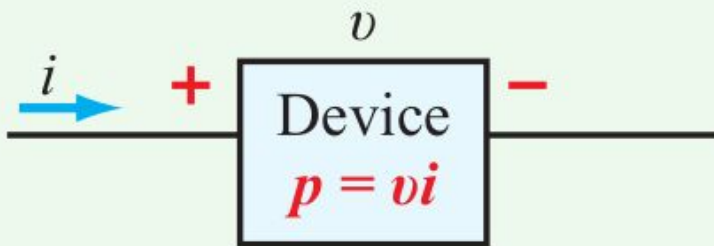


EECS 230
ENGINEERING ELECTROMAGNETICS
Leland Pierce

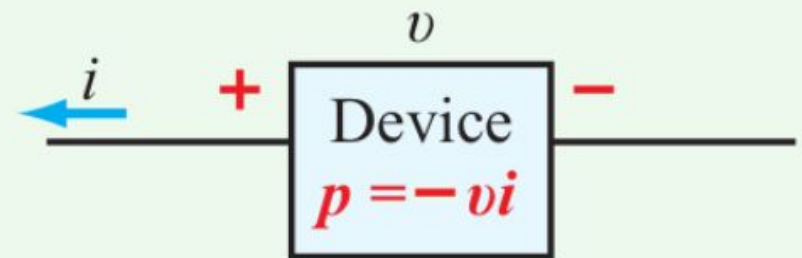
Transmission Lines 1

Power

Passive Sign Convention



Passive Sign Convention



$$p > 0$$

power delivered to device

$$p < 0$$

power supplied by device

Chapter 2 Overview

What is a transmission line?

Why study transmission lines?

model

diffeq

solve diffeq

wave propagation

lossless transmission line

microstrip lines

reflections

standing waves

impedance

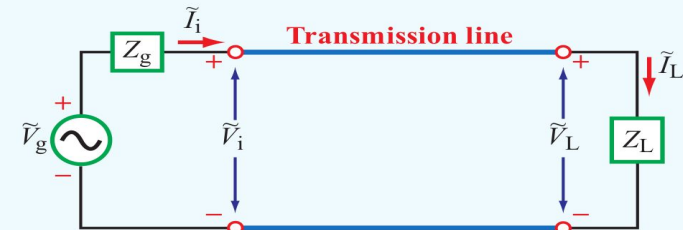
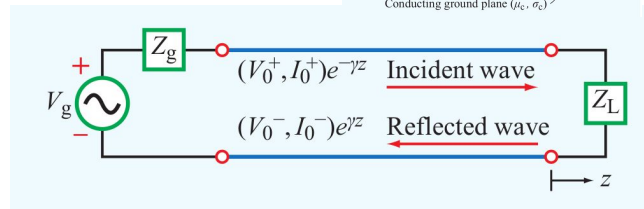
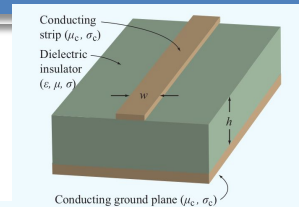
short, open

matching

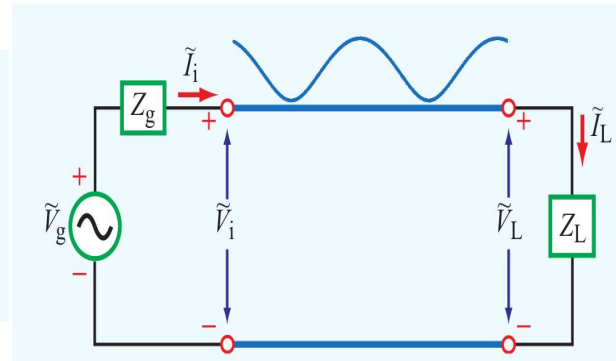
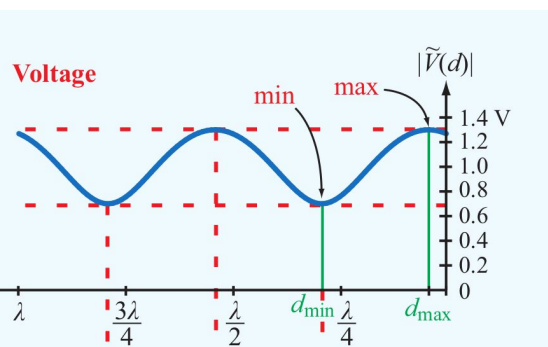
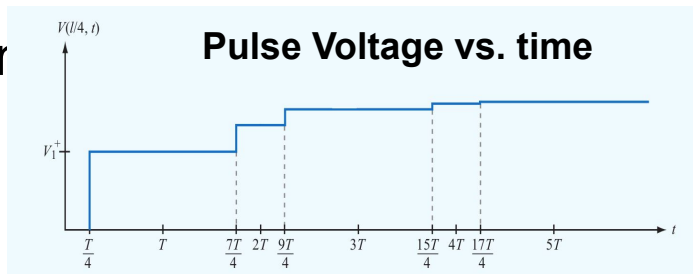
power flow

smith chart

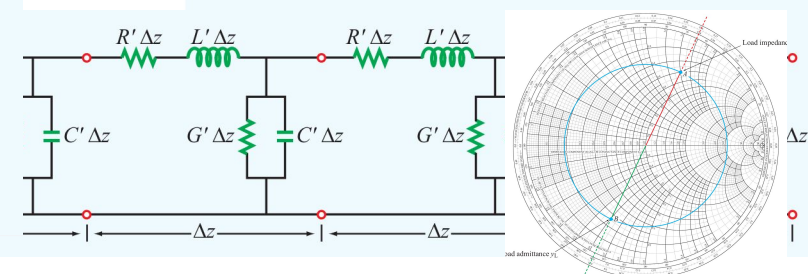
transients



Typical High-Frequency Circuit



Waves on line: old methods don't work



Chapter 2 Applications

Radios: cell phones, internet, broadcast radio/tv, GPS

Radar: civilian, aeronautical, military, vehicle collision sensors, weather, remote sensing

Heating: Microwave Ovens, Cancer Treatment

Relevant EECS Classes



411/413: RF circuits

330/430/455: Wireless circuits, systems

525: Advanced RF circuits

531: Antenna Design

532/632: Remote Sensing

Today's Lecture Coverage



Sections 1-6, 2-1, 2-2 of the book:

1-6: Review of complex numbers

2-1: What is a transmission line?
Why study transmission lines?

2-2: Lumped-Element Model

1-6 Complex Numbers

Already used complex numbers when discussing phasors last time.

This is a review of properties and best practices.

$$j = \sqrt{-1}$$

to avoid any confusion with current, i .

A complex number has a real-part and an imaginary-part:

$$z = x + jy$$

Operators to get these:

$$x = \Re(z), \quad y = \Im(z).$$

1-6 Complex Numbers

Rectangular Form: $z = x + jy$

Polar Form: $z = |z| e^{j\phi}$

alternate notation:

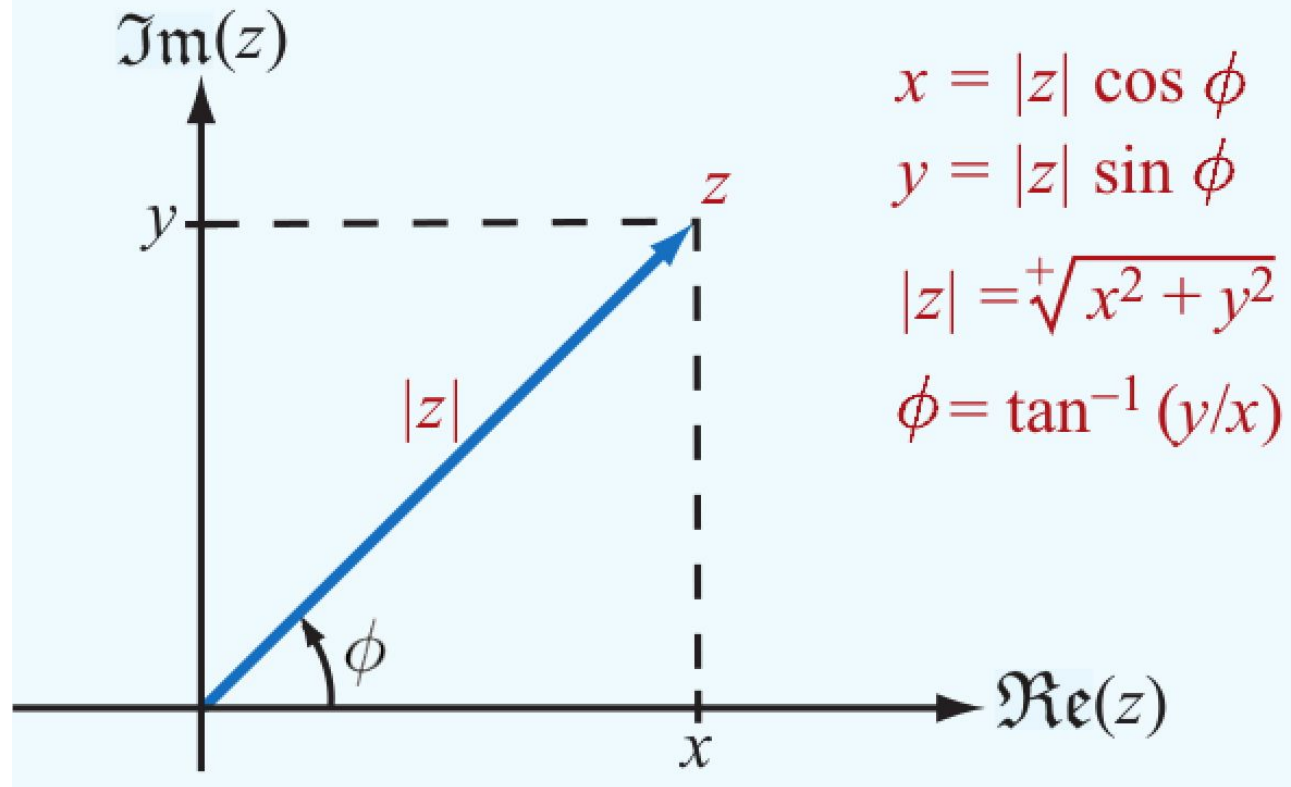
$$z = |z| \underline{\angle \phi}$$

Euler's identity: $e^{j\phi} = \cos \phi + j \sin \phi$

$$z = |z| e^{j\phi} = |z| \cos \phi + j |z| \sin \phi$$

1-6 Complex Numbers

Complex Plane:



1-6 Complex Numbers

Complex Conjugate: replace j with $-j$:

$$z^* = (x + jy)^* = x - jy = |z|e^{-j\theta} = |z|/\underline{-\theta}.$$

so:

$$|z| = \sqrt{zz^*}.$$

1-6 Complex Numbers

Addition/subtraction is easier with the rectangular form:

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

Multiplication is easier with the polar form:

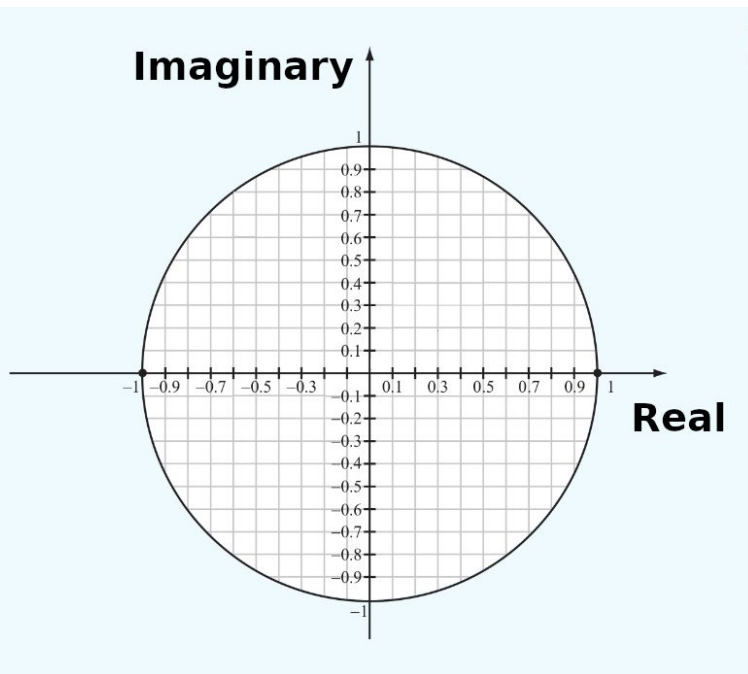
$$z_1 \times z_2 = |z_1||z_2| e^{j(\phi_1 + \phi_2)}$$

Powers are easier with the polar form:

$$\begin{aligned} z^n &= (|z|e^{j\theta})^n \\ &= |z|^n e^{jn\theta} \end{aligned}$$

