# EECS 230
## *ENGINEERING ELECTROMAGNETICS*
*Leland Pierce*

# Transmission Lines 2

University of Michigan - Fall 2024

# Chapter 2 Overview

**What is a transmission line?**

Why study transmission lines?

model

diffeq

solve diffeq

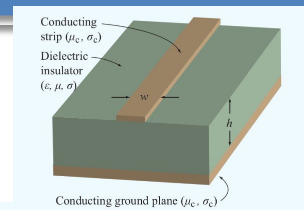
wave propagation

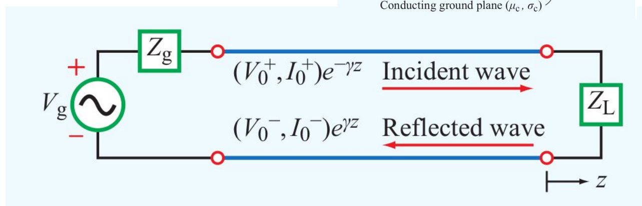
lossless xmission lir

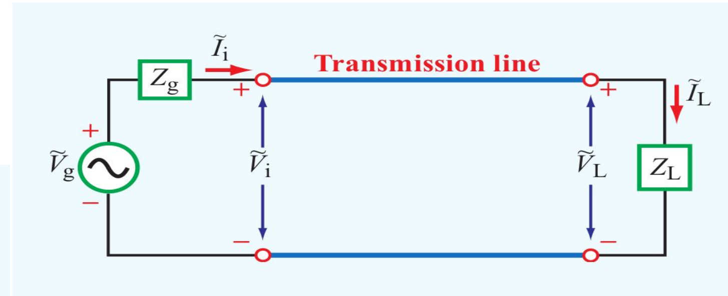
- microstrip lines
- reflections
- standing waves
- impedance
- short, open
- matching
- power flow

smith chart

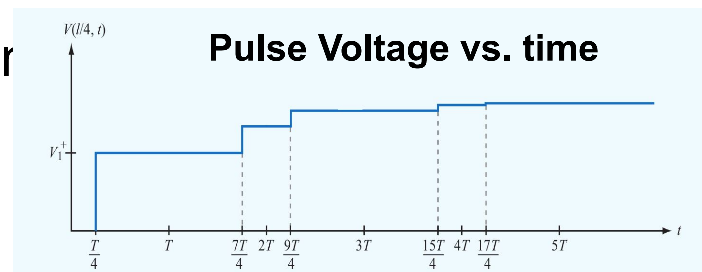
transients

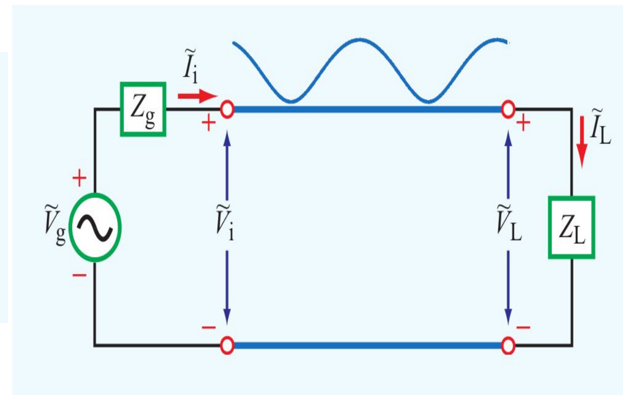


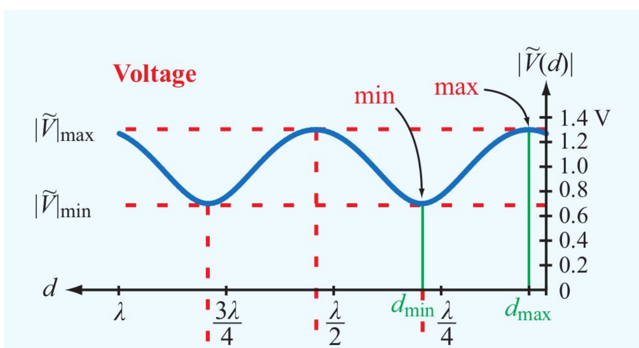



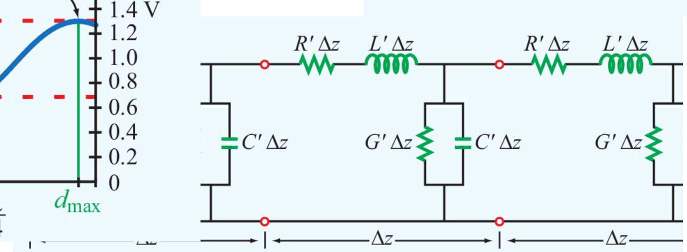

Typical High-Frequency Circuit

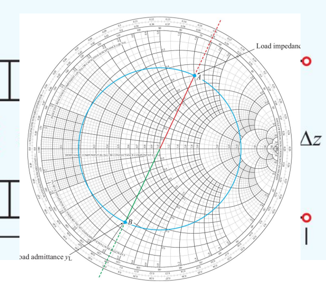



Waves on line: old methods don't work



# Chapter 2 Applications

**Radios:** cell phones, internet, broadcast radio/tv, GPS

**Radar:** civilian aeronautical, military, vehicle collision sensors, weather, remote sensing

**Heating:** Microwave Ovens, Cancer Treatment

# Today's Lecture Coverage

**Review Sections 2-1 and 2-2 of the book:**

**2-1**: What is a transmission line?
Why study transmission lines?
**2-2**: Lumped-Element Model

**Sections 2-3 through 2-5 of the book:**

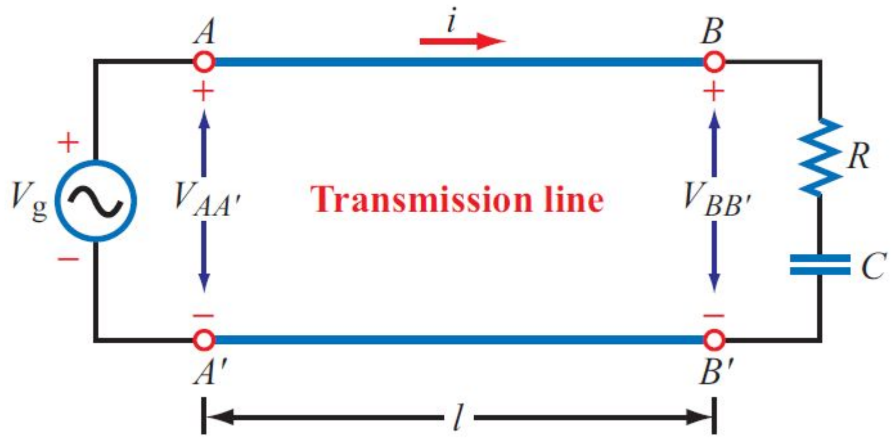
**2-3**: Governing Differential Eqns
**2-4**: Solve the Differential Equations
Properties of the solution: wave propagation
**2-5**: Lossless Microstrip Line

# Chapter 2 Review

- A transmission line connects a generator to a load.

