Lecture 3:
Cameras II

Administrative

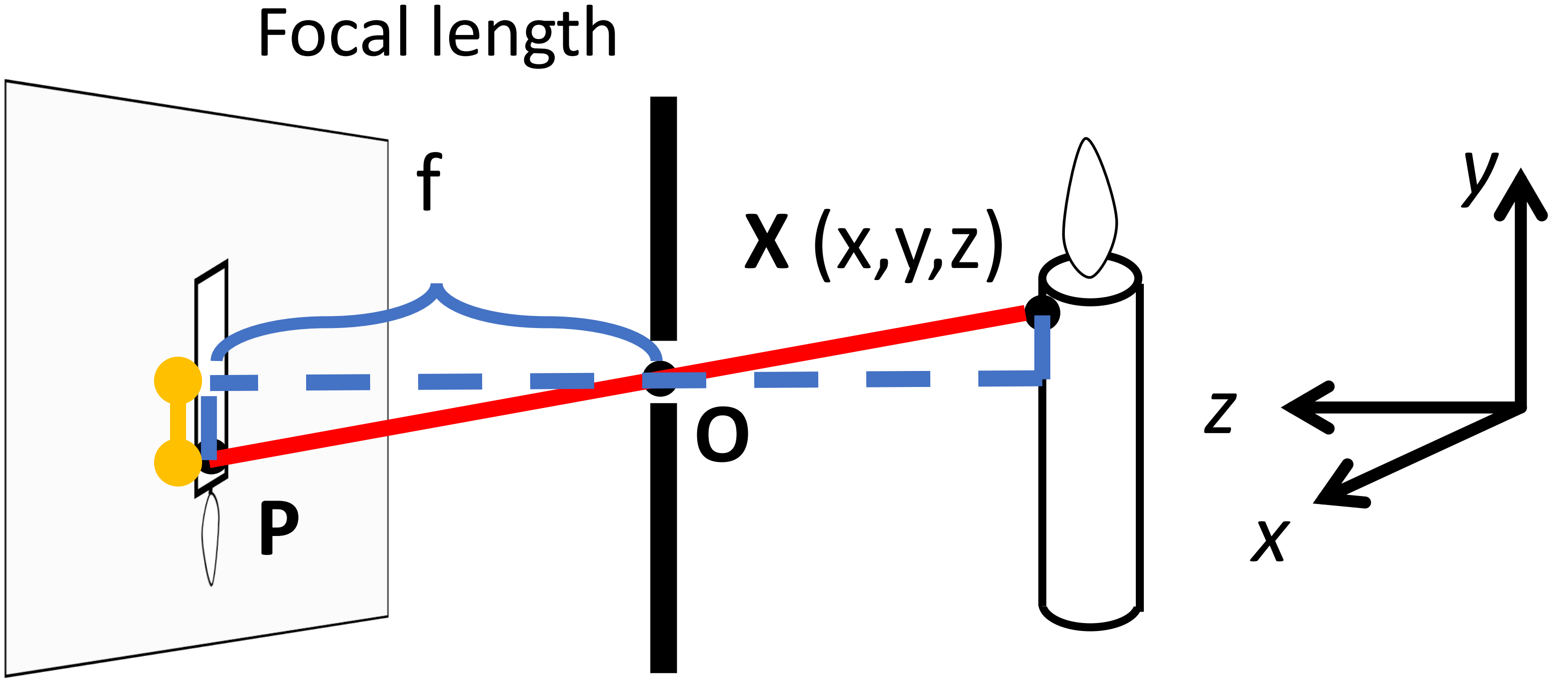
HW0 is released
will be due Friday 1/24 at 11:59pm

Administrative

HW0 is released
will be due ~~Friday 1/24~~ **Wednesday 1/29** at 11:59pm

(Had to split Cameras into 2 lectures; this makes HW0 due after linear algebra lectures)

Recap: Pinhole Camera Model



Coordinate system: **O** is origin, XY in image, Z sticks out.
XY is image plane, Z is optical axis.

(x,y,z) projects to (fx/z,fy/z) via similar triangles

Source: L Lazebnik

Recap: Homogenous Coordinates

Trick: add a dimension!

This also clears up lots of nasty special cases

Physical Point → Homogeneous Point → Physical Point

$$\begin{bmatrix} x \\ y \end{bmatrix} \xrightarrow[\text{w=1}]{\text{Concat}} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \xrightarrow[\text{by w}]{\text{Divide}} \begin{bmatrix} u/w \\ v/w \end{bmatrix}$$

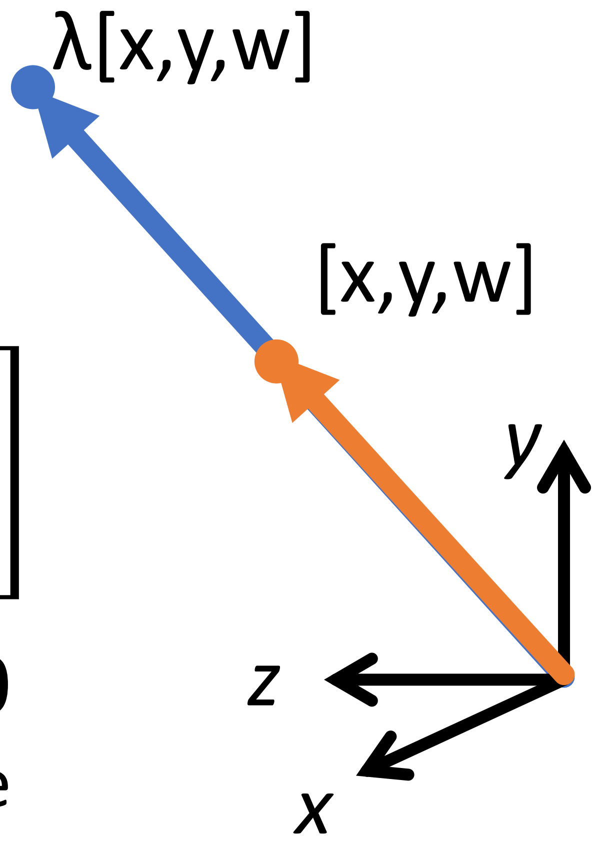
Recap: Homogenous Coordinates

Triple / Equivalent

Double / Equals

$$\begin{bmatrix}u\\v\\w\end{bmatrix} \boxed{\equiv} \begin{bmatrix}u'\\v'\\w'\end{bmatrix} \leftrightarrow \begin{bmatrix}u\\v\\w\end{bmatrix} \boxed{=} \lambda \begin{bmatrix}u'\\v'\\w'\end{bmatrix}$$

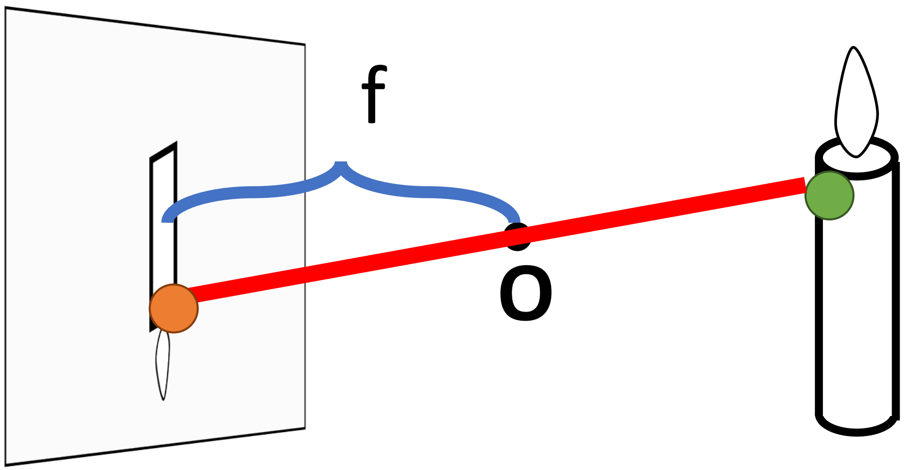
$$\lambda \neq 0$$



Two homogeneous coordinates are **equivalent** if they are proportional to each other. **Not = !**

Recap: Projection Matrix

Projection (x, y, z) -> (fx/z, fy/z) is matrix multiplication



$$\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \\ \boldsymbol{1} \end{bmatrix} = \begin{bmatrix} fx \\ fy \\ z \end{bmatrix} \equiv \begin{bmatrix} fx/z \\ fy/z \\ 1 \end{bmatrix}$$

3D homogenous point

2D homogenous point

Slide inspired from L. Lazebnik

Recap: Perspective Model

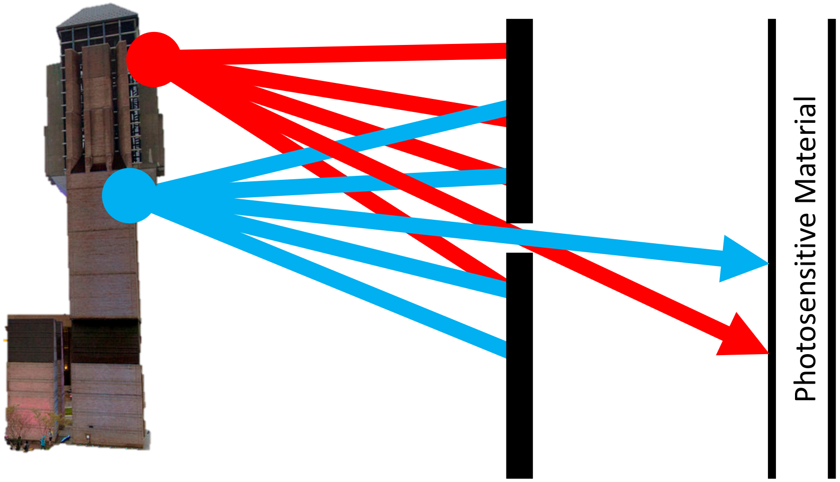
Intrinsic Matrix K **Extrinsic Matrix [R,t]**

$$\boldsymbol{P} \equiv \boxed{\begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix}} \ \boxed{\begin{bmatrix} \boldsymbol{R}_{3x3} & \boldsymbol{t}_{3x1} \end{bmatrix}} \ \boldsymbol{X}_{4x1}$$

$$\boldsymbol{P} \equiv \boldsymbol{K}[\boldsymbol{R} \mid \boldsymbol{t}]\boldsymbol{X} \equiv \boldsymbol{M}_{3x4}\boldsymbol{X}_{4x1}$$

Nice interactive demo: http://ksimek.github.io/2012/08/22/extrinsic/

Pinhole Model: Big Issue



Film captures all the rays going through a ***point*** (a *pencil of rays).*

How big is a point?

Slide inspired by S. Seitz; image from Michigan Engineering

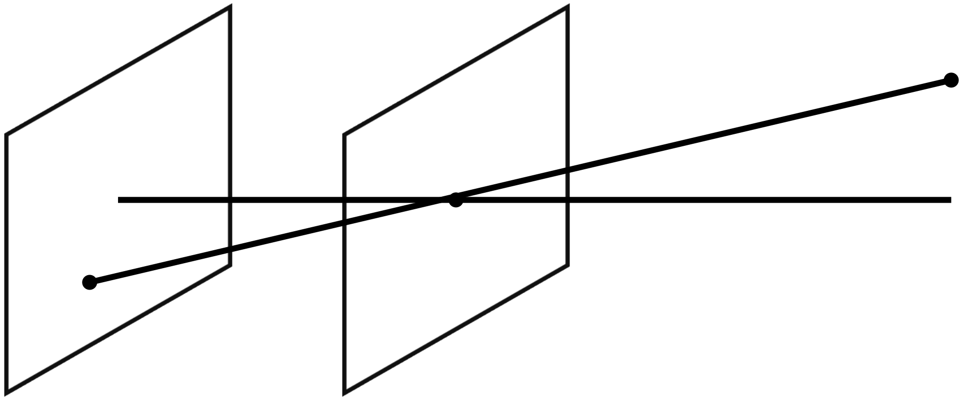
Math vs Reality

- Math: Any point projects to one point

- Reality
  - Don’t image points behind the camera / objects
  - Don’t have an infinite amount of sensor material

- Other issues
  - Light is limited
  - Spooky stuff happens with infinitely small holes