1-6 Complex Numbers

Useful Relations:



$$-1 = e^{j\pi} = e^{-j\pi} = 1/\underline{180^\circ},$$

$$j = e^{j\pi/2} = 1/\underline{90^\circ},$$

$$-j = -e^{j\pi/2} = e^{-j\pi/2} = 1/\underline{-90^\circ},$$

$$\sqrt{j} = (e^{j\pi/2})^{1/2} = \pm e^{j\pi/4} = \frac{\pm(1+j)}{\sqrt{2}},$$

$$\sqrt{-j} = \pm e^{-j\pi/4} = \frac{\pm(1-j)}{\sqrt{2}}.$$

Example 1-3

Given two complex numbers:

$$V = 3 - j4, \quad I = -(2 + j3),$$

Express each in polar form:

$$V = |V|e^{j\phi_V}$$

$$|V| = \sqrt{VV^*}$$

$$= \sqrt{(3 - j4)(3 + j4)}$$

$$= \sqrt{3^2 + 4^2}$$

$$= 5$$

$$\phi_V = \tan^{-1}(-4/3)$$

$$= -53.1^\circ$$

$$V = 5e^{-j53.1^\circ} = 5\angle -53.1^\circ$$

Example 1-3

Given two complex numbers:

$$V = 3 - j4, \quad I = -(2 + j3),$$

Express each in polar form:

$$I = |I|e^{j\phi_I}$$

$$|I| = \sqrt{II^*}$$

$$= \sqrt{(-2 - j3)(-2 + j3)}$$

$$= \sqrt{2^2 + 3^2}$$

$$= \sqrt{13}$$

$$= 3.61$$

$$\phi_I = \tan^{-1}(-3/-2)$$

Example 1-3

Given two complex numbers:

$$V = 3 - j4, \quad I = -(2 + j3),$$

Express each in polar form:

Calculators only give the correct arctan when in the 1st and 4th quadrants ($x > 0$),

since this is in the 3rd quadrant, use:

$$\phi_I = 180^\circ + \tan^{-1}(3/2)$$

$$\phi_I = 236.3^\circ$$

$$I = 3.61e^{j236.3^\circ} = 3.61 \angle 236.3^\circ$$

Example 1-3

Given two complex numbers:

$$V = 3 - j4, \quad I = -(2 + j3),$$

Multiply:

Use polar forms:

$$\begin{aligned} VI &= 5e^{-j53.1^\circ} \times 3.61e^{j236.3^\circ} \\ &= 18.03e^{j(236.3^\circ - 53.1^\circ)} \end{aligned}$$

$$VI = 18.03e^{j183.2^\circ}$$

Example 1-3

Given two complex numbers:

$$V = 3 - j4, \quad I = -(2 + j3),$$

Divide:

Use polar forms:

$$\begin{aligned} \frac{V}{I} &= \frac{5e^{-j53.1^\circ}}{3.61e^{j236.3^\circ}} \\ &= 1.39e^{-j289.4^\circ} \end{aligned}$$

$$\frac{V}{I} = 1.39e^{j70.6^\circ}$$

Example 1-3

Given two complex numbers:

$$V = 3 - j4, \quad I = -(2 + j3),$$

Square-root of I :

Use polar form:

$$\begin{aligned}\sqrt{I} &= \sqrt{3.61 e^{j236.3^\circ}} \\ &= \pm \sqrt{3.61} e^{j236.3^\circ/2}\end{aligned}$$

$$\sqrt{I} = \pm 1.90 e^{j118.15^\circ}$$

Note that there are 2 roots, as usual.

2-1 Transmission Lines

- Transfers energy (or information) between two points in a circuit (sort of conduit of EM signals).
- It can take different shapes and forms.

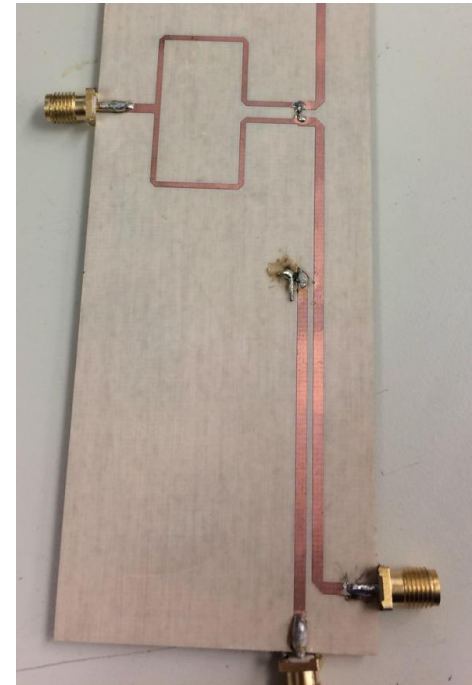
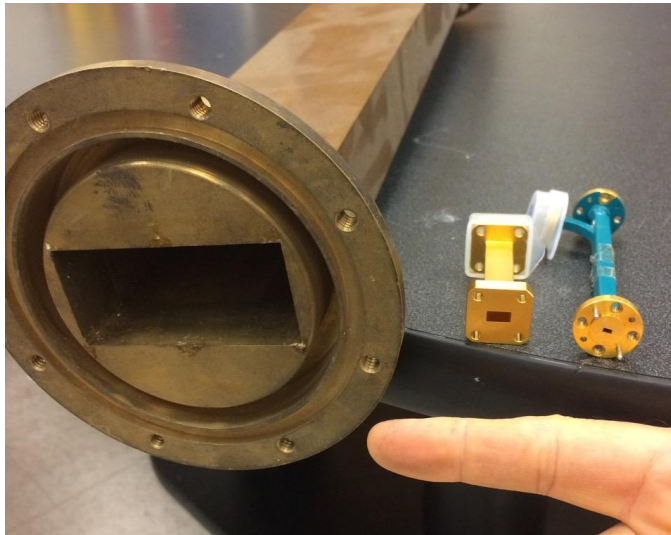
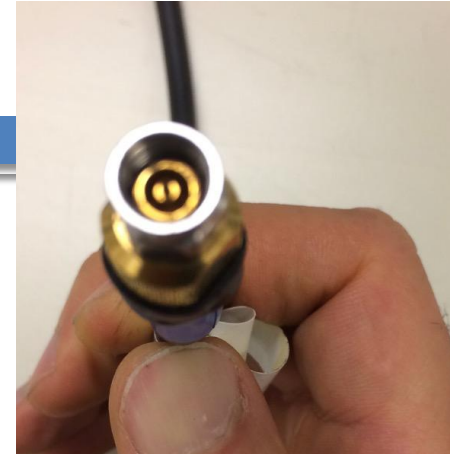


- A transmission line connects a **generator** to a **load**.

2-1 Transmission Lines

Transmission lines include:

- Two parallel wires
- Coaxial cable
- Microstrip line
- Optical fiber
- Waveguide
- etc.



2-1 Transmission Lines

Which transmission line to use?

Selection depends on many factors and priorities:

- **Frequency and signal bandwidth**
- **Transmitted power**
- **Acceptable signal loss / distortion**
- **Distance**
- **Available space (volume)**
- **Cost**
- **Convenience**
- **Standards compliance**

2-1 Transmission Line Effects

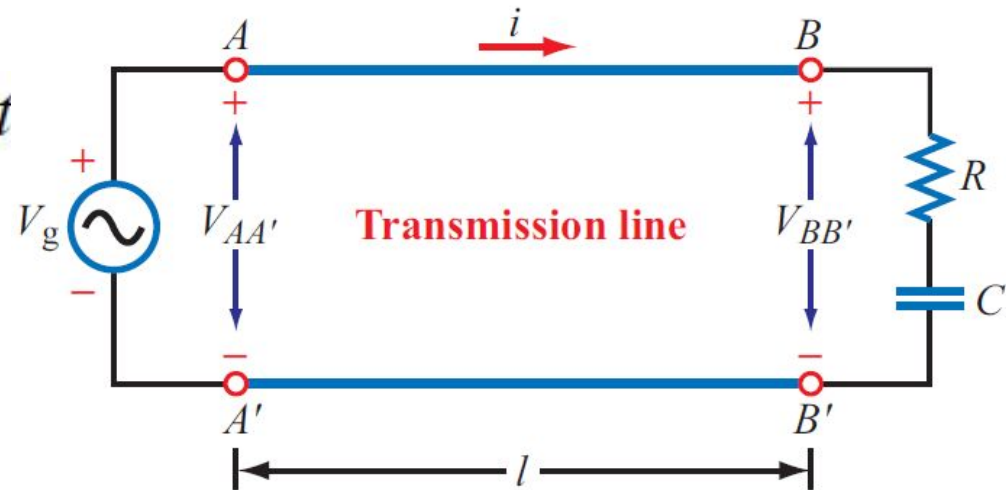
$$V_{AA'} = V_g(t) = V_0 \cos \omega t$$

For now, assume $u_p = c$.

At port B, the voltage is delayed by l/c :

$$\begin{aligned} V_{BB'}(t) &= V_{AA'}(t - l/c) \\ &= V_0 \cos [\omega(t - l/c)] \\ &= V_0 \cos(\omega t - \phi_0), \end{aligned}$$

$$\phi_0 = \frac{\omega l}{c} = \frac{2\pi f l}{c} = 2\pi \frac{l}{\lambda}$$



Phase delay is one important effect:
when length is a significant fraction of the wavelength.

2-1 Transmission Line Effects

Example 1: Given:

$$t = 0 \text{ sec}$$

$$f = 1 \text{ KHz}$$

$$l = 5 \text{ cm}$$

1

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8 \text{ m/sec}}{1 \times 10^3 \text{ Hz}}$$

$$= \frac{3 \times 10^8 \text{ m/sec}}{1 \times 10^3 \text{ cycles/sec}}$$

$$\lambda = 3 \times 10^5 \text{ m}$$

2

$$\phi_0 = 2\pi \frac{l}{\lambda}$$
$$= 2\pi \frac{0.05 \text{ m}}{3 \times 10^5 \text{ m}}$$

$$= 3.3 \times 10^{-7} \pi$$

$$\phi_0 = 1 \times 10^{-6} \text{ radians}$$

3

$$V_{BB'}(t) = V_0 \cos(\omega t - \phi_0)$$

$$V_{BB'}(t = 0) = V_0 \cos(\phi_0)$$

$$= V_0 \cos(1 \times 10^{-6} \text{ radians})$$

$$V_{BB'}(t = 0) = 0.9999999999 V_0$$

2-1 Transmission Line Effects

Example 2: Given:

$$t = 0 \text{ sec}$$

$$f = 1 \text{ KHz}$$

$$l = 20 \text{ Km}$$

1

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8 \text{ m/sec}}{1 \times 10^3 \text{ Hz}}$$

$$= \frac{3 \times 10^8 \text{ m/sec}}{1 \times 10^3 \text{ cycles/sec}}$$

$$\lambda = 3 \times 10^5 \text{ m}$$

2

$$\phi_0 = 2\pi \frac{l}{\lambda}$$
$$= 2\pi \frac{2 \times 10^4 \text{ m}}{3 \times 10^5 \text{ m}}$$

$$\phi_0 = 0.133 \pi = 0.42 \text{ radians}$$

3

$$V_{BB'}(t) = V_0 \cos(\omega t - \phi_0)$$

$$V_{BB'}(t = 0) = V_0 \cos(\phi_0)$$

$$= V_0 \cos(0.42 \text{ radians})$$

$$V_{BB'}(t = 0) = 0.91 V_0$$

2-1 Transmission Line Effects

Example 3: Given:

$$t = 0 \text{ sec}$$

$$f = 1.5 \text{ GHz}$$

$$l = 5 \text{ cm}$$

$$\begin{aligned} 2 \quad \phi_0 &= 2\pi \frac{l}{\lambda} \\ &= 2\pi \frac{0.05 \text{ m}}{0.2 \text{ m}} \\ \phi_0 &= \frac{\pi}{2} \end{aligned}$$

$$\begin{aligned} 1 \quad \lambda &= \frac{c}{f} = \frac{3 \times 10^8 \text{ m/sec}}{1.5 \times 10^9 \text{ Hz}} \\ &= \frac{3 \times 10^8 \text{ m/sec}}{1.5 \times 10^9 \text{ cycles/sec}} \\ \lambda &= 0.2 \text{ m} \end{aligned}$$

$$\begin{aligned} 3 \quad V_{BB'}(t) &= V_0 \cos(\omega t - \phi_0) \\ V_{BB'}(t = 0) &= V_0 \cos(\phi_0) \\ &= V_0 \cos\left(\frac{\pi}{2}\right) \\ V_{BB'}(t = 0) &= 0 \text{ Volts} \end{aligned}$$

2-1 Transmission Line Effects



So, line length has an effect on the voltage at the load.

This showed that the **phase** of the voltage is different.