# Chapter 2 Review

## Phase Delay due to length of transmission line:



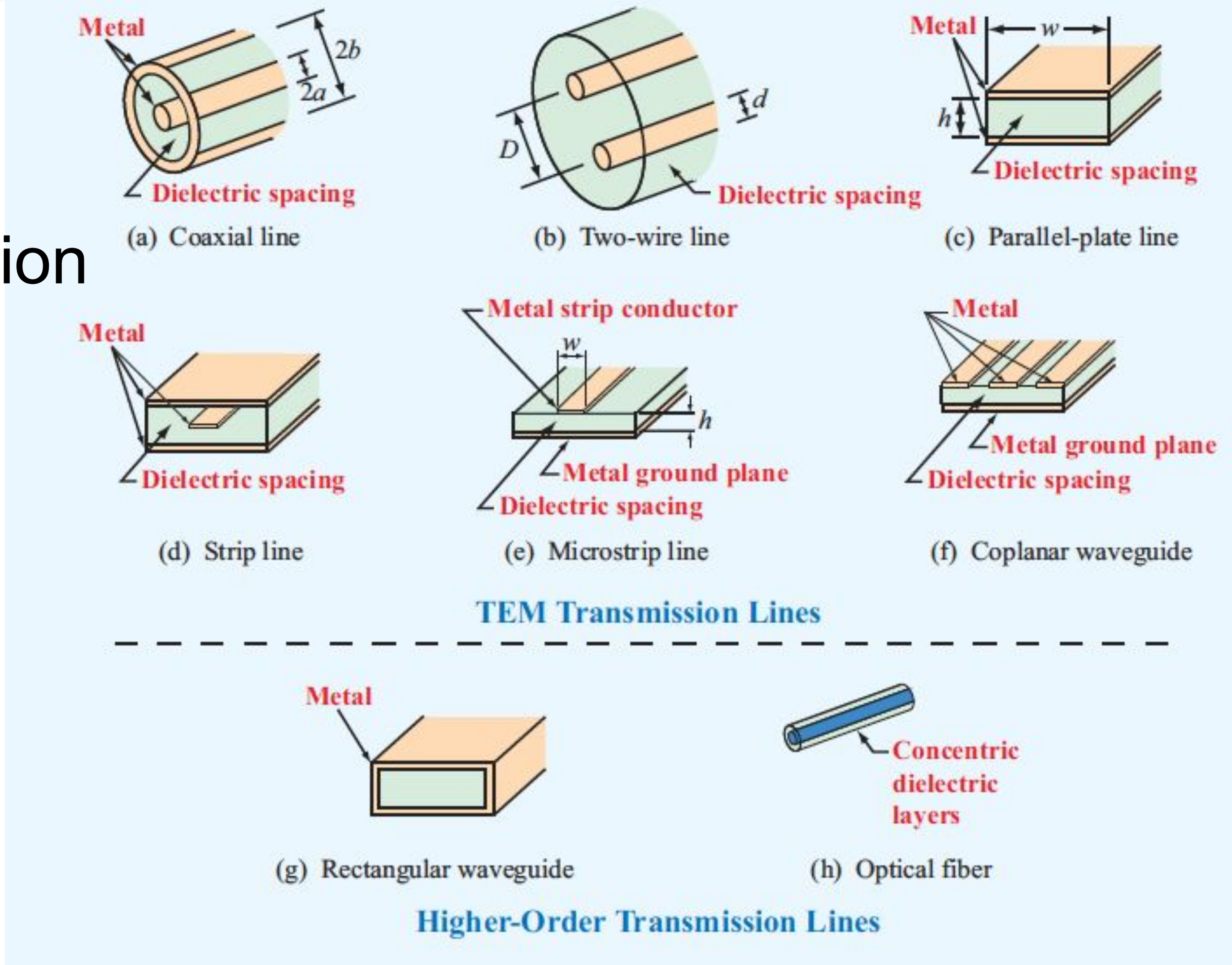

$$\phi_0 = \frac{\omega l}{c} = \frac{2\pi f l}{c} = 2\pi \frac{l}{\lambda} \quad \text{radians.}$$

$l/\lambda \lesssim 0.01$: Can ignore transmission-line effects

$l/\lambda \gtrsim 0.01$: Must deal with phase shift, and other effects...

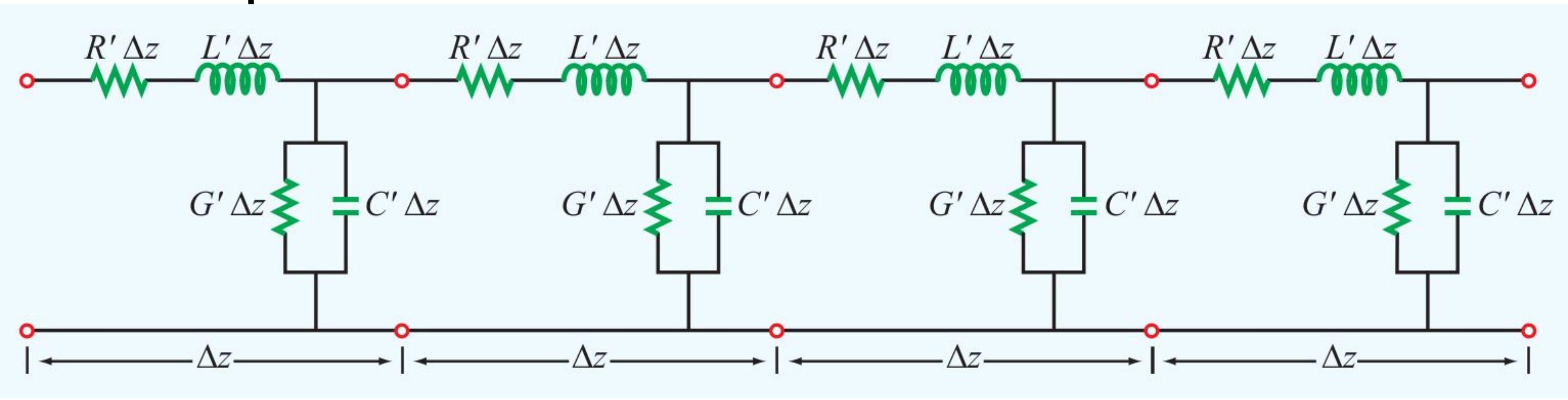
# Chapter 2 Review

Different geometries for transmission lines



# Chapter 2 Review

Lumped-Element Model:




**All parameters are "per unit length":**

**R': Combined Resistance of BOTH conductors: □/m**

**L': Combined Inductance of BOTH conductors, H/m**

**G': Conductance of insulation**
**between inner and outer conductor, S/m**

**C': Capacitance**
**between inner and outer conductors, F/m**

# Chapter 2 Review

Lumped-Element Values: geometry/materials/freq

**Table 2-1** **Transmission-line parameters $R'$, $L'$, $G'$, and $C'$ for three types of lines.**

| Parameter | Coaxial | Two-Wire | Parallel-Plate | Unit |
|---|---|---|---|---|
| $R'$ | $\frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$ | $\frac{2R_s}{\pi d}$ | $\frac{2R_s}{w}$ | Ω/m |
| $L'$ | $\frac{\mu}{2\pi}\ln(b/a)$ | $\frac{\mu}{\pi}\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]$ | $\frac{\mu h}{w}$ | H/m |
| $G'$ | $\frac{2\pi\sigma}{\ln(b/a)}$ | $\frac{\pi\sigma}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$ | $\frac{\sigma w}{h}$ | S/m |
| $C'$ | $\frac{2\pi\varepsilon}{\ln(b/a)}$ | $\frac{\pi\varepsilon}{\ln\left[(D/d)+\sqrt{(D/d)^2-1}\right]}$ | $\frac{\varepsilon w}{h}$ | F/m |

$$R_s = \sqrt{\pi f \mu_c / \sigma_c}$$

# Chapter 2 Review

Lumped-Element Values: geometry/materials/freq

Notice that for a coax:

$$L'C' = \left[\frac{\mu}{2\pi}\ln(b/a)\right]\left[\frac{2\pi\epsilon}{\ln(b/a)}\right]$$

$$\boxed{L'C' = \mu\epsilon}$$

This turns out to be true for all the transmission-lines we study.

# Chapter 2 Review

Lumped-Element Values: geometry/materials/freq

Also notice that for a coax:

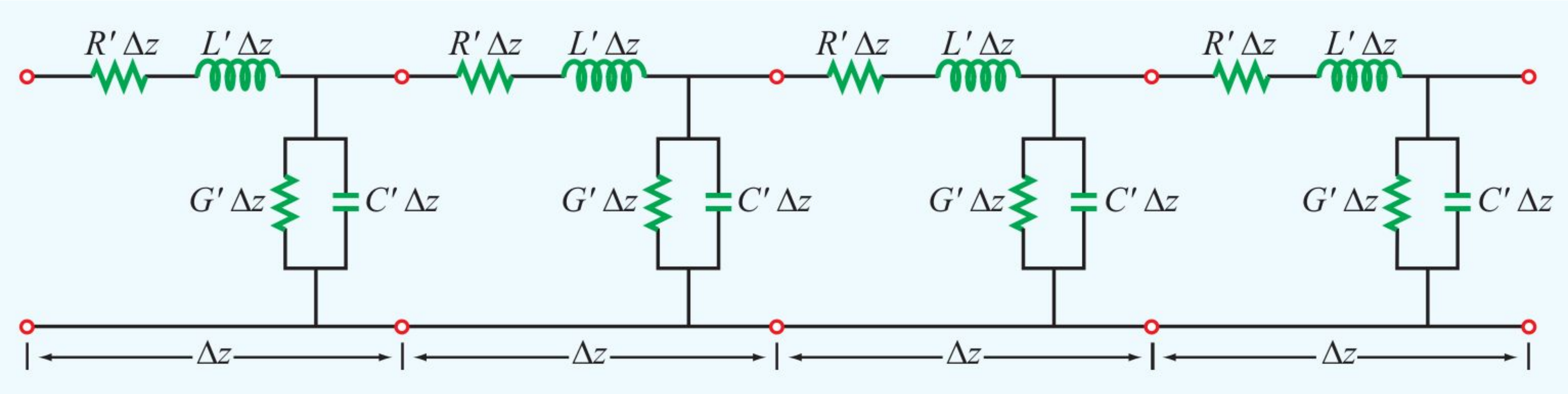
$$\frac{G'}{C'} = \frac{\left[\frac{2\pi\sigma}{\ln(b/a)}\right]}{\left[\frac{2\pi\epsilon}{\ln(b/a)}\right]}$$

$$\boxed{\frac{G'}{C'} = \frac{\sigma}{\epsilon}}$$

This also turns out to be true for all the transmission-lines we study.