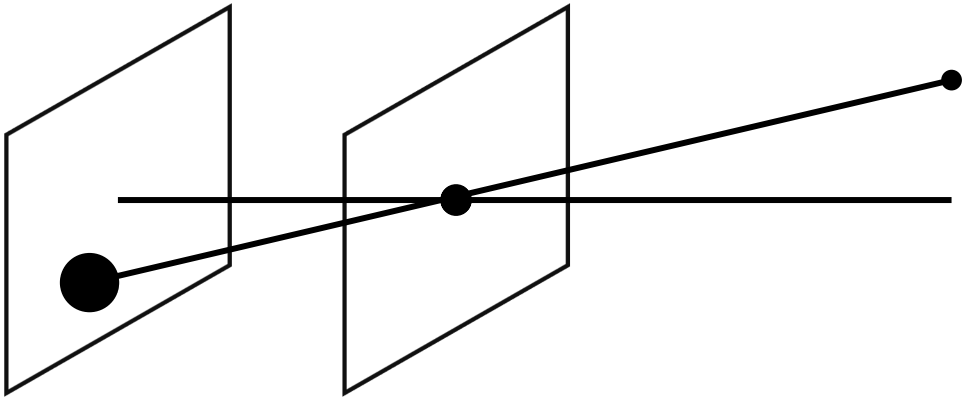
Limitations of Pinhole Model



Ideal Pinhole

1 point generates 1 image

Low-light levels



Finite Pinhole

1 point generates region

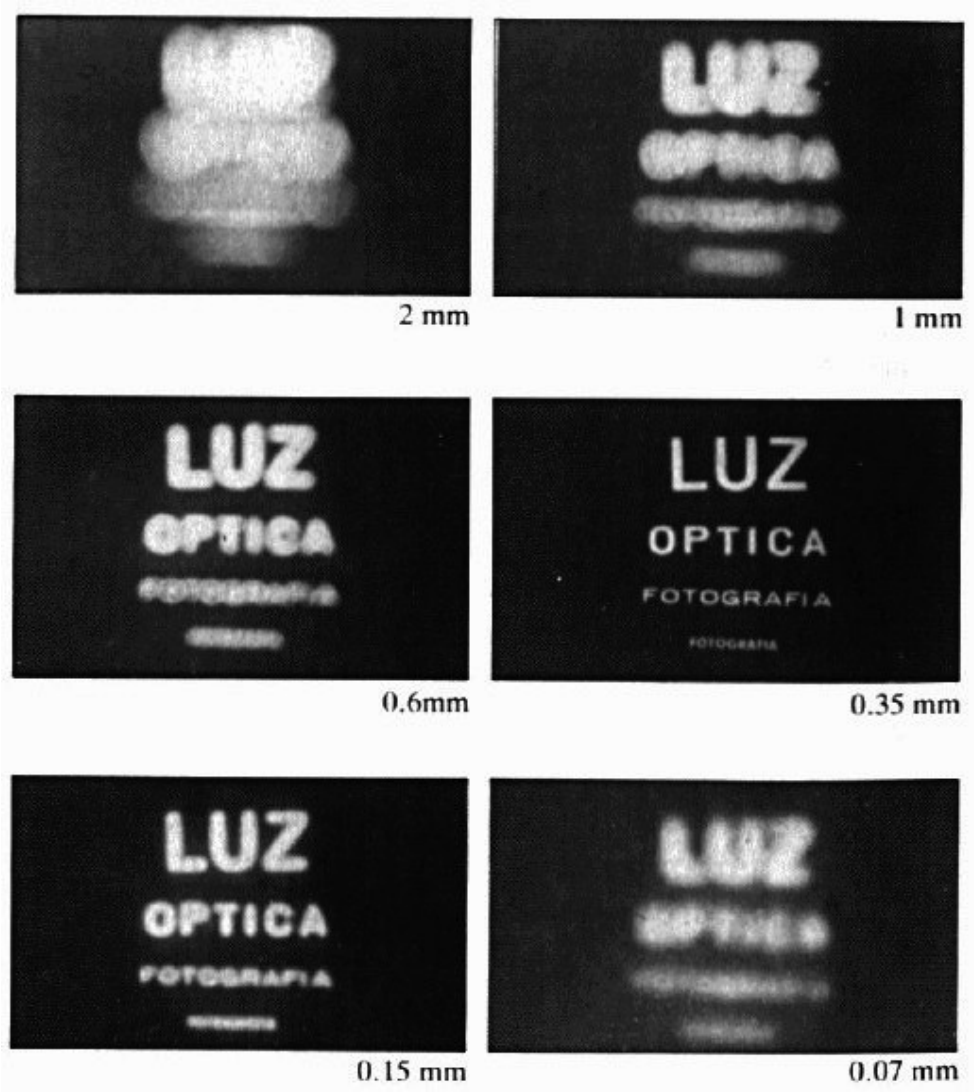
Blurry.

Why is it blurry?

Slide inspired by M. Hebert

Limitations of Pinhole Model

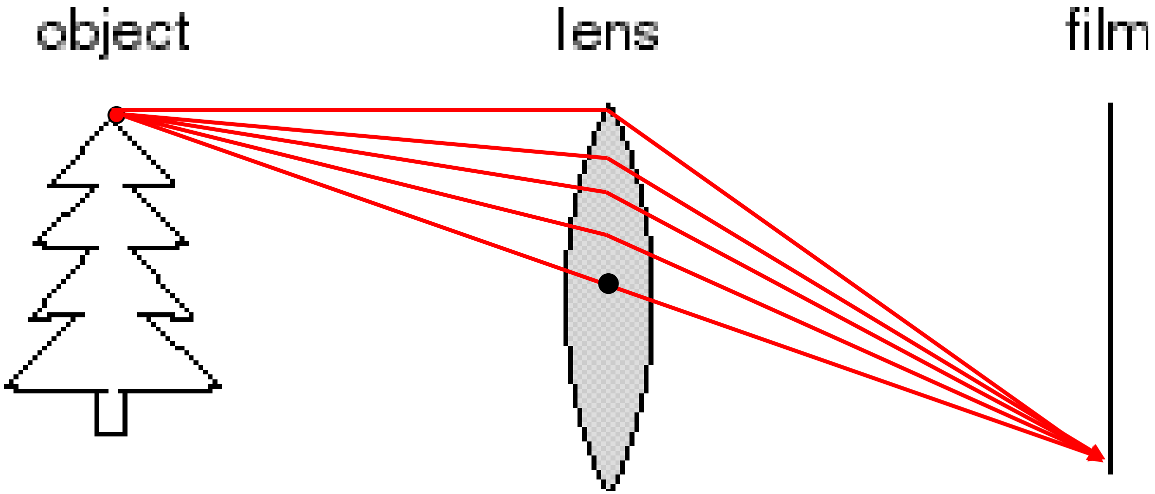
Small pinhole gives sharper image (but also needs longer exposure time)



When pinhole is too small, diffraction effects take over!

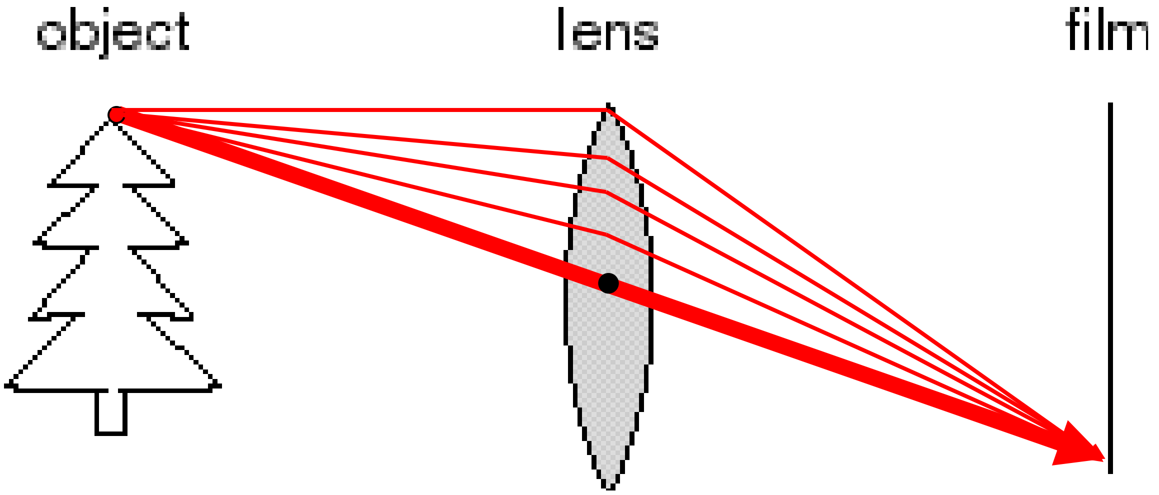
Slide Credit: S. Seitz

Adding a Lens



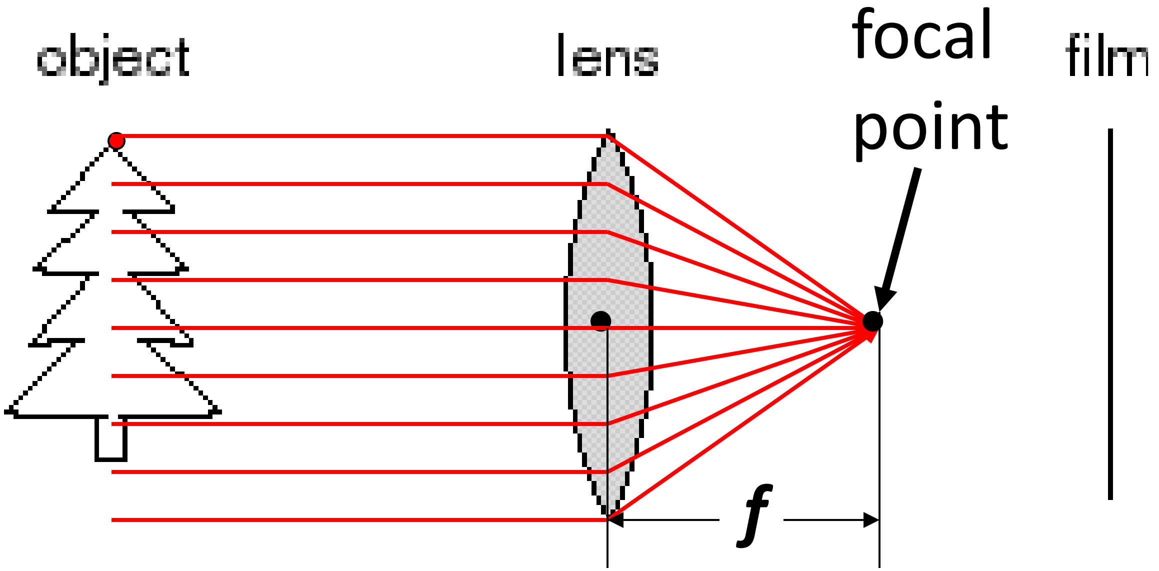
- A lens focuses light onto the film

Adding a Lens: Thin Lens Model



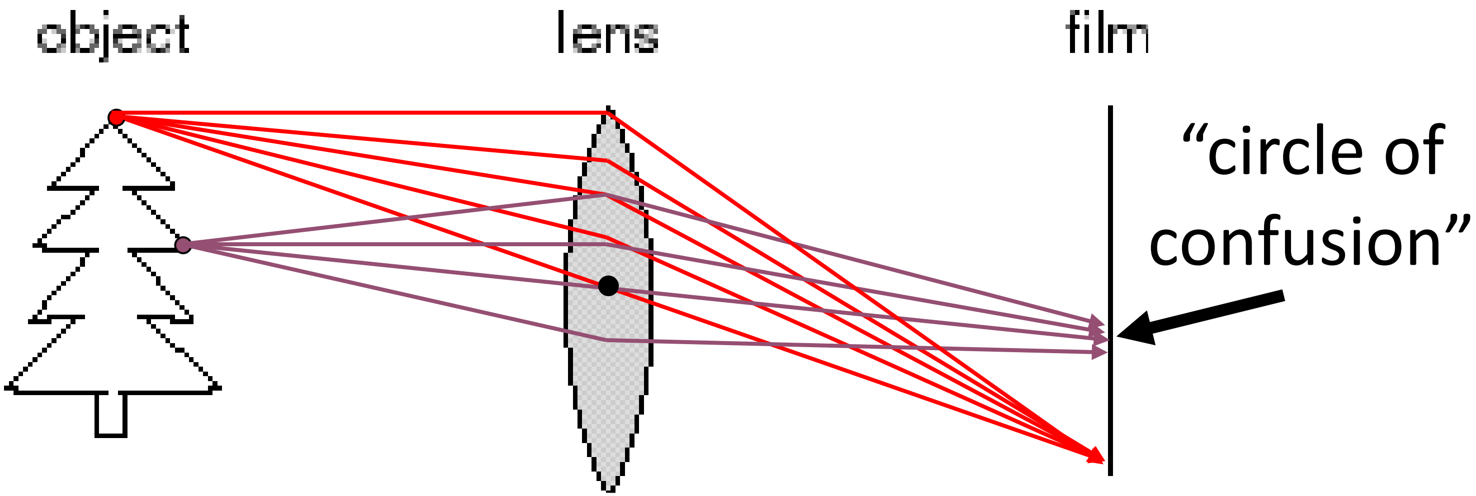
- A lens focuses light onto the film
- **Thin lens model**:
  - Rays passing through the center are not deviated (pinhole projection model still holds)

Adding a Lens: Thin Lens Model



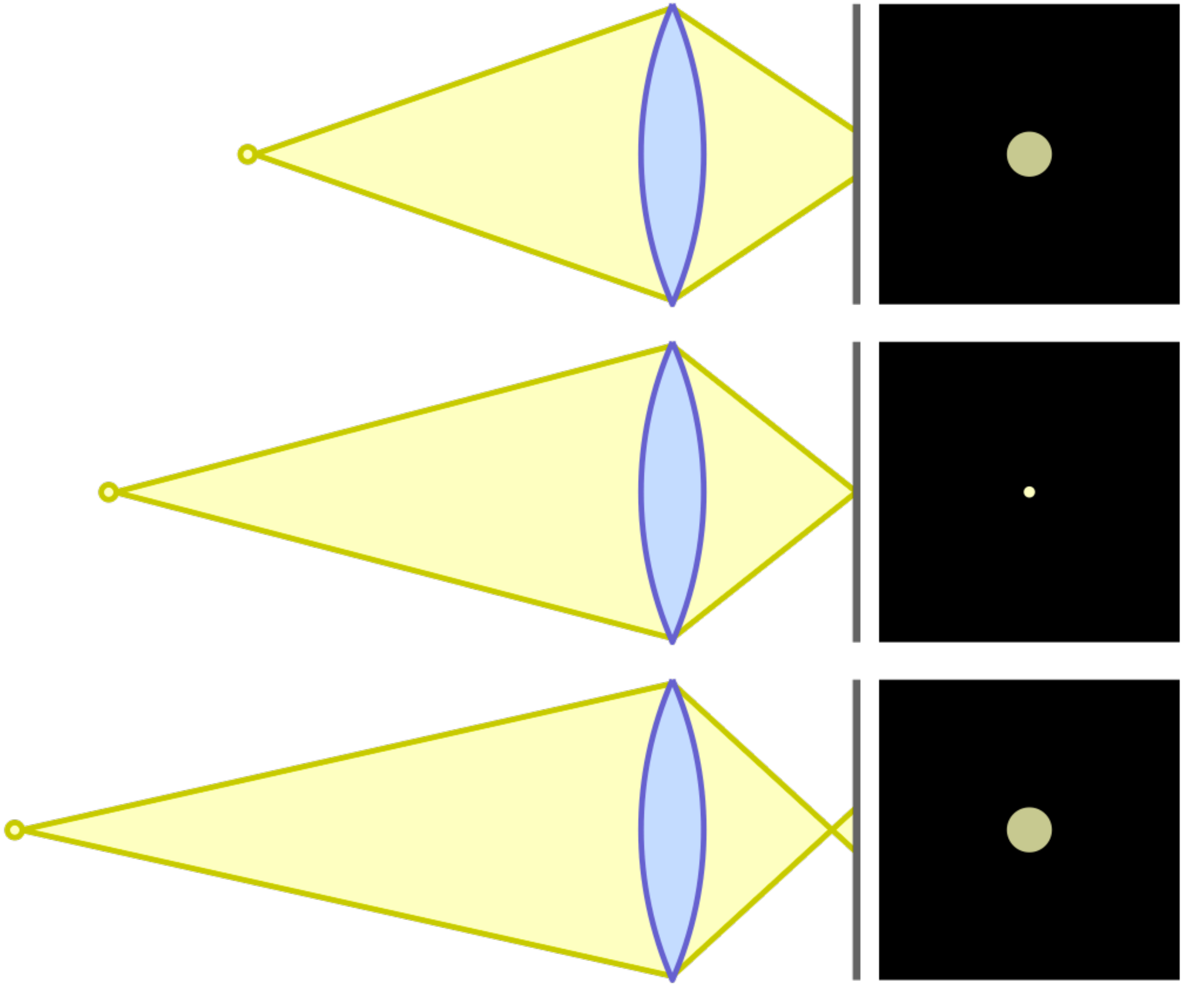
- A lens focuses light onto the film
- **Thin lens model**:
  - Rays passing through the center are not deviated (pinhole projection model still holds)
  - All rays parallel to the optical axis pass through the *focal point*

What’s the catch?



