

# Lecture 21:

# Reinforcement Learning

# Assignment 6: Generative Models

Generative Adversarial Networks  
Variational Autoencoders

Due on Wednesday, 12/9 11:59pm EST

# So far: Supervised Learning

## Supervised Learning

**Data:**  $(x, y)$

$x$  is data,  $y$  is label

**Goal:** Learn a *function* to map  $x \rightarrow y$

**Examples:** Classification, regression, object detection, semantic segmentation, image captioning, etc.

## Classification



Cat

[This image](#) is [CC0 public domain](#)

# So far: Unsupervised Learning

## Unsupervised Learning

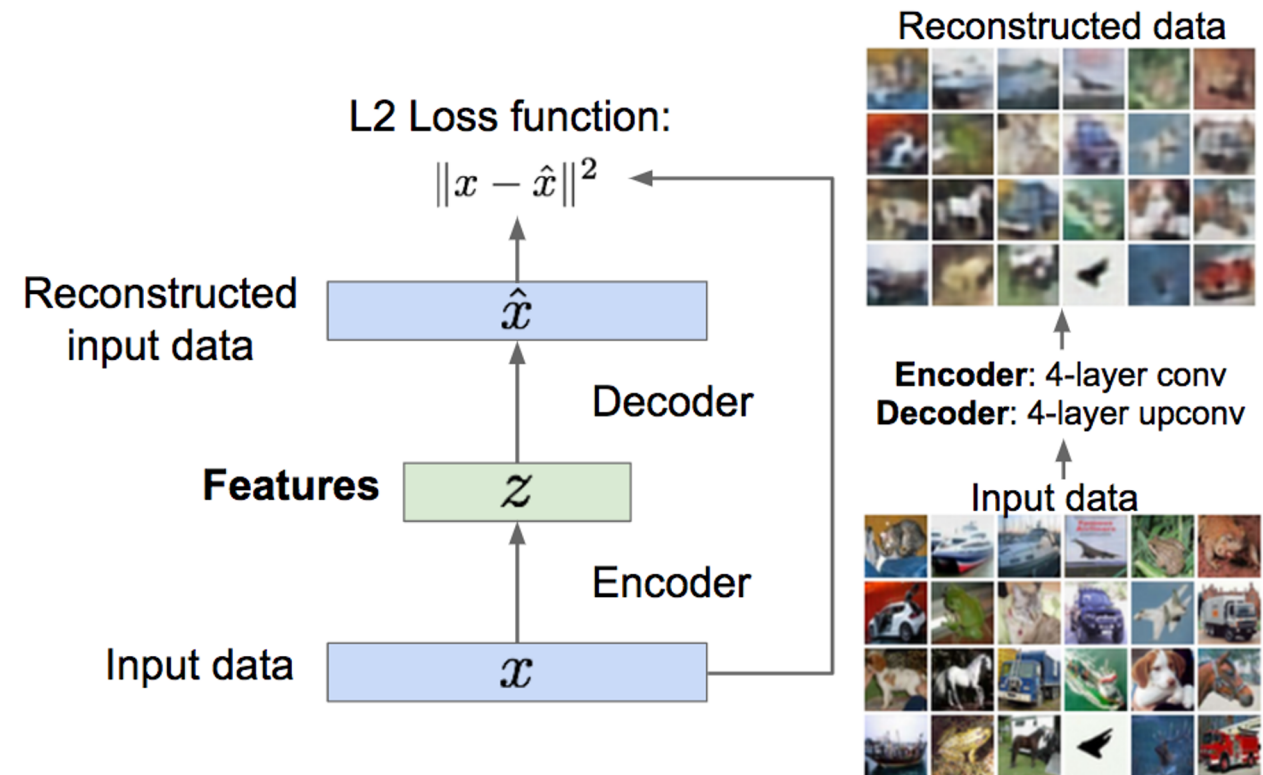
**Data:**  $x$

Just data, no labels!

**Goal:** Learn some underlying hidden *structure* of the data

**Examples:** Clustering, dimensionality reduction, feature learning, density estimation, etc.

## Feature Learning (e.g. autoencoders)

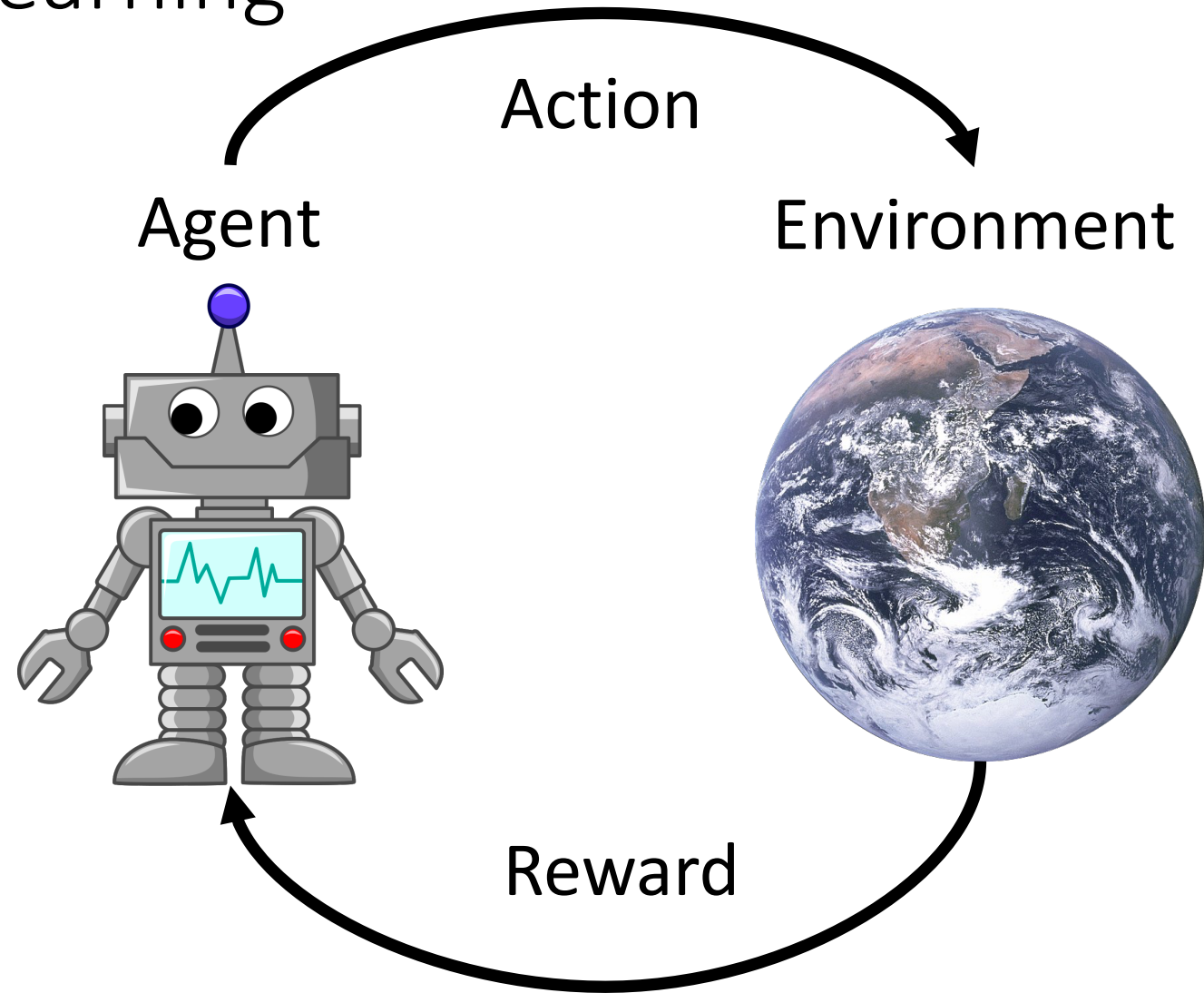




# Today: Reinforcement Learning

Problems where an **agent** performs **actions** in **environment**, and receives **rewards**

**Goal:** Learn how to take actions that maximize reward



[Earth photo](#) is in the public domain  
[Robot image](#) is in the public domain

# Overview

- What is reinforcement learning?
- Algorithms for reinforcement learning
  - Q-Learning
  - Policy Gradients

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This is just a taste! Can easily teach entire courses on (deep) RL:

- [UMich EECS 598-003](#)
- [Berkeley CS 285](#)
- [Stanford CS 234](#)
- [CMU 10-703](#)

# Reinforcement Learning

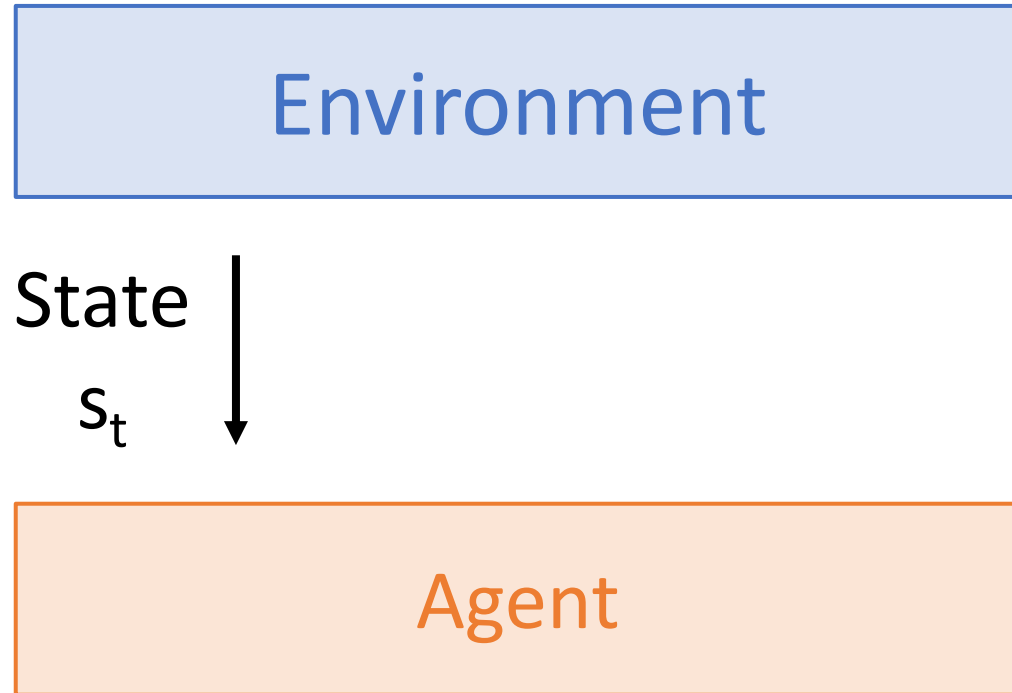


The diagram consists of two rectangular boxes. The top box is light blue with a dark blue border and contains the word 'Environment'. The bottom box is light orange with a dark orange border and contains the word 'Agent'. The boxes are positioned vertically, one above the other, with a significant gap between them.