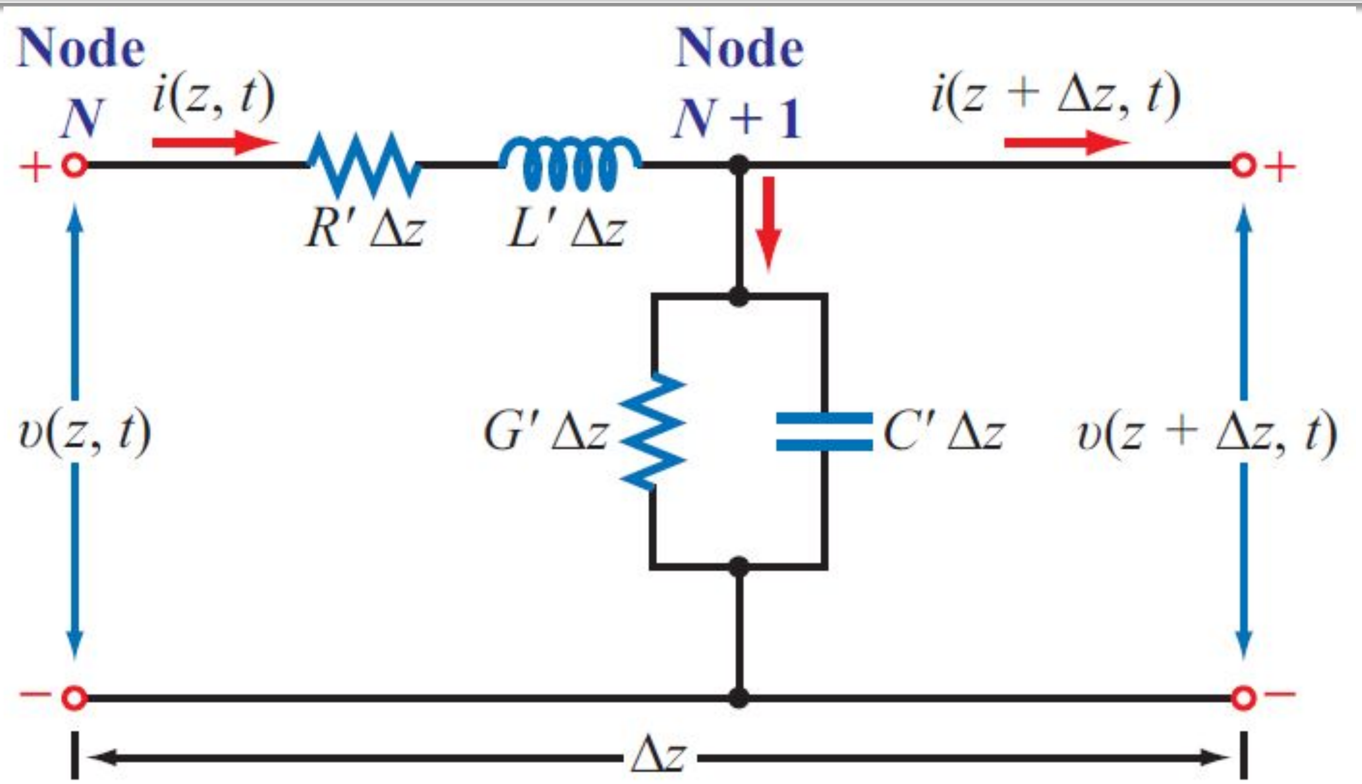
# 2-3 Transmission-Line Equations

Using our model:




Let's derive the governing differential equations for the transmission line.

# 2-3 Transmission-Line Equations




**Apply KVL clockwise around outer loop:**

$$-v(z,t) + R'\Delta z\, i(z,t) + L'\Delta z\,\frac{\partial i(z,t)}{\partial t} + v(z+\Delta z,t) = 0$$

# 2-3 Transmission-Line Equations

$$-v(z,t) + R'\Delta z\, i(z,t) + L'\Delta z\, \frac{\partial i(z,t)}{\partial t} + v(z+\Delta z,t) = 0$$

Divide by $\Delta z$ and rearrange:

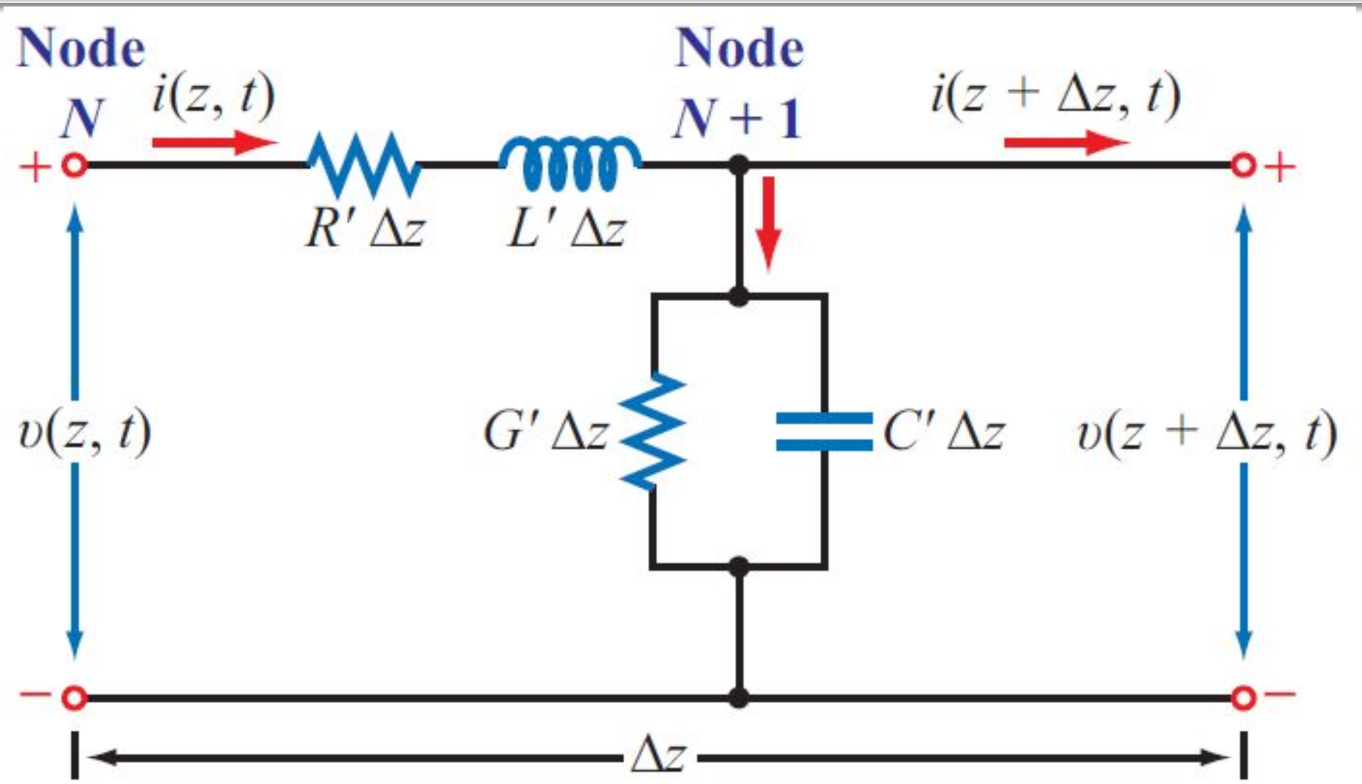
$$-\left[\frac{v(z+\Delta z,t) - v(z,t)}{\Delta z}\right] = R'\, i(z,t) + L'\, \frac{\partial i(z,t)}{\partial t}$$

Take the limit, $\Delta z \to 0$:

$$-\frac{\partial v(z,t)}{\partial z} = R'\, i(z,t) + L'\, \frac{\partial i(z,t)}{\partial t} \qquad (1)$$

Since we have 2 unknowns, $v$ and $i$, we need another equation.