But we'll see later that the **amplitude** of the voltage is also affected.

2-1 Transmission Line Effects

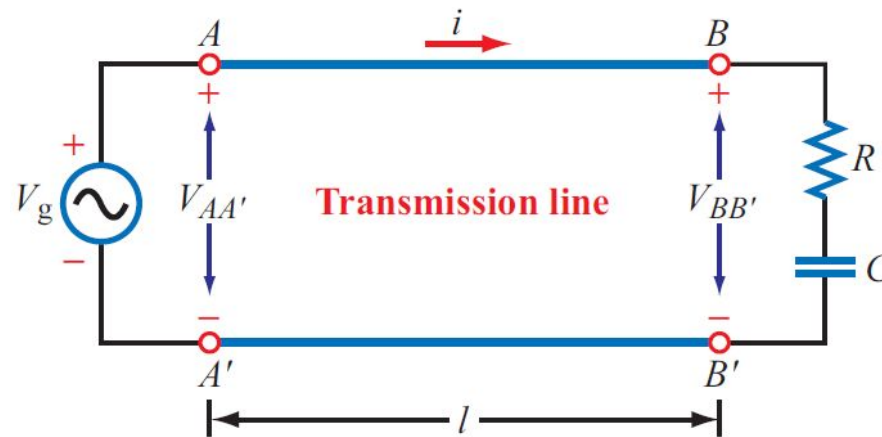
Common mistake:

When using calculator to compute $\cos()$:

Entering value in radians, when calculator assumes it's in degrees.

or vice-versa.

2-1 Transmission Line Effects



$$\phi_0 = \frac{\omega l}{c} = \frac{2\pi f l}{c} = 2\pi \frac{l}{\lambda} \quad \text{radians.}$$

$l/\lambda \lesssim 0.01$: Can ignore transmission-line effects

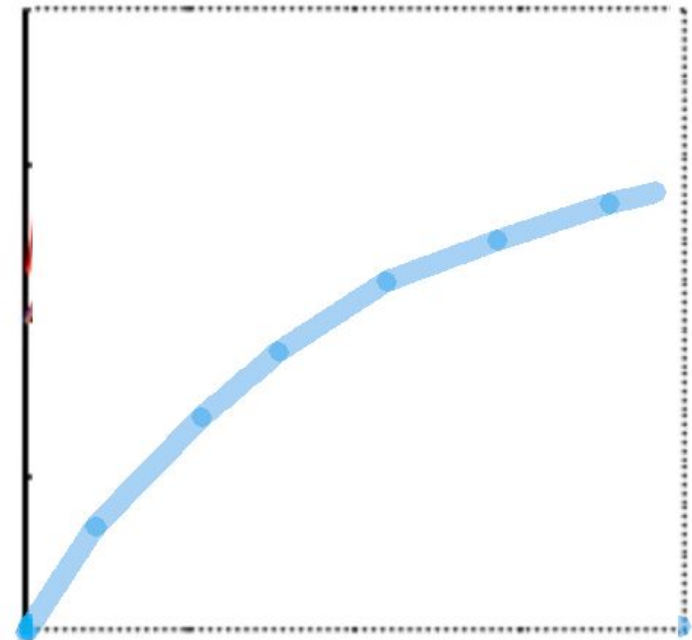
$l/\lambda \gtrsim 0.01$: Must deal with phase shift,
and other effects...

2-1 Dispersion

Velocity as a function of frequency:

Because the material parameters ϵ , μ , σ vary with frequency, so does the velocity of the signal in the transmission-line.

velocity vs. frequency

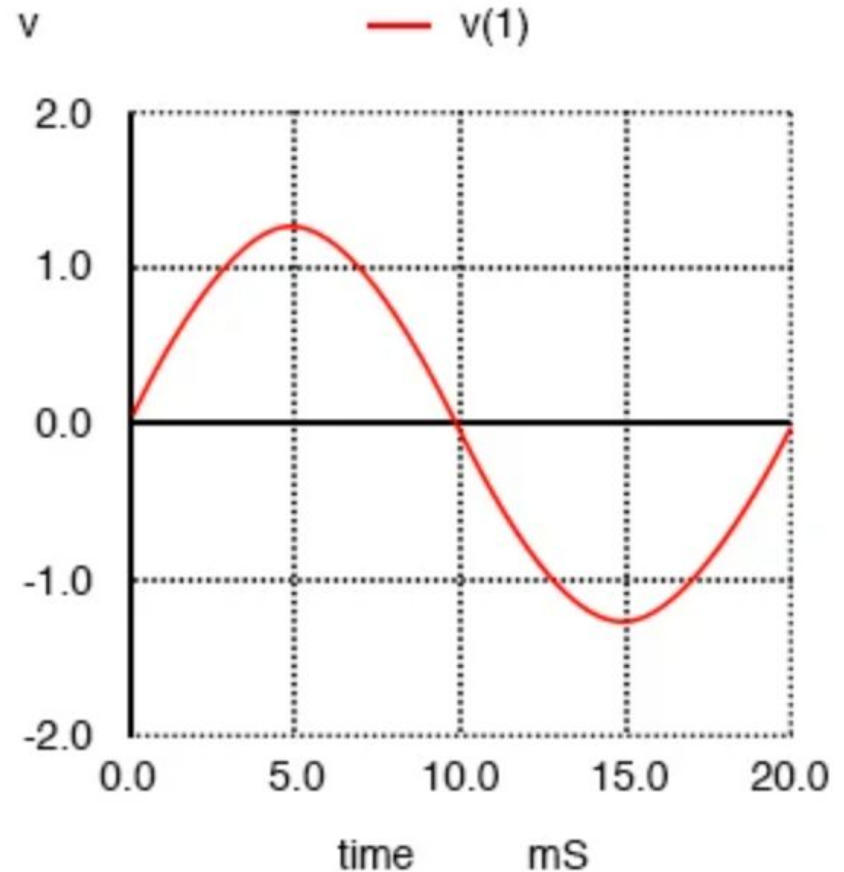


Frequency

2-1 Dispersion

A pulse is composed of a band of frequencies:

Add up several sinusoids with different amplitudes and phases:
can get a rectangular pulse

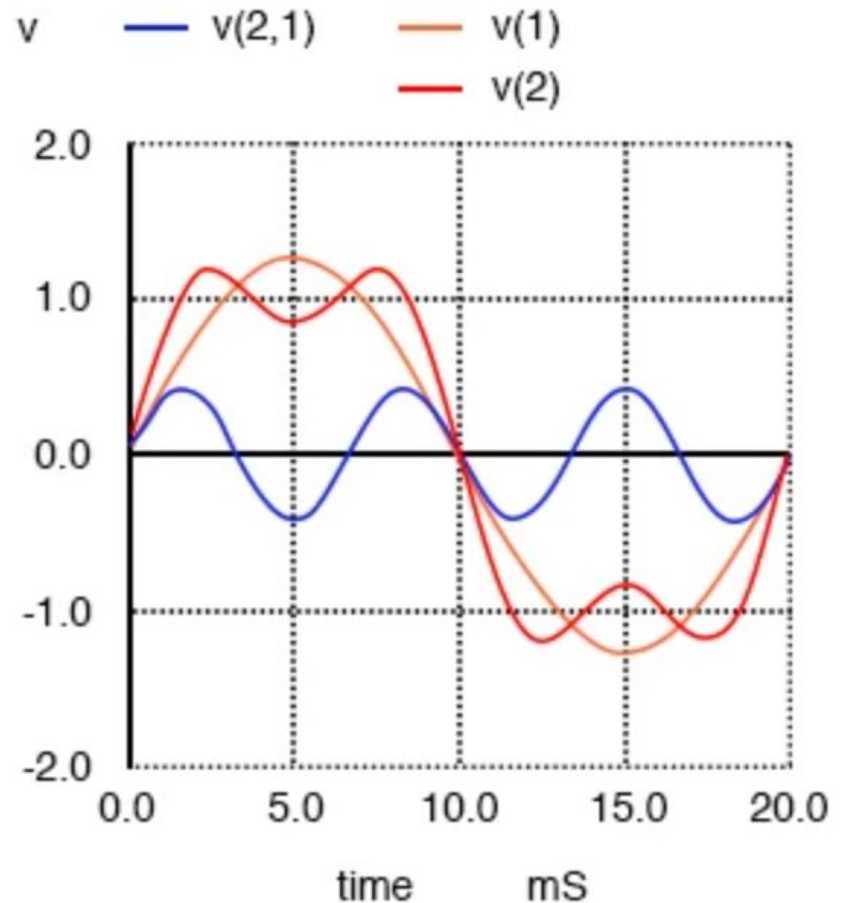


(allaboutcircuits.com)

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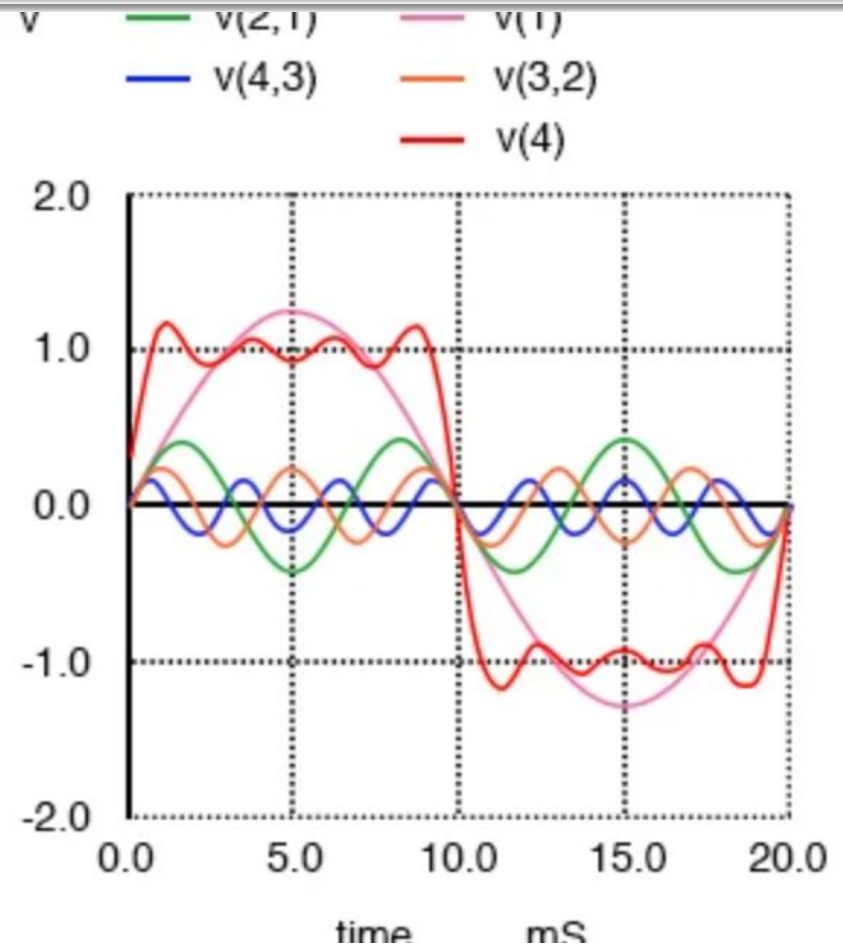


(allaboutcircuits.com)

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(allaboutcircuits.com)

2-1 Dispersion

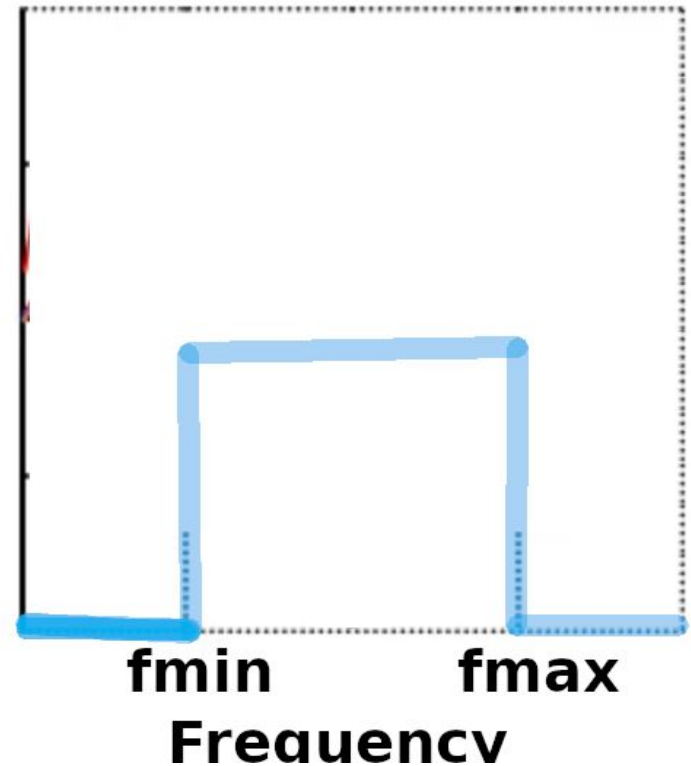
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Add up several sinusoids with different amplitudes and phases:

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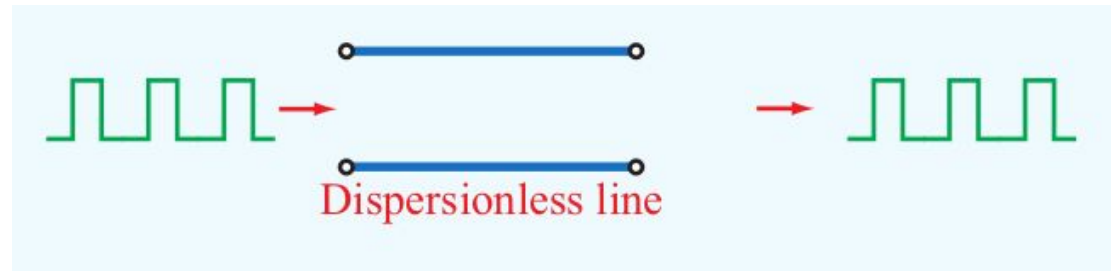
So will have all frequencies from some minimum to some maximum propagating in the transmission line.

