Lecture 3:
Linear Classifiers

Reminder: Assignment 1

- Due **Friday 1/14, 11:59pm EST**
- If you enroll late, you get a free extension for A1:
  - due_date = latest_day(original_due_date, your_enroll_date + 7 days)
- Make sure you submit the right .py file!
  - Make sure to **manually save** the .py file in Colab
  - After you download the .zip file, **check that the .py file is correct**

Office Hours

- Check Google Calendar (link also on website):
  https://calendar.google.com/calendar/b/0?cid=dW1pY2guZWR1X2cxMXJnNnZxNmd2YWNqOWRhZDRxOHVvZHNvQGdyb3VwLmNhbGVuZGFyLmdvb2dsZS5jb20
- Office hours may shift a bit from week to week (especially mine) – check Google Calendar for up-to-date info
- We'll use Umich office hours queue system; find link in the description of each calendar event

Last time: Image Classification

Input: image

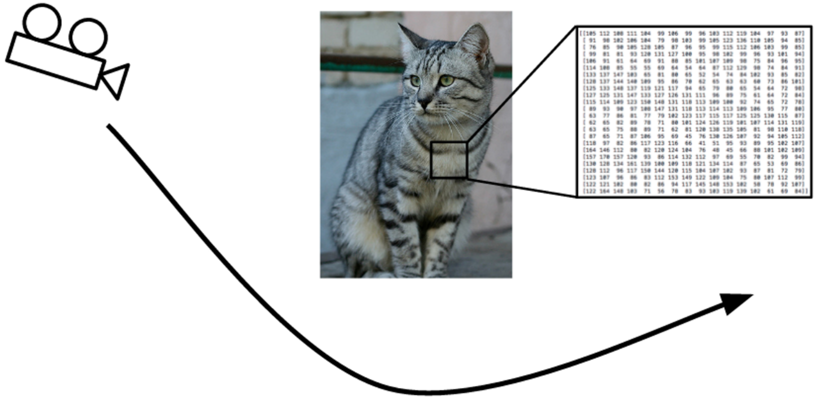
This image by Nikita is
licensed under CC-BY 2.0

Output: Assign image to one of a fixed set of categories

cat
bird
deer
dog
truck

Last Time: Challenges of Recognition

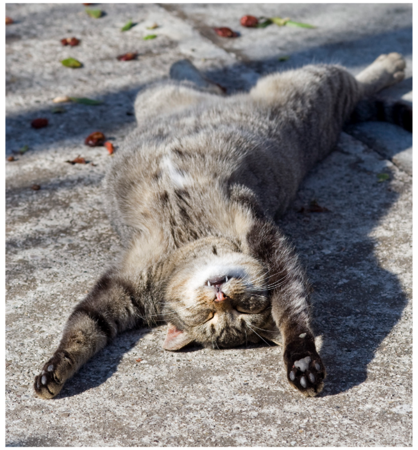
Viewpoint



Illumination

This image is CC0 1.0 public domain

Deformation



This image by Umberto Salvagnin is licensed under CC-BY 2.0

Occlusion

This image by jonsson is licensed under CC-BY 2.0

Clutter

This image is CC0 1.0 public domain

Intraclass Variation

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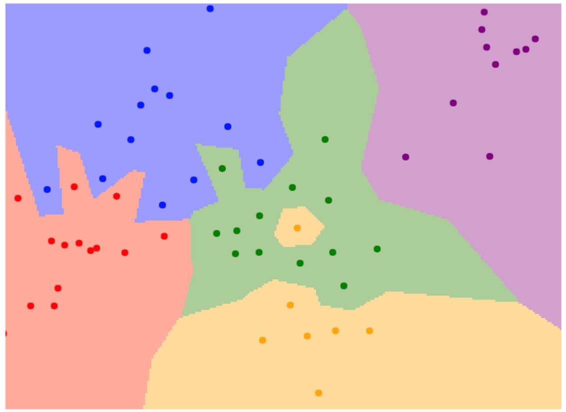
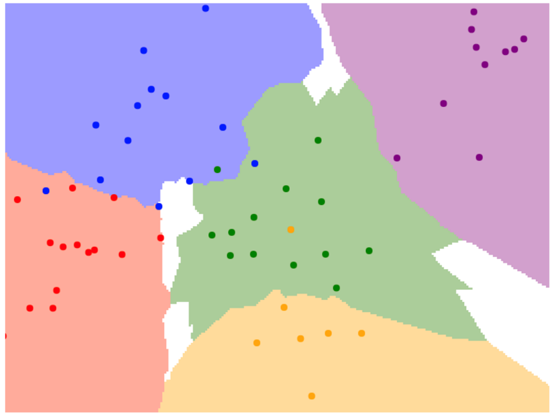
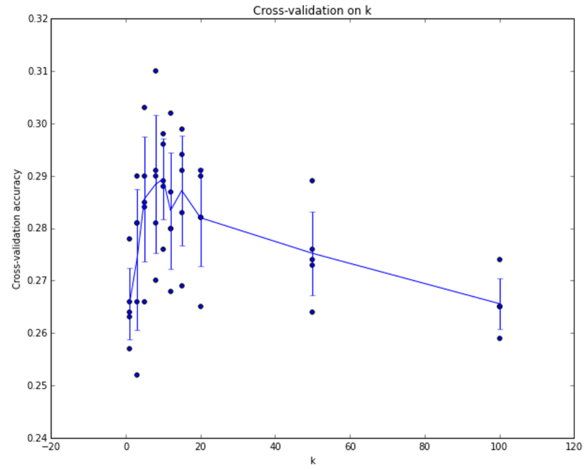
Last time: Data-Drive Approach, kNN

1-NN classifier



5-NN classifier





train	test

train	validation	test

Today: Linear Classifiers

Neural Network

Linear classifiers

This image is CC0 1.0 public domain

Recall CIFAR10

50,000 training images
each image is **32x32x3**

10,000 test images.

Parametric Approach

Image

Array of **32x32x3** numbers
(3072 numbers total)

f(**x**,**W**)

10 numbers giving
class scores

W
parameters
or weights

Parametric Approach: Linear Classifier

$$f(x,W) = Wx$$

Image

Array of **32x32x3** numbers
(3072 numbers total)

$f(\mathbf{x},\mathbf{W})$

10 numbers giving class scores

W
parameters or weights

Parametric Approach: Linear Classifier

$$\underbrace{\boxed{f(x,W)}}_{(10,)} = \underbrace{\boxed{W}}_{(10,\ 3072)}\overbrace{\boxed{x}}^{(3072,)}$$

Image

Array of **32x32x3** numbers
(3072 numbers total)

→ $f(\mathbf{x},\mathbf{W})$ → **10** numbers giving class scores

↑

W

parameters
or weights

Parametric Approach: Linear Classifier

$$\boxed{f(x,W)} = \boxed{W}\boxed{x} + \boxed{b}$$

(3072,)

(10,) (10, 3072) (10,)

Image

Array of **32x32x3** numbers
(3072 numbers total)

f(**x**,**W**)

10 numbers giving class scores

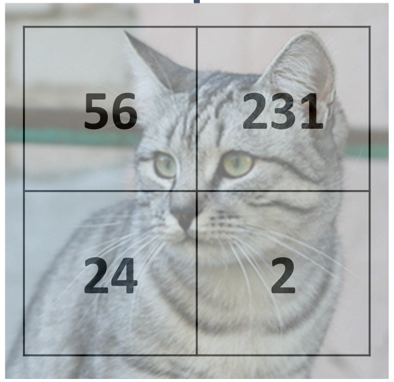
W

parameters
or weights

Example for 2x2 image, 3 classes (cat/dog/ship)

Stretch pixels into column

$f(x,W) = Wx + b$



Input image
(2, 2)

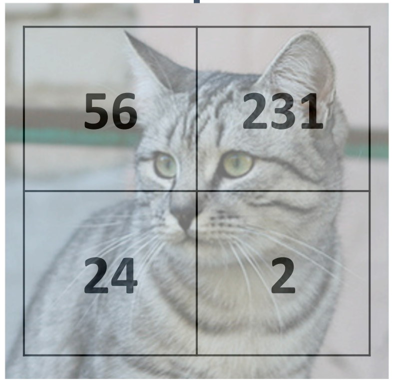
$$\begin{bmatrix} 56 \\ 231 \\ 24 \\ 2 \end{bmatrix}$$

(4,)

Example for 2x2 image, 3 classes (cat/dog/ship)

Stretch pixels into column

$$f(x,W) = Wx + b$$



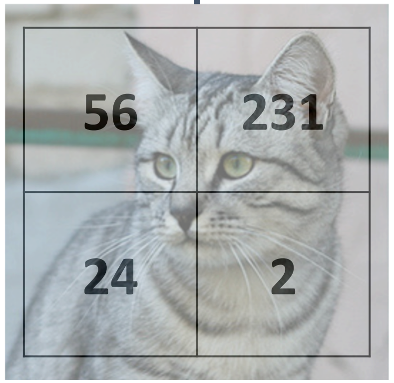
Input image
(2, 2)

$$\underbrace{\begin{bmatrix} 0.2 & -0.5 & 0.1 & 2.0 \\ 1.5 & 1.3 & 2.1 & 0.0 \\ 0 & 0.25 & 0.2 & -0.3 \end{bmatrix}}_{W\ (3,\,4)} \underbrace{\begin{bmatrix} 56 \\ 231 \\ 24 \\ 2 \end{bmatrix}}_{(4,)} + \underbrace{\begin{bmatrix} 1.1 \\ 3.2 \\ -1.2 \end{bmatrix}}_{b\ (3,)} = \underbrace{\begin{bmatrix} -96.8 \\ 437.9 \\ 61.95 \end{bmatrix}}_{(3,)}$$

Linear Classifier: Algebraic Viewpoint

Stretch pixels into column

$f(x,W) = Wx + b$



Input image

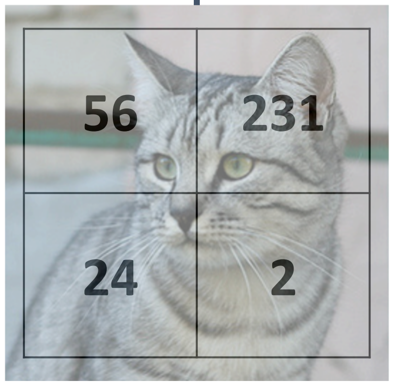
(2, 2)

$$\underbrace{\begin{bmatrix} 0.2 & -0.5 & 0.1 & 2.0 \\ 1.5 & 1.3 & 2.1 & 0.0 \\ 0 & 0.25 & 0.2 & -0.3 \end{bmatrix}}_{W \;(3,\,4)} \underbrace{\begin{bmatrix} 56 \\ 231 \\ 24 \\ 2 \end{bmatrix}}_{(4,)} + \underbrace{\begin{bmatrix} 1.1 \\ 3.2 \\ -1.2 \end{bmatrix}}_{b \;(3,)} = \underbrace{\begin{bmatrix} -96.8 \\ 437.9 \\ 61.95 \end{bmatrix}}_{(3,)}$$

Linear Classifier: Bias Trick

Add extra one to data vector; bias is absorbed into last column of weight matrix

Stretch pixels into column



Input image
(2, 2)

0.2	-0.5	0.1	2.0	1.1
1.5	1.3	2.1	0.0	3.2
0	0.25	0.2	-0.3	-1.2

W (3, 5)

56
231
24
2
1

(5,)

=

-96.8
437.9
61.95

(3,)

Linear Classifier: Predictions are Linear!

$$f(x, W) = Wx \quad \text{(ignore bias)}$$

$$f(cx, W) = W(cx) = c * f(x, W)$$

Linear Classifier: Predictions are Linear!

$$f(x, W) = Wx \quad \text{(ignore bias)}$$

$$f(cx, W) = W(cx) = c * f(x, W)$$

Image

Scores

-96.8
437.8
62.0

0.5 * Image

0.5 * Scores

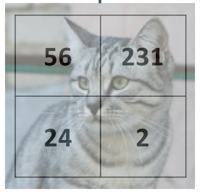
-48.4
218.9
31.0

Interpreting a Linear Classifier

Algebraic Viewpoint

$$f(x,W) = Wx + b$$

Stretch pixels into column



Input image
(2, 2)