# 2-3 Transmission-Line Equations




**Apply KCL at node N+1: "sum of the current into the node = 0"**

$$i(z,t) - G'\Delta z\, v(z+\Delta z,t) - C'\Delta z\, \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0$$

# 2-3 Transmission-Line Equations

$$i(z,t) - G'\Delta z\, v(z+\Delta z,t) - C'\Delta z\, \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0$$

Divide by $\Delta z$ and rearrange, and take the limit, $\Delta z \to 0$:

$$-\frac{\partial i(z,t)}{\partial z} = G'\, v(z,t) + C'\, \frac{\partial v(z,t)}{\partial t} \qquad (2)$$

For sinusoidal signals, convert to phasor notation, using:

$$\upsilon(z,t) = \mathfrak{Re}[\widetilde{V}(z)\, e^{j\omega t}],$$

$$i(z,t) = \mathfrak{Re}[\widetilde{I}(z)\, e^{j\omega t}],$$

# 2-3 Transmission-Line Equations

These are the governing differential equations for a transmission line in phasor form:

$$-\frac{d\widetilde{V}(z)}{dz} = (R' + j\omega L')\,\widetilde{I}(z),$$

$$-\frac{d\widetilde{I}(z)}{dz} = (G' + j\omega C')\,\widetilde{V}(z).$$

**(telegrapher's equations in phasor form)**

# 2-4 Transmission-Line Solutions

To solve these coupled differential equations, take the *z*-derivative of the first:

$$-\frac{d^2\widetilde{V}(z)}{dz^2} = (R' + j\omega L')\,\frac{d\widetilde{I}(z)}{dz}$$

Substitute in the equation for the derivative of the current:

$$-\frac{d^2\widetilde{V}(z)}{dz^2} = (R' + j\omega L')\left[-(G' + j\omega C')\,\widetilde{V}(z)\right]$$

$$\frac{d^2\widetilde{V}(z)}{dz^2} = (R' + j\omega L')\,(G' + j\omega C')\,\widetilde{V}(z)$$

$$\frac{d^2\widetilde{V}(z)}{dz^2} - (R' + j\omega L')\,(G' + j\omega C')\,\widetilde{V}(z) = 0$$

# 2-4 Transmission-Line Solutions

Simplify:

$$\frac{d^2\widetilde{V}(z)}{dz^2} - \gamma^2\,\widetilde{V}(z) = 0,$$

**(wave equation for $\widetilde{V}(z)$)**

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}\,.$$

**(propagation constant)**

# 2-4 Transmission-Line Solutions

Going through the same process for the current:

$$\frac{d^2\tilde{I}(z)}{dz^2} - \gamma^2\,\tilde{I}(z) = 0.$$

**(wave equation for $\widetilde{I}(z)$)**

# 2-4 Transmission-Line Solutions

Complex propagation constant:

$$\gamma = \alpha + j\beta$$

Where:

$\gamma$: Units of 1/m

$\alpha$: Attenuation constant, units of Np/m (>0 in this class)

$\beta$: Phase constant, units of rad/m

Np and radians are both "dimensionless", and are the conventional "units" for these parameters.

# 2-4 Transmission-Line Solutions

A solution to the wave equation is:

$$\widetilde{V} = Ae^{-\gamma z}$$

To show this: plug into original equation:

$$\frac{d\widetilde{V}}{dz} - \gamma^2 \widetilde{V} = 0$$

$$\frac{d^2}{dz^2}\left(Ae^{-\gamma z}\right) - \gamma^2 Ae^{-\gamma z} = 0$$

$$\frac{d}{dz}\left(-\gamma Ae^{-\gamma z}\right) - \gamma^2 Ae^{-\gamma z} = 0$$

$$\gamma^2 Ae^{-\gamma z} - \gamma^2 Ae^{-\gamma z} = 0$$

# 2-4 Transmission-Line Solutions

Another solution is:

$$\widetilde{V} = Be^{+\gamma z}$$

which means that:

$$\widetilde{V} = Ae^{-\gamma z} + Be^{+\gamma z}$$

is also a solution.

# 2-4 Transmission-Line Solutions

So, the solutions of these wave equations are:

$$\widetilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \qquad \text{(V)},$$

$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \qquad \text{(A)}.$$

These represent the sum of waves traveling in both directions on the line.

Need to solve for the 4 unknowns.

# 2-4 Transmission-Line Solutions

Let's find a relationship between the voltages and currents. We know:

$$-\frac{d\widetilde{V}(z)}{dz} = (R' + j\omega L')\,\tilde{I}(z),$$

and:

$$\widetilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

which has *z*-derivative of:

$$\frac{d\widetilde{V}(z)}{dz} = -\gamma V_0^+ e^{-\gamma z} + \gamma V_0^- e^{\gamma z}$$

# 2-4 Transmission-Line Solutions

Plug into first equation:

$$\gamma V_0^+ e^{-\gamma z} - \gamma V_0^- e^{\gamma z} = (R' + j\omega L')\, \tilde{I}(z)$$

Solve for the current:

$$\tilde{I}(z) = \frac{\gamma}{R' + j\omega L'} [V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}].$$

Equate like terms from this equation:

$$\tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

# 2-4 Transmission-Line Solutions

So we get:

$$I_0^+ = \frac{\gamma}{R' + j\omega L'} V_0^+$$
$$I_0^- = -\frac{\gamma}{R' + j\omega L'} V_0^-$$

Remembering Ohm's Law: $I = V/R$, write as:

$$I_0^+ = \frac{V_0^+}{Z_0}$$
$$I_0^- = -\frac{V_0^-}{Z_0}$$

# 2-4 Transmission-Line Solutions

Where:

$$Z_0 = \frac{R' + j\omega L'}{\gamma}$$

which is a complex resistance (impedance), in Ohms. This is called the Characteristic Impedance

# 2-4 Transmission-Line Solutions

Another way to express it is:

$$Z_0 = \frac{R' + j\omega L'}{\gamma}$$

$$Z_0 = \frac{R' + j\omega L'}{\sqrt{(R' + j\omega L')(G' + j\omega C')}}$$

$$Z_0 = \sqrt{\frac{(R' + j\omega L')(R' + j\omega L')}{(R' + j\omega L')(G' + j\omega C')}}$$

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

# 2-4 Transmission-Line Solutions

Using this relationship we get:

$$\widetilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \qquad \text{(V)},$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z} \qquad \text{(A)}.$$

which now has only 2 unknowns.

# 2-4 Transmission-Line Solutions

Each of these unknowns is complex:

$$V_0^+ = |V_0^+| e^{j\phi^+}$$

$$V_0^- = |V_0^-| e^{j\phi^-}$$

# 2-4 Transmission-Line Solutions

Express the solution, so far, in the time-domain:

$$\nu(z,t) = \Re e\left\{\widetilde{V}(z)\, e^{j\omega t}\right\}$$

$$\nu(z,t) = \Re e\left\{\left(V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}\right)\, e^{j\omega t}\right\}$$

$$\nu(z,t) = \Re e\left\{\left(|V_0^+| e^{j\phi^+} e^{-\gamma z} + |V_0^-| e^{j\phi^-} e^{+\gamma z}\right)\, e^{j\omega t}\right\}$$

$$\nu(z,t) = \Re e\left\{\left(|V_0^+| e^{j\phi^+} e^{-(\alpha+j\beta)z} + |V_0^-| e^{j\phi^-} e^{+(\alpha+j\beta)z}\right)\, e^{j\omega t}\right\}$$

$$\nu(z,t) = \Re e\left\{|V_0^+| e^{-\alpha z} e^{j(\phi^+ - \beta z + \omega t)} + |V_0^-| e^{+\alpha z} e^{j(\phi^- + \beta z + \omega t)}\right\}$$

$$\boxed{\nu(z,t) = |V_0^+| e^{-\alpha z} \cos(\omega t - \beta z + \phi^+) + |V_0^-| e^{+\alpha z} \cos(\omega t + \beta z + \phi^-)}$$

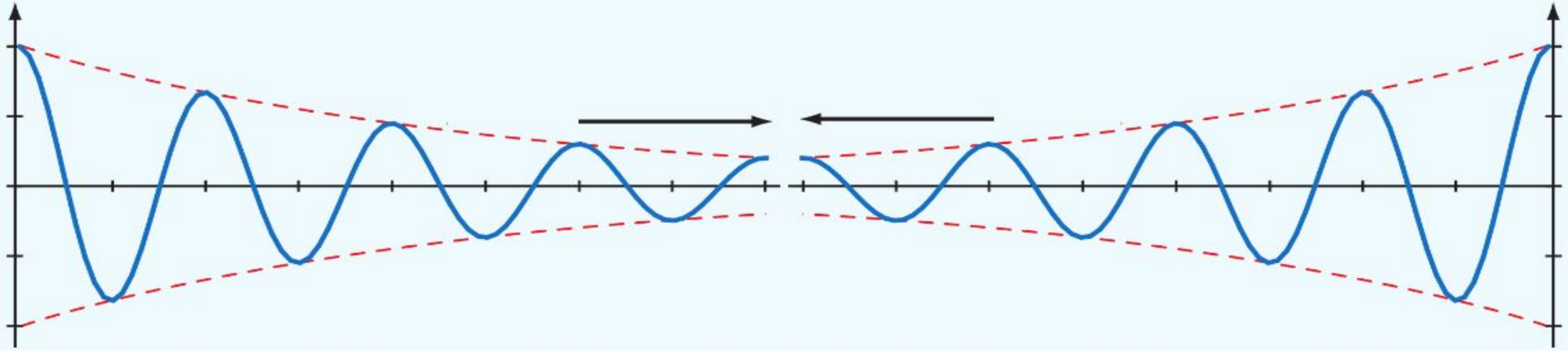
# 2-4 Transmission-Line Solutions

Express the solution, so far, in the time-domain:

$$\nu(z,t) = |V_0^+|e^{-\alpha z}\cos(\omega t - \beta z + \phi^+) + |V_0^-|e^{+\alpha z}\cos(\omega t + \beta z + \phi^-)$$