Power vs. Frequency



2-1 Dispersion

Dispersion for digital communications:

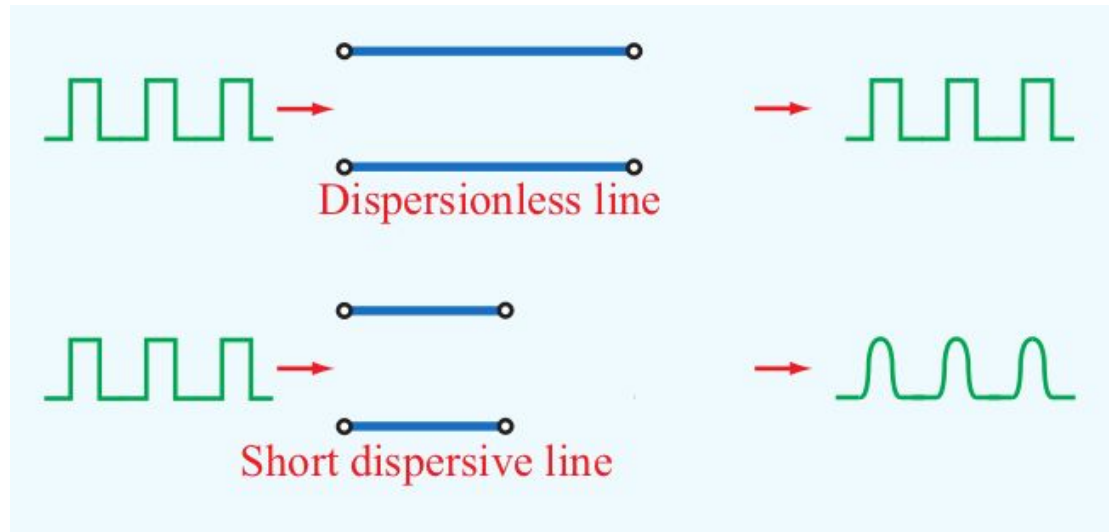


Dispersionless lines preserve the pulse train that was input.

2-1 Dispersion

Dispersion for digital communications:

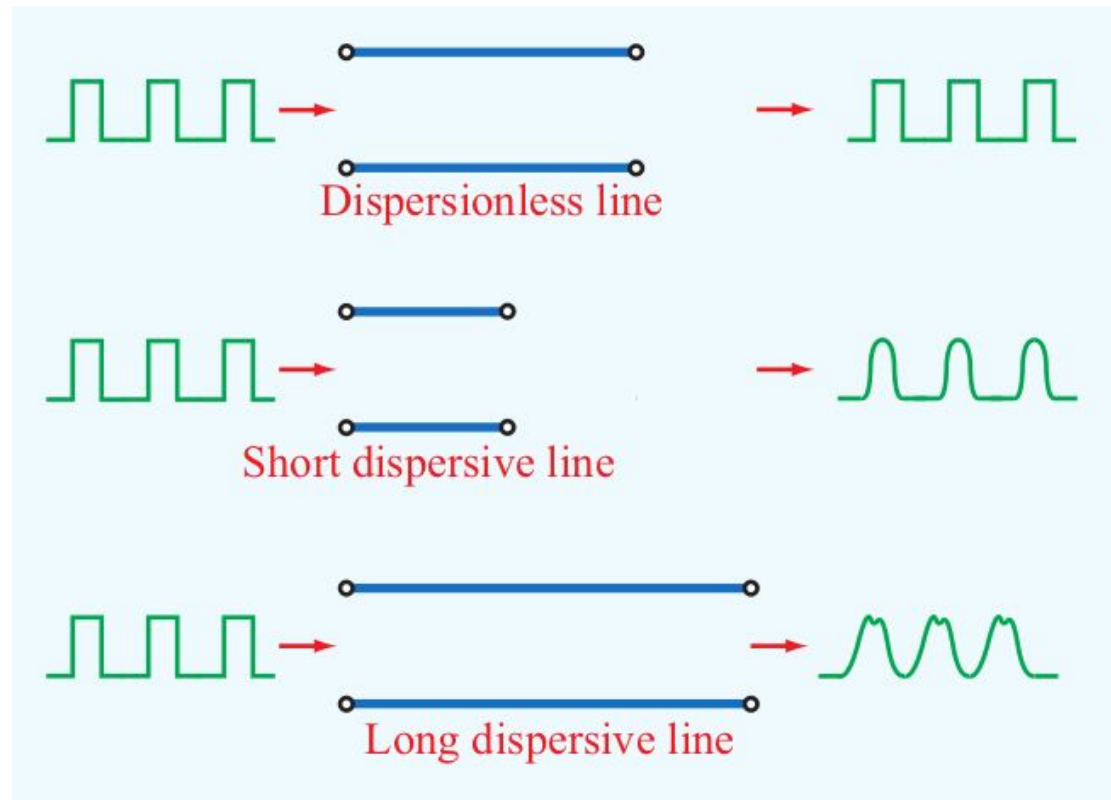
Short dispersive lines blur the pulse shape.



2-1 Dispersion

Dispersion for digital communications:

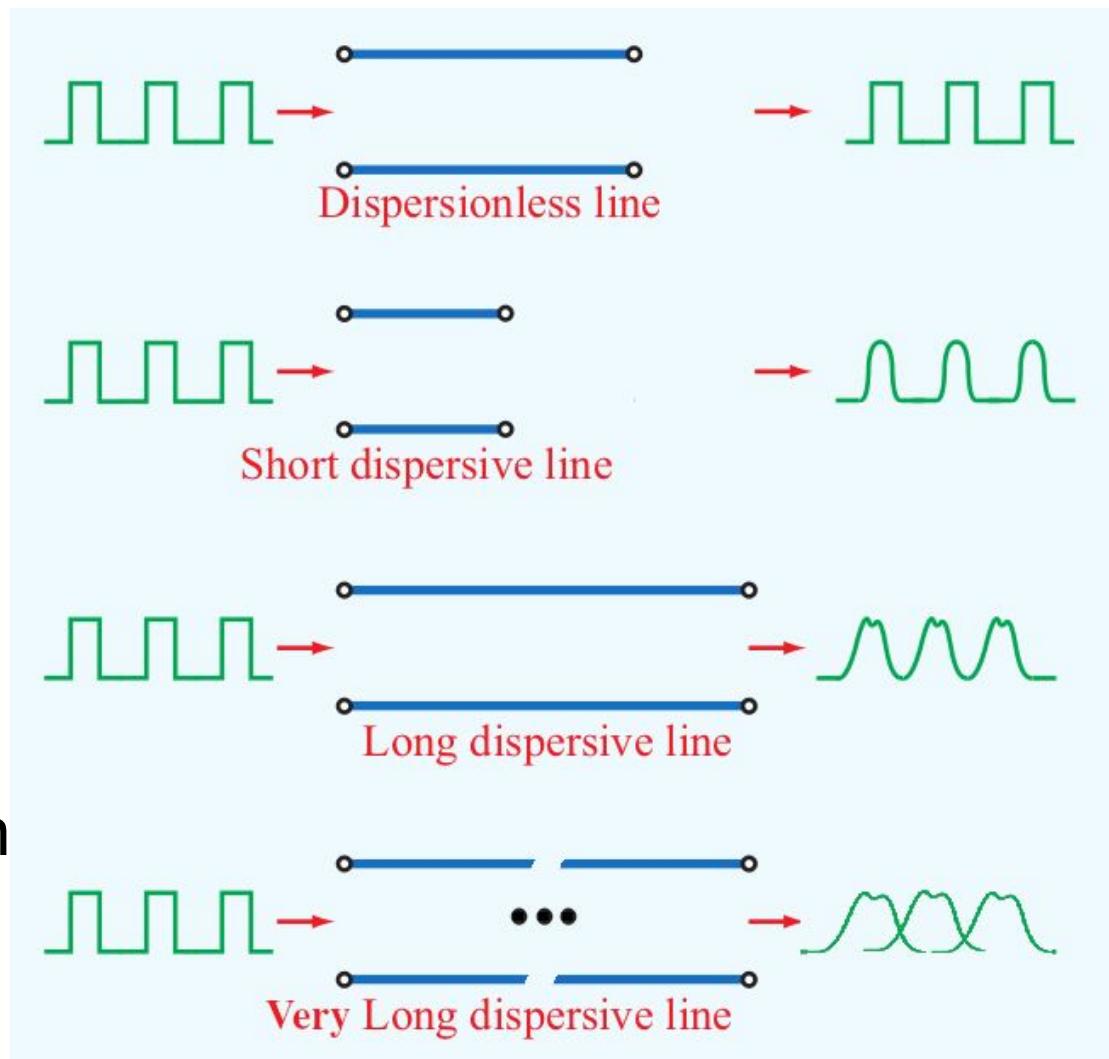
Longer dispersive lines noticeably distort the pulse shape.



2-1 Dispersion

Dispersion for digital communications:

Long dispersive lines blur each pulse so much that they overlap with each other.

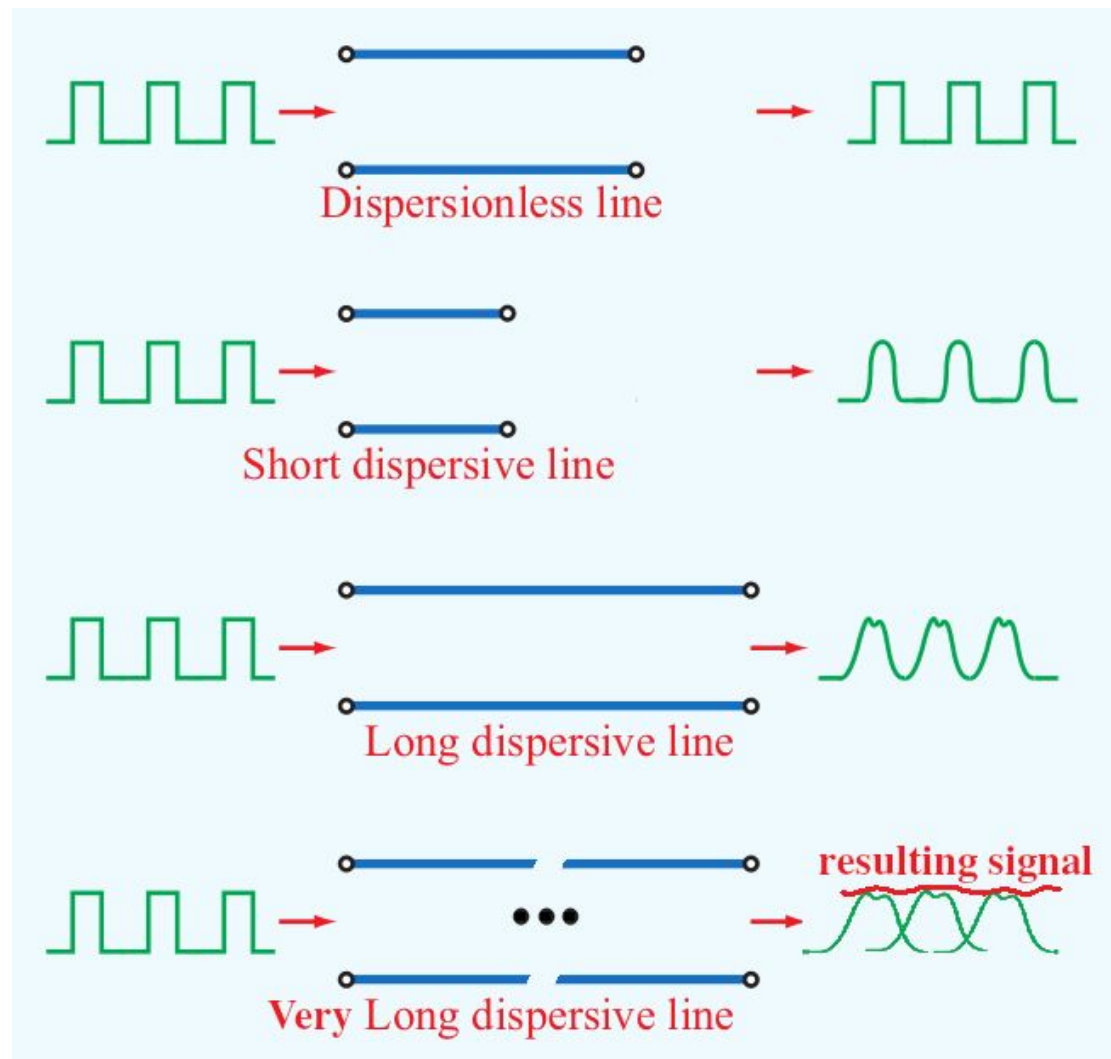


2-1 Dispersion

Dispersion for digital communications:

So the pulses are lost, and no information is transmitted.