Environment

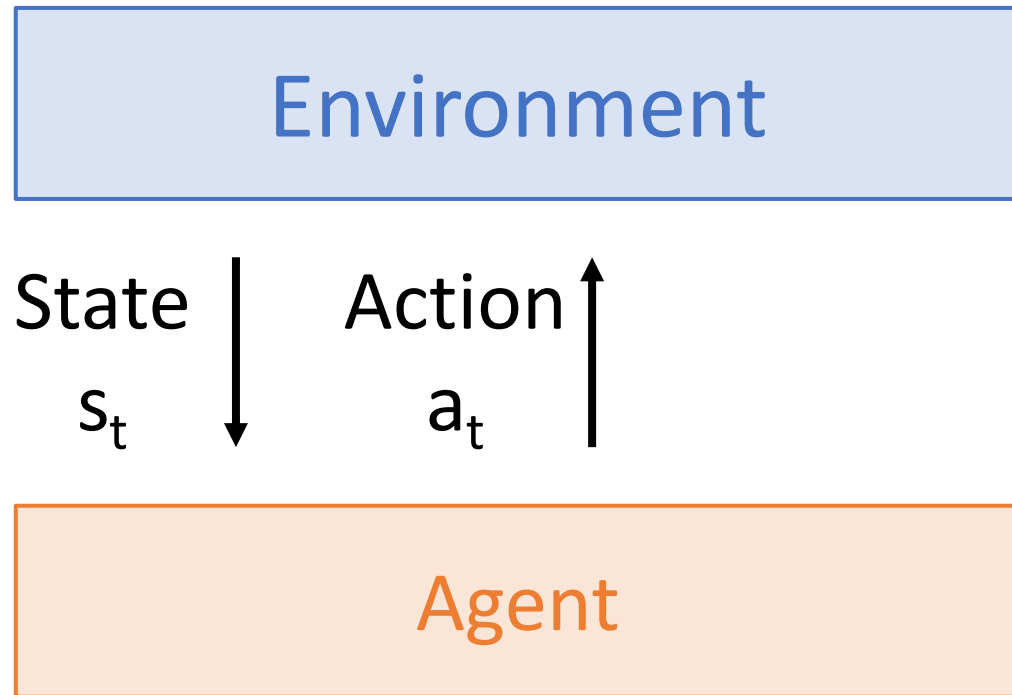
Agent

# Reinforcement Learning



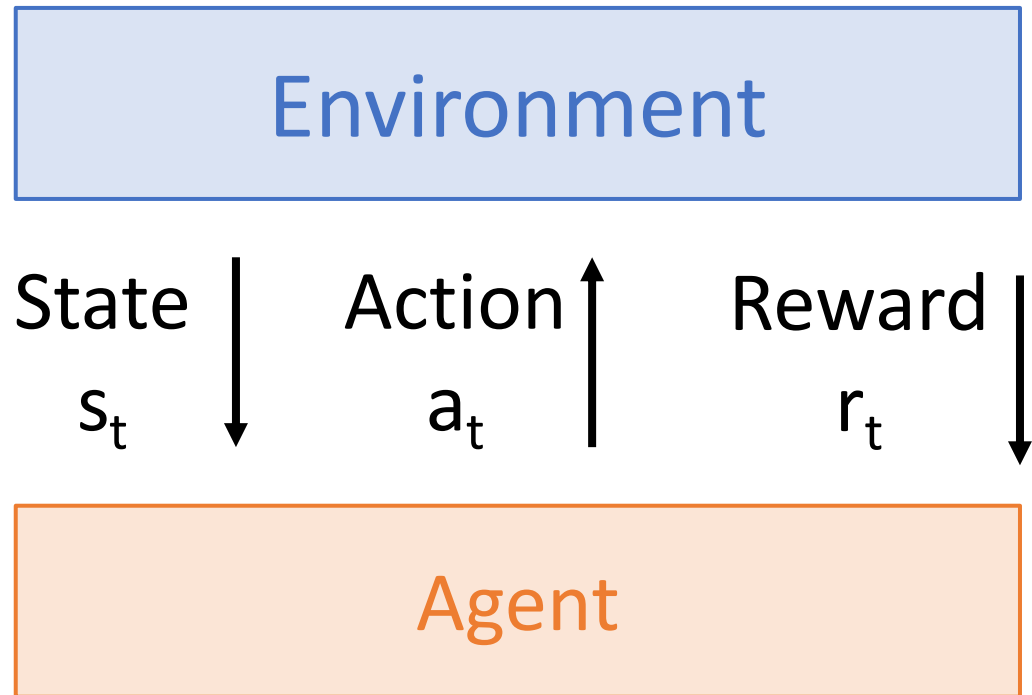
The agent sees a **state**; may be noisy or incomplete