- There’s a distance where objects are “in focus”
- Other points project to a “circle of confusion”

Circle of Confusion



Object is too close:
Point projects to circle
(blurry image)

Object is just right:
Point projects point
(sharp image)

Object is too far:
Point projects to circle
(blurry image)

Image Source: [Wikipedia]

Question: How can we tell if the object is just right?

Thin Lens Formula

Want relationship between y, D, D’, f that causes the object to be **in focus**

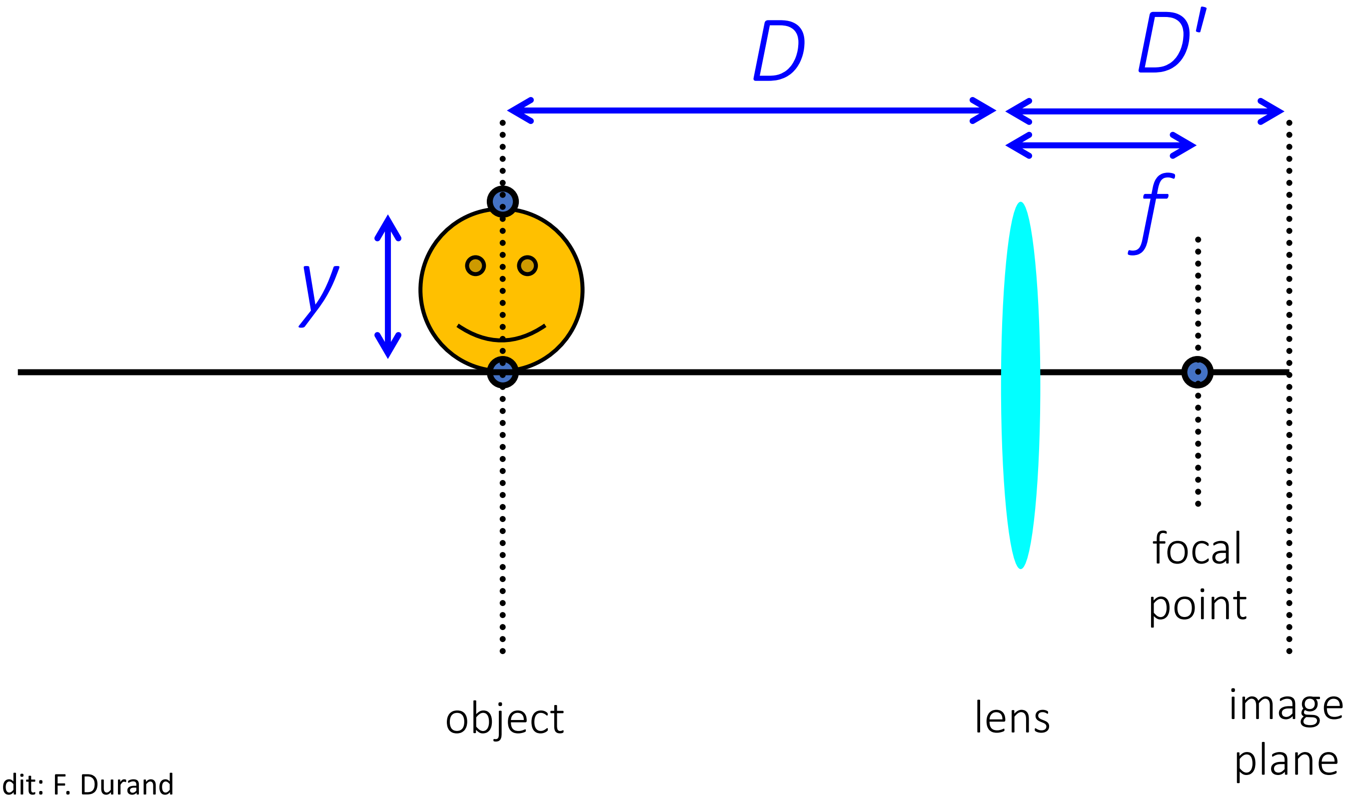


Diagram credit: F. Durand

Thin Lens Formula

Thin lens assumptions:

1. Rays through the lens center not deviated

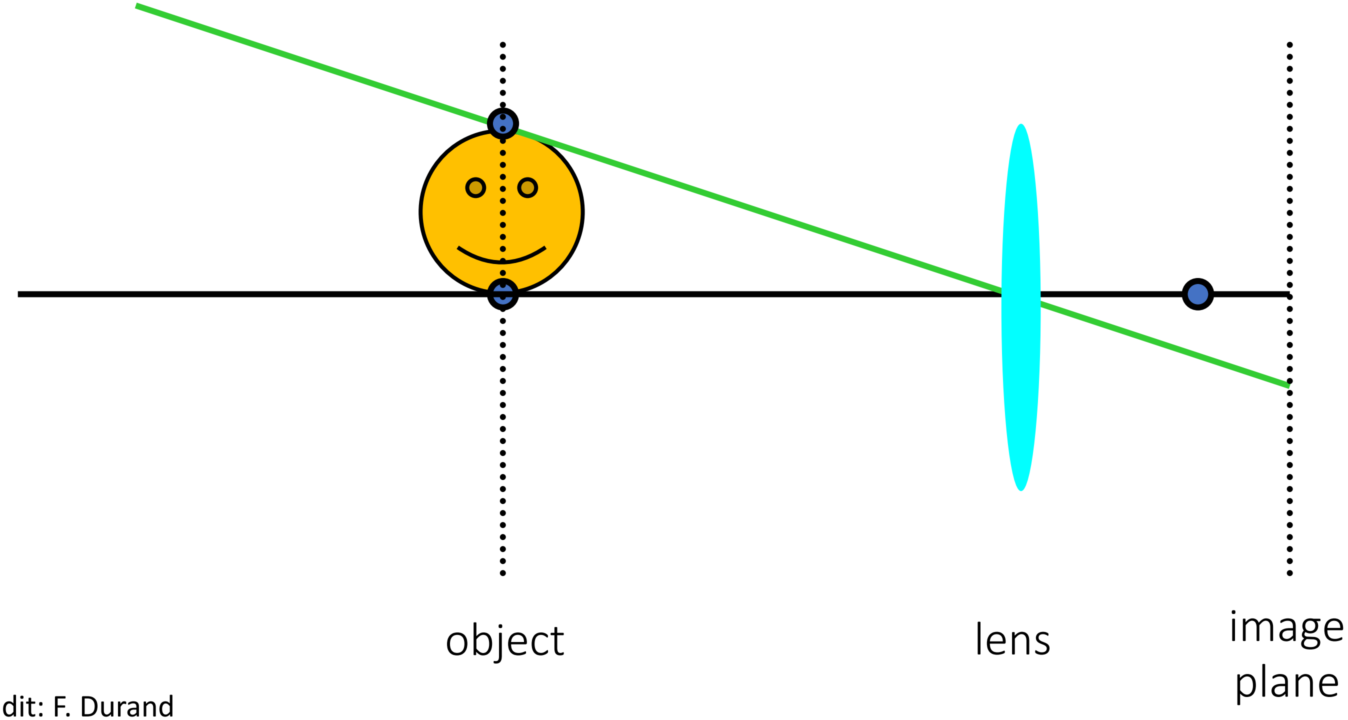


Diagram credit: F. Durand

Thin Lens Formula

Thin lens assumptions:

1. Rays through the lens center not deviated
2. Rays parallel to the optical axis pass through the focal point

The object is in focus when both rays intersect on the image plane

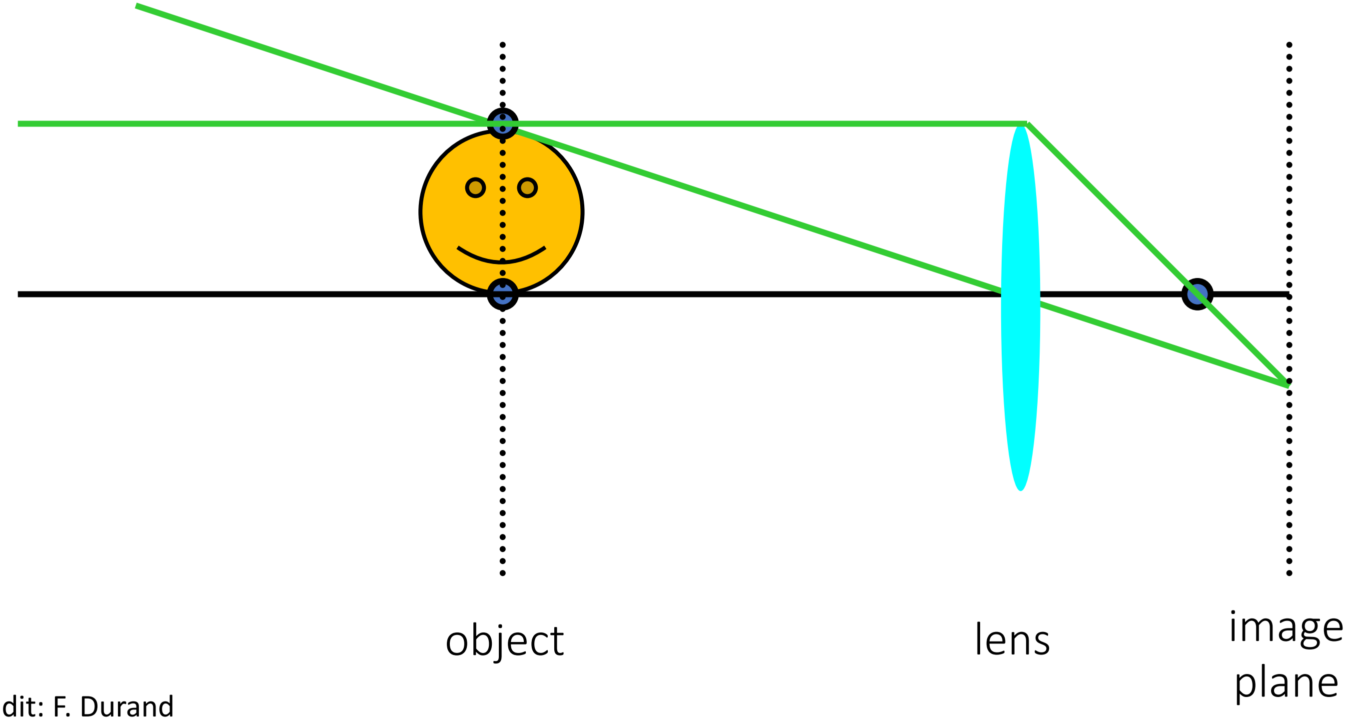


Diagram credit: F. Durand

Thin Lens Formula

Let’s derive the relationship between object distance D, image plane distance D’, and focal length f.

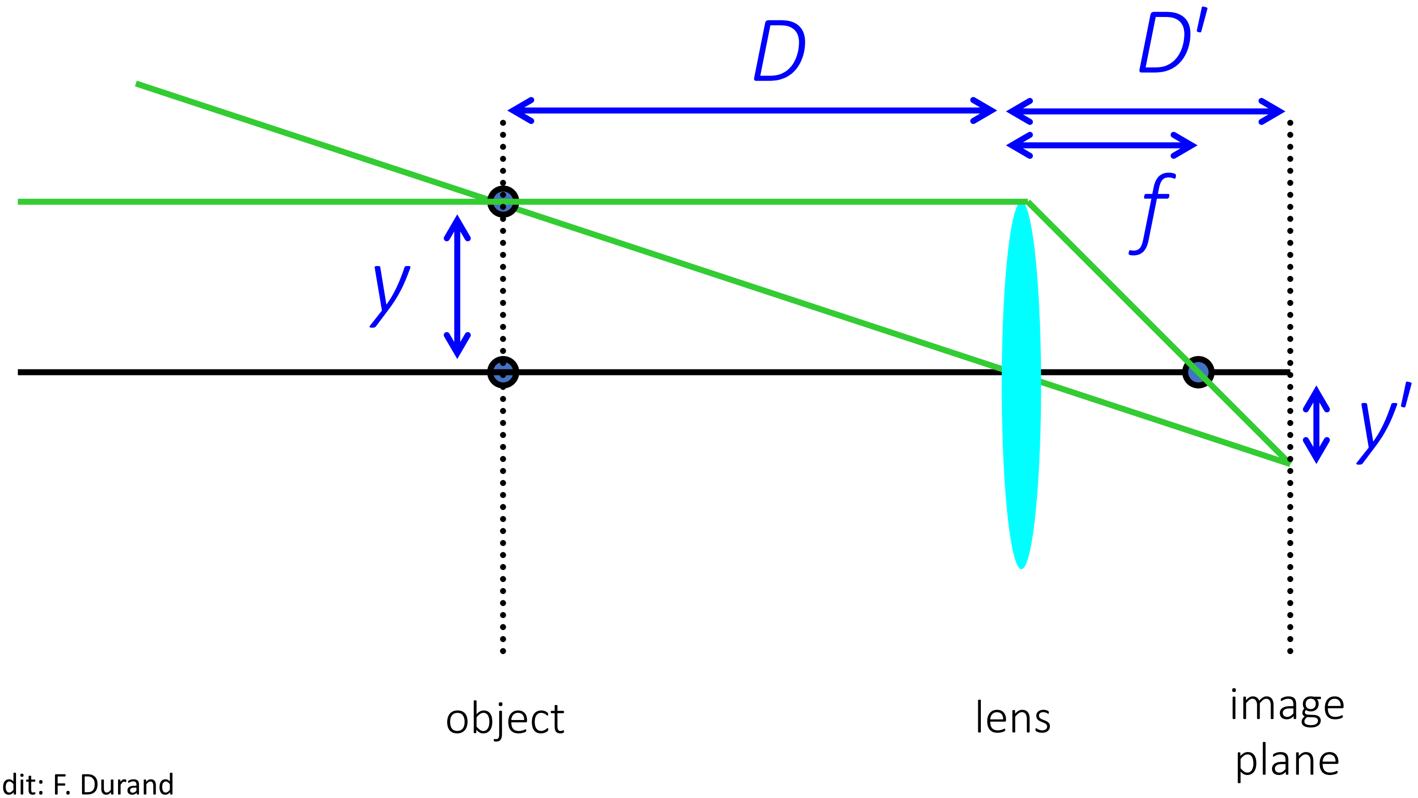


Diagram credit: F. Durand

Thin Lens Formula

One set of similar triangles:

$$\frac{y'}{D'-f} = \frac{y}{f} \rightarrow \frac{y'}{y} = \frac{D'-f}{f}$$

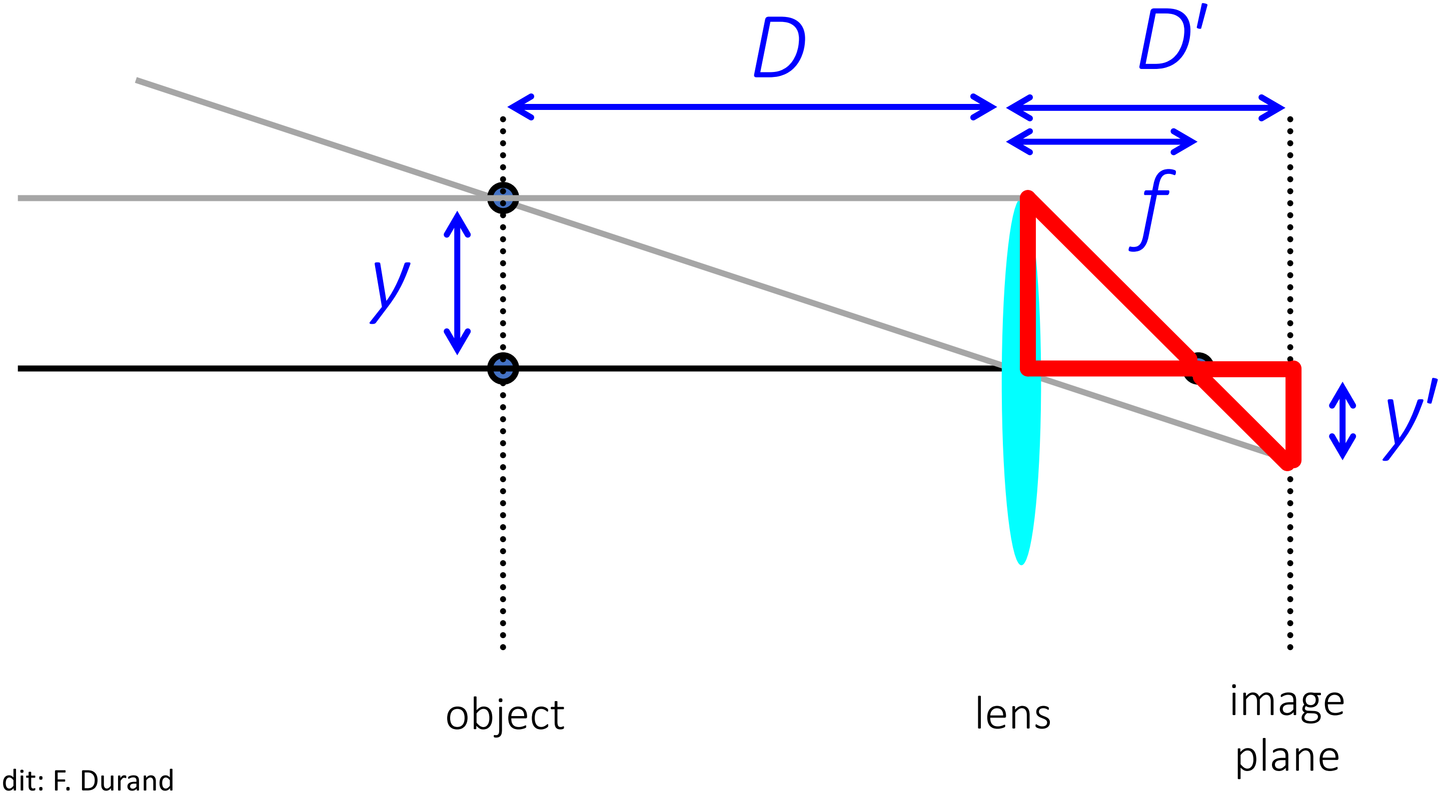
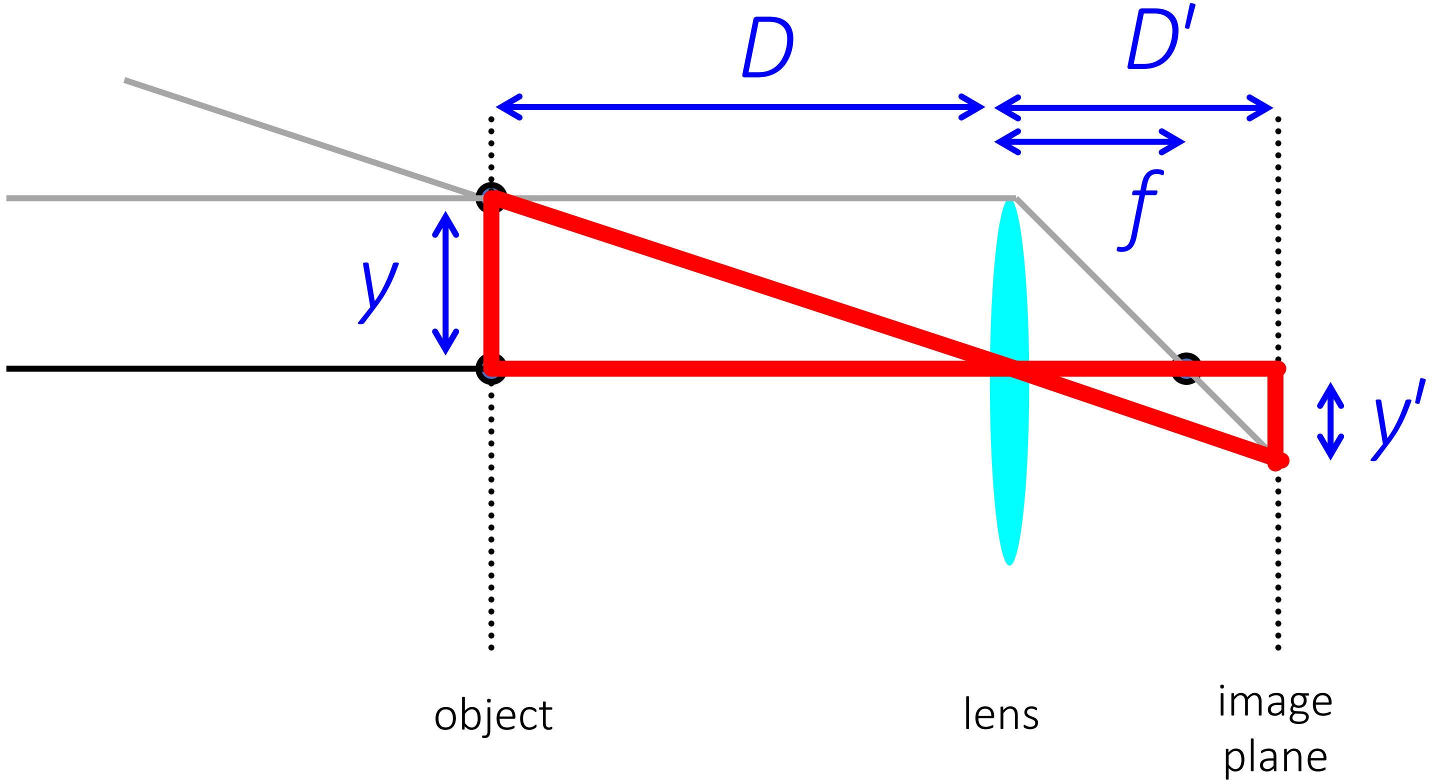


Diagram credit: F. Durand

Thin Lens Formula

Another set of similar triangles:

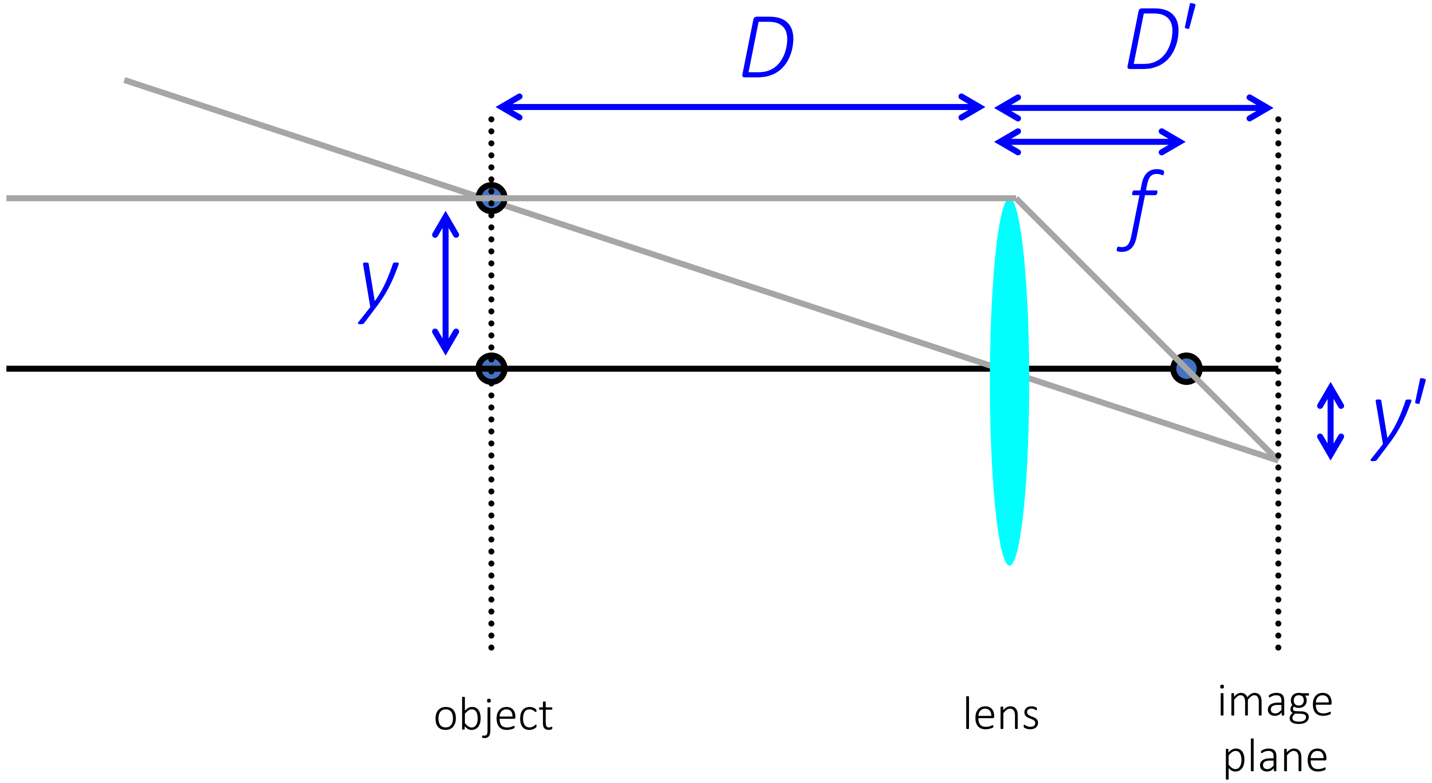
$$\frac{y'}{D'} = \frac{y}{D} \longrightarrow \frac{y'}{y} = \frac{D'}{D}$$



Thin Lens Formula

Set them equal:

$$\frac{D'}{D} = \frac{D' - f}{f} \longrightarrow \frac{1}{D} + \frac{1}{D'} = \frac{1}{f}$$



Thin Lens Formula

Suppose I want to take a picture of a lion with D big?
Which of D, D’, f **are fixed?**

How do we take pictures of things at different distances?