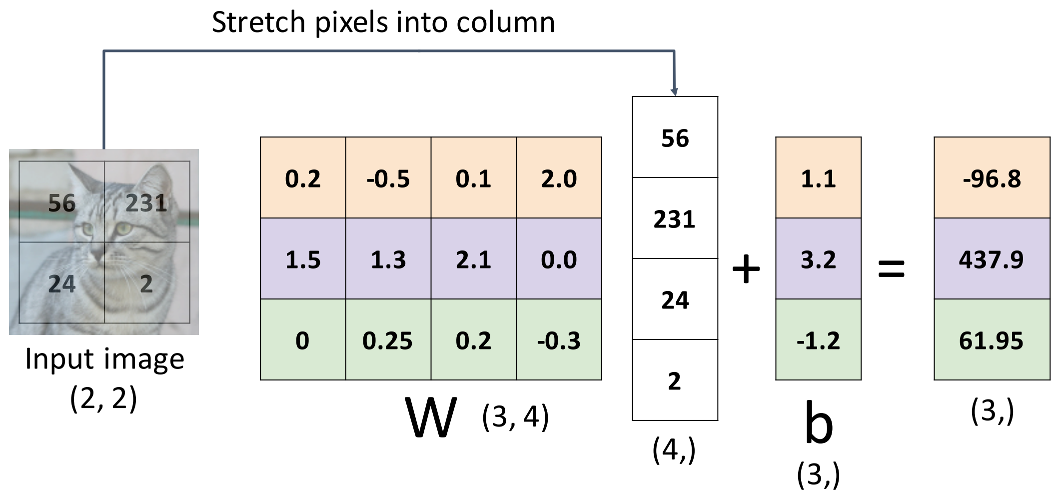
$$\underbrace{\begin{bmatrix} 0.2 & -0.5 & 0.1 & 2.0 \\ 1.5 & 1.3 & 2.1 & 0.0 \\ 0 & 0.25 & 0.2 & -0.3 \end{bmatrix}}_{W\ (3,\,4)} \underbrace{\begin{bmatrix} 56 \\ 231 \\ 24 \\ 2 \end{bmatrix}}_{(4,)} + \underbrace{\begin{bmatrix} 1.1 \\ 3.2 \\ -1.2 \end{bmatrix}}_{b\ (3,)} = \underbrace{\begin{bmatrix} -96.8 \\ 437.9 \\ 61.95 \end{bmatrix}}_{(3,)}$$

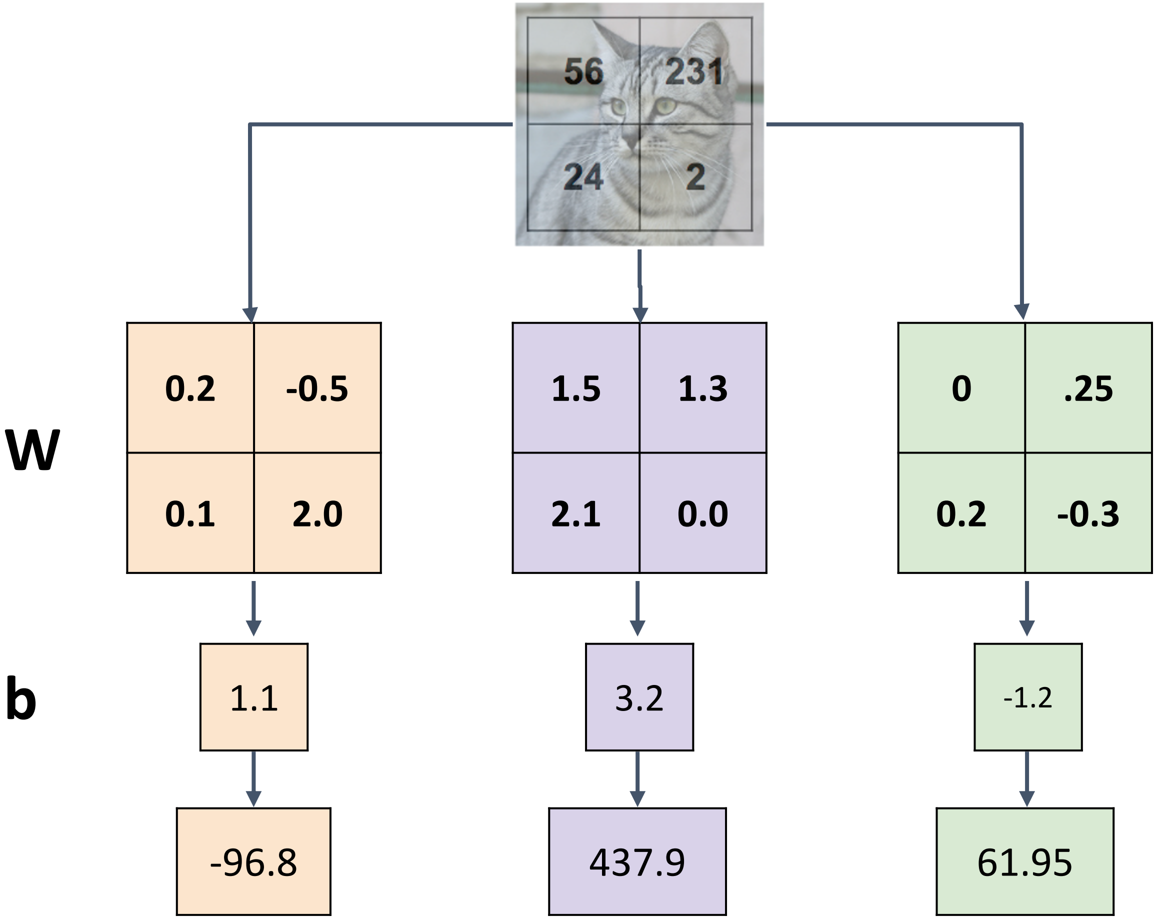
Interpreting a Linear Classifier

Instead of stretching pixels into columns, we can equivalently stretch rows of W into images!

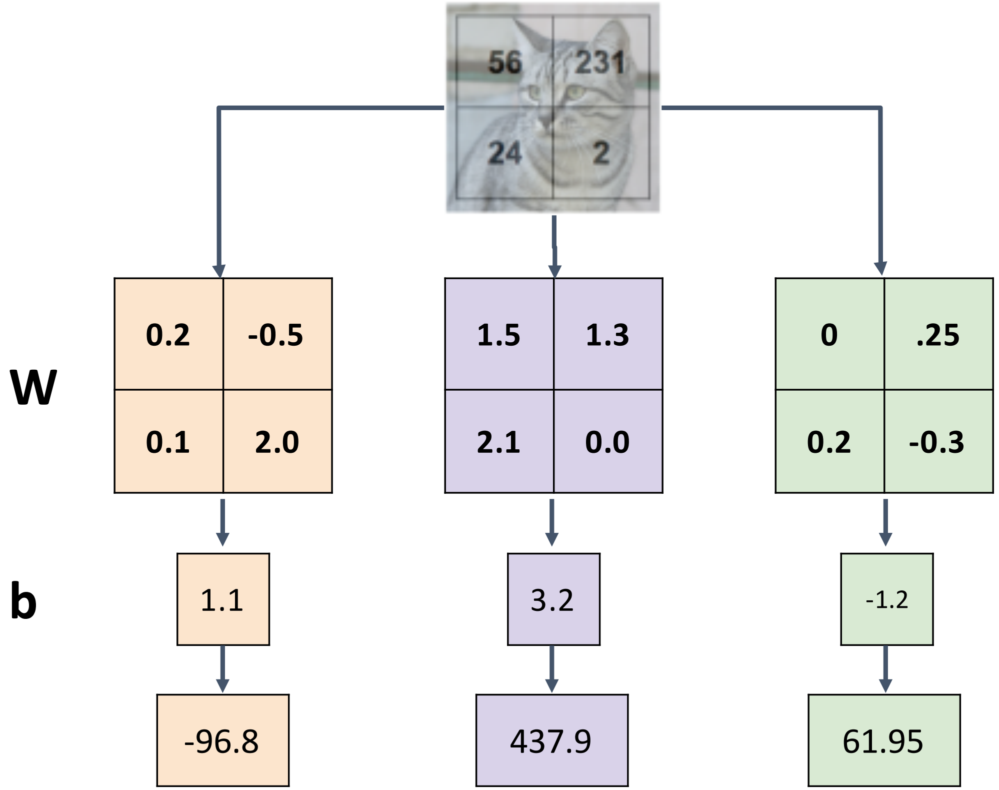
Algebraic Viewpoint

$$f(x,W) = Wx + b$$

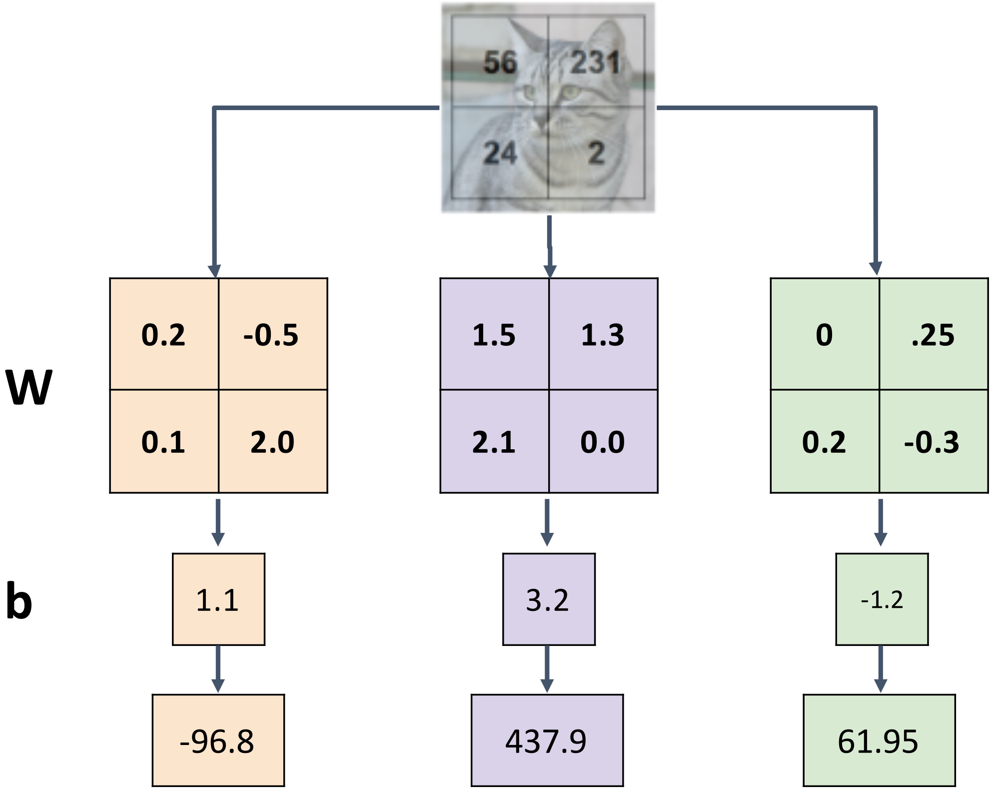


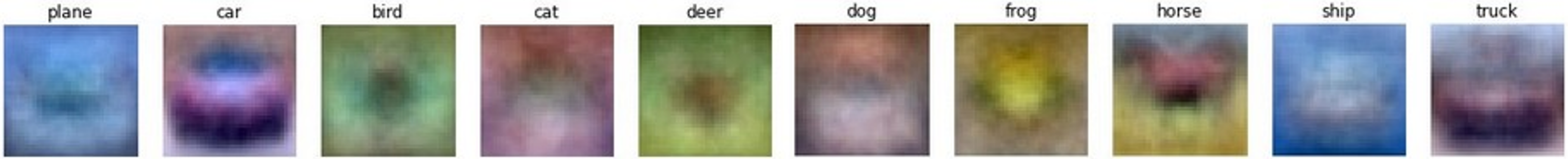


Interpreting an Linear Classifier



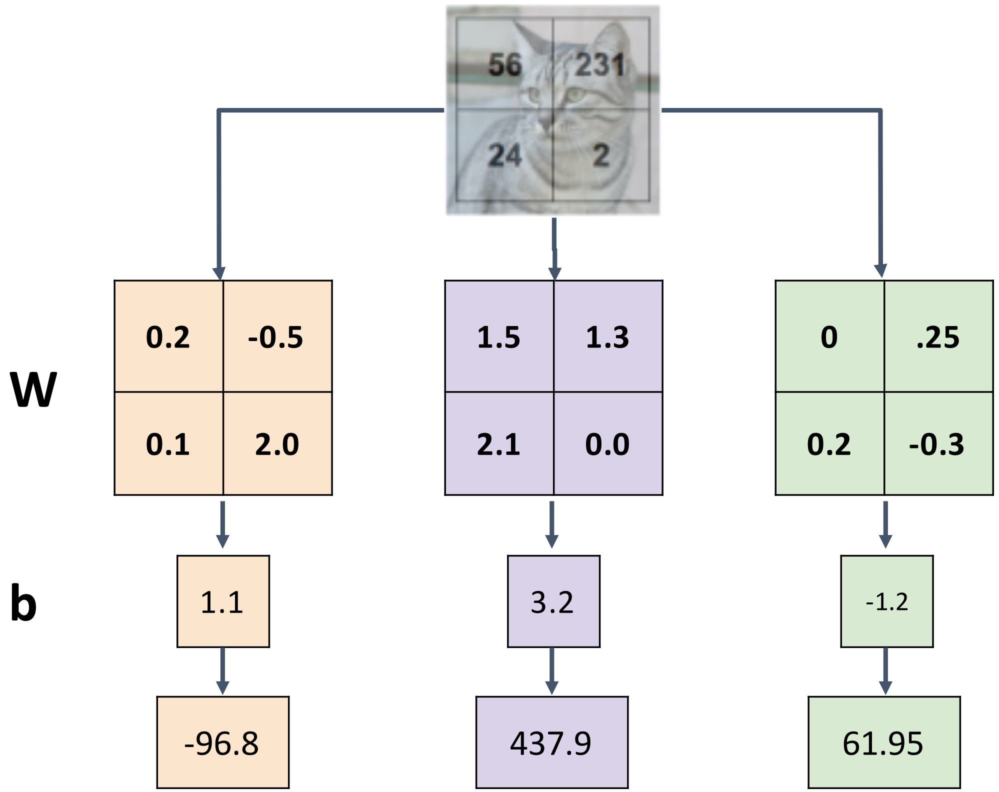
Interpreting an Linear Classifier: Visual Viewpoint





