j\tan^{-1}(19.1/167.8)}\,\Omega$$

$$Z_g + Z_{in} = 169e^{j6.5^\circ}\,\Omega$$

Example 4

Solution:

$$\widetilde{V}_i = \frac{\widetilde{V}_g Z_{\text{in}}}{Z_{\text{g}} + Z_{\text{in}}}$$

$$\widetilde{V}_i = \frac{(5e^{-j50^\circ}\text{ V})(26.1\,\Omega\, e^{j47.1^\circ})}{169e^{j6.5^\circ}\ \Omega}$$

$$\widetilde{V}_i = 0.77e^{j(-50^\circ+47.1^\circ-6.5^\circ)}\text{ V}$$

$$\widetilde{V}_i = 0.77e^{-j9.4^\circ}\text{ V}$$

Homework

Homework 6 is due tomorrow at midnight.

submit to gradescope via the canvas site.

Next Time

Sections 2-8, 2-9:

Lossless Line: Special Cases

Lossless Line: Power Flow