$$\frac{1}{D}+\frac{1}{D'}=\frac{1}{f}$$

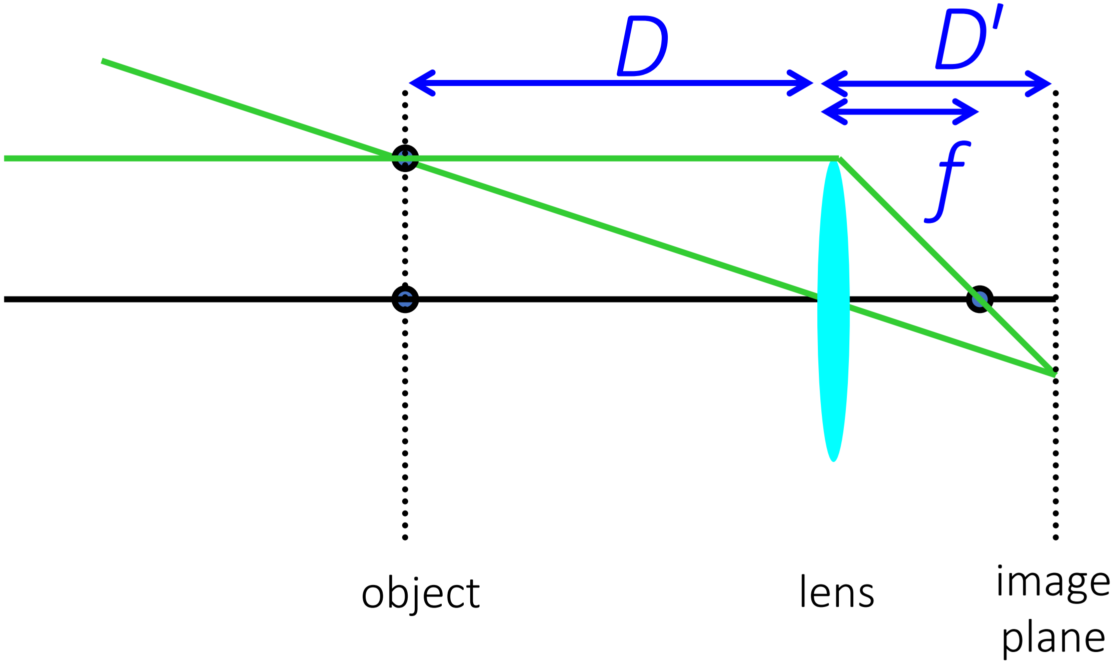


Diagram credit: F. Durand

Depth of Field

http://www.cambridgeincolour.com/tutorials/depth-of-field.htm

Slide Credit: A. Efros

Controlling Depth of Field

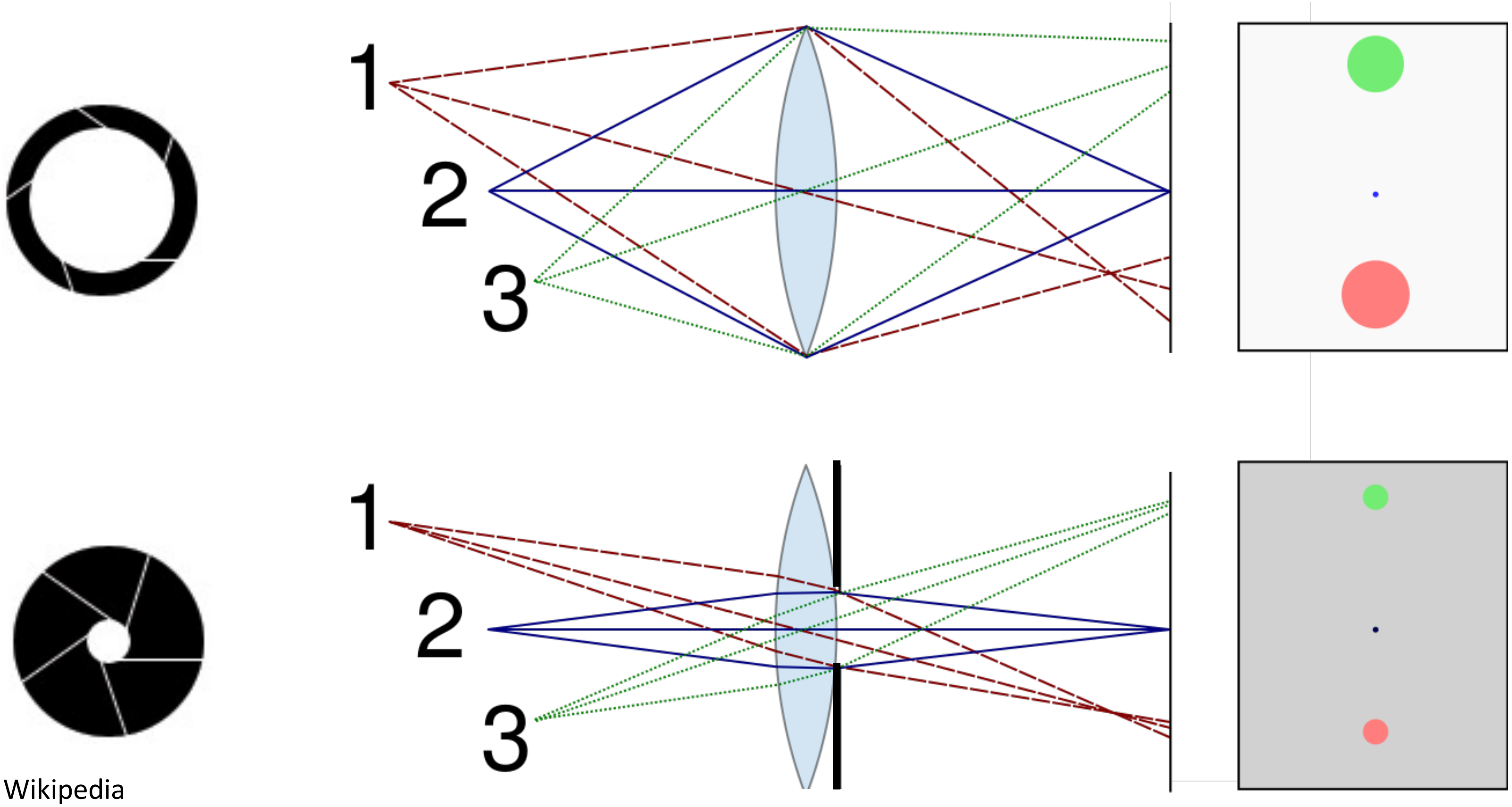


Diagram: Wikipedia

Changing the aperture size affects depth of field

A smaller aperture increases the range in which the object is approximately in focus

Controlling Depth of Field

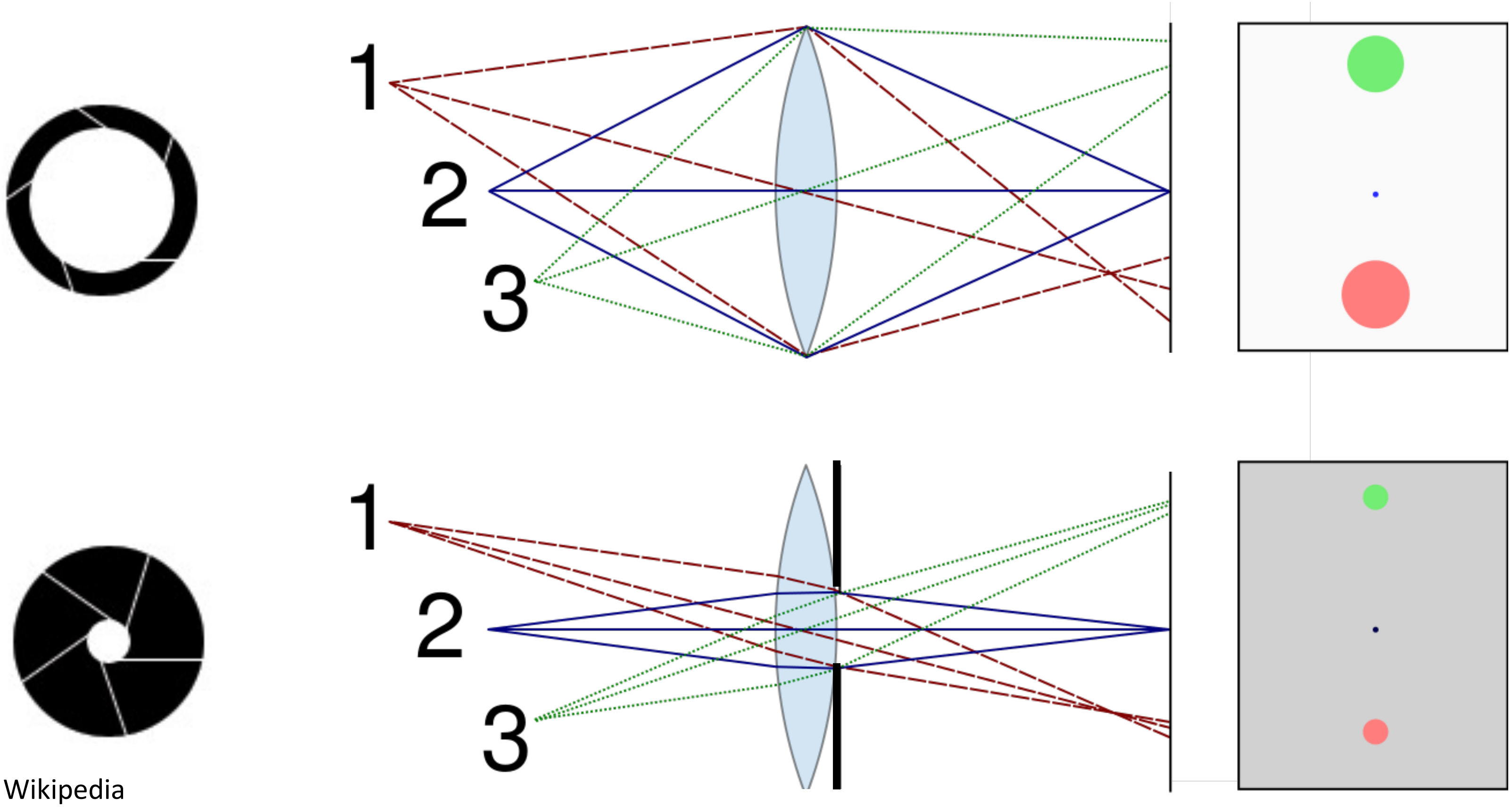


Diagram: Wikipedia

If a smaller aperture makes everything focused, why don't we just always use it?