Interpreting an Linear Classifier: Visual Viewpoint

Linear classifier has one "template" per category

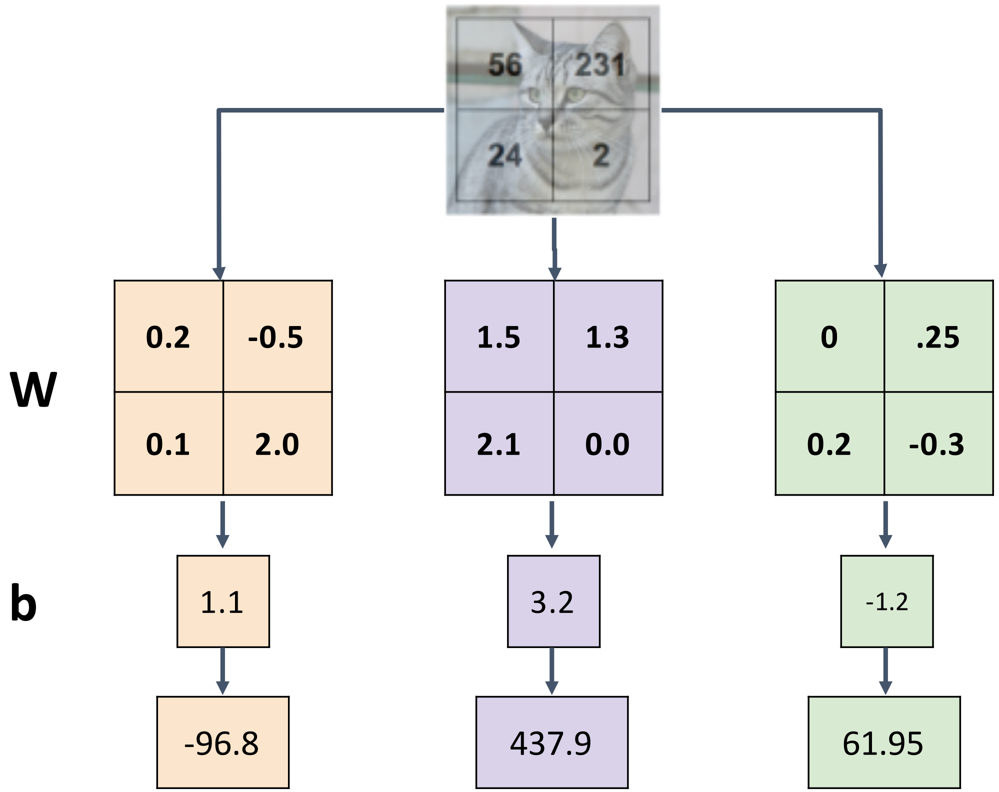


Interpreting an Linear Classifier: Visual Viewpoint

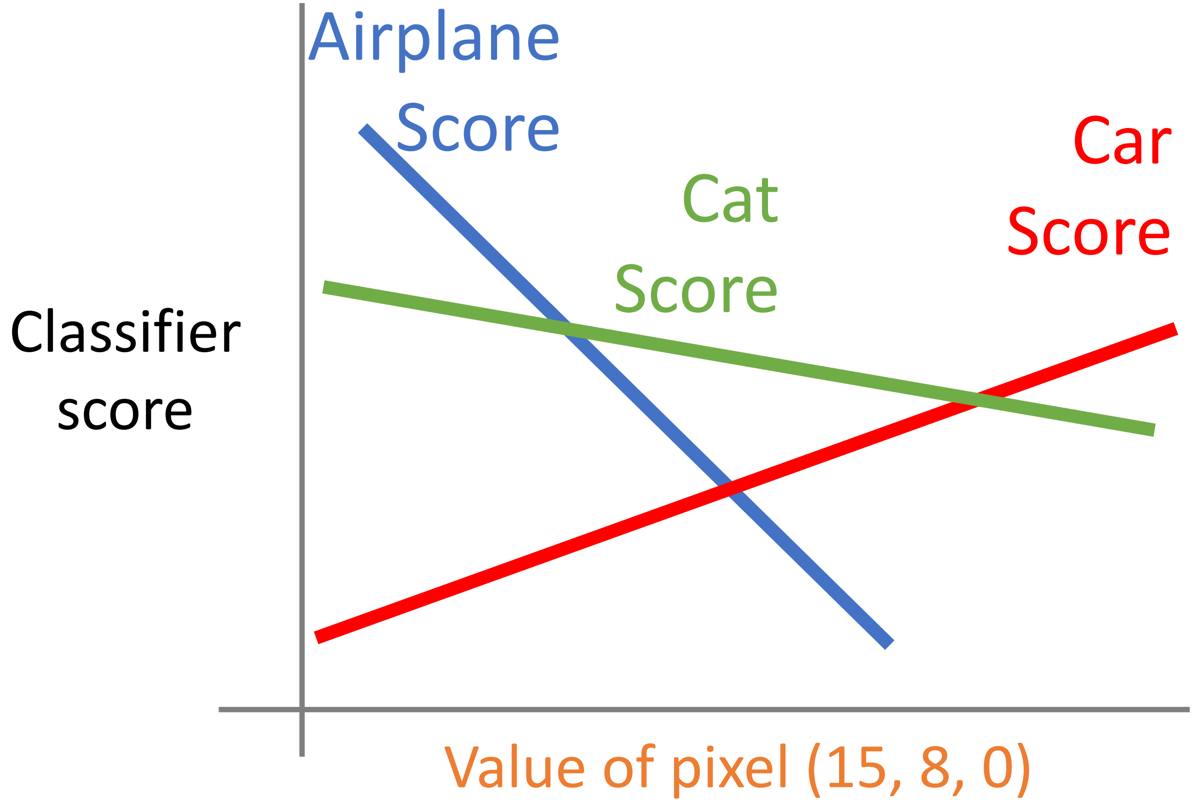
Linear classifier has one "template" per category

A single template cannot capture multiple modes of the data

e.g. horse template has 2 heads!



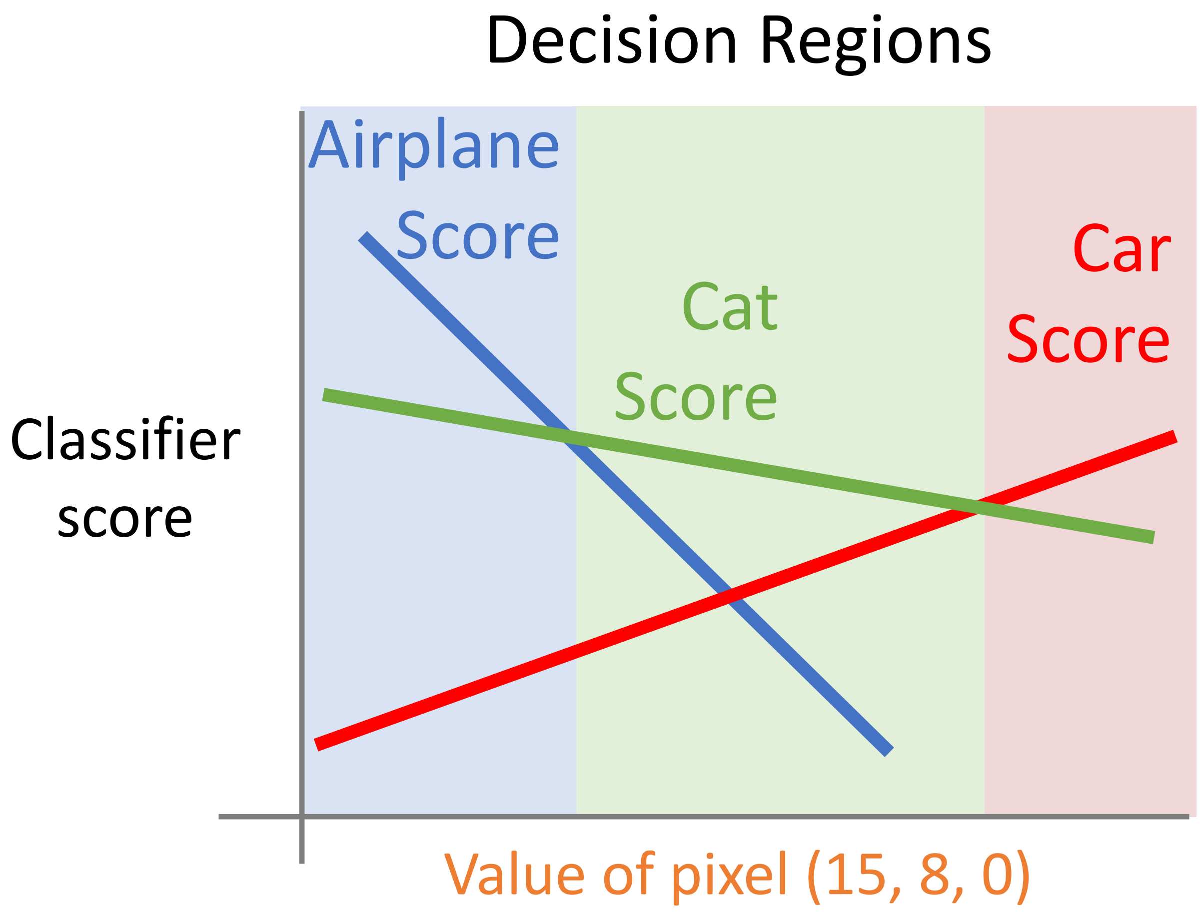
Interpreting a Linear Classifier: Geometric Viewpoint



$$f(x,W) = Wx + b$$

Array of **32x32x3** numbers
(3072 numbers total)