# Reinforcement Learning



The makes an **action**  
based on what it sees

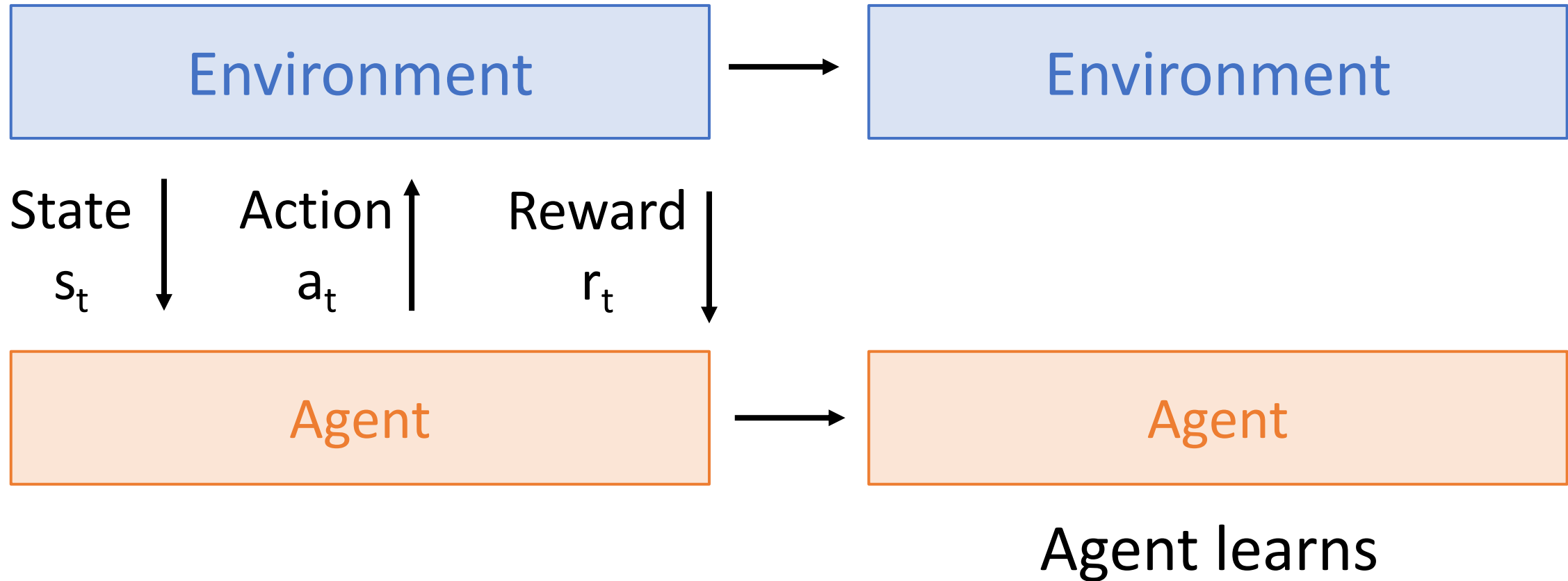
# Reinforcement Learning



**Reward** tells the agent how well it is doing

# Reinforcement Learning

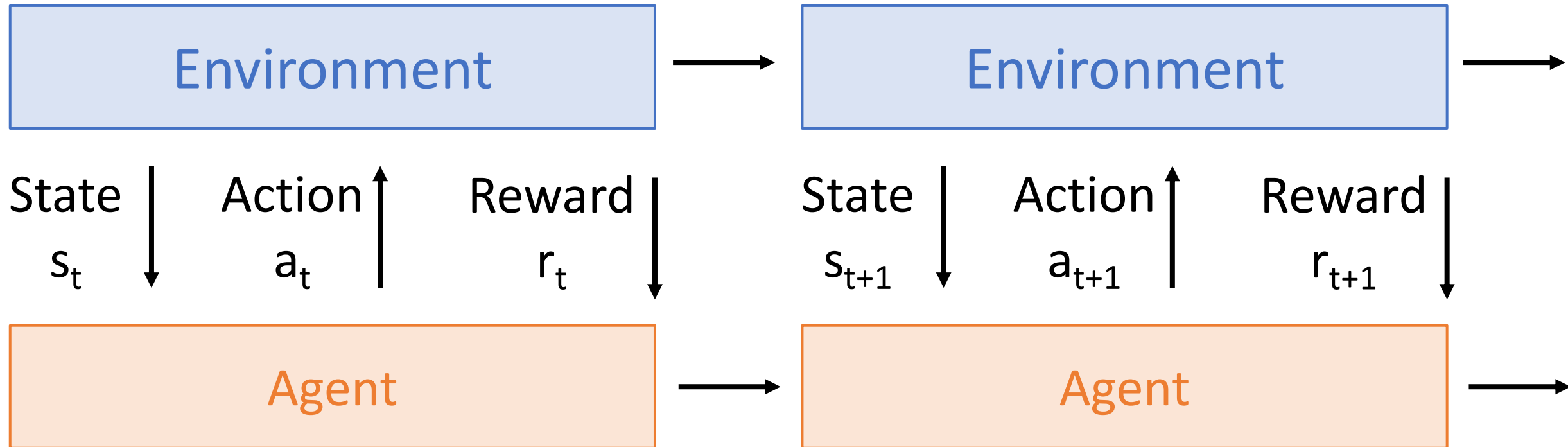
Action causes change  
to environment



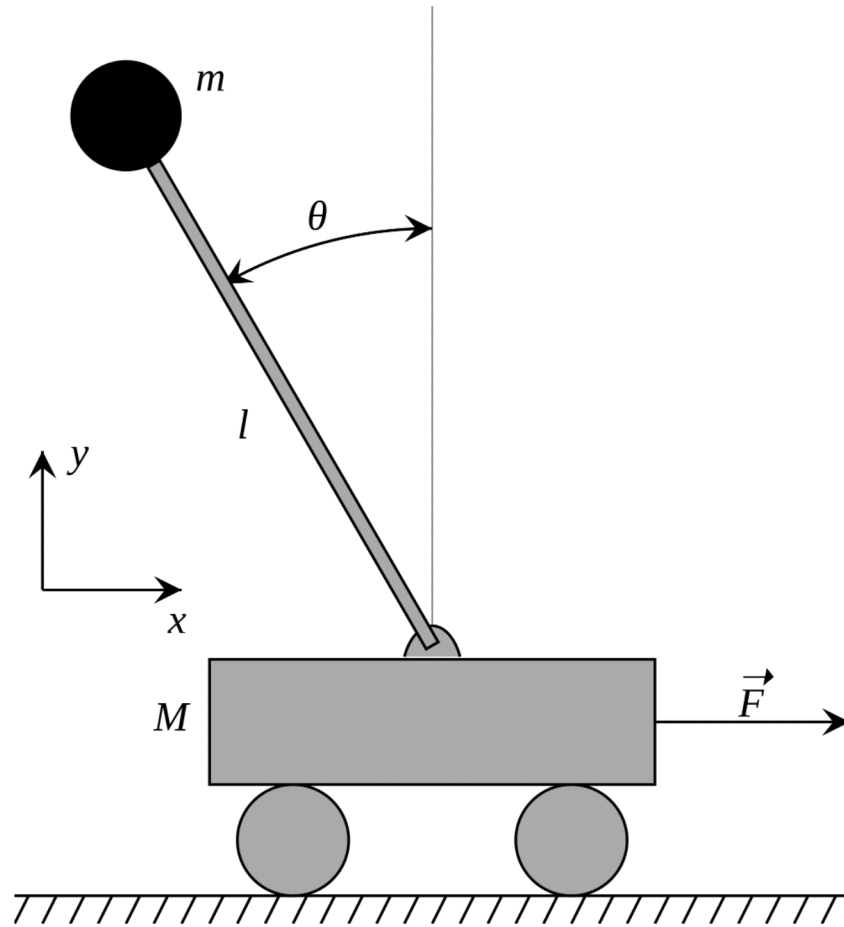


# Reinforcement Learning

Process repeats



# Example: Cart-Pole Problem



**Objective:** Balance a pole on top of a movable cart

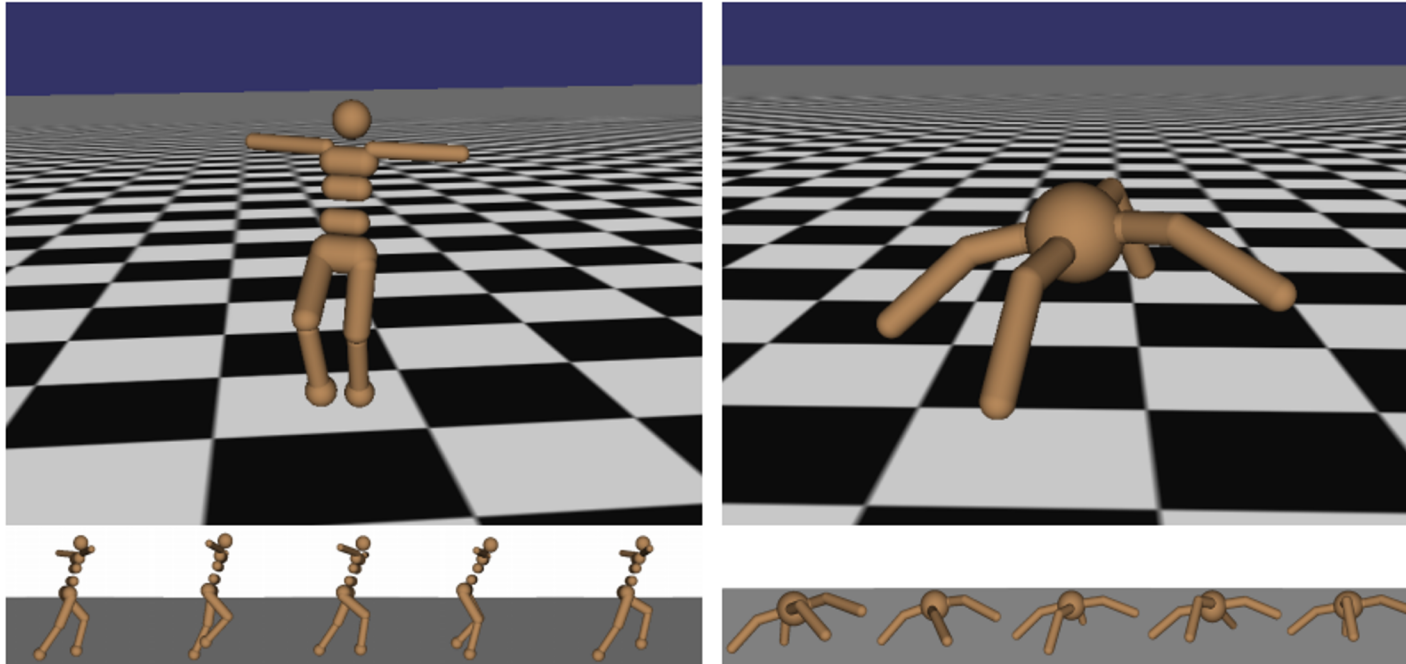
**State:** angle, angular speed, position, horizontal velocity

**Action:** horizontal force applied on the cart

**Reward:** 1 at each time step if the pole is upright

[This image](#) is [CC0 public domain](#)

# Example: Robot Locomotion



**Objective:** Make the robot move forward

**State:** Angle, position, velocity of all joints

**Action:** Torques applied on joints

**Reward:** 1 at each time step upright + forward movement

Figure from: Schulman et al, "High-Dimensional Continuous Control Using Generalized Advantage Estimation", ICLR 2016

# Example: Atari Games



**Objective:** Complete the game with the highest score

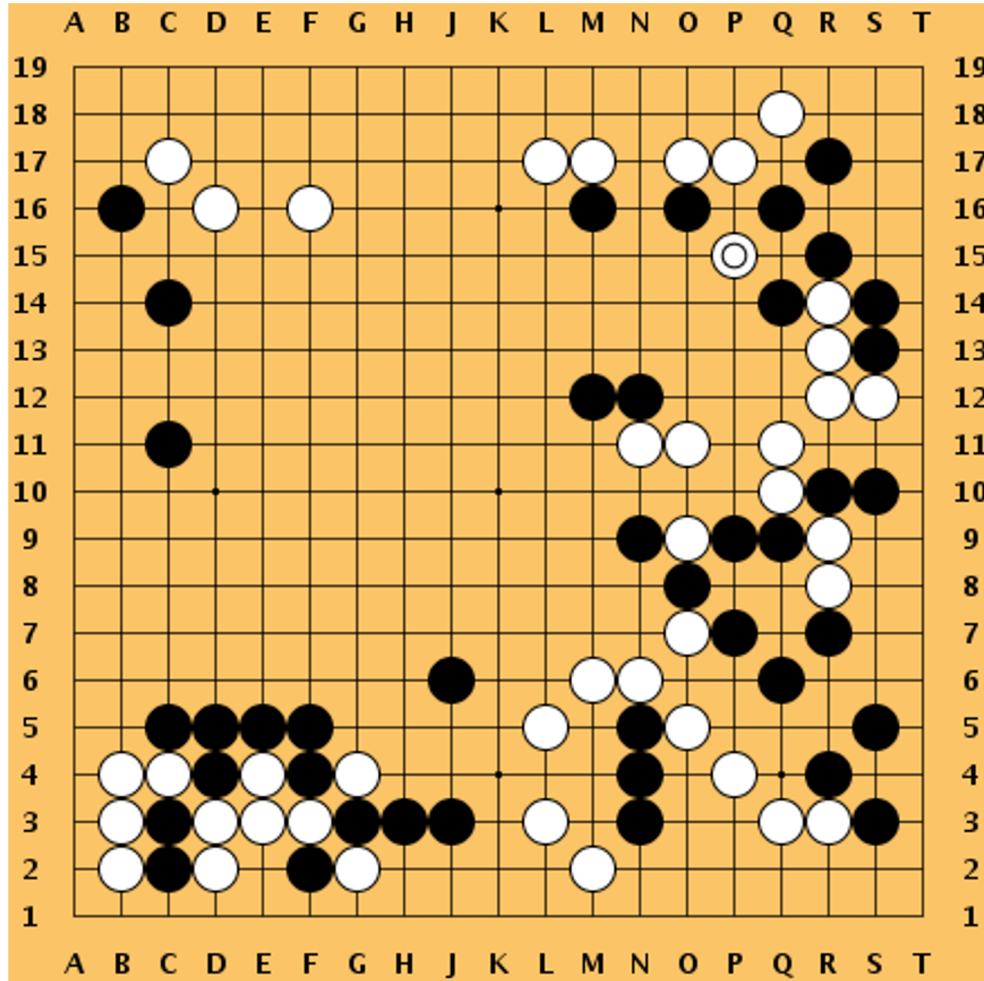
**State:** Raw pixel inputs of the game screen

**Action:** Game controls e.g. Left, Right, Up, Down

**Reward:** Score increase/decrease at each time step

Mnih et al, "Playing Atari with Deep Reinforcement Learning", NeurIPS Deep Learning Workshop, 2013

# Example: Go



[This image](#) is [CC0 public domain](#)

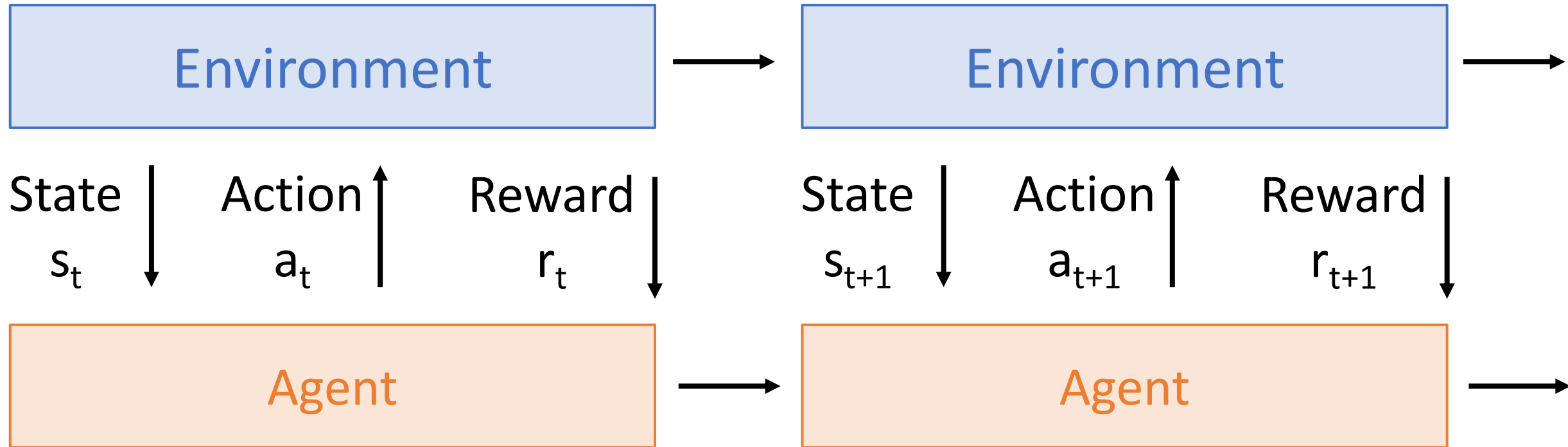
**Objective:** Win the game!

