EECS 230
ENGINEERING ELECTROMAGNETICS
Leland Pierce

Review of Circuits

Announcements

Use this email when asking questions to the staff: eecs230-staff@umich.edu

Any email directed personally to me or a TA will be forwarded to this list.

Lab 1 starts on Monday, Sept 16
Pre-Lab 1 is due Sunday, Sept 15, midnight

Announcements

Almost done grading homework 1.

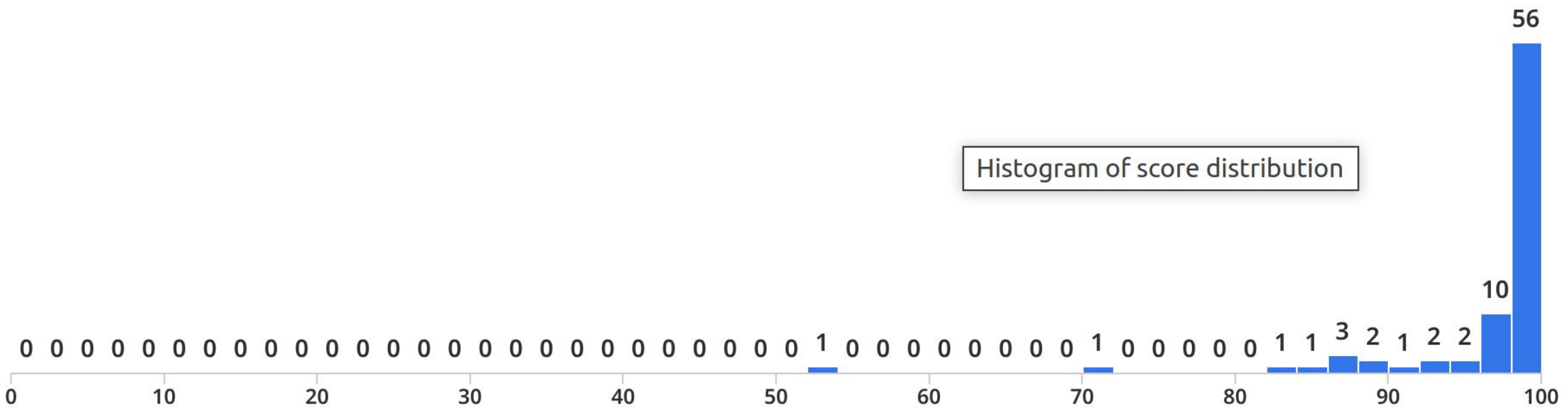
Did pretty well.

From now on: answer without work = 0 pts

Please try to make things neat and readable

I'm guessing that some grades would have been better if you had checked your work.

Announcements



Outline

- Static Circuits (d/dt=0)
  - Diagrams, Current, Voltage
  - Sign conventions, Ground
  - Open/Short circuit
  - Power, sign convention

  - Ohms Law, sign convention
  - Nodes, Loops
  - KVL, KCL
  - Resistors in parallel and series
  - Source Transformation

Outline

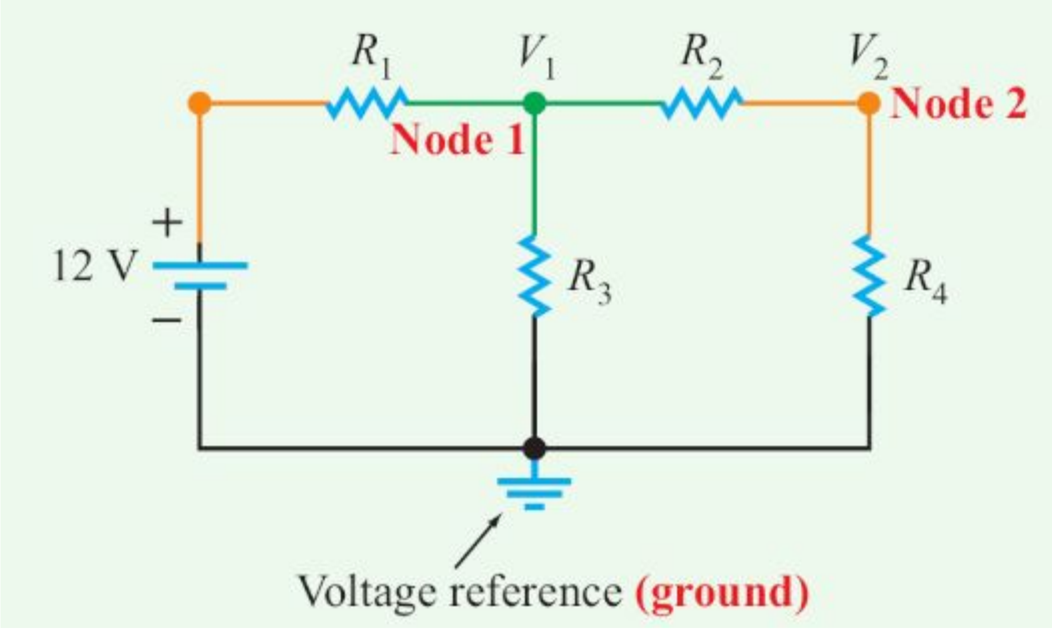
- Transient Circuits
  - Capacitors
  - Inductors

- Time-Harmonic Circuits
  - Phasors
  - Impedance
  - Parallel/Series
  - Circuit solution procedure

This involves topics from chapters 1,2, 5, 7 of the book: Circuit Analysis and Design by Ulaby, Maharbiz, Furse, that is used in EECS 215.

Static Circuits: Diagrams

- A circuit diagram represents a mathematical model for a circuit.
- Often, there is a correspondence between physical circuit elements and elements in the diagram.
- But there are times when this is not true.
- All that matters is that the mathematical model produces the "correct answer".



Static Circuits: Diagrams

- Over the years there have been very many problems solved using circuits, and each one has a particular mathematical model that is known to work.
- So, we use the **existing** circuit diagrams for these problems.
- New applications may require new models and new circuit diagrams in order to solve them and produce the "correct answer".

Static Circuits: Diagrams

- Another aspect that really matters is the actual physical layout of the circuit.

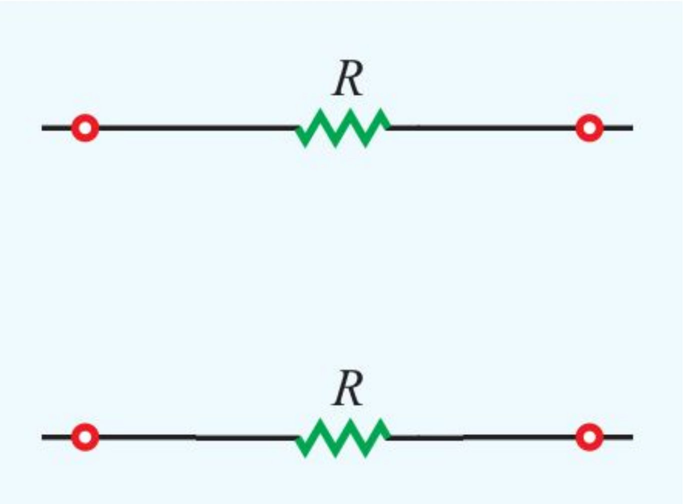
For Example:

- If you have 2 wires "far" from each other, those 2 wires don't interact: can draw the circuit one way.
- If you have 2 wires "close" to each other, those 2 wires interact: must draw the circuit another way
- It is required to model that interaction on order to get the "correct answer".

Static Circuits: Diagrams

- For Example:
- If you have 2 wires "far" from each other, those 2 wires don't interact: can draw the circuit one way.

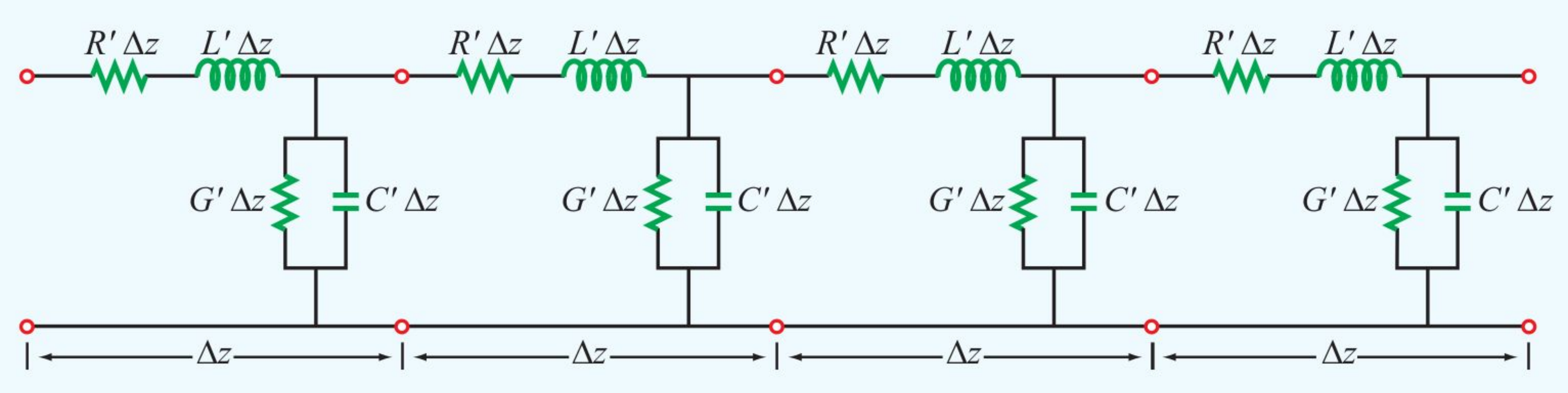
2 wires, each 1m long, separated by 100 m, carrying a current with frequency 1GHz:



Static Circuits: Diagrams

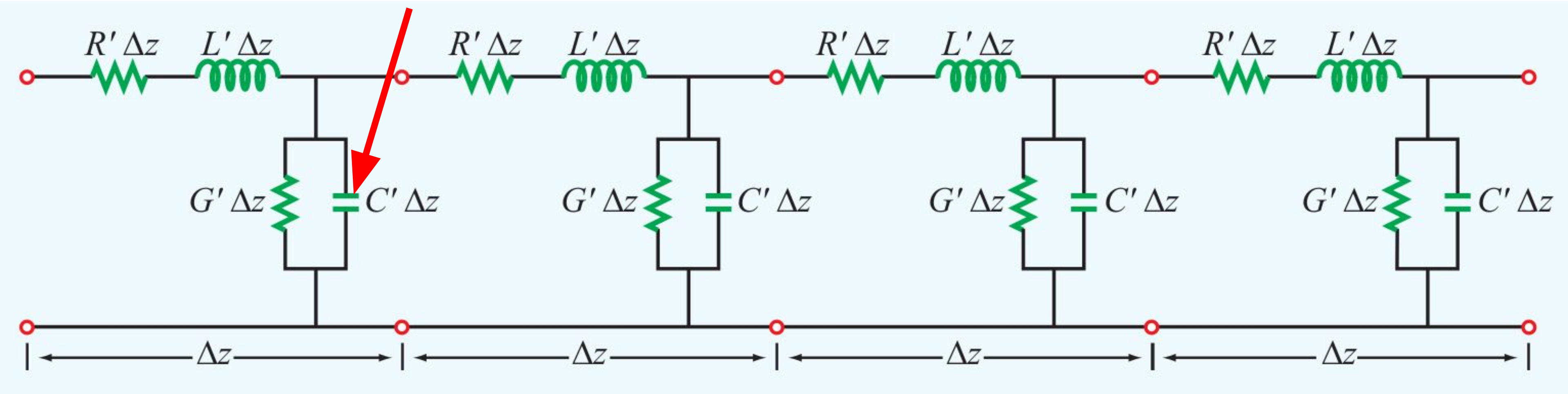
- For Example:
- If you have 2 wires "close" to each other, those 2 wires interact: must draw the circuit another way
-

2 wires, each 1m long, separated by 1 mm, carrying a current with frequency 1GHz:



Static Circuits: Diagrams

- Example of a model for a transmission line.
- Accounts for:
  - resistance in conductors (current)
  - inductance between conductors (magnetic field)
  - resistance through the insulation (current)
  - capacitance between the conductors (electric field)

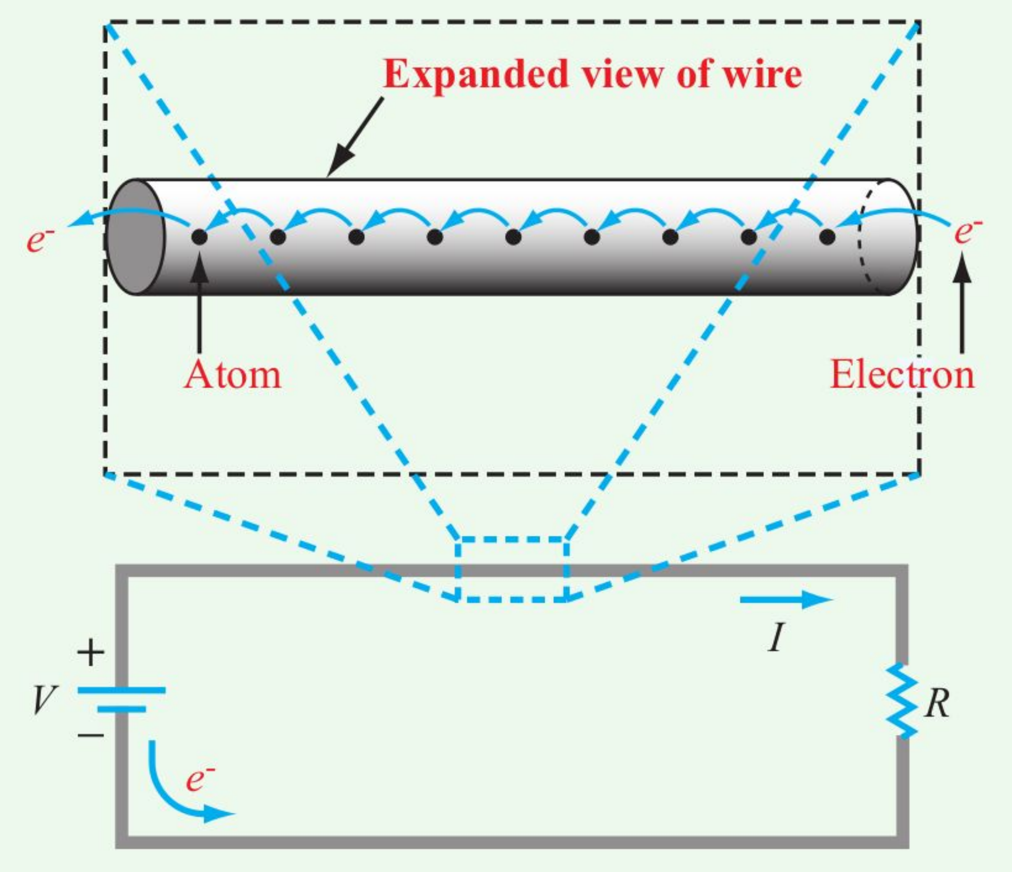


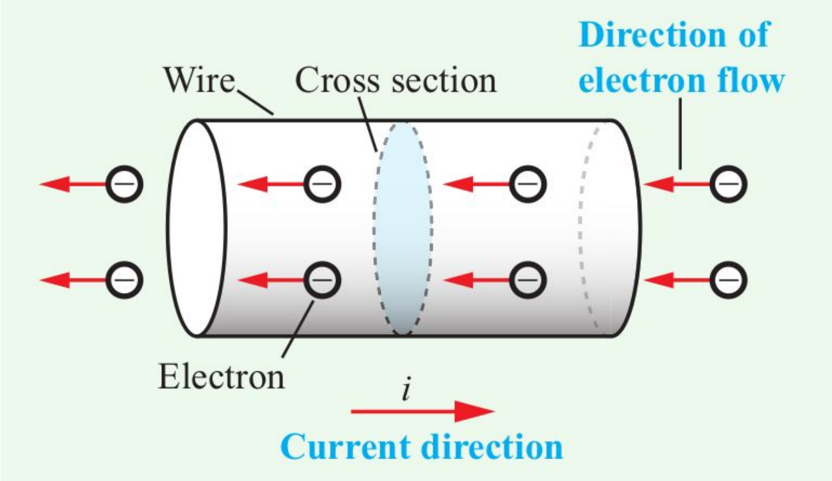
Static Circuits: Diagrams

- With more experience, can start creating new circuit diagrams for our own applications.

Static Circuits: Current

- Flow of **positive** charges
- Charge carriers are typically electrons (-)
- Current flow is opposite electron flow:





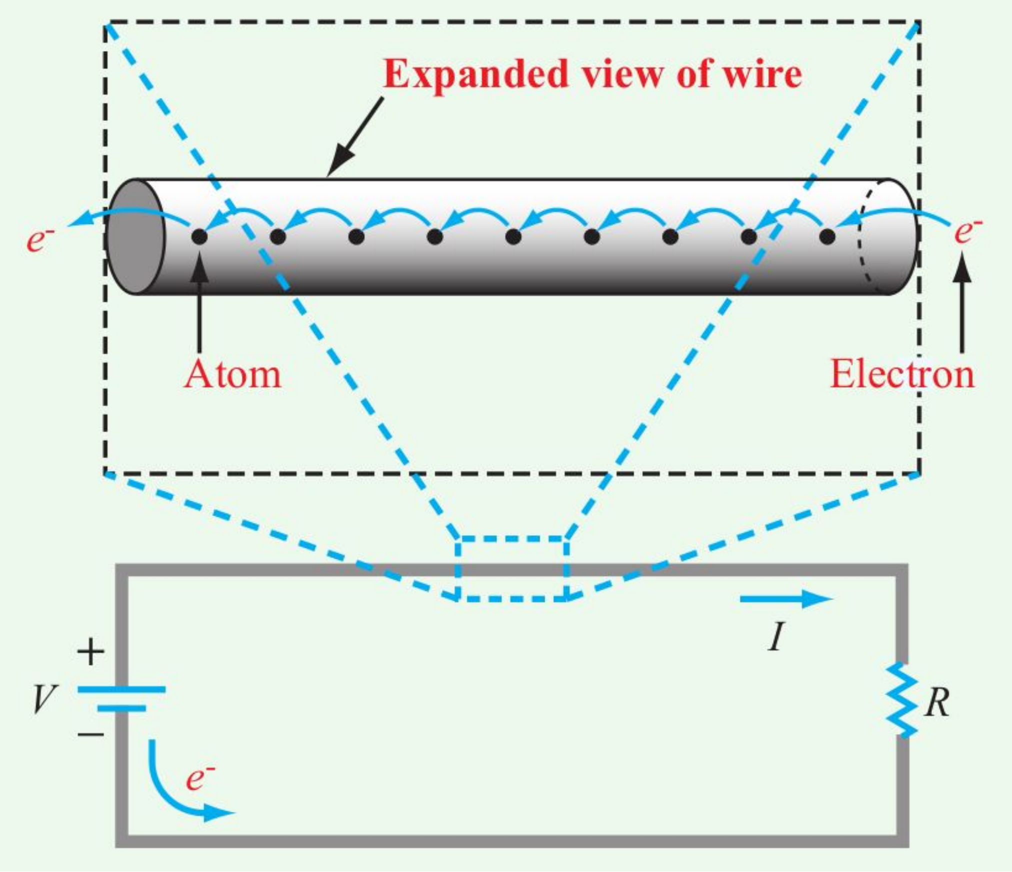
Static Circuits: Current

Current Units: ampere: A

The other electrical units are all defined in terms of the ampere:

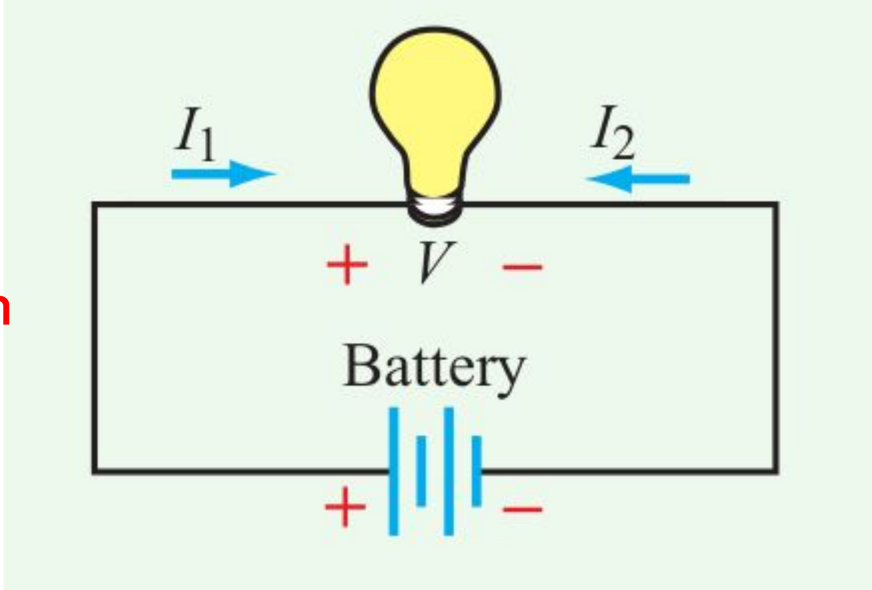
1 A = 1 Coloumb/sec

1 A = 1 Watt/Volt



Static Circuits: Current Sign Convention

The direction **defined** for a value of current determines its **sign**:
You choose the reference direction



We do not know the sign of the defined current until we have solved the circuit

You choose to work with I_1 or I_2 in the circuit (**not both**). The solution will give the sign, which then gives you the direction.

Static Circuits: Current Sign Convention

Lesson:

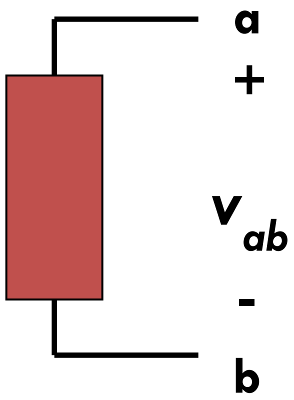
Draw the direction of all currents on a circuit diagram BEFORE start building equations.