Interpreting a Linear Classifier: Geometric Viewpoint



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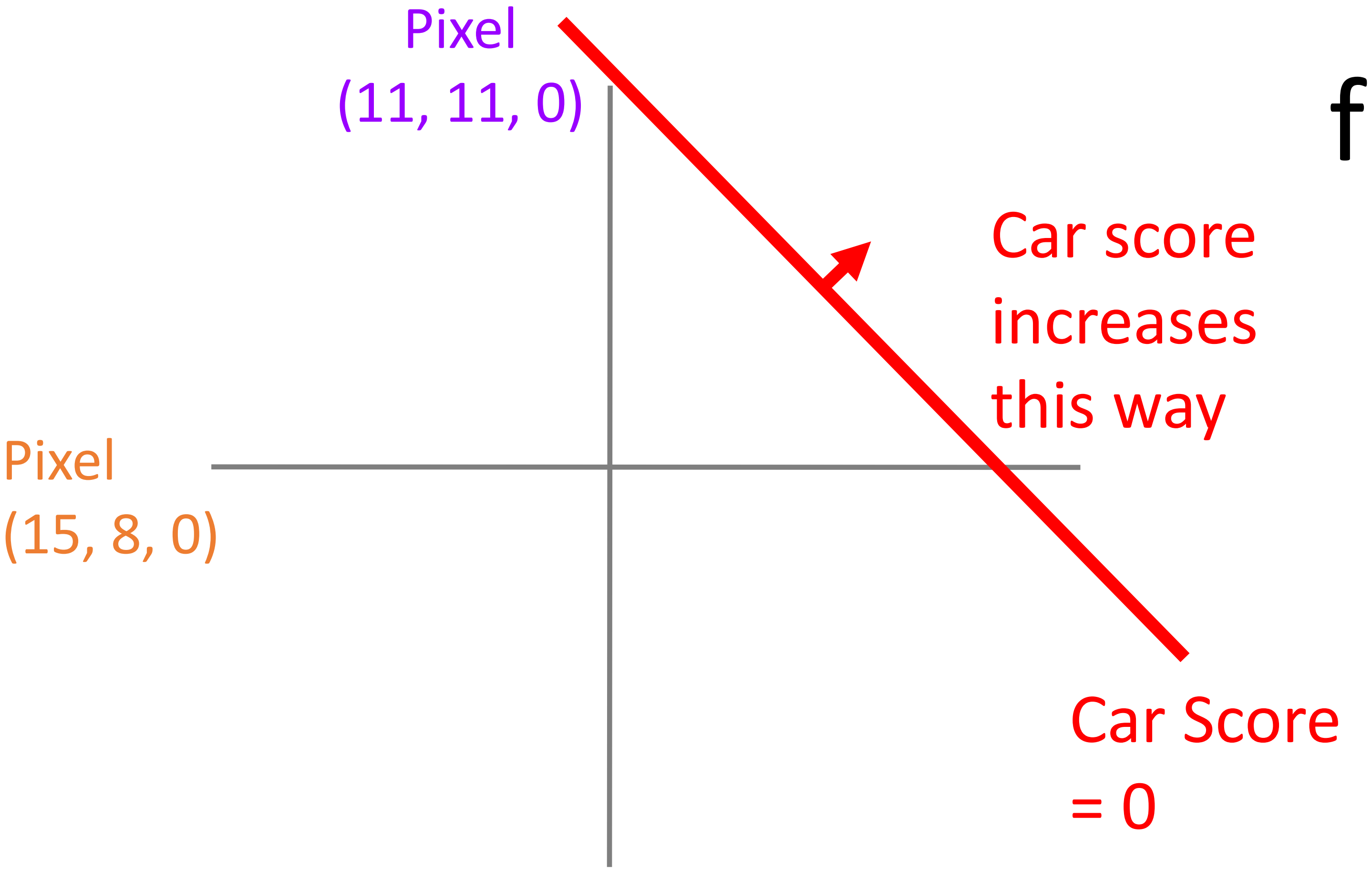
Interpreting a Linear Classifier: Geometric Viewpoint



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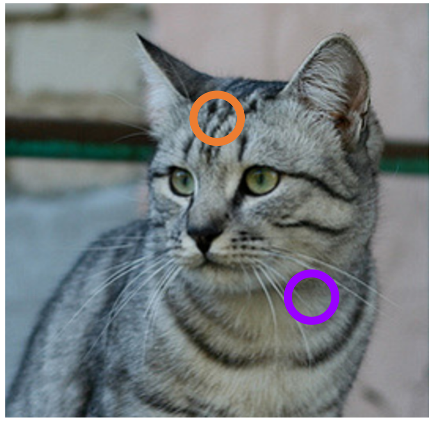

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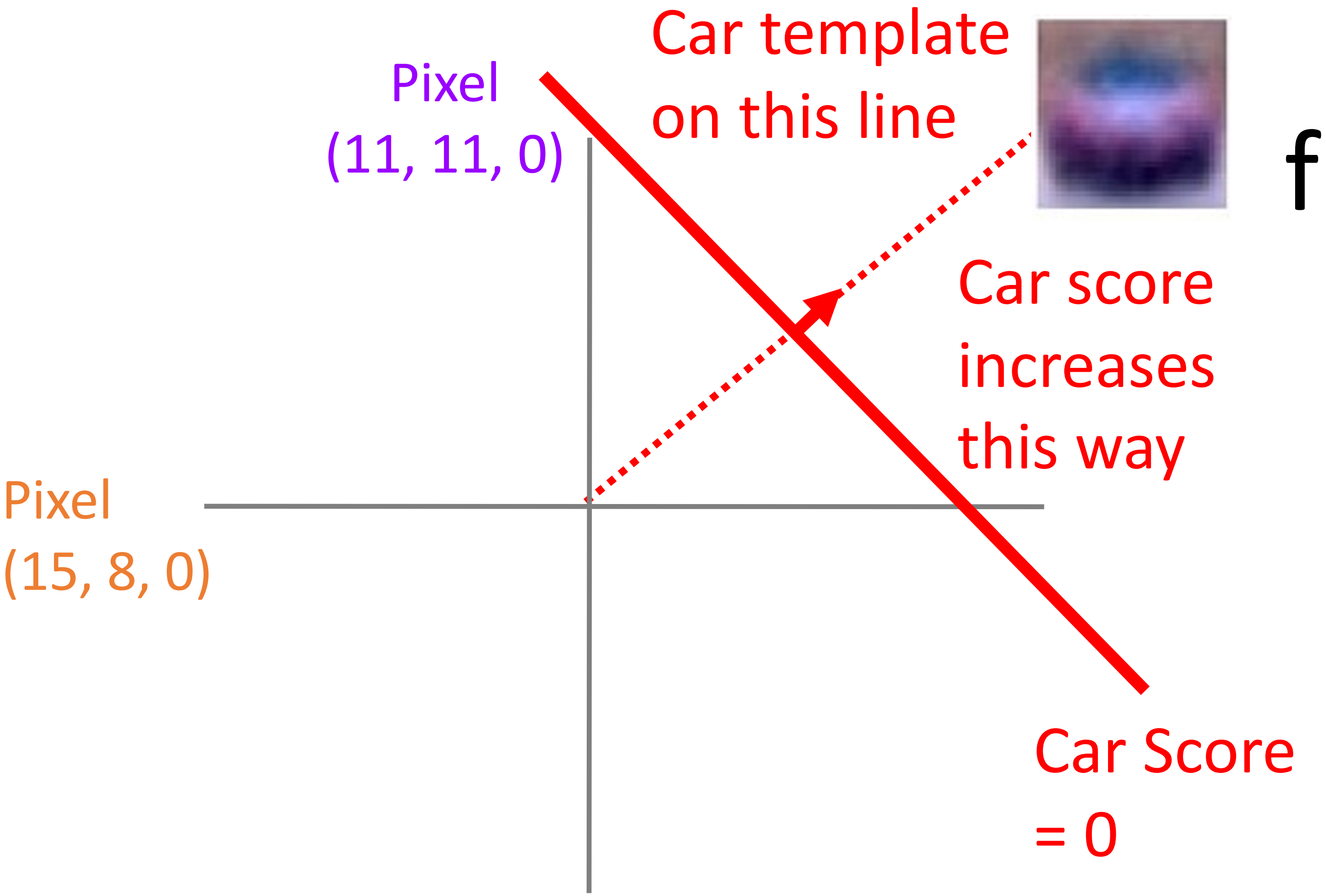
Interpreting a Linear Classifier: <u>Geometric Viewpoint</u>



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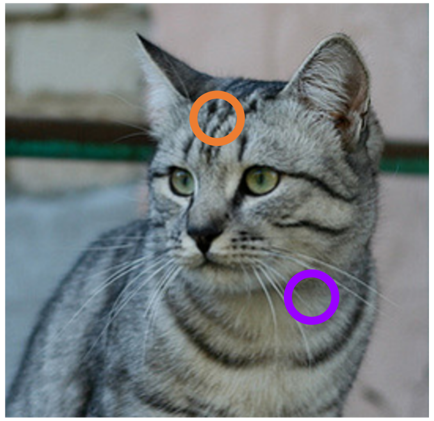

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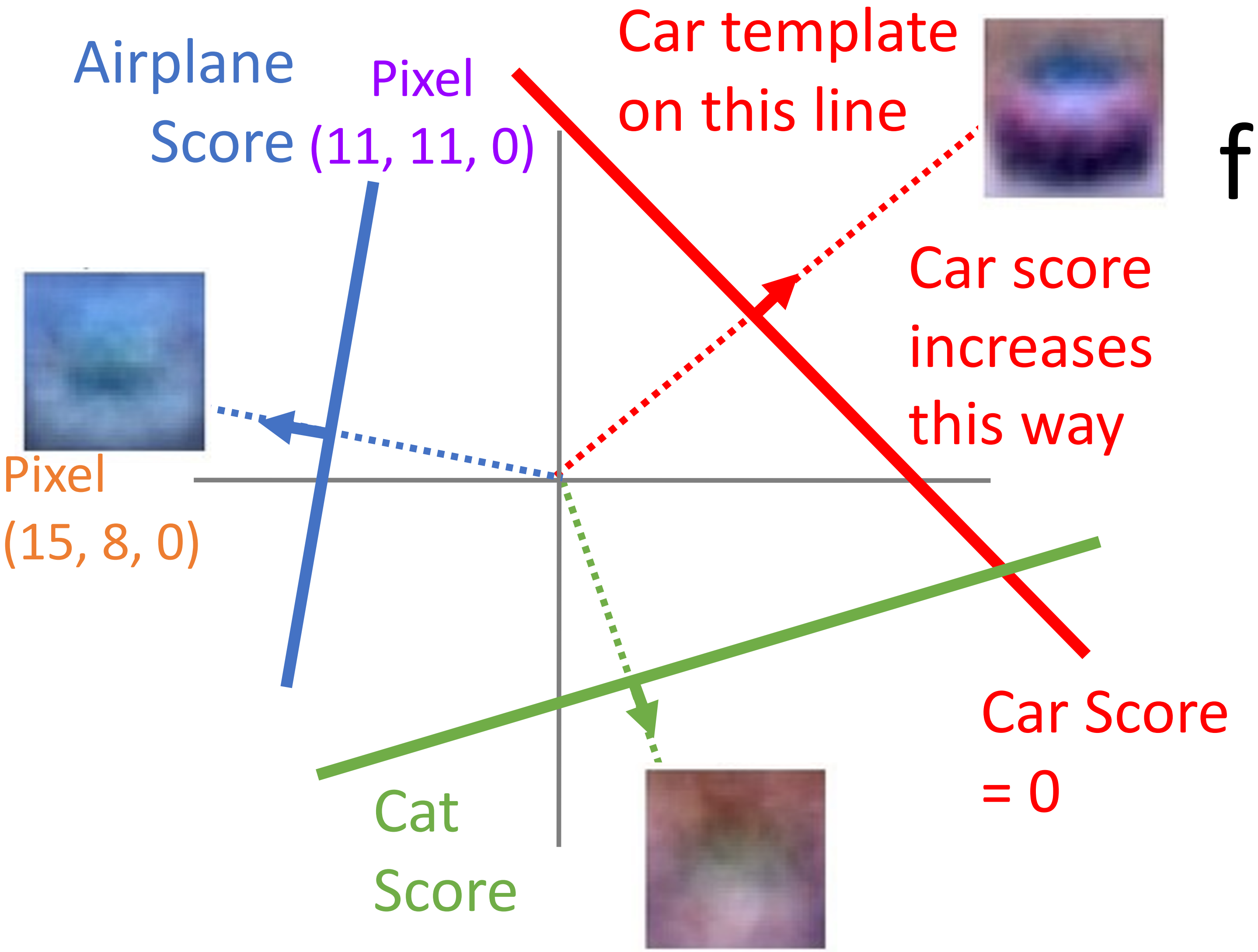
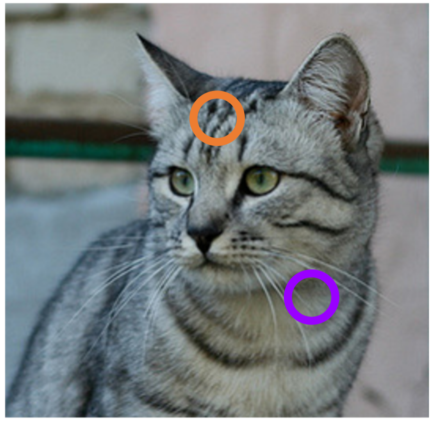
Interpreting a Linear Classifier: Geometric Viewpoint