This happens when the pulses spread by half a pulse-width in each direction.



2-1 Dispersion

Dispersion for digital communications:

The velocity-difference between the lowest-frequency and the upper-frequency causes this pulse spreading.

Assume these frequencies have different velocities:

$$u_1, u_2$$

The time it takes for each freq to propagate 1 meter is:

$$t_1 = 1\text{m}/u_1 \quad t_2 = 1\text{m}/u_2$$

The time offset is therefore:

$$\Delta t = t_1 - t_2 = 1\text{m}/u_1 - 1\text{m}/u_2$$

2-1 Dispersion

Dispersion for digital communications:

Transmission lines are characterized by a parameter called "delay", which is defined as:

$$\text{delay} = 1/u_1 - 1/u_2$$

with units of sec/m

For example: delay = 0.01 nsec/m

This is only relevant when the min and max frequency are also specified.

2-1 Dispersion

Dispersion for digital communications:

The maximum line length, L_{\max} , is defined as the length where the pulse spreading is half of the pulse width, w , where the spreading causes the pulses to become indistinguishable:

$$L_{\max} = 0.5w / \text{delay}$$

Example: Dispersion

Given: A digital transmission system.

Pulse width: 1 nsec

Line dispersion: delay: 0.003 nsec/m

Find: Maximum length of line

Solution: We know: $L_{\max} = 0.5w / \text{delay}$

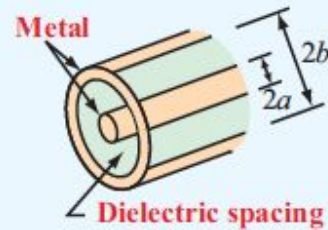
so: $L_{\max} = 0.5 \text{ nsec} / 0.003 \text{ nsec/m}$

$$L_{\max} = 167 \text{ m}$$

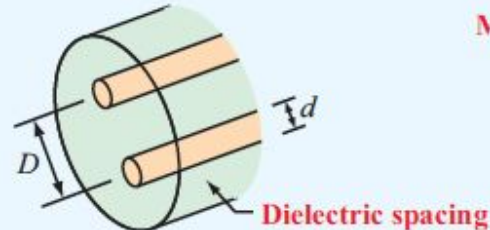
This is why certain interfaces have max cable-lengths

2-1 Types of Transmission Modes

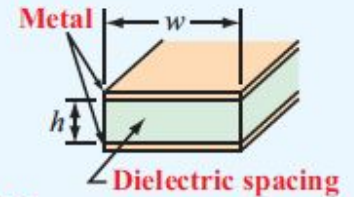
TEM (Transverse Electromagnetic):
Electric and magnetic fields are orthogonal to one another, and both are orthogonal to direction of propagation



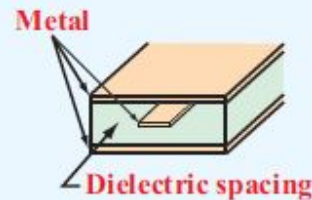
(a) Coaxial line



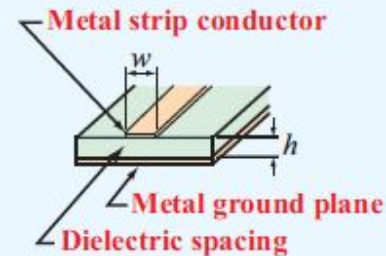
(b) Two-wire line



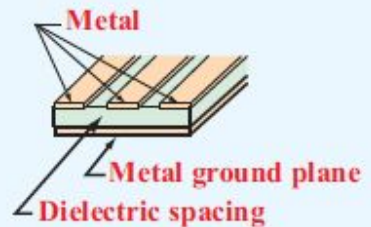
(c) Parallel-plate line



(d) Strip line

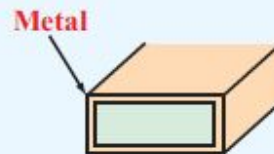


(e) Microstrip line

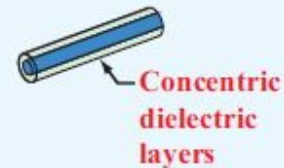


(f) Coplanar waveguide

TEM Transmission Lines



(g) Rectangular waveguide



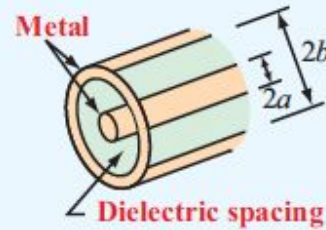
(h) Optical fiber

Higher-Order Transmission Lines

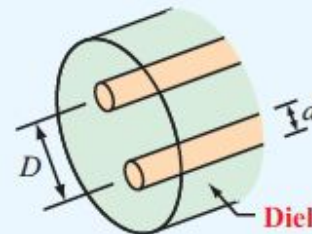
2-1 Types of Transmission Modes

TEM (Transverse Electromagnetic): Electric and magnetic fields are orthogonal to one another, and both are orthogonal to direction of propagation.

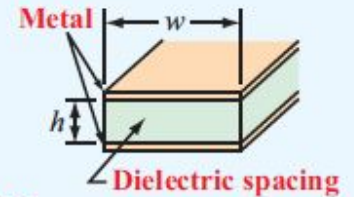
Deal with in later chapters



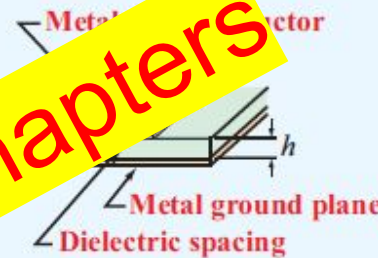
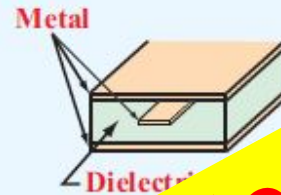
(a) Coaxial line



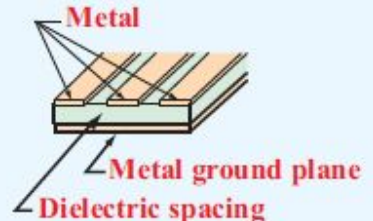
(b) Two-wire line



(c) Parallel-plate line

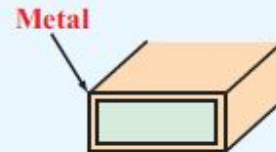


(e) Microstrip line

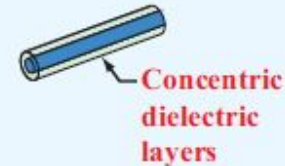


(f) Coplanar waveguide

TEM Transmission Lines



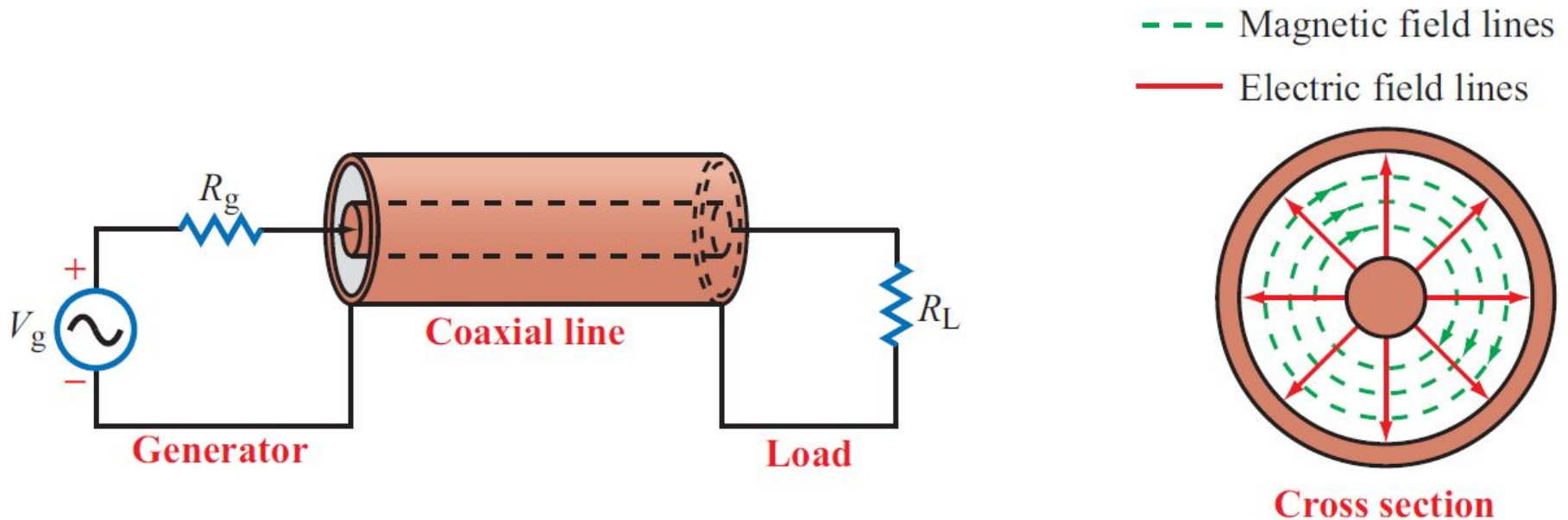
(g) Rectangular waveguide



(h) Optical fiber

Higher-Order Transmission Lines

2-1 Example of TEM Mode



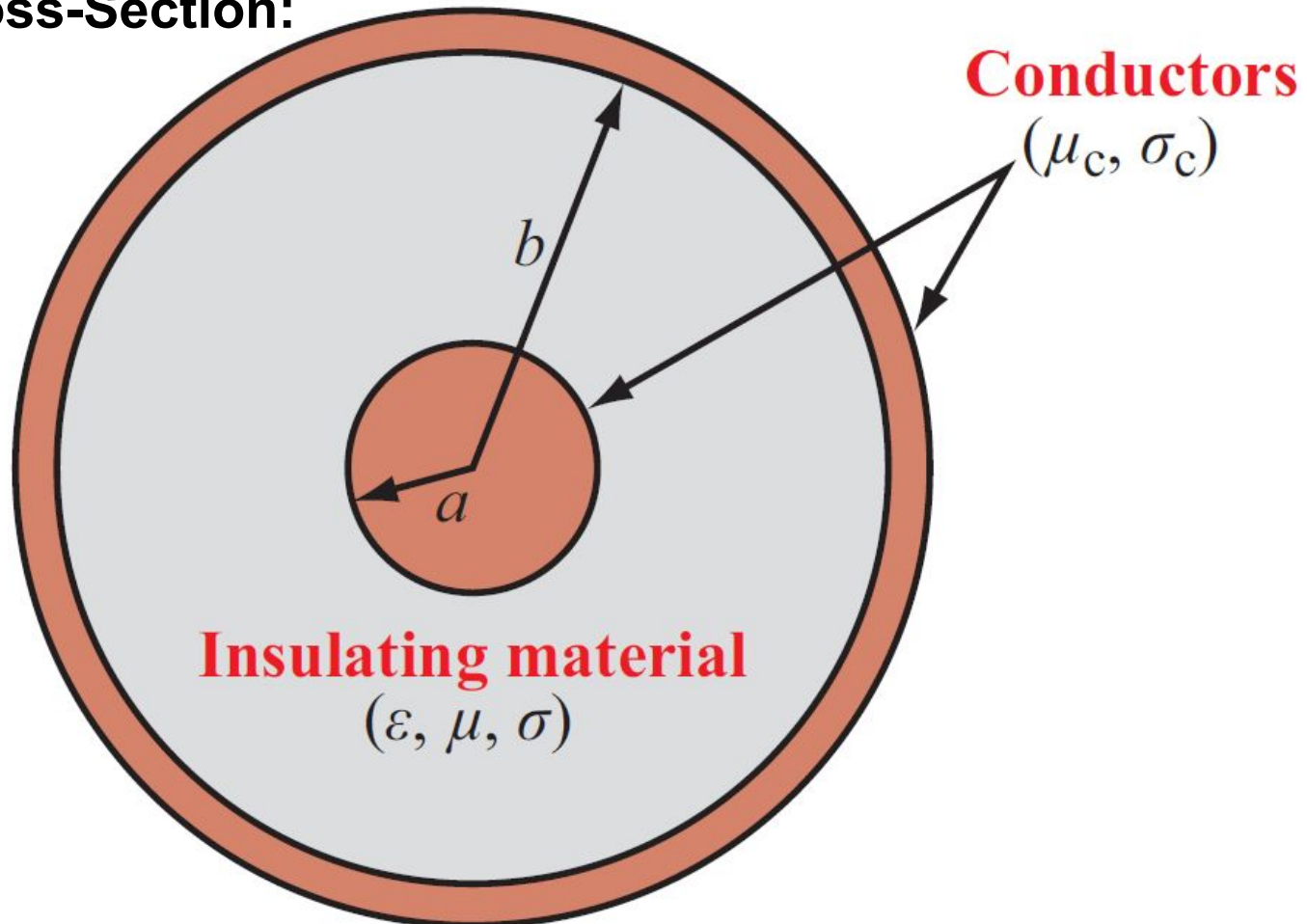
From Physics240:

Current along its length gives rise to **magnetic field**

Voltage between wire and sheath gives rise to **electric field**.