Varying the Aperture

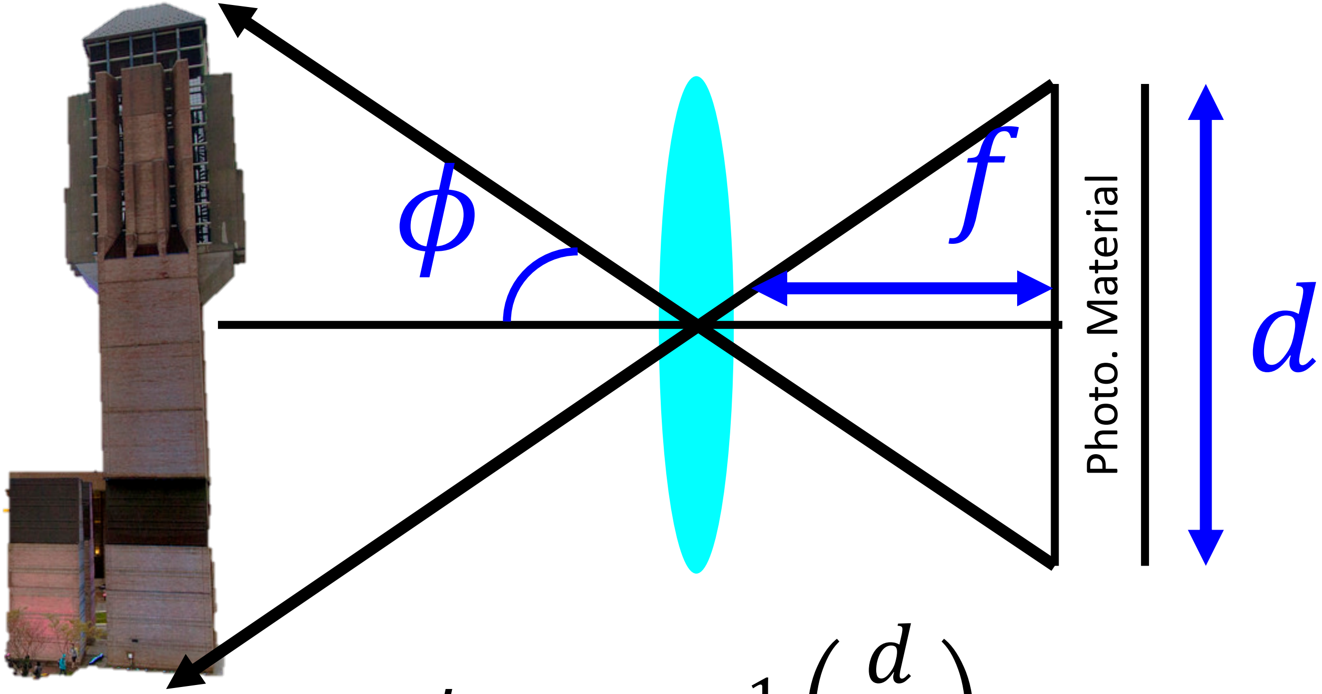
Small aperture = large DOF

Large aperture = small DOF

Slide Credit: A. Efros, Photo: Philip Greenspun

Varying the Aperture

Field of View

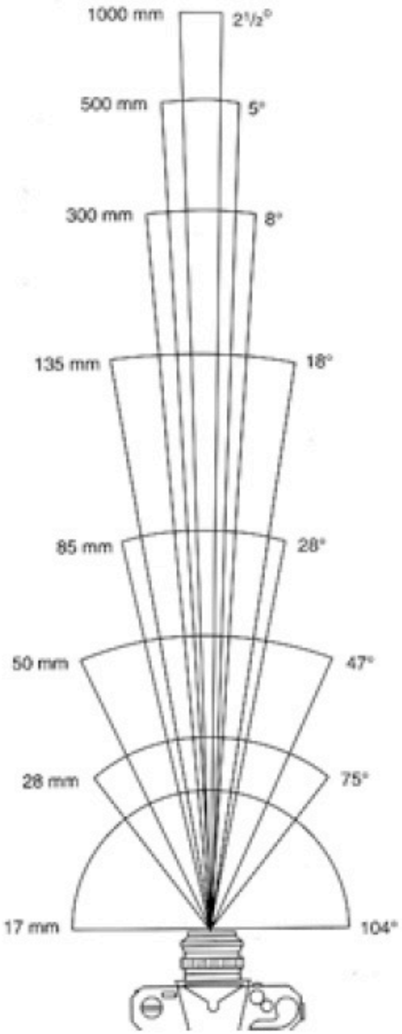


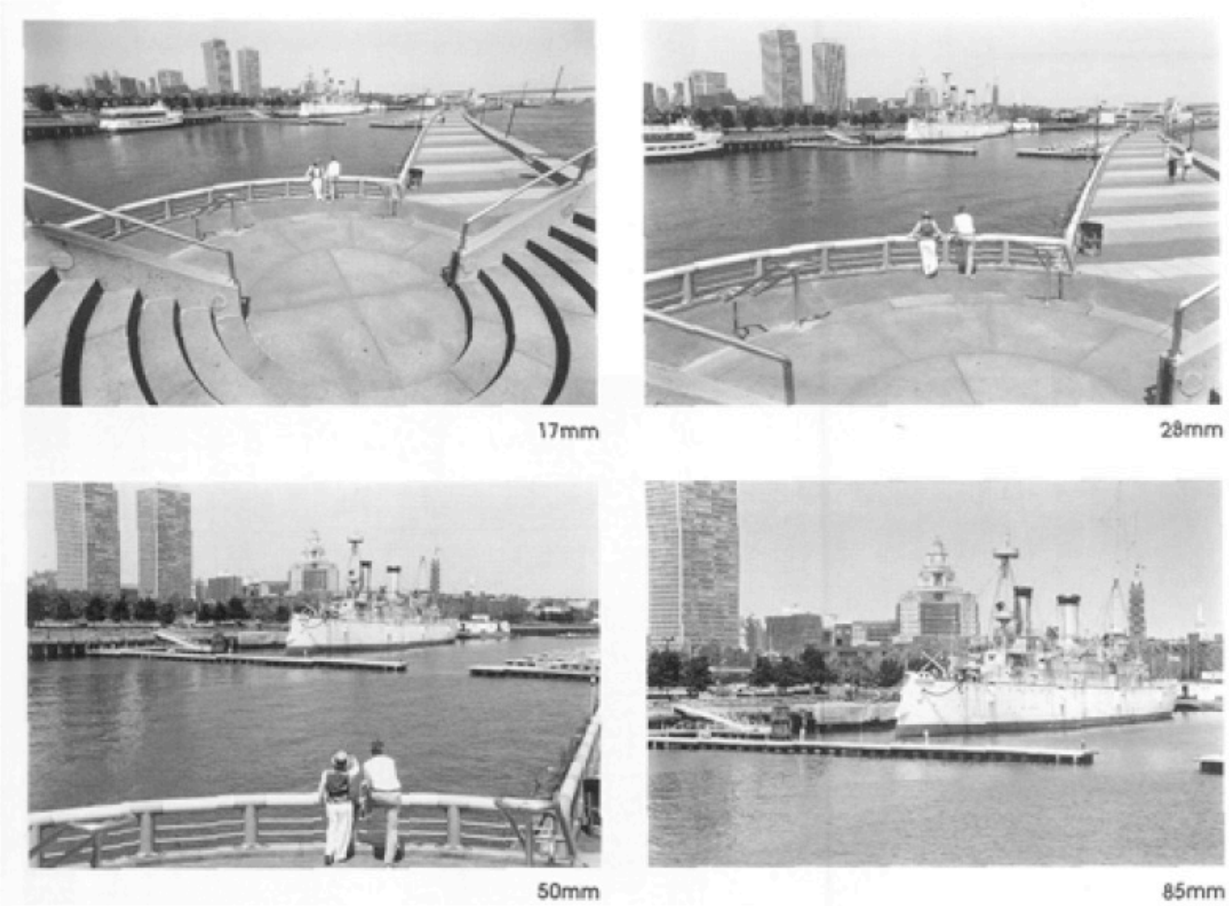
$$\phi = \tan^{-1}\left(\frac{d}{2f}\right)$$

tan^{-1} is monotonic increasing.

How can I get the FOV bigger?

Field of View




17mm

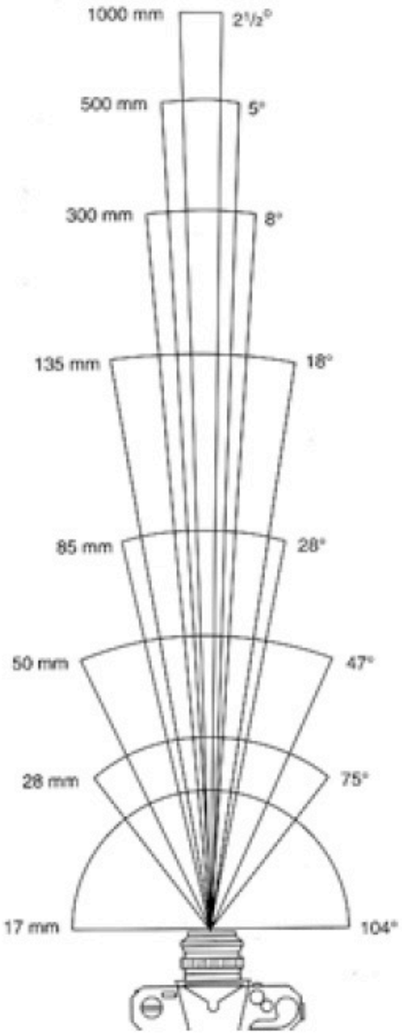
28mm

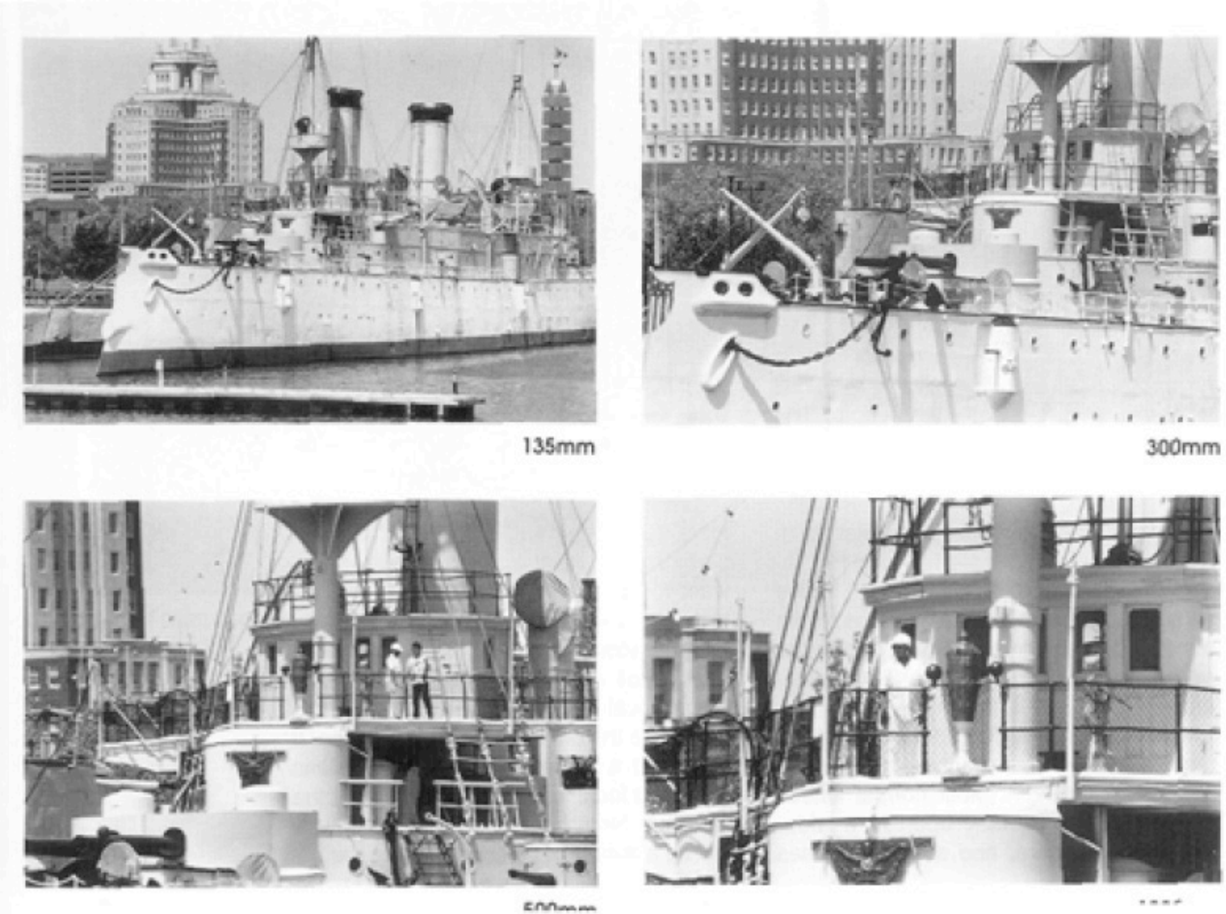
50mm

85mm

Slide Credit: A. Efros

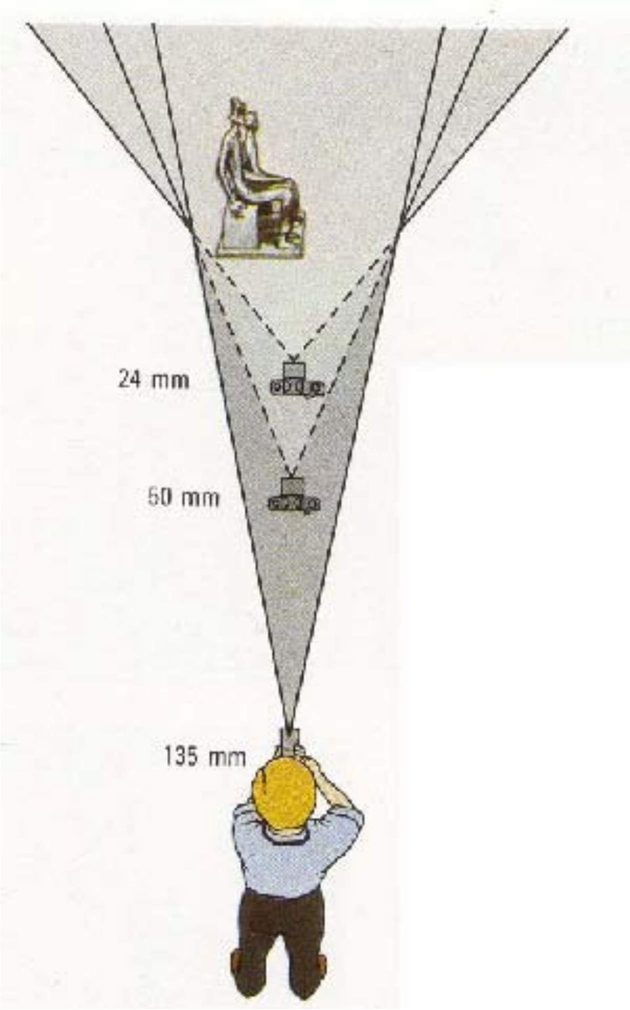
Field of View





Slide Credit: A. Efros

Field of View and Focal Length



Large FOV, small *f*
Camera close to car

Small FOV, large *f*
Camera far from the car

Slide Credit: A. Efros, F. Durand

Field of View and Focal Length

wide-angle

standard

telephoto

Slide Credit: F. Durand

Dolly Zoom

Change f and distance at the same time

Video Credit: Goodfellas 1990

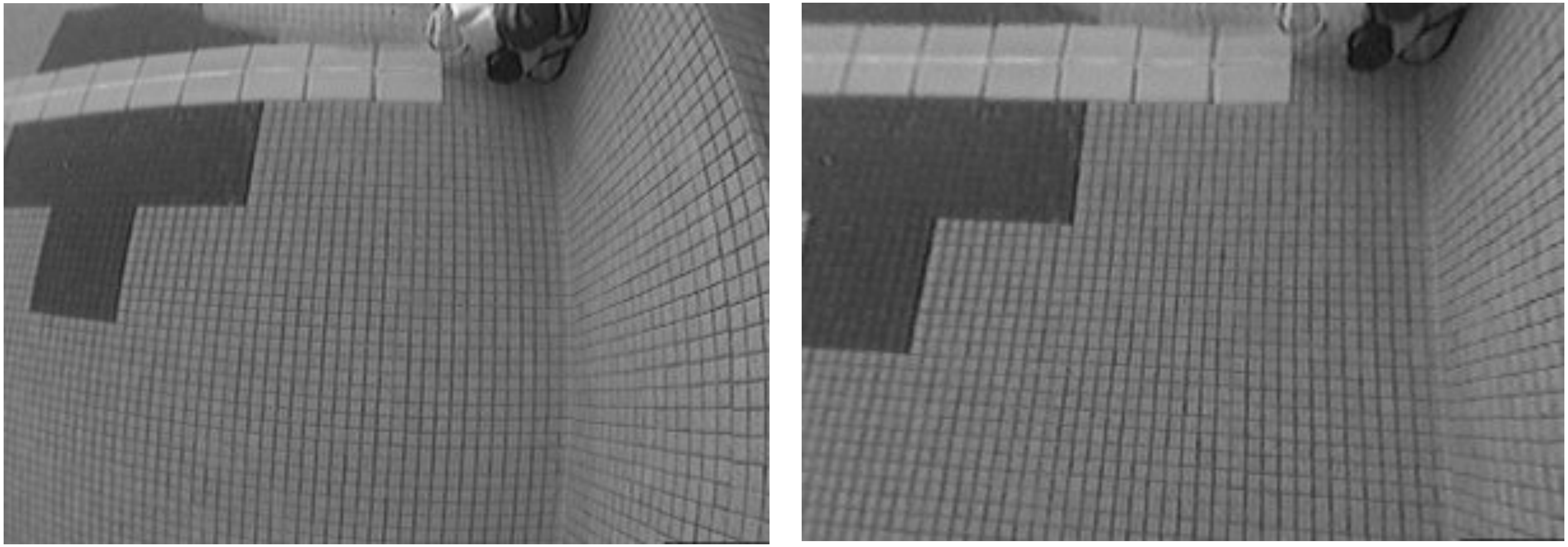
More Bad News

- First a pinhole…
- Then a thin lens model….

