EECS 230
ENGINEERING ELECTROMAGNETICS
Leland Pierce

Transmission Lines 3

Announcements

Lab 1 is next week
Pre-Lab is due sunday night

Obtain a compass now, so you have it for next week:

Chapter 2 Overview

What is a transmission line?

Why study transmission lines?

model

diffeq

solve diffeq

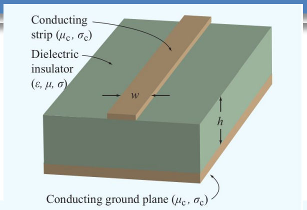
wave propagation

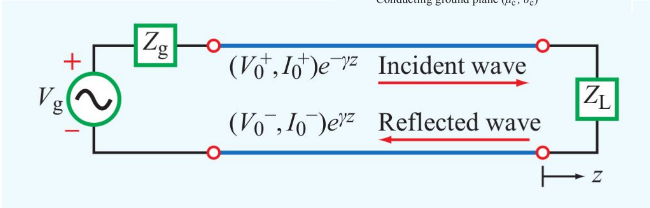
lossless xmission lir

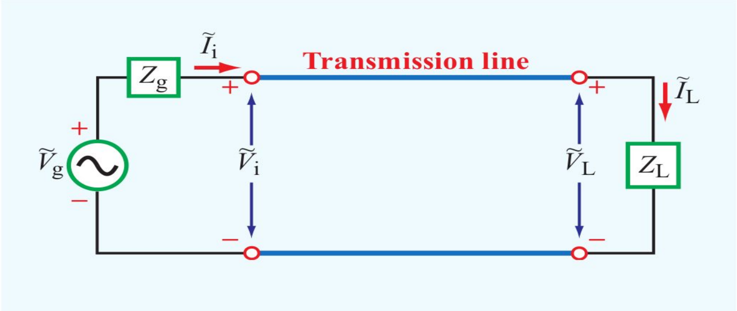
- microstrip lines
- reflections
- standing waves
- impedance
- short, open
- matching
- power flow

smith chart

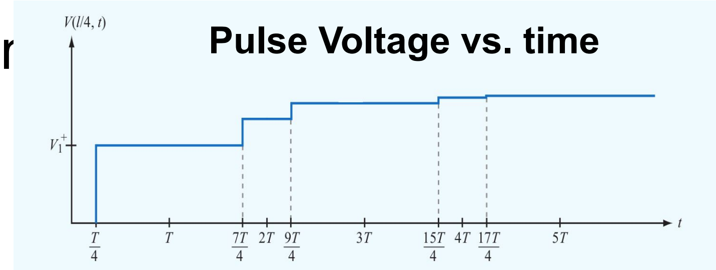
transients

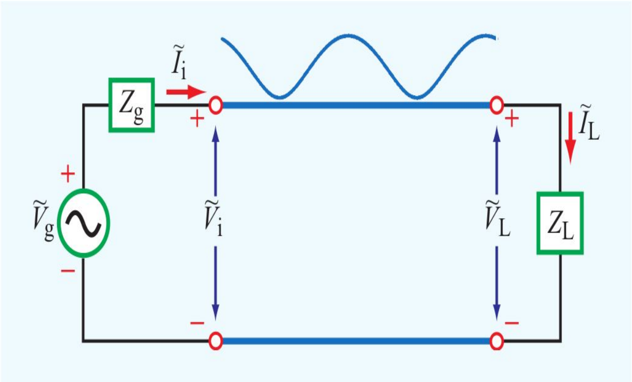




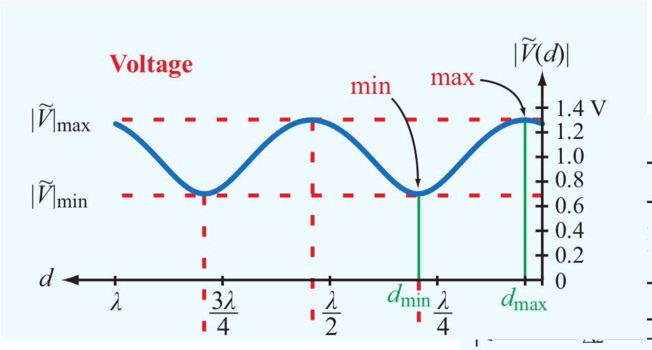


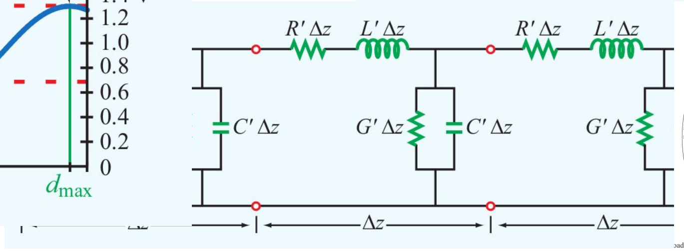
Typical High-Frequency Circuit

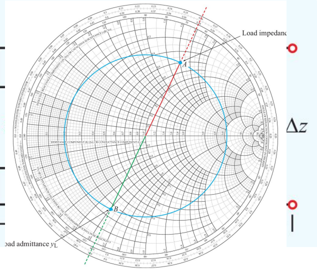




Waves on line: old methods don't work







Today's Lecture Coverage

Review Sections 2-1 through 2-5 of the book:

2-1: What is a transmission line?
Why study transmission lines?
2-2: Lumped-Element Model
2-3: Governing Differential Eqns
2-4: Solve the Differential Equations
Properties of the solution: wave propagation
2-5: Lossless Microstrip Line

Section 2-6 of the book:

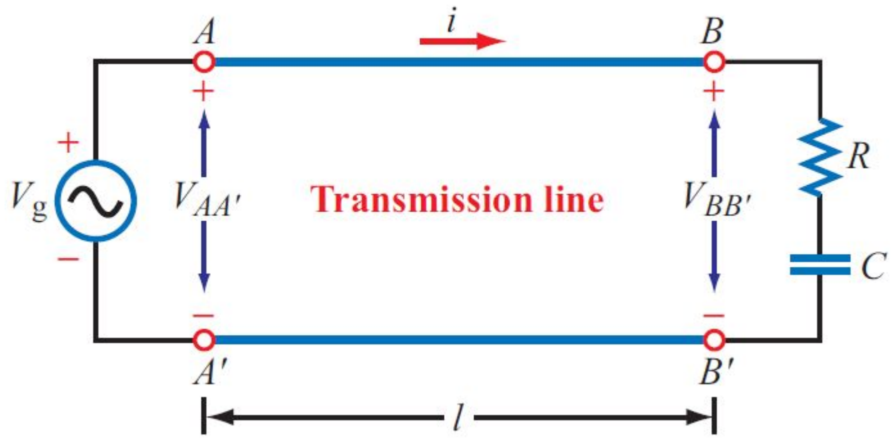
2-6: Lossless Transmission Lines

Chapter 2 Review

- A transmission line connects a generator to a load.

Chapter 2 Review

Phase Delay due to length of transmission line:



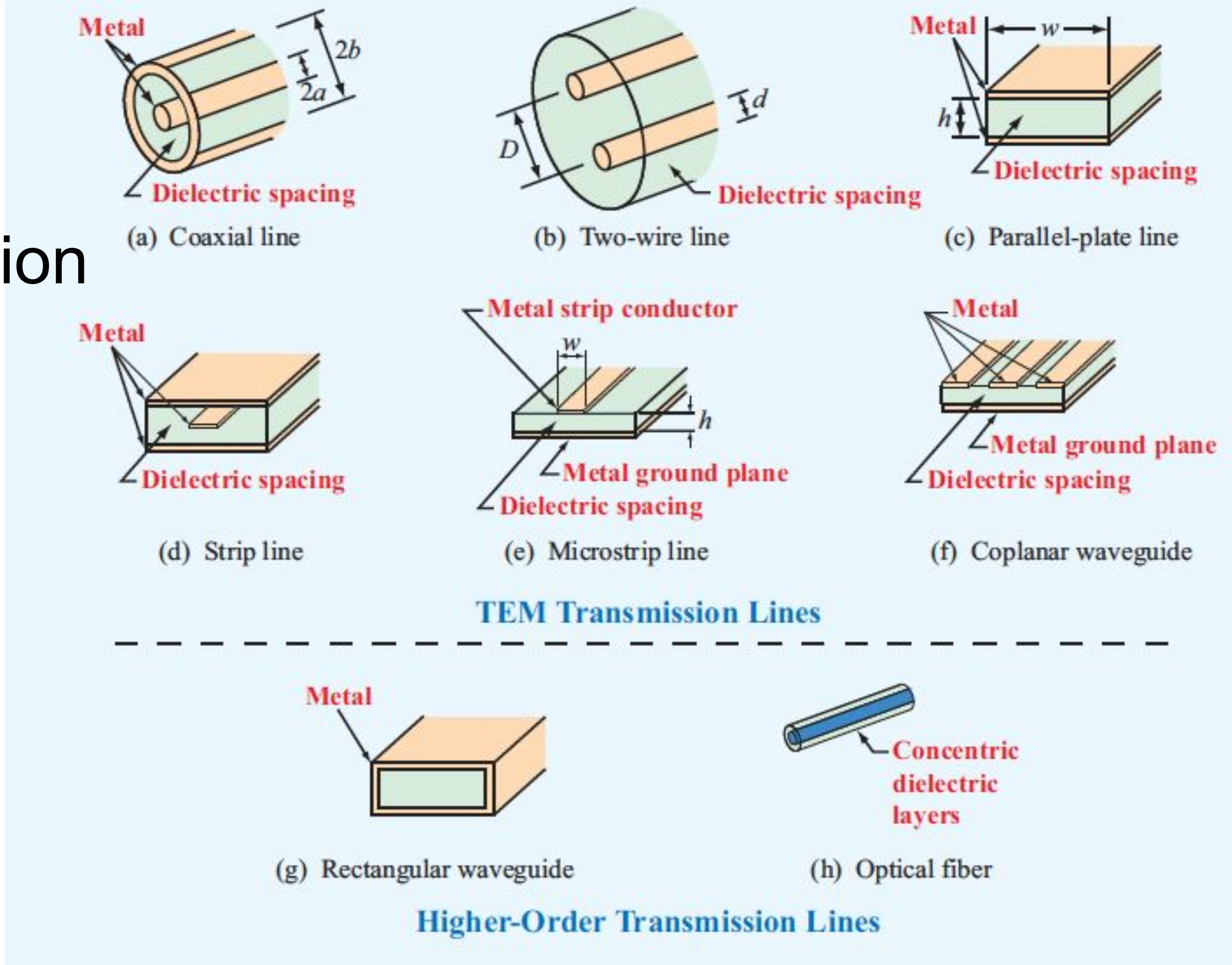
$$\phi_0 = \frac{\omega l}{c} = \frac{2\pi f l}{c} = 2\pi \frac{l}{\lambda} \quad \text{radians.}$$

$l/\lambda \lesssim 0.01$: Can ignore transmission-line effects

$l/\lambda \gtrsim 0.01$: Must deal with phase shift, and other effects...

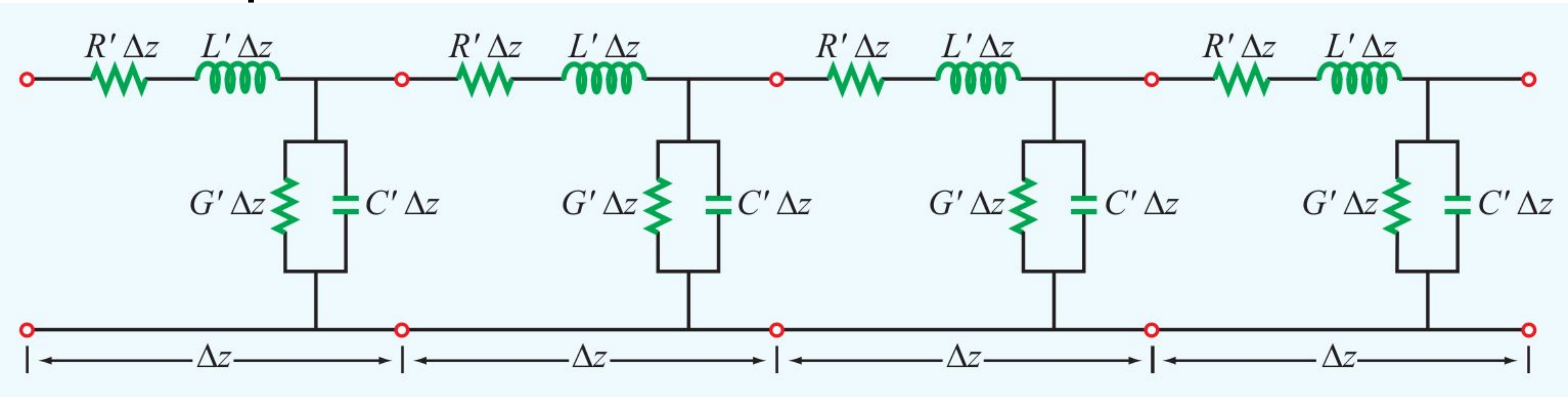
Chapter 2 Review

Different geometries for transmission lines



Chapter 2 Review

Lumped-Element Model:



All parameters are "per unit length":

R': Combined Resistance of BOTH conductors: □/m

L': Combined Inductance of BOTH conductors, H/m

G': Conductance of insulation
between inner and outer conductor, S/m

C': Capacitance
between inner and outer conductors, F/m

Chapter 2 Review

Lumped-Element Values: geometry/materials/freq

Table 2-1 **Transmission-line parameters R', L', G', and C' for three types of lines.**

Parameter	Coaxial	Two-Wire	Parallel-Plate	Unit
R'	$\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$\frac{2R_s}{\pi d}$	$\frac{2R_s}{w}$	Ω/m
L'	$\frac{\mu}{2\pi}\ln(b/a)$	$\frac{\mu}{\pi}\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]$	$\frac{\mu h}{w}$	H/m
G'	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\pi\sigma}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\sigma w}{h}$	S/m
C'	$\frac{2\pi\varepsilon}{\ln(b/a)}$	$\frac{\pi\varepsilon}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$	$\frac{\varepsilon w}{h}$	F/m

$$R_s = \sqrt{\pi f \mu_c / \sigma_c}$$

Chapter 2 Review

Lumped-Element Values: geometry/materials/freq

$$L'C' = \mu\epsilon$$

$$\frac{G'}{C'} = \frac{\sigma}{\epsilon}$$

This turns out to be true for all the transmission-lines we study.

Chapter 2 Review

Transmission-line governing Differential Equations

$$-\frac{d\widetilde{V}(z)}{dz} = (R' + j\omega L')\,\tilde{I}(z),$$

$$-\frac{d\tilde{I}(z)}{dz} = (G' + j\omega C')\,\widetilde{V}(z).$$

(telegrapher's equations in phasor form)

Chapter 2 Review

Transmission-line governing Differential Equation for *V*:

$$\frac{d^2\widetilde{V}(z)}{dz^2} - \gamma^2\,\widetilde{V}(z) = 0,$$

(wave equation for $\widetilde{V}(z)$)

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}\,.$$

(propagation constant)

Chapter 2 Review

Complex propagation constant:

$$\gamma = \alpha + j\beta$$

Where:

γ: Units of 1/m

α: Attenuation constant, units of Np/m (>0 in this class)

β: Phase constant, units of rad/m

Np and radians are both "dimensionless", and are the conventional "units" for these parameters.

Chapter 2 Review

Form of the solution: traveling waves, going in both directions:

$$\widetilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \qquad \text{(V)},$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z} \qquad \text{(A)}.$$

$$Z_0 = \frac{R' + j\omega L'}{\gamma} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \qquad (\Omega),$$

Chapter 2 Review

Solution in time-domain

$$\upsilon(z,t) = |V_0^+| e^{-\alpha z} \cos(\omega t - \beta z + \phi^+) + |V_0^-| e^{+\alpha z} \cos(\omega t + \beta z + \phi^-)$$

Remaining unknowns are determined via specification of source and load.

Chapter 2 Review

Microstrip Transmission-Line Parameters:

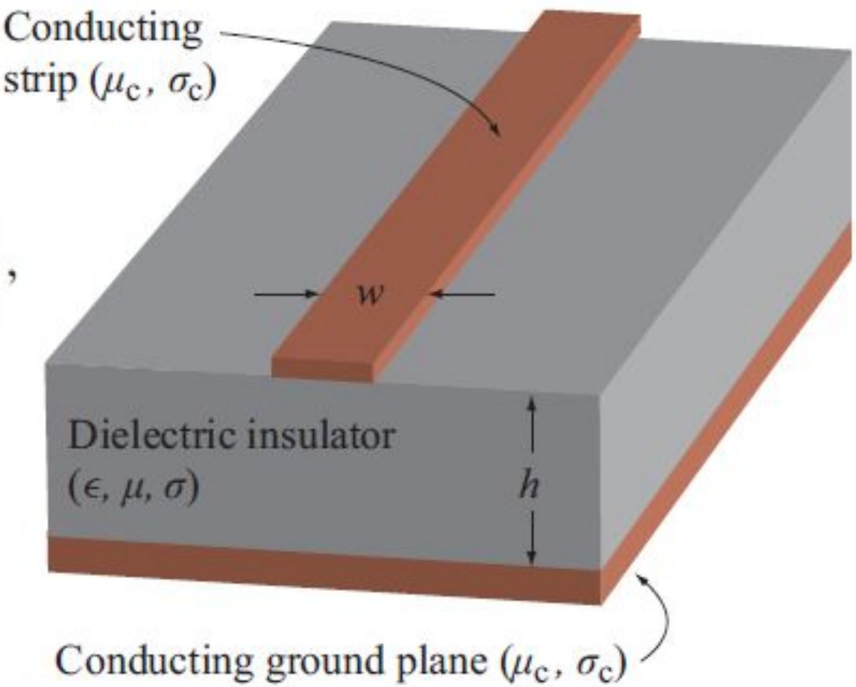
$$Z_0 = \frac{60}{\sqrt{\varepsilon_{\text{eff}}}} \ln \left\{ \frac{6 + (2\pi - 6)e^{-t}}{s} + \sqrt{1 + \frac{4}{s^2}} \right\},$$

$$t = \left(\frac{30.67}{s} \right)^{0.75} \qquad s = \frac{w}{h},$$

$$\varepsilon_{\text{eff}} = \frac{\varepsilon_r + 1}{2} + \left(\frac{\varepsilon_r - 1}{2} \right) \left(1 + \frac{10}{s} \right)^{-xy},$$

$$x = 0.56 \left[\frac{\varepsilon_r - 0.9}{\varepsilon_r + 3} \right]^{0.05},$$

$$y = 1 + 0.02 \ln \left(\frac{s^4 + 3.7 \times 10^{-4} s^2}{s^4 + 0.43} \right) + 0.05 \ln(1 + 1.7 \times 10^{-4} s^3).$$



Largely the result of fitting lots of data using functional forms based on intuition.