**State:** Position of all pieces

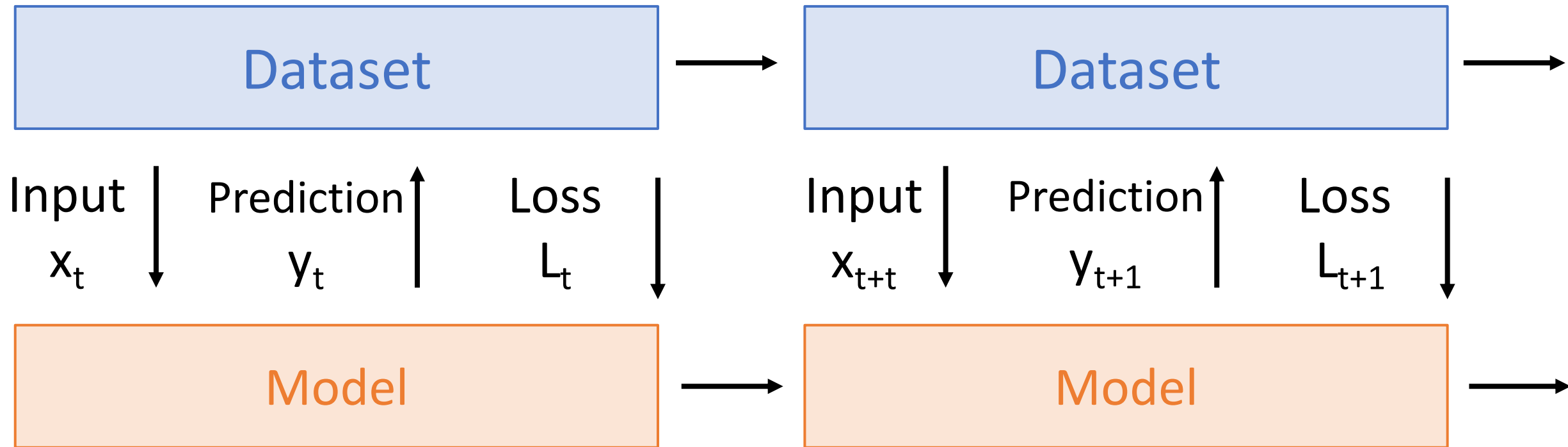
**Action:** Where to put the next piece down

**Reward:** On last turn: 1 if you won, 0 if you lost

# Reinforcement Learning vs Supervised Learning

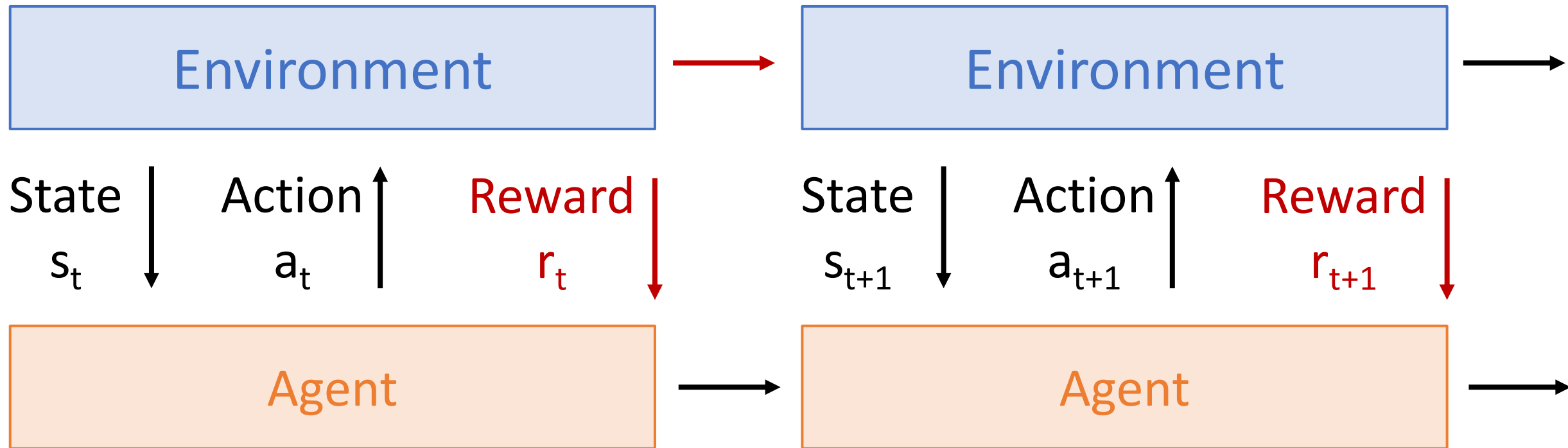


# Reinforcement Learning vs Supervised Learning



Why is RL different from normal supervised learning?

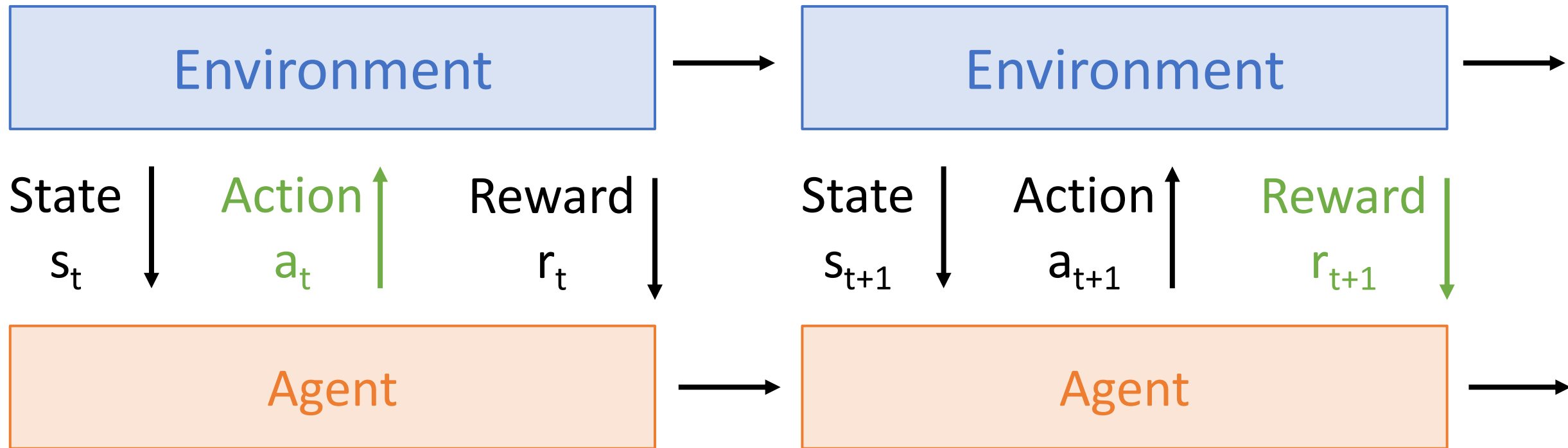
# Reinforcement Learning vs Supervised Learning



**Stochasticity:** Rewards and state transitions may be random

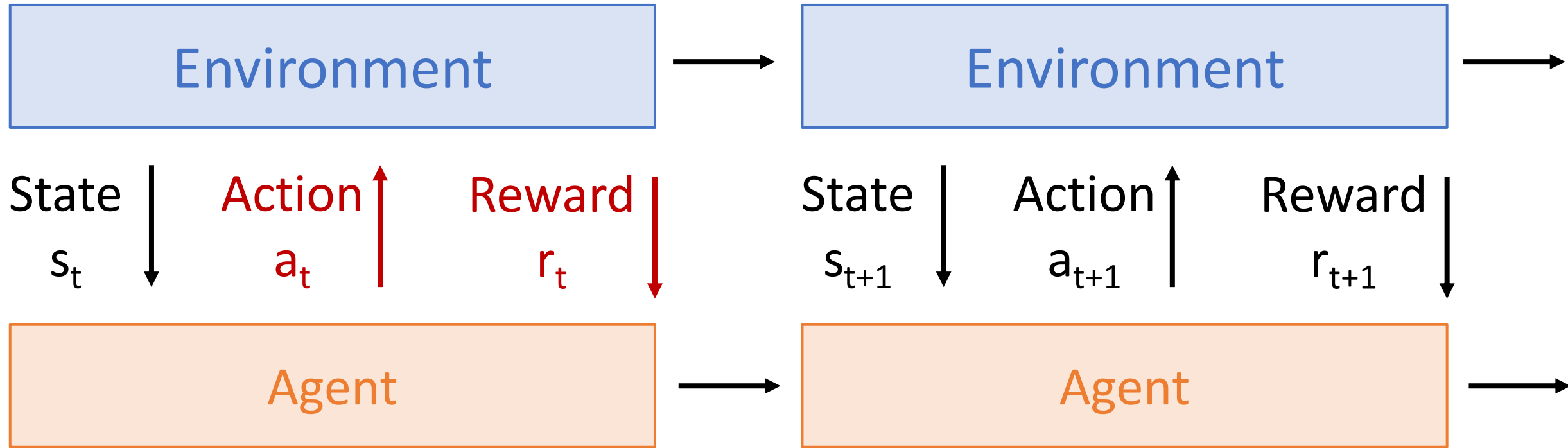


# Reinforcement Learning vs Supervised Learning



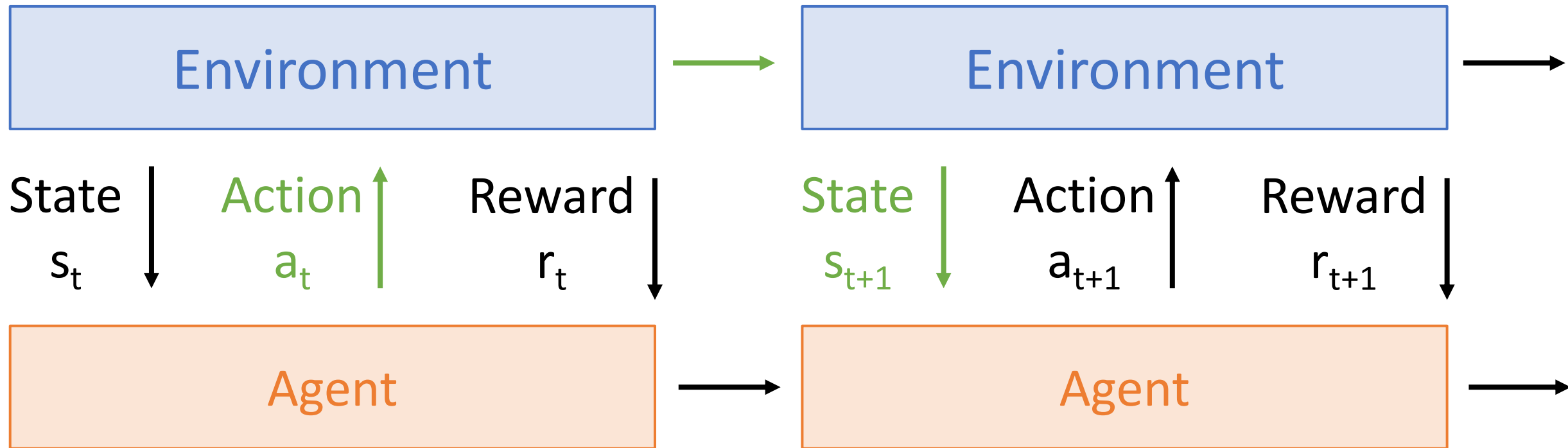
**Credit assignment:** Reward  $r_t$  may not directly depend on action  $a_t$

# Reinforcement Learning vs Supervised Learning



**Nondifferentiable:** Can't backprop through world; can't compute  $dr_t/da_t$

# Reinforcement Learning vs Supervised Learning



**Nonstationary:** What the agent experiences depends on how it acts

# Markov Decision Process (MDP)

Mathematical formalization of the RL problem: A tuple  $(S, A, R, P, \gamma)$

S: Set of possible states

A: Set of possible actions

R: Distribution of reward given (state, action) pair

P: Transition probability: distribution over next state given (state, action)

$\gamma$ : Discount factor (tradeoff between future and present rewards)

**Markov Property:** The current state completely characterizes the state of the world. Rewards and next states depend only on current state, not history.