Slide: L. Lazebnik

Radial Distortion

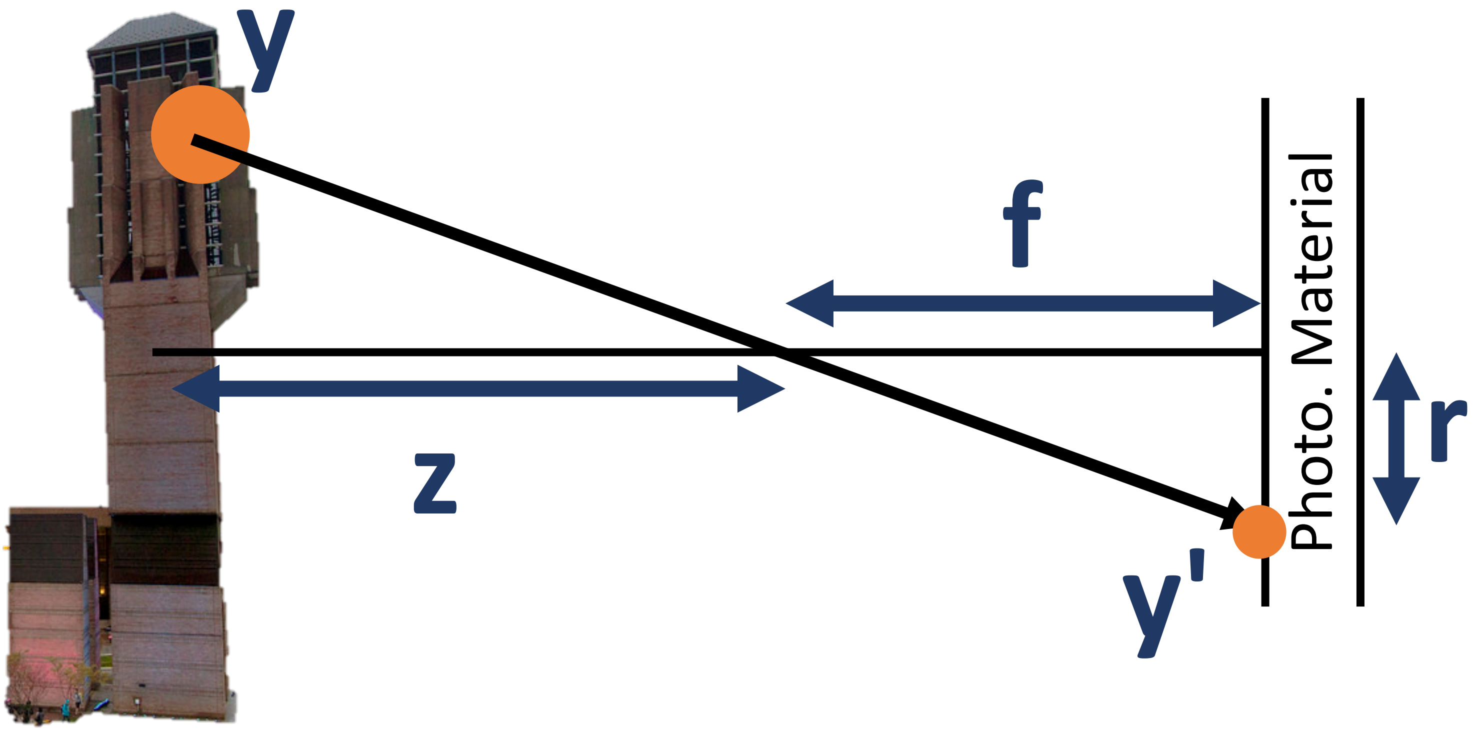
Lens imperfections cause distortions as a function of distance from optical axis



Less common these days in consumer devices

Photo: Mark Fiala, U. Alberta

Radial Distortion



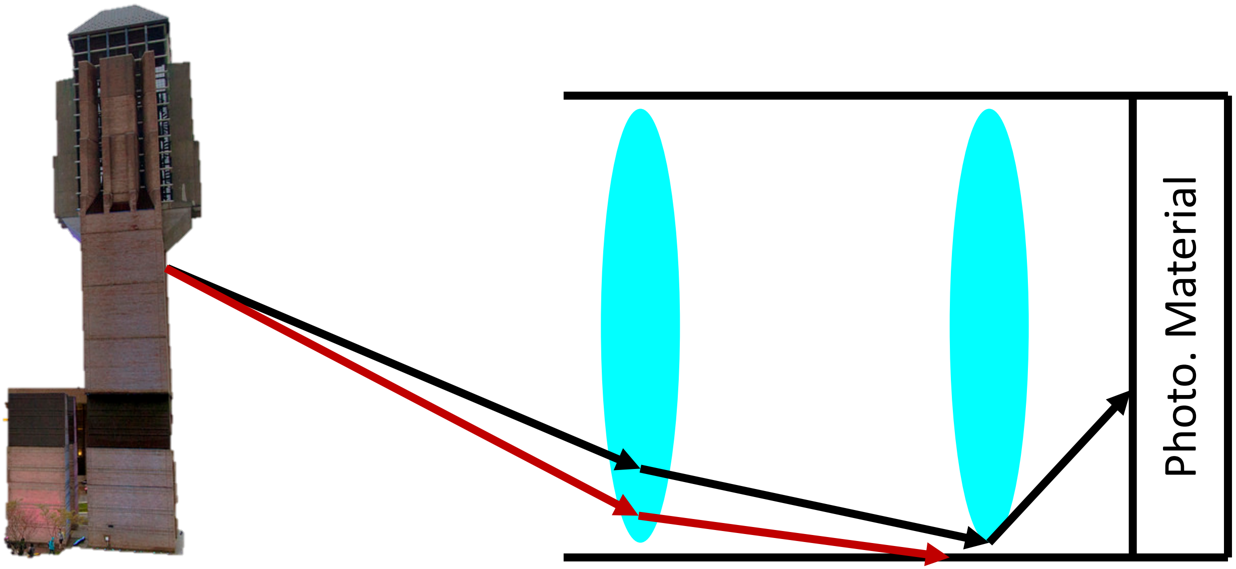
Ideal

$$y' = f\frac{y}{z}$$

Distorted

$$y' = (1 + k_1 r^2 + \cdots)\frac{y}{z}$$

Vignetting



What happens to the light between the black and red lines?

Doesn’t make it to the sensor!
Image darkens toward the edge

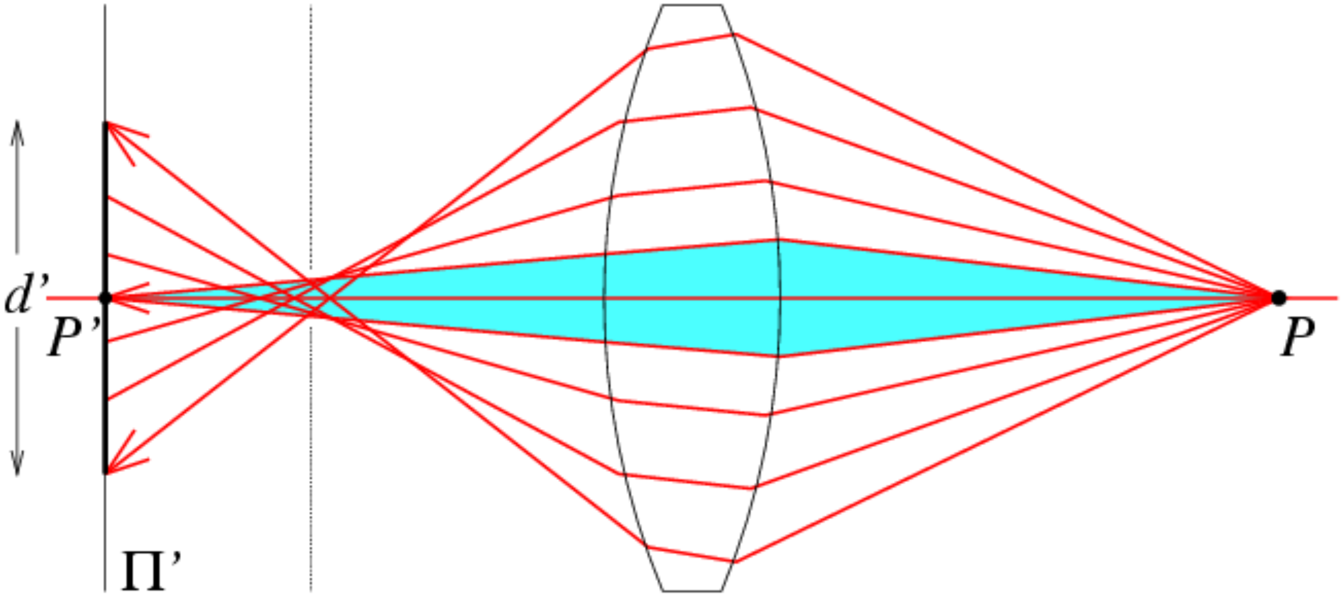
Slide inspired by L. Lazebnik Slide

Vignetting

Photo credit: Wikipedia (https://en.wikipedia.org/wiki/Vignetting)

Spherical Aberration

Lenses don't focus light perfectly!
Rays farther from the optical axis focus closer



Slide: L. Lazebnik

Chromatic Aberration

Lens refraction index is a function of the wavelength. Colors “fringe” or bleed

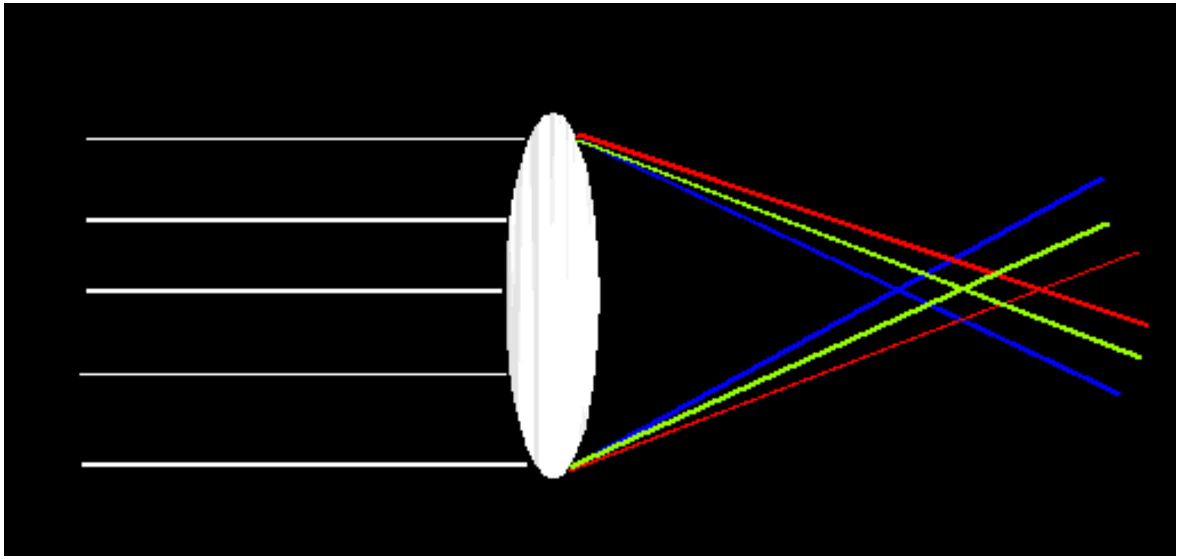
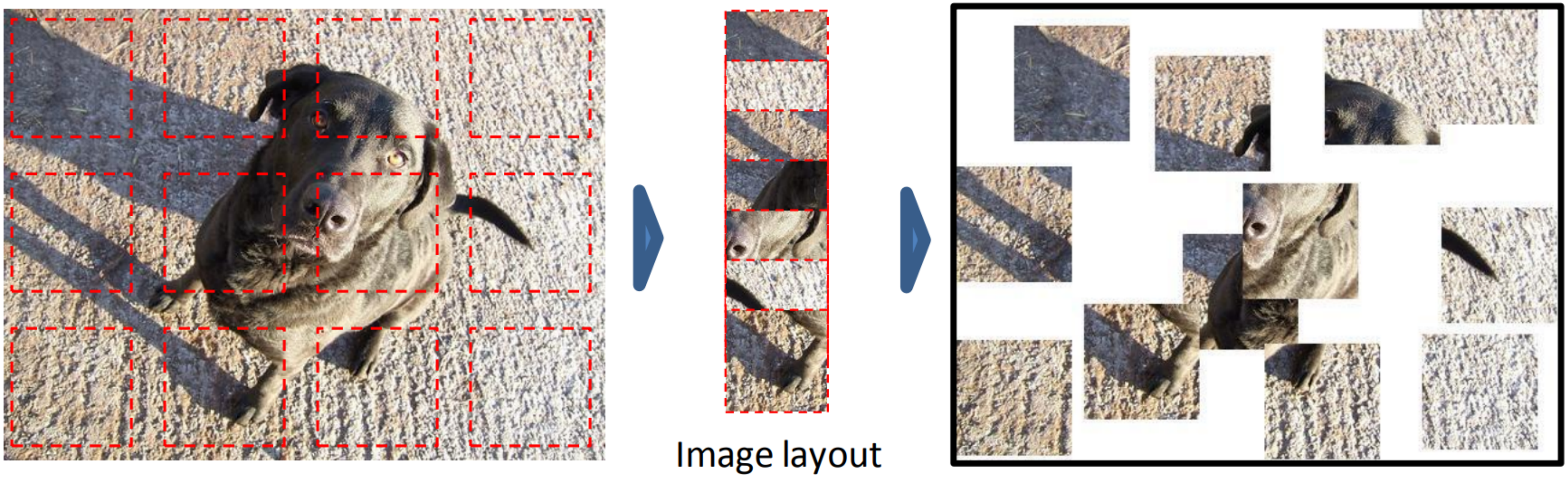


Image credits: L. Lazebnik, Wikipedia

Chromatic Aberration

Researchers tried teaching a network about objects by forcing it to assemble jigsaws.