2-2 Transmission Line Model

Coax Cross-Section:



2-2 Transmission Line Model

Modeling Expectations:

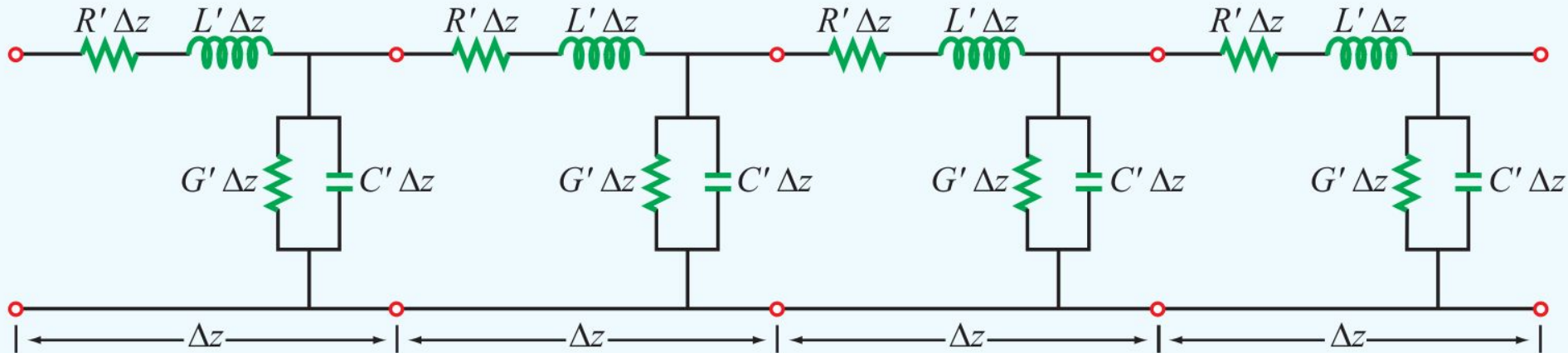
The metal conductors aren't perfect: so resistance

The current in the metal conductors creates a magnetic field, so there must be some inductance.

The insulator is not perfect, so we expect a small amount of leakage current: conductance.

There is some charge separation from one conductor to the other, hence there must be some capacitance.

2-2 Transmission Line Model



All parameters are "per unit length":

R' : Combined Resistance of BOTH conductors: Ω/m

L' : Combined Inductance of BOTH conductors, H/m

G' : Conductance of insulation

between inner and outer conductor, S/m

C' : Capacitance

between inner and outer conductors, F/m

2-2 Transmission Line Model

These expressions will be derived in later chapters:

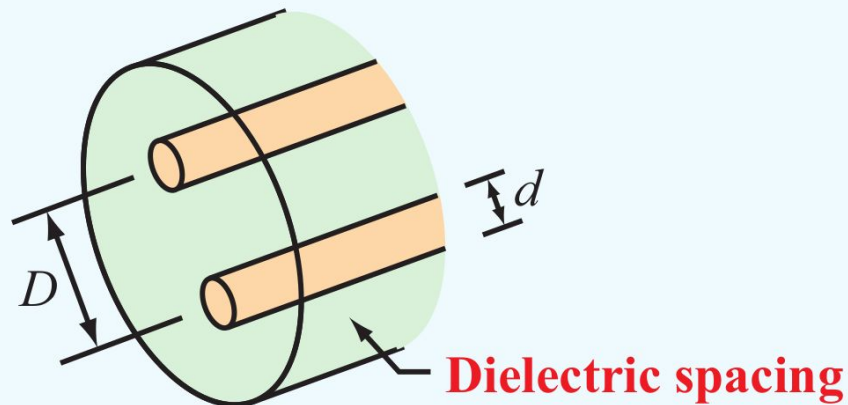
Table 2-1 Transmission-line parameters R' , L' , G' , and C' for three types of lines.

Parameter	Coaxial	Two-Wire	Parallel-Plate	Unit
R'	$\frac{R_s}{2\pi} \left(\frac{1}{a} + \frac{1}{b} \right)$	$\frac{2R_s}{\pi d}$	$\frac{2R_s}{w}$	Ω/m
L'	$\frac{\mu}{2\pi} \ln(b/a)$	$\frac{\mu}{\pi} \ln \left[(D/d) + \sqrt{(D/d)^2 - 1} \right]$	$\frac{\mu h}{w}$	H/m
G'	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\pi\sigma}{\ln \left[(D/d) + \sqrt{(D/d)^2 - 1} \right]}$	$\frac{\sigma w}{h}$	S/m
C'	$\frac{2\pi\epsilon}{\ln(b/a)}$	$\frac{\pi\epsilon}{\ln \left[(D/d) + \sqrt{(D/d)^2 - 1} \right]}$	$\frac{\epsilon w}{h}$	F/m

$$R_s = \sqrt{\pi f \mu_c / \sigma_c}$$

Exercise 2-1

A two-wire copper transmission line is embedded in a dielectric material with $\epsilon_r = 2.6$ and $\sigma = 2 \times 10^{-6}$ S/m. The two wires are separated by 3 cm and their radii are 1 mm each. Calculate the line parameters R' , L' , G' , and C' at 2 GHz.



$$R_s = \sqrt{\pi f \mu_c / \sigma_c}$$

Check Table B-2
(Appendix B) for
copper conductivity

Parameter	Two-Wire
R'	$\frac{2R_s}{\pi d}$
L'	$\frac{\mu}{\pi} \ln \left[(D/d) + \sqrt{(D/d)^2 - 1} \right]$
G'	$\frac{\pi \sigma}{\ln \left[(D/d) + \sqrt{(D/d)^2 - 1} \right]}$
C'	$\frac{\pi \epsilon}{\ln \left[(D/d) + \sqrt{(D/d)^2 - 1} \right]}$

Exercise 2-1

Solution: always do this first, converting as needed:

$$f = 2 \times 10^9 \text{ Hz},$$

$$d = 2 \times 10^{-3} \text{ m},$$

$$D = 3 \times 10^{-2} \text{ m},$$

$$\sigma_c = 5.8 \times 10^7 \text{ S/m (copper)},$$

$$\epsilon_r = 2.6,$$

$$\sigma = 2 \times 10^{-6} \text{ S/m},$$

$$\mu = \mu_c = \mu_0.$$

Exercise 2-1

Solution:

$$\begin{aligned}R_s &= \sqrt{\pi f \mu_c / \sigma_c} \\&= \left[\frac{\pi \times 2 \times 10^9 \text{ Hz} \times 4\pi \times 10^{-7} \text{ H/m}}{5.8 \times 10^7 \text{ S/m}} \right]^{1/2} \\&= 1.17 \times 10^{-2} \Omega\end{aligned}$$

Check the units: $\text{H} = \text{kg m}^2 \text{s}^{-2} \text{A}^{-2}$

$\Omega = \text{kg m}^2 \text{s}^{-3} \text{A}^{-2}$

so: $\text{H}\Omega\text{s}^{-1} = \Omega^2$ good.

Exercise 2-1

Solution:

$$\begin{aligned} R' &= \frac{2R_s}{\pi d} = \frac{2 \times 1.17 \times 10^{-2} \Omega}{\pi \times 2 \times 10^{-3} \text{ m}} \\ &= 3.71 \Omega/\text{m} \end{aligned}$$

Exercise 2-1

Solution:

For convenience, define:

$$\begin{aligned} m &:= \ln \left[(D/d) + \sqrt{(D/d)^2 - 1} \right] \\ &= \ln \left[15 + \sqrt{15^2 - 1} \right] \\ &= \ln [15 + 14.97] \\ &= 3.4 \quad (\text{unitless}) \end{aligned}$$

Exercise 2-1

Solution:

$$\begin{aligned}L' &= \frac{\mu}{\pi} m \\ &= \frac{4\pi \times 10^{-7} \text{ H/m}}{\pi} 3.4\end{aligned}$$

$$L' = 1.36 \times 10^{-6} \text{ H/m}$$

$$\begin{aligned}G' &= \frac{\pi\sigma}{m} \\ &= \frac{\pi \times 2 \times 10^{-6} \text{ S/m}}{3.4}\end{aligned}$$

$$G' = 1.85 \times 10^{-6} \text{ S/m}$$

Exercise 2-1

Solution:

$$\begin{aligned}C' &= \frac{\pi \epsilon}{m} = \frac{\pi \epsilon_r \epsilon_0}{m} \\ &= \frac{\pi \times 2.6 \times 8.85 \times 10^{-12} \text{ F/m}}{3.4} \\ C' &= 21.26 \times 10^{-12} \text{ F/m}\end{aligned}$$

Exercise 2-1

Solution: Summary

$$R' = 3.71 \, \Omega/\text{m}$$

$$L' = 1.36 \times 10^{-6} \, \text{H}/\text{m}$$

$$G' = 1.85 \times 10^{-6} \, \text{S}/\text{m}$$

$$C' = 21.26 \times 10^{-12} \, \text{F}/\text{m}$$

Exercise 2-1

Solution: Summary (with preferred use of prefixes)

$$R' = 3.71 \Omega/\text{m}$$

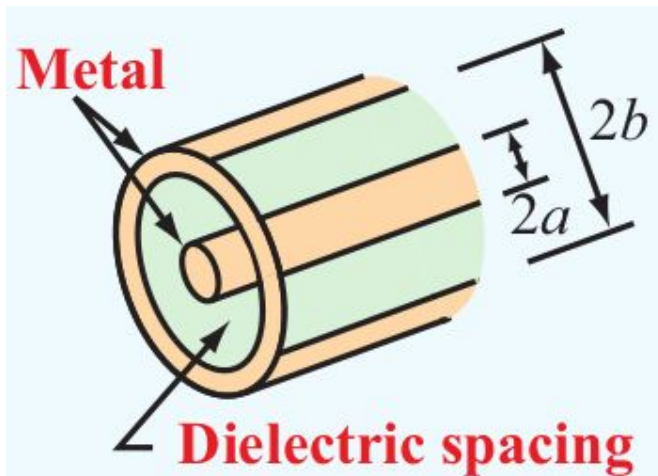
$$L' = 1.36 \mu\text{H}/\text{m}$$

$$G' = 1.85 \mu\text{S}/\text{m}$$

$$C' = 21.26 \text{pF}/\text{m}$$

Exercise 2-2

A coaxial copper transmission line uses air for a dielectric material. The inner conductor diameter is 0.6cm, the outer diameter is 1.2cm. Calculate the line parameters R' , L' , G' , and C' at 1 MHz.



$$R_s = \sqrt{\pi f \mu_c / \sigma_c}$$

Check Table B-2
(Appendix B) for
copper conductivity

Parameter	Coaxial
R'	$\frac{R_s}{2\pi} \left(\frac{1}{a} + \frac{1}{b} \right)$
L'	$\frac{\mu}{2\pi} \ln(b/a)$
G'	$\frac{2\pi\sigma}{\ln(b/a)}$
C'	$\frac{2\pi\epsilon}{\ln(b/a)}$