2-6 Lossless Transmission Line

Like the model used for Microstrip Lines,
many Transmission Lines can be made nearly lossless:

$R'=0$

$G'=0$

Hence:

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$

$$\gamma = \sqrt{(0 + j\omega L')(0 + j\omega C')}$$

$$\gamma = \sqrt{(j\omega L')(j\omega C')}$$

$$\boxed{\gamma = j\omega\sqrt{L'C'}}$$

2-6 Lossless Transmission Line

Hence, for lossless transmission lines:

$$\alpha + j\beta = j\omega\sqrt{L'C'}$$

$$\alpha = 0$$

$$\beta = \omega\sqrt{L'C'}$$

2-6 Lossless Transmission Line

Hence, for lossless transmission lines:

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

$$Z_0 = \sqrt{\frac{0 + j\omega L'}{0 + j\omega C'}}$$

$$Z_0 = \sqrt{\frac{j\omega L'}{j\omega C'}}$$

$$Z_0 = \sqrt{\frac{L'}{C'}}$$

Note that Z_0 is real for a lossless line

2-6 Lossless Transmission Line

Hence, for lossless transmission lines:

$$\beta = \frac{2\pi}{\lambda}$$

$$\lambda = \frac{2\pi}{\beta}$$

$$\boxed{\lambda = \frac{2\pi}{\omega\sqrt{L'C'}}}$$

2-6 Lossless Transmission Line

Hence, for lossless transmission lines:

$$u_{\mathrm{p}} = f\lambda$$

$$u_{\mathrm{p}} = \frac{\omega}{2\pi}\frac{2\pi}{\beta}$$

$$u_{\mathrm{p}} = \frac{\omega}{\beta}$$

$$u_{\mathrm{p}} = \frac{\omega}{\omega\sqrt{L'C'}}$$

$$\boxed{u_{\mathrm{p}} = \frac{1}{\sqrt{L'C'}}}$$

2-6 Lossless Transmission Line

Hence, for lossless transmission lines:

For the case where $\mu = \mu_0$, $\varepsilon_r > 1$:

$$u_p = \frac{1}{\sqrt{L'C'}}$$

$$u_p = \frac{1}{\sqrt{\mu\epsilon}}$$

$$u_p = \frac{1}{\sqrt{\mu_0\epsilon_0\epsilon_r}}$$

$$u_p = \frac{1}{\sqrt{\mu_0\epsilon_0}}\frac{1}{\sqrt{\epsilon_r}}$$

$$\boxed{u_p = \frac{c}{\sqrt{\epsilon_r}}}$$

2-6 Lossless Transmission Line

Hence, for lossless transmission lines:

For the case where $\mu=\mu_0$, $\varepsilon_r>1$:

$$u_{\mathrm{p}} = f\lambda$$

$$\lambda = \frac{u_{\mathrm{p}}}{f}$$

$$\lambda = \frac{c}{f}\frac{1}{\sqrt{\epsilon_r}}$$

$$\boxed{\lambda = \frac{\lambda_0}{\sqrt{\epsilon_r}}}$$

2-6 Lossless Transmission Line

Hence, for lossless transmission lines:

For the case where $\mu=\mu_0$, $\varepsilon_r>1$:

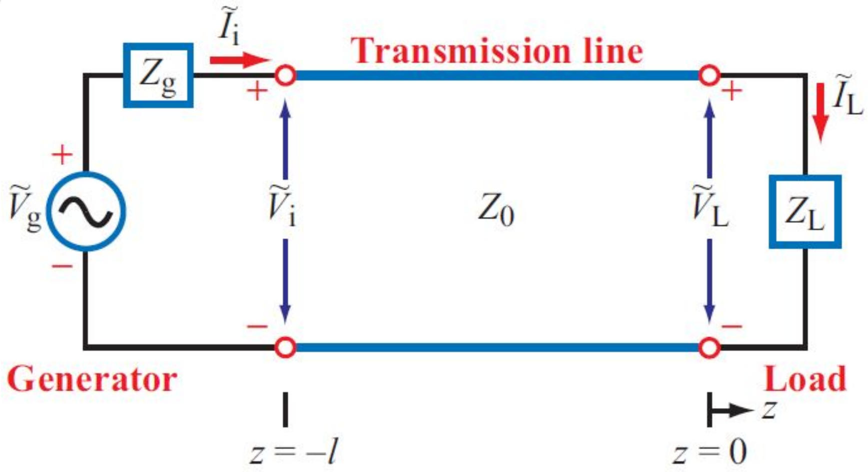
$$u_{\mathrm{p}} = \frac{c}{\sqrt{\epsilon_r}}$$

$$\lambda = \frac{\lambda_0}{\sqrt{\epsilon_r}}$$

Both phase velocity and wavelength are less than the free-space values when inside materials.

2-6 Voltage Reflection Coefficient

Recall the solution for the voltage on the line:



$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

For a lossless line:

$$\tilde{V}(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z},$$

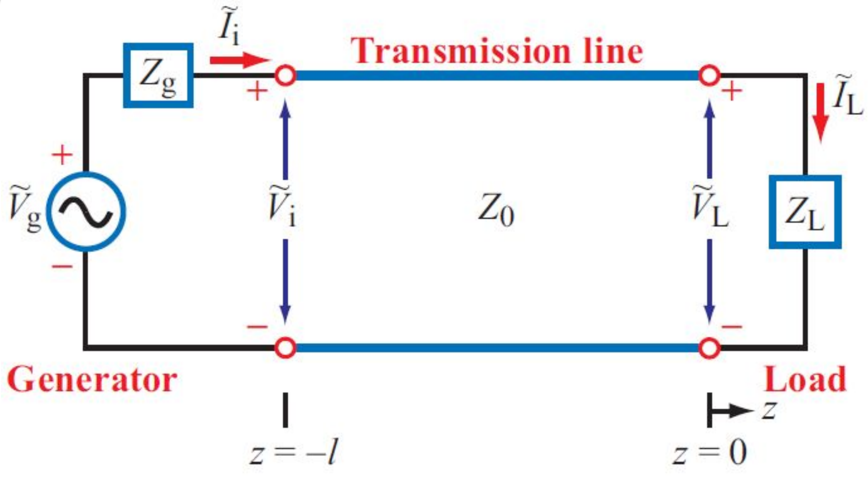
where γ has been replaced with $j\beta$

2-6 Voltage Reflection Coefficient

So for a lossless line:

$$\tilde{V}(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z},$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z}.$$



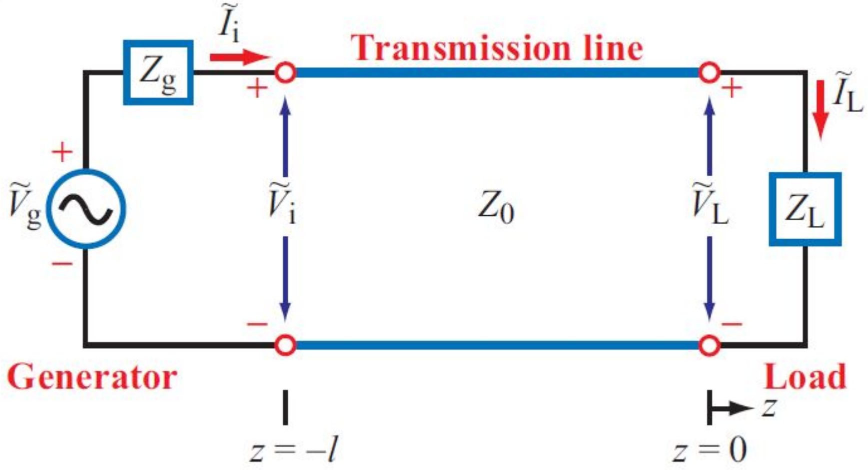
Solution is made up of 2 waves:
- one going to the right,
- the other going to the left.

This suggests that the left-going wave has been reflected from the load.

2-6 Voltage Reflection Coefficient

$$\widetilde{V}(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z},$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z}.$$



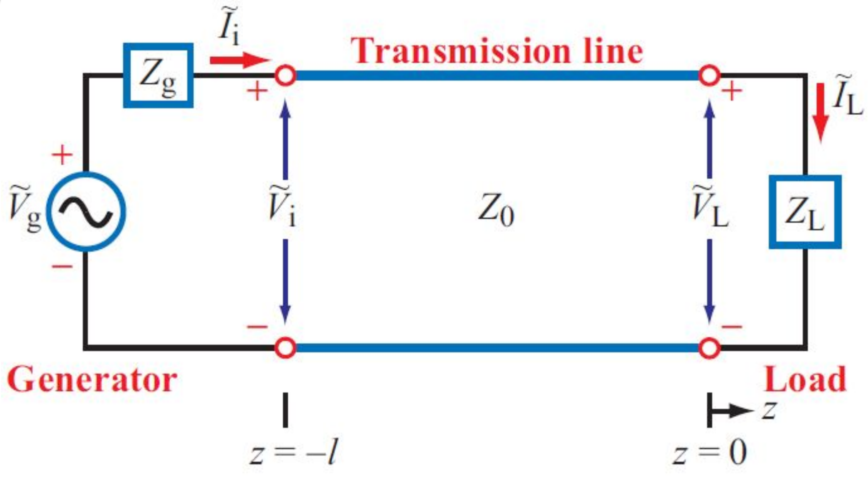
There are reflections at both ends: over-and-over.

The phasor solution provides a view of the steady-state: after all those reflections have died out.

2-6 Voltage Reflection Coefficient

$$\widetilde{V}(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z},$$

$$\widetilde{I}(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z}.$$



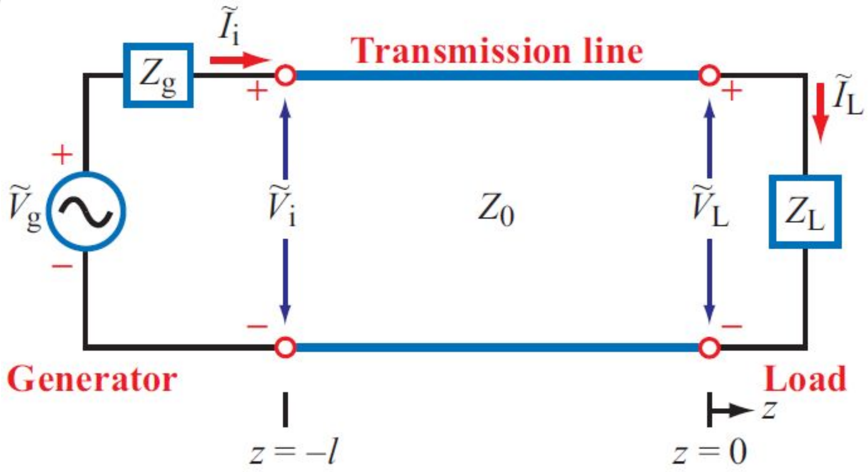
At the load (z=0):

$$\widetilde{V}_L = \widetilde{V}(z=0) = V_0^+ + V_0^-$$

$$\widetilde{I}_L = \widetilde{I}(z=0) = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}$$

Note that z is increasing to the right, and is set to zero at the load.

2-6 Voltage Reflection Coefficient



At the load (z = 0) define:

$$Z_L = \frac{\widetilde{V}_L}{\widetilde{I}_L} \quad \text{(Ohm's law)}$$

Since:

$$\widetilde{V}_L = \widetilde{V}(z=0) = V_0^+ + V_0^-,$$

$$\widetilde{I}_L = \widetilde{I}(z=0) = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}.$$

get:

$$Z_L = \left(\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \right) Z_0.$$

2-6 Voltage Reflection Coefficient

$$Z_L = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} Z_0$$

$$\frac{Z_L}{Z_0} = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}$$

$$\frac{Z_L}{Z_0} V_0^+ - \frac{Z_L}{Z_0} V_0^- = V_0^+ + V_0^-$$

$$V_0^- \left(-\frac{Z_L}{Z_0} - 1 \right) = V_0^+ \left(1 - \frac{Z_L}{Z_0} \right)$$

$$V_0^- \left(\frac{Z_L}{Z_0} + 1 \right) = V_0^+ \left(\frac{Z_L}{Z_0} - 1 \right)$$

2-6 Voltage Reflection Coefficient

$$V_0^- \left(\frac{Z_L}{Z_0} + 1 \right) = V_0^+ \left(\frac{Z_L}{Z_0} - 1 \right)$$

$$V_0^- = \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1} V_0^+$$

$$\boxed{V_0^- = \frac{Z_L - Z_0}{Z_L + Z_0} V_0^+}$$

So, the load impedance has helped us remove yet another unknown.

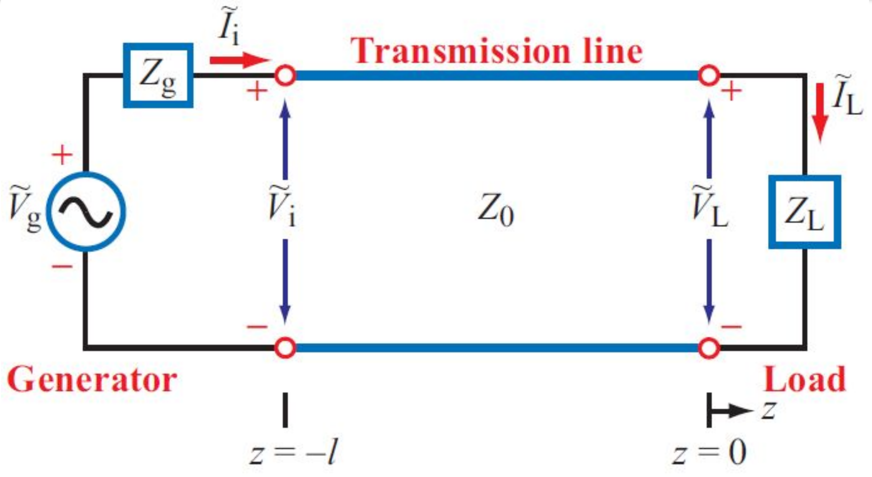
2-6 Voltage Reflection Coefficient

Voltage Reflection Coefficient:

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1}$$

$$= \frac{z_L - 1}{z_L + 1} \quad \textbf{(dimensionless)},$$

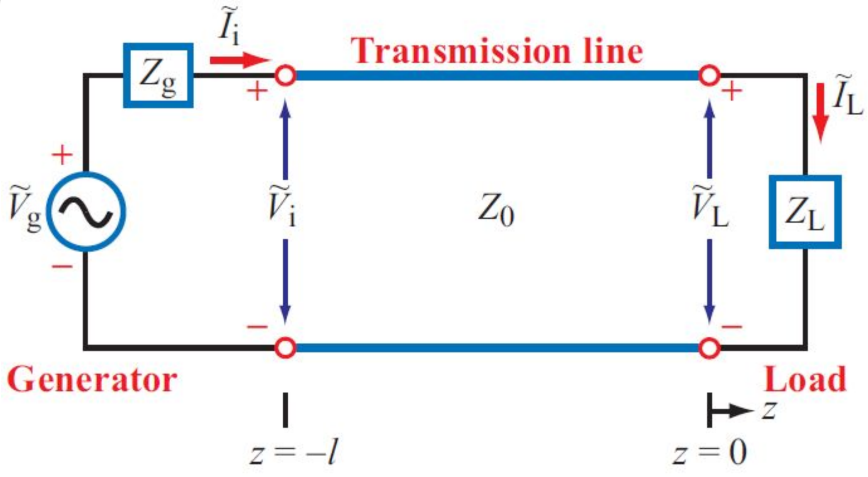


$$z_L = \frac{Z_L}{Z_0}$$

Normalized load impedance

2-6 Voltage Reflection Coefficient

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

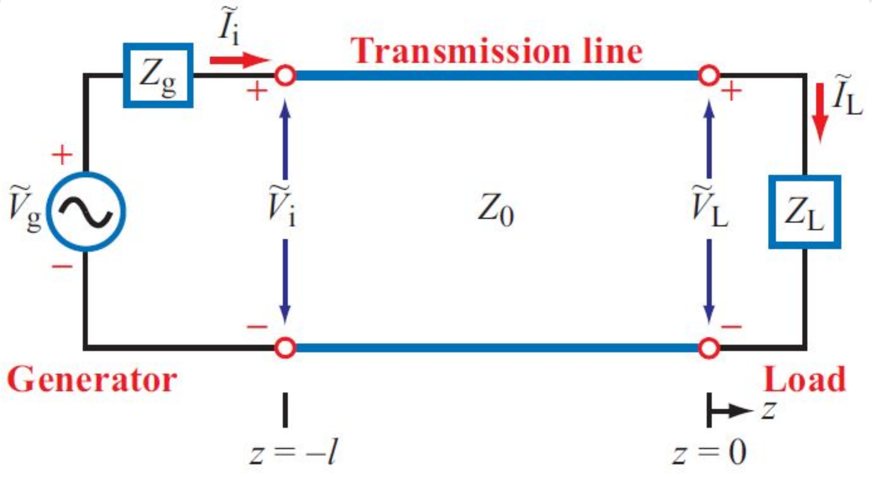


Special Cases:

1. $Z_L = 0$: $\Gamma = -1$
   reflection has opposing phase
2. $Z_L = \infty$: $\Gamma = +1$
   reflection has same phase
3. $Z_L = Z_0$: $\Gamma = 0$
   no reflection

2-6 Voltage Reflection Coefficient

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$



Special Cases:

4. $Z_L = jb$: $|\Gamma| = 1$

$$Z_0 = R$$

$$Z_L = jb$$

$$\Gamma = \frac{jb - R}{jb + R}$$

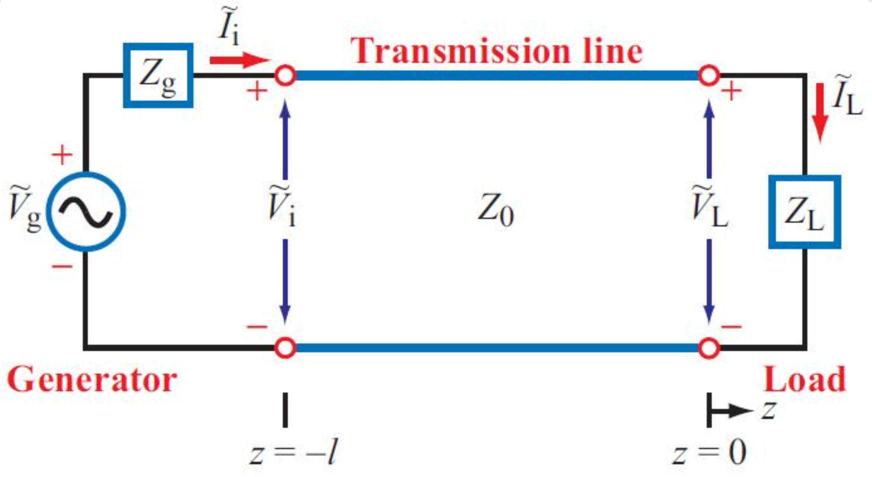
$$\Gamma = -\frac{R - jb}{R + jb}$$

$$\Gamma = -\frac{\sqrt{R^2 + b^2} e^{-j\phi}}{\sqrt{R^2 + b^2} e^{j\phi}}$$

$$|\Gamma| = 1$$

2-6 Voltage Reflection Coefficient

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$



Special Cases:

5. $Z_L = j\omega L$: $|\Gamma| = 1$

6. $Z_L = 1/(j\omega C)$: $|\Gamma| = 1$

2-6 Current Reflection Coefficient

$$\frac{I_0^-}{I_0^+} = -\frac{V_0^-}{V_0^+} = -\Gamma. \qquad (2.61)$$

Voltage ratio: +**Γ**
Current ratio: -**Γ**

2-6 Complex Reflection Coefficient

Since Γ is complex, we will also express it as:

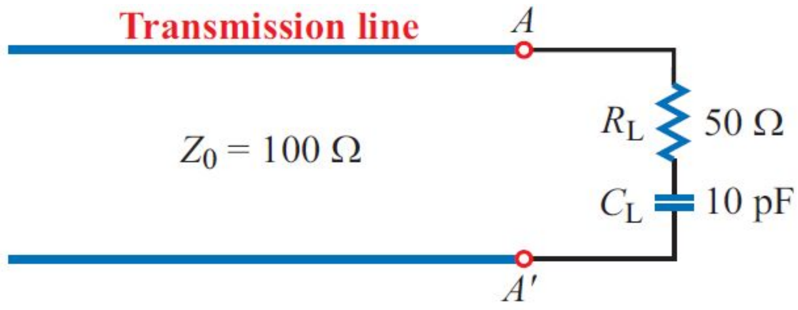
$$\Gamma = |\Gamma| e^{j\theta_r}$$

Note that $|\Gamma| \leq 1$

and Γ is dimensionless.