Static Circuits: Voltage

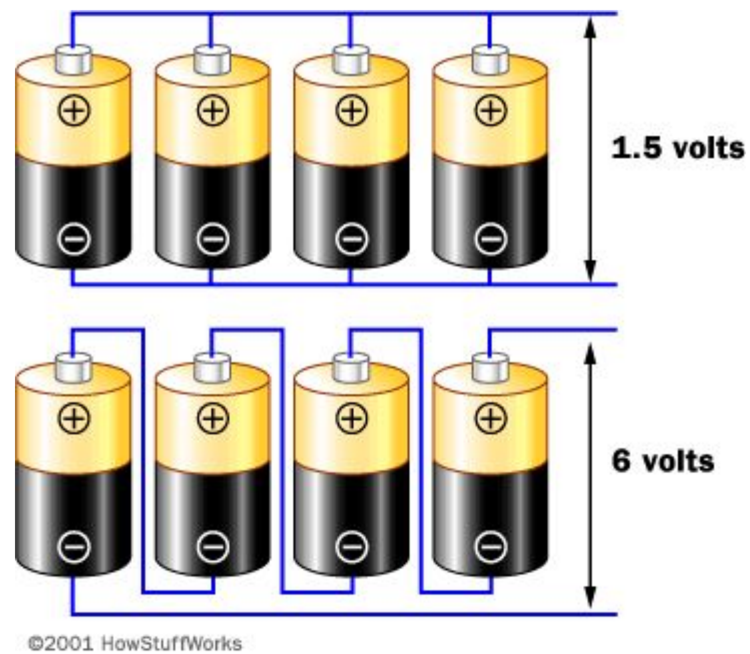
Voltage, potential difference ***between 2 points***

Energy required to move charge through an element

Work done = Force x distance = Charge x Potential



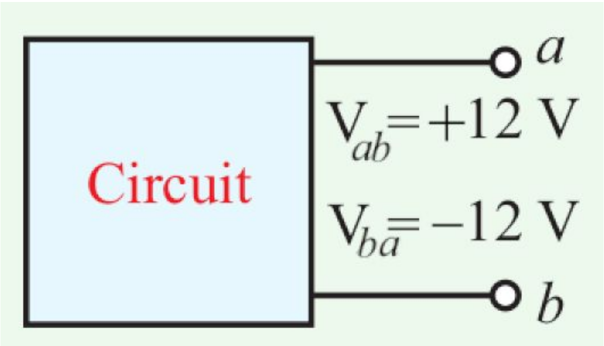
$v_{ab} = -v_{ba}$

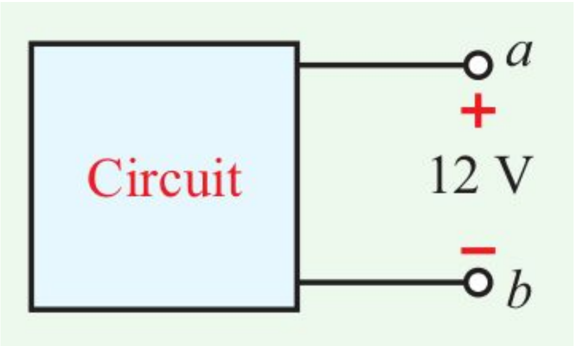


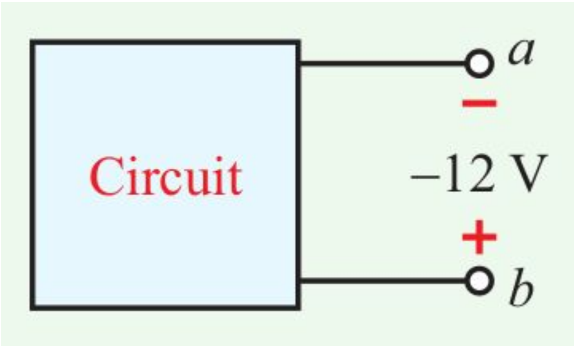
Units: Volts

Static Circuits: Voltage Sign Convention

- Signs Depend on ARBITRARY Choice of which terminal is "+" and which is "-"
- All 3 Pictures (and 4 cases) below have the SAME PHYSICAL MEANING

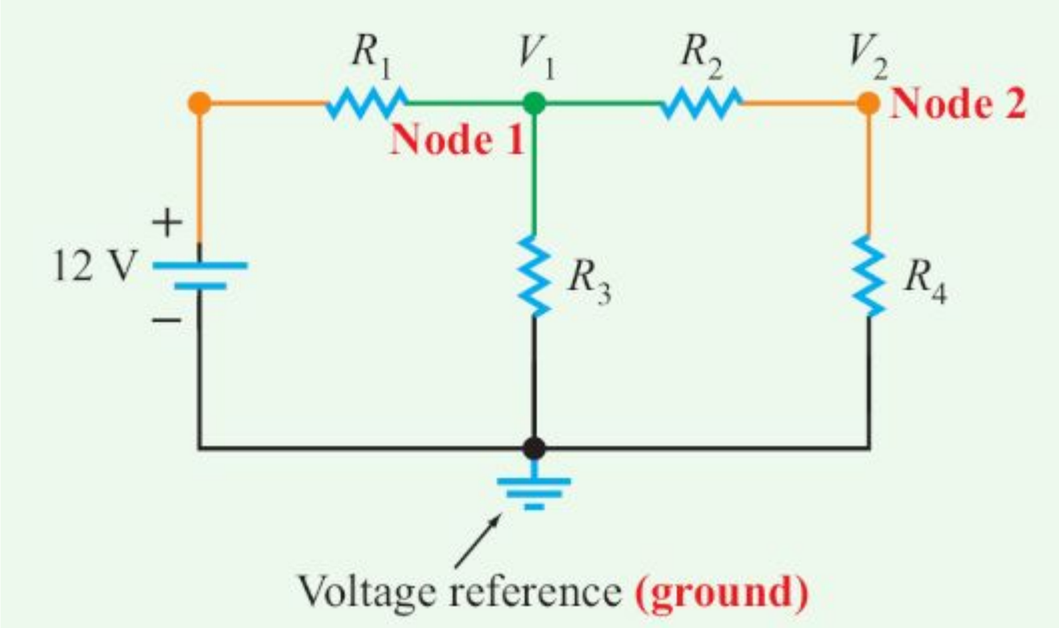






Static Circuits: Ground

- Choose reference point for potential
- Assign potential at reference = 0V, called ground
- Now all potentials can be specified relative to ground terminal



Static Circuits: Voltage Sign Convention

Lesson:

Choose one procedure:

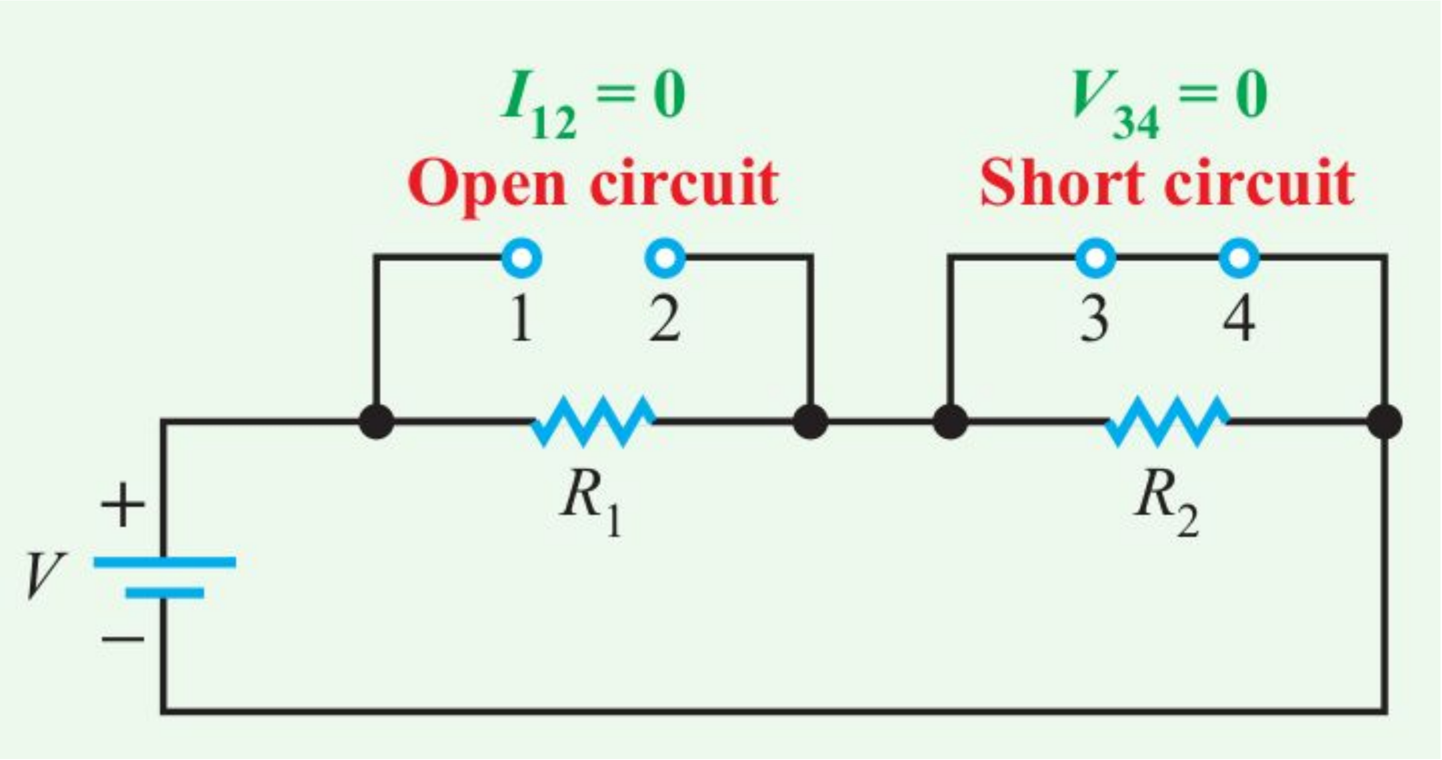
1. Pick a node and label it ground.
   Label all other voltages relative to that node.

OR

2. place + and - around each circuit element,
defining the polarity of each voltage variable.

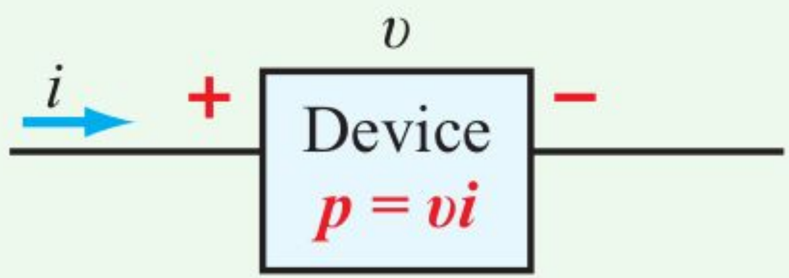
Static Circuits: Open and Short

- Open circuit: no path for current flow ($R=\infty$)
- Short circuit: no voltage difference ($R=0$)



Static Circuits: Power

Passive Sign Convention

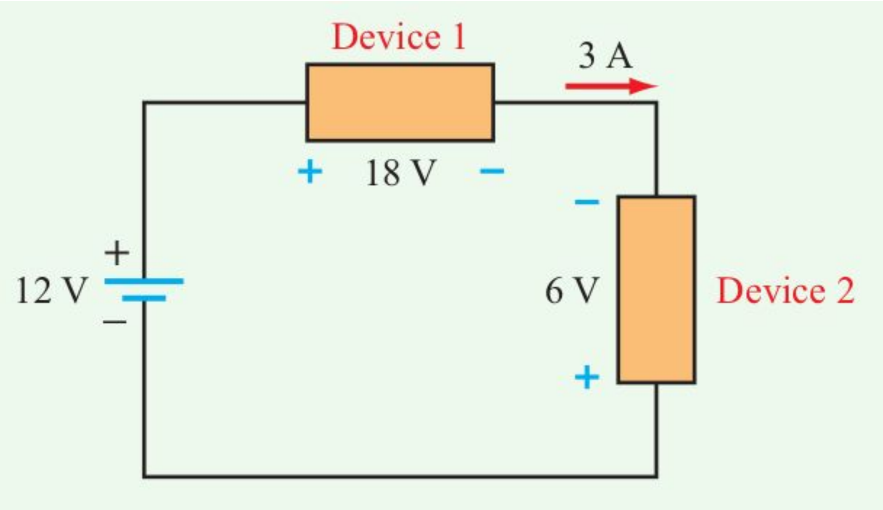


$p > 0$	power delivered to device
$p < 0$	power supplied by device

Note that i direction is defined as entering (+) side of v.

Example 1-5b: Conservation of Power

Determine the power for each device, and if it is supplying or consuming power.



Solution:

Device 1 has a current going into the positive voltage terminal.

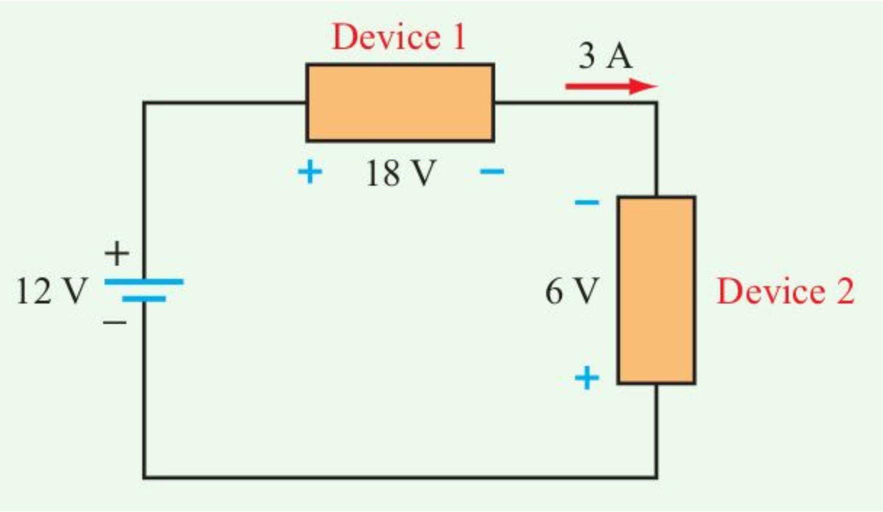
So, the passive sign convention is followed.

So the equation for power is:

$p = i\,v$ = (3A)(18V) = 54 W, consuming power since >0

Example 1-5b: Conservation of Power

Determine the power for each device, and if it is supplying or consuming power.



Solution:

Device 2 has a current going into the negative voltage terminal.

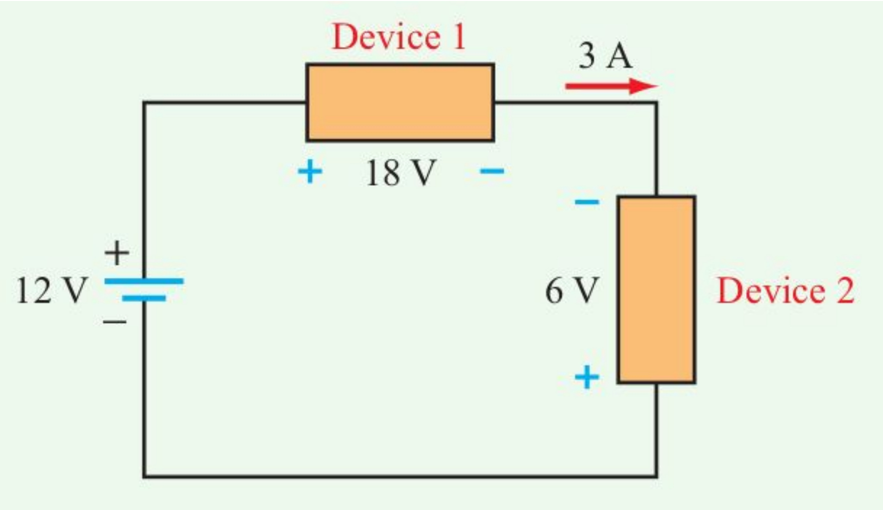
So, the passive sign convention is NOT followed.

So the equation for power is:

$p = - i\, v = - (3A)(6V) = -18$ W, supplying power since <0

Example 1-5b: Conservation of Power

Determine the power for each device, and if it is supplying or consuming power.



Solution:

The power supply has a current going into the negative voltage terminal.

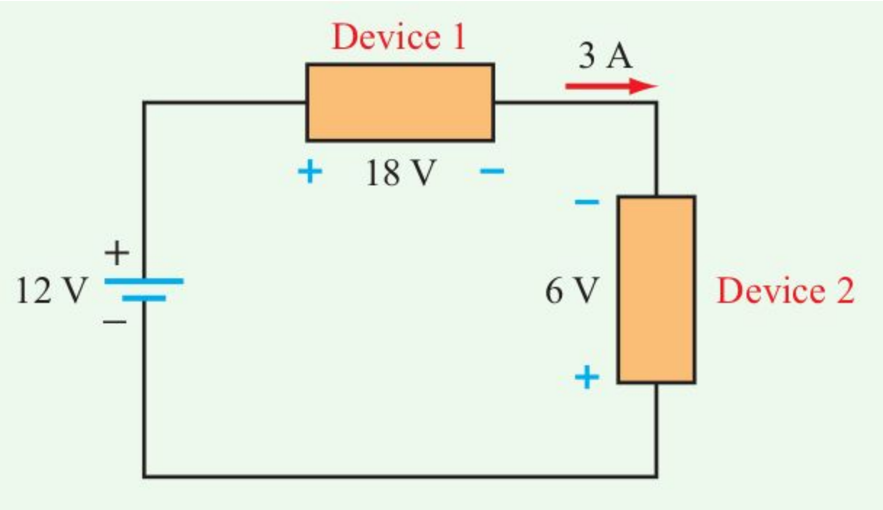
So, the passive sign convention is NOT followed.

So the equation for power is:

$p = -\,i\,v = -\,(3A)(12V) = -36\ W$, supplying power since <0

Example 1-5b: Conservation of Power

Determine the power for each device, and if it is supplying or consuming power.



Solution:

Check:

Is the law of Conservation of Power valid?

$$\sum_{k=1}^{n} p_k = 0,$$

require:

pwr(device1) + pwr(device2) +pwr(battery) = 0

get:

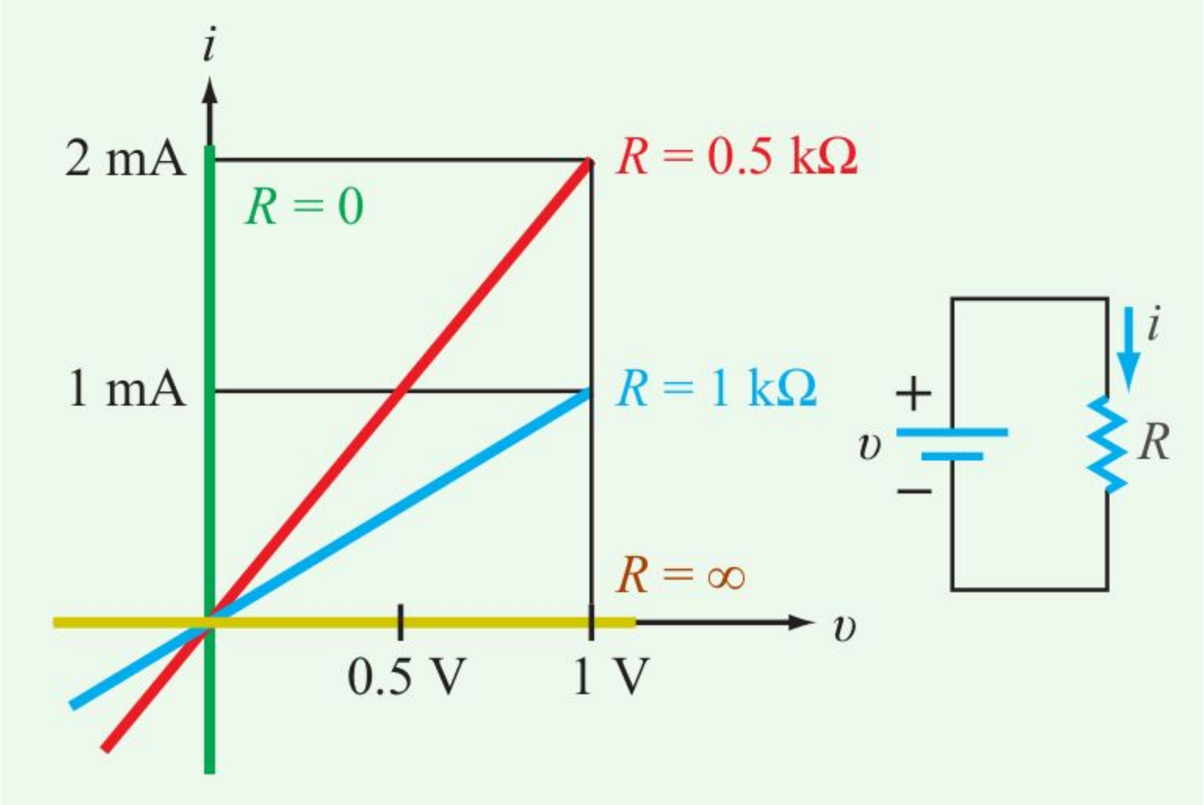
54W + -18W + -36W = 0 ✅

Static Circuits: Ohm's Law

Voltage across resistor is proportional to current

$$v = iR,$$

$$R = \frac{v}{i}$$



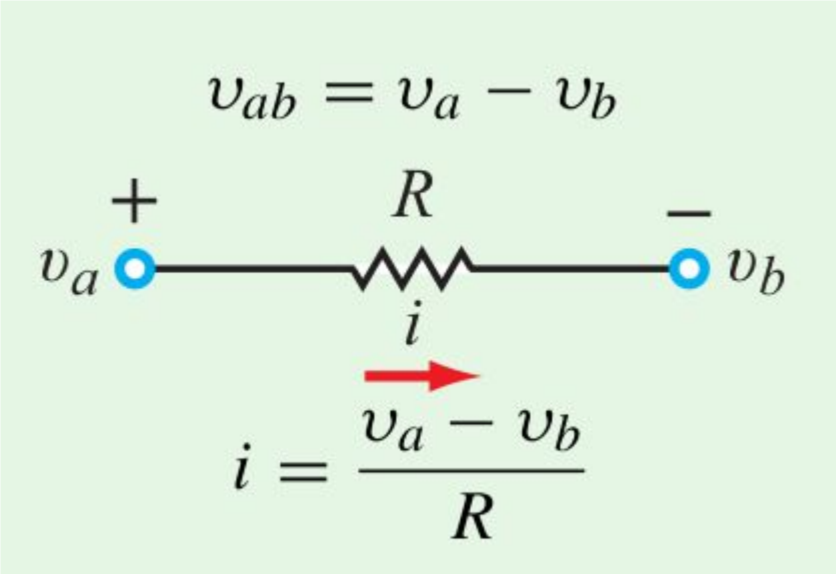
Resistance: ability to resist flow of electric current