Exercise 2-2

Solution:

$$f = 1 \times 10^6 \text{ Hz}$$

$$a = 0.3 \text{ cm} = 0.003 \text{ m}$$

$$b = 0.6 \text{ cm} = 0.006 \text{ m}$$

$$\varepsilon = \varepsilon_0$$

$$\mu = \mu_0$$

$$\sigma = 0$$

$$\mu_c = \mu_0$$

$$\sigma_c = 5.8 \times 10^7 \text{ S/m}$$

Exercise 2-2

Solution:

$$R_s = \sqrt{\pi f \mu_c / \sigma_c}$$

$$R_s = \left[\frac{\pi \times 1 \times 10^6 \text{ Hz} \times 4\pi \times 10^{-7} \text{ H/m}}{5.8 \times 10^7 \text{ S/m}} \right]^{1/2}$$

$$R_s = 2.6 \times 10^{-4} \Omega$$

Exercise 2-2

Solution:

$$R' = \frac{R_s}{2\pi} \left(\frac{1}{a} + \frac{1}{b} \right)$$

$$R' = \frac{2.6 \times 10^{-4} \Omega}{2\pi} \left(\frac{1}{0.003 \text{ m}} + \frac{1}{0.006 \text{ m}} \right)$$

$$R' = 2.07 \times 10^{-2} \Omega/\text{m}$$

Exercise 2-2

Solution:

$$L' = \frac{\mu}{2\pi} \ln(b/a)$$

$$L' = \frac{4\pi \times 10^{-7} \text{ H/m}}{2\pi} \ln(0.006 \text{ m}/0.003 \text{ m})$$

$$L' = 0.14 \mu\text{H/m}$$

Exercise 2-2

Solution:

$$G' = \frac{2\pi\sigma}{\ln(b/a)}$$

$$G' = 0$$

because $\sigma = 0$

Exercise 2-2

Solution:

$$C' = \frac{2\pi\epsilon}{\ln(b/a)}$$

$$C' = \frac{(2\pi)(8.85 \times 10^{-12} \text{ F/m})}{\ln 2}$$

$$C' = 80.3 \text{ pF/m}$$

Exercise 2-2

Solution:

$$R' = 20.7 \text{ m}\Omega/\text{m}$$

$$L' = 0.14 \text{ }\mu\text{H}/\text{m}$$

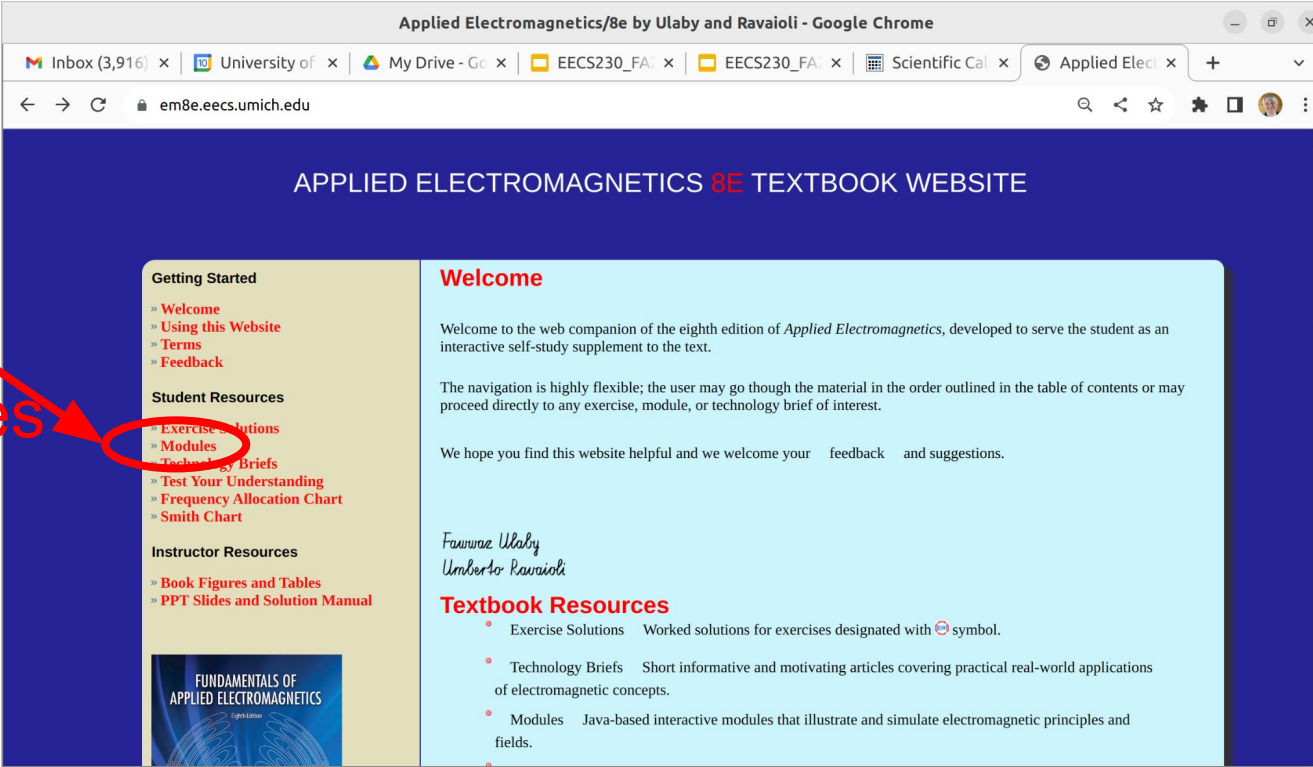
$$G' = 0$$

$$C' = 80.3 \text{ pF}/\text{m}$$

2-2 Transmission-Line Parameters

On-Line Educational materials:

<https://em8e.eecs.umich.edu/>



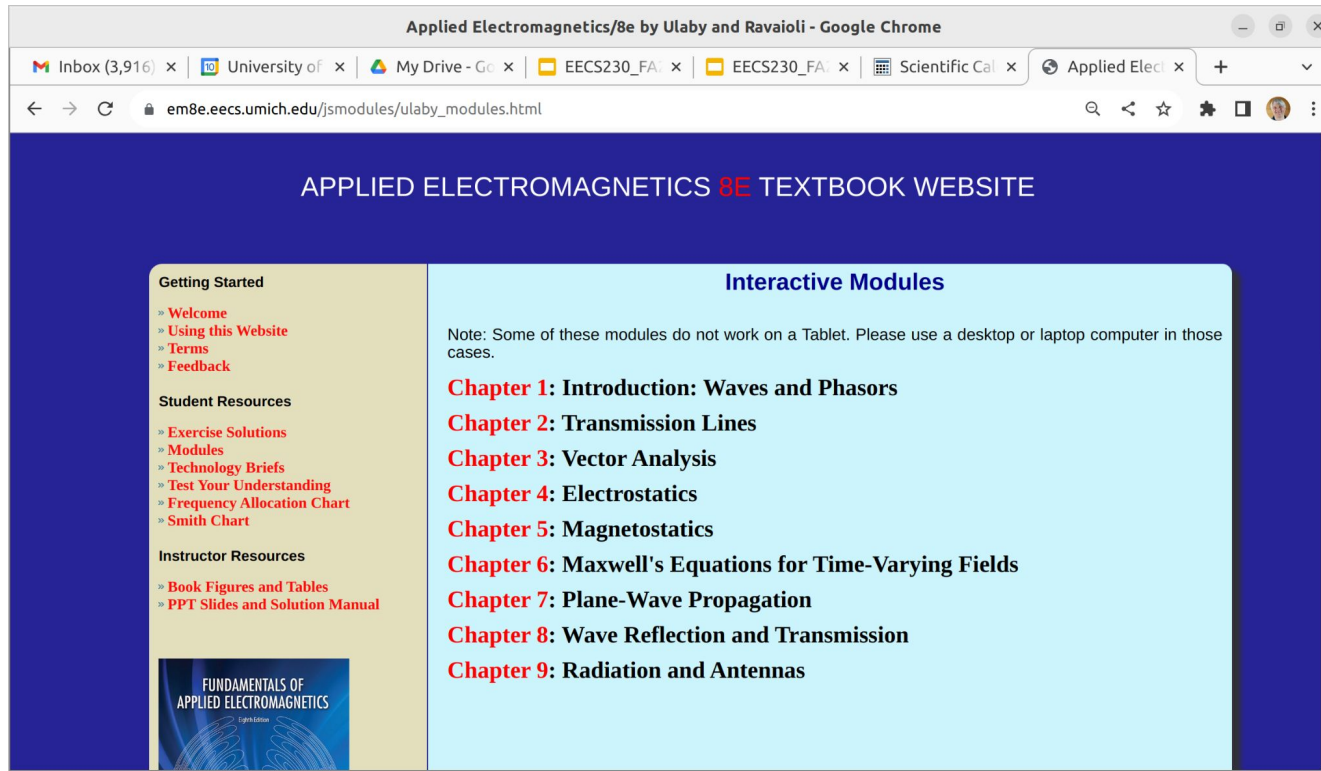
The screenshot shows a web browser window with the title "Applied Electromagnetics/8e by Ulaby and Ravaioli - Google Chrome". The address bar shows "em8e.eecs.umich.edu". The main content area is titled "APPLIED ELECTROMAGNETICS 8E TEXTBOOK WEBSITE". On the left, there is a navigation menu with sections: "Getting Started" (Welcome, Using this Website, Terms, Feedback), "Student Resources" (Exercise Solutions, Modules, Technology Briefs, Test Your Understanding, Frequency Allocation Chart, Smith Chart), and "Instructor Resources" (Book Figures and Tables, PPT Slides and Solution Manual). A red arrow points from the text "CLICK on Modules" to the "Modules" link in the Student Resources section. The main content area has a "Welcome" message, navigation instructions, and a list of "Textbook Resources" including Exercise Solutions, Technology Briefs, and Modules.

CLICK
on
Modules

2-2 Transmission-Line Parameters

On-Line Educational materials:

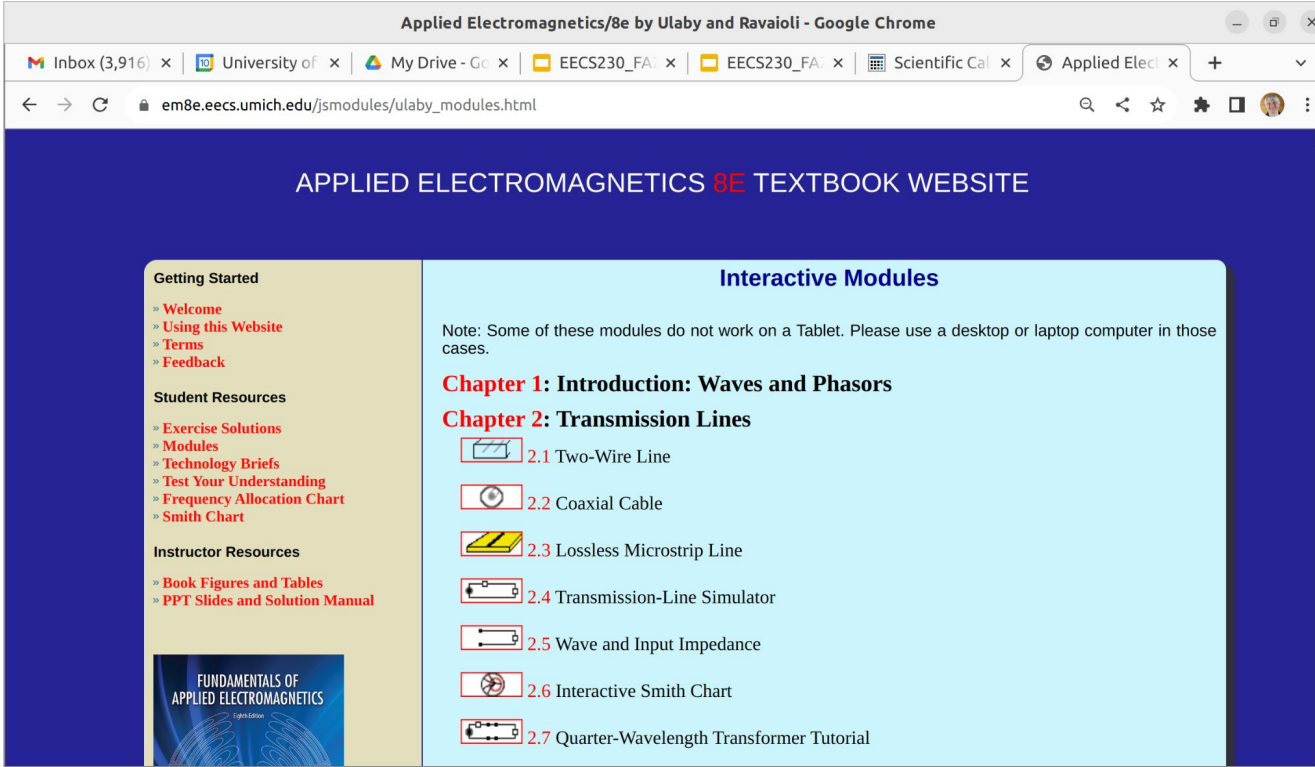
<https://em8e.eecs.umich.edu/>



The screenshot shows a web browser window displaying the website for the 8th edition of the textbook "Applied Electromagnetics" by Ulaby and Ravaioli. The browser's address bar shows the URL em8e.eecs.umich.edu/jsmodules/ulaby_modules.html. The website has a dark blue header with the text "APPLIED ELECTROMAGNETICS 8E TEXTBOOK WEBSITE". The main content area is divided into two columns. The left column, titled "Getting Started", contains links for "Welcome", "Using this Website", "Terms", "Feedback", "Student Resources" (including "Exercise Solutions", "Modules", "Technology Briefs", "Test Your Understanding", "Frequency Allocation Chart", and "Smith Chart"), and "Instructor Resources" (including "Book Figures and Tables" and "PPT Slides and Solution Manual"). At the bottom of this column is a small image of the textbook cover. The right column, titled "Interactive Modules", contains a note about device compatibility and a list of chapters: "Chapter 1: Introduction: Waves and Phasors", "Chapter 2: Transmission Lines", "Chapter 3: Vector Analysis", "Chapter 4: Electrostatics", "Chapter 5: Magnetostatics", "Chapter 6: Maxwell's Equations for Time-Varying Fields", "Chapter 7: Plane-Wave Propagation", "Chapter 8: Wave Reflection and Transmission", and "Chapter 9: Radiation and Antennas".