Example 2-3: Γ of Series RC Load

Given:



$$R_L = 50\ \Omega, \qquad C_L = 10\ \text{pF} = 10^{-11}\ \text{F},$$

$$Z_0 = 100\ \Omega, \qquad f = 100\ \text{MHz} = 10^8\ \text{Hz}.$$

Find: reflection coefficient at the load.

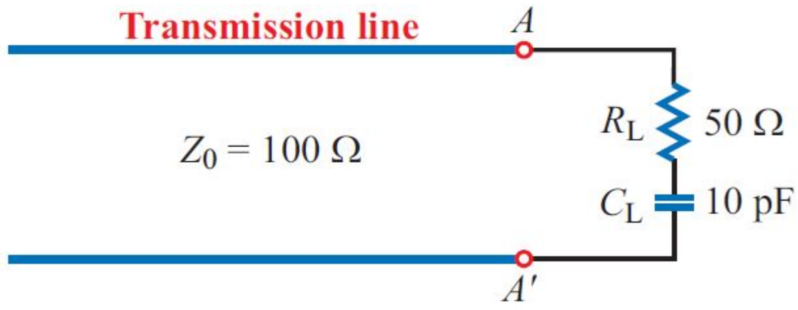
Solution: We know:

$$\Gamma = \frac{z_L - 1}{z_L + 1}$$

So, first find z_L.

Example 2-3: Γ of Series RC Load

Solution:



The normalized load impedance is

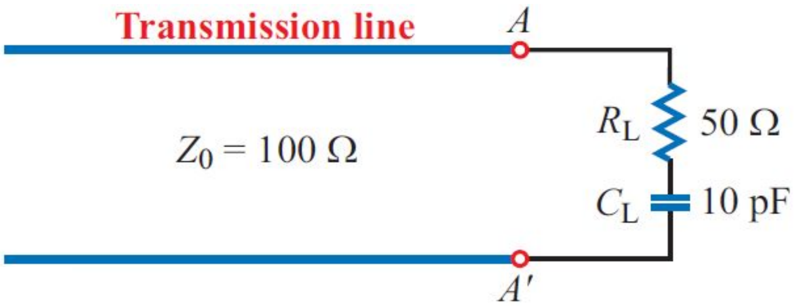
$$z_L = \frac{Z_L}{Z_0} = \frac{R_L - j/(\omega C_L)}{Z_0}$$

$$= \frac{1}{100}\left(50 - j\frac{1}{2\pi \times 10^8 \times 10^{-11}}\right)$$

$$= (0.5 - j1.59)\ \Omega.$$

Example 2-3: Γ of Series RC Load

Solution:



$$\Gamma = \frac{z_L - 1}{z_L + 1} = \frac{0.5 - j1.59 - 1}{0.5 - j1.59 + 1} = \frac{-0.5 - j1.59}{1.5 - j1.59} = \frac{-1.67e^{j72.6^\circ}}{2.19e^{-j46.7^\circ}} = -0.76e^{j119.3^\circ}.$$

replace the minus sign with e^{-j180°

$$\Gamma = 0.76e^{j119.3^\circ} e^{-j180^\circ} = 0.76e^{-j60.7^\circ} = 0.76\underline{/-60.7^\circ},$$

$$|\Gamma| = 0.76, \qquad \theta_r = -60.7^\circ.$$

Example 2-3: Γ of Series RC Load

Solution:

$$|\Gamma| = 0.76, \qquad \theta_r = -60.7^\circ.$$

Magnitude must be positive.
Angle must be between -180° and -180° (or $-\pi$ to $+\pi$)

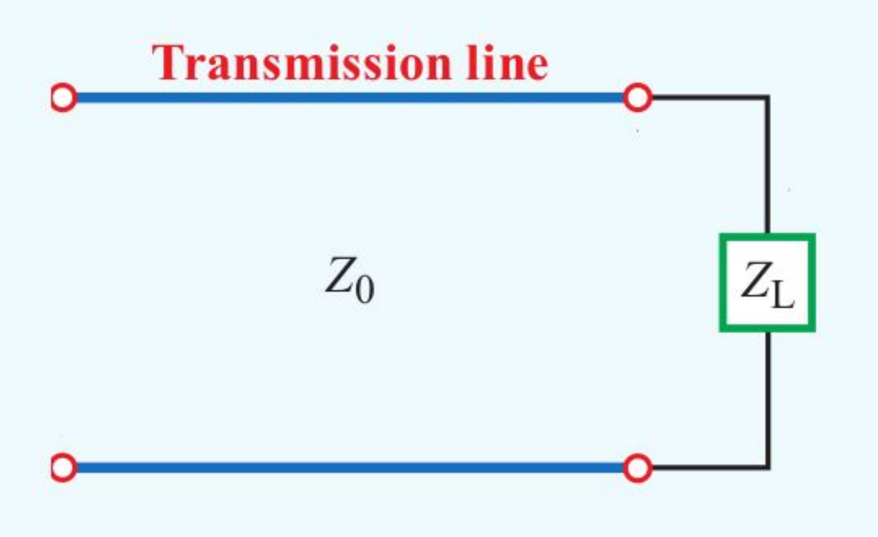
Can always add or subtract 360° (or 2π)

Exercise 2-8 Reflection Coefficient

Given: $Z_0 = 50\ \Omega$,

$Z_L = 30 - j200\ \Omega$

Find: Γ, in polar format



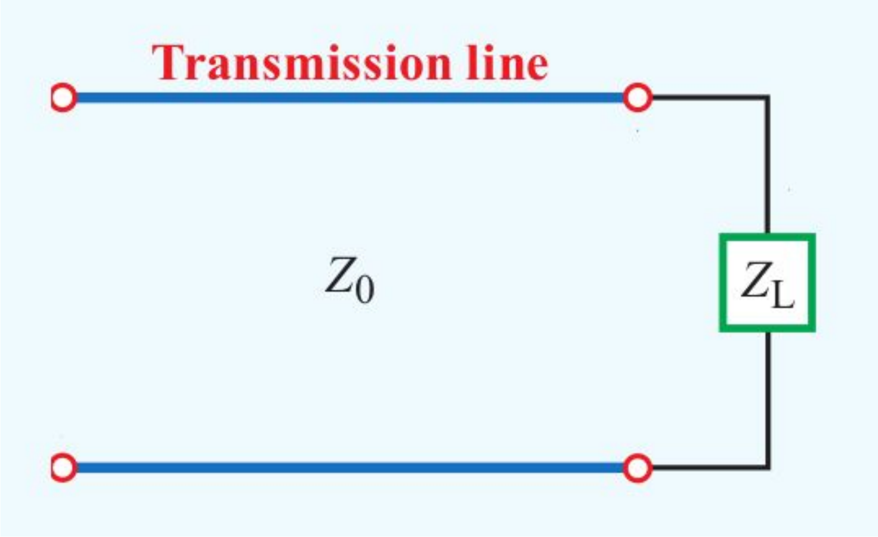
Solution:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{30 - j200 - 50}{(30 - j200) + 50}$$

Exercise 2-8 Reflection Coefficient

Solution:



$$\Gamma = \frac{30 - j200 - 50}{(30 - j200) + 50}$$

$$= \frac{-20 - j200}{80 - j200}$$

$$\Gamma = \frac{\sqrt{20^2 + 200^2} e^{j(\tan^{-1}(200/20) - 180^\circ)}}{\sqrt{80^2 + 200^2} e^{j \tan^{-1}(-200/80)}}$$

$$\Gamma = \frac{201.e^{j(84.3^\circ - 180^\circ)}}{215.4e^{-j68.2^\circ}}$$

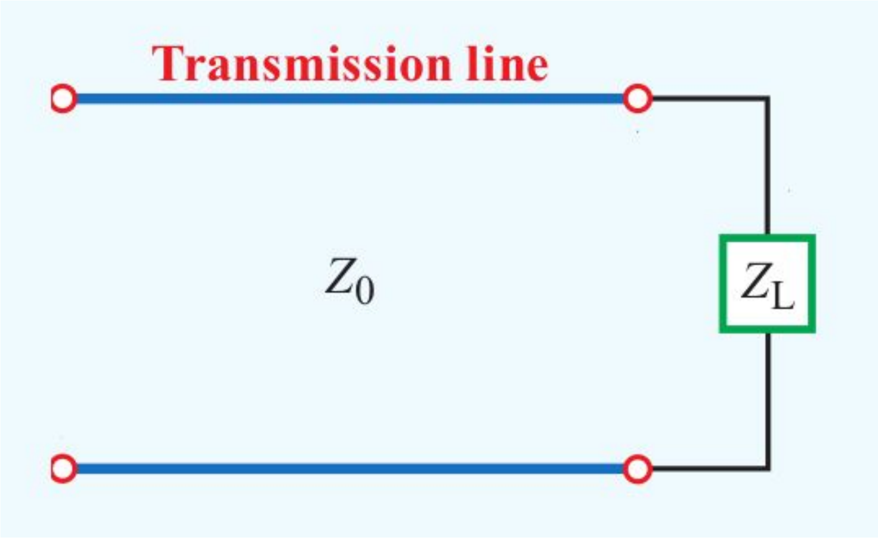
Exercise 2-8 Reflection Coefficient

Solution:

$$\Gamma = \frac{201.e^{j(84.3^\circ - 180^\circ)}}{215.4e^{-j68.2^\circ}}$$

$$\Gamma = 0.933e^{j(-95.7^\circ + 68.2^\circ)}$$

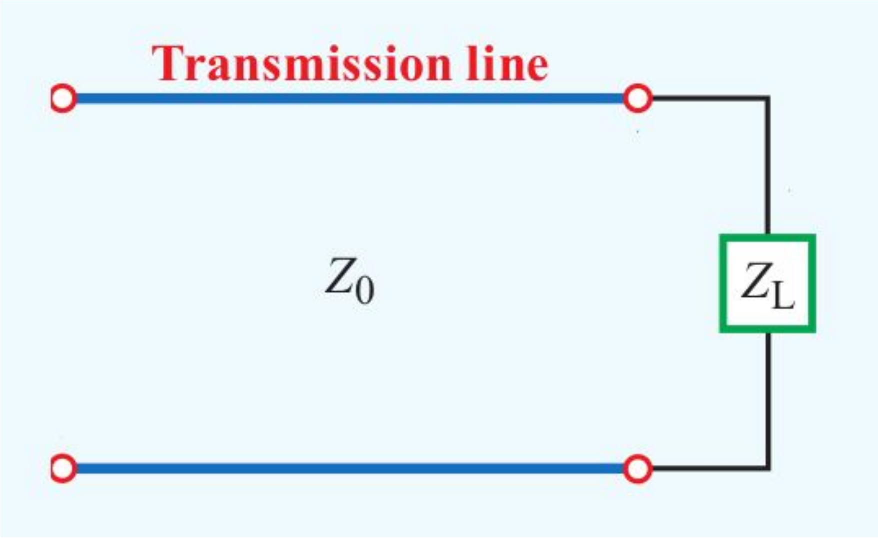
$$\boxed{\Gamma = 0.933e^{-j27.5^\circ}}$$



Exercise 2-10 Reflection Coefficient

Given: $\Gamma = 0.6 - j0.3$

Find: normalized load impedance: z_L, in rectangular format



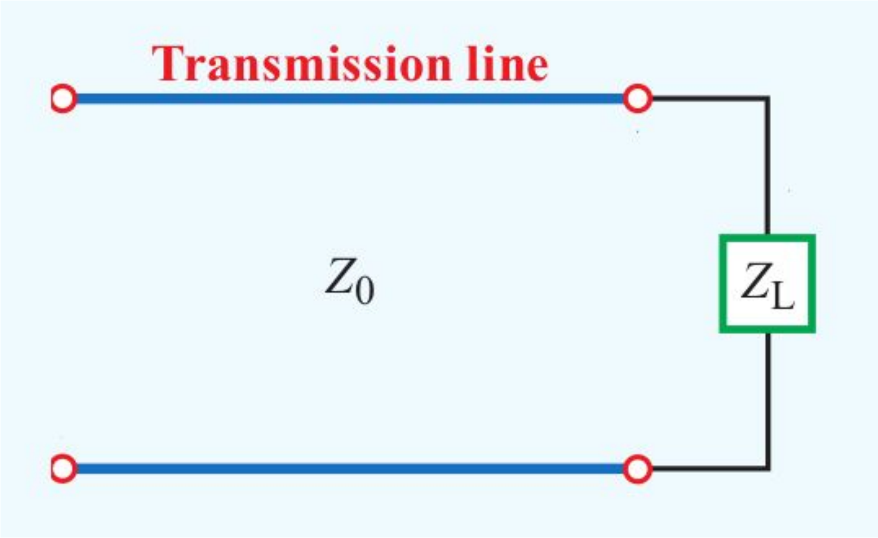
Solution: we know:

$$\Gamma = \frac{z_L - 1}{z_L + 1}$$

derive equation for z_L...

Exercise 2-10 Reflection Coefficient

Solution:



$$\Gamma = \frac{z_L - 1}{z_L + 1}$$

$$\Gamma(z_L + 1) = z_L - 1$$

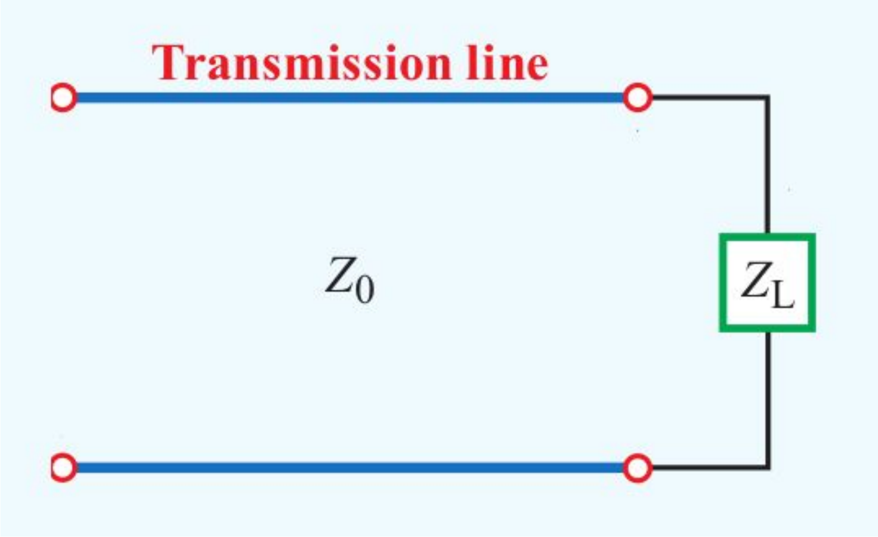
$$z_L(\Gamma - 1) = -\Gamma - 1$$

$$z_L = -\frac{\Gamma + 1}{\Gamma - 1}$$

$$\boxed{z_L = \frac{1 + \Gamma}{1 - \Gamma}}$$

Exercise 2-10 Reflection Coefficient

Solution:



$$z_L = \frac{1+\Gamma}{1-\Gamma}$$

$$z_L = \frac{1+(0.6-j0.3)}{1-(0.6-j0.3)}$$

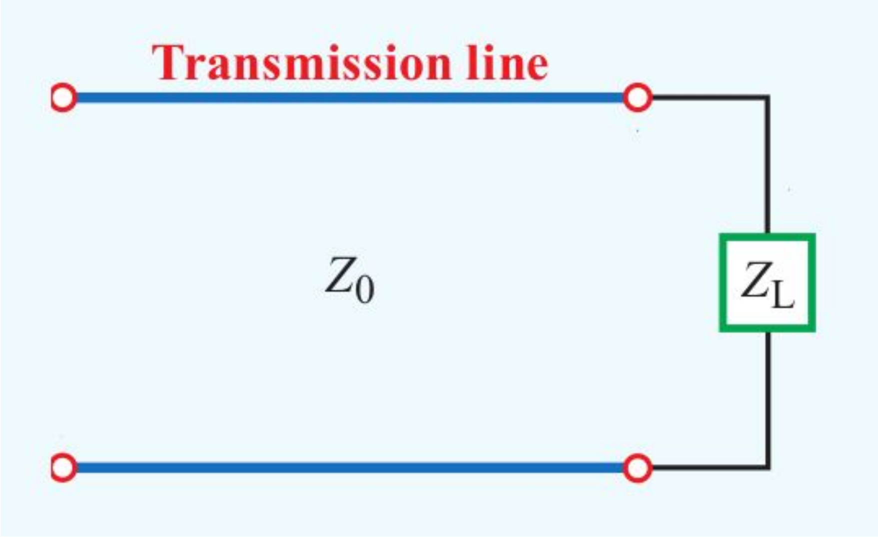
$$z_L = \frac{1.6-j0.3}{0.4+j0.3}$$

$$z_L = \frac{\sqrt{0.3^3+1.6^2}e^{j\tan^{-1}(-0.3/1.6)}}{\sqrt{0.3^2+0.4^2}e^{j\tan^{-1}(0.3/0.4)}}$$

$$z_L = \frac{1.63e^{-j10.62^\circ}}{0.5e^{j36.87^\circ}}$$

Exercise 2-10 Reflection Coefficient

Solution:



$$z_L = \frac{1.63e^{-j10.62^\circ}}{0.5e^{j36.87^\circ}}$$

$$z_L = 3.26e^{j(-10.62^\circ - 36.87^\circ)}$$

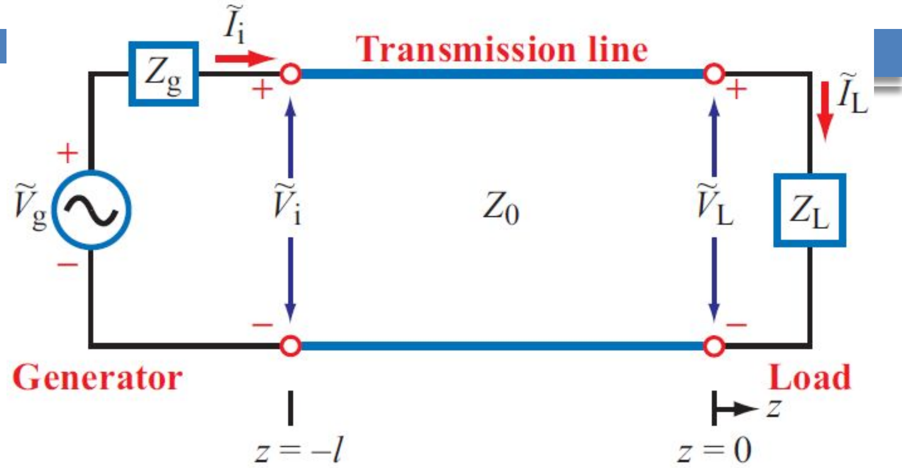
$$z_L = 3.26e^{-j47.5^\circ}$$

$$z_L = 3.26\cos(-47.5^\circ) + j3.26\sin(-47.5^\circ)$$

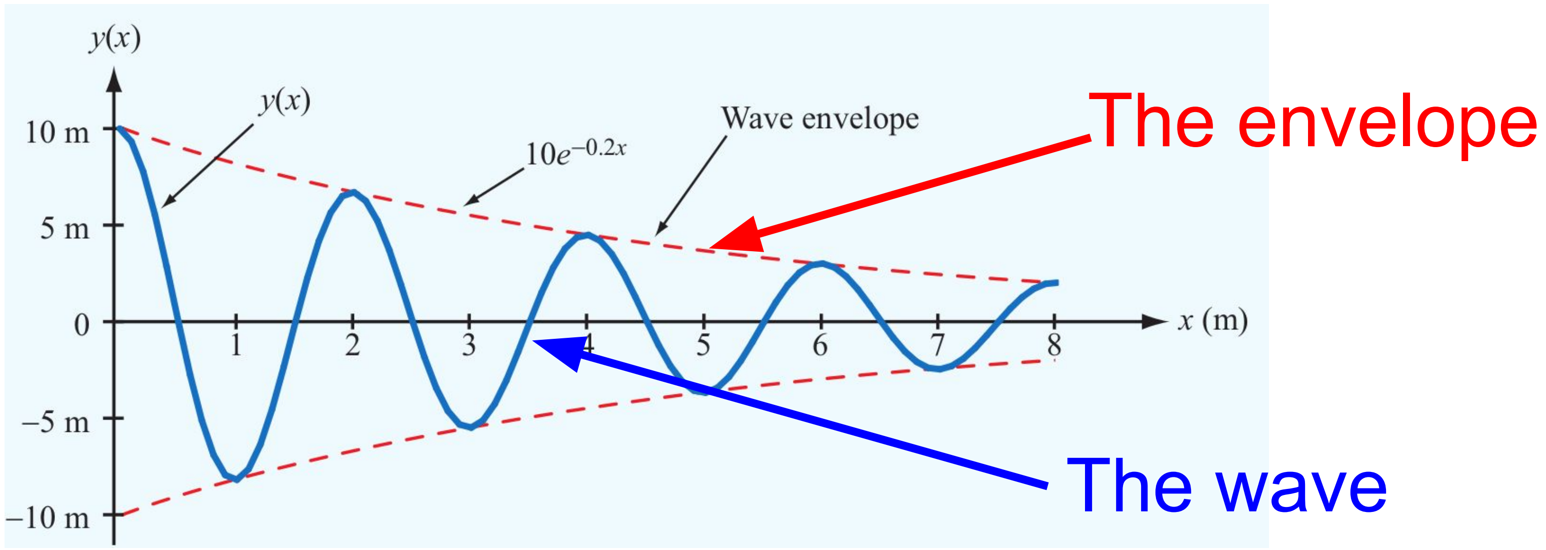
$$\boxed{z_L = 2.2 - j2.4}$$

2-6 Standing Waves

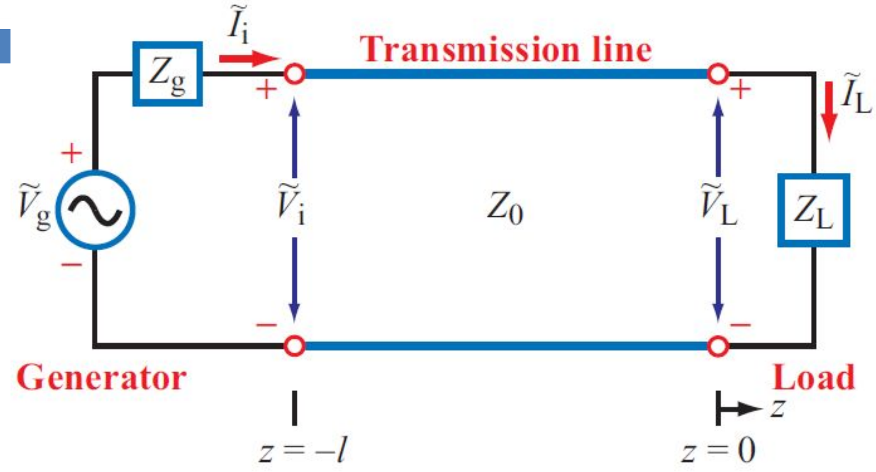
Recall: For a travelling wave in a lossy material we have:



$$y(x,t) = Ae^{-\alpha x}\cos(\omega t - \beta x + \phi_0)$$



2-6 Standing Waves



For 2 travelling waves expect to have a wave and an envelope as well.

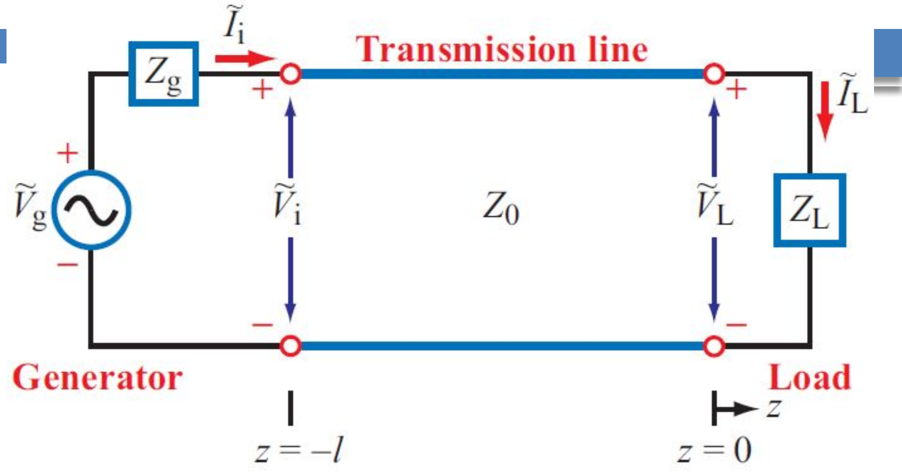
Start with the envelope: Find an expression for the magnitude of the Voltage on the line.

2-6 Standing Waves

Using the relation $V_0^- = \Gamma V_0^+$ yields

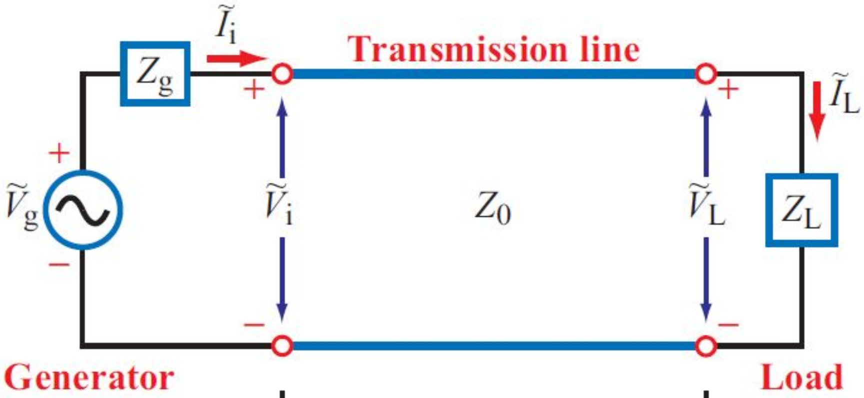
$$\tilde{V}(z) = V_0^+(e^{-j\beta z} + \Gamma e^{j\beta z}),$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0}(e^{-j\beta z} - \Gamma e^{j\beta z}).$$



Remember: using $j\beta$ and not γ (lossless case)

2-6 Standing Waves



Using the relation $V_0^- = \Gamma V_0^+$ yields

$$\widetilde{V}(z) = V_0^+ (e^{-j\beta z} + \Gamma e^{j\beta z}),$$

$$\widetilde{I}(z) = \frac{V_0^+}{Z_0} (e^{-j\beta z} - \Gamma e^{j\beta z}).$$

Since: $|\widetilde{V}(z)| = [\widetilde{V}(z) \widetilde{V}^*(z)]^{1/2}$

Recall: complex conjugate means replace *j* with *-j*

$$|\widetilde{V}(z)| = \left\{ \left[V_0^+ (e^{-j\beta z} + |\Gamma| e^{j\theta_r} e^{j\beta z}) \right] \cdot \left[(V_0^+)^* (e^{j\beta z} + |\Gamma| e^{-j\theta_r} e^{-j\beta z}) \right] \right\}^{1/2}$$

2-6 Standing Waves

$$|\tilde{V}(z)| = \{V_0^+ \left[e^{-j\beta z} + |\Gamma| e^{+j\theta_r} e^{+j\beta z}\right]$$

$$(V_0^+)^* \left[e^{+j\beta z} + |\Gamma| e^{-j\theta_r} e^{-j\beta z}\right]\}^{1/2}$$

$$= \{|V_0^+|^2\}^{1/2} \{e^{-j\beta z} e^{+j\beta z} +$$

$$e^{-j\beta z} |\Gamma| e^{-j\theta_r} e^{-j\beta z} +$$

$$|\Gamma| e^{+j\theta_r} e^{+j\beta z} e^{+j\beta z} +$$

$$|\Gamma| e^{+j\theta_r} e^{+j\beta z} |\Gamma| e^{-j\theta_r} e^{-j\beta z}\}^{1/2}$$

2-6 Standing Waves

$$= \left\{|V_0^+|^2\right\}^{1/2} \left\{e^{-j\beta z}e^{+j\beta z} + e^{-j\beta z}|\Gamma|e^{-j\theta_r}e^{-j\beta z} + |\Gamma|e^{+j\theta_r}e^{+j\beta z}e^{+j\beta z} + |\Gamma|e^{+j\theta_r}e^{+j\beta z}|\Gamma|e^{-j\theta_r}e^{-j\beta z}\right\}^{1/2}$$

$$= |V_0^+|\left\{1 + |\Gamma|e^{-j2\beta z}e^{-j\theta_r} + |\Gamma|e^{+j2\beta z}e^{+j\theta_r} + |\Gamma|^2\right\}^{1/2}$$

2-6 Standing Waves

$$= |V_0^+| \left\{1 + |\Gamma| e^{-j2\beta z} e^{-j\theta_r} + |\Gamma| e^{+j2\beta z} e^{+j\theta_r} + |\Gamma|^2\right\}^{1/2}$$

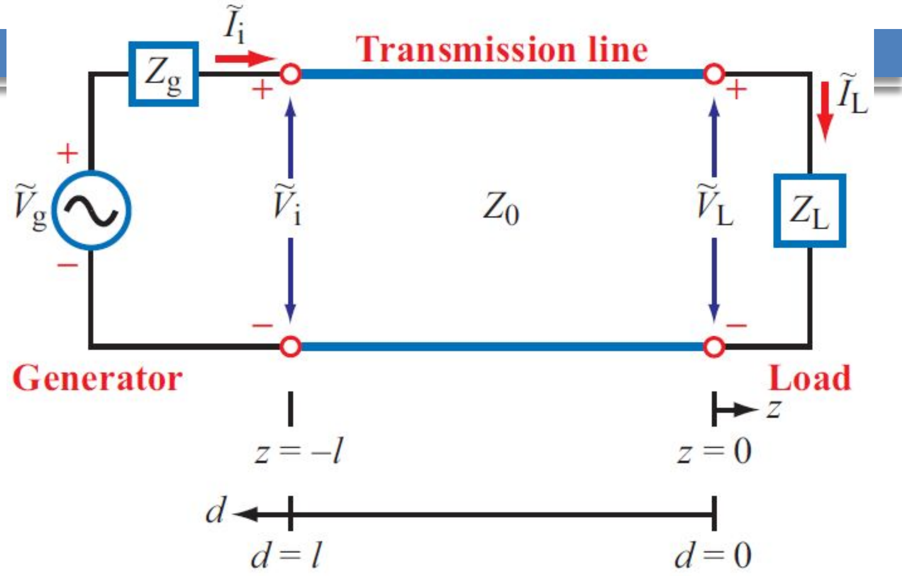
$$= |V_0^+| \left\{1 + |\Gamma|^2 + 2|\Gamma| \cos(2\beta z + \theta_r)\right\}^{1/2}$$

Recall from math:

$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$

2-6 Standing Waves

Define the coordinate *d* to be zero at the load, increasing toward the generator.



Replace *z* with -*d*:

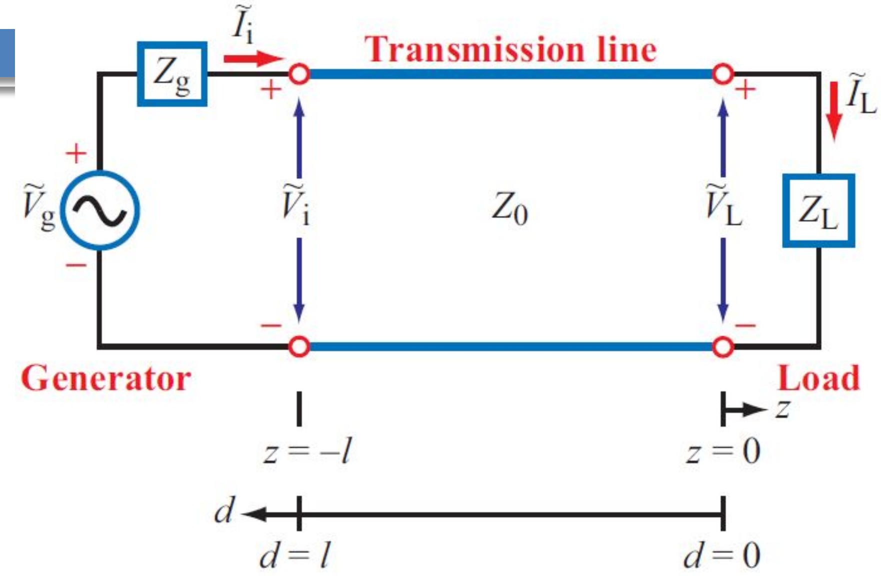
$$\cos(2\beta(-d) + \theta_r)$$

$$\cos(-[2\beta d - \theta_r])$$

$$\cos(2\beta d - \theta_r)$$

2-6 Standing Waves

Define the coordinate *d* to be zero at the load, increasing toward the generator.



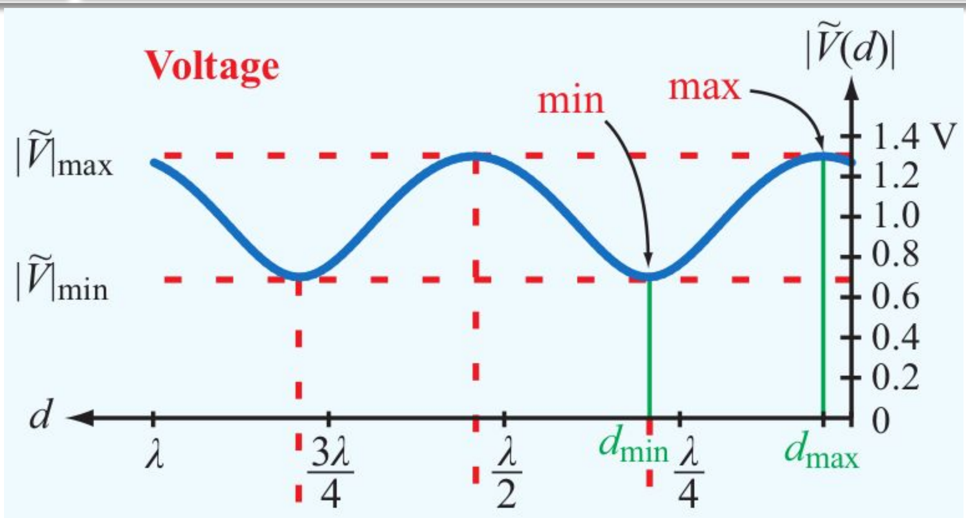
Replace *z* with -*d*:

$$|\tilde{V}(d)| = |V_0^+| \left[1 + |\Gamma|^2 + 2|\Gamma|\cos(2\beta d - \theta_r)\right]^{1/2}.$$

Similarly:

$$|\tilde{I}(d)| = \frac{|V_0^+|}{Z_0} \left[1 + |\Gamma|^2 - 2|\Gamma|\cos(2\beta d - \theta_r)\right]^{1/2}$$

2-6 Standing-Wave Pattern



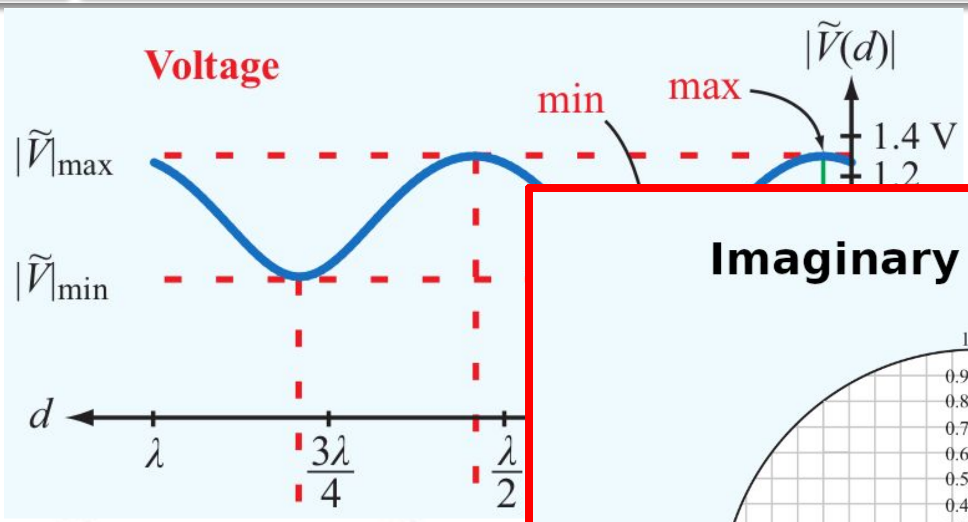
Magnitude of the phasor Voltage:
(the "envelope")

$$|\widetilde{V}(d)| = |V_0^+| \left[1 + |\Gamma|^2 + 2|\Gamma| \cos(2\beta d - \theta_r)\right]^{1/2}. \quad (2.66)$$