This is two travelling waves:

one moving in the +z direction    another in the -z direction



# 2-4 Transmission-Line Solutions

For the current, we use: $Z_0 = |Z_0| e^{j\phi_z}$

$$i(z,t) = \mathfrak{Re}(\widetilde{I}(z)\, e^{j\omega t}) = \mathfrak{Re}\left[\frac{|V_0^+|}{|Z_0|}\, e^{j\phi^+} e^{-j\phi_z} e^{j\omega t} e^{-(\alpha+j\beta)z} - \frac{|V_0^-|}{|Z_0|}\, e^{j\phi^-} e^{-j\phi_z} e^{j\omega t} e^{(\alpha+j\beta)z}\right],$$

which yields

$$i(z,t) = \frac{|V_0^+|}{|Z_0|}\, e^{-\alpha z} \cos(\omega t - \beta z + \phi^+ - \phi_z) - \frac{|V_0^-|}{|Z_0|}\, e^{+\alpha z} \cos(\omega t + \beta z + \phi^- - \phi_z) \qquad (2.35)$$

# 2-4 Transmission-Line Solutions

$$v(z,t) = |V_0^+| e^{-\alpha z} \cos(\omega t - \beta z + \phi^+) + |V_0^-| e^{+\alpha z} \cos(\omega t + \beta z + \phi^-) \quad (2.32)$$

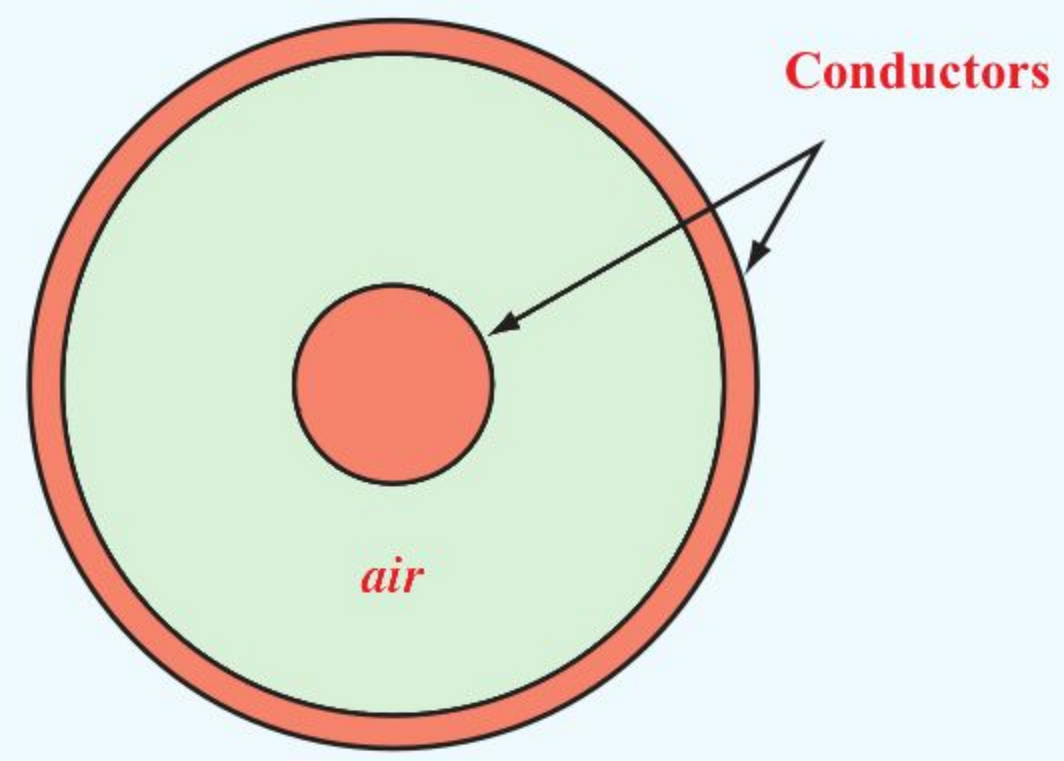
$$i(z,t) = \frac{|V_0^+|}{|Z_0|} e^{-\alpha z} \cos(\omega t - \beta z + \phi^+ - \phi_z) - \frac{|V_0^-|}{|Z_0|} e^{+\alpha z} \cos(\omega t + \beta z + \phi^- - \phi_z) \quad (2.35)$$

Current has an extra phase term due to $Z_0$,
Current magnitudes are basically *V/R*
Reflected *i* has opposite sign compared with $v$

# Example 2-1 Air Line

Given: Air separates the 2 conductors, so:
$G' = 0$
Conductors are very good: $R' = 0$

$Z_0 = 50\ \Omega$,
$\beta = 20$ rad/m,
$f = 700$ MHz




Find: $L'$, $C'$

Solution: Since we know $Z_0$ and $\beta$, use those equations

# Example 2-1 Air Line

Solution: Since we know $Z_0$ and $\beta$, use those equations

$$\beta = \Im\mathfrak{m}(\gamma)$$
$$= \Im\mathfrak{m}\left(\sqrt{(R' + j\omega L')(G' + j\omega C')}\right)$$

$$\beta = \Im\mathfrak{m}\left[\sqrt{(j\omega L')(j\omega C')}\right]$$
$$= \Im\mathfrak{m}\left(j\omega\sqrt{L'C'}\right)$$
$$= \omega\sqrt{L'C'}$$

# Example 2-1 Air Line

Solution: Since we know $Z_0$ and $\beta$, use those equations

$$Z_0 = \frac{R' + j\omega L'}{\gamma} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

$$Z_0 = \sqrt{\frac{j\omega L'}{j\omega C'}} = \sqrt{\frac{L'}{C'}}.$$

Notice that: $\beta / Z_0 = \dfrac{\omega\sqrt{L'C'}}{\sqrt{\frac{L'}{C'}}} = \omega C'$

# Example 2-1 Air Line

Solution: So: $\omega C' = \beta / Z_0$

$$C' = \frac{\beta}{\omega Z_0} = \frac{\beta}{2\pi f Z_0}$$

$$C' = \frac{20 \text{ rad/m}}{2\pi(700 \times 10^6 \text{ Hz})(50\,\Omega)}$$

$$C' = 9.09 \times 10^{-11} \text{ F/m}$$

$$\boxed{C' = 90.9 \text{ pF/m}}$$

# Example 2-1 Air Line

Solution: Given: $Z_0 = \sqrt{\frac{L'}{C'}}$

$$L' = Z_0^2 C'$$

$$= (50)^2 \times 90.9 \times 10^{-12}$$

$$= 2.27 \times 10^{-7} \text{ (H/m)}$$

$$L' = 227 \text{ (nH/m)}$$

# Exercise 2-4 Two-Wire Air Line

Given: $R' = 0.404$ mΩ/m

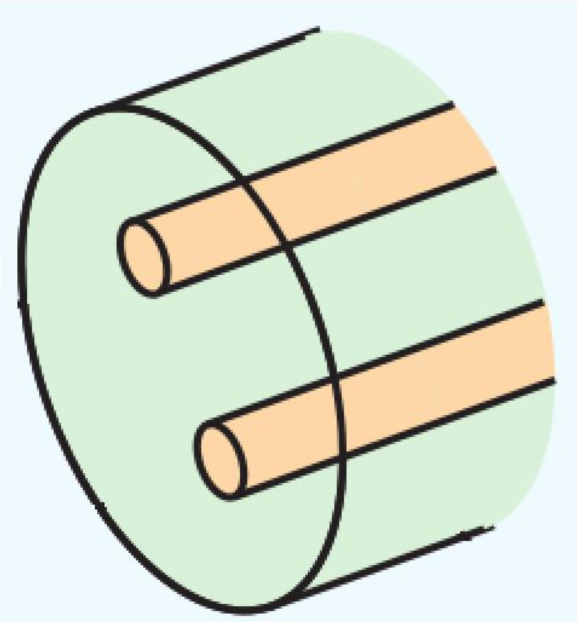
$L' = 2.0$ μH/m

$G' = 0$

$C' = 5.56$ pF/m

$f = 5$ kHz

Find: $\alpha$, $\beta$, $u_p$, $Z_0$



# Exercise 2-4 Two-Wire Air Line

Solution:

$$\alpha = \mathfrak{Re}\left\{[(R' + j\omega L')(G' + j\omega C')]^{1/2}\right\}$$

$$= \mathfrak{Re}\left\{[(0.404 \times 10^{-3} + j2\pi \times 5 \times 10^{3} \times 2 \times 10^{-6})\right.$$

$$\left. \cdot (0 + j2\pi \times 5 \times 10^{3} \times 5.56 \times 10^{-12})]^{1/2}\right\}$$

$$= \mathfrak{Re}[3.37 \times 10^{-7} + j1.05 \times 10^{-4}]$$

$$\alpha = 3.37 \times 10^{-7} \quad (\text{Np/m}).$$

Note the units!