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Agent executes a **policy**  $\pi$  giving distribution of actions conditioned on states

**Goal:** Find policy  $\pi^*$  that maximizes cumulative discounted reward:  $\sum_t \gamma^t r_t$

# Markov Decision Process (MDP)

- At time step  $t=0$ , environment samples initial state  $s_0 \sim p(s_0)$
- Then, for  $t=0$  until done:
  - Agent selects action  $a_t \sim \pi(a \mid s_t)$
  - Environment samples reward  $r_t \sim R(r \mid s_t, a_t)$
  - Environment samples next state  $s_{t+1} \sim P(s \mid s_t, a_t)$
  - Agent receives reward  $r_t$  and next state  $s_{t+1}$

# A simple MDP: Grid World

## Actions:

1. Right
2. Left
3. Up
4. Down

## States

★			
			★

## Reward

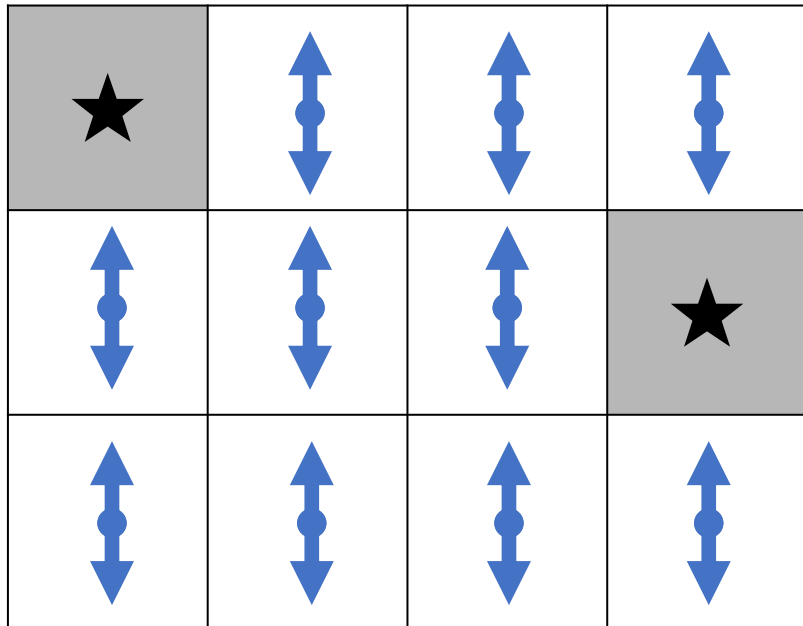
Set a negative  
“reward” for  
each transition  
(e.g.  $r = -1$ )

**Objective:** Reach one of the terminal states in as few moves as possible

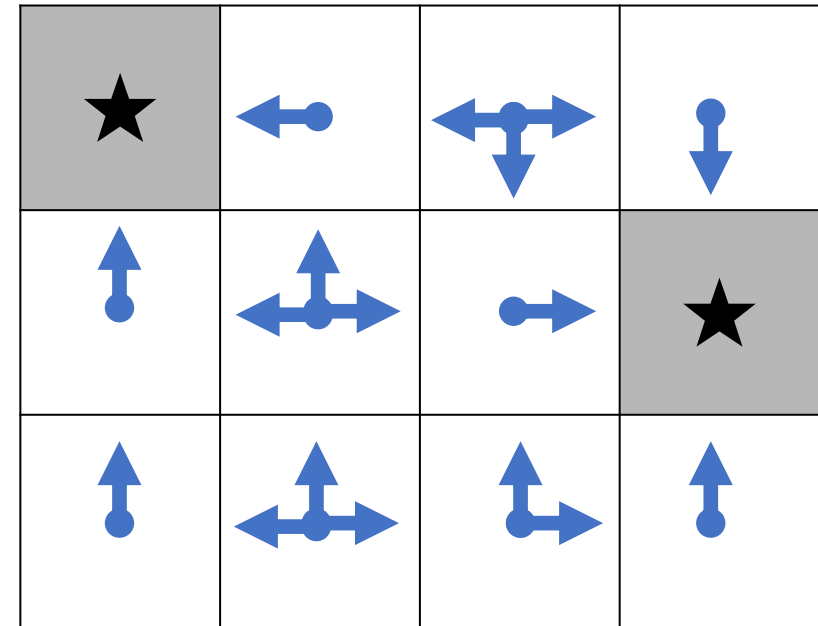


# A simple MDP: Grid World

# Bad policy



# Optimal Policy



# Finding Optimal Policies

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**Solution:** Maximize the expected sum of rewards

$$\pi^* = \arg \max_{\pi} \mathbb{E} \left[ \sum_{t \geq 0} \gamma^t r_t \mid \pi \right]$$

$$s_0 \sim p(s_0)$$

$$a_t \sim \pi(a \mid s_t)$$

$$s_{t+1} \sim P(s \mid s_t, a_t)$$

# Value Function and Q Function

Following a policy  $\pi$  produces **sample trajectories** (or paths)  $s_0, a_0, r_0, s_1, a_1, r_1, \dots$

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**How good is a state?** The **value function** at state  $s$ , is the expected cumulative reward from following the policy from state  $s$ :

$$V^\pi(s) = \mathbb{E} \left[ \sum_{t \geq 0} \gamma^t r_t \mid s_0 = s, \pi \right]$$

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**How good is a state-action pair?** The **Q function** at state  $s$  and action  $a$ , is the expected cumulative reward from taking action  $a$  in state  $s$  and then following the policy:

$$Q^\pi(s, a) = \mathbb{E} \left[ \sum_{t \geq 0} \gamma^t r_t \mid s_0 = s, a_0 = a, \pi \right]$$

# Bellman Equation

**Optimal Q-function:**  $Q^*(s, a)$  is the Q-function for the optimal policy  $\pi^*$   
It gives the max possible future reward when taking action  $a$  in state  $s$ :

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**Bellman Equation:**  $Q^*$  satisfies the following recurrence relation:

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**Intuition:** After taking action  $a$  in state  $s$ , we get reward  $r$  and move to a new state  $s'$ . After that, the max possible reward we can get is  $\max_{a'} Q^*(s', a')$

# Solving for the optimal policy: Value Iteration

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$$Q_{i+1}(s, a) = \mathbb{E}_{r, s'} \left[ r + \gamma \max_{a'} Q_i(s', a') \right]$$

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**Solution:** Approximate  $Q(s, a)$  with a neural network, use Bellman Equation as loss!



# Solving for the optimal policy: Deep Q-Learning

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Use the Bellman Equation to tell what  $Q$  should output for a given state and action:

$$y_{s,a,\theta} = \mathbb{E}_{r, s'} \left[ r + \gamma \max_{a'} Q(s', a'; \theta) \right]$$

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Use this to define the loss for training  $Q$ :  $L(s, a) = (Q(s, a; \theta) - y_{s,a,\theta})^2$

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**Problem:** How to sample batches of data for training?

# Case Study: Playing Atari Games



**Objective:** Complete the game with the highest score

**State:** Raw pixel inputs of the game screen

**Action:** Game controls e.g. Left, Right, Up, Down

**Reward:** Score increase/decrease at each time step

Mnih et al, "Playing Atari with Deep Reinforcement Learning", NeurIPS Deep Learning Workshop, 2013

# Case Study: Playing Atari Games

## Network output:

Q-values for all actions

FC-A (Q-values)

FC-256

Conv(16→32, 4x4, stride 2)

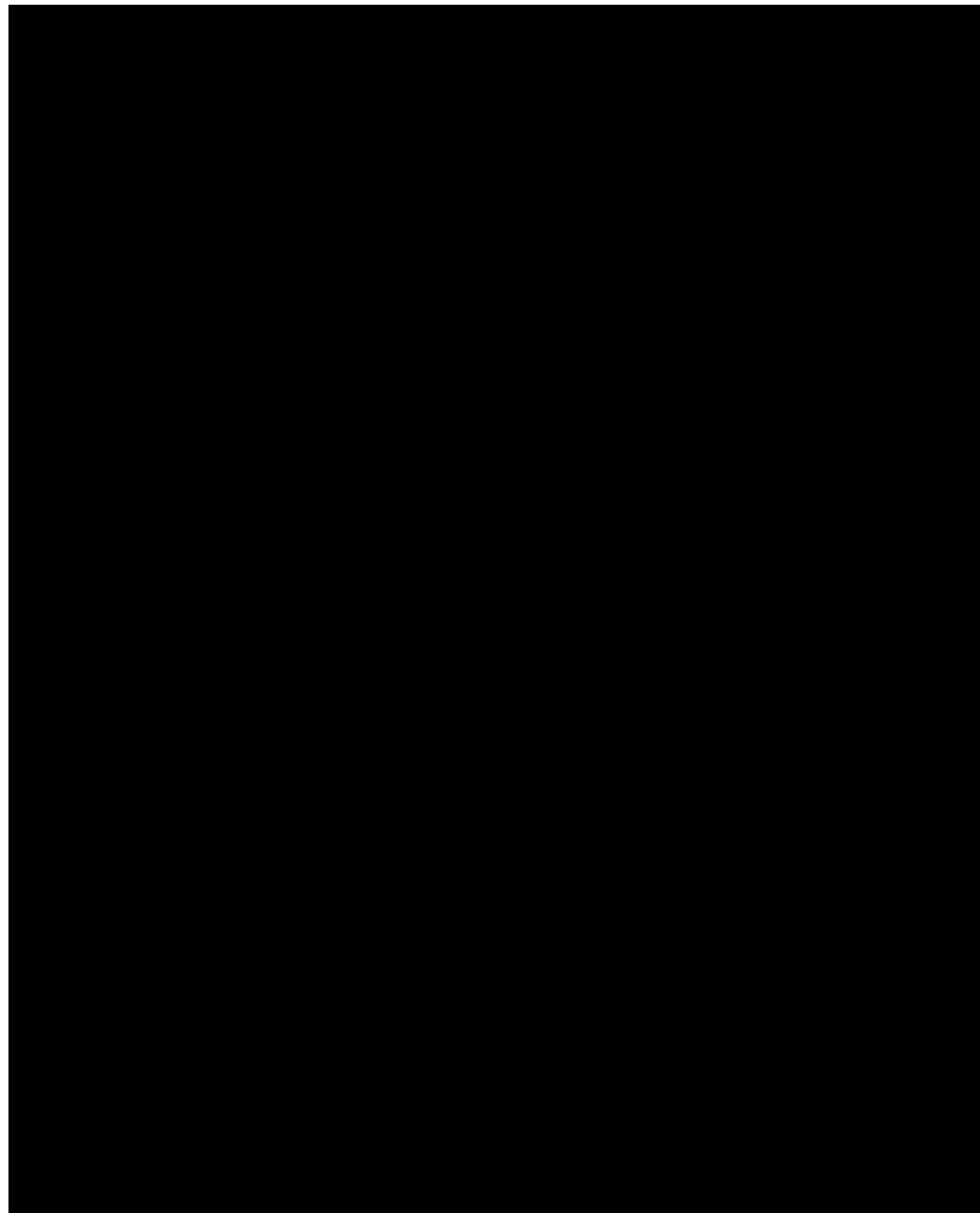
Conv(4→16, 8x8, stride 4)

With 4 actions: last layer gives values  $Q(s_t, a_1)$ ,  $Q(s_t, a_2)$ ,  $Q(s_t, a_3)$ ,  $Q(s_t, a_4)$

$Q(s, a; \theta)$   
Neural network  
with weights  $\theta$



**Network input: state  $s_t$ : 4x84x84 stack of last 4 frames**  
(after RGB→grayscale conversion, downsampling, and cropping)



<https://www.youtube.com/watch?v=V1eYniJORnk>



# Q-Learning

**Q-Learning:** Train network  $Q_{\theta}(s, a)$  to estimate future rewards for every (state, action) pair

**Problem:** For some problems this can be a hard function to learn.

For some problems it is easier to learn a mapping from states to actions

# Q-Learning vs Policy Gradients

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**Policy Gradients:** Train a network  $\pi_\theta(a | s)$  that takes state as input, gives distribution over which action to take in that state

**Objective function:** Expected future rewards when following policy  $\pi_\theta$ :

$$J(\theta) = \mathbb{E}_{r \sim p_\theta} \left[ \sum_{t \geq 0} \gamma^t r_t \right]$$

Find the optimal policy by maximizing:  $\theta^* = \arg \max_{\theta} J(\theta)$  (Use gradient ascent!)