Voltage magnitude is **maximum:**
cos()=+1:

$$2\beta d_{max} - \theta_r = 2n\pi,$$

2-6 Standing-Wave Pattern

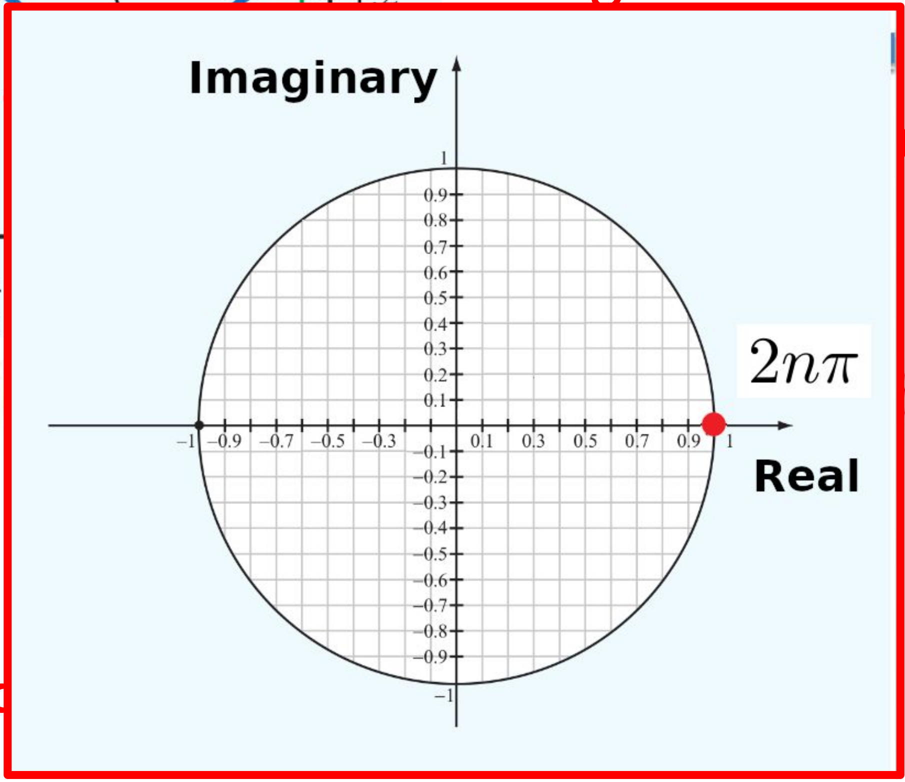


Magnitude of the phasor

e")

$$|\tilde{V}(d)| = |V_0^+| \left[1 + \right.$$

5)

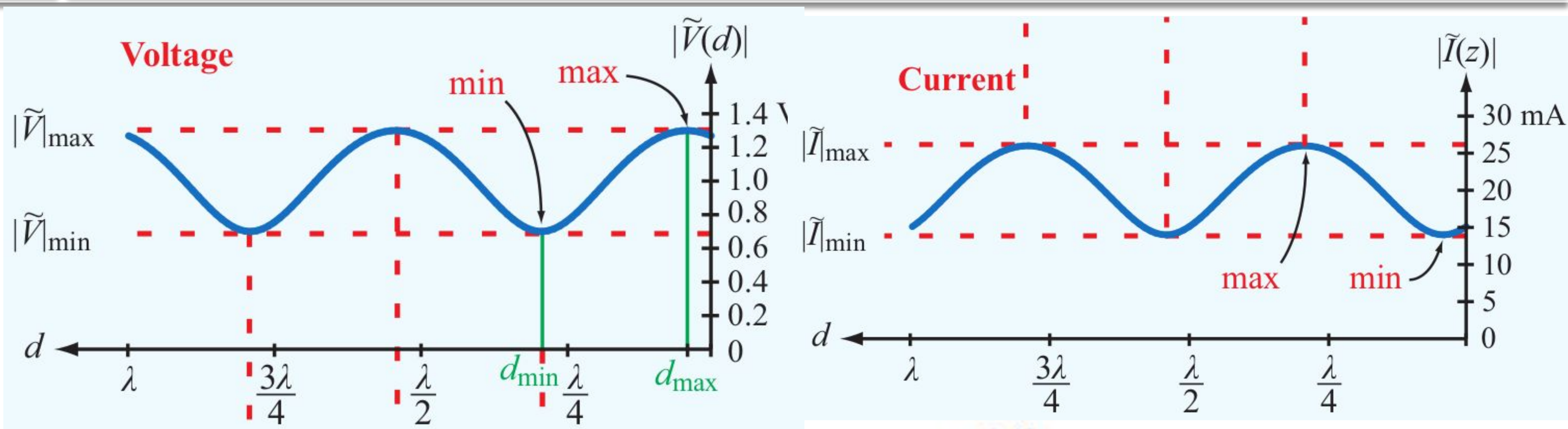


Voltage magnitu

cos()=+1:

$$2\beta d_{max} - \theta_r = 2n\pi,$$

2-6 Standing-Wave Pattern



$$|\widetilde{V}(d)| = |V_0^+|\left[1 + |\Gamma|^2 + 2|\Gamma|\cos(2\beta d - \theta_r)\right]^{1/2}. \quad (2.66)$$

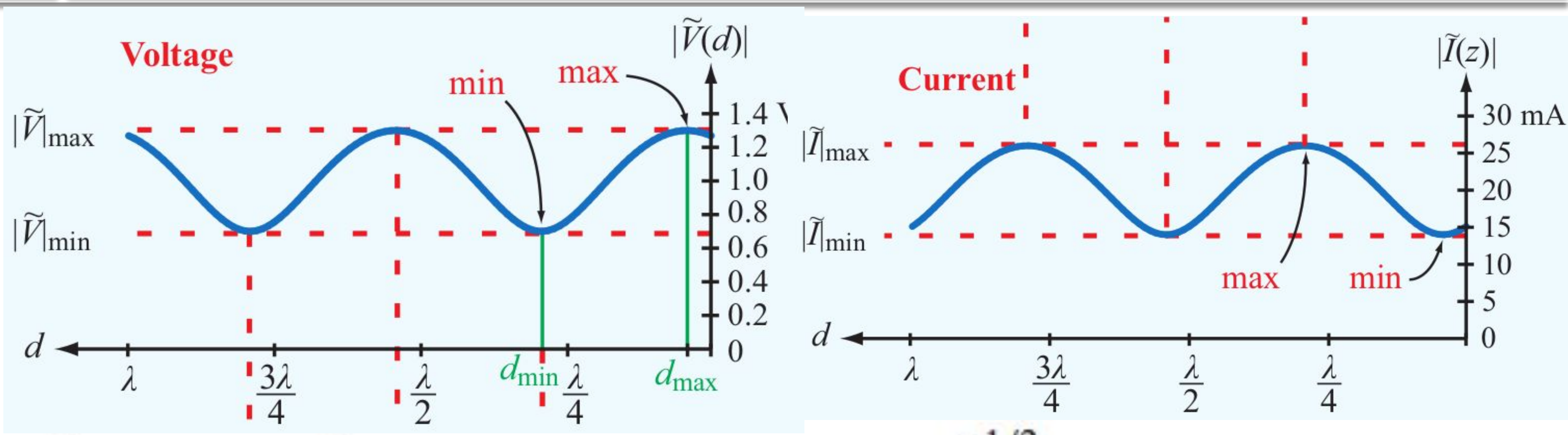
$$|\widetilde{I}(d)| = \left|\frac{V_0^+}{Z_0}\right| [1 + |\Gamma|^2 - 2|\Gamma|\cos(2\beta d - \theta_r)]^{1/2}$$

Voltage magnitude is **maximum**:
cos()=+1:

$$2\beta d_{max} - \theta_r = 2n\pi,$$

When:
voltage is a maximum,
current is a minimum,
and vice versa

2-6 Standing-Wave Pattern



$$|\widetilde{V}(d)| = |V_0^+| \left[1 + |\Gamma|^2 + 2|\Gamma| \cos(2\beta d - \theta_r)\right]^{1/2}. \quad (2.66)$$

$$|\widetilde{I}(d)| = \left|\frac{V_0^+}{Z_0}\right| [1 + |\Gamma|^2 - 2|\Gamma| \cos(2\beta d - \theta_r)]^{1/2}$$

Voltage magnitude is **minimum**:

cos()=-1:

$$2\beta d_{min} - \theta_r = (2n+1)\pi$$

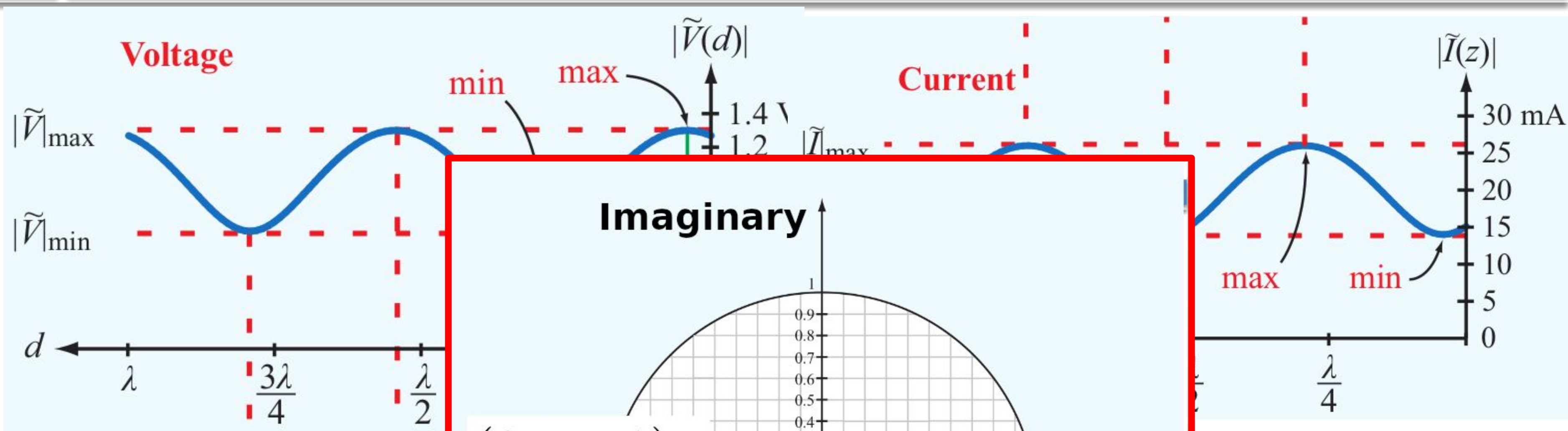
When:

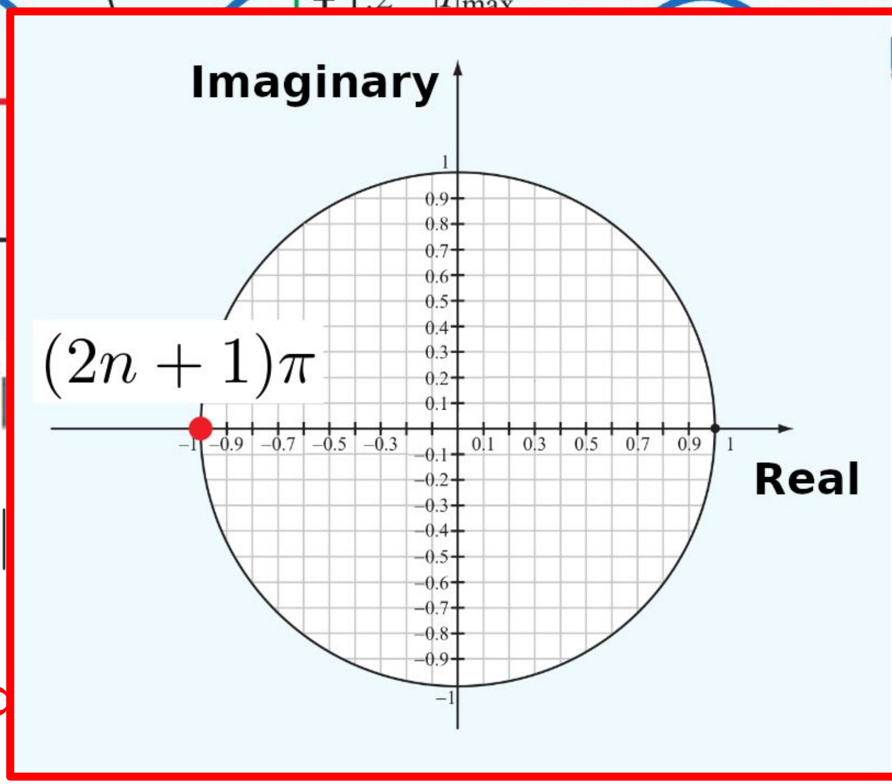
voltage is a maximum,

current is a minimum,

and vice versa

2-6 Standing-Wave Pattern





$$|\widetilde{V}(d)| = |V_0^+| \left[1 + | \right.$$

$$|\widetilde{I}(d)| = \left|\frac{V_0^+}{Z_0}\right| [1 + |$$

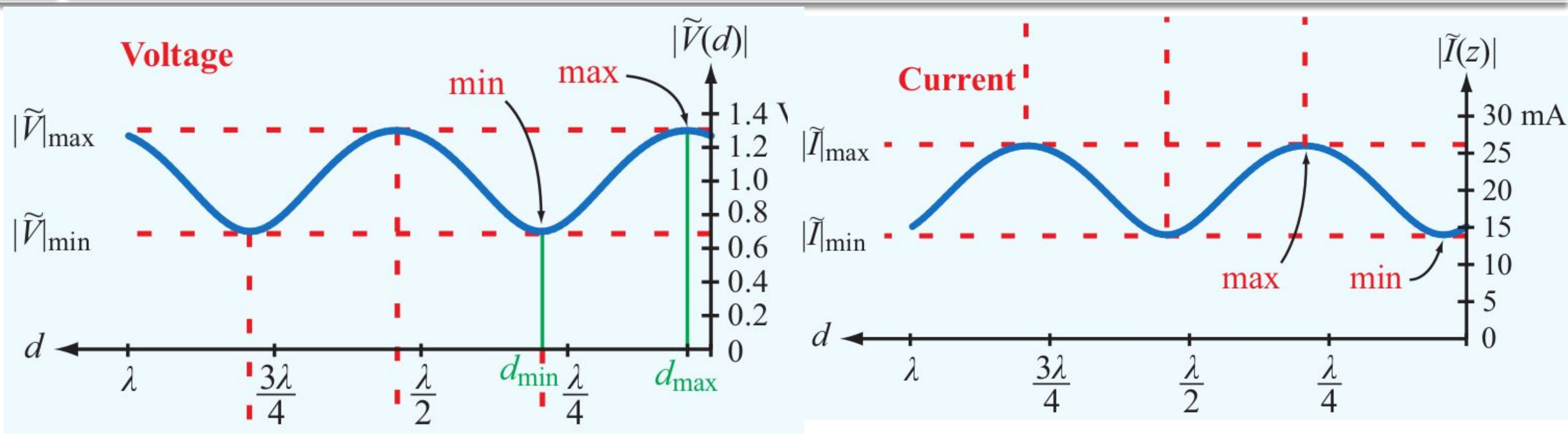
Voltage magnitud … maximum,

cos()=-1:

$$2\beta d_{\min} - \theta_r = (2n+1)\pi$$

current is a minimum, and vice versa

2-6 Standing-Wave Pattern



What is the period of the Standing wave (envelope)?

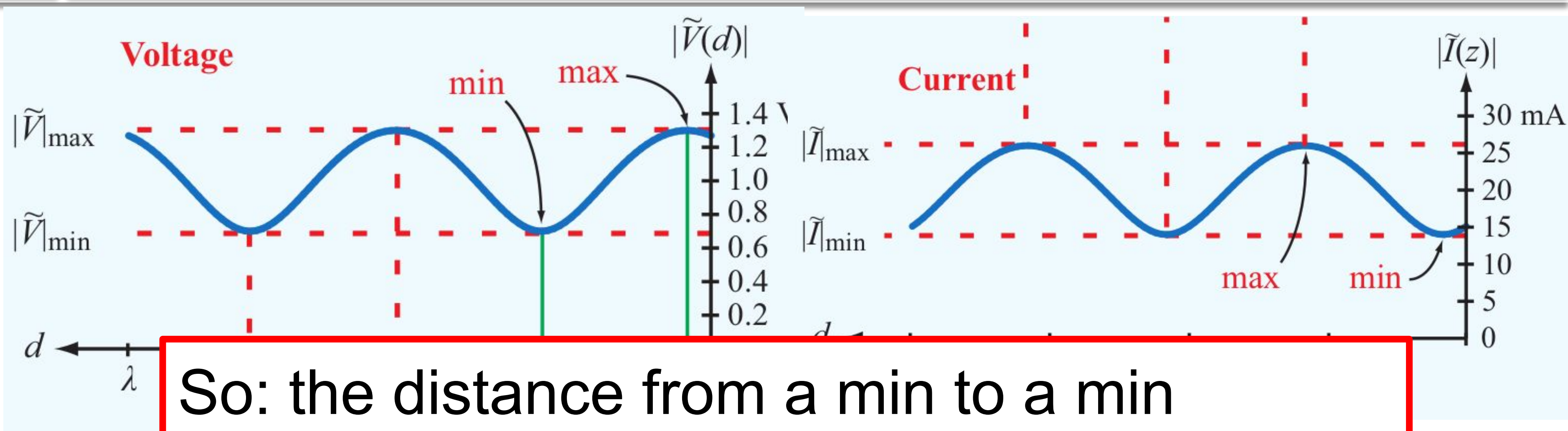
$$2\beta d_n = 2\pi n + \theta_r$$

$$d_n = \frac{n\lambda}{2} + \frac{\lambda\theta_r}{4\pi}$$

$$2\frac{2\pi}{\lambda} d_n = 2\pi n + \theta_r$$

$$d_2 - d_1 = \frac{\lambda}{2}$$

2-6 Standing-Wave Pattern



What …)?