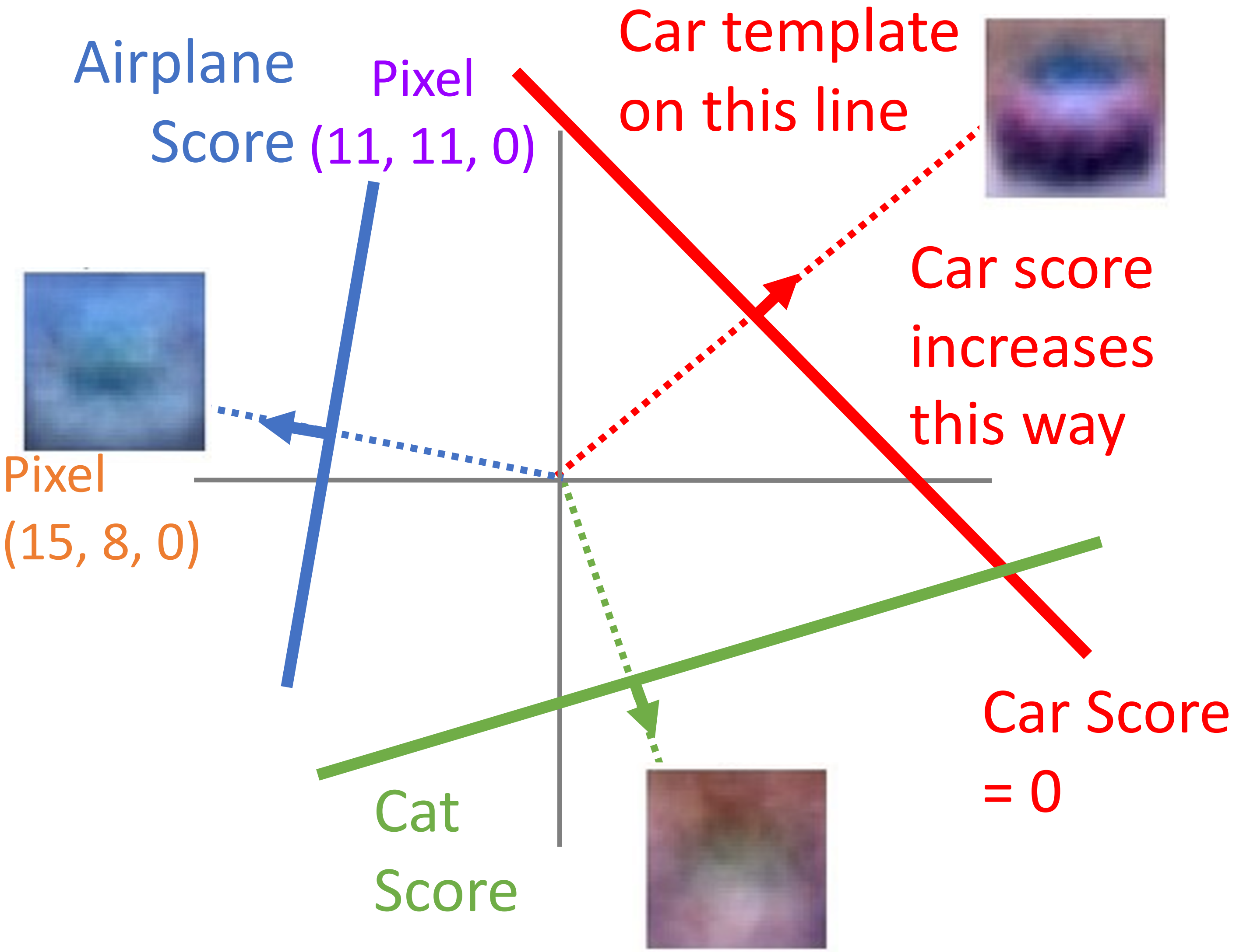


$$f(x,W) = Wx + b$$

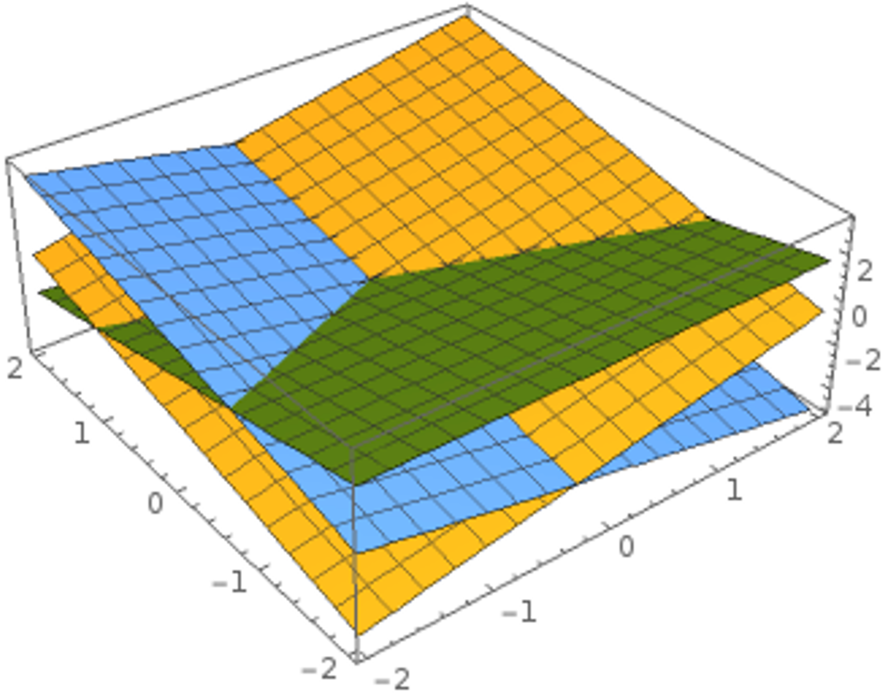


Array of **32x32x3** numbers
(3072 numbers total)