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**Problem:** Nondifferentiability! Don't know how to compute  $\frac{\partial J}{\partial \theta}$

# Policy Gradients

**Objective function:** Expected future rewards when following policy  $\pi_\theta$ :

$$J(\theta) = \mathbb{E}_{r \sim p_\theta} \left[ \sum_{t \geq 0} \gamma^t r_t \right]$$

Find the optimal policy by maximizing:  $\theta^* = \arg \max_{\theta} J(\theta)$  (Use gradient ascent!)

**Problem:** Nondifferentiability! Don't know how to compute  $\frac{\partial J}{\partial \theta}$

**General formulation:**  $J(\theta) = \mathbb{E}_{x \sim p_\theta} [f(x)]$  Want to compute  $\frac{\partial J}{\partial \theta}$

# Policy Gradients: REINFORCE Algorithm

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Approximate the expectation via sampling!

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Sequence of states  
and actions when  
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$$J(\theta) = \mathbb{E}_{x \sim p_{\theta}}[f(x)]$$

$$\frac{\partial J}{\partial \theta} = \mathbb{E}_{x \sim p_{\theta}} \left[ \textcolor{green}{f(x)} \sum_{t \geq 0} \frac{\partial}{\partial \theta} \log \pi_{\theta}(a_t | s_t) \right]$$

**Reward we get from  
state sequence  $x$**

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**Gradient of predicted action scores with respect to model weights. Backprop through model  $\pi_{\theta}$ !**

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1. Initialize random weights  $\theta$

Expected reward under  $\pi_{\theta}$ :

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3. Compute  $dJ/d\theta$

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4. Gradient ascent step on  $\theta$
5. GOTO 2

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**Intuition:**

When  $f(x)$  is high: Increase the probability of the actions we took.

When  $f(x)$  is low: Decrease the probability of the actions we took.

# So far: Q-Learning and Policy Gradients

**Q-Learning:** Train network  $Q_\theta(s, a)$  to estimate future rewards for every (state, action) pair  
Use Bellman Equation to define loss function for training Q:

$$y_{s,a,\theta} = \mathbb{E}_{r,s'} \left[ r + \gamma \max_{a'} Q(s', a'; \theta) \right] \quad \text{Where } r \sim R(s, a), s' \sim P(s, a)$$
$$L(s, a) = \left( Q(s, a; \theta) - y_{s,a,\theta} \right)^2$$

**Policy Gradients:** Train a network  $\pi_\theta(a | s)$  that takes state as input, gives distribution over which action to take in that state. Use REINFORCE Rule for computing gradients:

$$J(\theta) = \mathbb{E}_{x \sim p_\theta} [f(x)] \quad \frac{\partial J}{\partial \theta} = \mathbb{E}_{x \sim p_\theta} \left[ f(x) \sum_{t \geq 0} \frac{\partial}{\partial \theta} \log \pi_\theta(a_t | s_t) \right]$$

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Improving policy gradients: Add **baseline** to reduce variance of gradient estimator



# Other approaches

**Actor-Critic:** Train an actor that predicts actions (like policy gradient) and a critic that predicts the future rewards we get from taking those actions (like Q-Learning)

Sutton and Barto, “Reinforcement Learning: An Introduction”, 1998; Degris et al, “Model-free reinforcement learning with continuous action in practice”, 2012; Mnih et al, “Asynchronous Methods for Deep Reinforcement Learning”, ICML 2016

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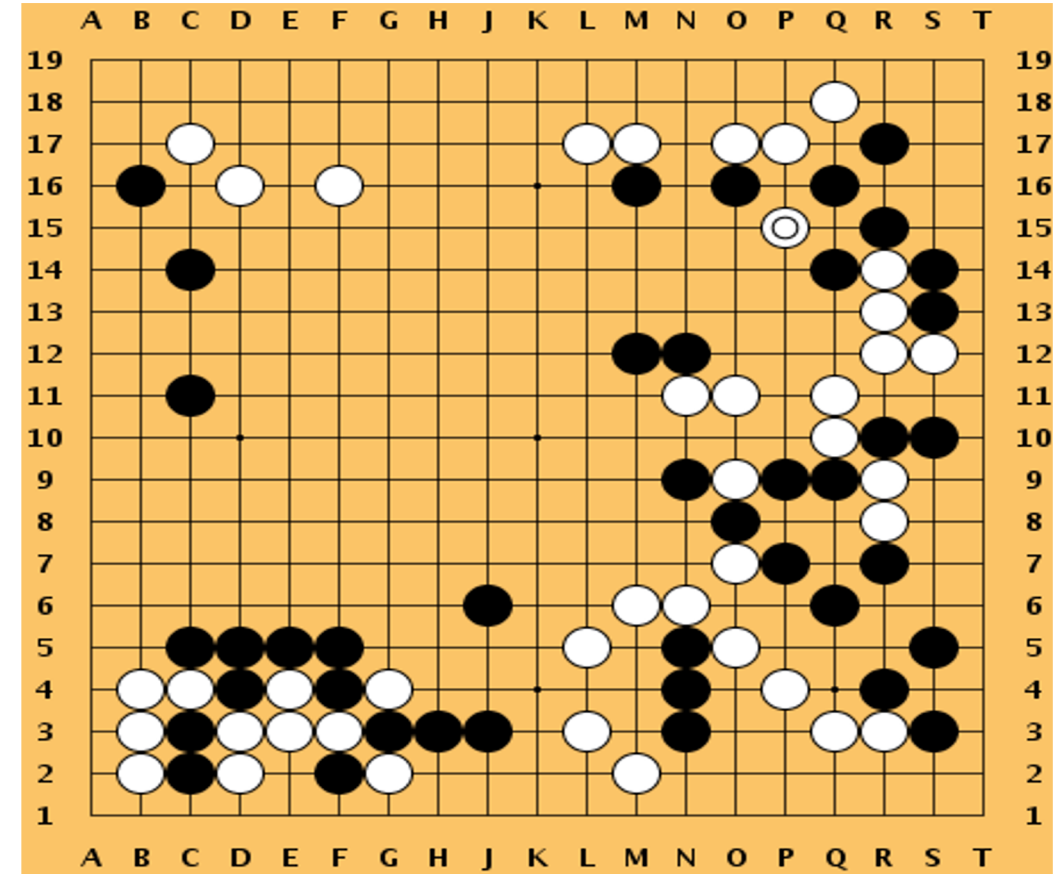
**Adversarial Learning:** Learn to fool a discriminator that classifies actions as real/fake

Ho and Ermon, “Generative Adversarial Imitation Learning”, NeurIPS 2016

# Case Study: Playing Games

## AlphaGo: (January 2016)

- Used imitation learning + tree search + RL
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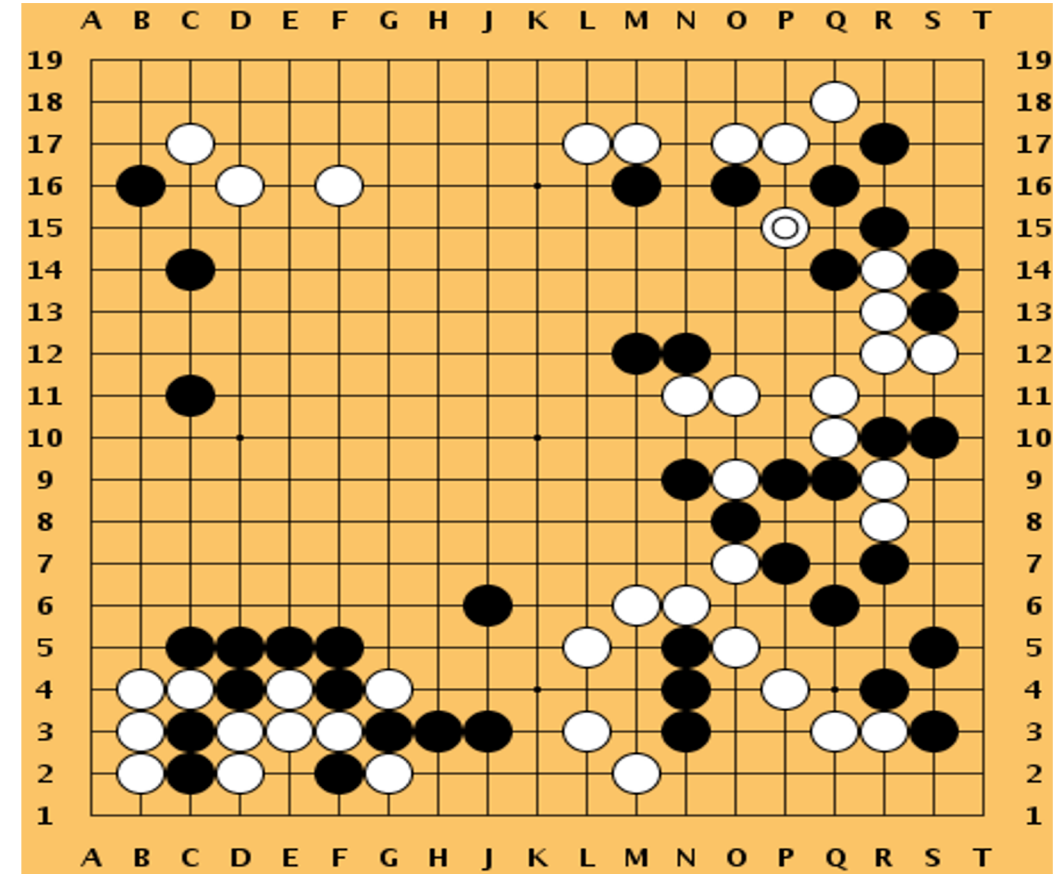
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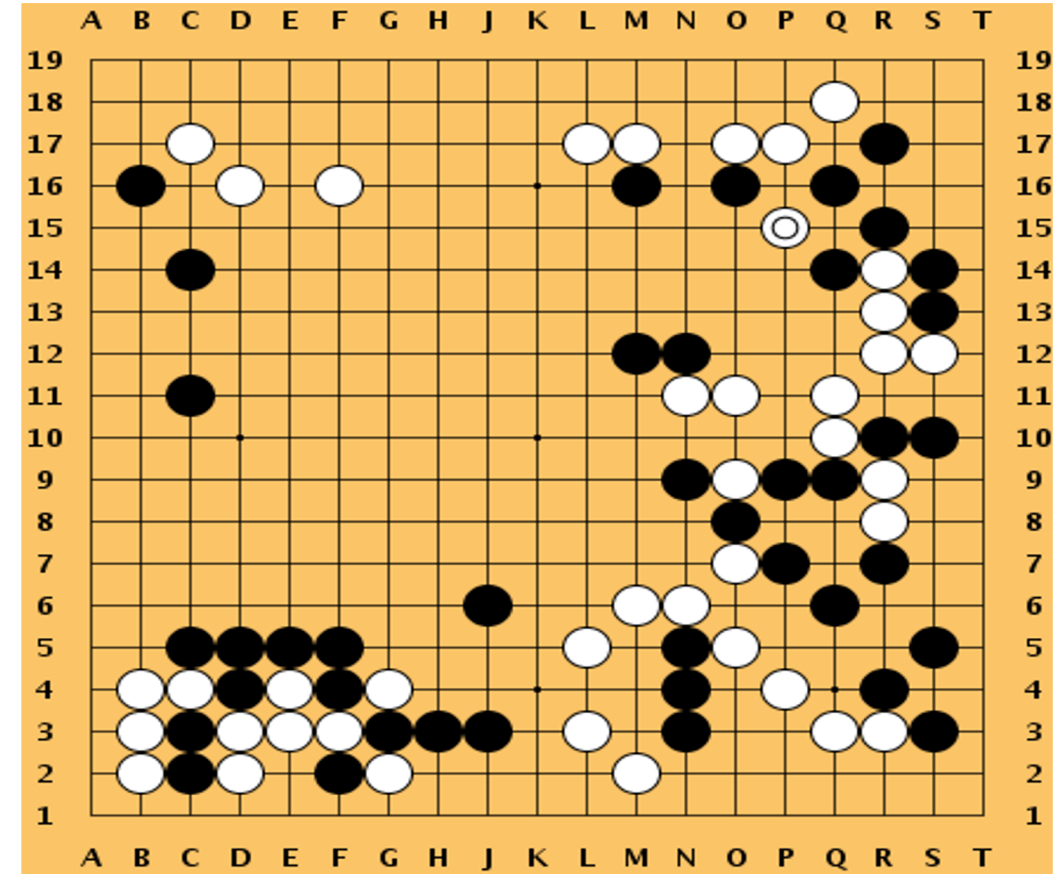
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- Generalized to other games: Chess and Shogi