Static Circuits: Ohm's Law Sign Convention

- Resistors are not polarized

  Can flip their direction and they work the same

- But the polarities of current and voltage definitions are important for $V = IR$ to hold



Conductance

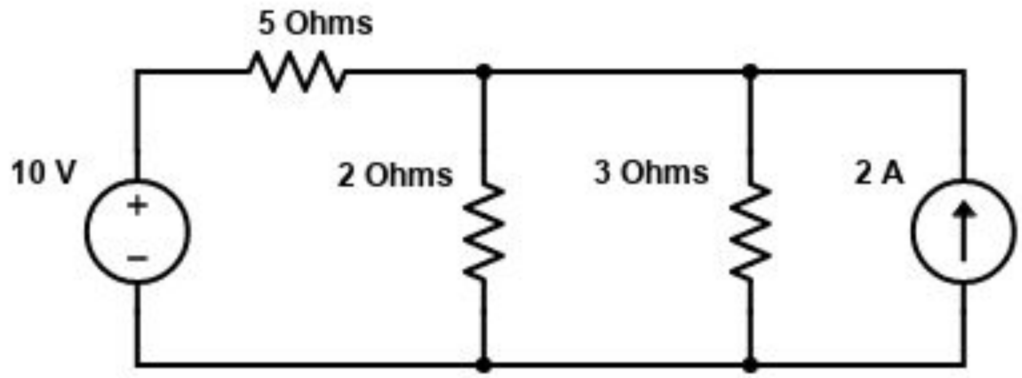
$$G = \frac{1}{R} = \frac{i}{v}$$

Static Circuits: Nodes, Loops

Branch: single element, such as a resistor or source

Node: connection point between two or more branches

Loop: closed path in a circuit



Static Circuits: Nodes, Loops

Branch: single element, such as a resistor or source

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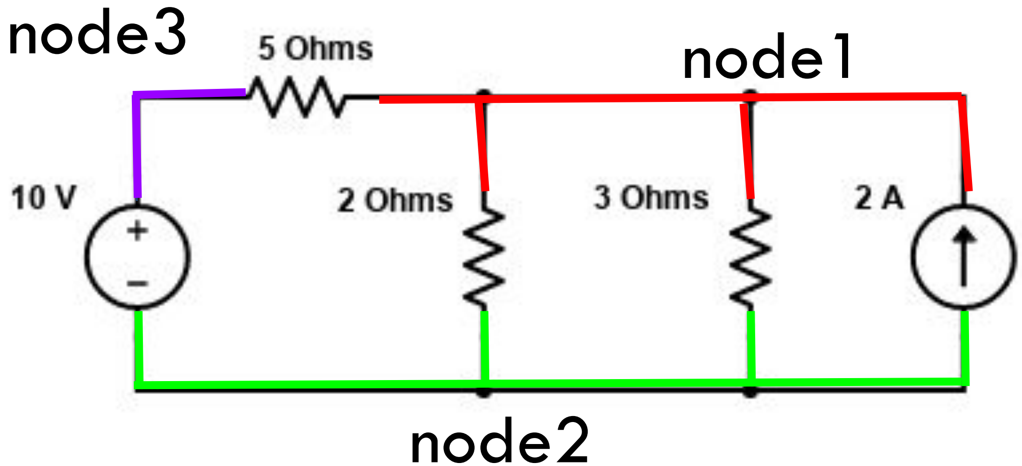
Loop: closed path in a circuit



Static Circuits: Nodes, Loops

Branch: single element, such as a resistor or source

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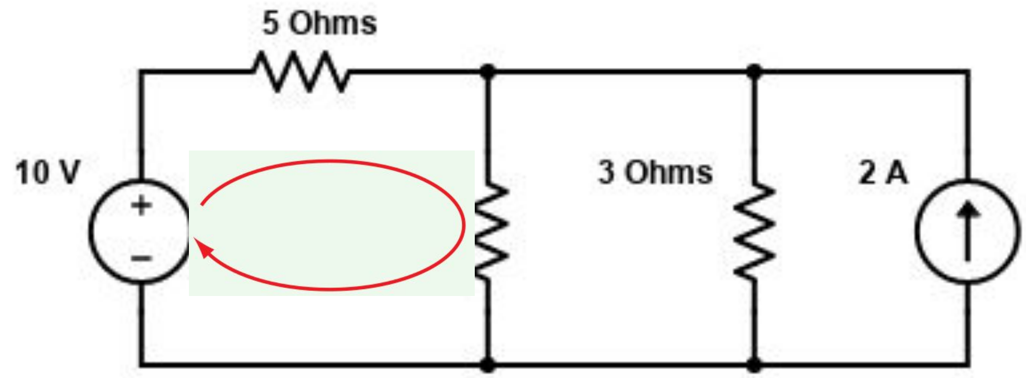


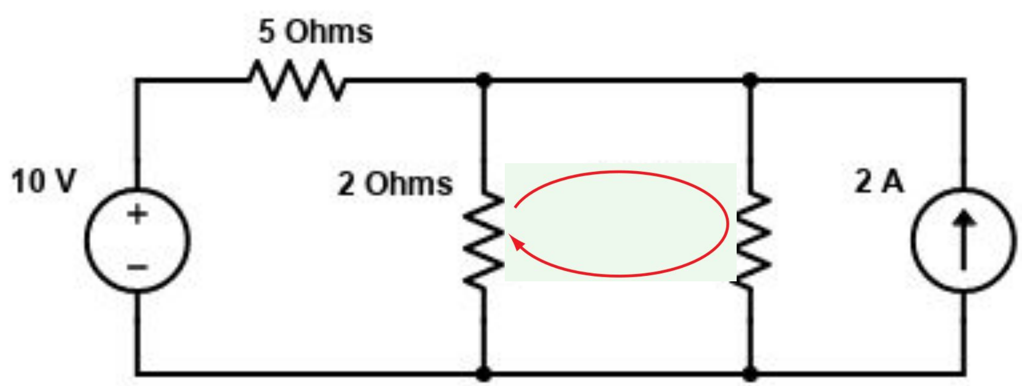
loop1

Static Circuits: Nodes, Loops

Branch: single element, such as a resistor or source

Node: connection point between two or more branches

Loop: closed path in a circuit



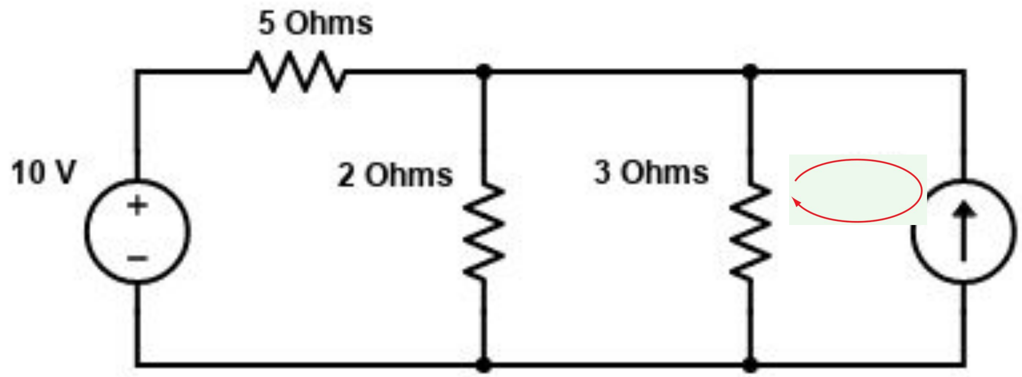
loop2

Static Circuits: Nodes, Loops

Branch: single element, such as a resistor or source

Node: connection point between two or more branches

Loop: closed path in a circuit



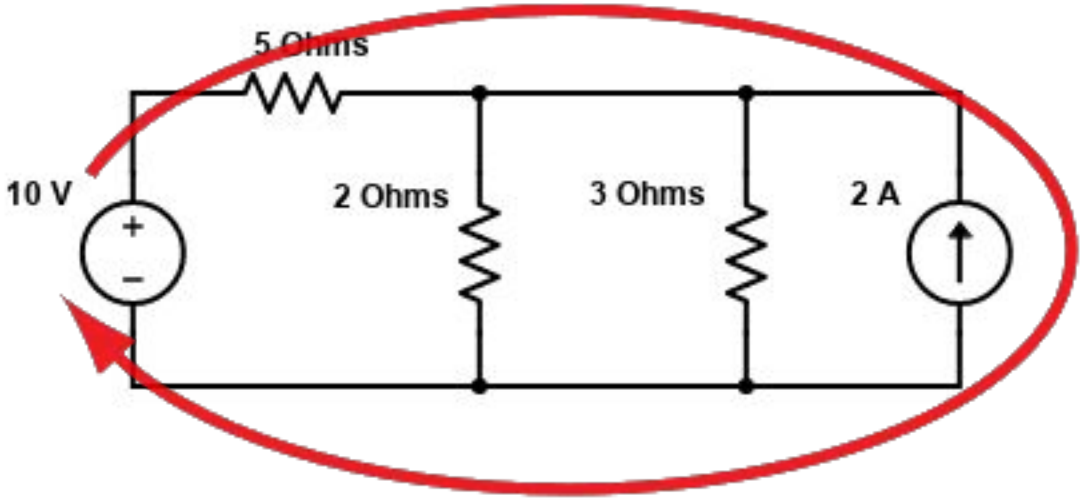
loop3

Static Circuits: Nodes, Loops

Branch: single element, such as a resistor or source

Node: connection point between two or more branches

Loop: closed path in a circuit



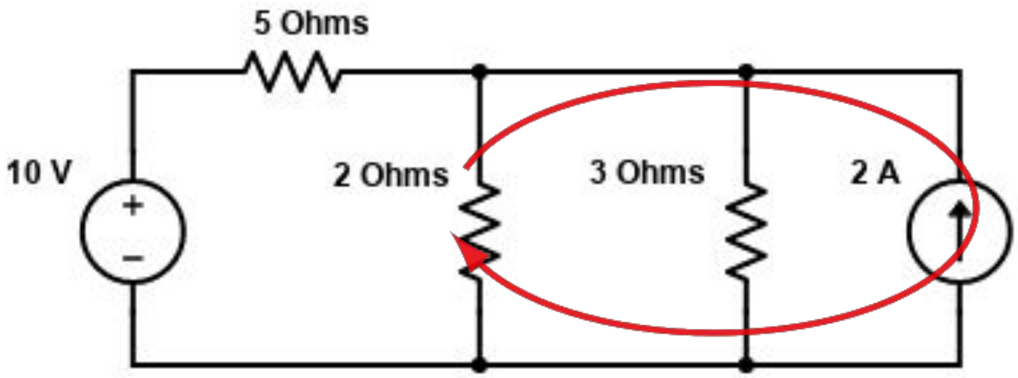
loop4

Static Circuits: Nodes, Loops

Branch: single element, such as a resistor or source

Node: connection point between two or more branches

Loop: closed path in a circuit



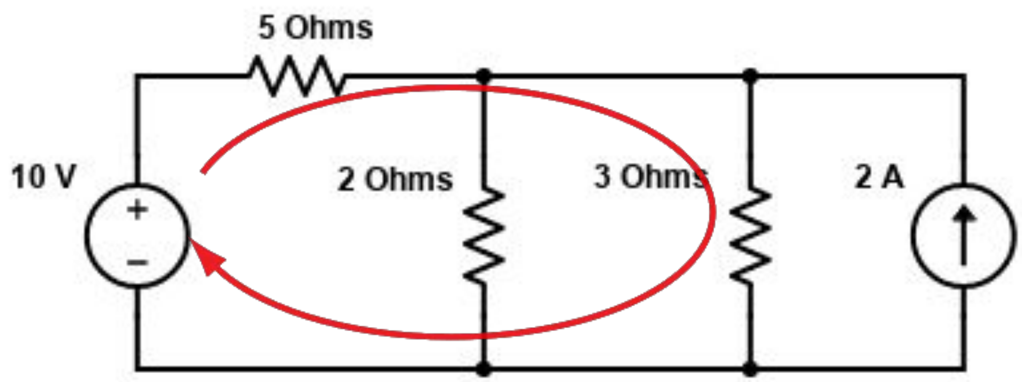
loop5

Static Circuits: Nodes, Loops

Branch: single element, such as a resistor or source

Node: connection point between two or more branches

Loop: closed path in a circuit



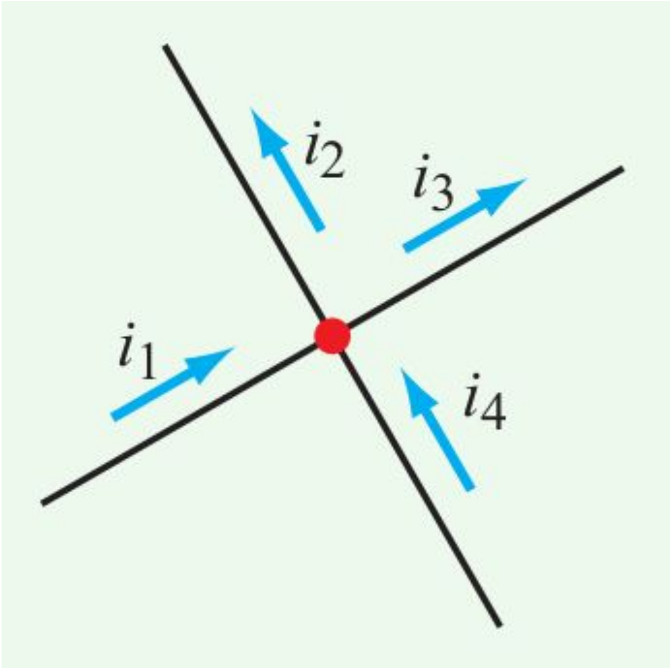
loop6

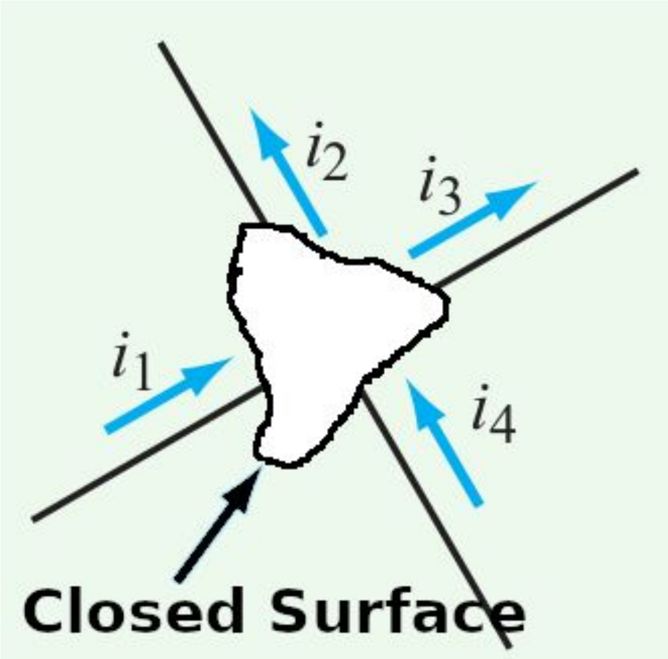
Static Circuits: KCL

Sum of all currents <u>into</u> a node is zero

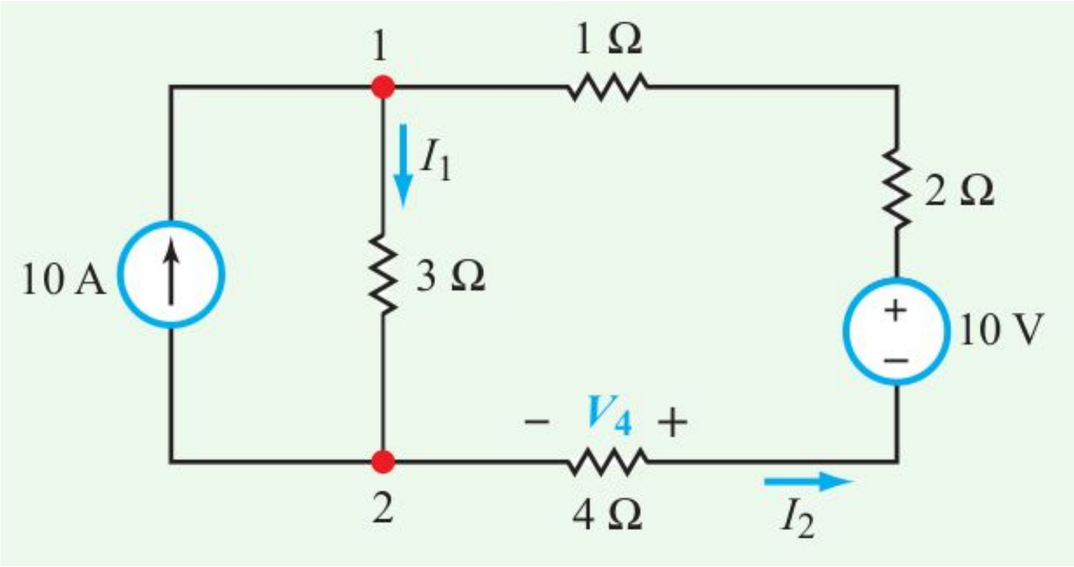
- This also holds for any closed boundary

$$\sum_n I_n = 0$$





Example 2-4: Applying KCL



Given: The circuit with

V_4 = 8 V.

Find: I_1 and I_2

Solution:

Start with Ohms law:

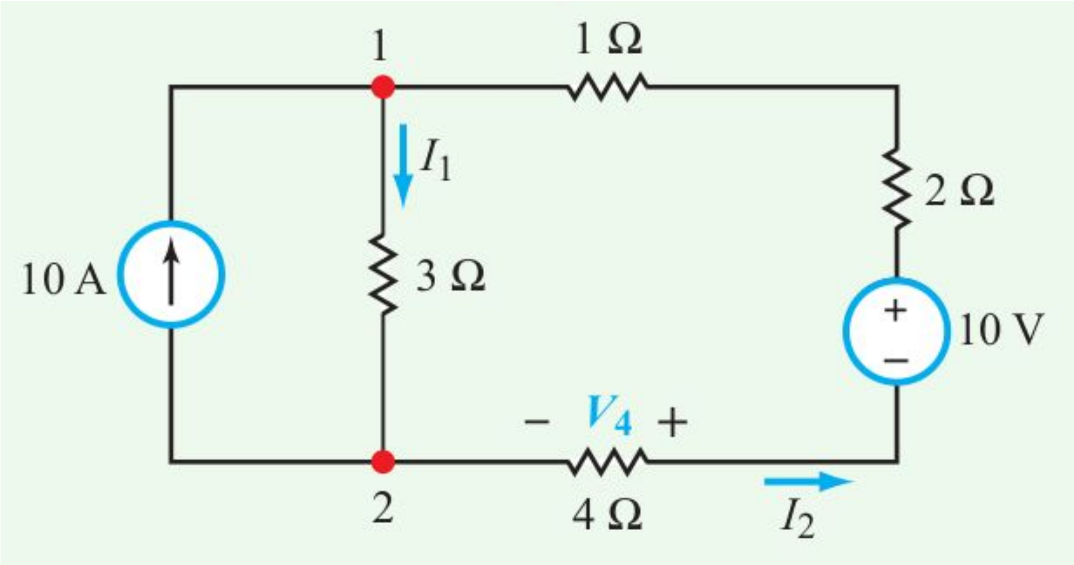
$v = i\,R$

true when passive sign convention is followed.