# Exercise 2-4 Two-Wire Air Line

Solution:

from part (a):

$$\beta = \Im\mathfrak{m}\left\{\left[(R' + j\omega L')(G' + j\omega C')\right]^{1/2}\right\}$$

$$\beta = 1.05 \times 10^{-4} \quad \text{(rad/m)}.$$

$$u_\mathrm{p} = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \frac{2\pi(5 \times 10^3\ \mathrm{Hz})}{1.05 \times 10^{-4}\ \mathrm{rad/m}}$$

$$u_\mathrm{p} = 3 \times 10^8\ \mathrm{m/sec}$$

# Exercise 2-4 Two-Wire Air Line

Solution:

$$Z_0 = \frac{R' + j\omega L'}{\gamma}$$

$$Z_0 = \frac{R' + j\omega L'}{\alpha + j\beta}$$

$$Z_0 = \frac{R' + j(2\pi f)L'}{\alpha + j\beta}$$

$$Z_0 = \frac{(0.404 \times 10^{-3}\,\Omega/\mathrm{m} + j2\pi(5 \times 10^{3}\,\mathrm{Hz})(2 \times 10^{-6}\,\mathrm{H/m})}{3.37 \times 10^{-7}\,\mathrm{m}^{-1} + j1.05 \times 10^{-4}\,\mathrm{m}^{-1}}$$

# Exercise 2-4 Two-Wire Air Line

Solution:

$$Z_0 = \frac{0.404 \times 10^{-3} + j0.062832}{3.37 \times 10^{-7} + j1.05 \times 10^{-4}}$$

$$Z_0 = \frac{\sqrt{(0.404 \times 10^{-3})^2 + (0.062832)^2} e^{j \tan^{-1}(.062832/0.404 \times 10^{-3})}}{\sqrt{(3.37 \times 10^{-7})^2 + (1.05 \times 10^{-4})^2} e^{j \tan^{-1}(1.05 \times 10^{-4}/3.37 \times 10^{-7})}}$$

$$Z_0 = \frac{\sqrt{1.63216 \times 10^{-7} + 0.0039478} e^{j \tan^{-1}(155.44)}}{\sqrt{1.13569 \times 10^{-13} + 1.1025 \times 10^{-8}} e^{j \tan^{-1}(311.57)}}$$

$$Z_0 = \frac{\sqrt{0.003948} e^{j89.631^\circ}}{\sqrt{1.10251136 \times 10^{-8}} e^{j89.816^\circ}}$$

# Exercise 2-4 Two-Wire Air Line

Solution:

$$Z_0 = \frac{0.06283315}{0.000105} e^{j(89.631^\circ - 89.816^\circ)}$$

$$Z_0 = 598.4 e^{-j0.1846^\circ}\ \Omega$$

$$Z_0 = 598.4 \cos(-0.1846^\circ) + j598.4 \sin(-0.1846^\circ)$$

$$Z_0 = 598.4 - j1.928\,\Omega$$

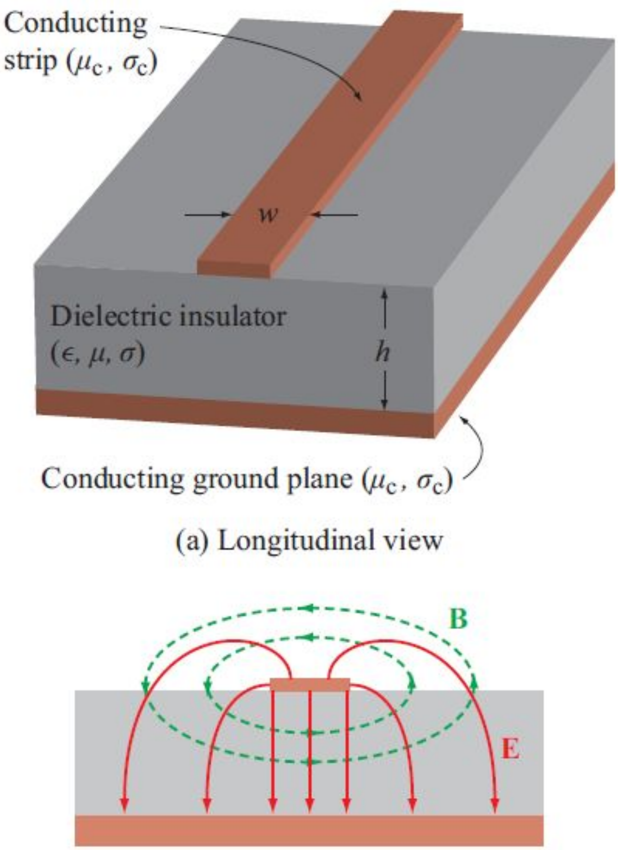
# 2-5 Lossless Microstrip Line

Designed to be compatible with usual Printed-Circuit Board (PCB) technology:

Narrow metal strip on top

Large ground plane on bottom

Dielectric between




(b) Cross-sectional view with **E** and **B** field lines

# 2-5 Lossless Microstrip Line

Modeled Transmission-Line Parameters:

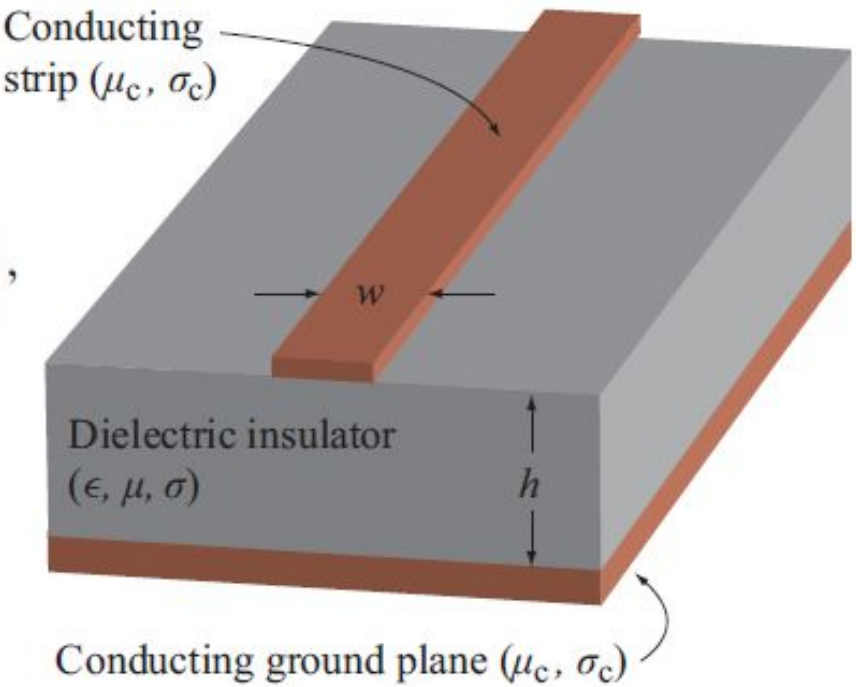
$$Z_0 = \frac{60}{\sqrt{\varepsilon_{\text{eff}}}} \ln\left\{\frac{6+(2\pi-6)e^{-t}}{s} + \sqrt{1+\frac{4}{s^2}}\right\},$$

$$t = \left(\frac{30.67}{s}\right)^{0.75} \qquad s = \frac{w}{h},$$

$$\varepsilon_{\text{eff}} = \frac{\varepsilon_r+1}{2} + \left(\frac{\varepsilon_r-1}{2}\right)\left(1+\frac{10}{s}\right)^{-xy},$$

$$x = 0.56\left[\frac{\varepsilon_r-0.9}{\varepsilon_r+3}\right]^{0.05},$$

$$y = 1 + 0.02\ln\left(\frac{s^4+3.7\times10^{-4}s^2}{s^4+0.43}\right) + 0.05\ln(1+1.7\times10^{-4}s^3).$$




Largely the result of fitting lots of data using functional forms based on intuition.