Silver et al, "Mastering the game of Go with deep neural networks and tree search", Nature 2016

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# Case Study: Playing Games

## AlphaGo: (January 2016)

- Used imitation learning + tree search + RL
- Beat 18-time world champion Lee Sedol

## AlphaGo Zero (October 2017)

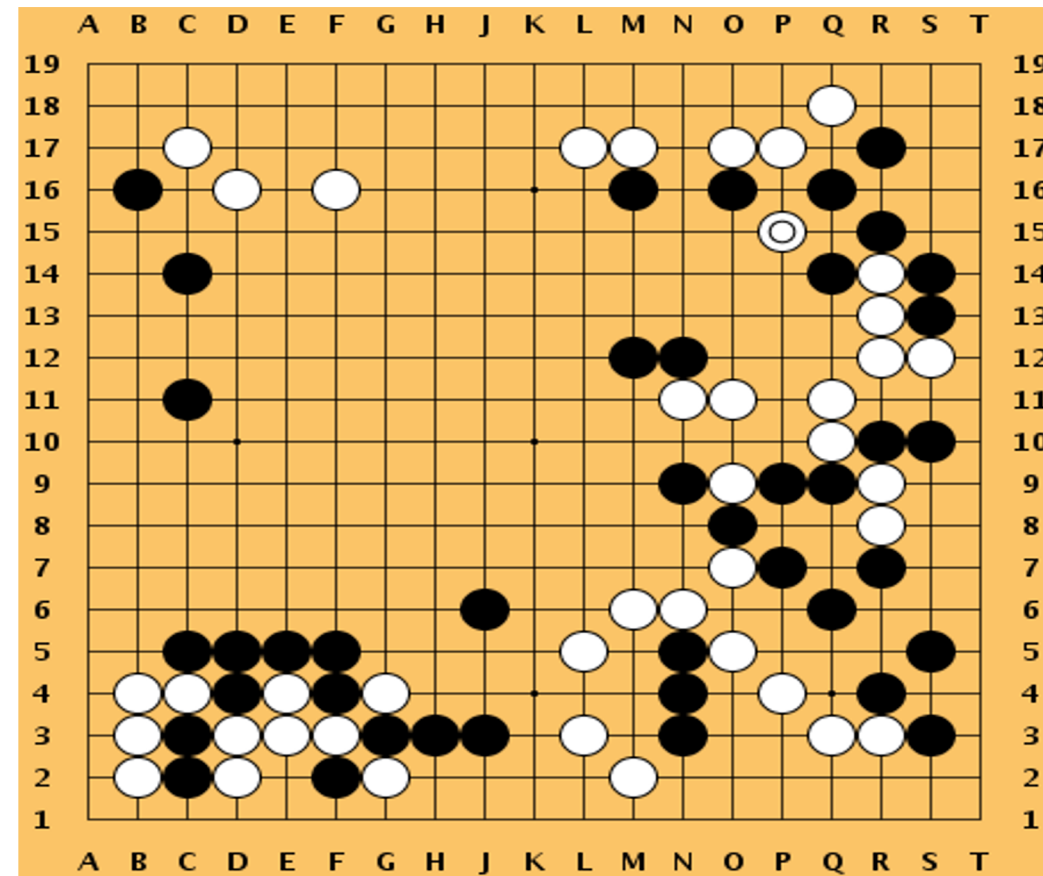
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November 2019: Lee Sedol  
announces retirement



“With the debut of AI in Go games, I've realized that I'm not at the top even if I become the number one through frantic efforts”

“Even if I become the number one, there is an entity that cannot be defeated”

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Quotes from: <https://en.yna.co.kr/view/AEN20191127004800315>

[Image of Lee Sedol](#) is licensed under [CC BY 2.0](#)

# More Complex Games

## **StarCraft II: AlphaStar**

(October 2019)

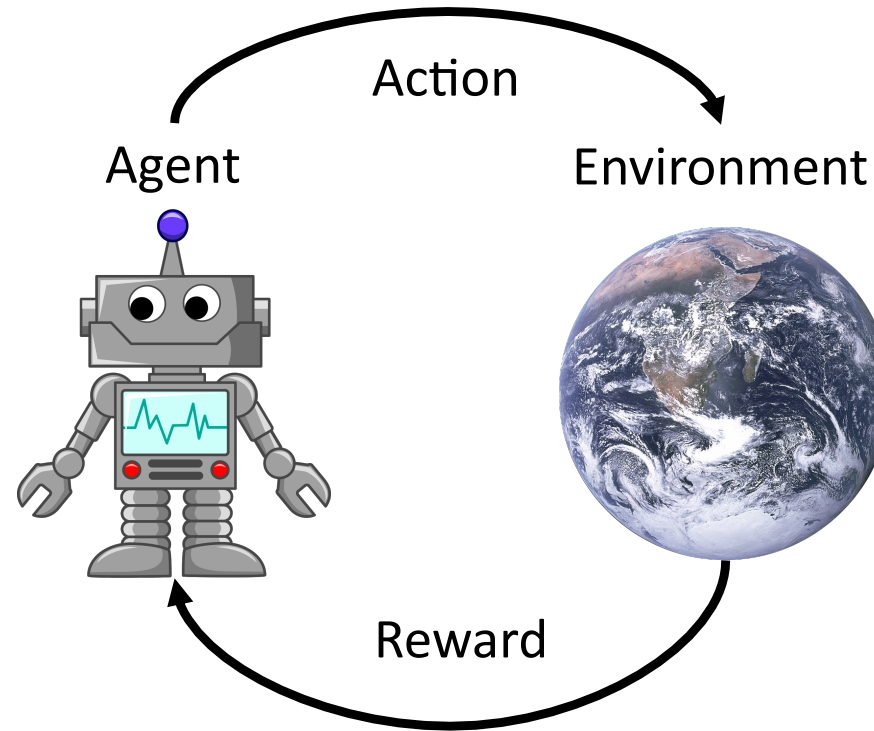
Vinyals et al, “Grandmaster level in StarCraft II using multi-agent reinforcement learning”, Science 2018

## **Dota 2: OpenAI Five (April 2019)**

Dota 2 with Large Scale Deep Reinforcement Learning

<https://arxiv.org/abs/1912.06680>

# Reinforcement Learning: Interacting With World



Normally we use RL to train **agents** that interact with a (noisy, nondifferentiable) **environment**

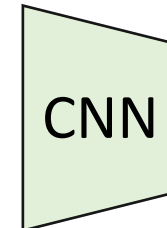
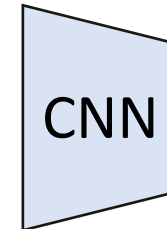
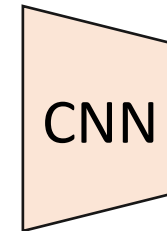
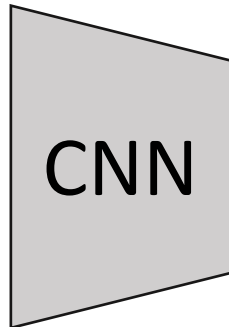
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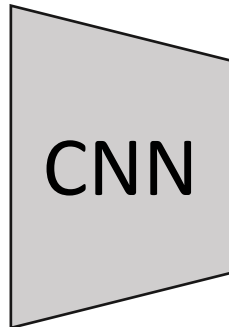
Example: Small “routing” network sends image to one of K networks



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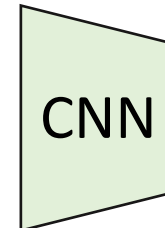
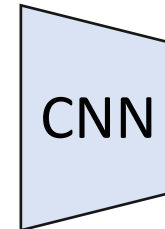
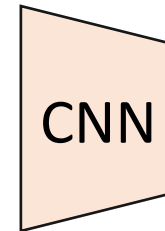


Which network  
to use?