Because I_2 is opposite to this convention, we have:

$V_4 = -I_2\,(4\ \Omega) = 8\ \text{V}$ so: $I_2 = -2\ \text{A}$

Example 2-4: Applying KCL



Solution:

Apply KCL at Node 2: sum of currents flowing IN = 0:

$$\sum i_{in} = 0$$

I_1 is flowing in

I_2 is flowing out: $-I_2$ is flowing in

10 A is flowing out: -10 A is flowing in

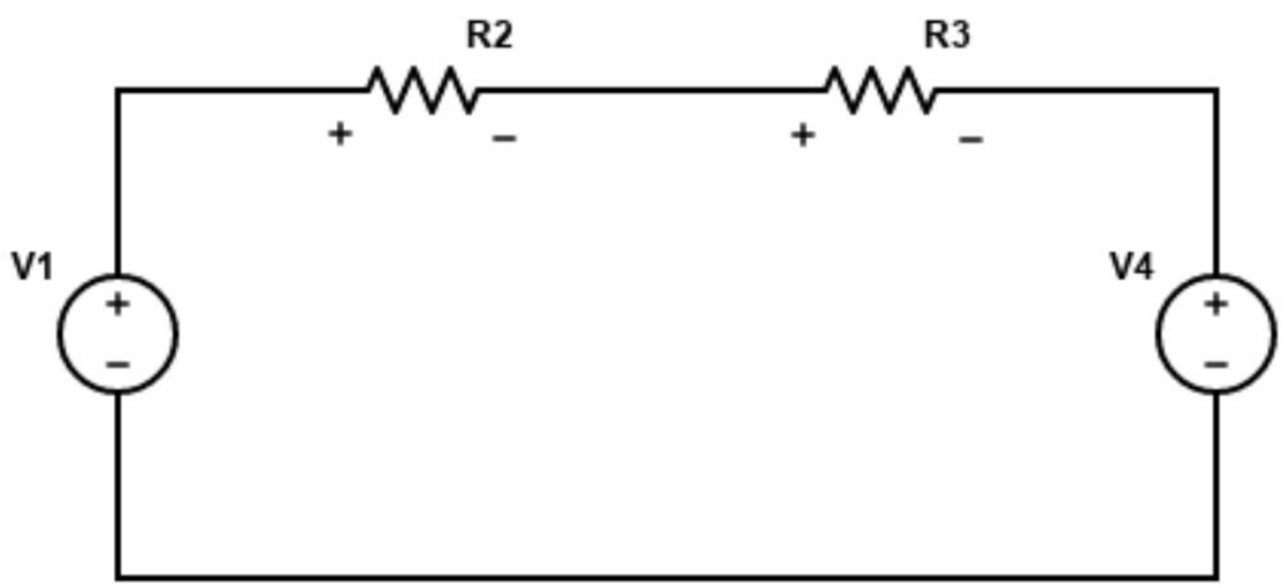
so: $I_1 + -I_2 + -10\text{ A} = 0 = I_1 + 2\text{ A} + -10\text{ A} = 0$

Hence: $I_1 = 8\text{ A}$

Static Circuits: KVL

Sum of voltages around a closed path is zero

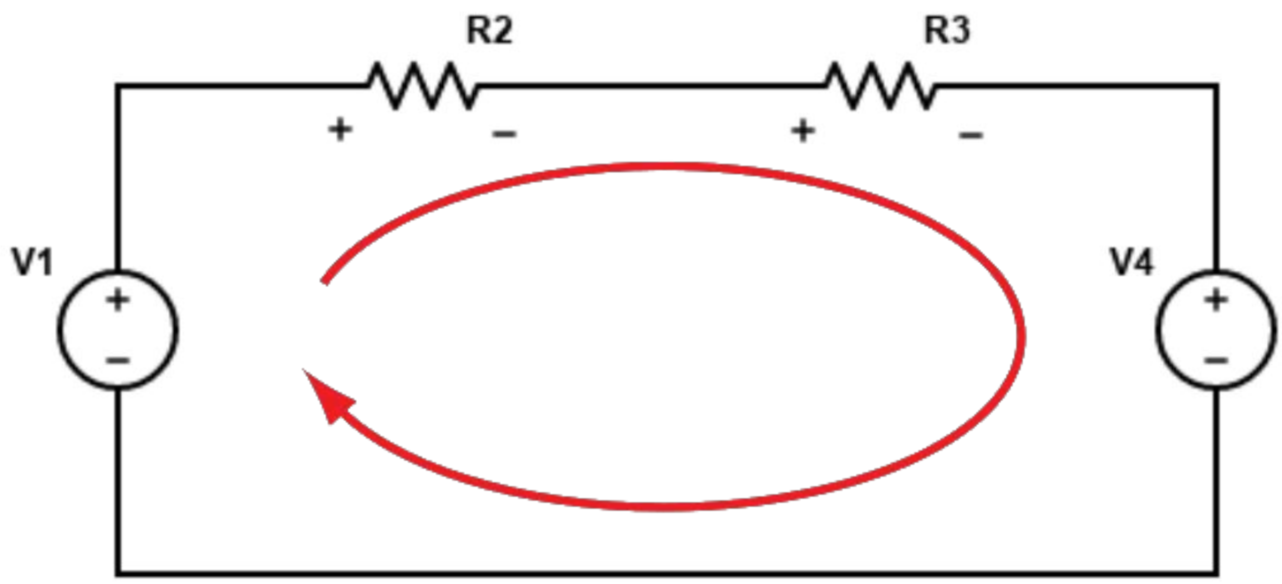
$$\sum_{n} V_n = 0$$



Static Circuits: KVL

EVERY circuit component must have +- voltage identified BEFORE you can apply this rule:
Sign of the voltage is the first sign you see when traversing the loop:

$-V1 + V2 + V3 + V4 = 0$

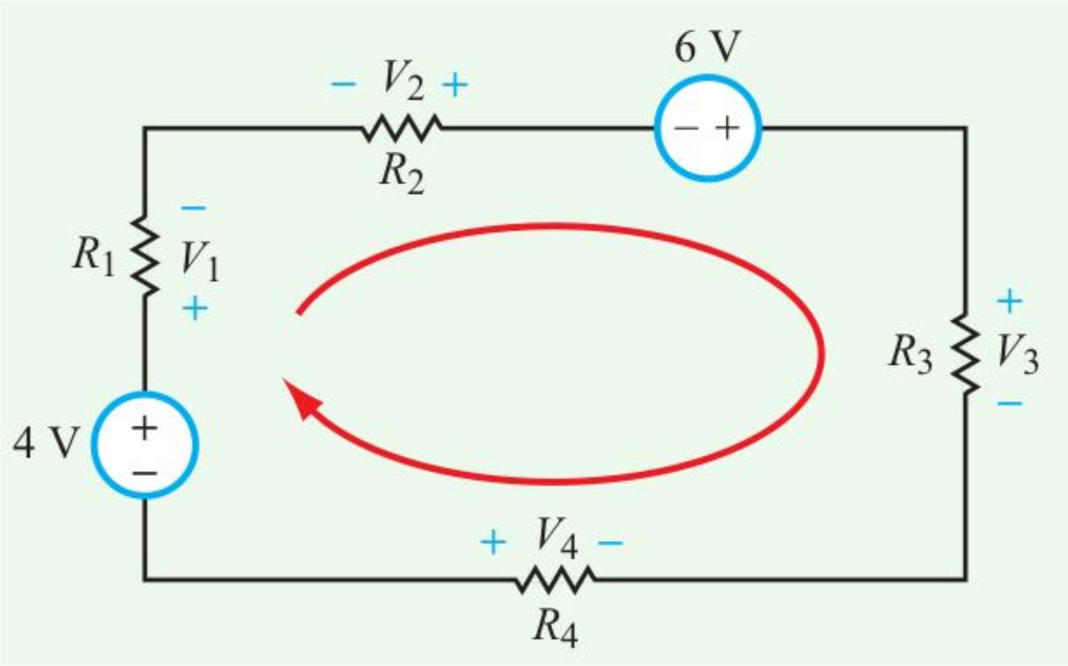


Example: Applying KVL

Given: The circuit with specified voltages.

Find: KVL equation

Solution:
Follow the arc starting from bottom left:



$$-4\ V + V_1 - V_2 - 6V + V_3 - V_4 = 0$$

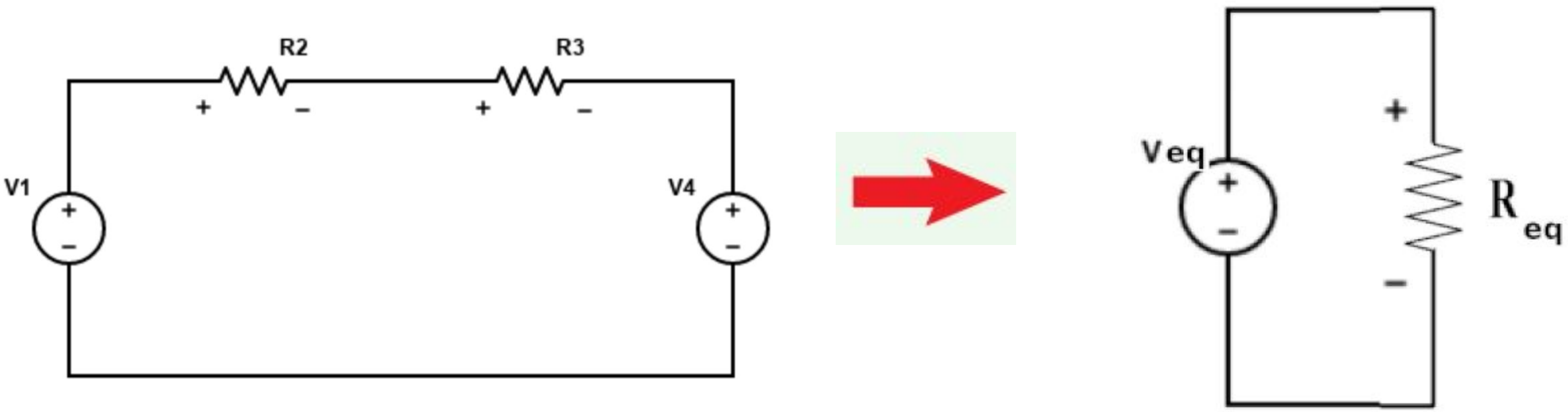
Static Circuits: Series

In series: **current** is the same

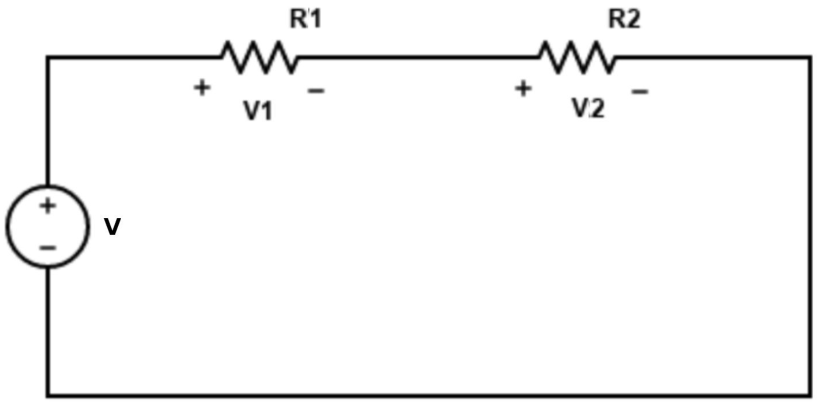
Equivalent resistance (series) is the sum of resistances

$$R_{eq} = \sum R_n = R_1 + R_2 + \ldots + R_N$$

$$R_{eq} = R_1 + R_2$$



Static Circuits: Voltage Divider



$$v_1 = \frac{R_1}{R_1 + R_2} v$$

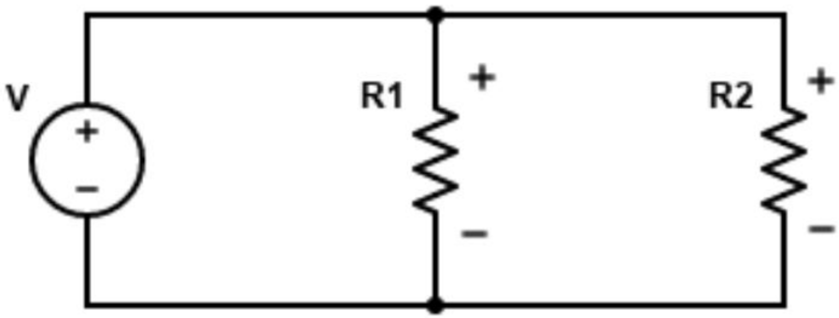
$$v_2 = \frac{R_2}{R_1 + R_2} v$$

Static Circuits: Parallel

In parallel: **voltage** is the same

R_{eq} (parallel) is inverted sum of resistances

$$\frac{1}{R_{eq}} = \sum \frac{1}{R_n} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}$$

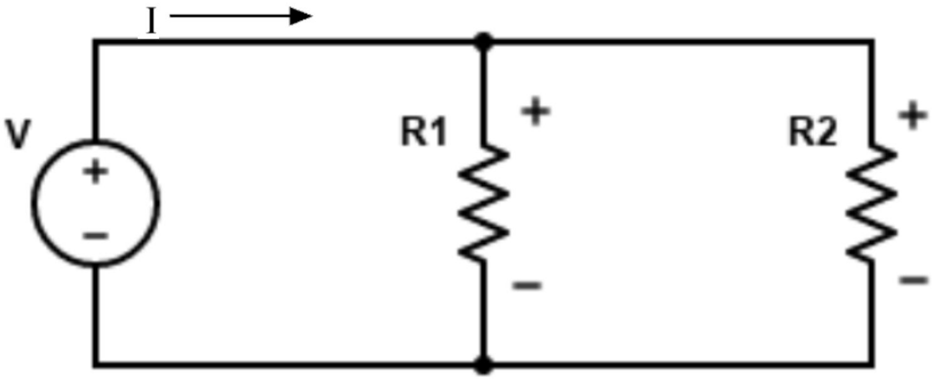


$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

Static Circuits: Current Divider

Current divided over resistors

$$i_n = \frac{v}{R_n} = i\,\frac{R_{eq}}{R_n}$$

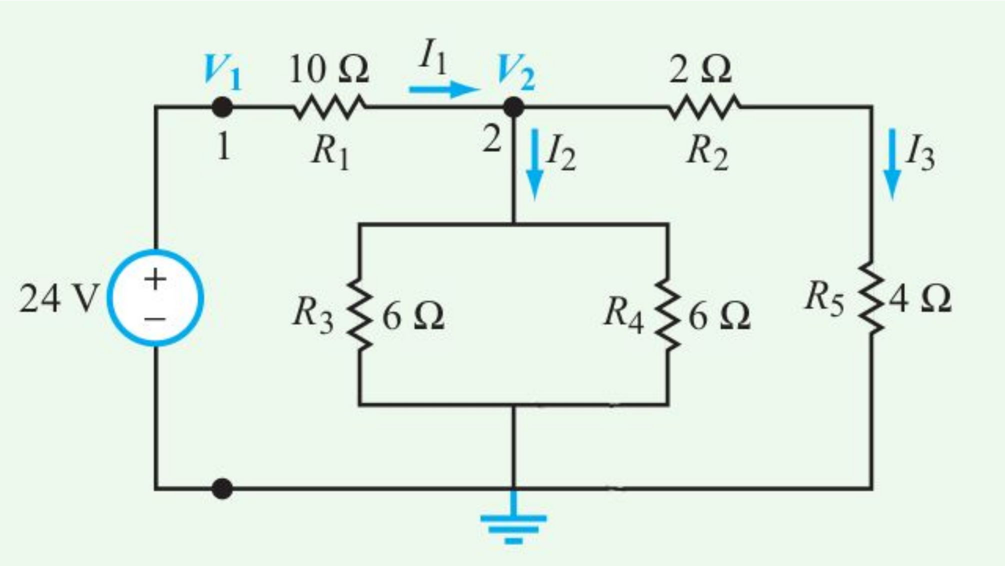


$$i_1 = i\,\frac{R_2}{R_1 + R_2}$$

$$i_2 = i\,\frac{R_1}{R_1 + R_2}$$

Example 2-12: Equivalent Circuits

Given: The circuit shown:



Find: V_2, I_1, I_2, I_3

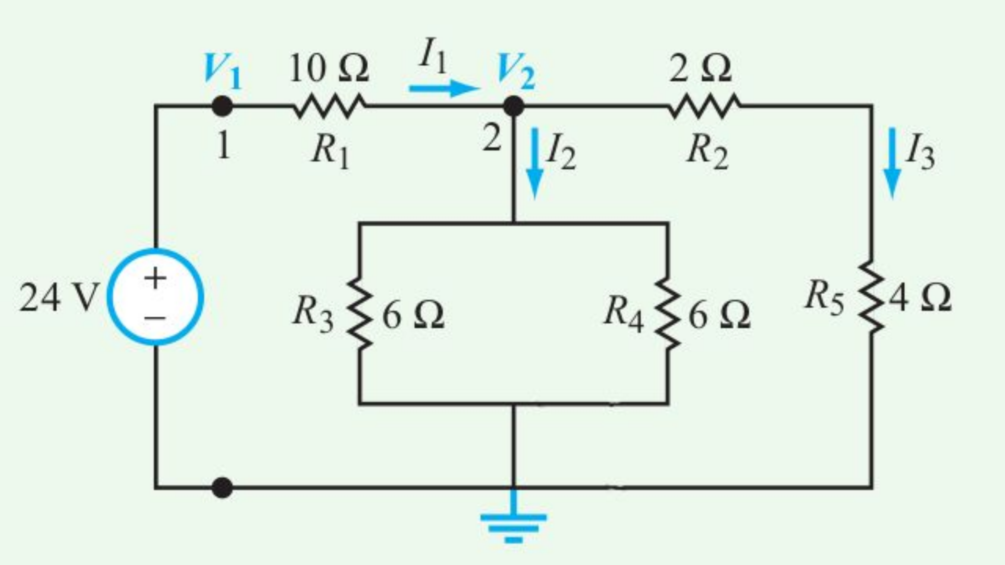
V_2 is defined wrt ground

Use equiv-resistance approach

Solution:

The equivalent-resistance approach means to successively simplify the circuit with combinations of series and parallel resistors until you can solve it.

Example 2-12: Equivalent Circuits



Solution:

Step 1:

replace R_3 and R_4 with their parallel equivalent:

$$R_a = R_3 \,||\, R_4$$

$$R_a = 1/\left(1/R_3 + 1/R_4\right)$$

$$R_a = 1/\left(1/6\Omega + 1/6\Omega\right)$$

$$R_a = 1/\left(2/6\Omega\right)$$

$$R_a = 6\Omega/2 = 3\Omega$$

Example 2-12: Equivalent Circuits

Solution:

Step 1:

replace R_3 and R_4 with their parallel equivalent:

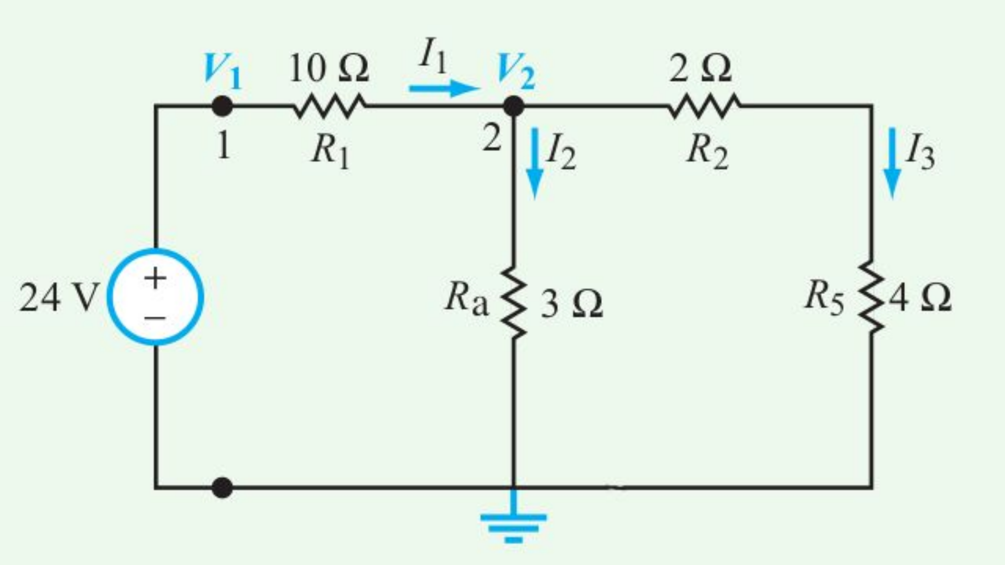
$$R_a = R_3 \parallel R_4$$

$$R_a = 1/\left(1/R_3 + 1/R_4\right)$$

$$R_a = 1/\left(1/6\Omega + 1/6\Omega\right)$$

$$R_a = 1/\left(2/6\Omega\right)$$

$$R_a = 6\Omega/2 = 3\Omega$$



Example 2-12: Equivalent Circuits

Solution:

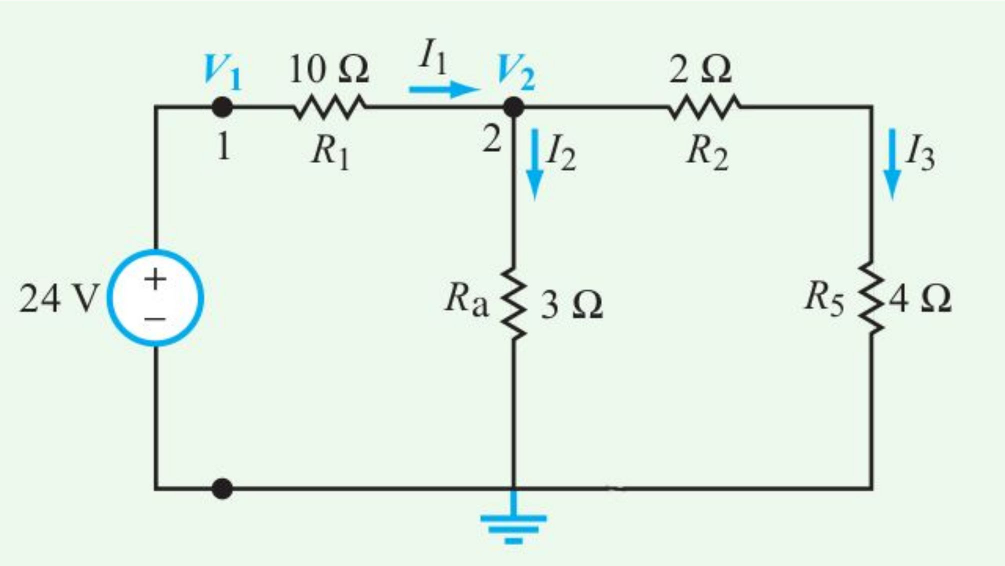
Step 2:

replace R_2 and R_5 with their series equivalent:

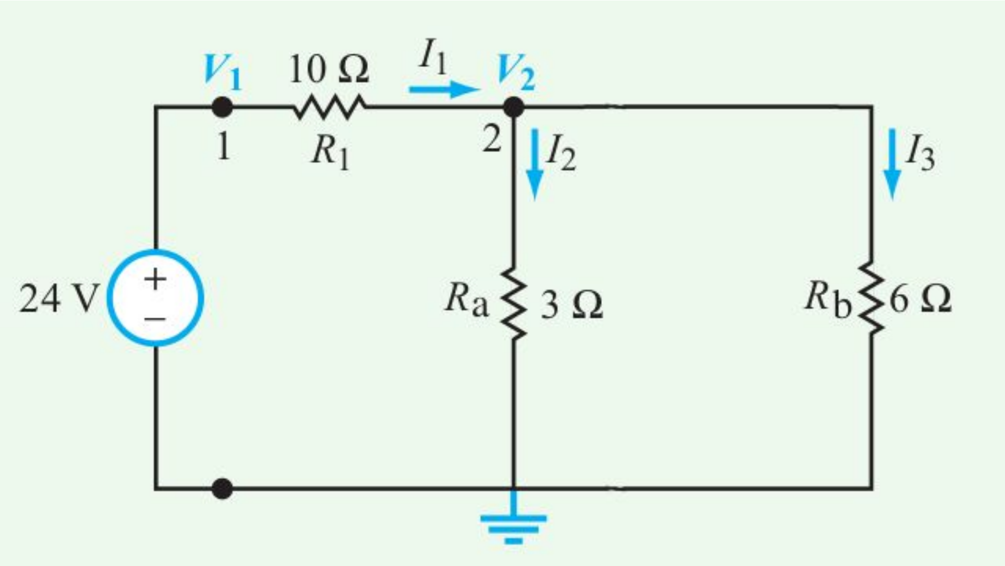
$$R_b = R_2 + R_5$$

$$R_b = 2\Omega + 4\Omega$$

$$R_b = 6\Omega$$



Example 2-12: Equivalent Circuits



Solution:

Step 2:

replace R_2 and R_5 with their series equivalent:

$$R_b = R_2 + R_5$$

$$R_b = 2\Omega + 4\Omega$$

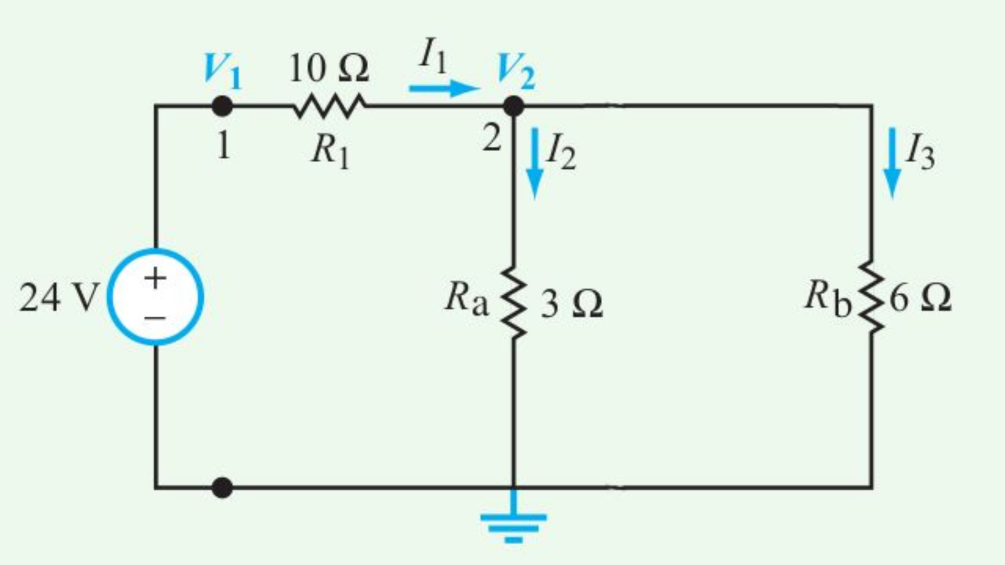
$$R_b = 6\Omega$$

Example 2-12: Equivalent Circuits

Solution:

Step 3:

replace R_a and R_b with their parallel equivalent:



$$R_c = R_a \parallel R_b$$

$$R_c = 1/\left(1/R_a + 1/R_b\right)$$

$$R_c = 1/\left(1/3\Omega + 1/6\Omega\right)$$

$$R_c = 1/\left(3/6\Omega\right)$$

$$R_c = 6\Omega/3 = 2\Omega$$

Example 2-12: Equivalent Circuits

Solution:

Step 3:

replace R_a and R_b with their parallel equivalent:

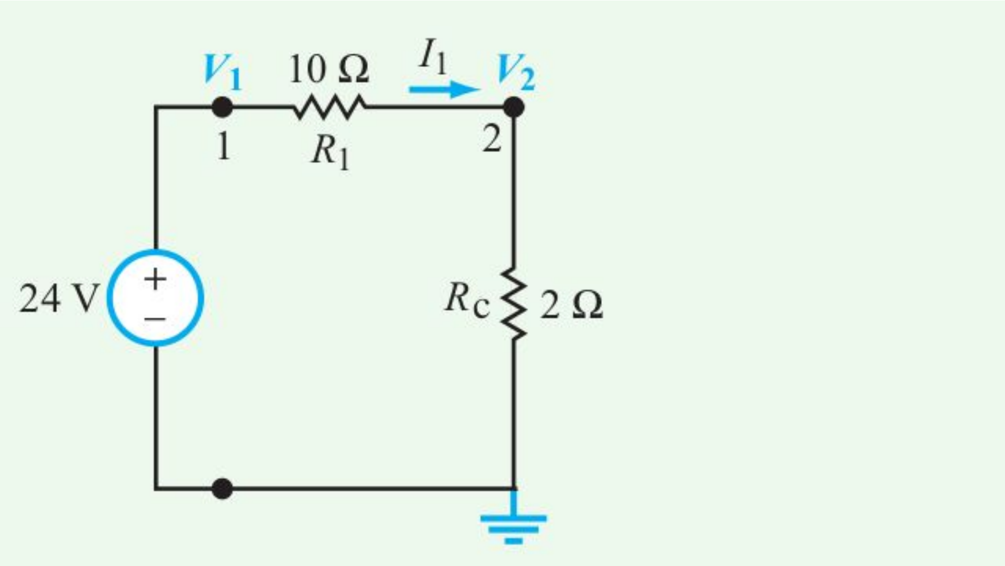
$$R_c = R_a \,||\, R_b$$

$$R_c = 1/\,(\,1/R_a + 1/R_b)$$

$$R_c = 1/\,(\,1/3\Omega + 1/6\Omega)$$

$$R_c = 1/\,(\,3/6\Omega)$$

$$R_c = 6\Omega/3 = 2\Omega$$



Example 2-12: Equivalent Circuits

Solution:

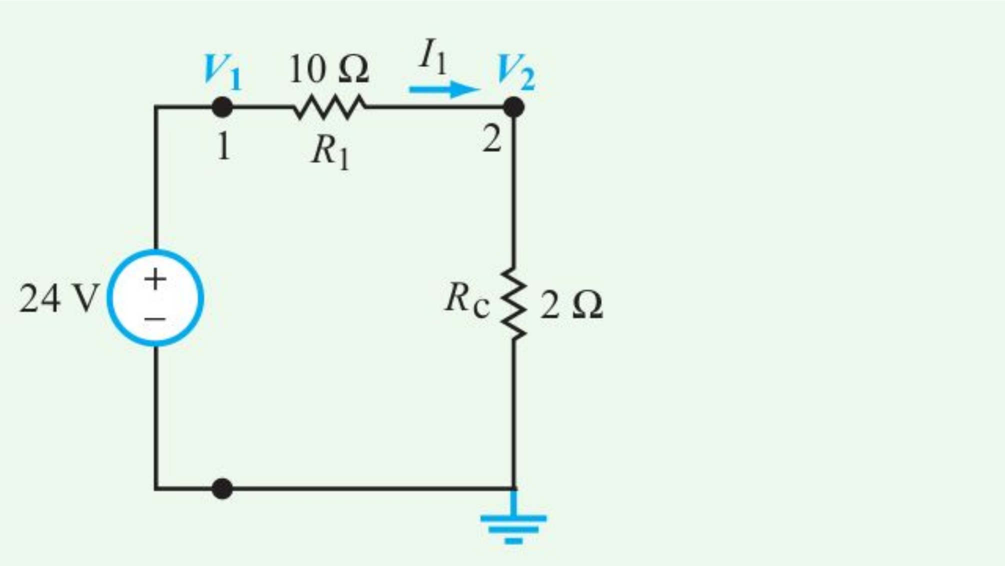
Step 4:

replace R_1 and R_c with their series equivalent:

$$R_d = R_1 + R_c$$

$$R_d = 10\Omega + 2\Omega$$

$$R_d = 12\Omega$$

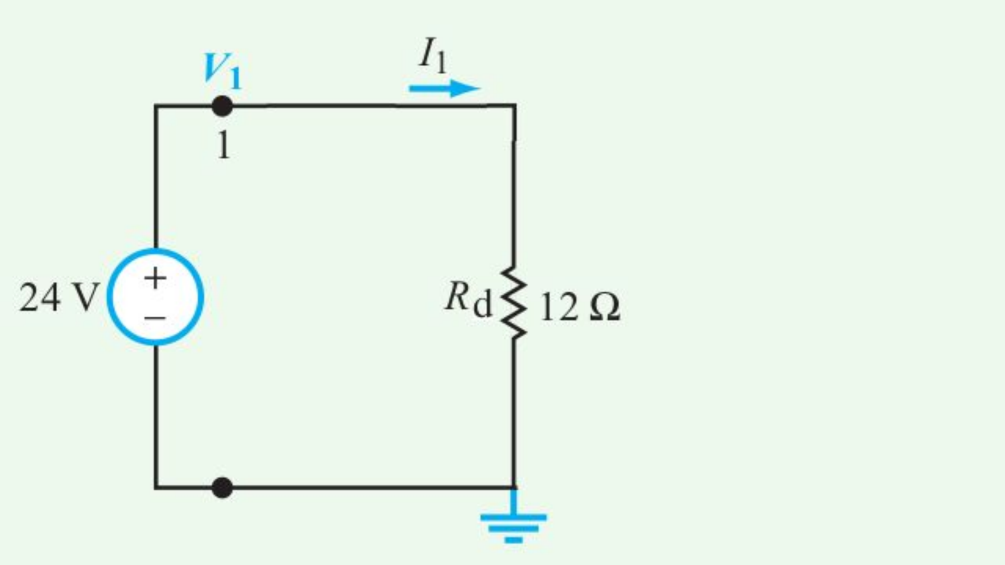


Example 2-12: Equivalent Circuits

Solution:

Step 4:

replace R_1 and R_c with their series equivalent:



$$R_d = R_1 + R_c$$

$$R_d = 10\Omega + 2\Omega$$

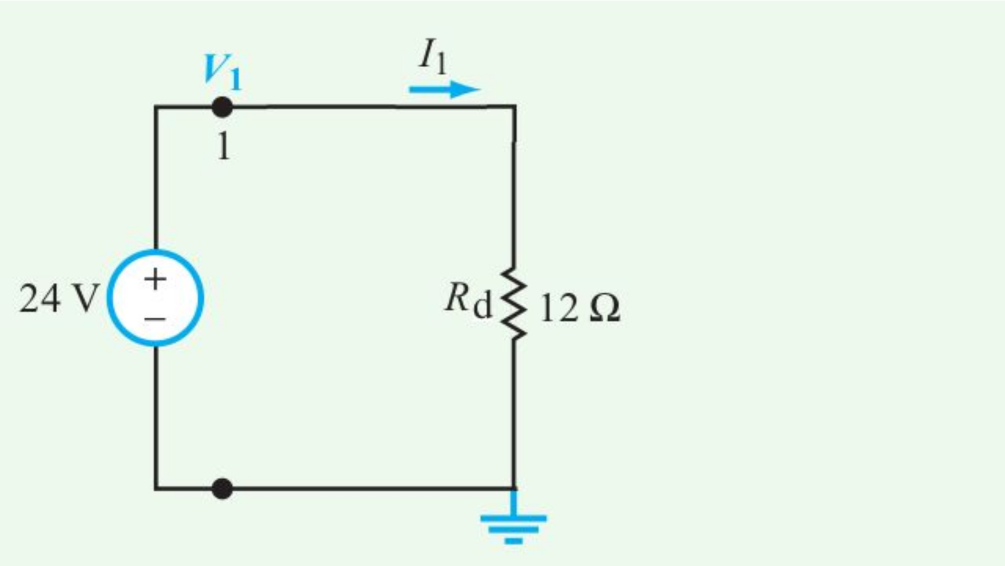
$$R_d = 12\Omega$$

Example 2-12: Equivalent Circuits

Solution:

Step 5:

solve for I_1 using Ohm's Law



$$V = I\,R$$

(passive sign convention is followed)

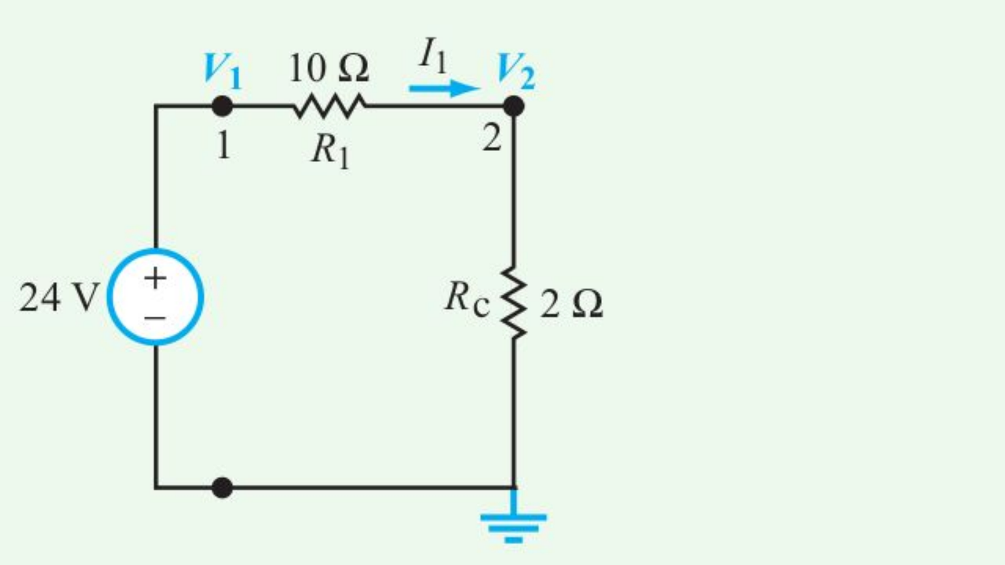
$$I_1 = V_1 / R_d$$

$$I_1 = 24V / 12\Omega$$

$$I_1 = 2\text{ A}$$

Example 2-12: Equivalent Circuits

Solution:
Step 6:
Using a previous circuit:



solve for V_2 using Ohm's Law

$$V = I\,R$$

(passive sign convention is followed)

$$V_2 = I_1\,R_c$$

$$V_2 = (2\text{ A})\,(2\ \Omega)$$

$$V_2 = 4\text{ V}$$

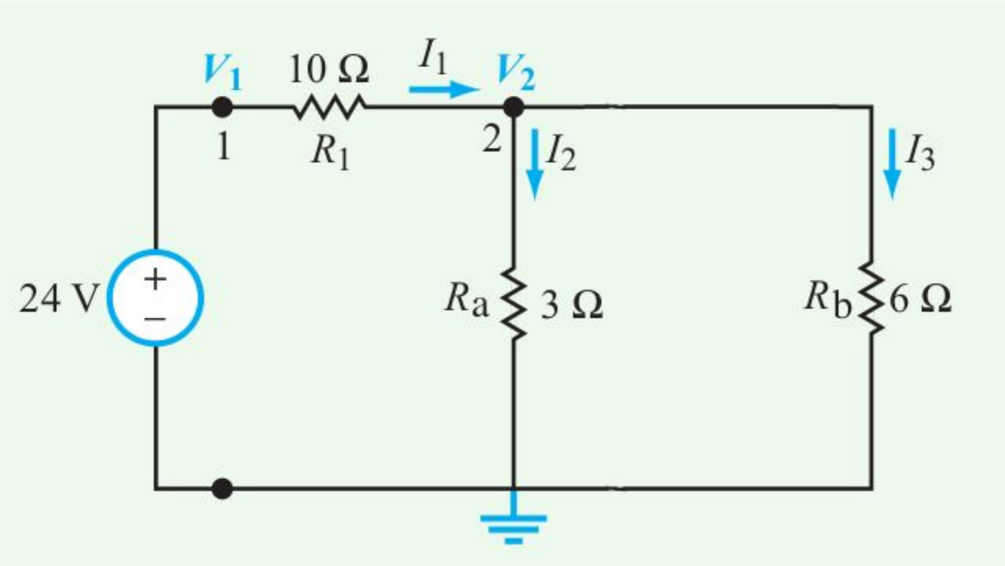
Example 2-12: Equivalent Circuits

Solution:

Step 7:

Using a previous circuit:

solve for I_2 and I_3 using Ohm's Law



$$V = I\,R$$

(passive sign convention is followed)

$I_2 = V_2 / R_a$

$I_2 = 4\text{ V} / 3\ \Omega$

$I_2 = 1.33\text{ A}$

$I_3 = V_2 / R_b$

$I_3 = 4\text{ V} / 6\ \Omega$

$I_3 = 0.67\text{ A}$

Example 2-12: Equivalent Circuits

Solution:

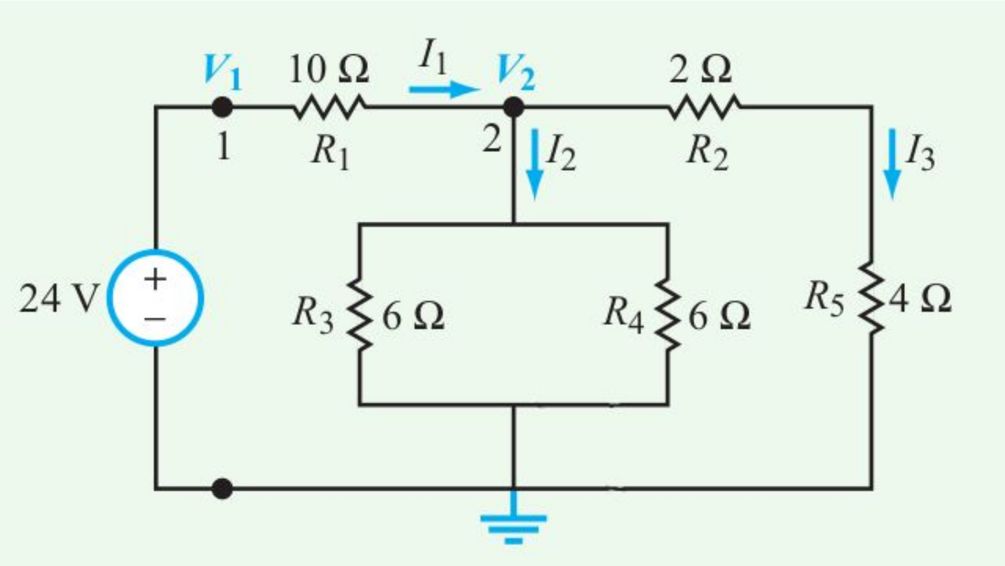
Summary:

$V_2 = 4$ V

$I_1 = 2$ A

$I_2 = 1.33$ A

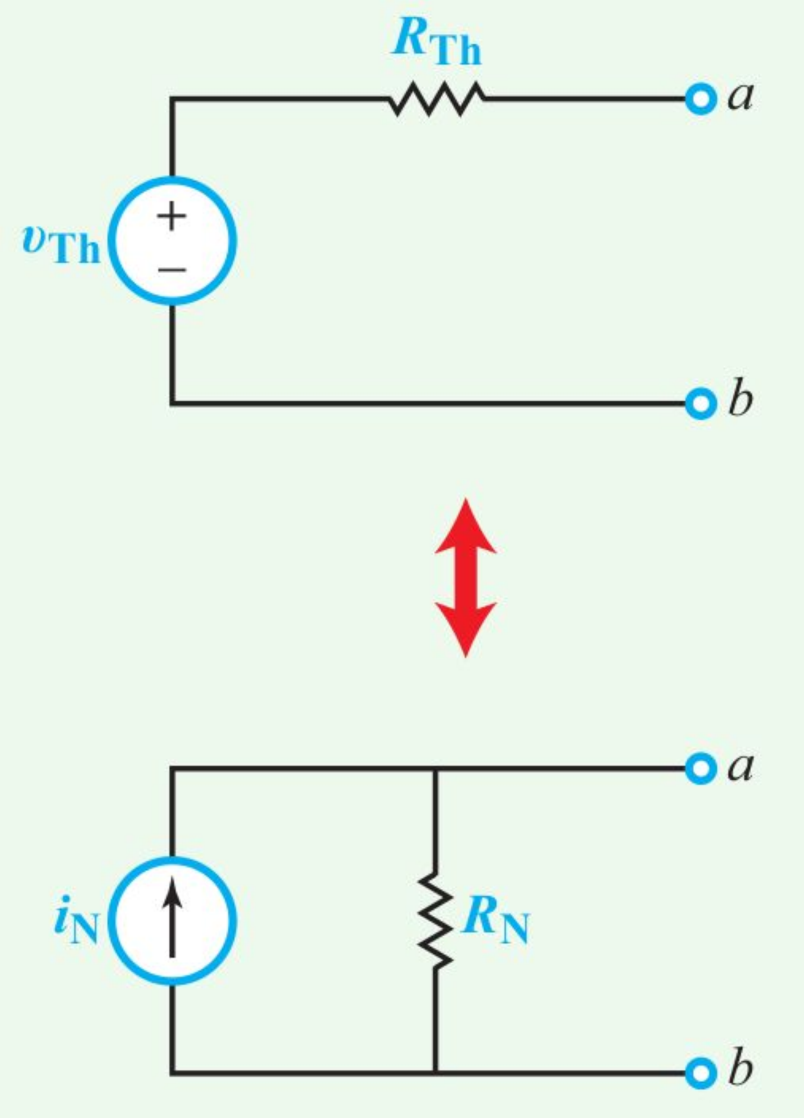
$I_3 = 0.67$ A



Static Circuits: Source Transformation

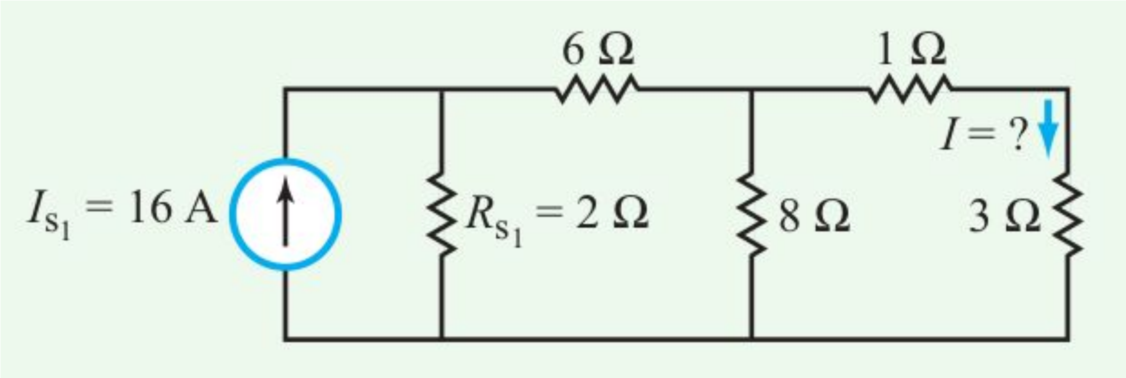
- Viewed from the output terminals a&b, the two circuits are *equivalent*
- Replacement of a circuit by an equivalent can make some problems easier to solve

$$i_N = v_{Th} / R_{Th}$$
$$R_N = R_{Th}$$



Example 2-13: Source Transformation

Given: The circuit shown:



Find: The current *I* using source transformation

Solution: Could use the method of equivalent resistances, going right-to-left

Instead: use source transformation by starting on the left.
Use equivalent resistances as needed.