# 2-5 Lossless Microstrip Line

Modeled Transmission-Line Parameters:

$$R' = 0 \qquad \text{(because } \sigma_c = \infty\text{)},$$

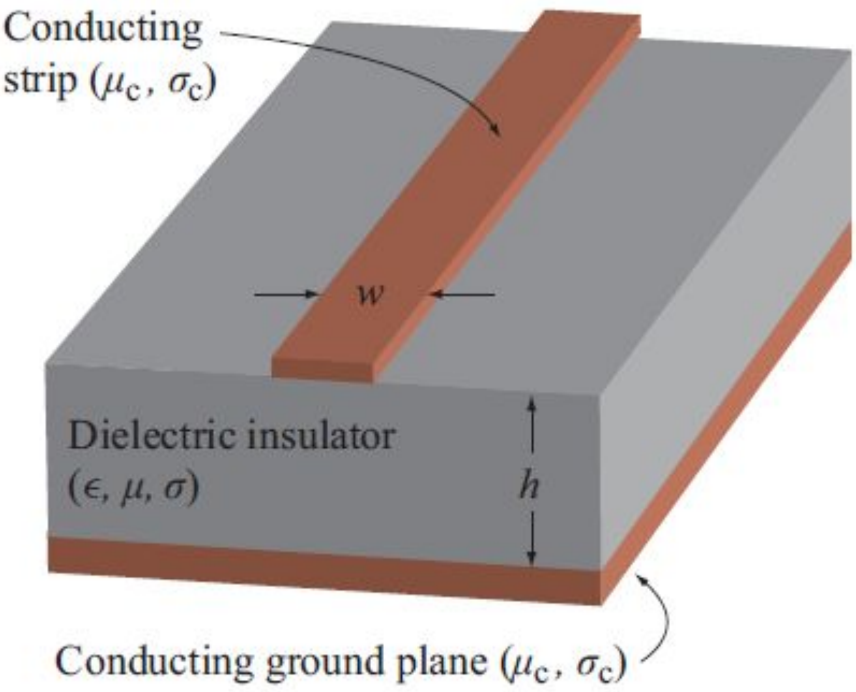
$$G' = 0 \qquad \text{(because } \sigma = 0\text{)},$$

$$C' = \frac{\sqrt{\varepsilon_{\text{eff}}}}{Z_0 c},$$

$$L' = Z_0^2 C',$$

$$\alpha = 0 \qquad \text{(because } R' = G' = 0\text{)},$$

$$\beta = \frac{\omega}{c} \sqrt{\varepsilon_{\text{eff}}} .$$



# 2-5 Lossless Microstrip Line

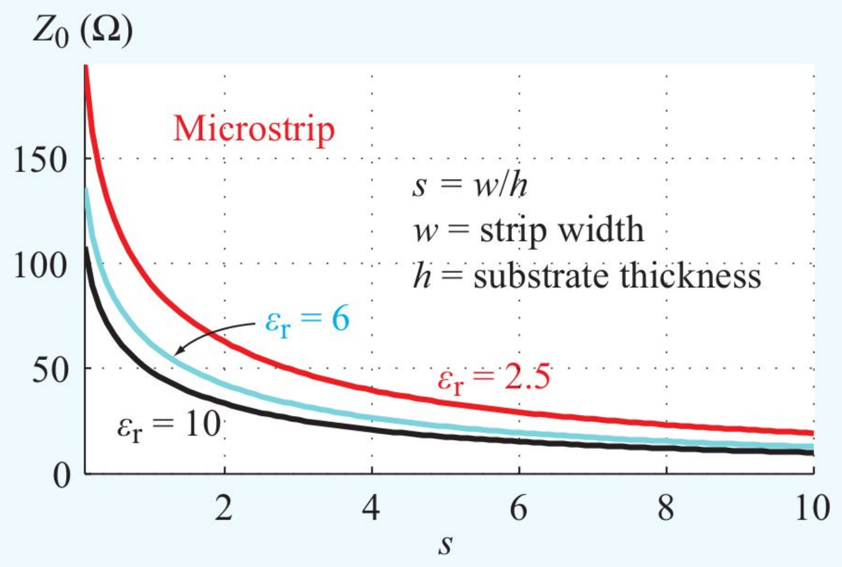
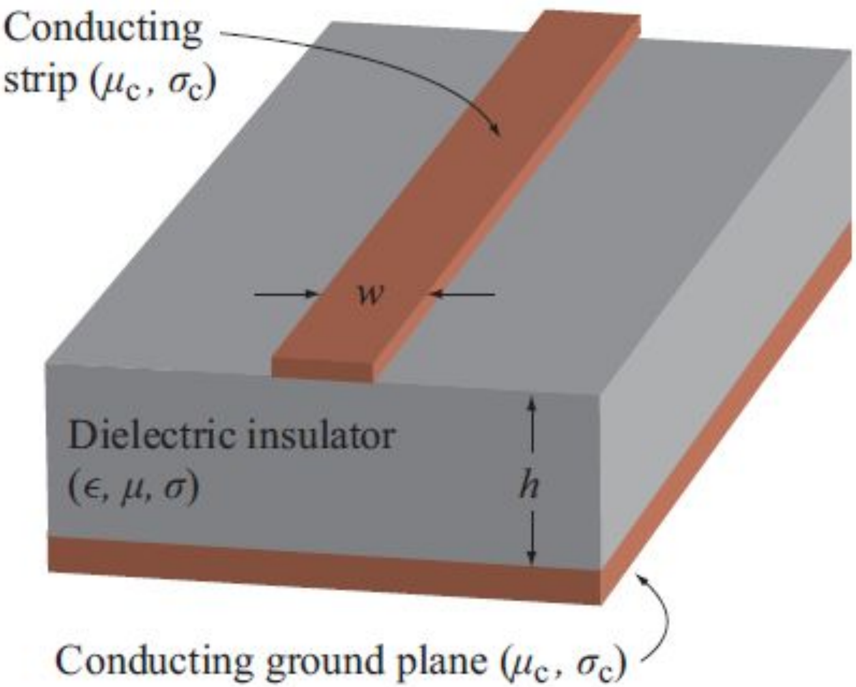



**Figure 2-11** Plots of $Z_0$ as a function of $s$ for various types of dielectric materials.



# 2-5 Lossless Microstrip Line

Use a curve-fit of the previous plots in order to solve for *s*, given other params.

**(a)** For $Z_0 \leq (44 - 2\varepsilon_r)\ \Omega$,

$$s = \frac{w}{h} = \frac{2}{\pi}\left\{(q-1) - \ln(2q-1) + \frac{\varepsilon_r - 1}{2\varepsilon_r}\left[\ln(q-1) + 0.29 - \frac{0.52}{\varepsilon_r}\right]\right\}$$

with

$$q = \frac{60\pi^2}{Z_0\sqrt{\varepsilon_r}},$$

# 2-5 Lossless Microstrip Line

Use a curve-fit of the previous plots in order to solve for *s*, given other params.

**(b)** for $Z_0 \geq (44 - 2\varepsilon_r)\ \Omega$,

$$s = \frac{w}{h} = \frac{8e^p}{e^{2p} - 2},$$

with

$$p = \sqrt{\frac{\varepsilon_r + 1}{2}}\frac{Z_0}{60} + \left(\frac{\varepsilon_r - 1}{\varepsilon_r + 1}\right)\left(0.23 + \frac{0.12}{\varepsilon_r}\right)$$

# Example 2-2 Microstrip Line Design

Given: Have a substrate: $h$=0.5mm thick, $\varepsilon_r$=9

Find: strip width, $w$, to get $Z_0$ = 50 Ω

Solution: 44- 2$\varepsilon_r$= 44 - 2(9) = 32, so use:

$$s = \frac{w}{h} = \frac{8e^p}{e^{2p} - 2},$$

with

$$p = \sqrt{\frac{\varepsilon_r + 1}{2}}\,\frac{Z_0}{60} + \left(\frac{\varepsilon_r - 1}{\varepsilon_r + 1}\right)\left(0.23 + \frac{0.12}{\varepsilon_r}\right)$$