Interpreting a Linear Classifier: Geometric Viewpoint



Hyperplanes carving up a high-dimensional space

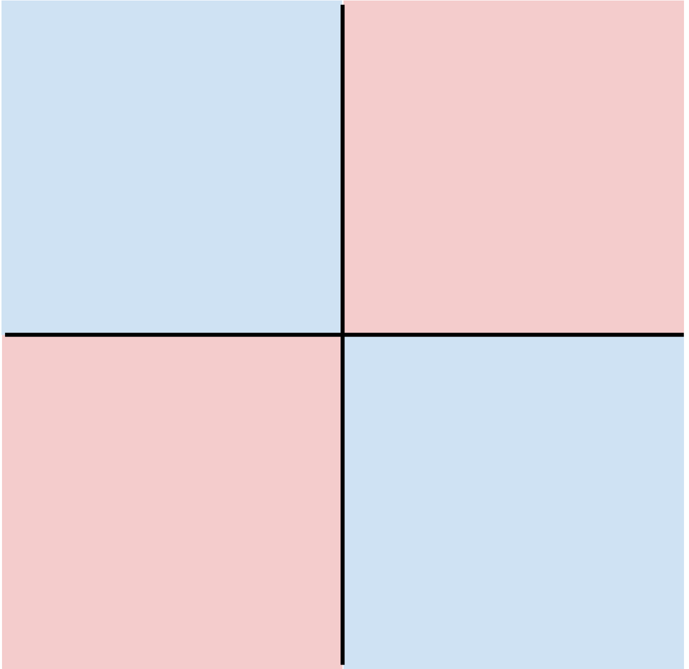


Plot created using Wolfram Cloud

Hard Cases for a Linear Classifier

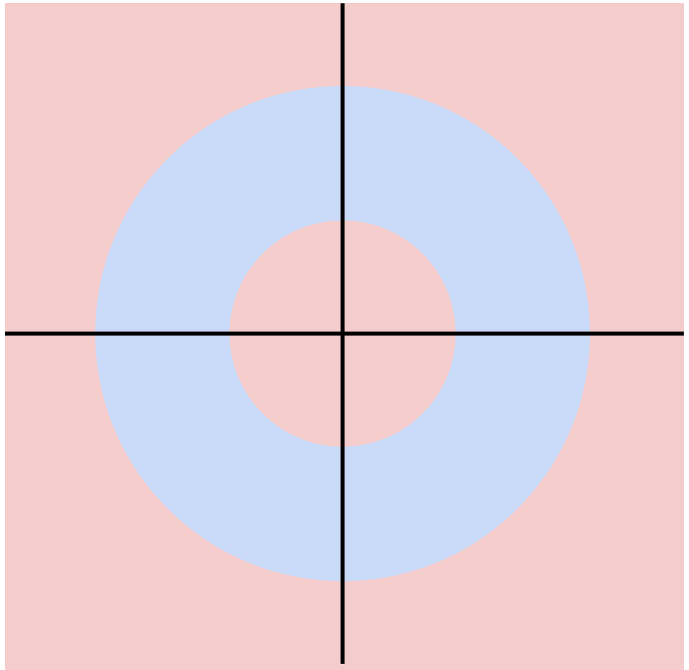
Class 1:
First and third quadrants

Class 2:
Second and fourth quadrants



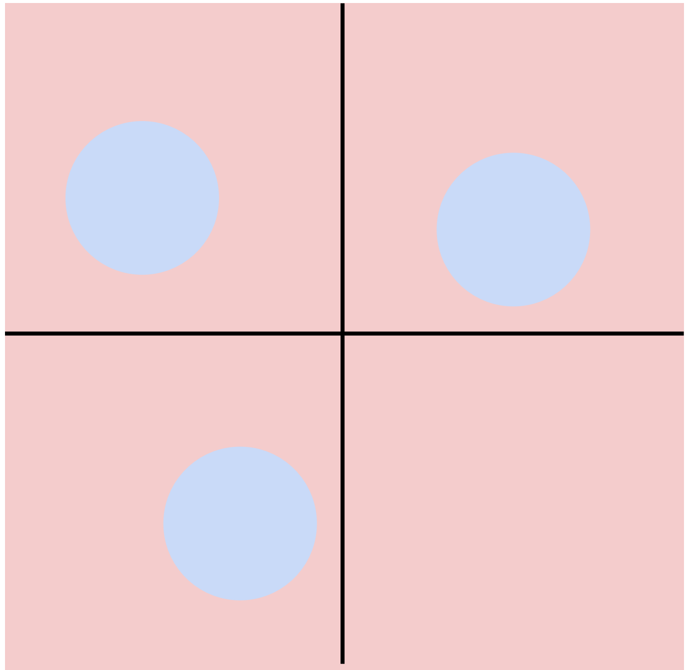
Class 1:
1 <= L2 norm <= 2

Class 2:
Everything else



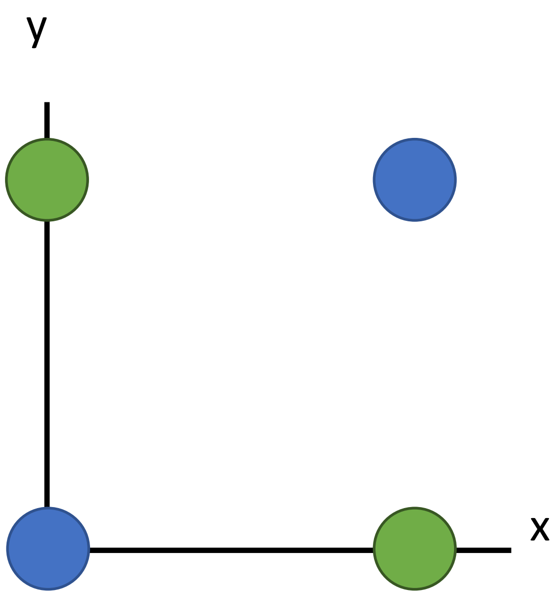
Class 1:
Three modes

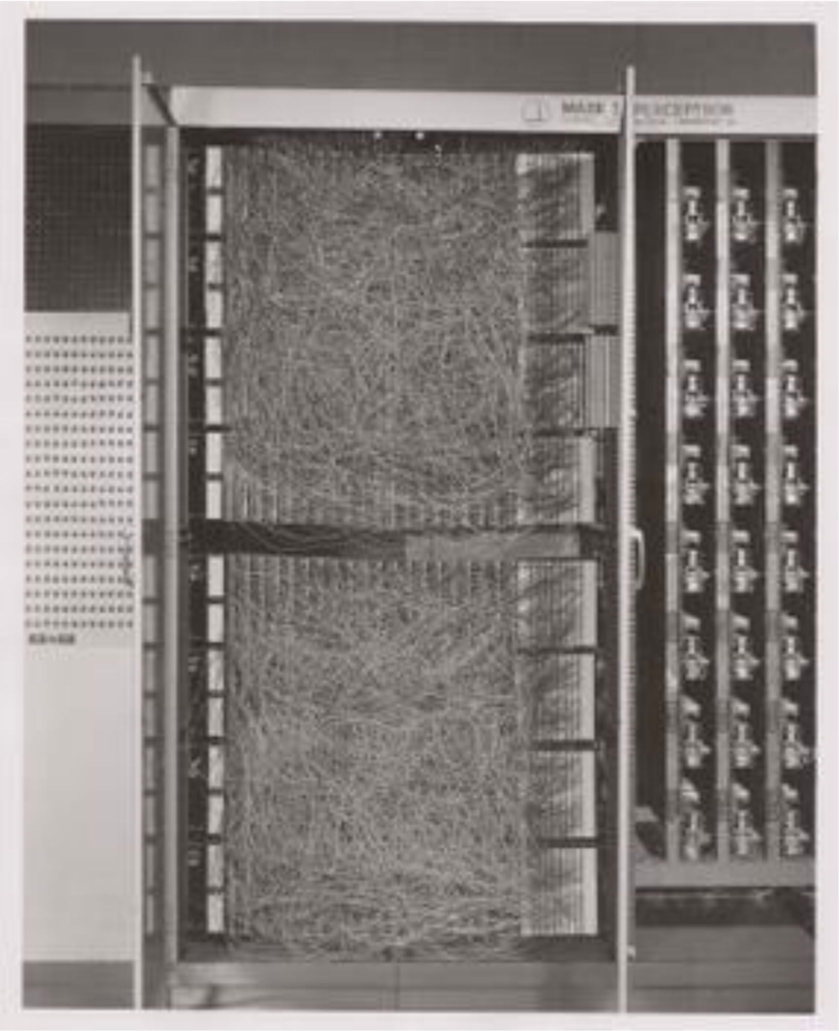
Class 2:
Everything else



Recall: Perceptron couldn't learn XOR

X	Y	F(x,y)
0	0	0
0	1	1
1	0	1
1	1	0

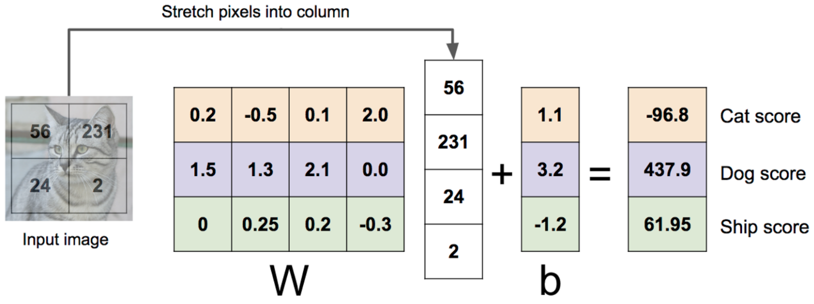




Linear Classifier: Three Viewpoints

Algebraic Viewpoint

$f(x,W) = Wx$

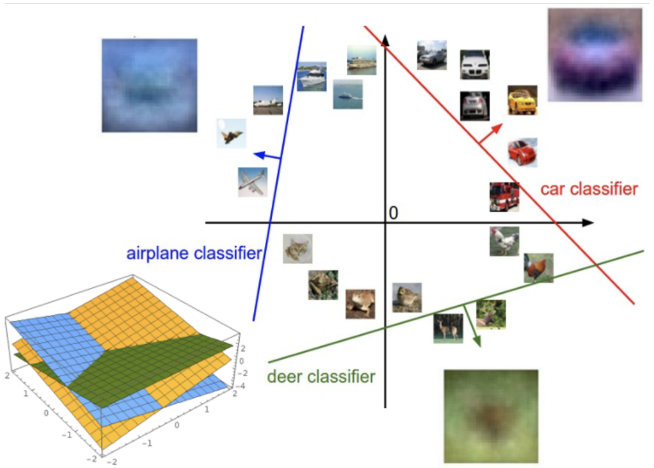


Visual Viewpoint

One template
per class

Geometric Viewpoint

Hyperplanes
cutting up space



So Far: Defined a linear score function

$f(x,W) = Wx + b$



airplane	-3.45	-0.51	3.42
automobile	-8.87	**6.04**	4.64
bird	0.09	5.31	2.65
cat	**2.9**	-4.22	5.1
deer	4.48	-4.19	2.64
dog	8.02	3.58	5.55
frog	3.78	4.49	**-4.34**
horse	1.06	-4.37	-1.5
ship	-0.36	-2.09	-4.79
truck	-0.72	-2.93	6.14

Given a W, we can compute class scores for an image x.

But how can we actually choose a good W?

Cat image by Nikita is licensed under CC-BY 2.0; Car image is CC0 1.0 public domain; Frog image is in the public domain

Choosing a good W

$$f(x,W) = Wx + b$$



airplane	-3.45	-0.51	3.42
automobile	-8.87	**6.04**	4.64
bird	0.09	5.31	2.65
cat	**2.9**	-4.22	5.1
deer	4.48	-4.19	2.64
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horse	1.06	-4.37	-1.5
ship	-0.36	-2.09	-4.79
truck	-0.72	-2.93	6.14

TODO:

1. Use a **loss function** to quantify how good a value of W is

2. Find a W that minimizes the loss function (**optimization**)

Loss Function

A **loss function** tells how good our current classifier is

Low loss = good classifier
High loss = bad classifier

(Also called: **objective function**; **cost function**)

Loss Function

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Low loss = good classifier
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Negative loss function sometimes called **reward function**, **profit function**, **utility function**, **fitness function**, etc

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Low loss = good classifier
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(Also called: **objective function**; **cost function**)

Negative loss function sometimes called **reward function**, **profit function**, **utility function**, **fitness function**, etc

Given a dataset of examples

$$\{(x_i, y_i)\}_{i=1}^{N}$$

Where x_i is image and
y_i is (integer) label

Loss Function

A **loss function** tells how good our current classifier is

Low loss = good classifier
High loss = bad classifier

(Also called: **objective function**; **cost function**)

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Given a dataset of examples

$$\{(x_i, y_i)\}_{i=1}^{N}$$

Where x_i is image and
y_i is (integer) label

Loss for a single example is

$$L_i(f(x_i, W), y_i)$$

Loss Function

A **loss function** tells how good our current classifier is

Low loss = good classifier
High loss = bad classifier

(Also called: **objective function**; **cost function**)

Negative loss function sometimes called **reward function**, **profit function**, **utility function**, **fitness function**, etc

Given a dataset of examples

$$\{(x_i, y_i)\}_{i=1}^{N}$$

Where x_i is image and
y_i is (integer) label

Loss for a single example is

$$L_i(f(x_i, W), y_i)$$

Loss for the dataset is average of per-example losses:

$$L = \frac{1}{N}\sum_i L_i(f(x_i, W), y_i)$$

Cross-Entropy Loss (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

cat	**3.2**
car	5.1
frog	-1.7

Cross-Entropy Loss (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W) \qquad P(Y = k \mid X = x_i) = \frac{\exp(s_k)}{\sum_j \exp(s_j)}$$

Softmax function

cat	**3.2**
car	5.1
frog	-1.7

Cross-Entropy Loss (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$\boxed{s = f(x_i; W)}$$

$$\boxed{P(Y = k \mid X = x_i) = \frac{\exp(s_k)}{\sum_j \exp(s_j)}}$$

Softmax function

cat	**3.2**
car	5.1
frog	-1.7

Unnormalized log-probabilities / logits

Cross-Entropy Loss (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k \mid X = x_i) = \frac{\exp(s_k)}{\sum_j \exp(s_j)}$$

Softmax function

Probabilities must be >= 0

cat	**3.2**	exp →	**24.5**
car	5.1		164.0
frog	-1.7		0.18

Unnormalized log-probabilities / logits

unnormalized probabilities

Cross-Entropy Loss (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$\boxed{s = f(x_i; W)}$$

$$\boxed{P(Y = k \mid X = x_i) = \frac{\exp(s_k)}{\sum_j \exp(s_j)}}$$ Softmax function

Probabilities must be >= 0

Probabilities must sum to 1

	Unnormalized log-probabilities / logits	exp →	unnormalized probabilities	normalize →	probabilities
cat	**3.2**		**24.5**		**0.13**
car	5.1		164.0		0.87
frog	-1.7		0.18		0.00

Cross-Entropy Loss (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k \mid X = x_i) = \frac{\exp(s_k)}{\sum_j \exp(s_j)}$$ Softmax function

$$L_i = -\log P(Y = y_i \mid X = x_i)$$

Probabilities must be >= 0

Probabilities must sum to 1

	Unnormalized log-probabilities / logits	exp → unnormalized probabilities	normalize → probabilities
cat	**3.2**	**24.5**	**0.13**
car	5.1	164.0	0.87
frog	-1.7	0.18	0.00

$L_i = -\log(0.13)$
$= \mathbf{2.04}$

Cross-Entropy Loss (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k \mid X = x_i) = \frac{\exp(s_k)}{\sum_j \exp(s_j)}$$ Softmax function

$$L_i = -\log P(Y = y_i \mid X = x_i)$$

Probabilities must be >= 0

Probabilities must sum to 1

		exp →		normalize →	
cat	**3.2**		**24.5**		**0.13**
car	5.1		164.0		0.87
frog	-1.7		0.18		0.00
	Unnormalized log-probabilities / logits		unnormalized probabilities		probabilities

$L_i = -\log(0.13)$
$= \mathbf{2.04}$

Maximum Likelihood Estimation
Choose weights to maximize the likelihood of the observed data
(See EECS 445 or EECS 545)

Cross-Entropy Loss (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k \mid X = x_i) = \frac{\exp(s_k)}{\sum_j \exp(s_j)}$$ Softmax function

$$L_i = -\log P(Y = y_i \mid X = x_i)$$

Probabilities must be >= 0

Probabilities must sum to 1

	Unnormalized log-probabilities / logits	exp →	unnormalized probabilities	normalize →	probabilities	→ Compare ←	Correct probs
cat	**3.2**		**24.5**		**0.13**		**1.00**
car	5.1		164.0		0.87		0.00
frog	-1.7		0.18		0.00		0.00

Cross-Entropy Loss (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$\boxed{s = f(x_i; W)}$$

$$\boxed{P(Y = k \mid X = x_i) = \frac{\exp(s_k)}{\sum_j \exp(s_j)}}$$ Softmax function

Probabilities must be >= 0

Probabilities must sum to 1

$$\boxed{L_i = -\log P(Y = y_i \mid X = x_i)}$$

	Unnormalized log-probabilities / logits	exp →	unnormalized probabilities	normalize →	probabilities		Correct probs
cat	**3.2**		**24.5**		**0.13**	→ Compare ←	**1.00**
car	5.1		164.0		0.87		0.00
frog	-1.7		0.18		0.00		0.00

Kullback–Leibler divergence

$$D_{KL}(\boldsymbol{P} \,||\, \boldsymbol{Q}) = \sum_y \boldsymbol{P}(\boldsymbol{y}) \log \frac{\boldsymbol{P}(\boldsymbol{y})}{\boldsymbol{Q}(\boldsymbol{y})}$$

Cross-Entropy Loss (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k \mid X = x_i) = \frac{\exp(s_k)}{\sum_j \exp(s_j)}$$ Softmax function

$$L_i = -\log P(Y = y_i \mid X = x_i)$$

Probabilities must be >= 0

Probabilities must sum to 1

	Unnormalized log-probabilities / logits		unnormalized probabilities		probabilities		Correct probs
cat	**3.2**	exp →	**24.5**	normalize →	**0.13**	→ Compare ←	**1.00**
car	5.1		164.0		0.87	*Cross Entropy*	0.00
frog	-1.7		0.18		0.00		0.00

$$H(P, Q) = H(P) + D_{KL}(P \,||\, Q)$$

Cross-Entropy Loss (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k \mid X = x_i) = \frac{\exp(s_k)}{\sum_j \exp(s_j)}$$

Softmax function

cat	**3.2**
car	5.1
frog	-1.7

Maximize probability of correct class

$$L_i = -\log P(Y = y_i \mid X = x_i)$$

Putting it all together:

$$L_i = -\log\left(\frac{\exp(s_{y_i})}{\sum_j \exp(s_j)}\right)$$

Cross-Entropy Loss (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

cat	**3.2**
car	5.1
frog	-1.7

$$\boxed{s = f(x_i; W)}$$

$$\boxed{P(Y = k \mid X = x_i) = \frac{\exp(s_k)}{\sum_j \exp(s_j)}}$$

Softmax function

Maximize probability of correct class

$$\boxed{L_i = -\log P(Y = y_i \mid X = x_i)}$$

Putting it all together:

$$\boxed{L_i = -\log\left(\frac{\exp(s_{y_i})}{\sum_j \exp(s_j)}\right)}$$

Q: What is the min / max possible loss L_i?