So: the distance from a min to a min (or max to a max) is $\lambda/2$: half the wavelength of the voltage wave.

$$2\beta d_n = 2\pi n + \theta_r$$

$$d_n = \frac{n\lambda}{2} + \frac{\theta_r \lambda}{4\pi}$$

$$2\frac{2\pi}{\lambda} d_n = 2\pi n + \theta_r$$

$$d_2 - d_1 = \frac{\lambda}{2}$$

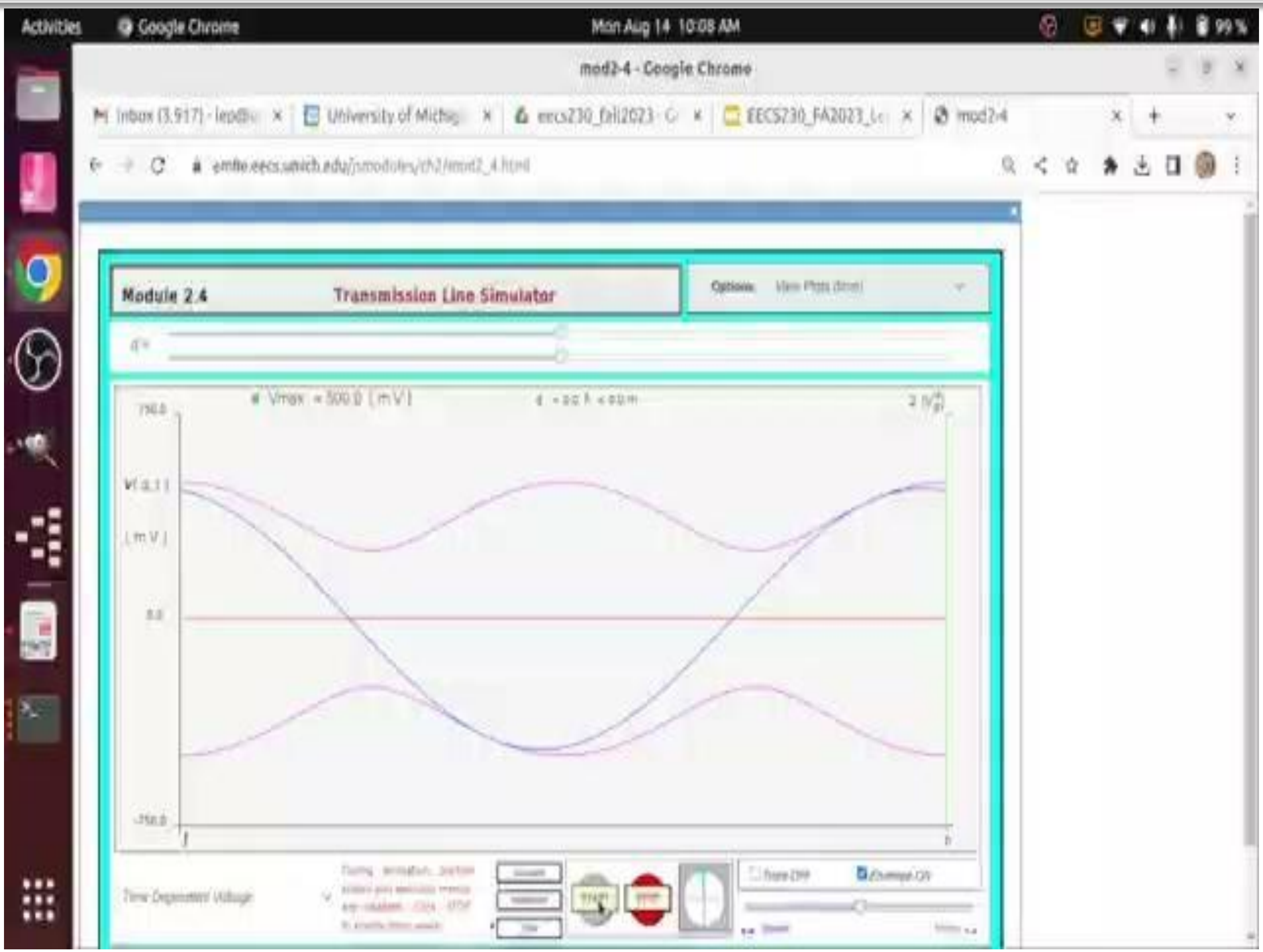
2-6 Standing-Wave Pattern

The amplitude of the forward-propagating phasor voltage is a sinusoid with wavelength λ

The amplitude of the total phasor voltage is usually a standing wave (envelope) with wavelength $\lambda/2$

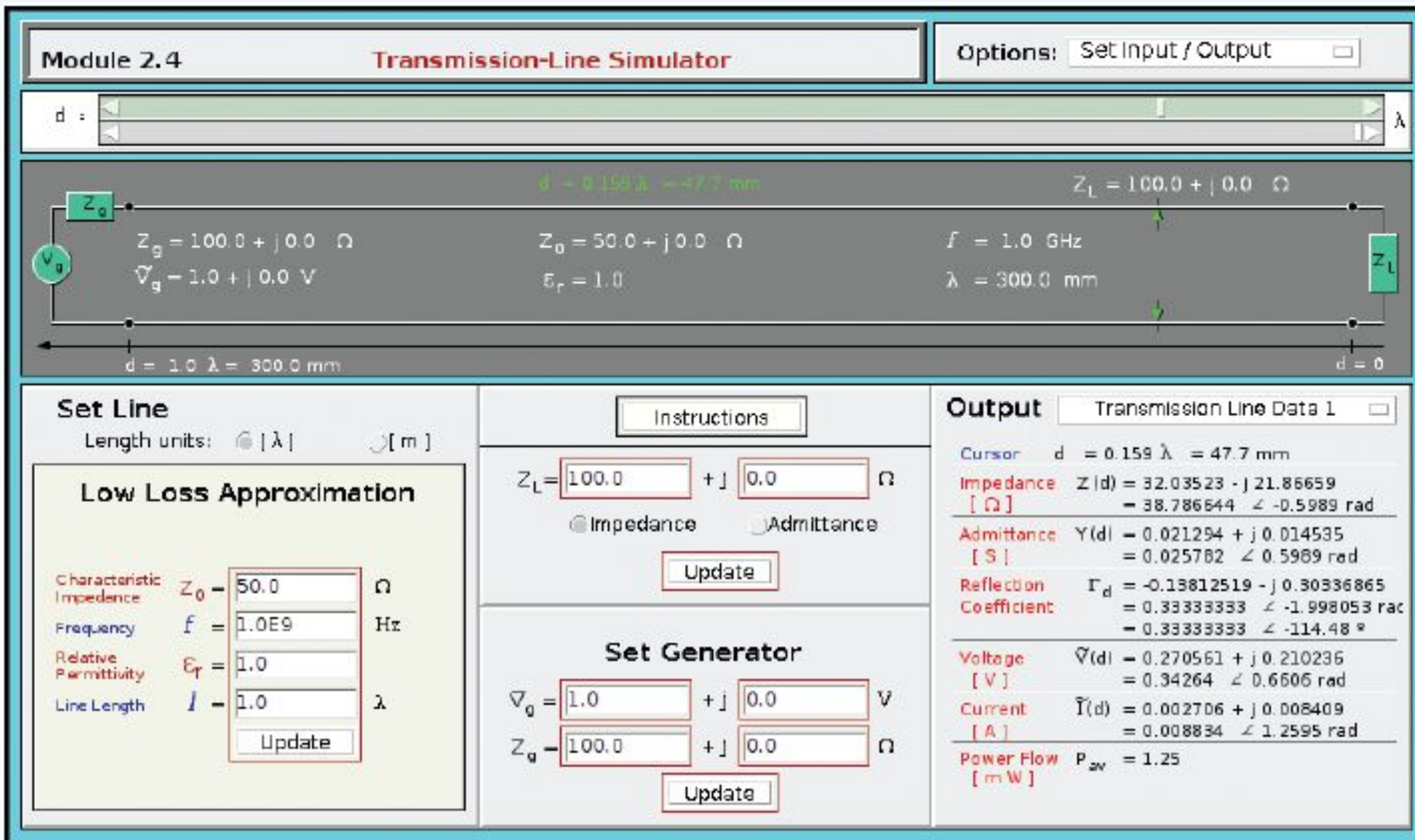
The actual voltage waveform oscillates within this envelope:

2-6 Standing-Wave Pattern



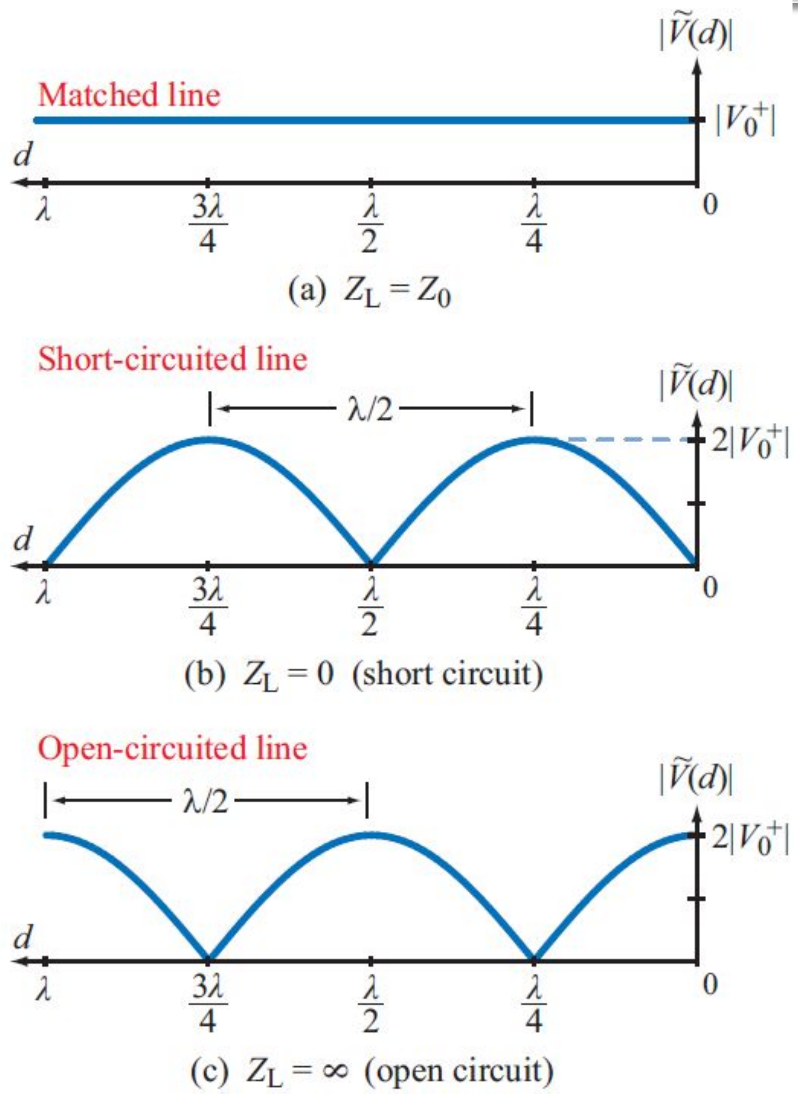
2-6 Standing-Wave Pattern

Module 2.4 Transmission-Line Simulator Upon specifying the requisite input data—including the load impedance at $d = 0$ and the generator voltage and impedance at $d = l$—this module provides a wealth of output information about the voltage and current waveforms along the trasmission line. You can view plots of the standing wave patterns for voltage and current, the time and spatial variations of the instantaneous voltage $v(d, t)$ and current $i(d, t)$, and other related quantities.



2-6 Standing Wave Patterns: 3 Types of Loads

Magnitude of the phasor voltage (envelope)

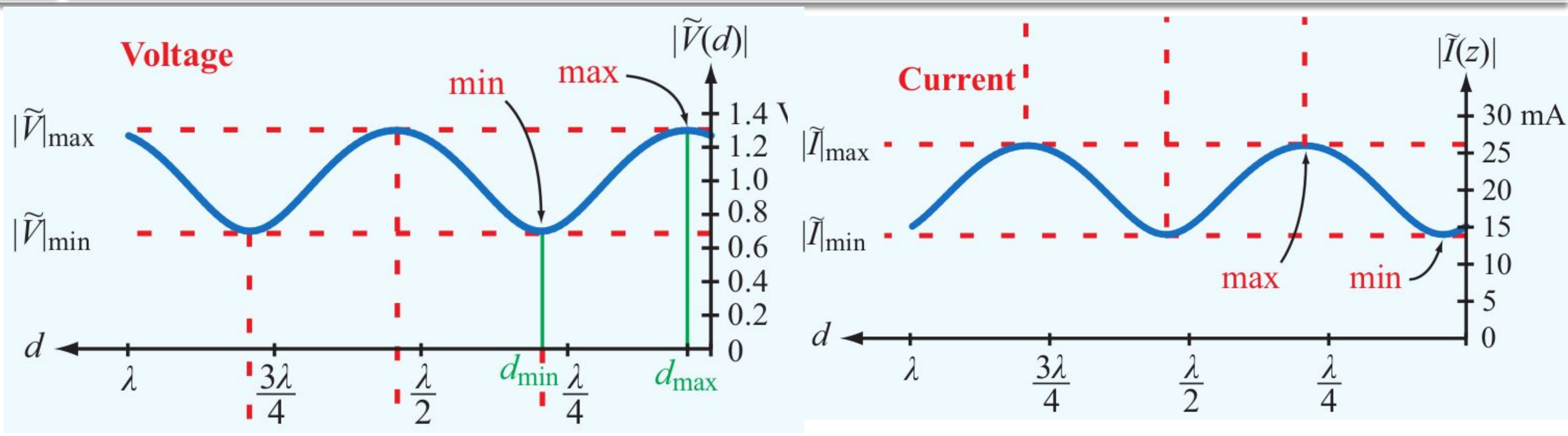


No reflection: no standing wave

Standing wave: zero voltage at short.

Standing wave: Max voltage at open.

2-6 Standing-Wave Pattern

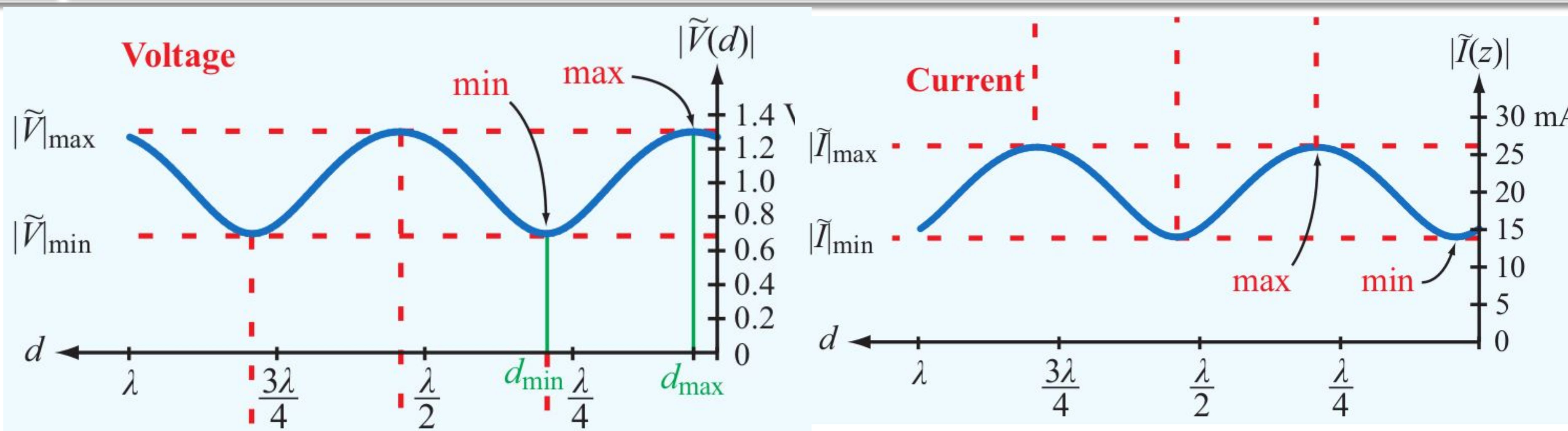


So, we've figured out the positions of the Max and Min on the line,

Next: the values of the Max and Min:

we know they occur at cos()=±1

2-6 Standing-Wave Pattern



Values of Max and Min? at cos()=±1

$$|\widetilde{V}(d)| = |V_0^+| \left[1 + |\Gamma|^2 + 2|\Gamma| \cos(2\beta d - \theta_r)\right]^{1/2}. \quad (2.66)$$

$$|\widetilde{V}|_{\max} = |V_0^+| \left[1 + |\Gamma|^2 + 2|\Gamma|\right]^{1/2} = |V_0^+| \left[(1 + |\Gamma|)^2\right]^{1/2} = |V_0^+| [1 + |\Gamma|]$$

$$|\widetilde{V}|_{\min} = |V_0^+| \left[1 + |\Gamma|^2 - 2|\Gamma|\right]^{1/2} = |V_0^+| \left[(1 - |\Gamma|)^2\right]^{1/2} = |V_0^+| [1 - |\Gamma|]$$

2-6 Standing-Wave Pattern

Define: Voltage Standing Wave Ratio (VSWR)

$$S = \frac{|\widetilde{V}|_{\max}}{|\widetilde{V}|_{\min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \qquad \text{(dimensionless)}$$

Special Cases:

Matched Load: $|\Gamma|=0$, so $S=1$
Short, Open, or purely reactive load: $|\Gamma|=1$, so $S=\infty$

Range of S: 1 to ∞

2-6 Standing-Wave Pattern

VDefine: oltage Standing Wave Ratio (VSWR)

$$S = \frac{|\widetilde{V}|_{\max}}{|\widetilde{V}|_{\min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \quad \text{(dimensionless)}$$

Reactive refers to the imaginary part of an impedance:
Purely Reactive is L or C: no R

Special Cases:

Matched Load: $|\Gamma|=0$, so $S=1$
Short, Open, or purely reactive load: $|\Gamma|=1$, so $S=\infty$

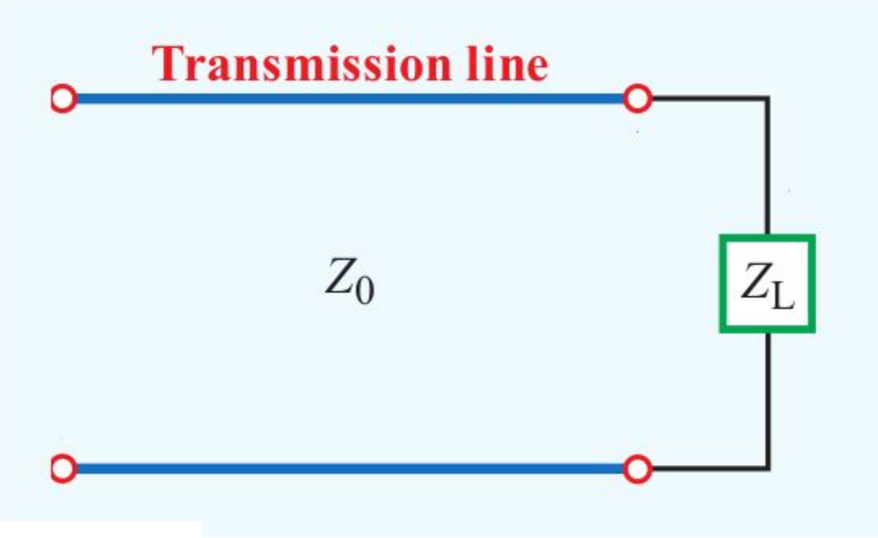
Range of S: 1 to ∞

Example 2-5 Standing-Wave Ratio

Given: $Z_0 = 50\ \Omega$

$Z_L = 100 + j50\ \Omega$

Find: Γ (in polar format), S



Solution:

$$\Gamma = \frac{z_L - 1}{z_L + 1}$$

$$\Gamma = \frac{2 + j1 - 1}{2 + j1 + 1}$$

$$\Gamma = \frac{1 + j1}{3 + j1}$$

Example 2-5 Standing-Wave Ratio

Solution:

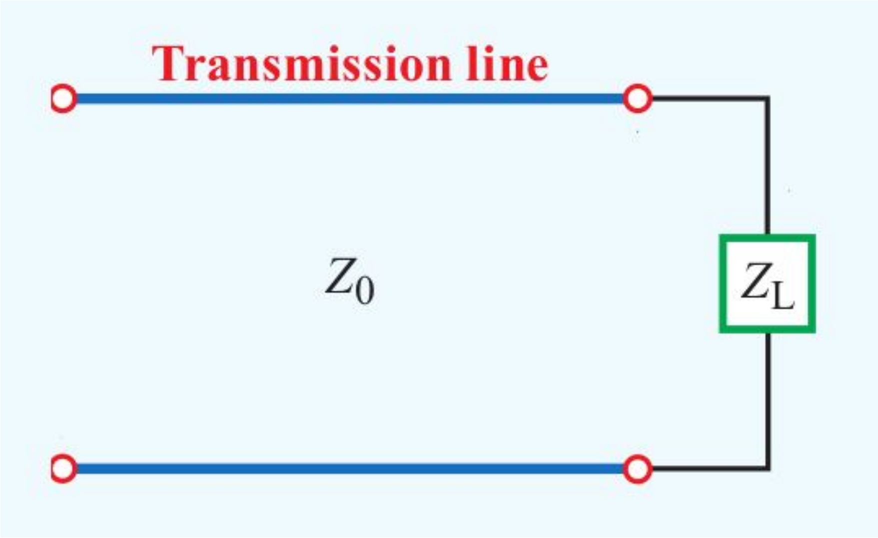
$$\Gamma = \frac{1 + j1}{3 + j1}$$

$$\Gamma = \frac{\sqrt{2}e^{j\tan^{-1}(1)}}{\sqrt{3^2 + 1}e^{j\tan^{-1}(1/3)}}$$

$$\Gamma = \frac{1.41e^{j45^\circ}}{3.16e^{j18.4^\circ}}$$

$$\Gamma = 0.446e^{j(45^\circ - 18.4^\circ)}$$

$$\Gamma = 0.446e^{j26.6^\circ}$$



Example 2-5 Standing-Wave Ratio

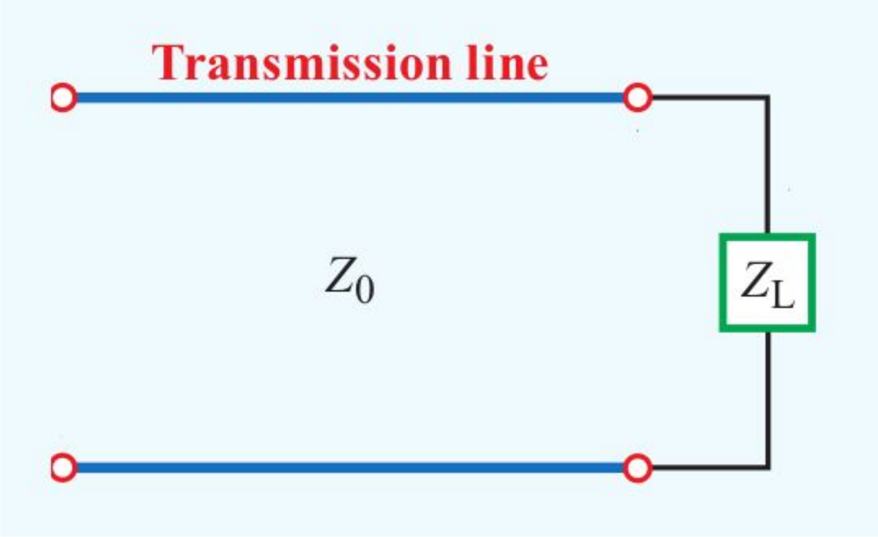
Solution:

$$\Gamma = 0.446e^{j26.6^\circ}$$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$S = \frac{1 + 0.446}{1 - 0.446}$$

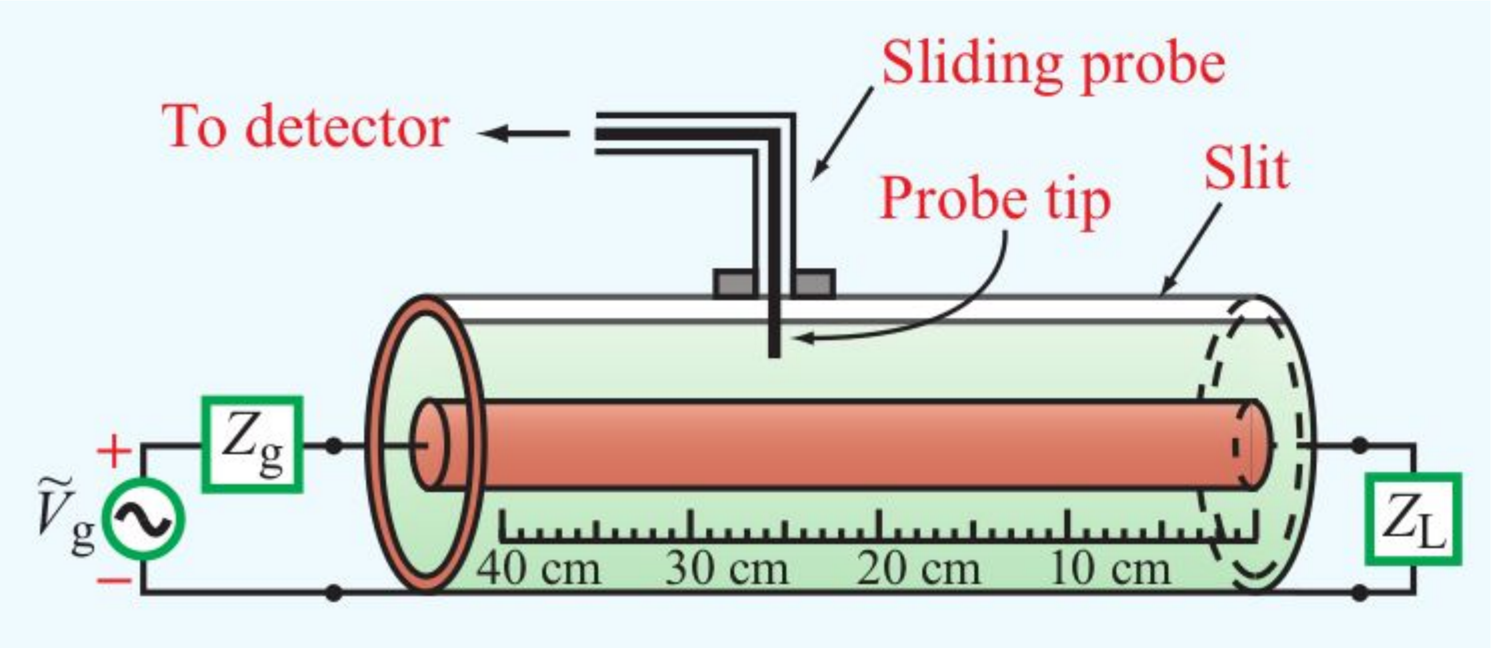
$$S = 2.6$$



Example 2-6 Slotted Line

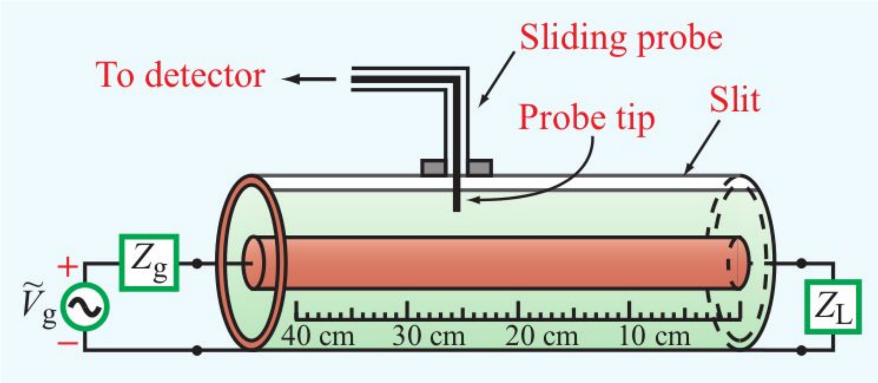
A slotted line can be used to measure the positions and magnitudes of the voltage min and max.

These can then be used to determine Z_L



Example 2-6 Slotted Line

Given: Z_0 = 50 Ω, S = 3
1st Vmin at d = 12cm
dist betw V mins=30cm



Find: Z_L (rectangular format)

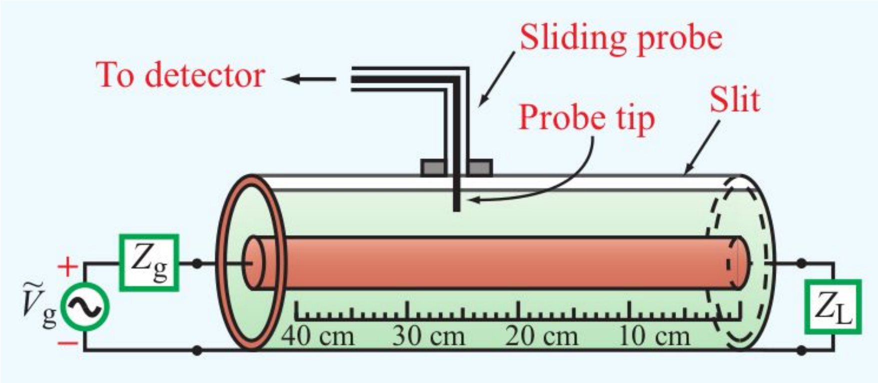
Solution:

$$d_{min} = 12\text{cm}$$

$$\lambda/2 = 30\text{cm}, \quad \text{so: } \lambda = 60\text{cm} = 0.6\text{m}$$

Example 2-6 Slotted Line

Given: Z_0 = 50 Ω, S = 3
1st Vmin at d = 12cm
dist betw V mins=30cm



Find: Z_L (rectangular format)

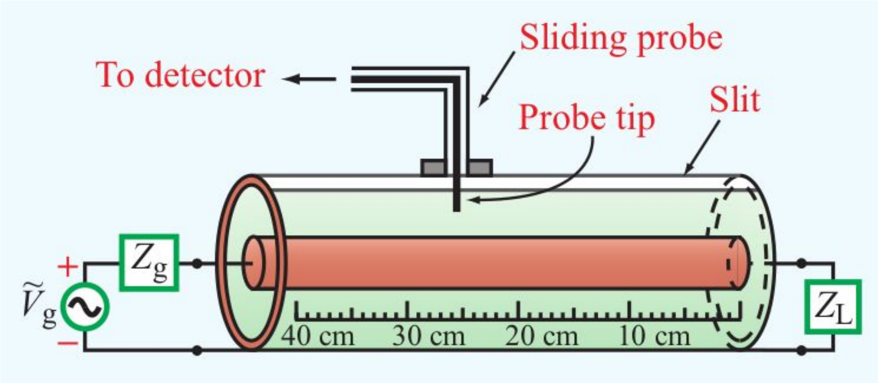
Solution:

We can use this equation to get Z_L:

$$Z_L = Z_0 \left[\frac{1+\Gamma}{1-\Gamma}\right]$$

So, need to find Γ first.

Example 2-6 Slotted Line



Solution:

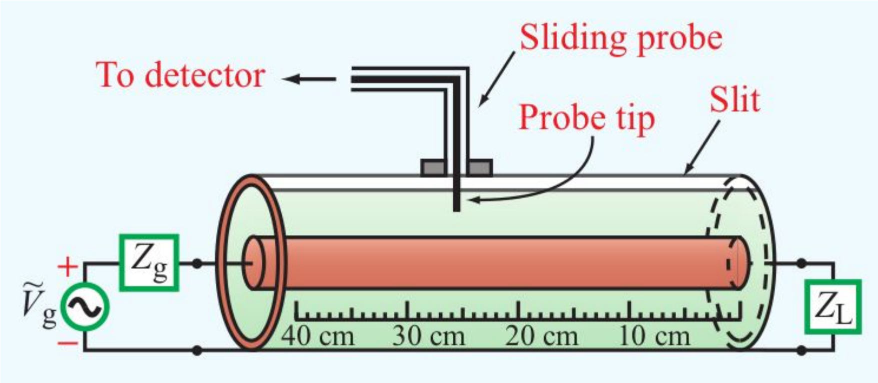
Can get |Γ| using *S*:

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Get phase, θ_r, from:

$$2\beta d_{\min} - \theta_r = (2n + 1)\pi$$

Example 2-6 Slotted Line



Solution:

Given first Vmin distance:

d_{min} = 12cm = 0.12m

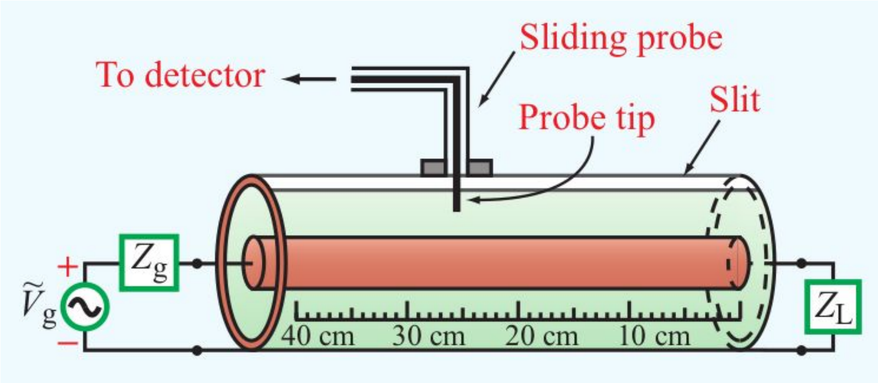
Given min-to-min distance:

$\lambda/2$ = 30cm, so: λ = 60cm = 0.6m

Need β to get θ_r:

$\beta = 2\pi / \lambda = 2\pi / 0.6\text{m} = 10.47$ rad/m

Example 2-6 Slotted Line



Solution:

determine an eqn for |Γ|(*S*):

$$S = \frac{1+|\Gamma|}{1-|\Gamma|}$$

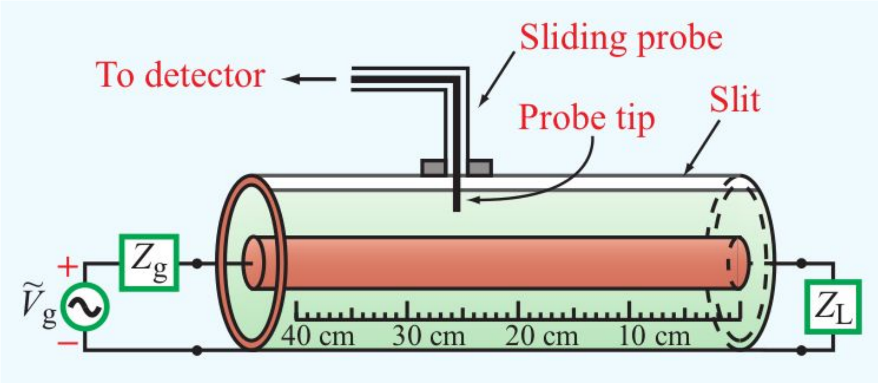
$$S(1-|\Gamma|) = 1+|\Gamma|$$

$$|\Gamma|(-S-1) = 1-S$$

$$|\Gamma| = \frac{1-S}{-1-S}$$

$$|\Gamma| = \frac{S-1}{S+1}$$

Example 2-6 Slotted Line



Solution:

$$|\Gamma| = \frac{S-1}{S+1}$$

$$|\Gamma| = \frac{3-1}{3+1}$$

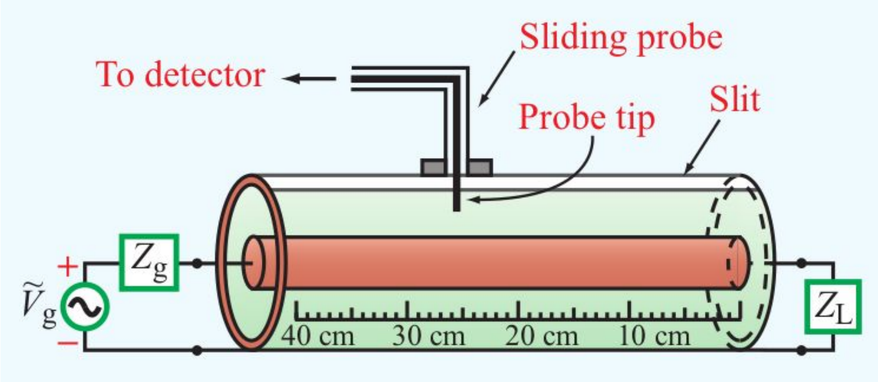
$$|\Gamma| = 0.5$$

we know for d_{min}: $2\beta d_{\min} - \theta_r = (2n+1)\pi$

so for the first minimum: $2\beta d_{\min} - \theta_r = \pi$

Example 2-6 Slotted Line

Solution:



$$2\beta d_{\min} - \theta_r = \pi$$

hence:

$$\theta_r = 2\beta d_{\min} - \pi$$

$$\theta_r = 2(10.47 \text{ rad/m})(0.12 \text{ m}) - \pi$$

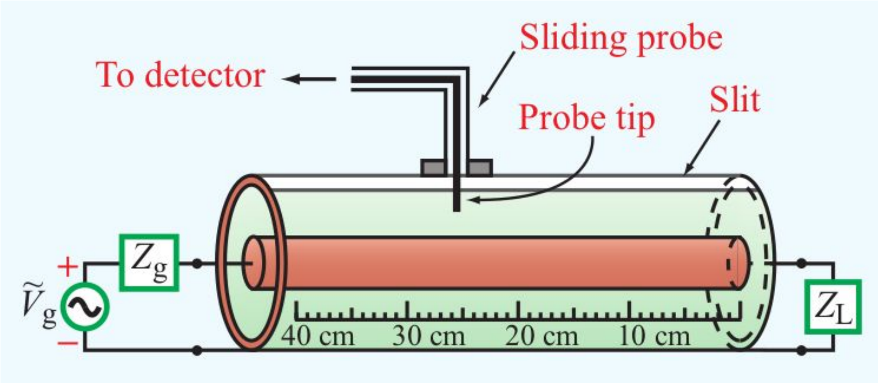
$$\theta_r = 2.5128 \text{ rad} - 3.1415 \text{ rad}$$

$$\theta_r = -0.63 \text{ rad}$$

$$\theta_r = -36^\circ$$

Example 2-6 Slotted Line

Solution:



$$\Gamma = |\Gamma| e^{j\theta_r}$$

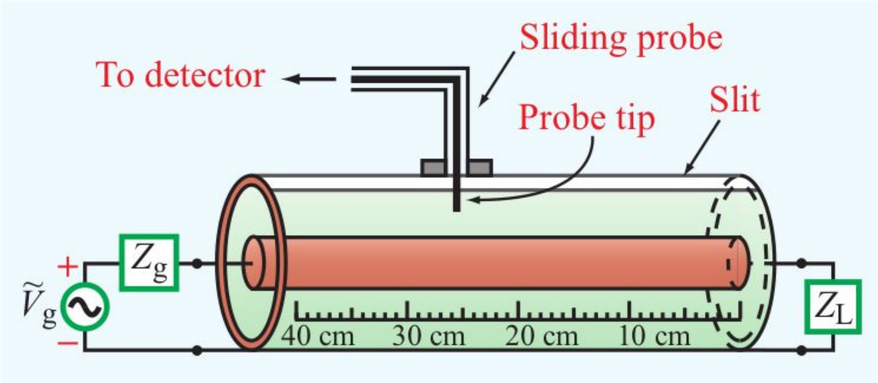
$$\Gamma = 0.5 e^{-j36°}$$

$$\Gamma = 0.5\cos(-36°) + j0.5\sin(-36°)$$

$$\Gamma = 0.405 - j0.294$$

Example 2-6 Slotted Line

Solution:



$$Z_L = Z_0 \left[\frac{1+\Gamma}{1-\Gamma}\right]$$

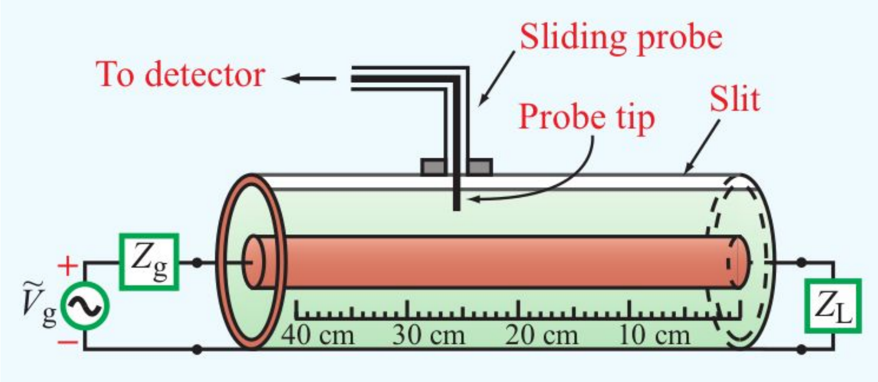
$$Z_L = (50\,\Omega)\left[\frac{1+0.405-j0.294}{1-(0.405-j0.294)}\right]$$

$$Z_L = (50\,\Omega)\left[\frac{1.405-j0.294}{0.595+j0.294}\right]$$

$$Z_L = (50\,\Omega)\left[\frac{\sqrt{1.405^2+0.294^2}e^{j\tan^{-1}(-0.294/1.405)}}{\sqrt{0.595^2+0.294^2}e^{j\tan^{-1}(0.294/0.595)}}\right]$$