Example 2-13: Source Transformation

Solution:

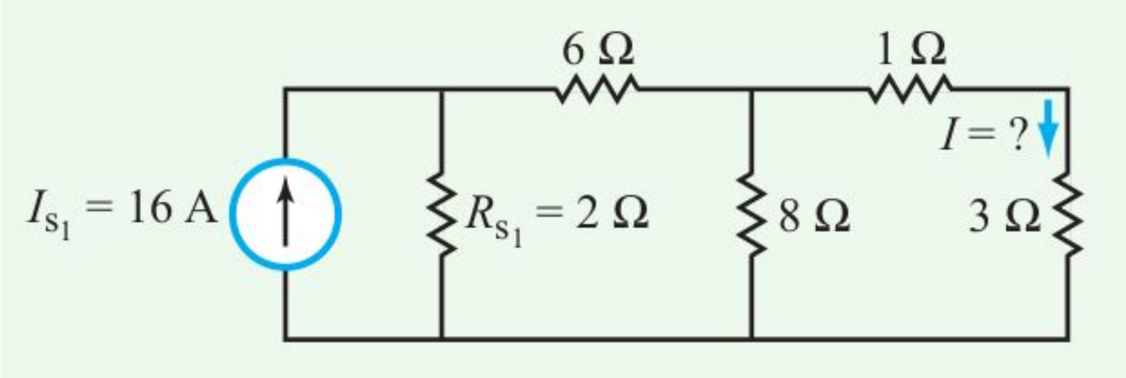
Step 1:

Convert Current source S_1 to a Voltage source:

$V_{s1} = I_{s1} R_{s1}$

$V_{s1} = (16\ A)\ (2\ \Omega)$

$V_{s1} = 32\ V$



Example 2-13: Source Transformation

Solution:

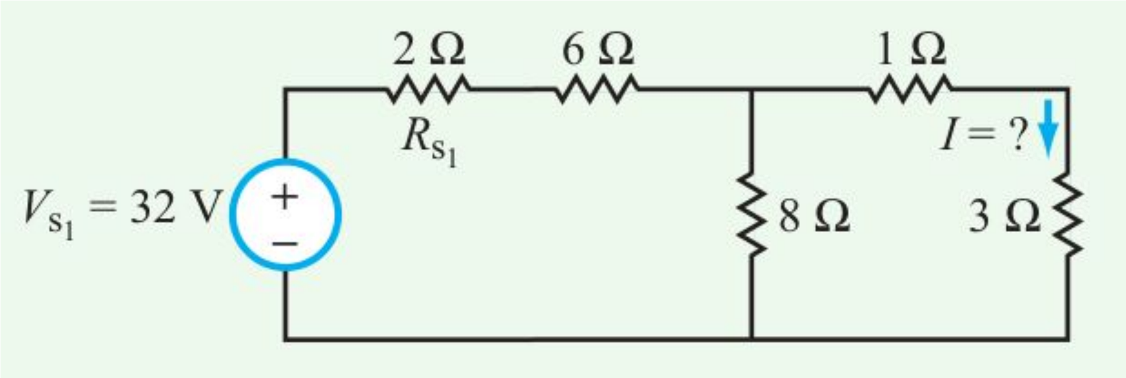
Step 1:

Convert Current source S_1 to a Voltage source:

$V_{s1} = I_{s1} R_{s1}$

$V_{s1} = (16\ \text{A})\ (2\ \Omega)$

$V_{s1} = 32\ \text{V}$

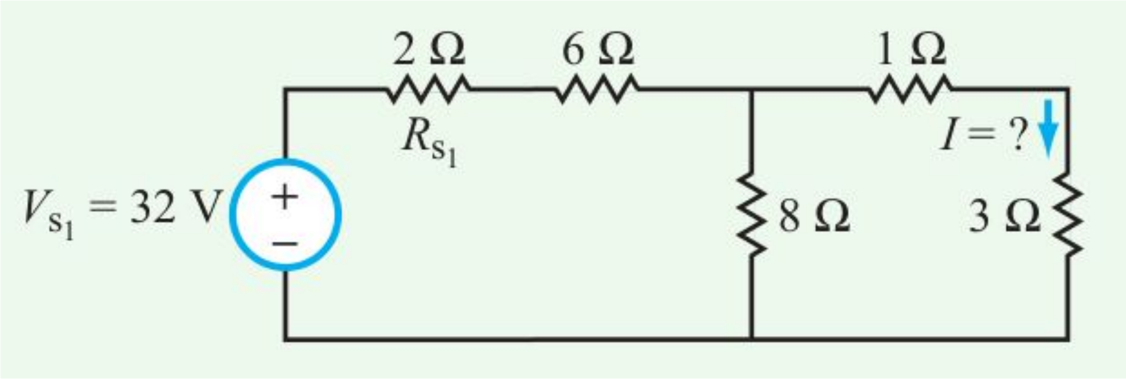


Example 2-13: Source Transformation

Solution:

Step 2:

Series resistance:



$$R_{S2} = 2\ \Omega + 6\ \Omega$$

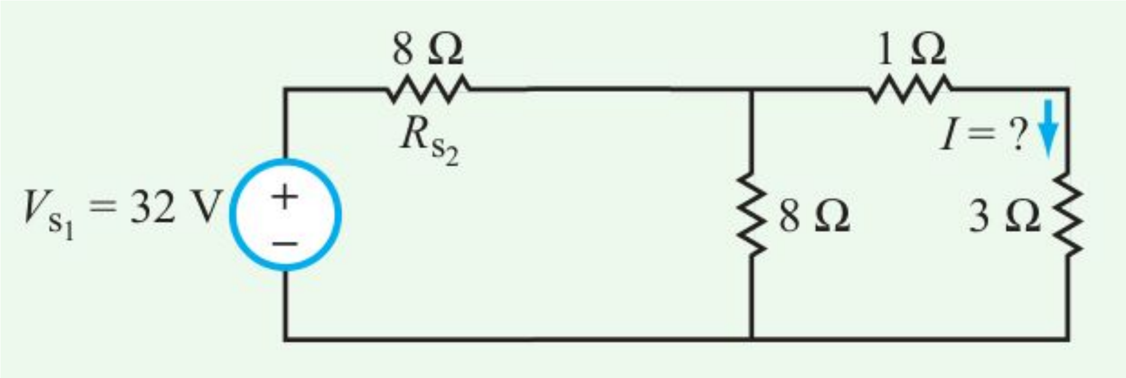
$$R_{S2} = 8\ \Omega$$

Example 2-13: Source Transformation

Solution:

Step 2:

Series resistance:



$R_{S2} = 2\ \Omega + 6\ \Omega$

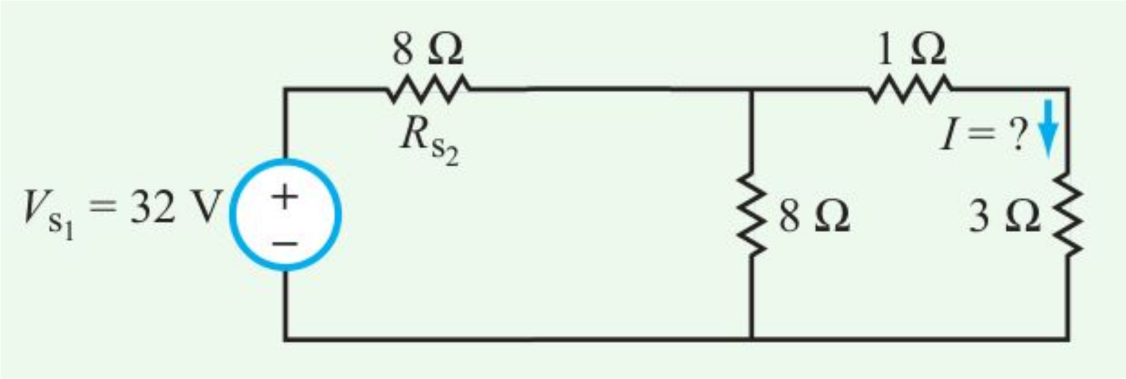
$R_{S2} = 8\ \Omega$

Example 2-13: Source Transformation

Solution:

Step 3:

Convert Voltage source to Current source.



$I_{s2} = V_{s1} / R_1$

$I_{S2} = 32\text{ V} / 8\ \Omega$

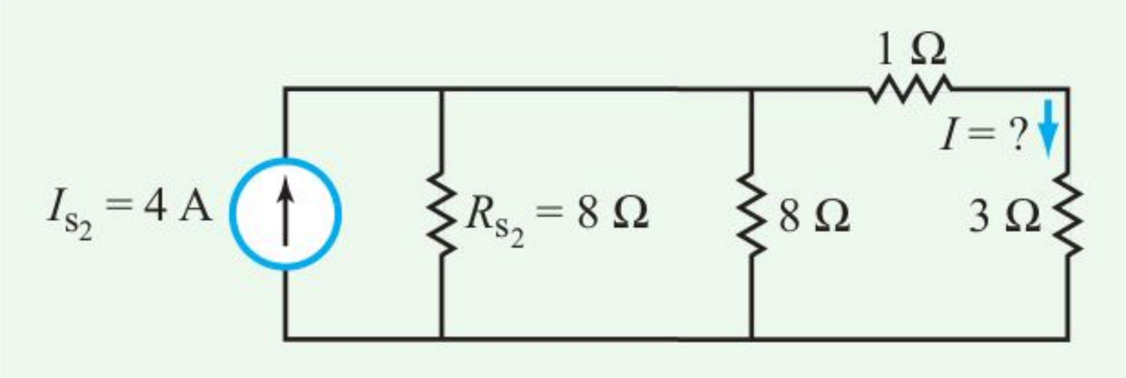
$I_{S2} = 4\text{ A}$

Example 2-13: Source Transformation

Solution:

Step 3:

Convert Voltage source to Current source.



$I_{s2} = V_{s1} / R_1$

I_{S2} = 32 V / 8 Ω

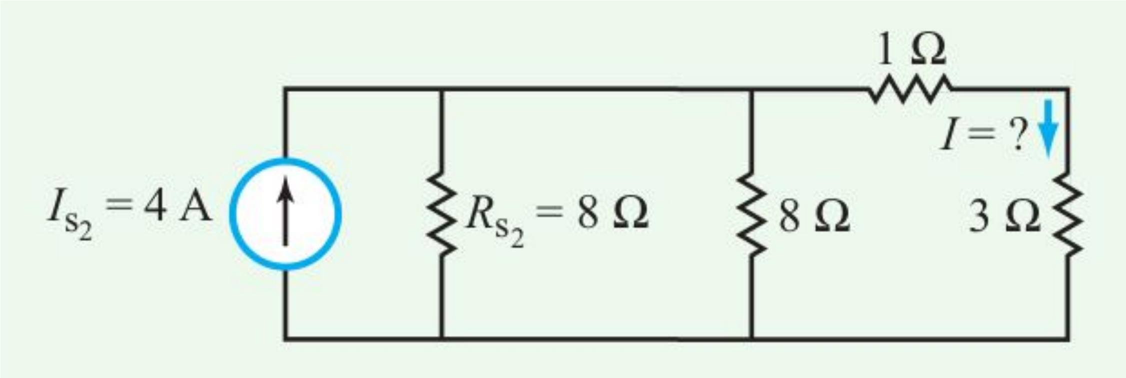
I_{S2} = 4 A

Example 2-13: Source Transformation

Solution:

Step 4:

Parallel resistors:



R_{S3} = 1/(1/8Ω + 1/8Ω)

R_{S3} = 1/(2/8Ω)

R_{S3} = 8Ω / 2

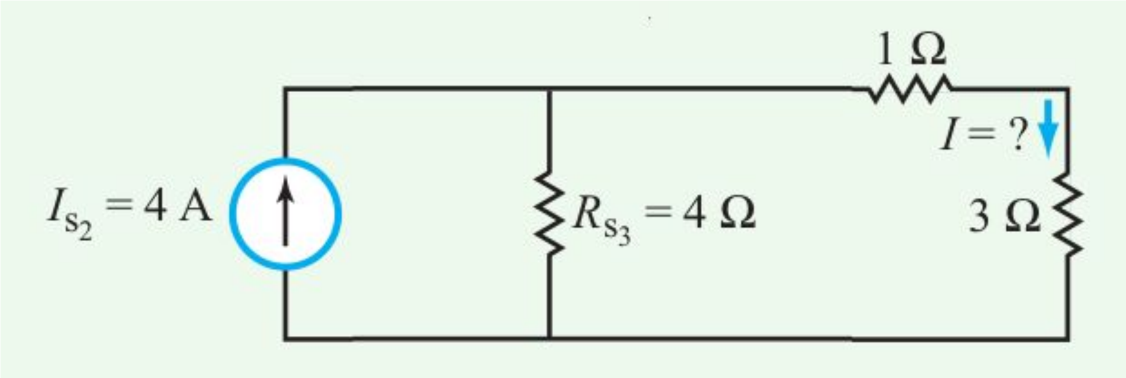
R_{S3} = 4Ω

Example 2-13: Source Transformation

Solution:

Step 4:

Parallel resistors:



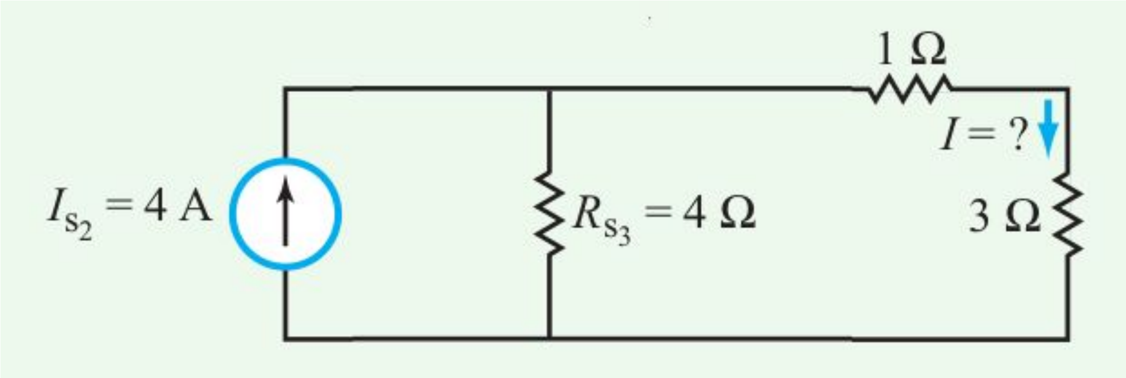
R_{S3} = 1/(1/8Ω + 1/8Ω)

R_{S3} = 1/(2/8Ω)

R_{S3} = 8Ω / 2

R_{S3} = 4Ω

Example 2-13: Source Transformation



Solution:

Step 5:

Convert Current source S_2 to a Voltage source:

$V_{s2} = I_{s2}R_{s3}$

$V_{s2} = (4\text{ A})\ (4\ \Omega)$

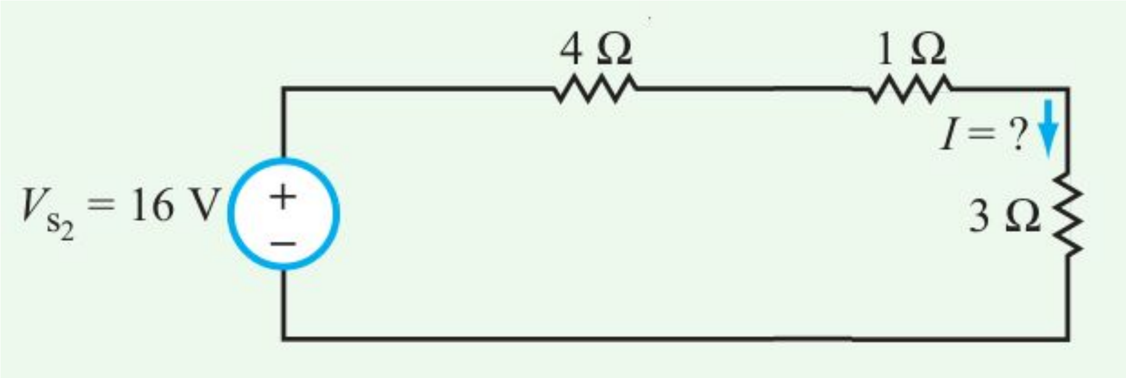
$V_{s2} = 16\text{ V}$

Example 2-13: Source Transformation

Solution:

Step 5:

Convert Current source S_2 to a Voltage source:



$V_{s2} = I_{s2}R_{s3}$

$V_{s2} = (4\ A)\ (4\ \Omega)$

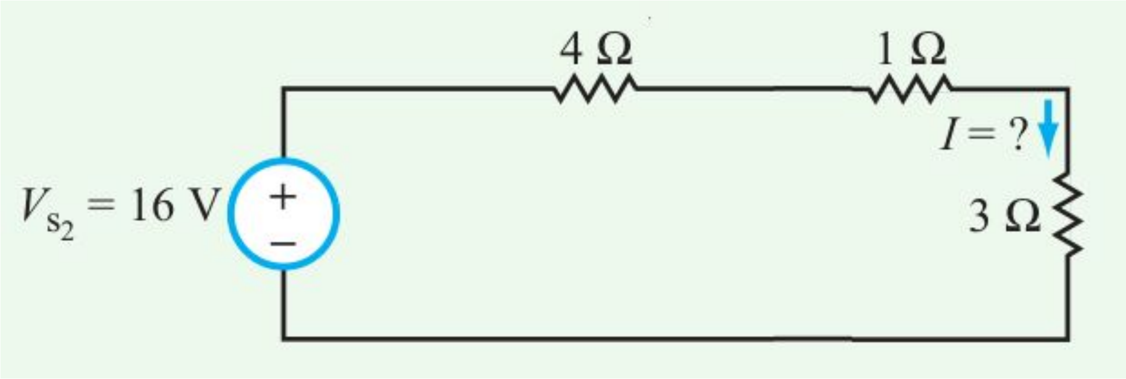
$V_{s2} = 16\ V$

Example 2-13: Source Transformation

Solution:

Step 6:

Series Resistors:



$R = 4\ \Omega + 1\ \Omega + 3\ \Omega$

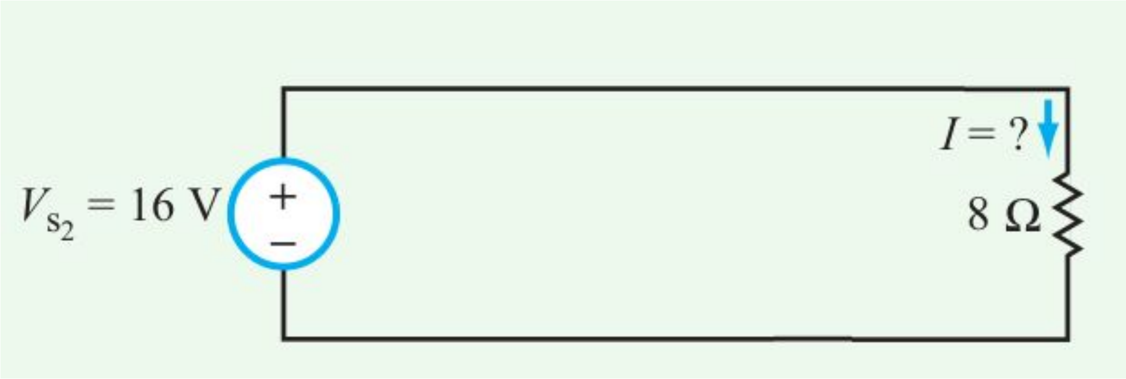
$R = 8\ \Omega$

Example 2-13: Source Transformation

Solution:

Step 6:

Series Resistors:



$R = 4\ \Omega + 1\ \Omega + 3\ \Omega$

$R = 8\ \Omega$

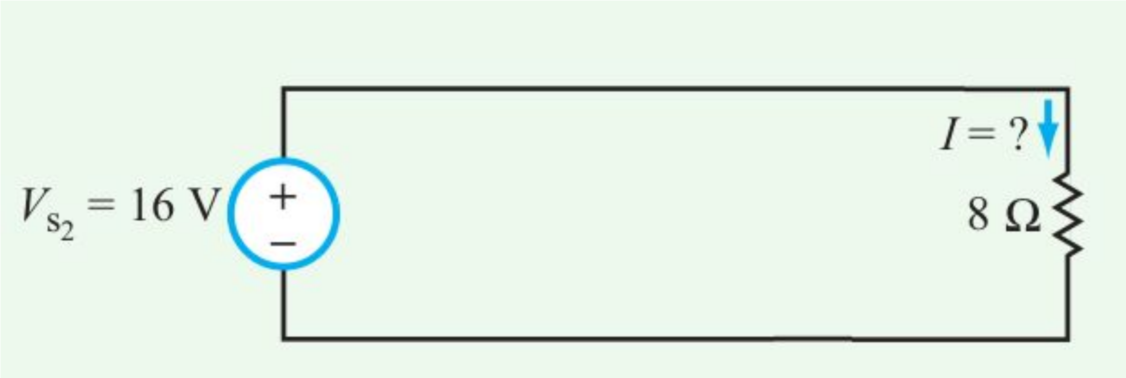
Example 2-13: Source Transformation

Solution:

Step 7:

Ohm's Law:

$I = V_{s2} / R$

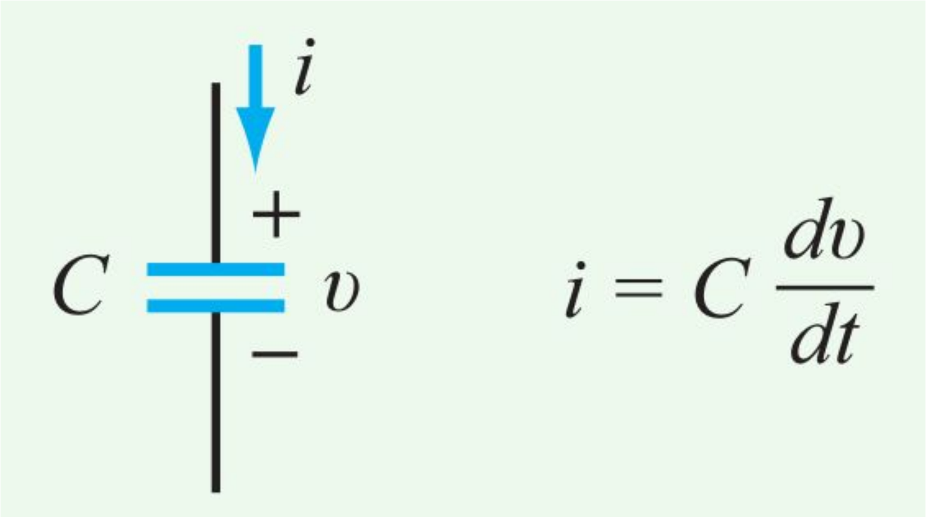


(passive sign convention followed)

I = 16 V / 8 Ω

I = 2 A

Transient Circuits: Capacitors



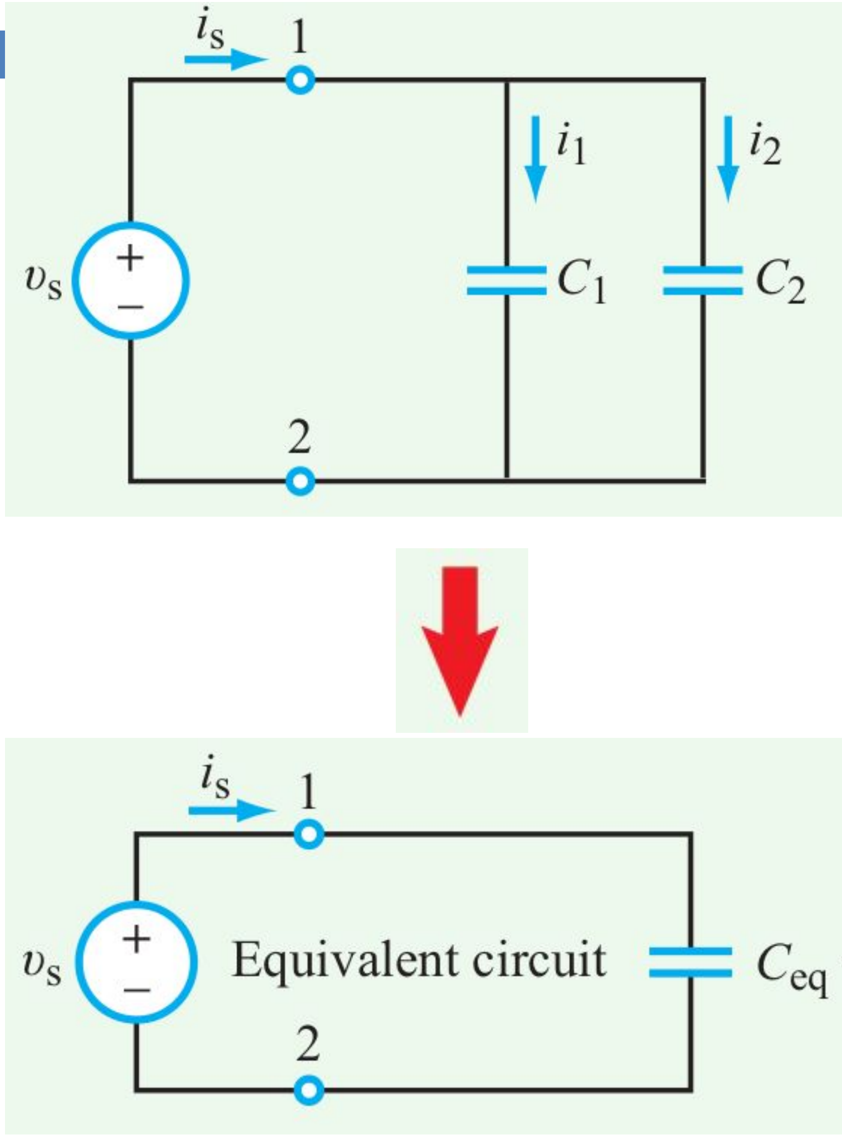
C in Farads 1F = 1 C/V

v must be continuous function of time t.
Otherwise i is infinite (& power is infinite).

Voltage across a capacitor cannot change instantly !

At DC (nothing changing in time):
i=0→ Capacitor looks like OPEN Circuit

Transient Circuits: Capacitors



Capacitors in Parallel:

Both capacitors share same voltage:

KCL: $i_s = i_1 + i_2 = C_1\, dv_s/ dt + C_2\, dv_s/ dt$

For the equivalent circuit:

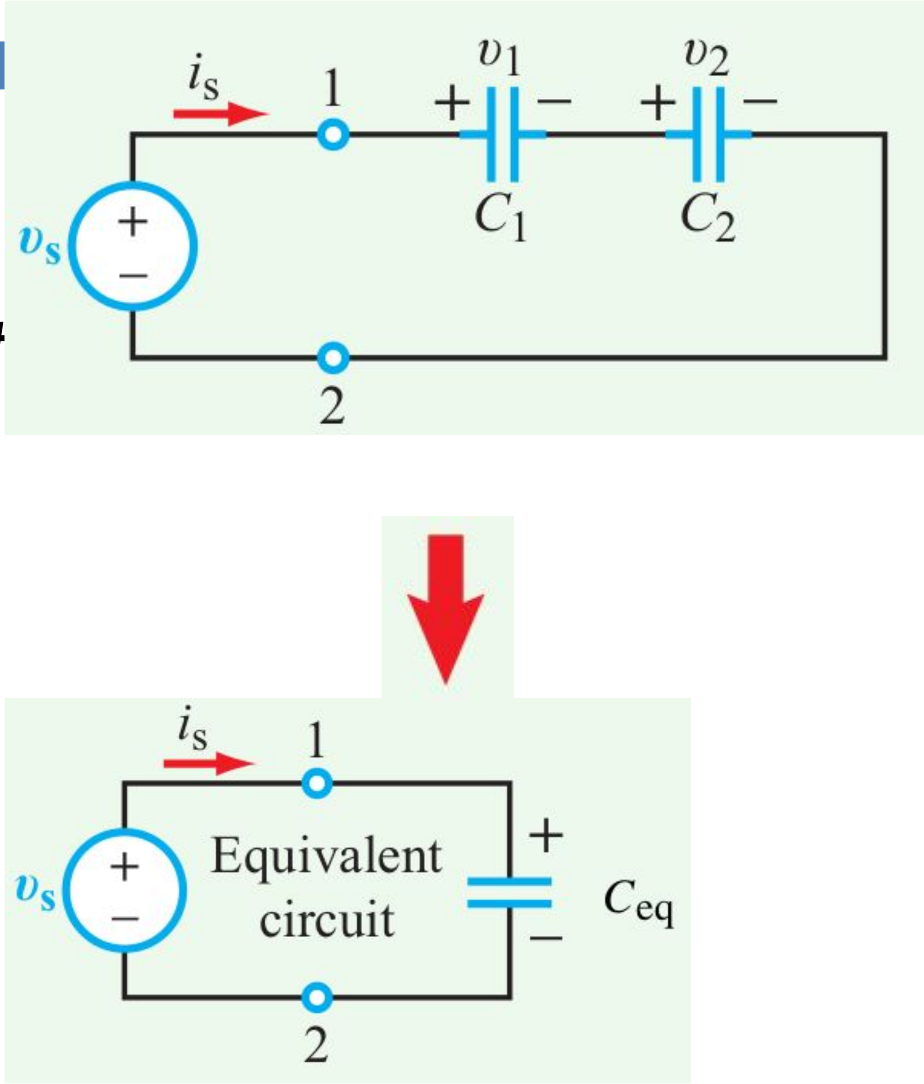
$$i_s = C_{eq}\, dv_s/ dt$$

$$i_s = (C_1 + C_2)\, dv_s/ dt$$

So: $C_{eq} = C_1 + C_2$

$$C_{eq} = \sum_{i=1}^{N} C_i$$

Transient Circuits: Capacitors



Capacitors in Series:

Since: $i_s = C_{eq}\, d\,(\nu_1 + \nu_2) \,/\, dt$

and: $i_s = C_1\, d\nu_1/dt = C_2\, d\nu_2/dt$

then: $i_s = C_{eq}\, [\, i_s / C_1 + i_s / C_2 \,]$

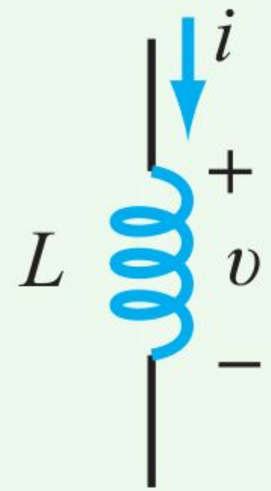
$1 = C_{eq}\, [\, 1 / C_1 + 1 / C_2 \,]$

$1 \,/\, C_{eq} = 1 / C_1 + 1 / C_2$

So:

$$\frac{1}{C_{eq}} = \sum_{i=1}^{N} \frac{1}{C_i}$$

Transient Circuits: Inductors



$$v = L\frac{di}{dt}$$

L in Henries 1H= 1V-s/A

i must be continuous function of time *t*.
Otherwise *v* is infinite (& power is infinite).

Current through an inductor cannot change instantly!

At DC (nothing changing in time):

$v = 0 \rightarrow$ Inductor looks like SHORT Circuit

Transient Circuits: Inductors

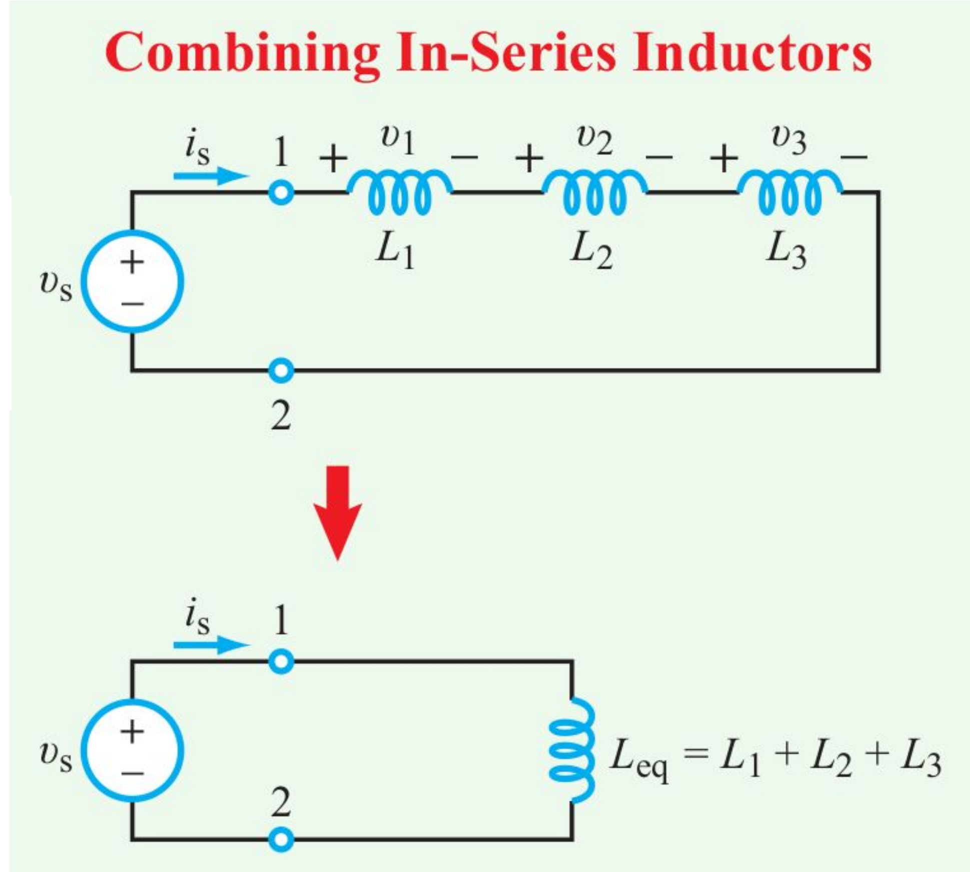
Applying KVL:

$$v_s = v_1 + v_2 + v_3$$
$$= L_1 \frac{di_s}{dt} + L_2 \frac{di_s}{dt} + L_3 \frac{di_s}{dt}$$
$$= (L_1 + L_2 + L_3) \frac{di_s}{dt},$$

and for the equiv ckt:

$$v_s = L_{eq} \frac{di_s}{dt}$$

hence: $L_{eq} = L_1 + L_2 + L_3.$



$$L_{eq} = \sum_{i=1}^{N} L_i$$

Transient Circuits: Inductors

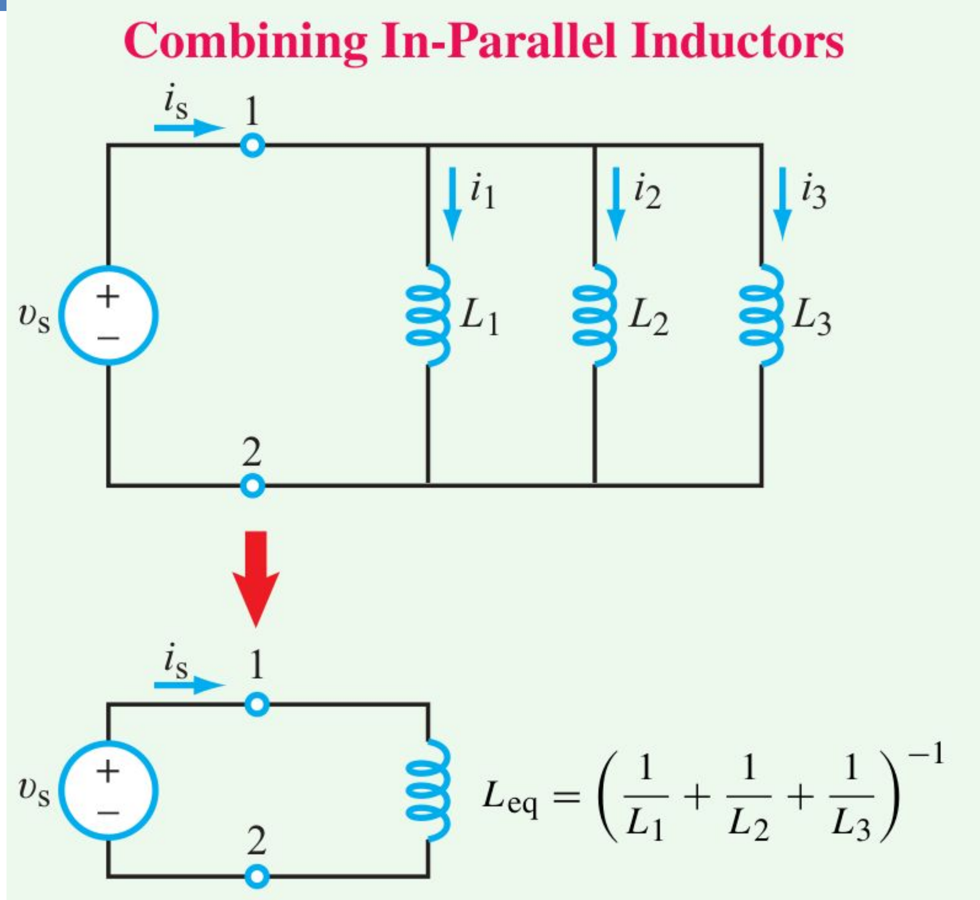
$$v_s = L_{eq}\frac{d(i_1+i_2+i_3)}{dt}$$

$$v_s = L_1\frac{di_1}{dt} = L_2\frac{di_2}{dt} = L_3\frac{di_3}{dt}$$

$$v_s = L_{eq}\left[\frac{v_s}{L_1}+\frac{v_s}{L_2}+\frac{v_s}{L_3}\right]$$

$$1 = L_{eq}\left[\frac{1}{L_1}+\frac{1}{L_2}+\frac{1}{L_3}\right]$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1}+\frac{1}{L_2}+\frac{1}{L_3}$$



$$\frac{1}{L_{eq}} = \sum_{i=1}^{N}\frac{1}{L_i}$$

Time-Harmonic Circuits: Phasors

Assume **ALL** voltages and currents in a circuit are time-harmonic: **no transients.** For example:

$$v(t) = V_0 \cos(\omega t + \phi)$$

ω: radian frequency: radians/sec

$\omega = 2\pi f$: f is frequency: cycles/sec, or Hz

$f = 1/T$: T is the period: sec

ϕ: phase offset: radians

Time-Harmonic Circuits: Phasors

Assume **ALL** voltages and currents in a circuit are time-harmonic: **no transients.** For example:

$$v(t) = V_0 \cos(\omega t + \phi)$$

Recall from math:

$$e^{jx} = \cos x + j \sin x$$

So we can rewrite:

$$v(t) = \text{Real-Part-of}\{V_0 e^{j(\omega t + \phi)}\}$$

$$v(t) = \Re\{V_0 e^{+j\phi} e^{j\omega t}\}$$

$$v(t) = \Re\{\tilde{V} e^{j\omega t}\}$$

Time-Harmonic Circuits: Phasors

$$v(t) = V_0 \cos(\omega t + \phi) \qquad v(t) = \Re e\{\tilde{V} e^{j\omega t}\}$$

$\tilde{V}$ is called the "Phasor Representation of V"

It is a complex number, $V_0 e^{i\phi}$
with an amplitude V_0 and a phase ϕ

EECS 215 book uses ***V* (bold-face)** for notation of the phasor for *v*(t)

Since students can't make bold handwritten letters,
the tilde is preferred for handwritten work.

Time-Harmonic Circuits: Phasors

Why do this? Look at an inductor in the time-domain:

$$v(t) = L\frac{di(t)}{dt}$$

Assuming that everything is time-harmonic, we must have:

$$v(t) = V\cos(\omega t + \phi_v)$$
$$i(t) = I\cos(\omega t + \phi_i)$$

Time-Harmonic Circuits: Phasors

Rewrite:

$$v(t) = L\frac{di(t)}{dt}$$

$$\Re\mathfrak{e}\{Ve^{j\phi_v}e^{j\omega t}\} = L\frac{d}{dt}(\Re\mathfrak{e}\{Ie^{j\phi_i}e^{j\omega t}\})$$

$$\Re\mathfrak{e}\{Ve^{j\phi_v}e^{j\omega t}\} = L\Re\mathfrak{e}\{j\omega Ie^{j\phi_i}e^{j\omega t}\}$$

Now, work with the complex quantities, and remember to come back at the end and apply the real-part operator.

Time-Harmonic Circuits: Phasors

Rewrite:

$$V e^{j\phi_v} e^{j\omega t} = L j\omega I e^{j\phi_i} e^{j\omega t}$$

$$V e^{j\phi_v} = j\omega L I e^{j\phi_i}$$

$$\tilde{V} = j\omega L \tilde{I}$$

where: $\tilde{V}$ $\tilde{I}$

are the Phasor Representations of *v*(*t*) and *i*(*t*)

Time-Harmonic Circuits: Phasors

So now we can write the impedance of an inductor:

$$\tilde{Z} = \frac{\tilde{V}}{\tilde{I}} = j\omega L$$

similarly for a capacitor:

$$\tilde{Z} = \frac{1}{j\omega C}$$

So instead of using time-derivatives and forming a **differential equation** to solve a circuit:
Use the phasor representation of the impedances and sources, and form an **algebraic equation** to solve a circuit.