Cross-Entropy Loss (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k \mid X = x_i) = \frac{\exp(s_k)}{\sum_j \exp(s_j)}$$

Softmax function

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$$L_i = -\log P(Y = y_i \mid X = x_i)$$

Putting it all together:

$$L_i = -\log\left(\frac{\exp(s_{y_i})}{\sum_j \exp(s_j)}\right)$$

Q: What is the min / max possible loss L_i?

A: Min 0, max +infinity

Cross-Entropy Loss (Multinomial Logistic Regression)

cat	**3.2**
car	5.1
frog	-1.7

Want to interpret raw classifier scores as **probabilities**

$$\boxed{s = f(x_i; W)}$$

$$\boxed{P(Y = k \mid X = x_i) = \frac{\exp(s_k)}{\sum_j \exp(s_j)}}$$

Softmax function

Maximize probability of correct class

$$\boxed{L_i = -\log P(Y = y_i \mid X = x_i)}$$

Putting it all together:

$$\boxed{L_i = -\log\left(\frac{\exp(s_{y_i})}{\sum_j \exp(s_j)}\right)}$$

Q: If all scores are small random values, what is the loss?

Cross-Entropy Loss (Multinomial Logistic Regression)

cat	**3.2**
car	5.1
frog	-1.7

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k \mid X = x_i) = \frac{\exp(s_k)}{\sum_j \exp(s_j)}$$

Softmax function

Maximize probability of correct class

$$L_i = -\log P(Y = y_i \mid X = x_i)$$

Putting it all together:

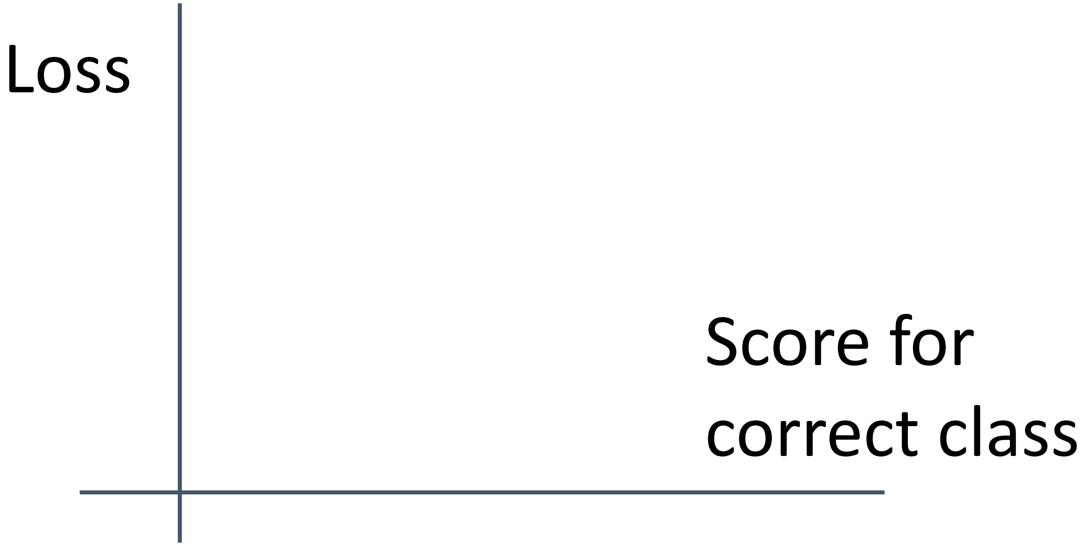
$$L_i = -\log\left(\frac{\exp(s_{y_i})}{\sum_j \exp(s_j)}\right)$$

Q: If all scores are small random values, what is the loss?

A: -log(1/C)
log(10) ≈ 2.3

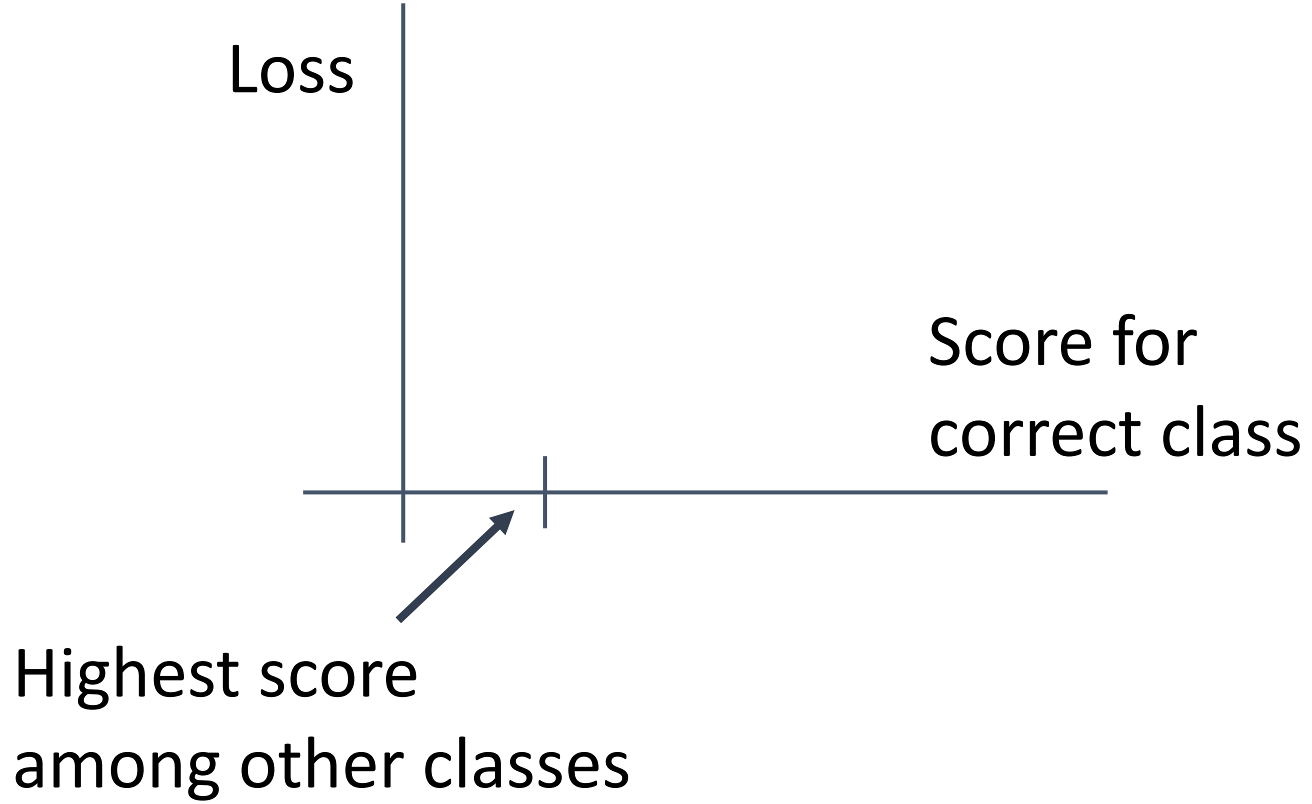
Multiclass SVM Loss

"The score of the correct class should be higher than all the other scores"



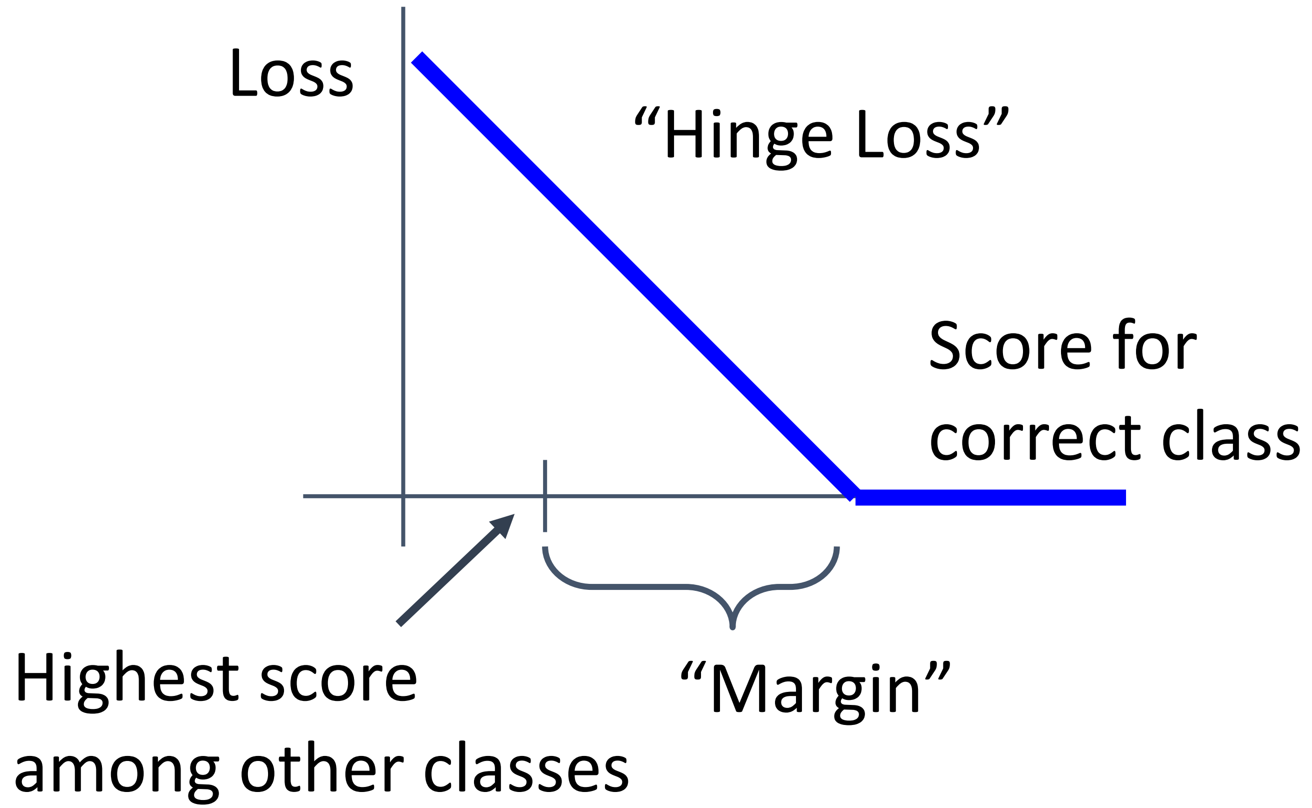
Multiclass SVM Loss

”The score of the correct class should be higher than all the other scores”



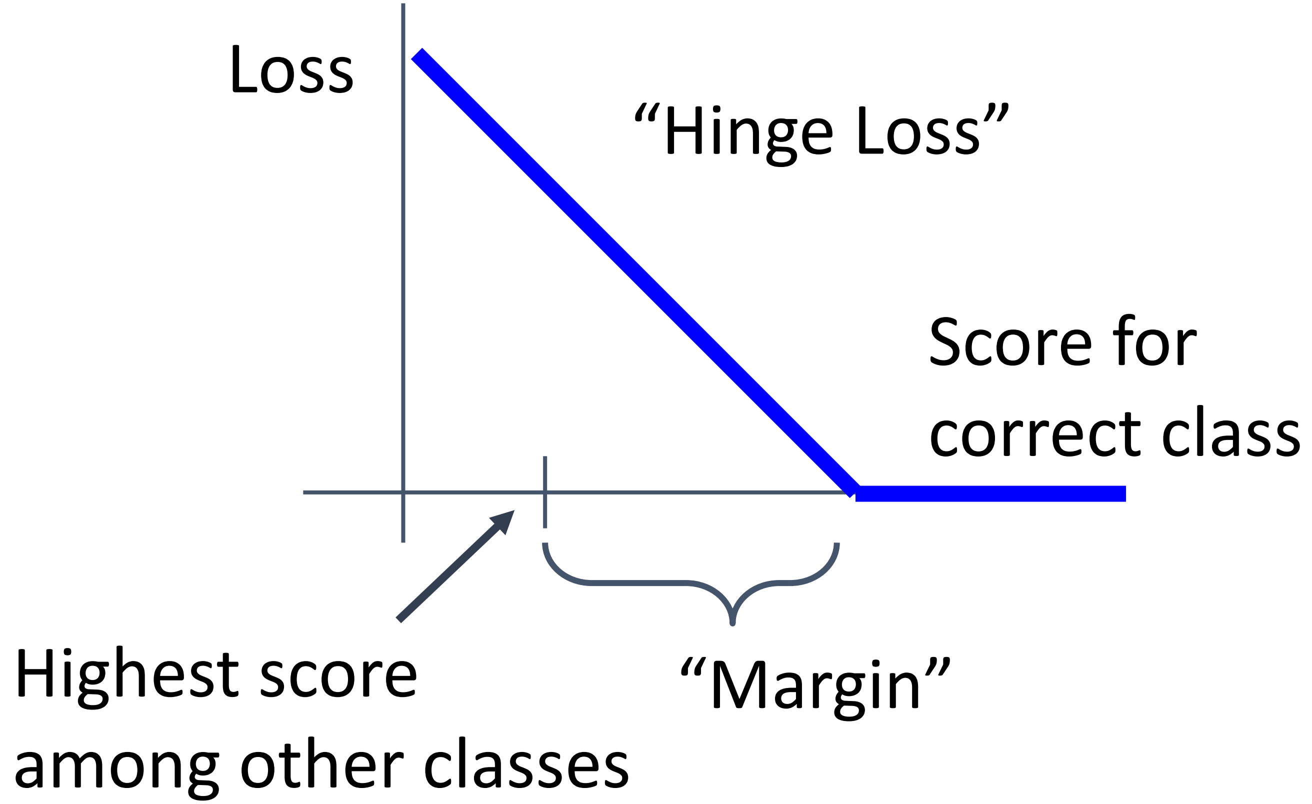
Multiclass SVM Loss

”The score of the correct class should be higher than all the other scores”