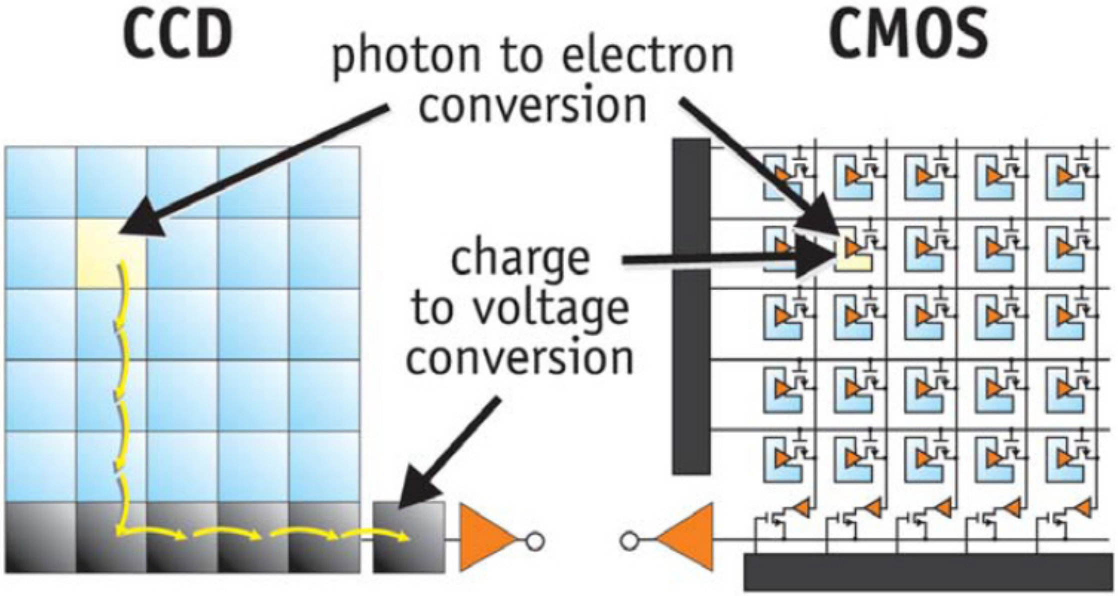
Initial layout, with sampled patches in red

Image layout is discarded

We can recover image layout automatically

Slide Credit: C. Doersch

From Photon to Photo

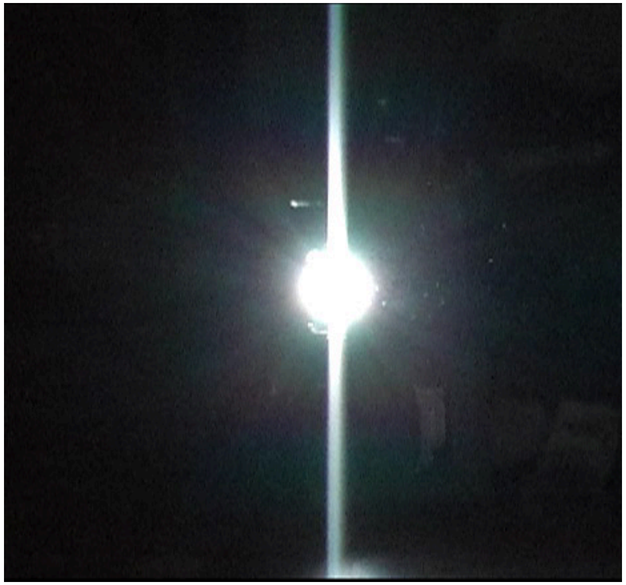


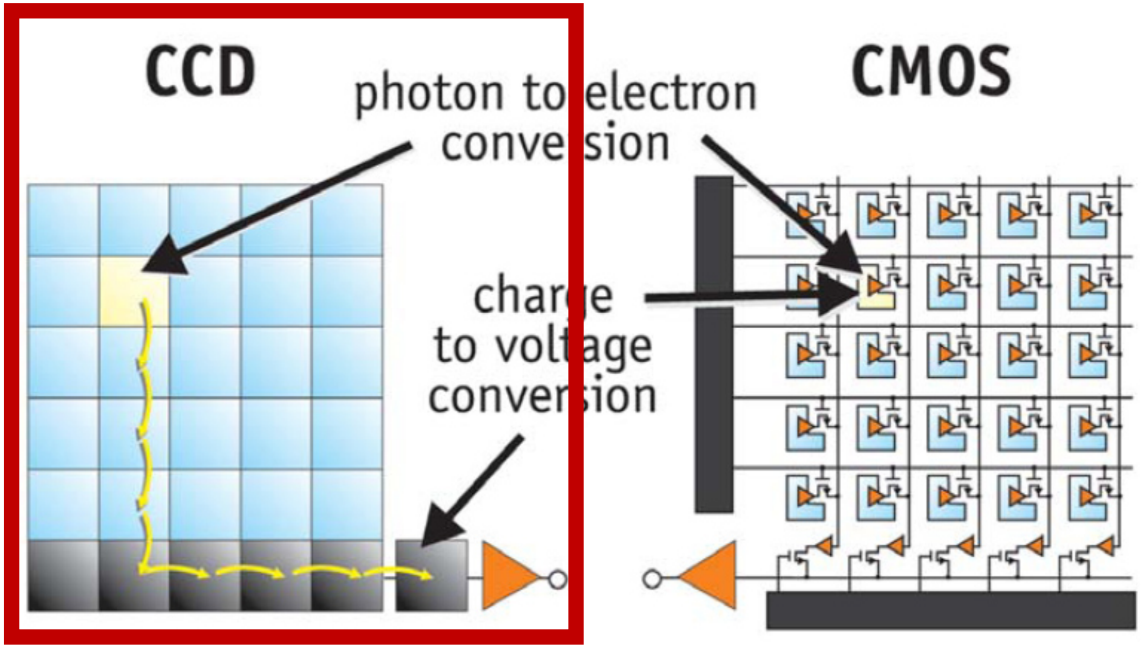
CCDs move photogenerated charge from pixel to pixel and convert it to voltage at an output node. CMOS imagers convert charge to voltage inside each pixel.

- Each cell in a sensor array is a light-sensitive diode that converts photons to electrons
  - Dominant in the past: **Charge Coupled Device (CCD)**
  - Dominant now: **Complementary Metal Oxide Semiconductor (CMOS)**

Slide Credit: L. Lazebnik, Photo Credit: Wikipedia, Stefano Meroli

From Photon to Photo



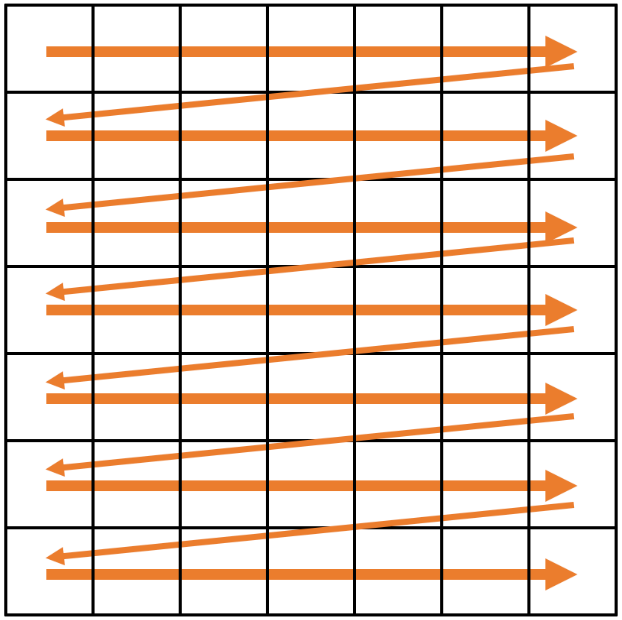


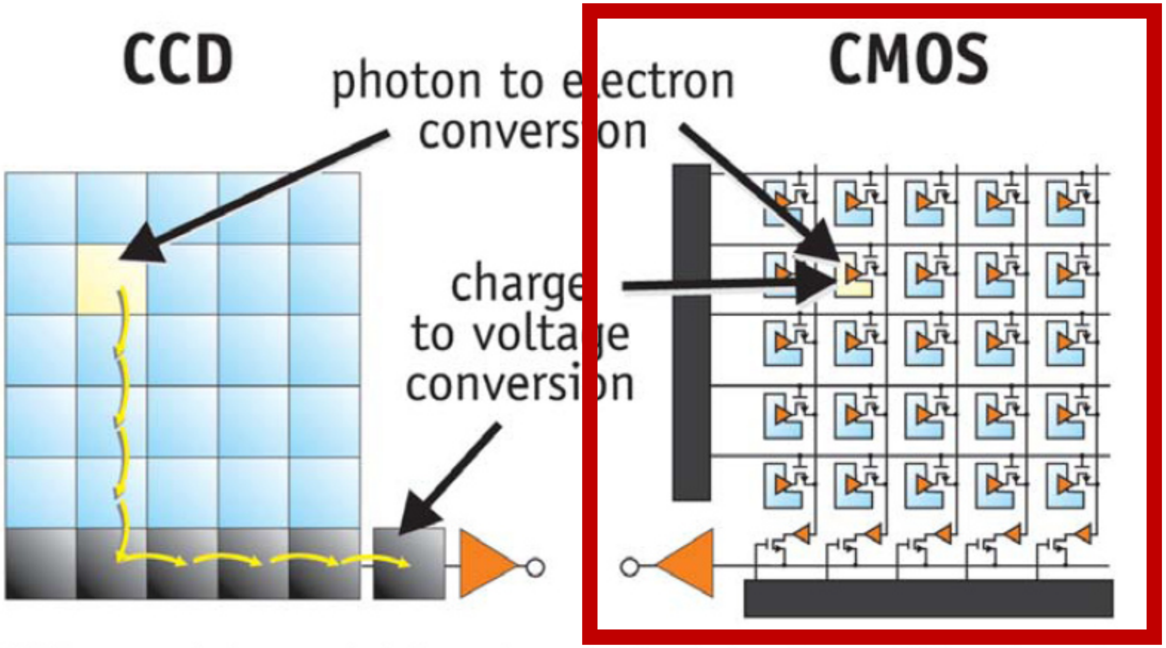
CCDs move photogenerated charge from pixel to pixel and convert it to voltage at an output node. CMOS imagers convert charge to voltage inside each pixel.

- CCD Problem: Vertical Smear

Slide Credit: L. Lazebnik, Photo Credit: Wikipedia, Stefano Meroli

From Photon to Photo





CCDs move photogenerated charge from pixel to pixel and convert it to voltage at an output node. CMOS imagers convert charge to voltage inside each pixel.

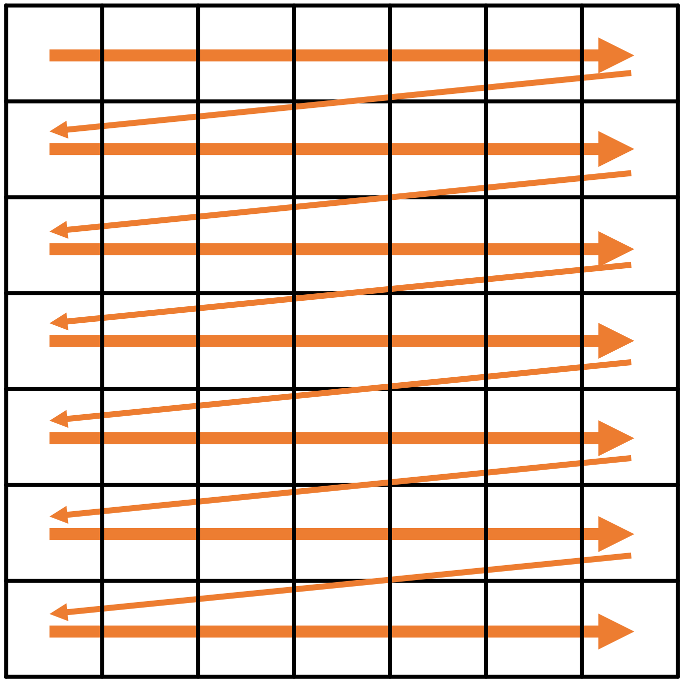
- CMOS problem: Rolling Shutter

Slide Credit: L. Lazebnik, Photo Credit: Wikipedia, Stefano Meroli

From Photon to Photo

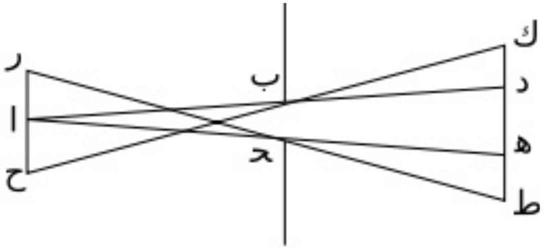
Rolling Shutter: pixels read in sequence

Can get global reading, but $$$



Historic Milestones

- **Pinhole model:** Mozi (470-390 BCE), Aristotle (384-322 BCE)
- **Principles of optics (including lenses):** Alhacen (965-1039 CE)
- **Camera obscura:** Leonardo da Vinci (1452-1519), Johann Zahn (1631-1707)
- **First photo:** Joseph Nicephore Niepce (1822)
- **Daguerréotypes** (1839)
- **Photographic film** (Eastman, 1889)
- **Cinema** (Lumière Brothers, 1895)
- **Color Photography** (Lumière Brothers, 1908)
- **Television** (Baird, Farnsworth, Zworykin, 1920s)
- **First consumer camera with CCD** Sony Mavica (1981)
- **First fully digital camera:** Kodak DCS100 (1990)



Alhacen's notes

Niepce, “La Table Servie,” 1822

Old television camera

Slide Credit: S. Lazebnik

First Digitally Scanned Photgraph

- 1957, 176 x 176 pixels

Slide Credit: http://listverse.com/history/top-10-incredible-early-firsts-in-photography/

Historic Milestone

Sergey Prokudin-Gorskii (1863-1944)
Photographs of the Russian empire (1909-1916)

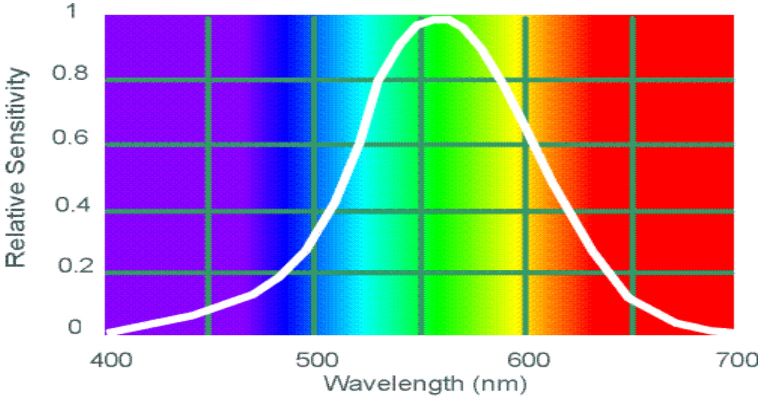
Slide Credit: S. Maji

Historic Milestone

Slide Credit: S. Maji

Your Milestone: HW1

Your job in homework 1:
Make the left look like the right.



Human Luminance Sensitivity Function

Next Time: Light, Color