Time-Harmonic Circuits: Parallel/Series

Note that impedances can be combined in a circuit just like resistances:

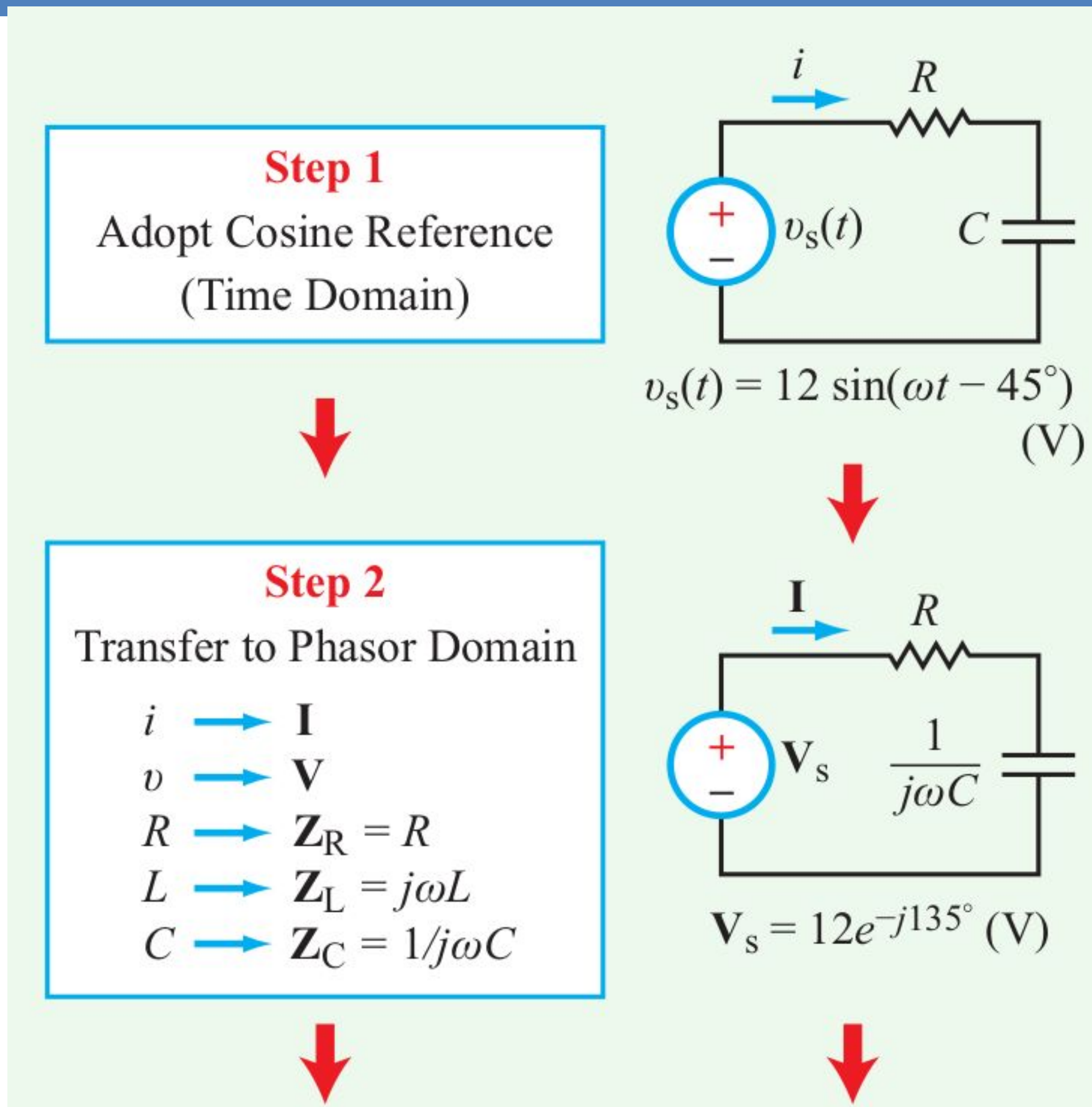
Impedances in series: add

$$\mathbf{Z}_{\text{eq}} = \sum_{i=1}^{N} \mathbf{Z}_i$$

Impedances in parallel: add reciprocally:

$$\mathbf{Z}_{\text{eq}} = \left[\sum_{i=1}^{N} \frac{1}{\mathbf{Z}_i} \right]^{-1}$$

Time-Harmonic Circuits: Solution Procedure



Time-Harmonic Circuits: Solution Procedure

Step 3
Cast Equations in Phasor Form

$$\mathbf{I}\left(R + \frac{1}{j\omega C}\right) = \mathbf{V}_s$$

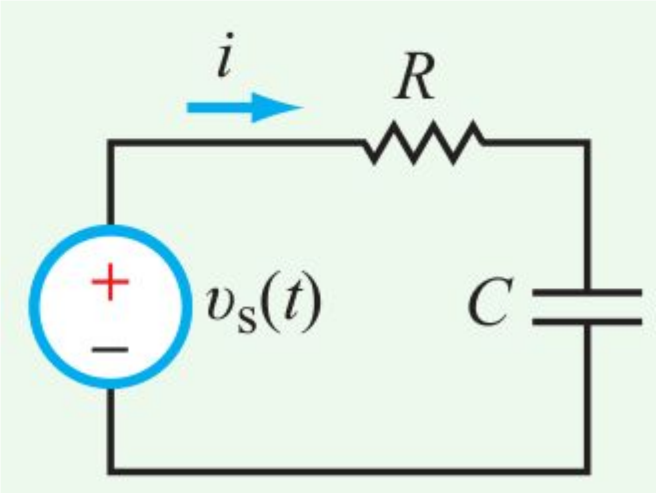
Step 4
Solve for Unknown Variable (Phasor Domain)

$$\mathbf{I} = \frac{\mathbf{V}_s}{R + \frac{1}{j\omega C}}$$

Step 5
Transform Solution Back to Time Domain

$$i(t) = \mathfrak{Re}[\mathbf{I}e^{j\omega t}] = 6\cos(\omega t - 105^\circ) \text{ (mA)}$$

Time-Harmonic Circuits: Solution Procedure



Given: The circuit shown:

$$v_s(t) = 12 \sin(\omega t - 45°) \text{ V}$$

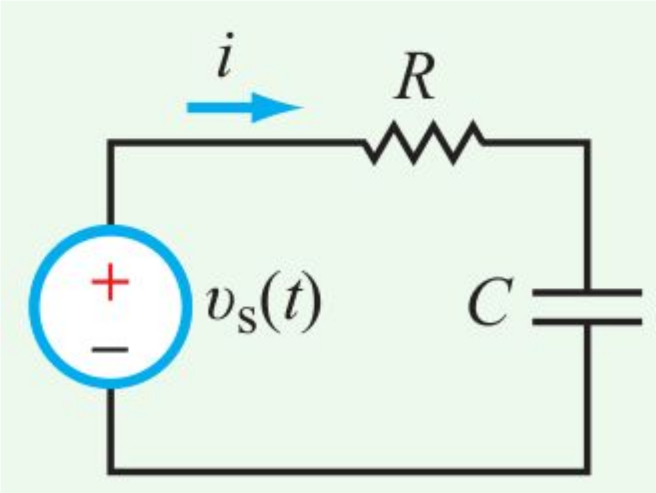
$$R = \sqrt{3} \text{ k}\Omega$$

$$C = 1 \ \mu\text{F}$$

$$\omega = 1 \times 10^3 \text{ rad/sec}$$

Find: An expression for the current, *i*, in the time-domain.

Time-Harmonic Circuits: Solution Procedure



Solution: Apply the 5-step phasor-based process:

1. Adopt cosine reference:

$$\upsilon_s(t) = 12\sin(\omega t - 45^\circ) \text{ V}$$
$$= 12\cos(\omega t - 45^\circ - 90^\circ) \text{ V}$$
$$= 12\cos(\omega t - 135^\circ) \text{ V.}$$

$$\widetilde{V}_s = 12e^{-j135^\circ} \text{ V}$$

Time-Harmonic Circuits: Solution Procedure

Solution: Apply the 5-step phasor-based process:

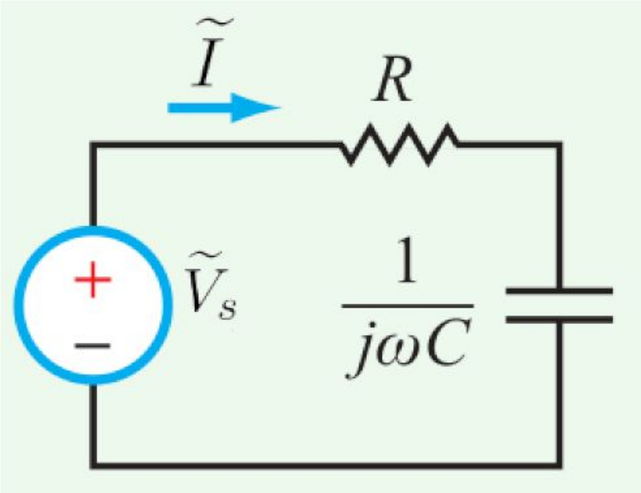
2. Transform circuit to phasor domain:

$$\widetilde{V}_s = 12e^{-j135^\circ} \text{ V}$$

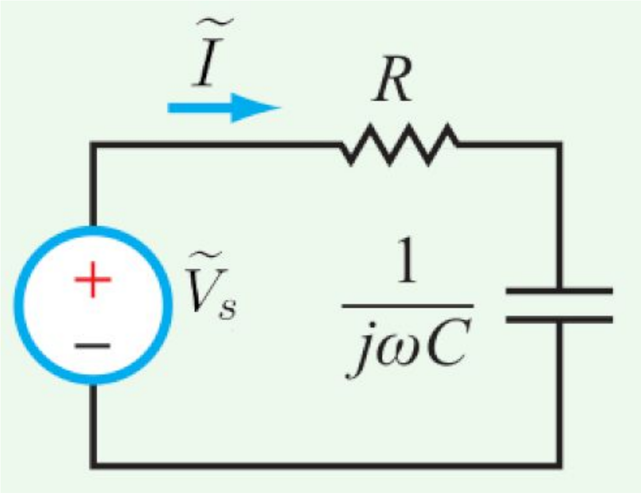
$$i(t) = \Re\text{e}\{\widetilde{I}e^{j\omega t}\}$$

$$Z_R = R$$

$$Z_C = \frac{1}{j\omega C}$$



Time-Harmonic Circuits: Solution Procedure



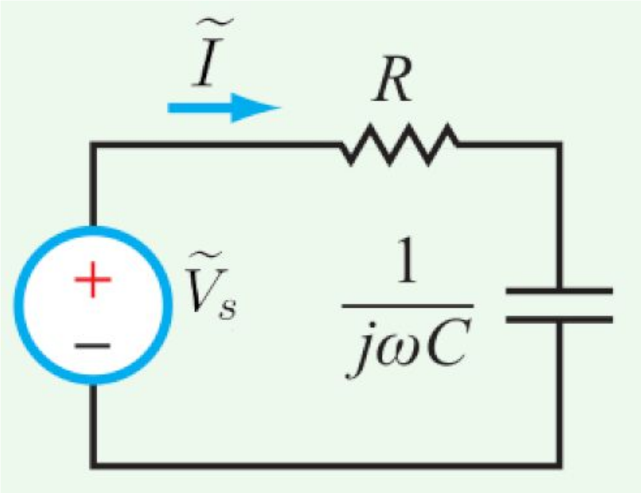
Solution: Apply the 5-step phasor-based process:

3. Use KCL/KVL: Apply KVL:

$$-\widetilde{V}_s + \widetilde{I}R + \widetilde{I}\frac{1}{j\omega C} = 0$$

$$\widetilde{I}\left\{R + \frac{1}{j\omega C}\right\} = 12e^{-j135^\circ} \text{ V}$$

Time-Harmonic Circuits: Solution Procedure



Solution: Apply the 5-step phasor-based process:

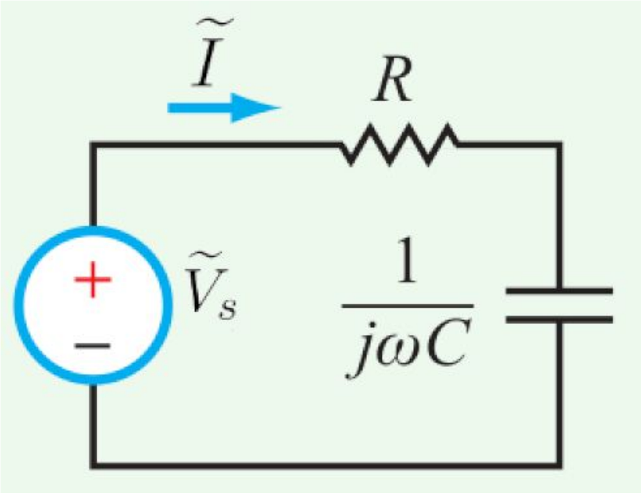
3. Solve for the unknown(s):

$$\widetilde{I}\left\{R + \frac{1}{j\omega C}\right\} = 12e^{-j135^\circ}\ \mathrm{V}$$

$$\widetilde{I} = \frac{12e^{-j135^\circ}\ \mathrm{V}}{R + \frac{1}{j\omega C}}$$

$$\widetilde{I} = \frac{j12\omega C e^{-j135^\circ}\ \mathrm{V}}{j\omega RC + 1}$$

Time-Harmonic Circuits: Solution Procedure



Solution: Apply the 5-step phasor-based process:

3. Solve for the unknown(s):

$$\widetilde{I} = \frac{j12\omega C e^{-j135^\circ} \text{ V}}{j\omega RC + 1}$$

plug in given values:

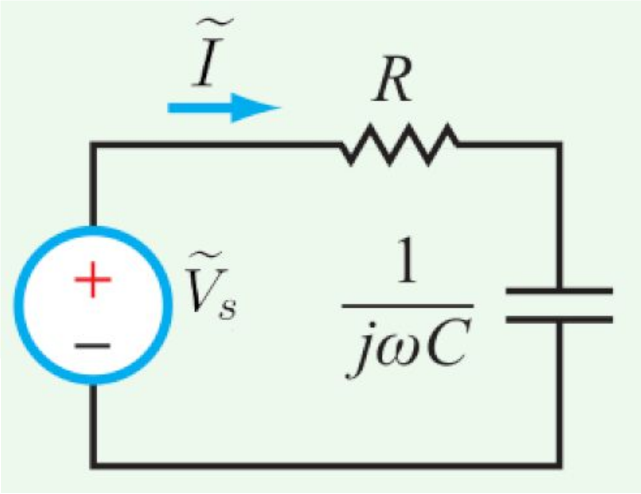
$$R = \sqrt{3}\ \Omega$$

$$C = 1\ \mu\text{F}$$

$$\omega = 1 \times 10^3 \text{ rad/sec}$$

$$\widetilde{I} = \frac{j12(1 \times 10^3 \text{ rad/sec})(1\ \mu\text{F})e^{-j135^\circ} \text{ V}}{j(1 \times 10^3 \text{ rad/sec})(\sqrt{3}\ \text{k}\Omega)(1\ \mu\text{F}) + 1}$$

Time-Harmonic Circuits: Solution Procedure



Solution: Apply the 5-step phasor-based process:

3. Solve for the unknown(s):

$$\widetilde{I} = \frac{j12(1 \times 10^3 \text{ rad/sec})(1\ \mu\text{F})e^{-j135^\circ}\ \text{V}}{j(1 \times 10^3 \text{ rad/sec})(\sqrt{3}\ \text{k}\Omega)(1\ \mu\text{F}) + 1}$$

Check units: ωC has units of Ω^{-1},

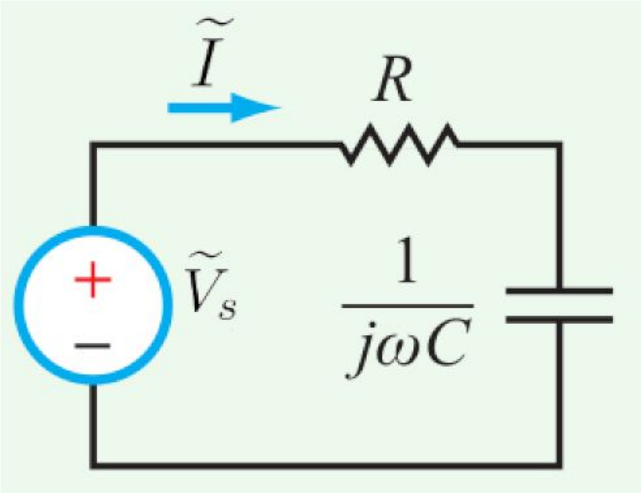
numerator units: A, denominator units: none

good

Simplify:

$$\widetilde{I} = \frac{j12e^{-j135^\circ}}{1 + j\sqrt{3}}\ \text{mA}$$

Time-Harmonic Circuits: Solution Procedure



Solution: Apply the 5-step phasor-based process:

3. Solve for the unknown(s):

$$\widetilde{I} = \frac{j12e^{-j135^\circ}}{1+j\sqrt{3}} \text{ mA}$$

division of complex numbers: use mag/phase:

$$1+j\sqrt{3} = \sqrt{1+3}e^{j\tan^{-1}(\sqrt{3}/1)}$$ (arctan ok: in region 1)

$$= 2e^{j60^\circ}$$

plug in: $$\widetilde{I} = \frac{12e^{-j135^\circ}e^{j90^\circ}}{2e^{j60^\circ}} \text{ mA}$$

Time-Harmonic Circuits: Solution Procedure

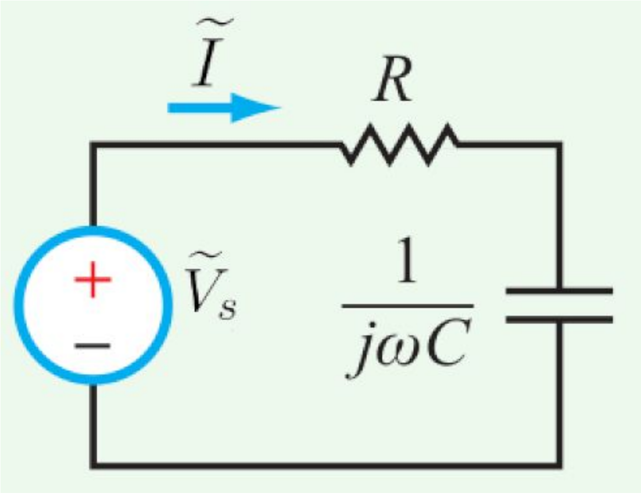
Solution: Apply the 5-step phasor-based process:

3. Solve for the unknown(s):

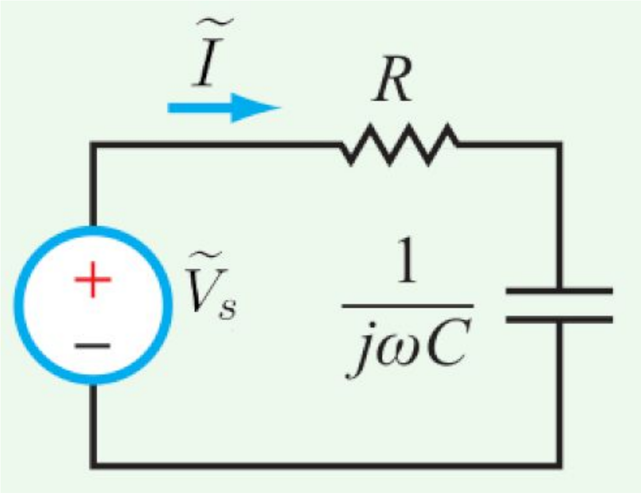
$$\widetilde{I} = \frac{12e^{-j135^\circ}e^{j90^\circ}}{2e^{j60^\circ}} \text{ mA}$$

$$\widetilde{I} = 6e^{j(-135^\circ+90^\circ-60^\circ)} \text{ mA}$$

$$\widetilde{I} = 6e^{-j105^\circ} \text{ mA}$$



Time-Harmonic Circuits: Solution Procedure



Solution: Apply the 5-step phasor-based process:

4. Transform solution to time-domain:

$$i(t) = \Re\mathrm{e}\{\widetilde{I}e^{j\omega t}\}$$

$$i(t) = \Re\mathrm{e}\{6e^{-j105^\circ}e^{j\omega t}\}\ \mathrm{mA}$$

$$\boxed{i(t) = 6\cos(\omega t - 105^\circ)\ \mathrm{mA}}$$

note: could fill in the value of ω, or not. Either is fine.

Time-Harmonic Circuits: Traveling Waves

What is the phasor representation of a travelling wave?

$$
\begin{aligned}
y(x,t) &= Ae^{-\alpha x}\cos(\omega t - \beta x - \phi_0) \\
&= \Re\mathrm{e}\left\{Ae^{-\alpha x}e^{j(\omega t - \beta x - \phi_0)}\right\} \\
&= \Re\mathrm{e}\left\{Ae^{-\alpha x}e^{-j\beta x}e^{j\phi_0}e^{j\omega t}\right\}
\end{aligned}
$$

Hence:

$$
\tilde{Y} = Ae^{j\phi_0}e^{-(\alpha + j\beta)x}
$$

Time-Harmonic Circuits: Traveling Waves

Given: The phasor travelling-wave:

$$\widetilde{Y} = 10e^{j10^\circ} e^{-(0.1\ \text{Np/m} + j3\ \text{rad/m})x}\ \text{mA}$$

Find: The time-domain expression

Solution:

$$y(t) = \Re\text{e}\{\widetilde{Y} e^{j\omega t}\}$$

$$y(t) = \Re\text{e}\{10e^{j10^\circ} e^{-(0.1\ \text{Np/m} + j3\ \text{rad/m})x} e^{j\omega t}\}\ \text{mA}$$

$$\boxed{y(t) = 10e^{-(0.1\ \text{Np/m})x} \cos(\omega t - (3\ \text{rad/m})x + 10^\circ)\ \text{mA}}$$

Homework

Homework 2 is due tomorrow at midnight.

submit to gradescope via the canvas site.

Next Time

Sections 2-1 through 2-4:

What is a transmission line?
Why study transmission lines?
model
diffeq
solve diffeq
wave propagation eqns