2-2 Transmission-Line Parameters

On-Line Educational materials:
<https://em8e.eecs.umich.edu/>



The screenshot shows a web browser window with the title "Applied Electromagnetics/8e by Ulaby and Ravaioli - Google Chrome". The address bar shows the URL "em8e.eecs.umich.edu/jsmodules/ulaby_modules.html". The page content is titled "APPLIED ELECTROMAGNETICS 8E TEXTBOOK WEBSITE".

Getting Started

- » [Welcome](#)
- » [Using this Website](#)
- » [Terms](#)
- » [Feedback](#)

Student Resources

- » [Exercise Solutions](#)
- » [Modules](#)
- » [Technology Briefs](#)
- » [Test Your Understanding](#)
- » [Frequency Allocation Chart](#)
- » [Smith Chart](#)

Instructor Resources








- » [Book Figures and Tables](#)
- » [PPT Slides and Solution Manual](#)

Interactive Modules

Note: Some of these modules do not work on a Tablet. Please use a desktop or laptop computer in those cases.

Chapter 1: Introduction: Waves and Phasors

Chapter 2: Transmission Lines

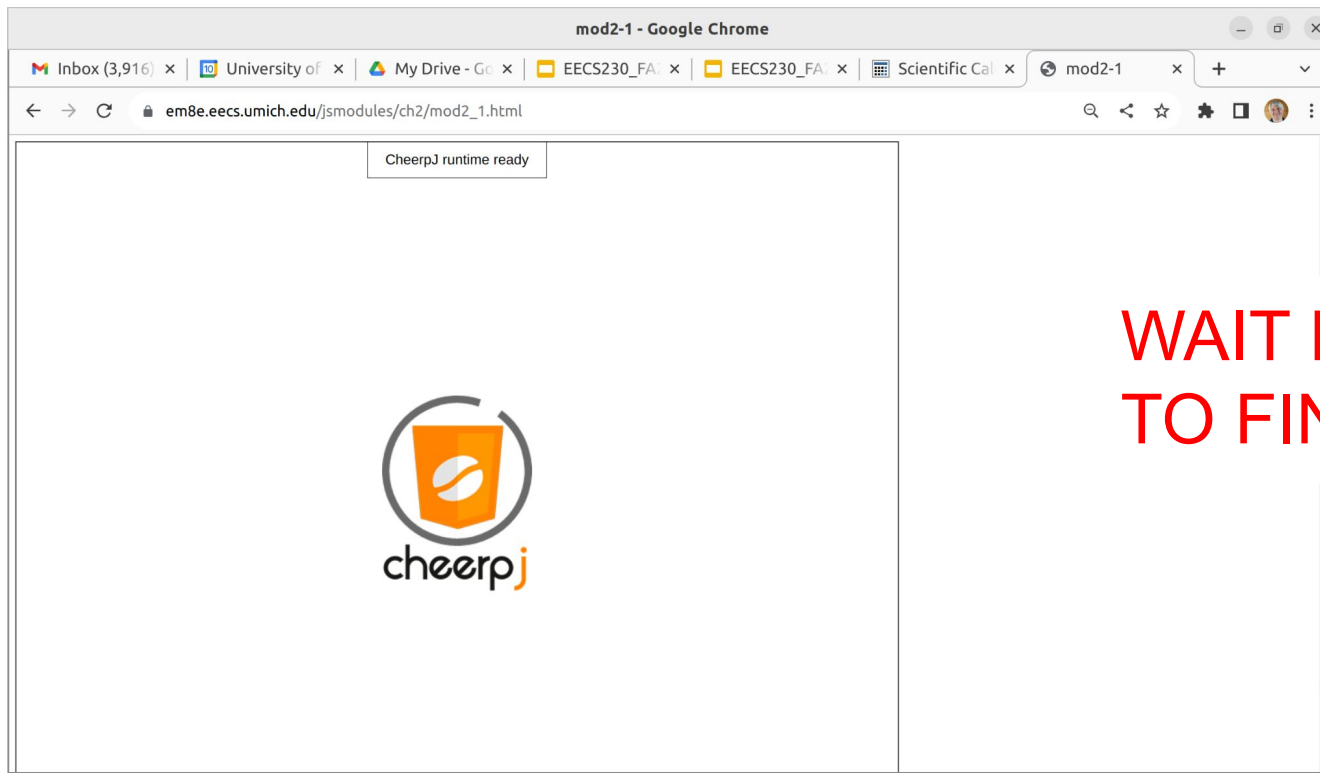
-  [2.1 Two-Wire Line](#)
-  [2.2 Coaxial Cable](#)
-  [2.3 Lossless Microstrip Line](#)
-  [2.4 Transmission-Line Simulator](#)
-  [2.5 Wave and Input Impedance](#)
-  [2.6 Interactive Smith Chart](#)
-  [2.7 Quarter-Wavelength Transformer Tutorial](#)

FUNDAMENTALS OF APPLIED ELECTROMAGNETICS
EIGHTH EDITION

2-2 Transmission-Line Parameters

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**WAIT FOR THIS
TO FINISH**

2-2 Transmission-Line Parameters

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The screenshot shows a web browser window titled "mod2-1 - Google Chrome" with the URL "em8e.eecs.umich.edu/jsmodules/ch2/mod2_1.html". The main content area displays "Module 2.1 Two-Wire Line".

Diagram: A 3D perspective view of two parallel wires of diameter d separated by a distance D between their centers. They are mounted on a substrate with relative permittivity $\epsilon_r = 2.3$ and conductivity $\sigma = 0.0$ [S/m]. The wires have a conductivity $\sigma_c = 5.8E7$ [S/m].

Graph: A plot titled "Real Part of Characteristic Impedance" showing Z_0 [Ω] on the y-axis (ranging from 0.0 to 213.0) versus D [mm] on the x-axis (ranging from 1.0 to 11.0). A yellow curve shows the relationship, with a red dot at approximately $D = 10.5$ mm and $Z_0 = 213.0$ Ω .

Instructions:

Input

- d = wire diameter
- D = distance between wires' centers
- ϵ_r = relative permittivity of substrate
- σ = conductivity of substrate
- σ_c = conductivity of wire material
- $\mu = \mu_0$ for both substrate and wires

Displayed Information

Line Parameters C' , L' , R' , G'
 Z_0 = characteristic impedance
 α = attenuation constant
 β = wave number
 $\lambda = \lambda_0 / \sqrt{\epsilon_r}$

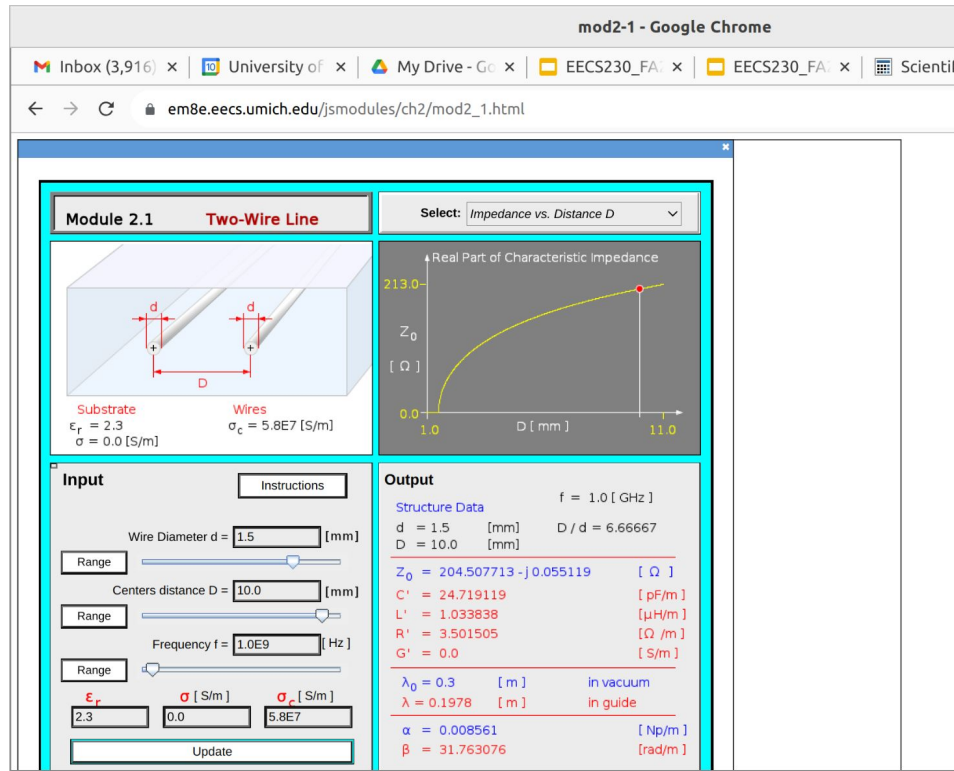
Application Design: Umberto Ravaioli
Interactive Java™ platform: www.amanogawa.com
All Rights Reserved

A "CLOSE" button is visible at the bottom right of the module window.

A Fast, Easy way to calculate the transmission-line parameters for a 2-wire line

Transmission-Line Parameters

On-Line Educational materials:
<https://em8e.eecs.umich.edu/>



Vary the geometry
and material
properties with
sliders:
Immediately see
the R, L, G, C
params of the
resulting line.

Transmission-Line Parameters

On-Line Educational materials:
<https://em8e.eecs.umich.edu/>

For a coax too.

The screenshot shows a web browser window displaying an educational module titled "Module 2.2 Coaxial Cable". The interface is divided into several sections:

- Diagram:** A cross-sectional diagram of a coaxial cable with inner radius a and outer radius b . Parameters shown include $\sigma = 0.0$ S/m, $\epsilon_r = 2.3$, and frequency $f = 1.0$ GHz.
- Select:** A dropdown menu set to "Impedance vs. Radius b".
- Graph:** A plot titled "Real Part of Characteristic Impedance" showing Z_0 [Ω] on the y-axis (0.0 to 103.0) versus b [mm] on the x-axis (1.0 to 20.0). A red dot marks the current value of $b = 10.0$ mm.
- Input Section:** Fields for "Inner radius a = 1.5 [mm]", "Shield radius b = 10.0 [mm]", and "Frequency f = 1.0E9 [Hz]". Material properties $\epsilon_r = 2.3$, $\sigma = 0.0$ [S/m], and $\sigma_c = 5.8E7$ [S/m] are also shown.
- Output Section:** "Structure Data" showing $a = 1.5$ [mm], $b = 10.0$ [mm], and $b/a = 6.66667$. Calculated parameters include:
 - $Z_0 = 75.055434 - j0.01584674$ [Ω]
 - $C' = 67.353556$ [pF/m]
 - $L' = 379.423997$ [nH/m]
 - $R' = 1.006683$ [Ω/m]
 - $G' = 0.0$ [S/m]
 - $\lambda_0 = 0.3$ [m] in vacuum
 - $\lambda = 0.1978$ [m] in guide
 - $\alpha = 0.006706$ [Np/m]
 - $\beta = 31.763075$ [rad/m]

Homework

74

Homework 3 is due tomorrow at midnight.

submit to gradescope via the canvas site.

Next Time

Sections 2-3, 2-4, and 2-5:

Form the governing differential equation

Get the form of the solution as traveling waves.

Specify **source** and **load** and solve for the 2 unknowns, giving us a **complete solution**.