# Example 2-2 Microstrip Line Design

Solution:

$$p = \sqrt{\frac{\varepsilon_r + 1}{2} \times \frac{Z_0}{60}} + \left(\frac{\varepsilon_r - 1}{\varepsilon_r + 1}\right)\left(0.23 + \frac{0.12}{\varepsilon_r}\right)$$

$$= \sqrt{\frac{9+1}{2} \times \frac{50}{60}} + \left(\frac{9-1}{9+1}\right)\left(0.23 + \frac{0.12}{9}\right) = 2.06,$$

$$s = \frac{w}{h} = \frac{8e^p}{e^{2p} - 2} = \frac{8e^{2.06}}{e^{4.12} - 2} = 1.056$$

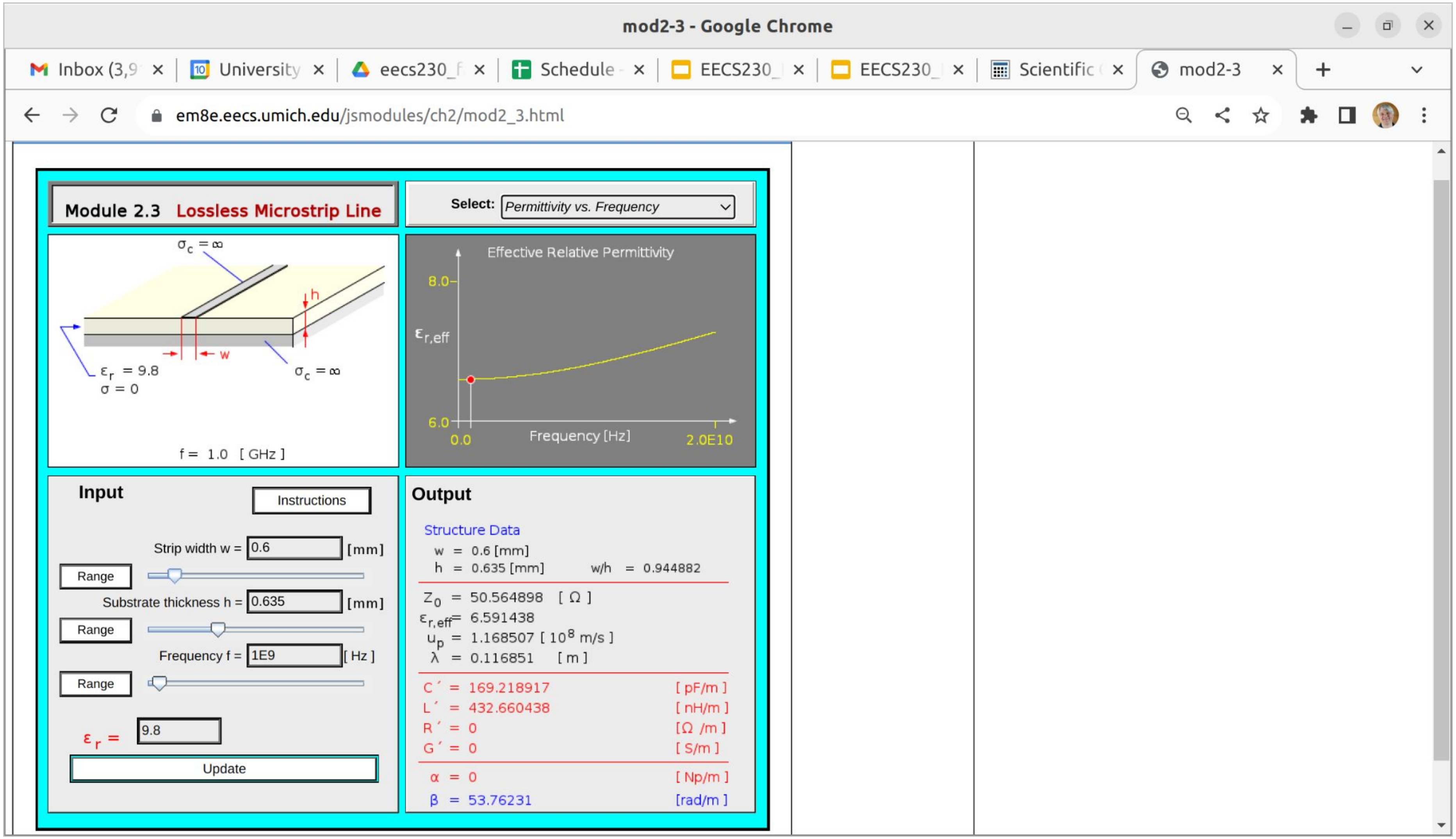
# Example 2-2 Microstrip Line Design

Solution:

$$w = sh = 1.056 \times 0.5\ \text{mm} = \boxed{0.53\ \text{mm}.}$$

Microwave CAD design codes use this for an initial design.

# 2-5 Lossless Microstrip Line



# 2-5 Lossless Microstrip Line

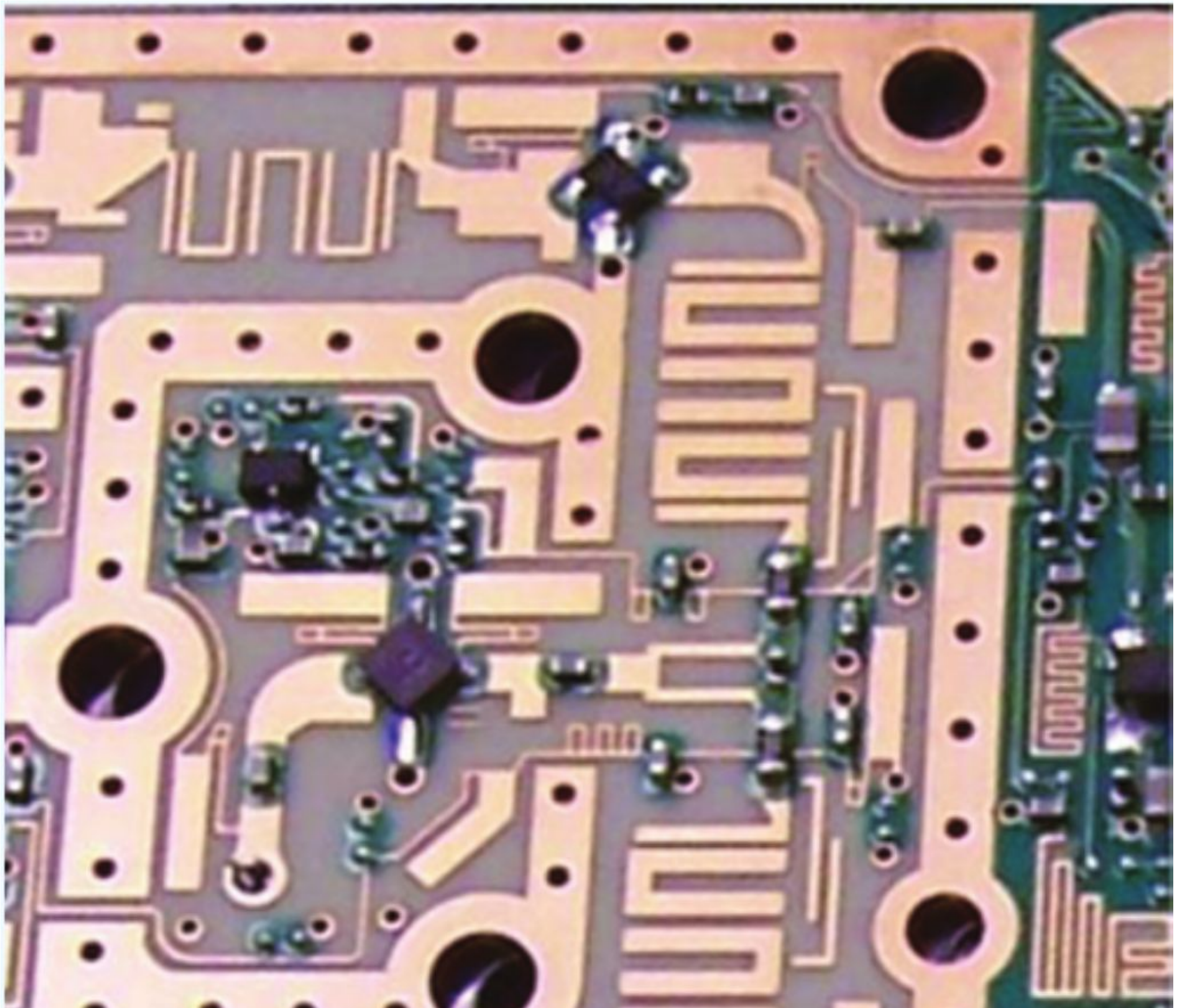


Photo of a microstrip circuit

# Homework



**Homework 4 is due tomorrow at midnight.**

**submit to gradescope via the canvas site.**

# Next Time

**Section 2-6:**

Simplify by assuming lossless line.

Investigate various parameters that characterize the solution.