Multiclass SVM Loss

”The score of the correct class should be higher than all the other scores”



Given an example (x_i, y_i)
(x_i is image, y_i is label)

Let $s = f(x_i, W)$ be scores

Then the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Multiclass SVM Loss

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**

Given an example (x_i, y_i)
(x_i is image, y_i is label)

Let $s = f(x_i, W)$ be scores

Then the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Multiclass SVM Loss



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Loss	2.9		

Given an example (x_i, y_i)
(x_i is image, y_i is label)

Let $s = f(x_i, W)$ be scores

Then the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

= max(0, 5.1 - 3.2 + 1)
+ max(0, -1.7 - 3.2 + 1)
= max(0, 2.9) + max(0, -3.9)
= 2.9 + 0
= 2.9

Multiclass SVM Loss

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Loss	2.9	0	

Given an example (x_i, y_i)
(x_i is image, y_i is label)

Let $s = f(x_i, W)$ be scores

Then the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

= max(0, 1.3 - 4.9 + 1)
+max(0, 2.0 - 4.9 + 1)
= max(0, -2.6) + max(0, -1.9)
= 0 + 0
= 0

Multiclass SVM Loss

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Loss	2.9	0	12.9

Given an example (x_i, y_i)
(x_i is image, y_i is label)

Let $s = f(x_i, W)$ be scores

Then the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

= max(0, 2.2 - (-3.1) + 1)
+max(0, 2.5 - (-3.1) + 1)
= max(0, 6.3) + max(0, 6.6)
= 6.3 + 6.6
= 12.9

Multiclass SVM Loss

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Loss	2.9	0	12.9

Given an example (x_i, y_i)
(x_i is image, y_i is label)

Let $s = f(x_i, W)$ be scores

Then the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Loss over the dataset is:

L = (2.9 + 0.0 + 12.9) / 3
= 5.27

Multiclass SVM Loss

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Loss	2.9	0	12.9

Given an example (x_i, y_i)
(x_i is image, y_i is label)

Let $s = f(x_i, W)$ be scores

Then the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q: What happens to the loss if the scores for the car image change a bit?

Multiclass SVM Loss

			
cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Loss	2.9	0	12.9

Given an example (x_i, y_i)
(x_i is image, y_i is label)

Let $s = f(x_i, W)$ be scores

Then the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q2: What are the min and max possible loss?

Multiclass SVM Loss

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Loss	2.9	0	12.9

Given an example (x_i, y_i)
(x_i is image, y_i is label)

Let $s = f(x_i, W)$ be scores

Then the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q3: If all the scores were random, what loss would we expect?

Multiclass SVM Loss



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Loss	2.9	0	12.9

Given an example (x_i, y_i)
(x_i is image, y_i is label)

Let $s = f(x_i, W)$ be scores

Then the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q4: What would happen if the sum were over all classes? (including $i = y_i$)

Multiclass SVM Loss

			
cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Loss	2.9	0	12.9

Given an example (x_i, y_i)
(x_i is image, y_i is label)

Let $s = f(x_i, W)$ be scores

Then the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q5: What if the loss used a mean instead of a sum?

Multiclass SVM Loss



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Loss	2.9	0	12.9

Given an example (x_i, y_i)
(x_i is image, y_i is label)

Let $s = f(x_i, W)$ be scores

Then the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q6: What if we used this loss instead?

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)^2$$

Cross-Entropy vs SVM Loss

$$L_i = -\log\left(\frac{\exp(s_{y_i})}{\sum_j \exp(s_j)}\right)$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

assume scores:

[10, -2, 3]

[10, 9, 9]

[10, -100, -100]

and $\boxed{y_i = 0}$

Q: What is cross-entropy loss?

What is SVM loss?

Cross-Entropy vs SVM Loss

$$L_i = -\log\left(\frac{\exp(s_{y_i})}{\sum_j \exp(s_j)}\right)$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

assume scores:

[10, -2, 3]

[10, 9, 9]

[10, -100, -100]

and $\boxed{y_i = 0}$

Q: What is cross-entropy loss?
What is SVM loss?

A: Cross-entropy loss > 0
SVM loss = 0

Cross-Entropy vs SVM Loss

$$L_i = -\log\left(\frac{\exp(s_{y_i})}{\sum_j \exp(s_j)}\right)$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

assume scores:

[10, -2, 3]

[10, 9, 9]

[10, -100, -100]

and $\boxed{y_i = 0}$

Q: What happens to each loss if I slightly change the scores of the last datapoint?

Cross-Entropy vs SVM Loss

$$L_i = -\log\left(\frac{\exp(s_{y_i})}{\sum_j \exp(s_j)}\right)$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

assume scores:

[10, -2, 3]

[10, 9, 9]

[10, -100, -100]

and $\boxed{y_i = 0}$

Q: What happens to each loss if I slightly change the scores of the last datapoint?

A: Cross-entropy loss will change; SVM loss will stay the same

Cross-Entropy vs SVM Loss

$$L_i = -\log\left(\frac{\exp(s_{y_i})}{\sum_j \exp(s_j)}\right)$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

assume scores:

[10, -2, 3]

[10, 9, 9]

[10, -100, -100]

and $y_i = 0$

Q: What happens to each loss if I double the score of the correct class from 10 to 20?

Cross-Entropy vs SVM Loss

$$L_i = -\log\left(\frac{\exp(s_{y_i})}{\sum_j \exp(s_j)}\right)$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

assume scores:

[10, -2, 3]

[10, 9, 9]

[10, -100, -100]

and $\boxed{y_i = 0}$

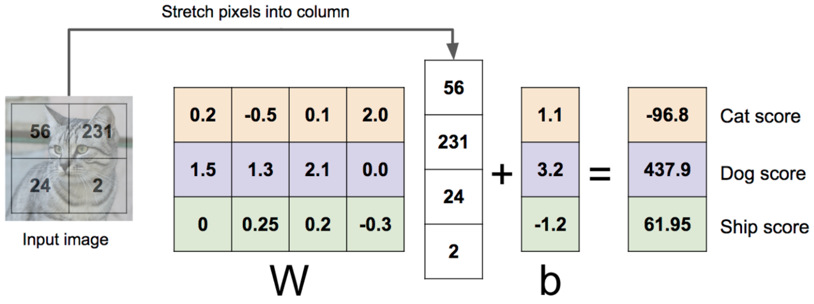
Q: What happens to each loss if I double the score of the correct class from 10 to 20?

A: Cross-entropy loss will decrease, SVM loss still 0

Recap: Three ways to think about linear classifiers

Algebraic Viewpoint

$f(x,W) = Wx$

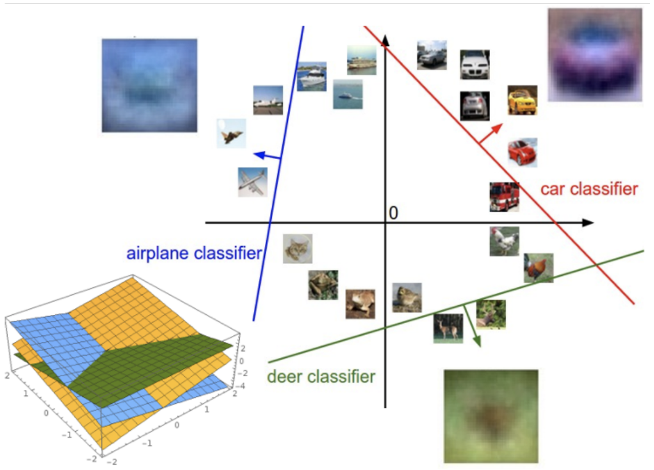


Visual Viewpoint

One template per class

Geometric Viewpoint

Hyperplanes cutting up space



Recap: Loss Functions quantify preferences

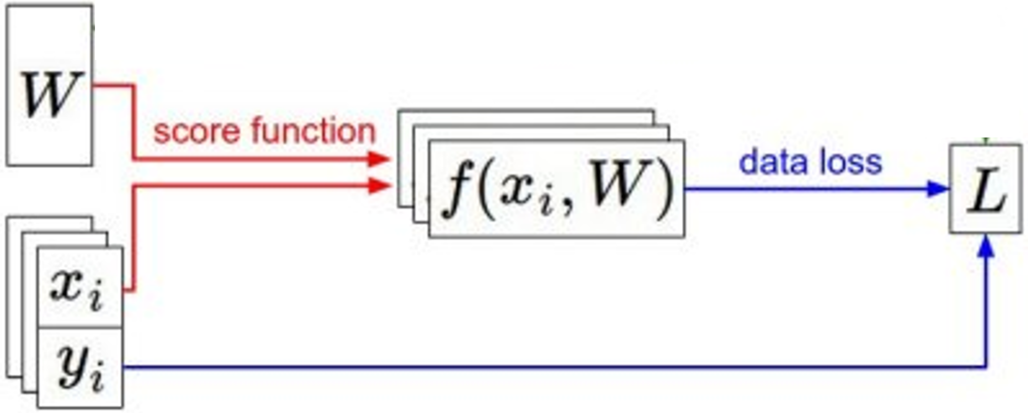
- We have some dataset of (x, y)
- We have a **score function:**
- We have a **loss function**:

$$s = f(x; W, b) = Wx + b$$

Linear classifier

Softmax: $L_i = -\log\left(\frac{\exp(s_{y_i})}{\sum_j \exp(s_j)}\right)$

SVM: $L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$



Recap: Loss Functions quantify preferences

- We have some dataset of (x, y)
- We have a **score function:**
- We have a **loss function**:

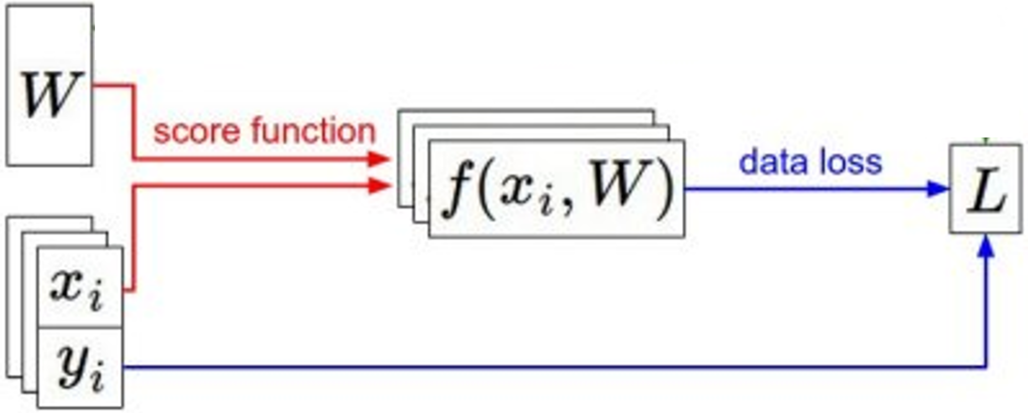
Q: How do we find the best W, b?

$$s = f(x; W, b) = Wx + b$$

Linear classifier

Softmax: $L_i = -\log\left(\frac{\exp(s_{y_i})}{\sum_j \exp(s_j)}\right)$

SVM: $L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$



Next time:
Regularization + Optimization