Example 2-6 Slotted Line

Solution:



$$Z_L = (50\,\Omega)\left[\frac{1.44e^{-j11.82^\circ}}{0.664e^{j26.29^\circ}}\right]$$

$$Z_L = (50\,\Omega)\left[2.17e^{-j(11.82^\circ+26.29^\circ)}\right]$$

$$Z_L = (108.43\,\Omega)e^{-j38.11^\circ}$$

$$Z_L = (108.43\,\Omega)\cos(-38.11^\circ) + j(108.43\,\Omega)\sin(-38.11^\circ)$$

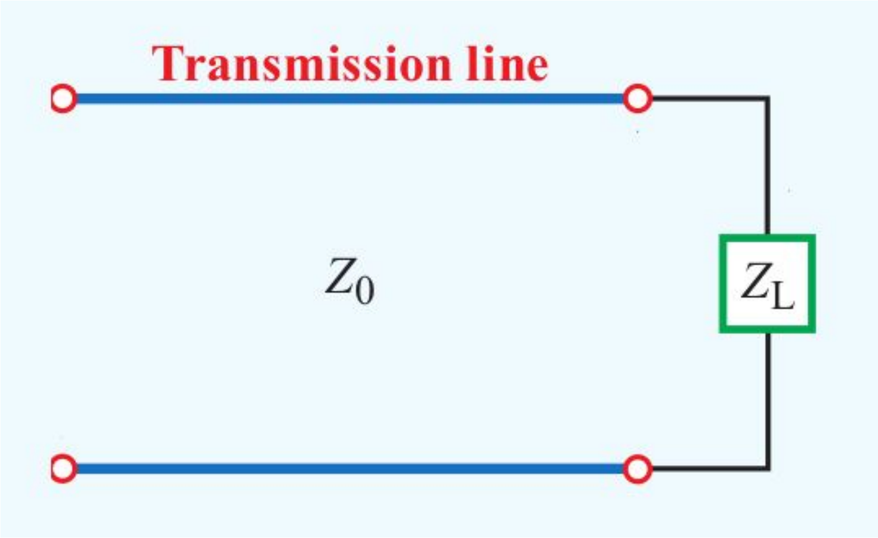
$$\boxed{Z_L = 85.31 - j66.9\,\Omega}$$

Exercise 2-12 Maxima/Minima

Given: $\Gamma = 0.5 \angle -60^\circ$,

$\lambda = 24$ cm

Find: distances of the voltage max and min nearest the load.



Solution:

We know:

$$2\beta d_{\max} - \theta_r = 2n\pi,$$

$$2\beta d_{\min} - \theta_r = (2n+1)\pi$$

need to find β

Exercise 2-12 Maxima/Minima

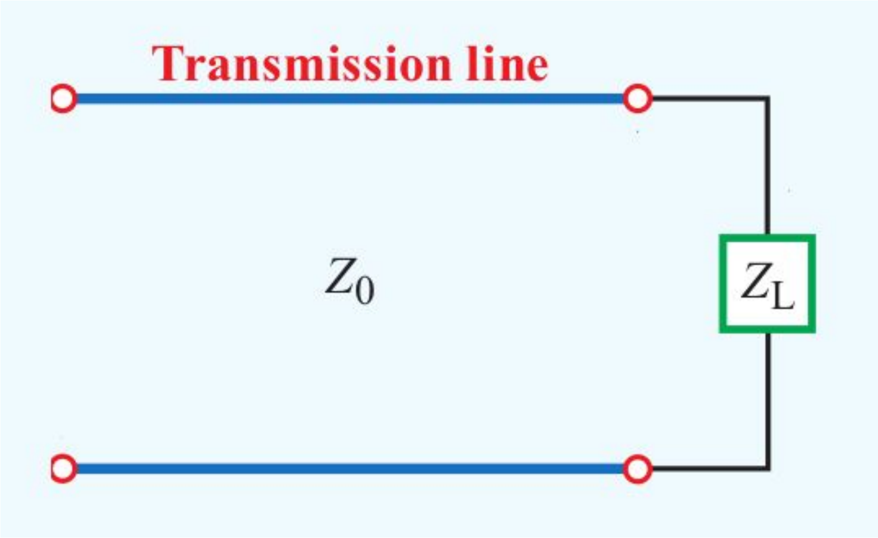
Solution:

$\beta = 2\pi / \lambda = 2\pi / 0.24\text{m}$

$\beta = 26.18$ rad/m

$\theta_r = -60^o = -\pi/3$

$\theta_r = -1.047$ rad



Exercise 2-12 Maxima/Minima

Solution:

$$2\beta d_{\mathrm{max}} - \theta_{\mathrm{r}} = 2n\pi$$

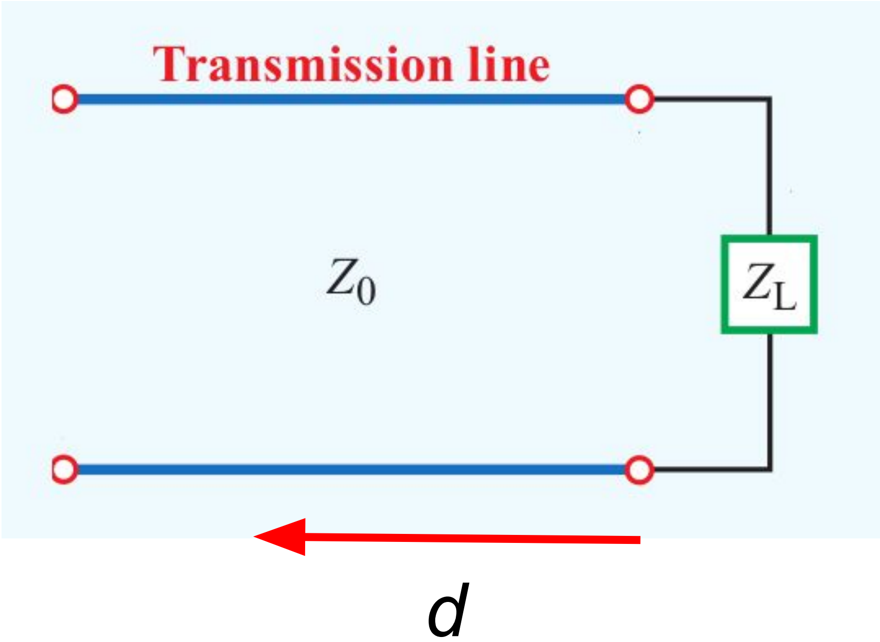
$$2\beta d_{\mathrm{max}} = 2n\pi + \theta_{\mathrm{r}}$$

$$d_{\mathrm{max}} = \frac{2n\pi + \theta_{\mathrm{r}}}{2\beta}$$

$$d_{\mathrm{max}} = \frac{0 + \theta_{\mathrm{r}}}{2\beta}$$

$$d_{\mathrm{max}} = \frac{-1.047\,\mathrm{rad}}{2(26.18\,\mathrm{rad/m})}$$

$$d_{\mathrm{max}} = -0.2\,\mathrm{m}$$



But wait!
d can't be negative!

Exercise 2-12 Maxima/Minima

Solution:

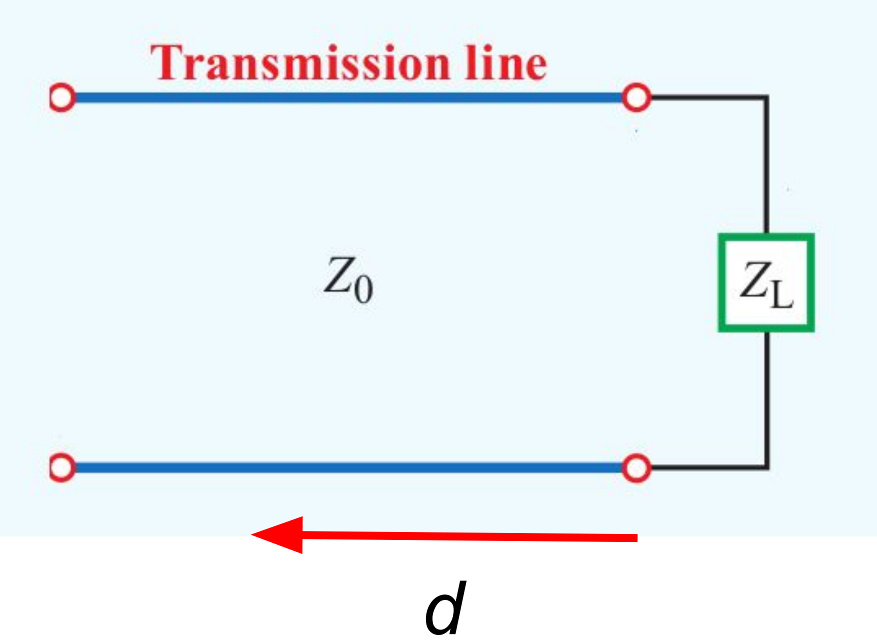
Try again, with n=1:

$$d_{\max} = \frac{2\pi + \theta_r}{2\beta}$$

$$d_{\max} = \frac{6.283\,\text{rad} - 1.047\,\text{rad}}{2(26.18\,\text{rad/m})}$$

$$d_{\max} = \frac{5.236\,\text{rad}}{2(26.18\,\text{rad/m})}$$

$$d_{\max} = 0.1\,\text{m}$$



Exercise 2-12 Maxima/Minima

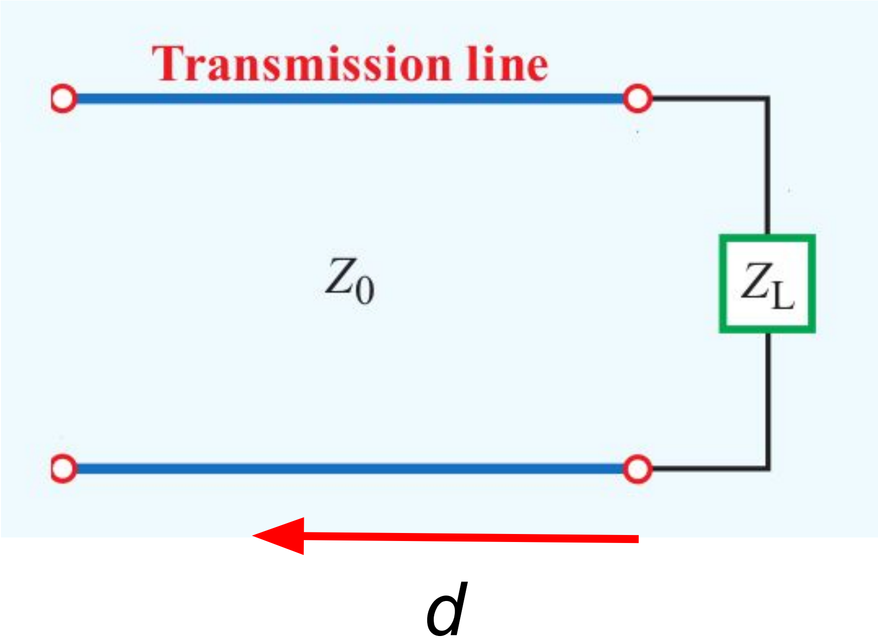
Solution:

$$2\beta d_{\min} - \theta_r = (2n+1)\pi$$

$$2\beta d_{\min} = (2n+1)\pi + \theta_r$$

$$d_{\min} = \frac{(2n+1)\pi + \theta_r}{2\beta}$$

$$d_{\min} = \frac{\pi + \theta_r}{2\beta}$$



Exercise 2-12 Maxima/Minima

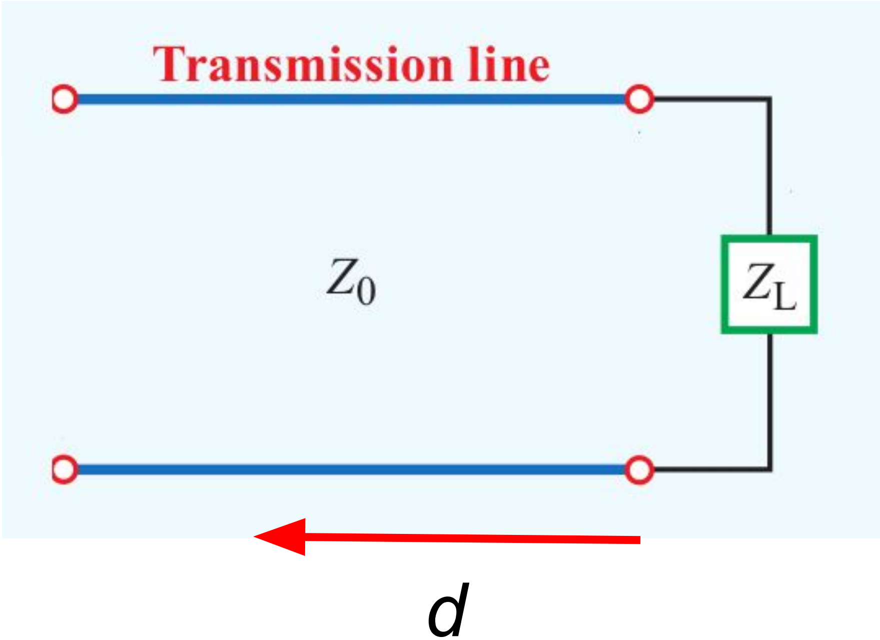
Solution:

$$d_{\min} = \frac{\pi + \theta_r}{2\beta}$$

$$d_{\min} = \frac{3.14159 + -1.047\,\text{rad}}{2(26.18\,\text{rad/m})}$$

$$d_{\min} = \frac{2.0946\,\text{rad}}{2(26.18\,\text{rad/m})}$$

$$\boxed{d_{\min} = 0.04\,\text{m}}$$



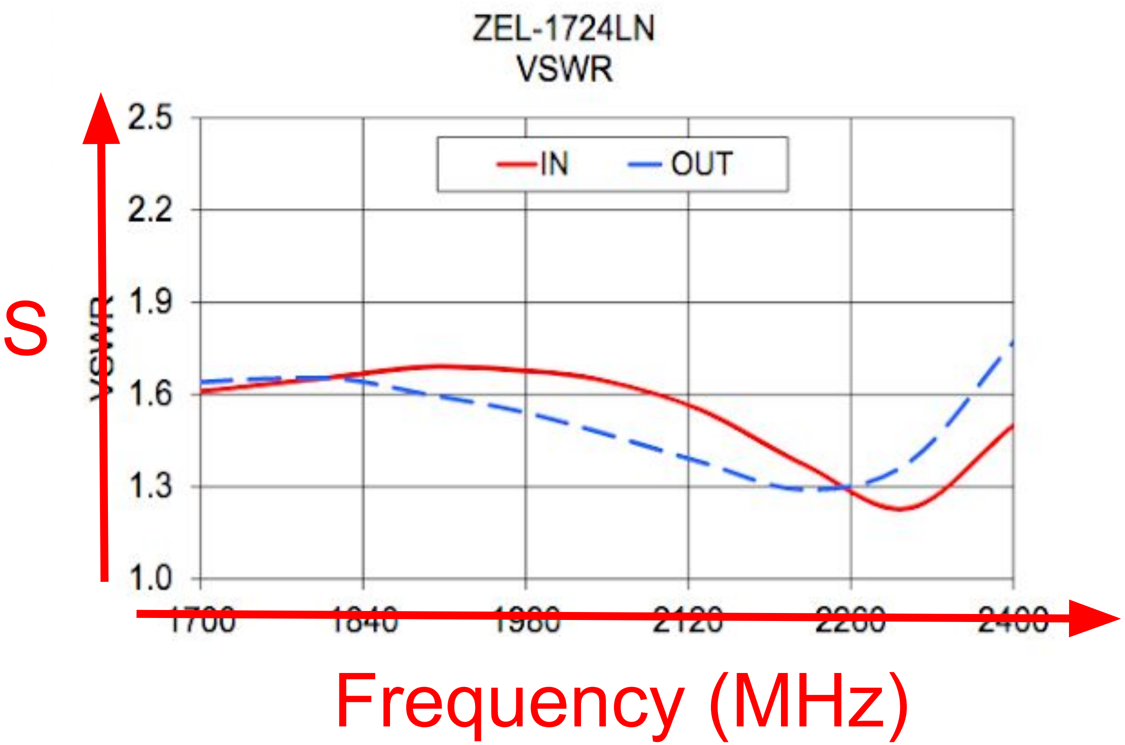
2-6 VSWR of a Real Amplifier

Coaxial

Low Noise Amplifier

ZEL-1724LN+

50Ω 1700 to 2400 MHz



Generic photo used for illustration purposes only

Case Style: EEE132

Connectors	Model
SMA	ZEL-1724LN+

+RoHS Compliant
The +Suffix identifies RoHS Compliance. See our web site for RoHS Compliance methodologies and qualifications

Shows how well matched across the frequency band

Homework

Homework 5 is due tomorrow at midnight.

submit to gradescope via the canvas site.

Next Time

Section 2-7:

Lossless Line: Wave Impedance