**P(orange) = 0.2**

**P(blue) = 0.1**

**P(green) = 0.7**

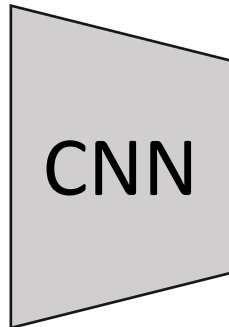




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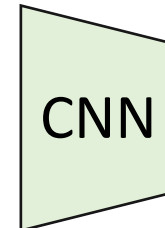
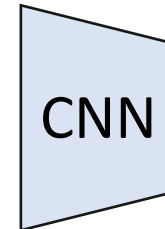
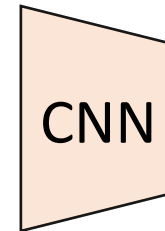
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Sample:  
**Green**

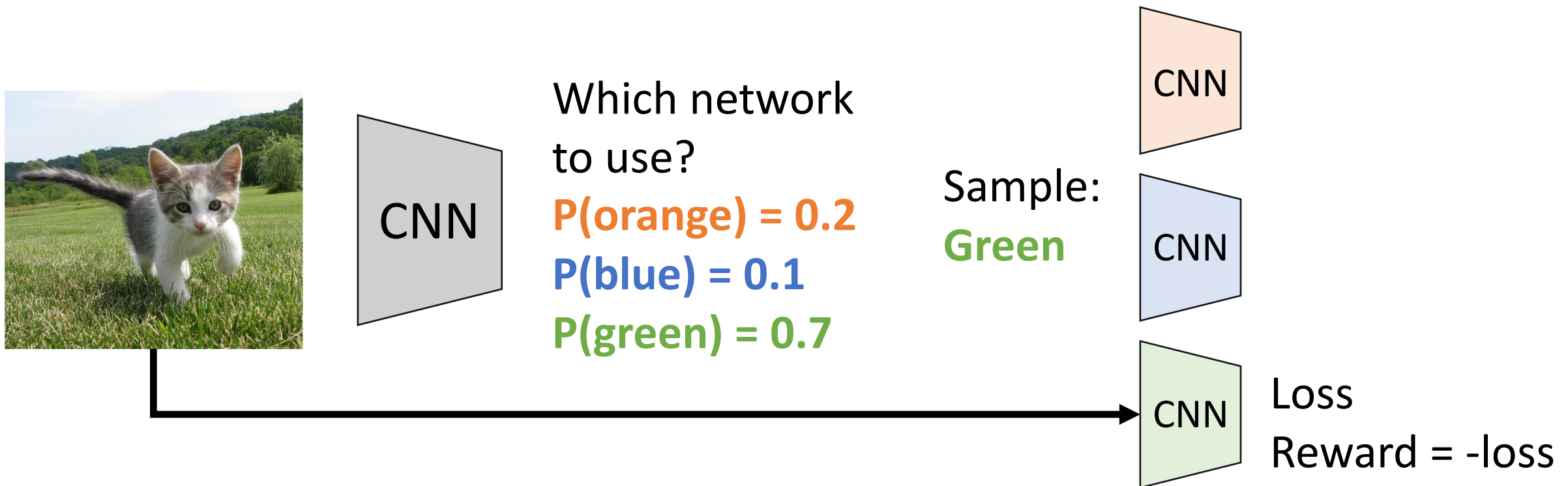




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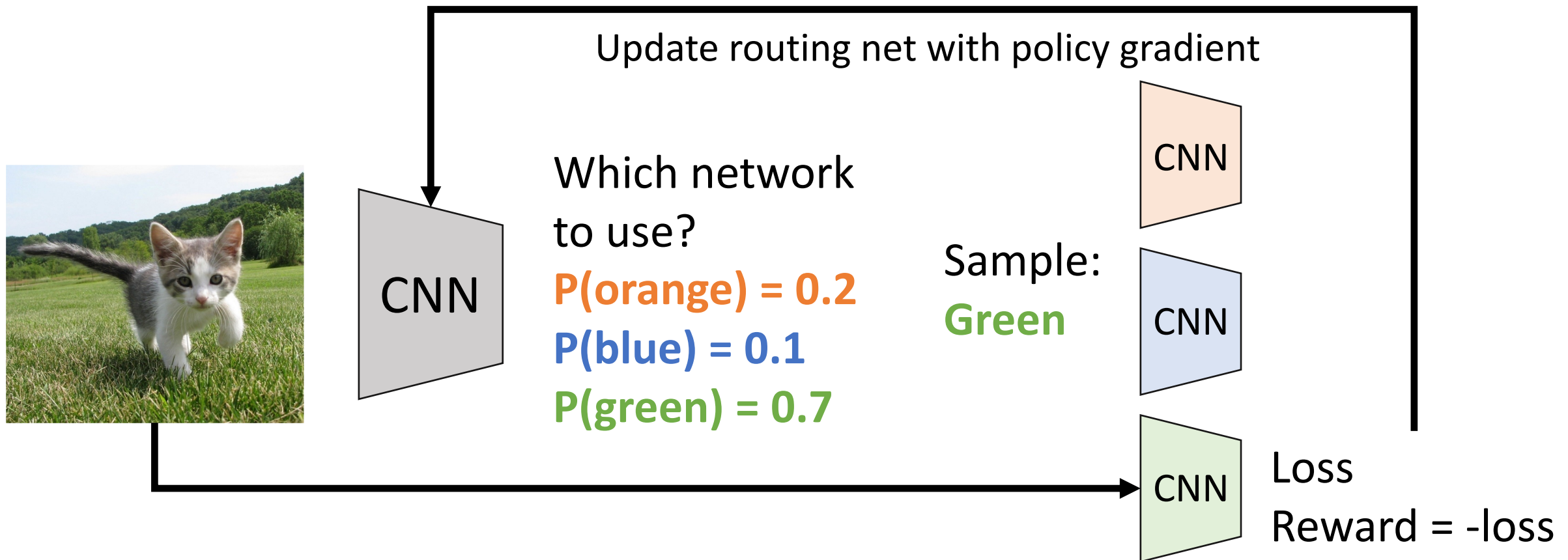
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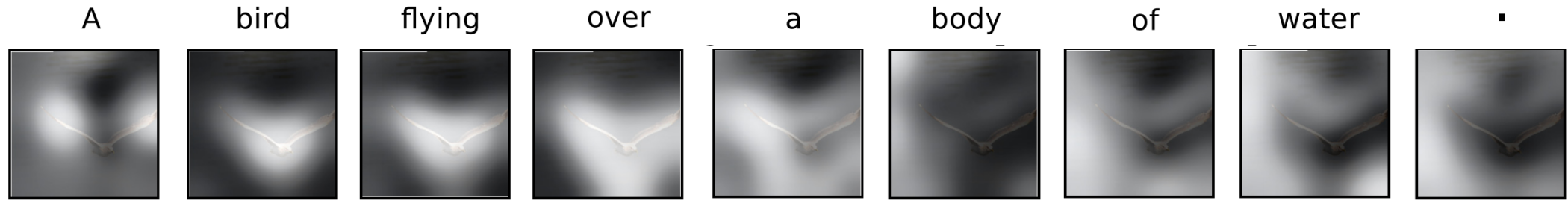
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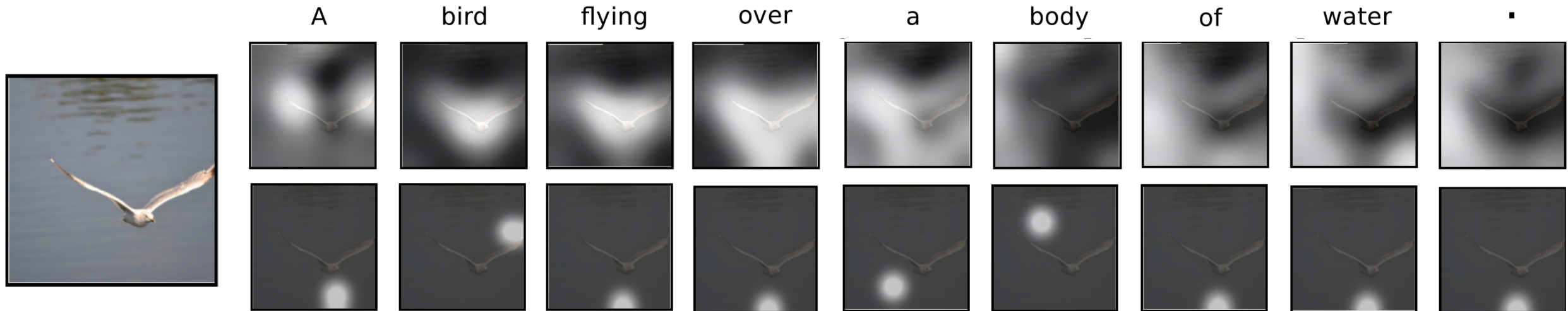
# Stochastic Computation Graphs: Attention

**Recall:** Image captioning with attention. At each timestep use a weighted combination of features from different spatial positions  
(Soft Attention)



# Stochastic Computation Graphs: Attention

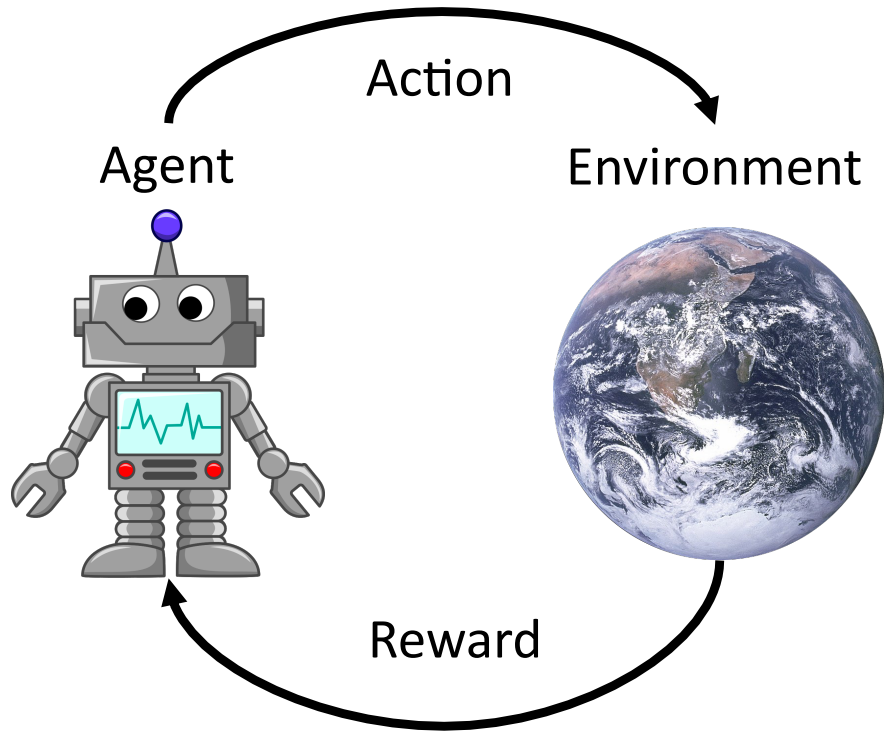
**Recall:** Image captioning with attention. At each timestep use a weighted combination of features from different spatial positions (Soft Attention)



**Hard Attention:** At each timestep, select features from exactly one spatial location. Train with policy gradient.

# Summary: Reinforcement Learning

RL trains **agents** that interact with an **environment** and learn to maximize **reward**



**Q-Learning:** Train network  $Q_{\theta}(s, a)$  to estimate future rewards for every (state, action) pair. Use Bellman Equation to define loss function for training Q

**Policy Gradients:** Train a network  $\pi_{\theta}(a | s)$  that takes state as input, gives distribution over which action to take in that state. Use REINFORCE Rule for computing gradients

Next Time:  
Course Recap  
Open Problems in Computer Vision